Electromagnetic Strength Distributions from the IT-NCSM

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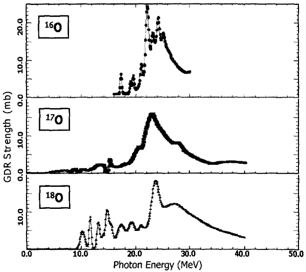
Motivation

- testing ground for theoretical models
  - electromagnetic transitions sensitive to detailed form of nuclear wave function
  - strength distributions can be easily related to experiments

Tryggestadt et al.: Nucl. Phys. A 687 (2001) 231



- IT-NCSM + Lanczos strength functions
  - conceptually simple and efficient method for calculation of strength distributions
  - validate strength distributions from approximate approaches

Strength Functions from Simple Lanczos Algorithm

- aim: $$S(E) = \sum_f |\langle \Psi_f | \mathbf{O} | \Psi_i \rangle|^2 \delta(E - E_f)$$
- simple Lanczos algorithm:
  - start from normalized pivot vector $|\Sigma_1\rangle$
  - construct tridiagonal matrix T iteratively via basis transformation of Hamilton matrix
  - diagonalize T

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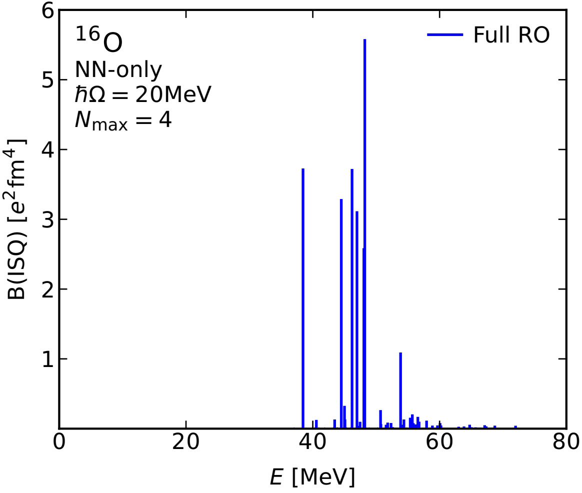
- $U_{1,j} = \langle\Sigma_1|\Psi_j\rangle$ is overlap of pivot vector with eigenstate of T
- choose special pivot: $|\Sigma_1\rangle = \frac{1}{\sqrt{\tilde{S}}}\mathbf{O}|\Psi_i\rangle$, $\quad \tilde{S} = \langle\Psi_i|\mathbf{O}^\dagger\mathbf{O}|\Psi_i\rangle$
- fast converging approximation of $S(E)$ with number of iterations

Effects of Finite-Precision Arithmetics



- reorthogonalization of Lanczos basis would render method impractical

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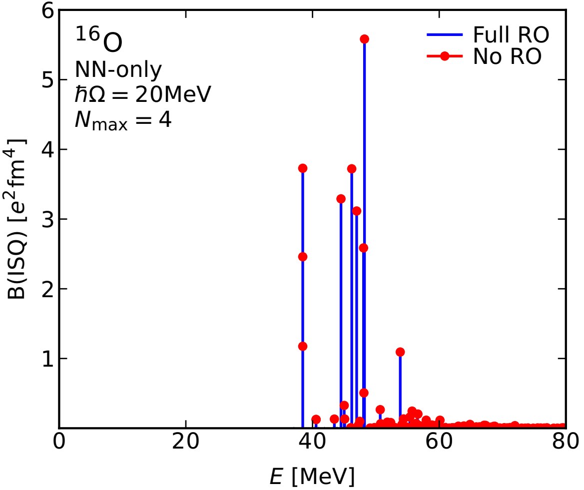

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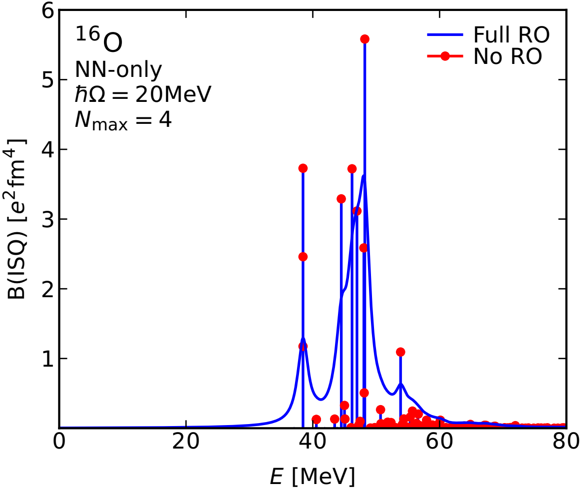

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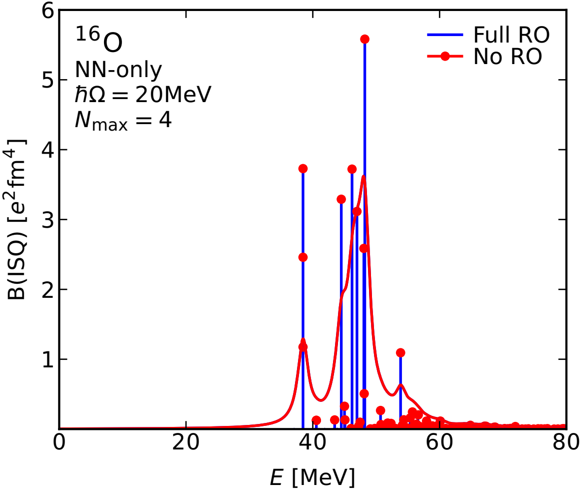

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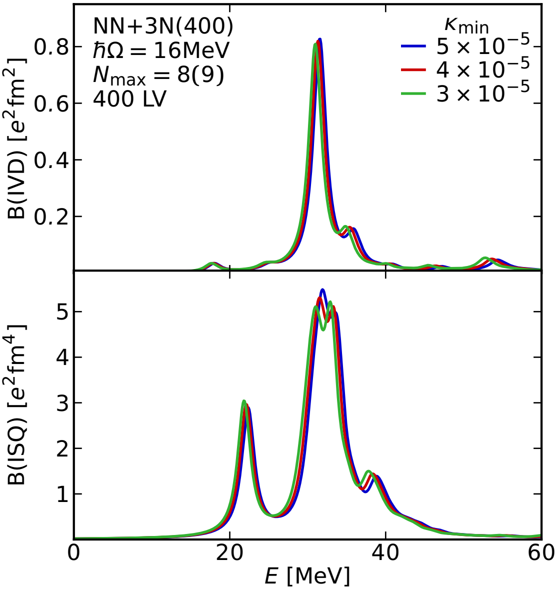
⇒ orthogonality loss no issue

⇒ efficiency of Lanczos strength functions approach

Convergence Benchmark for ^{16}O



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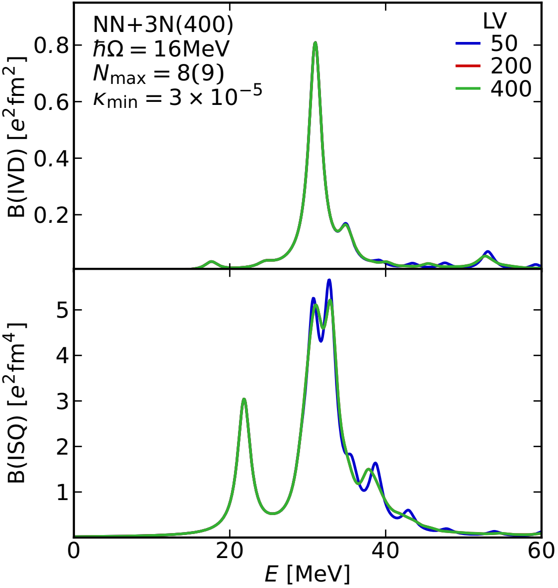

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- very fast convergence w.r.t. Lanczos basis size

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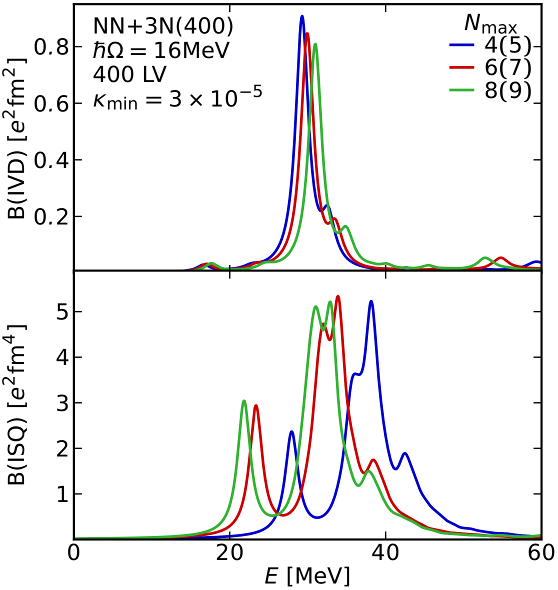

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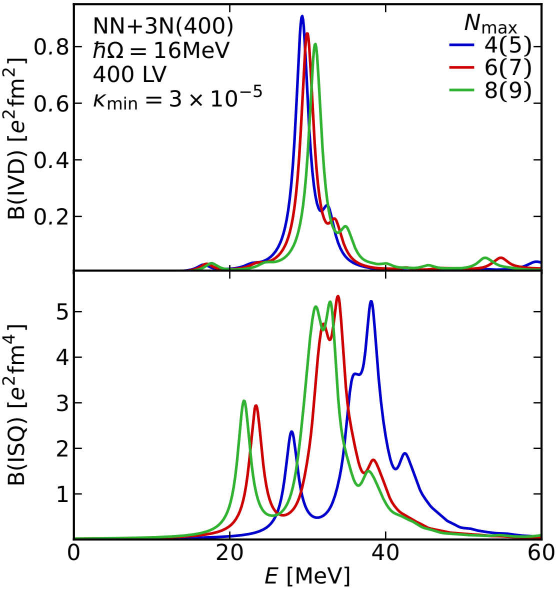

- negligible effect of importance truncation
- very fast convergence w.r.t. Lanczos basis size
- model-space convergence as for spectra
- same convergence behavior for other modes