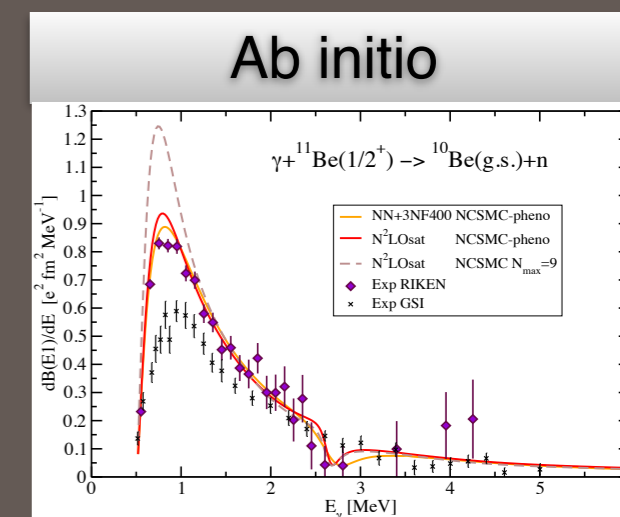
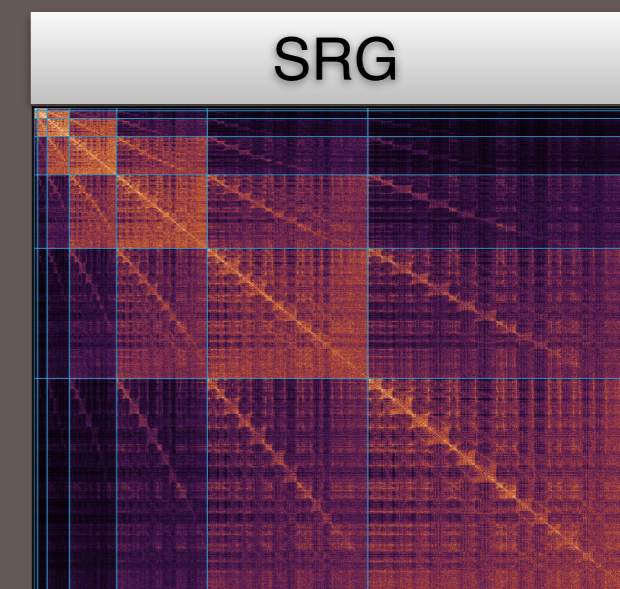
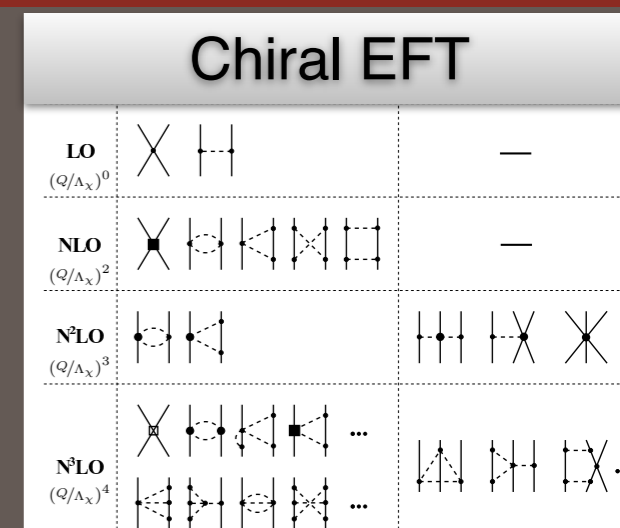


# Probing Nuclear Forces in *ab initio* Nuclear Reactions

Progress in Ab Initio Techniques in Nuclear Physics

March 1 2017, TRIUMF, Vancouver

Angelo Calci | TRIUMF

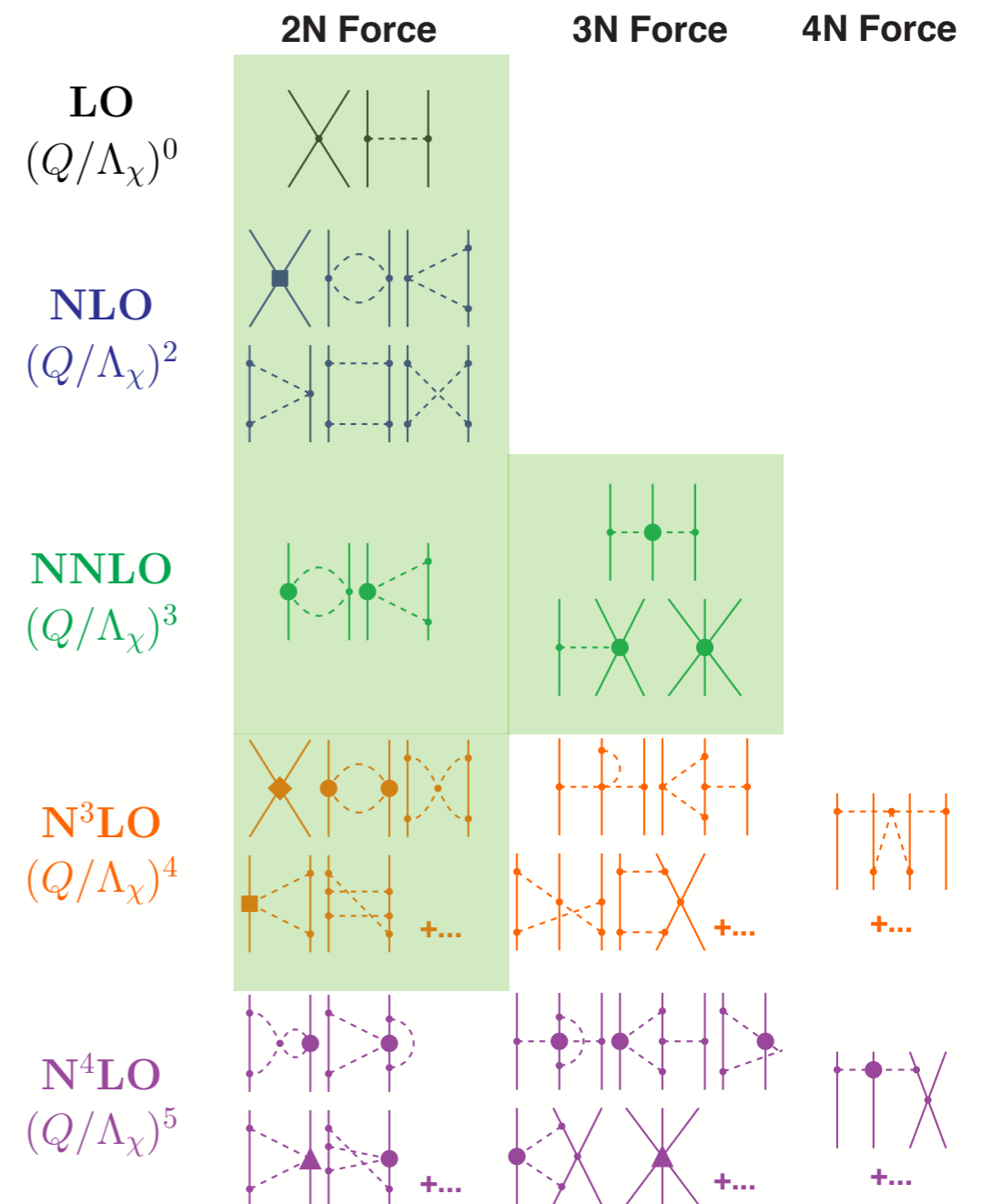


# Chiral NN+3N Interactions

Weinberg, van Kolck, Machleidt, Entem, Meissner, Epelbaum, Krebs, Bernard,...

## ● standard interaction:

- NN @ N<sup>3</sup>LO: Entem & Machleidt, 500MeV cutoff
- 3N @ N<sup>2</sup>LO: Navrátil, local cutoff, modifications of cutoff



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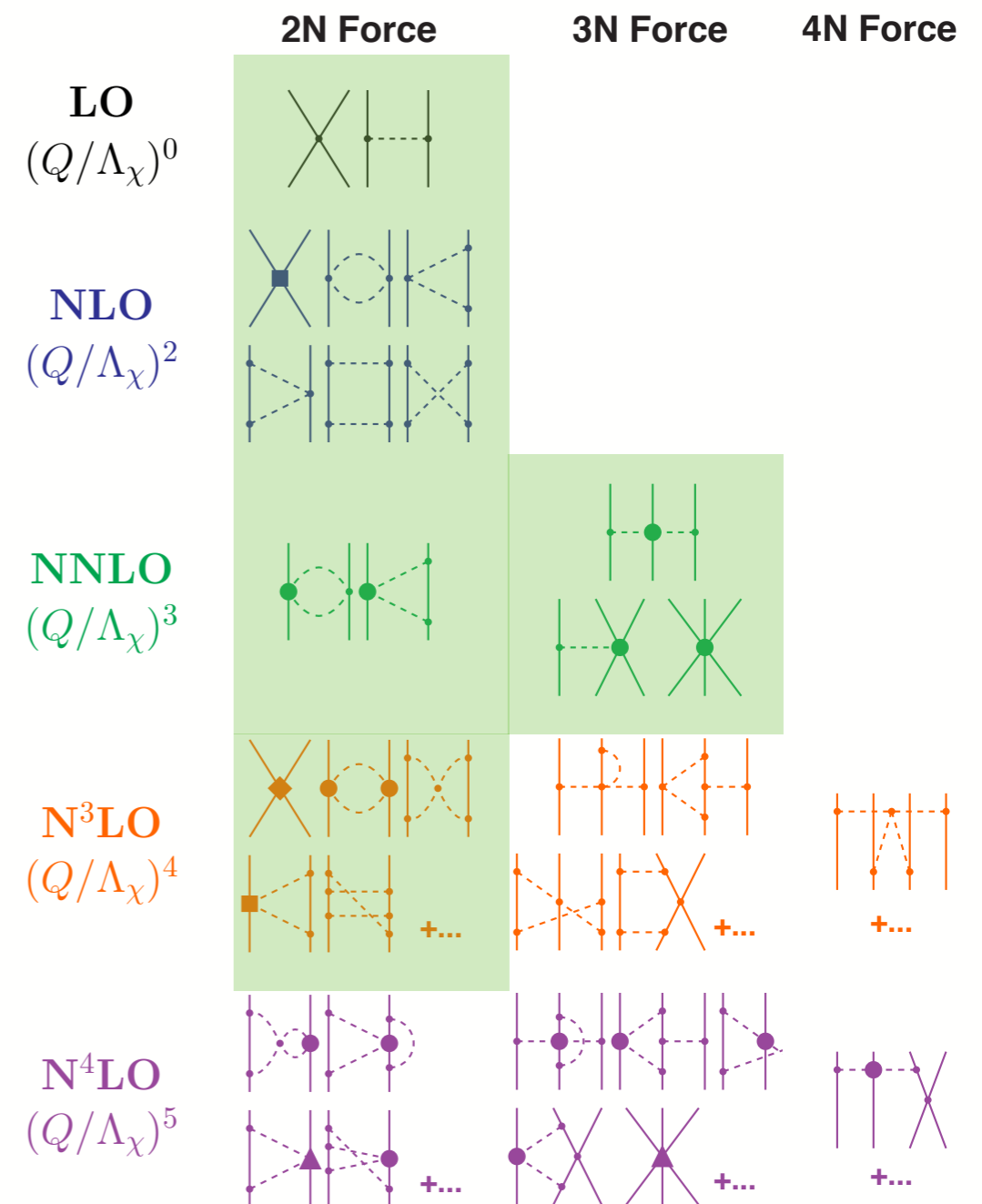
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## chiral interactions are not unique:

- chiral order
- regularization
- fit of LECs



# Chiral NN+3N Interactions

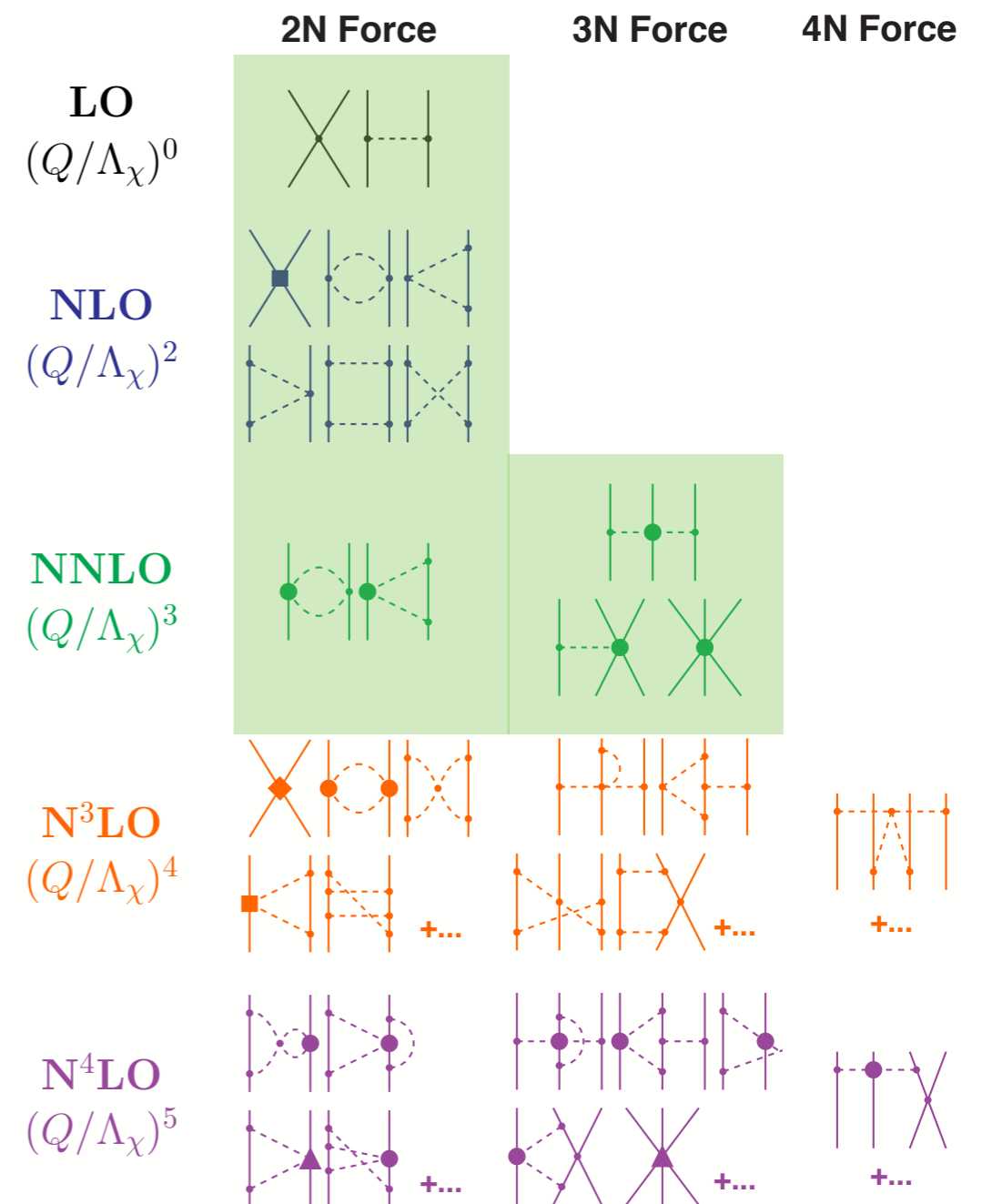
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- NN+3N: Ekström et al., nonlocal 450MeV cutoff, simultaneous fit to NN data and selected many-body observables



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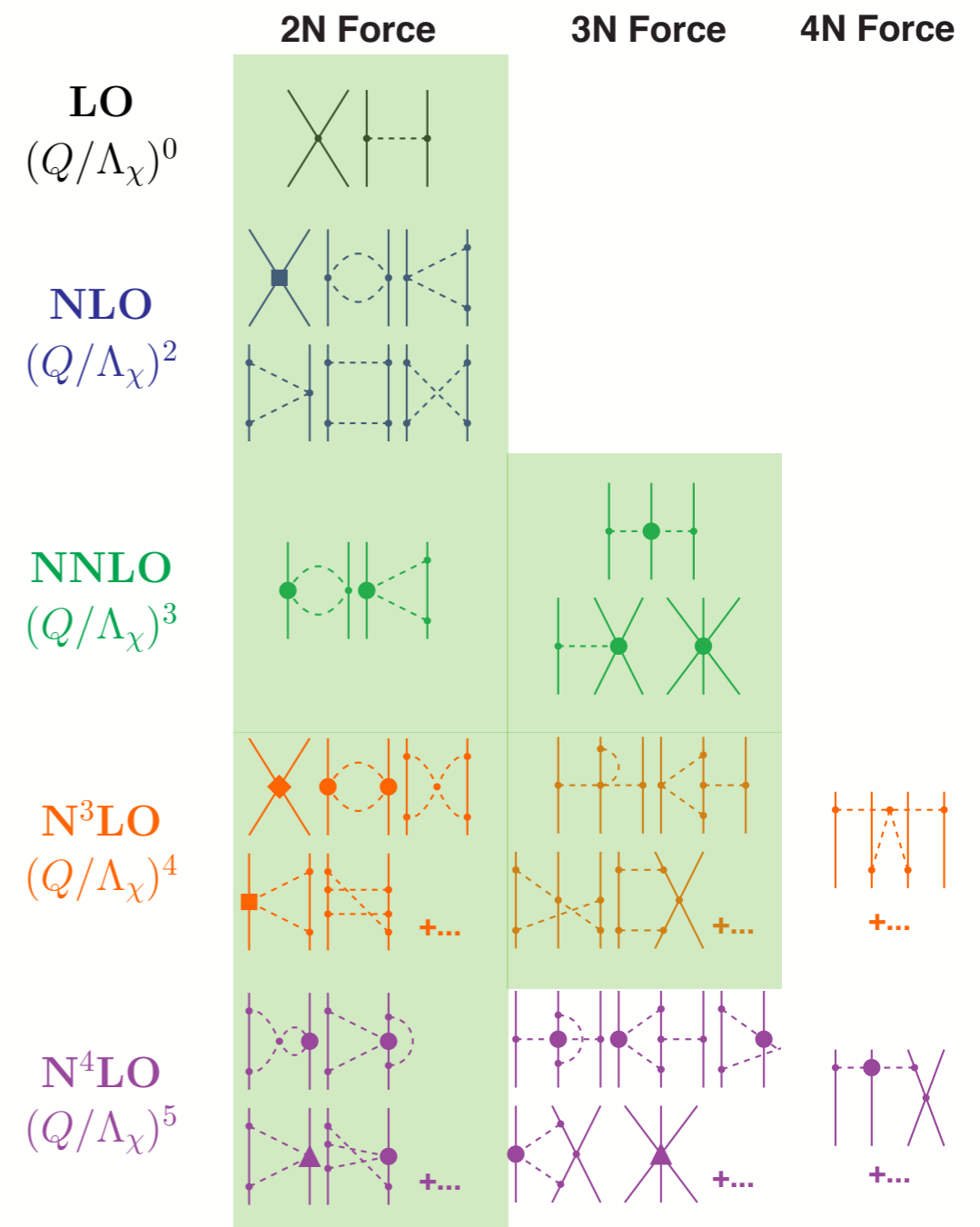
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- **LENPIC interaction:**

- NN up to N<sup>4</sup>LO: Epelbaum et al., semi-local cutoff
- 3N up to N<sup>3</sup>LO: under construction



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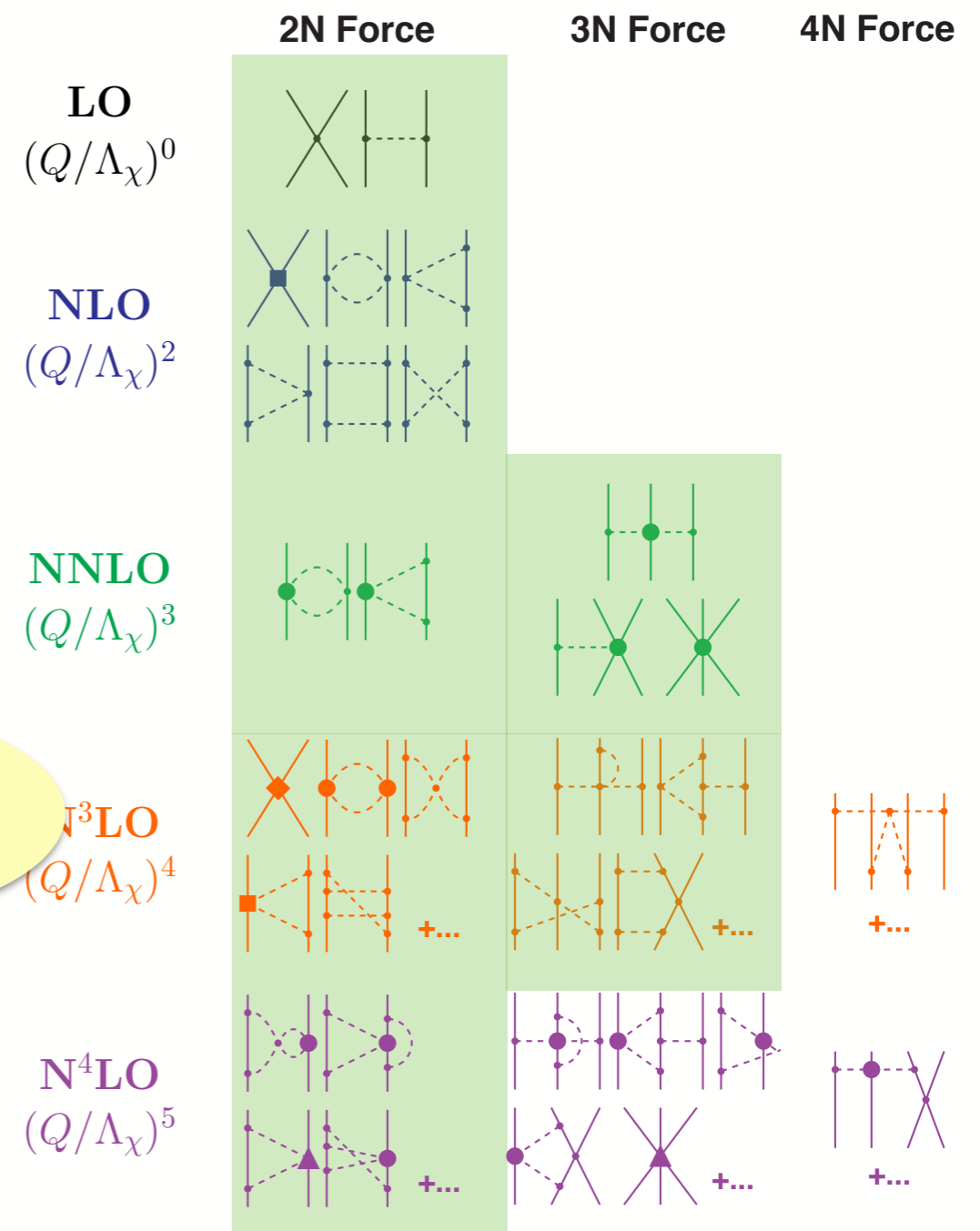
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talk by Kai Hebeler



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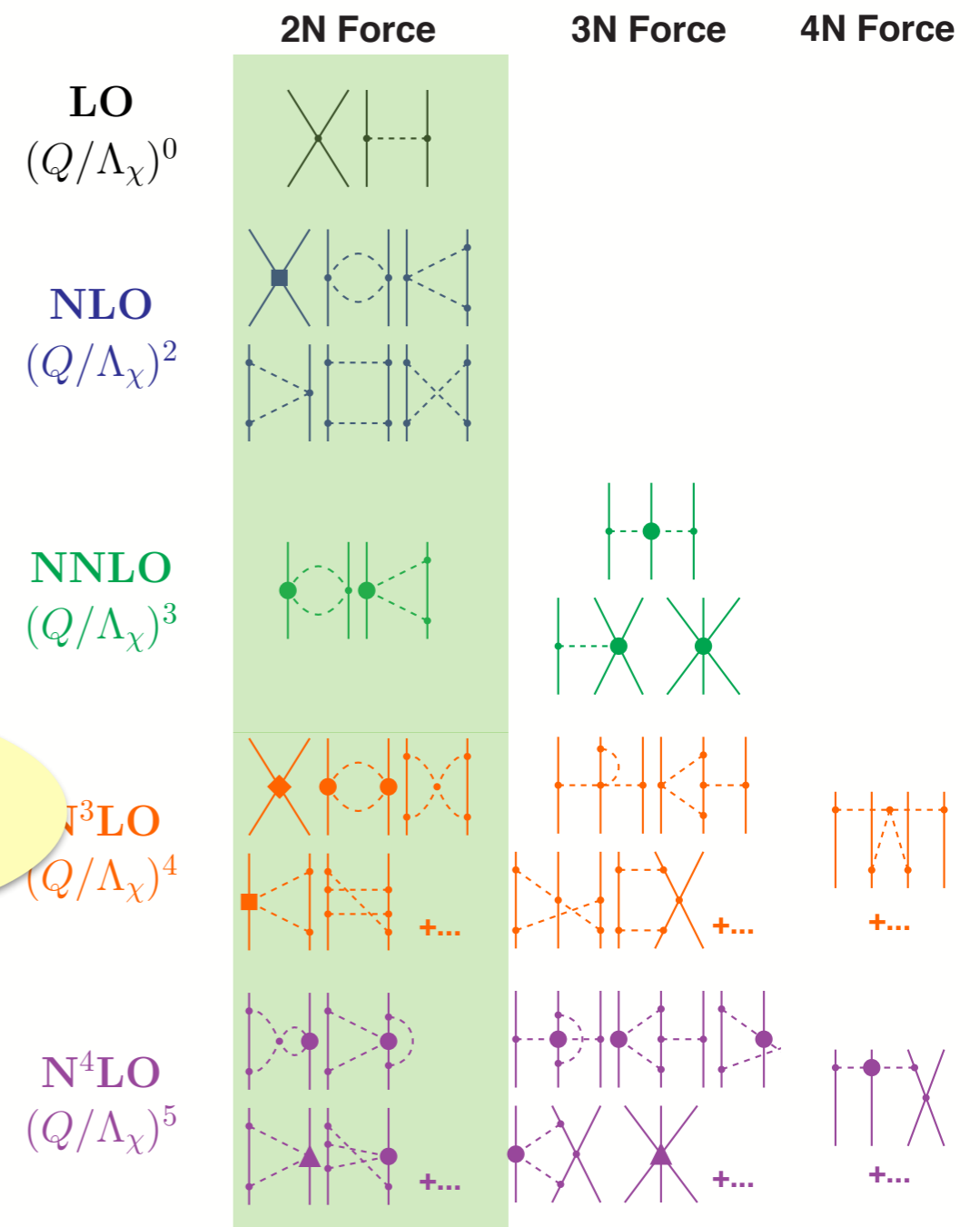
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talk by Kai Hebeler

- **N<sup>4</sup>LO(500):**

- NN @ N<sup>4</sup>LO: Machleidt et al., 500MeV cutoff



# Similarity Renormalization Group (SRG)

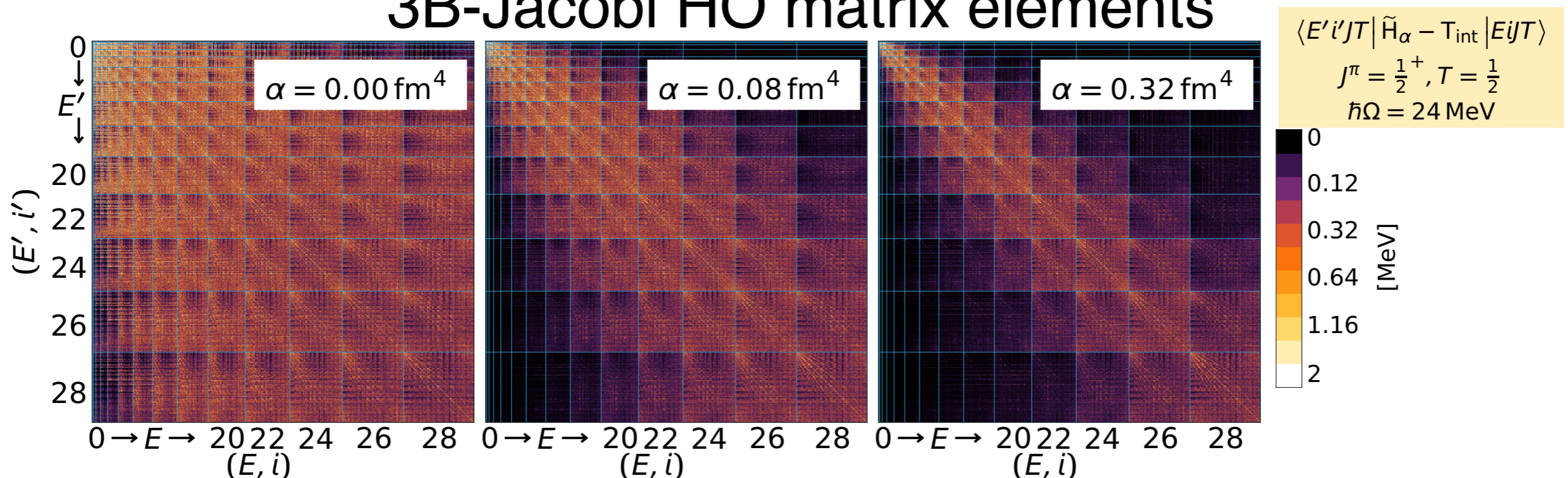
**accelerate** convergence by **pre-diagonalizing** the Hamiltonian with respect to the many-body basis

- unitary transformation leads to evolution equation**

$$\frac{d}{d\alpha} \tilde{H}_\alpha = [\eta_\alpha, \tilde{H}_\alpha] \quad \text{with} \quad \eta_\alpha = (2\mu)^2 [T_{\text{int}}, \tilde{H}_\alpha] = -\eta_\alpha^\dagger$$

advantages of SRG: **flexibility** and **simplicity**

## 3B-Jacobi HO matrix elements





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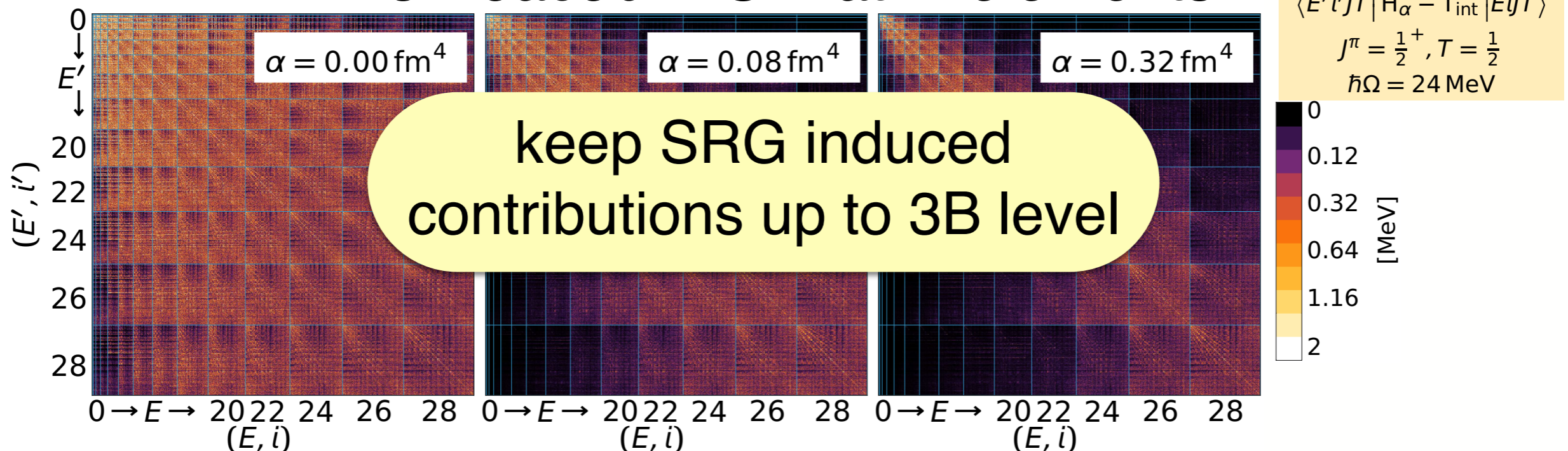
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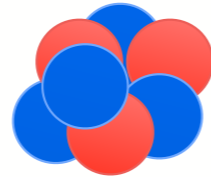


# Continuum effects in Nuclei

*Baroni, Navrátil, Quaglioni*

*Phys. Rev. Lett. 110, 022505 (2013)*

**ab initio description of nuclei**



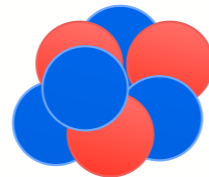
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ab initio description of  
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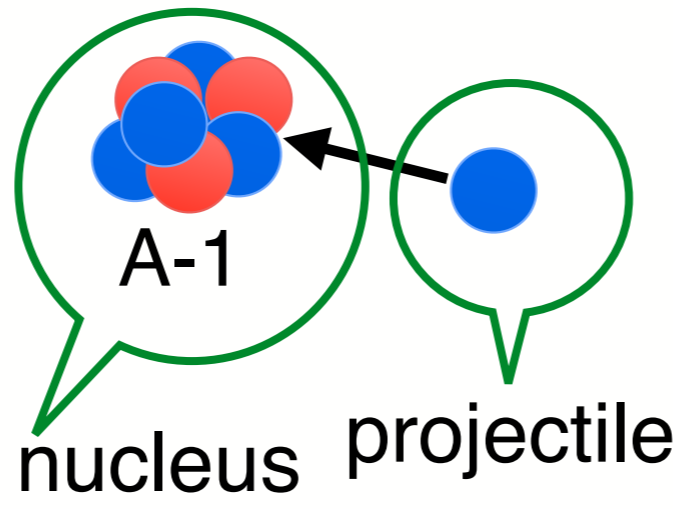
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resonances &  
scattering states



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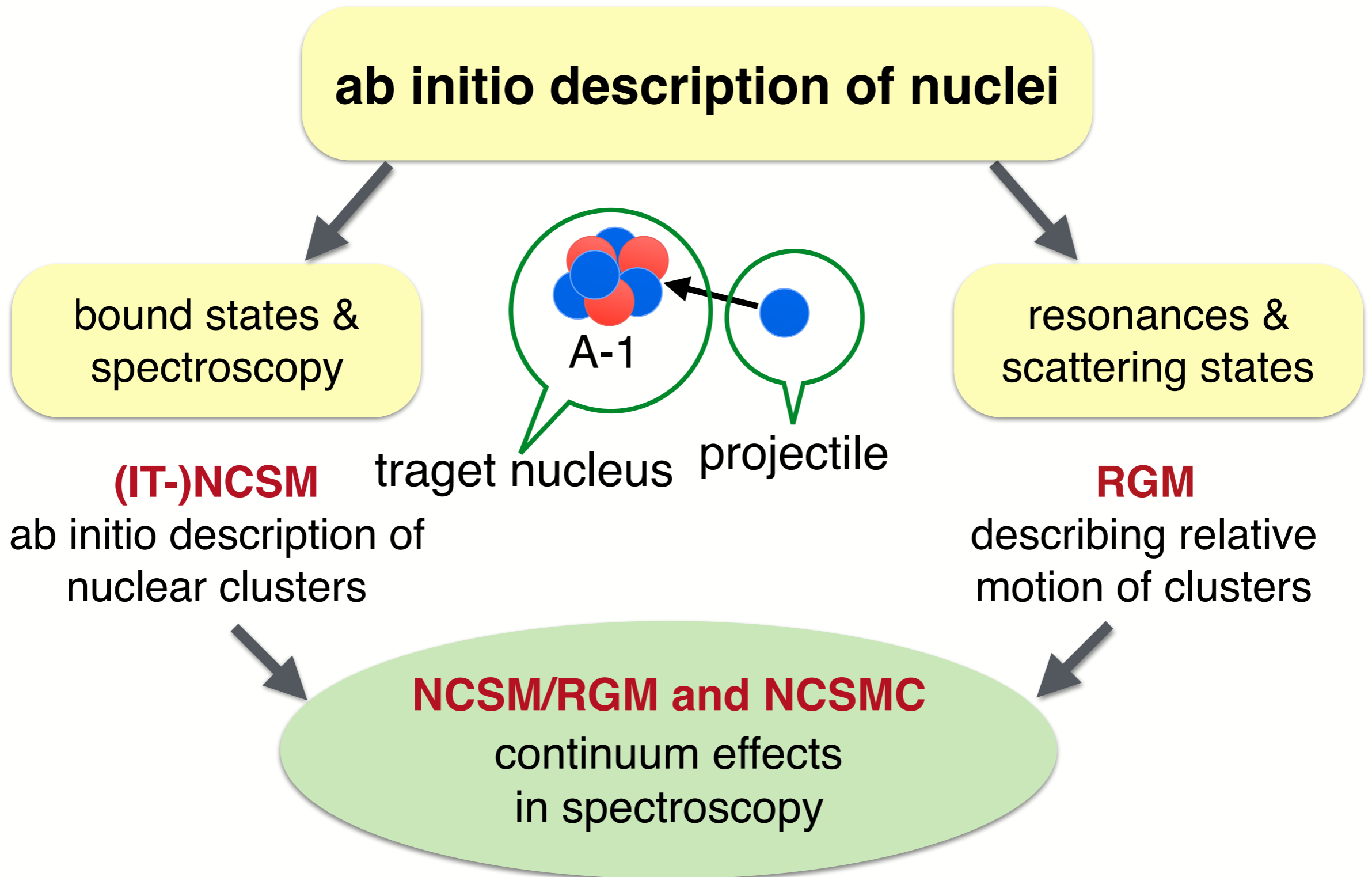
**RGM**

describing relative  
motion of clusters

# Continuum effects in Nuclei

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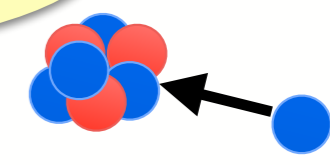
# NCSM with Continuum (NCSMC)

- representing  $H |\psi^{J\pi T}\rangle = E |\psi^{J\pi T}\rangle$  using the **over-complete basis**

$$|\psi^{J\pi T}\rangle = \sum_{\lambda} c_{\lambda} |\Psi_{A E_{\lambda} J^{\pi T}}\rangle + \sum_{\nu} \int dr r^2 \frac{\chi_{\nu}(r)}{r} |\xi_{\nu r}^{J\pi T}\rangle$$



expansion in A-body  
NCSM eigenstates



relative motion of clusters  
NCSM/RGM expansion

- leads to NCSMC equation

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contains NCSM/RGM  
Hamiltonian kernel

# Continuum effects and Nuclear Reactions

with

P. Navrátil, S. Quaglioni,  
R. Roth, J. Dohet-Eraly G. Hupin



## Can *Ab Initio* Theory Explain the Phenomenon of Parity Inversion in $^{11}\text{Be}$ ?

Angelo Calci,<sup>1,\*</sup> Petr Navrátil,<sup>1,†</sup> Robert Roth,<sup>2</sup> Jérémy Dohet-Eraly,<sup>1,‡</sup> Sofia Quaglioni,<sup>3</sup> and Guillaume Hupin<sup>4,5</sup>

<sup>1</sup>TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

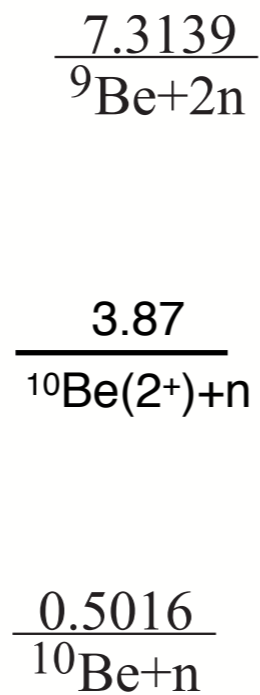
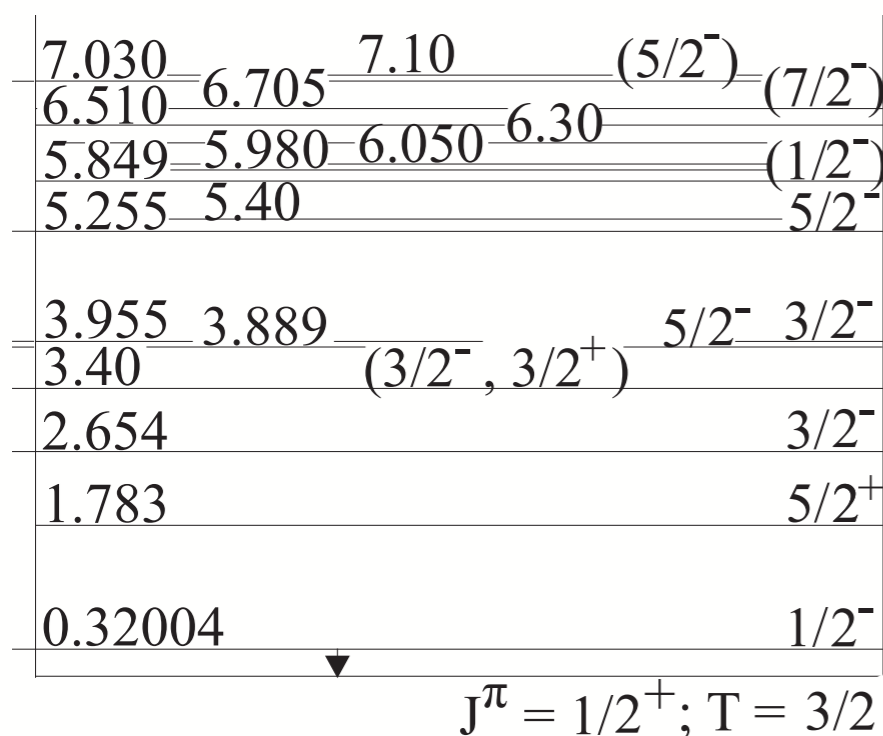
<sup>2</sup>Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany

<sup>3</sup>Lawrence Livermore National Laboratory, P.O. Box 808, L-414, Livermore, California 94551, USA

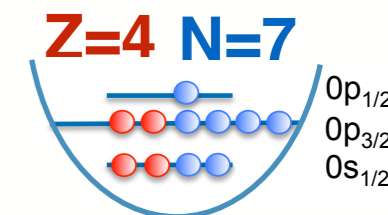
<sup>4</sup>Institut de Physique Nucléaire, Université Paris-Sud, IN2P3/CNRS, F-91406 Orsay Cedex, France

<sup>5</sup>CEA, DAM, DIF, F-91297 Arpajon, France

## Spectrum



- **parity inversion**  
shell model predicts  
g.s. to be  $J^{\pi}=1/2^{-}$



- **Halo structure**  
weakly bound  $J=1/2$  states  
spectrum dominated by  $n-^{10}\text{Be}$

# Neutron-rich halo Nucleus $^{11}\text{Be}$

PRL 117, 242501 (2016)

PHYSICAL REVIEW LETTERS

week ending  
9 DECEMBER 2016

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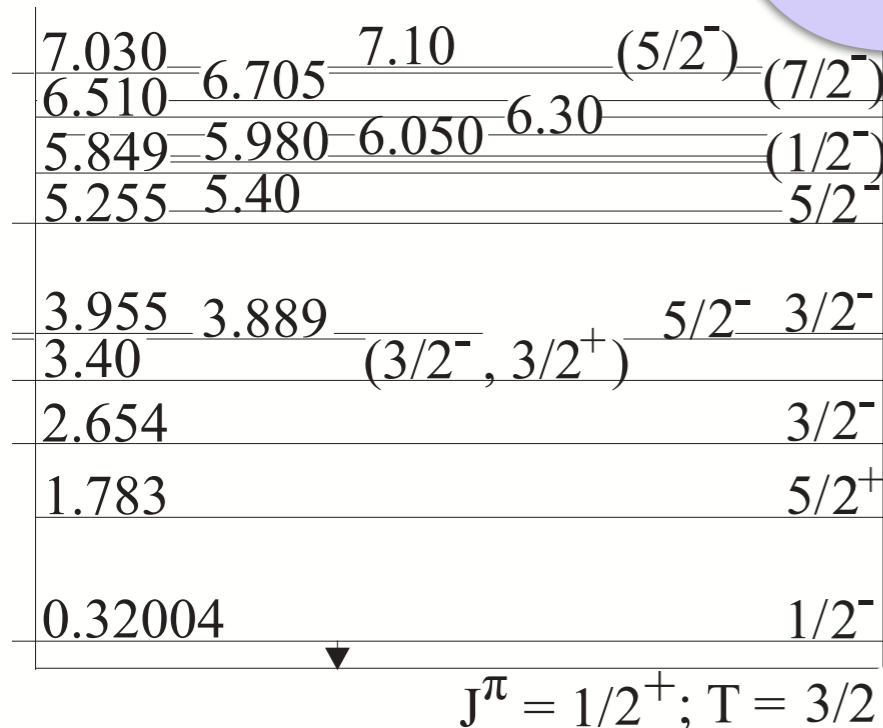
<sup>1</sup>TRIUMF, 4004 Westweek Mall, Vancouver, British Columbia V6T 2A3, Canada

<sup>2</sup>Institut für Experimentelle und Angewandte Physik, Universität zu Köln, 50939 Köln, Germany

<sup>3</sup>Lawrence Livermore National Laboratory, Livermore, California 94551, USA

<sup>4</sup>Institut de Physique Nucléaire, Université Paris-Saclay, 91191 Gif-sur-Yvette Cedex, France

Can **ab initio** theory describe this complicated system?



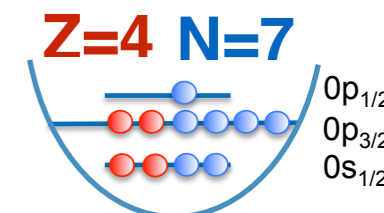
$^9\text{Be}+2n$

$\frac{3.87}{^{10}\text{Be}(2^+)+n}$

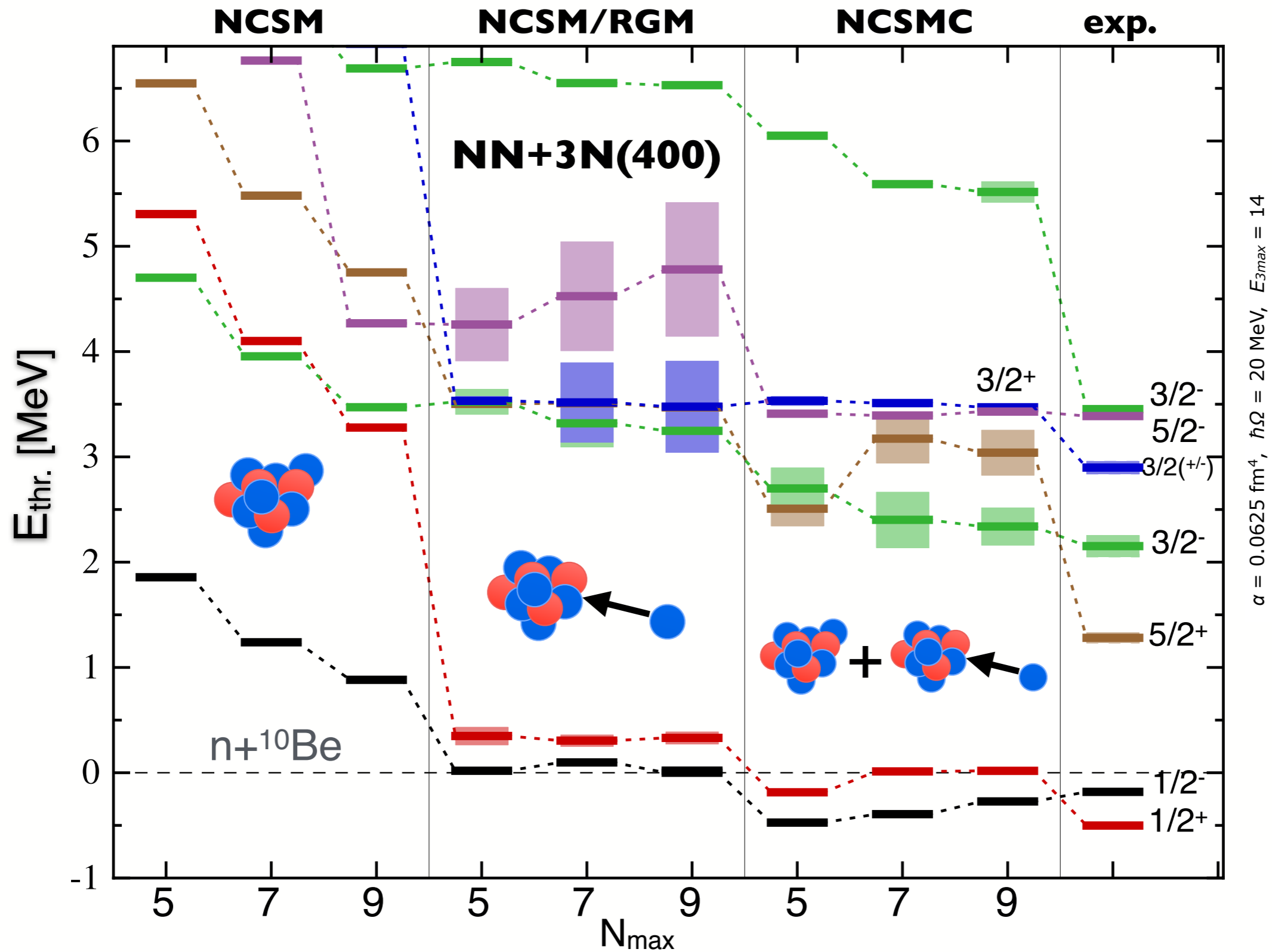
$\frac{0.5016}{^{10}\text{Be}+n}$

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shell model predicts  
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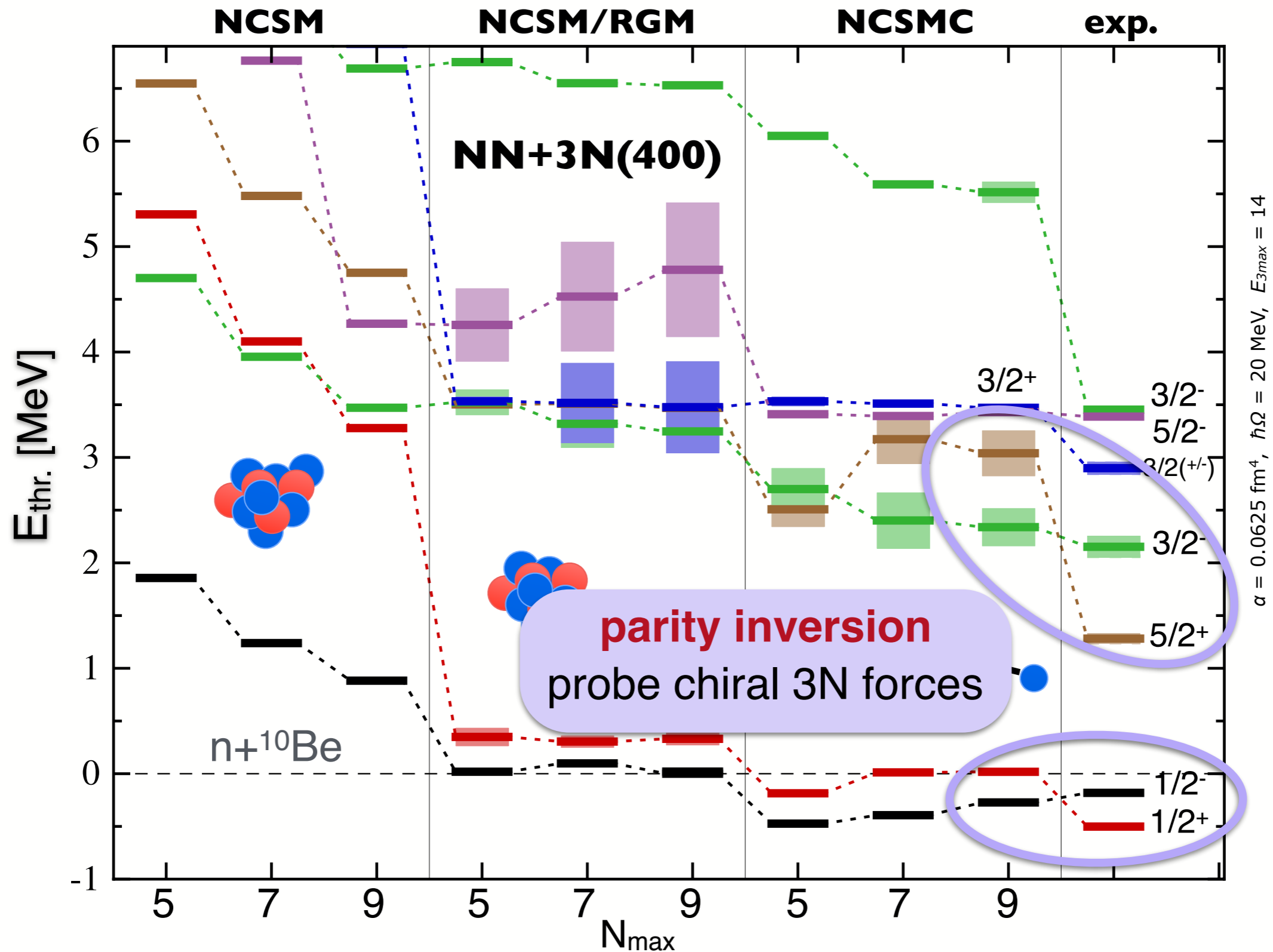
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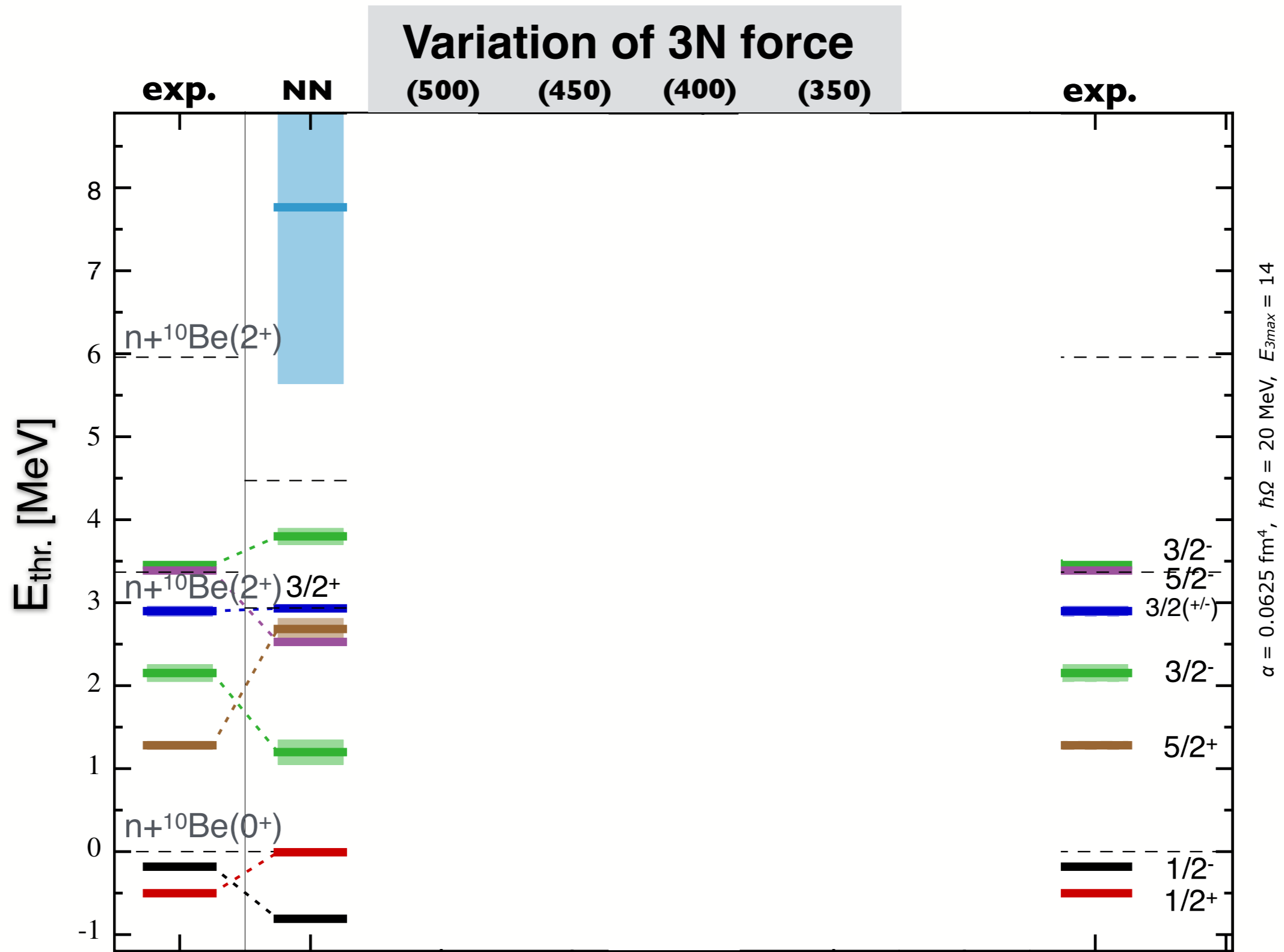
# $^{11}\text{Be}$ excitation spectrum



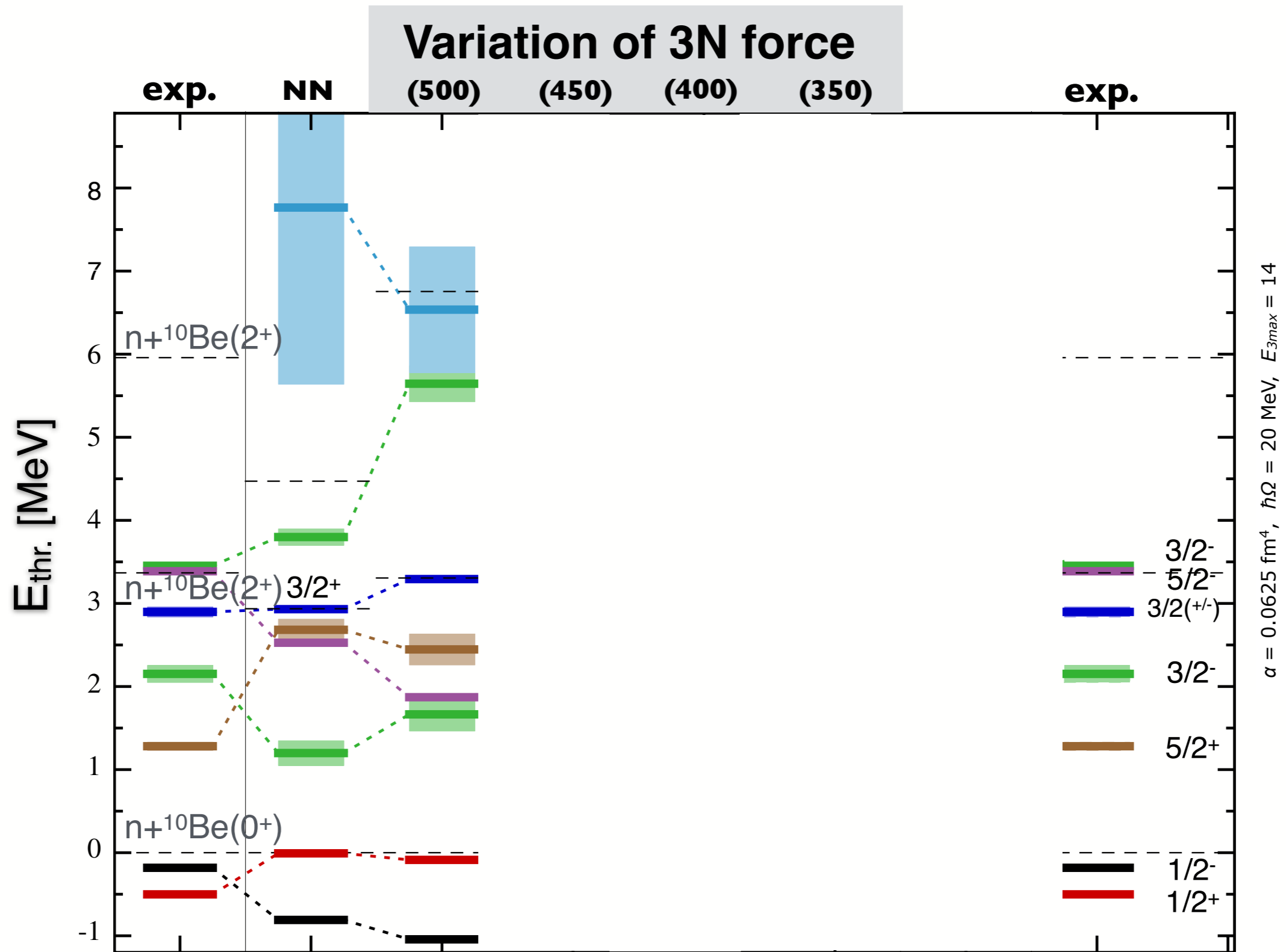
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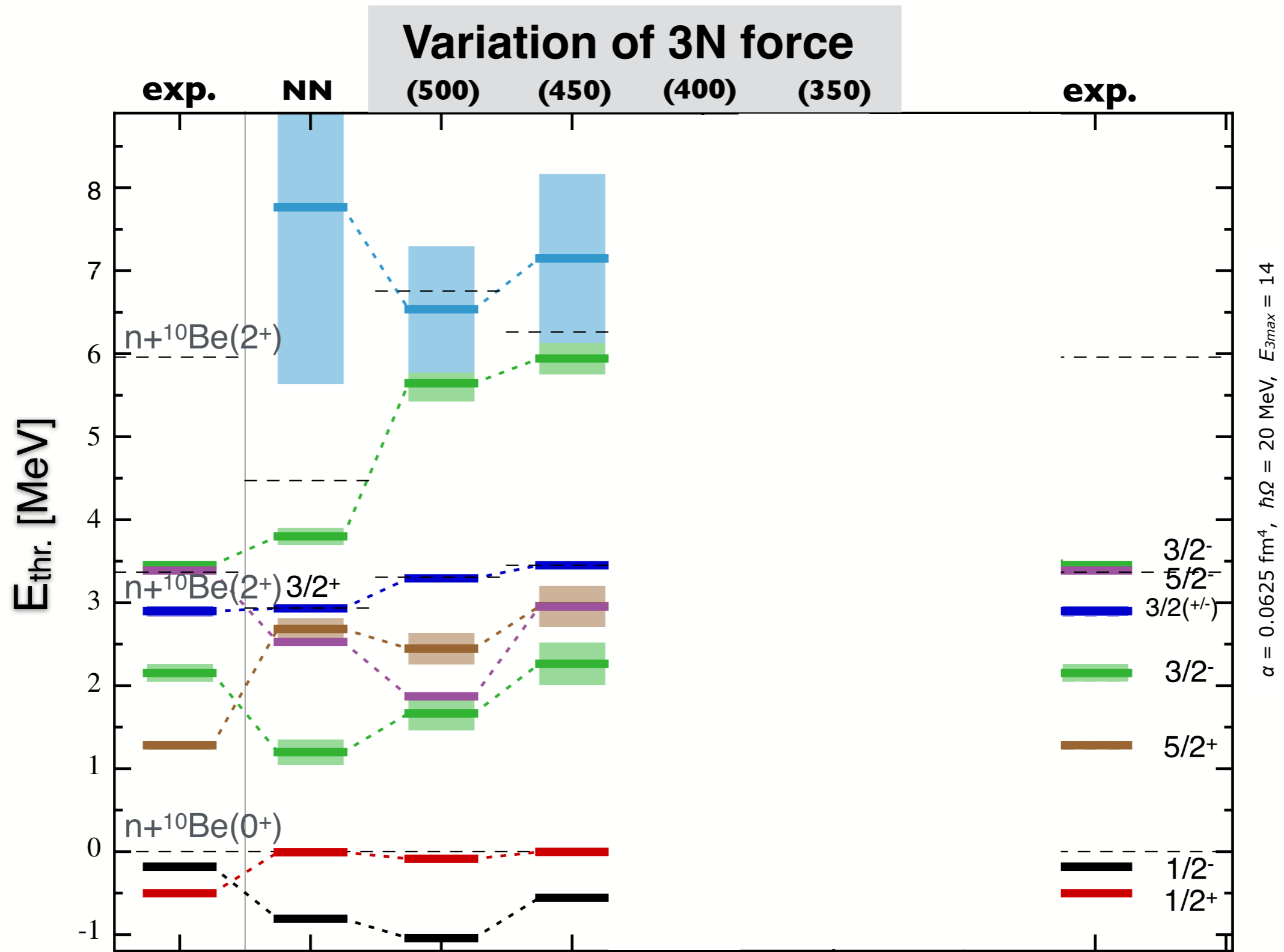
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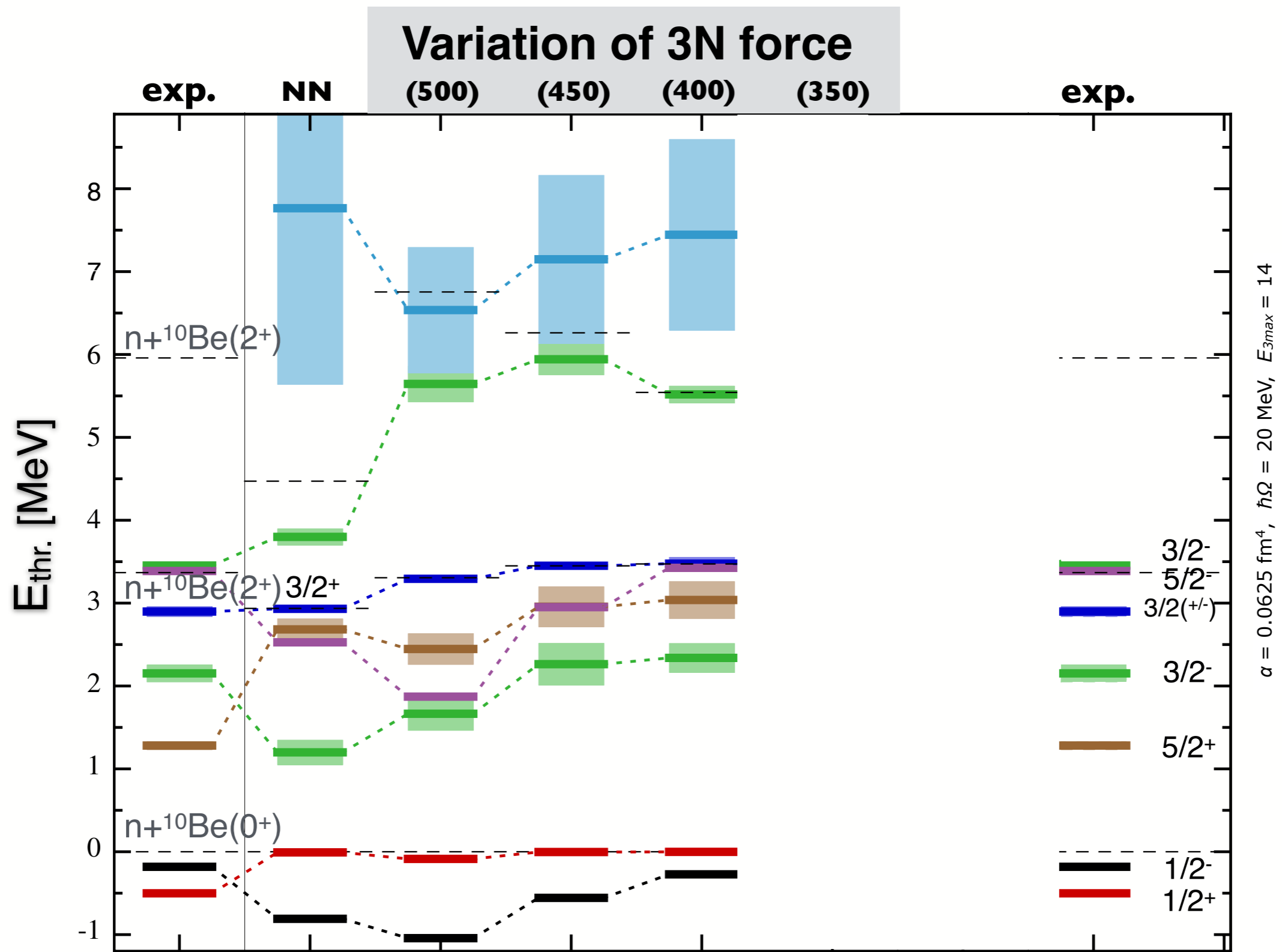
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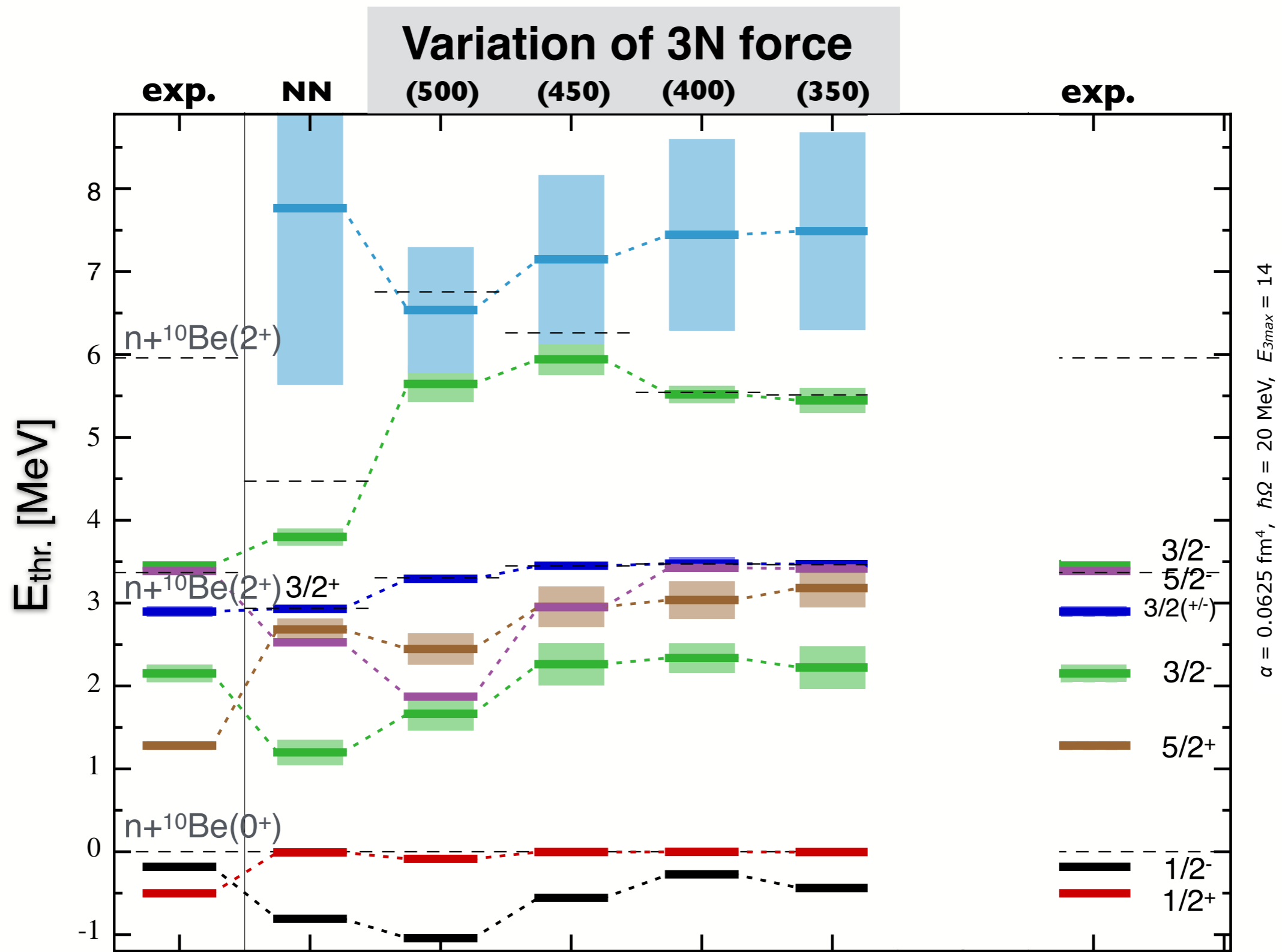


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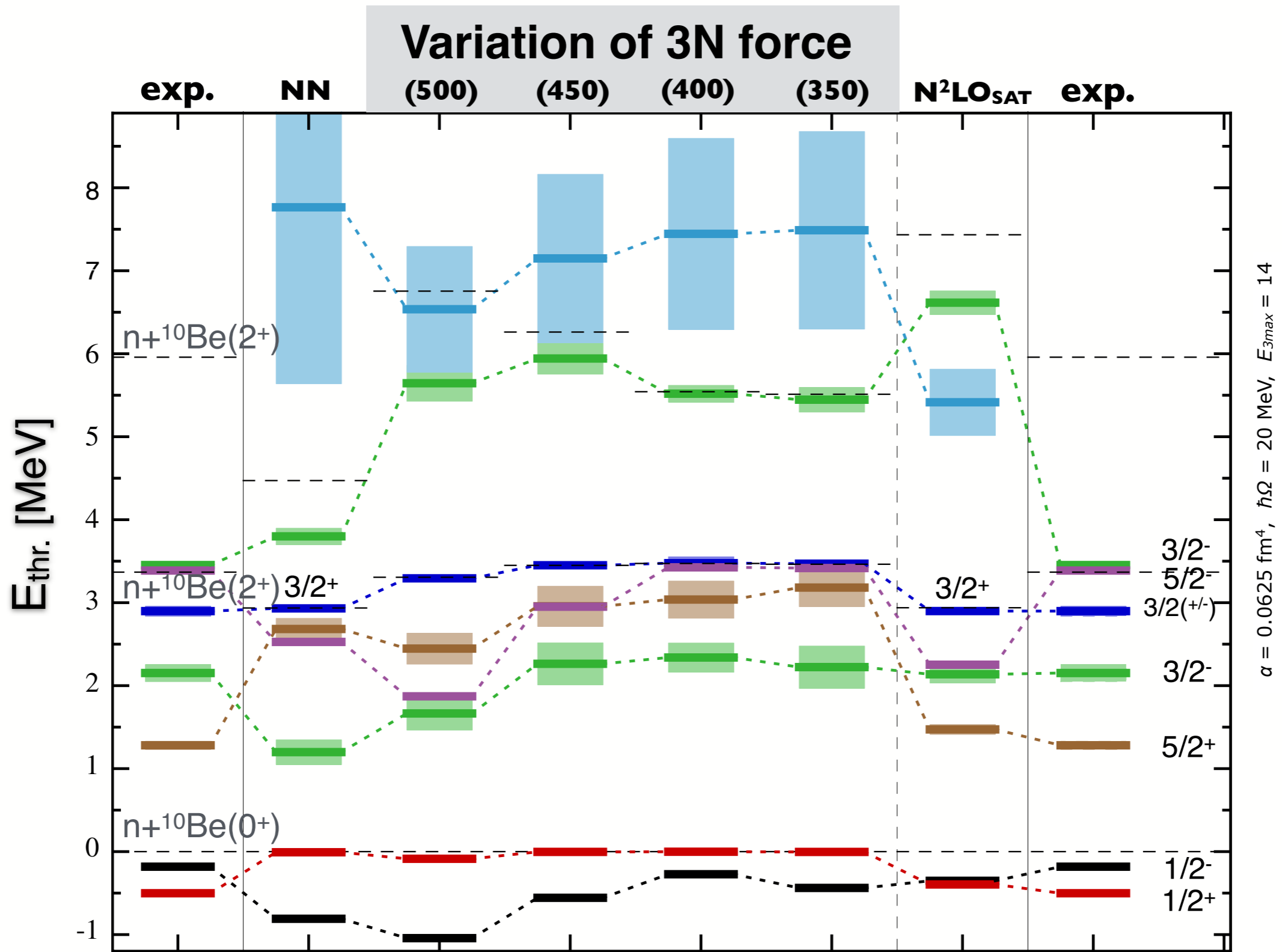




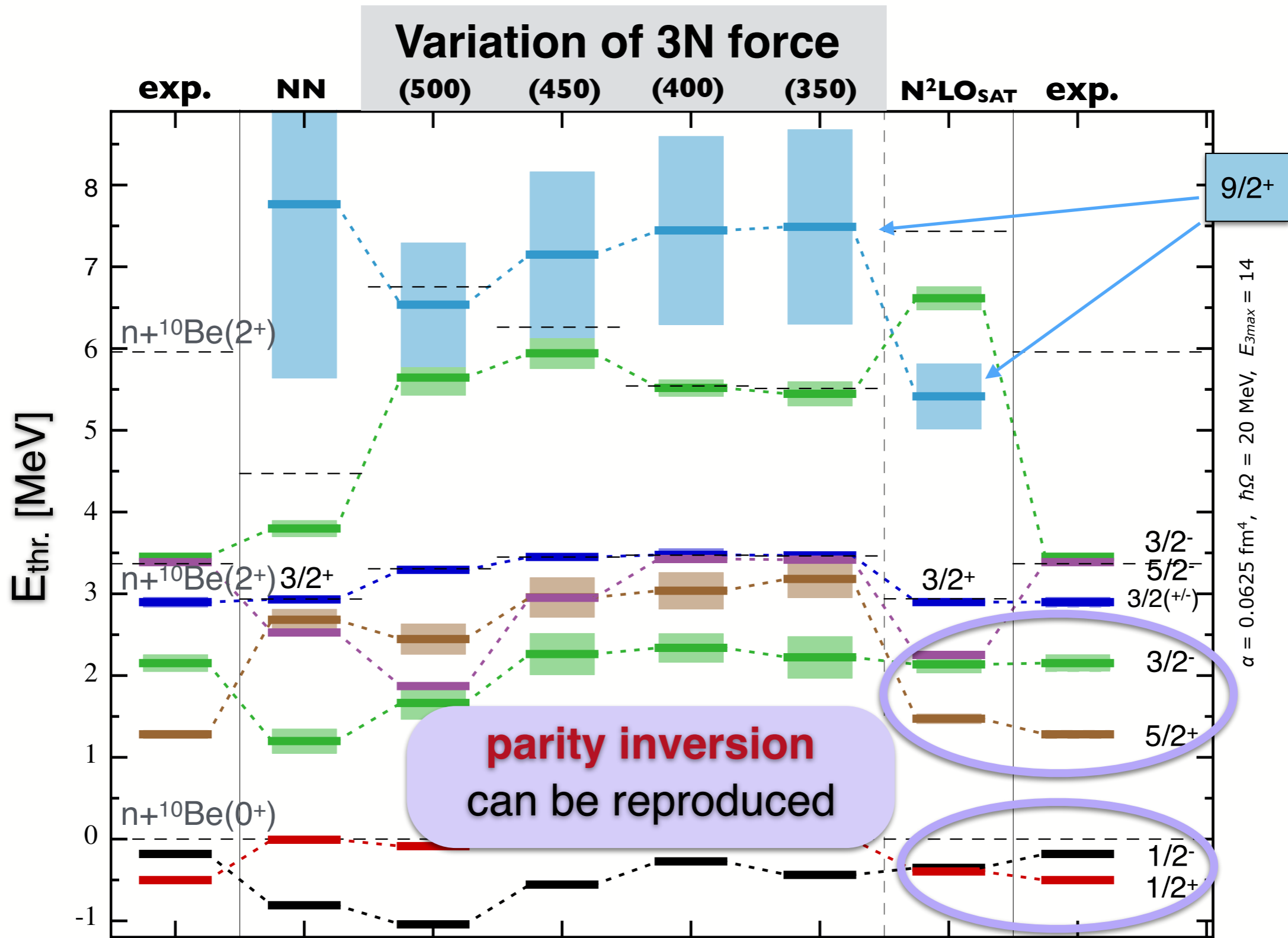
# $^{11}\text{Be}$ excitation spectrum



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# $^{11}\text{Be}$ : Photodisintegration process & E1 transition

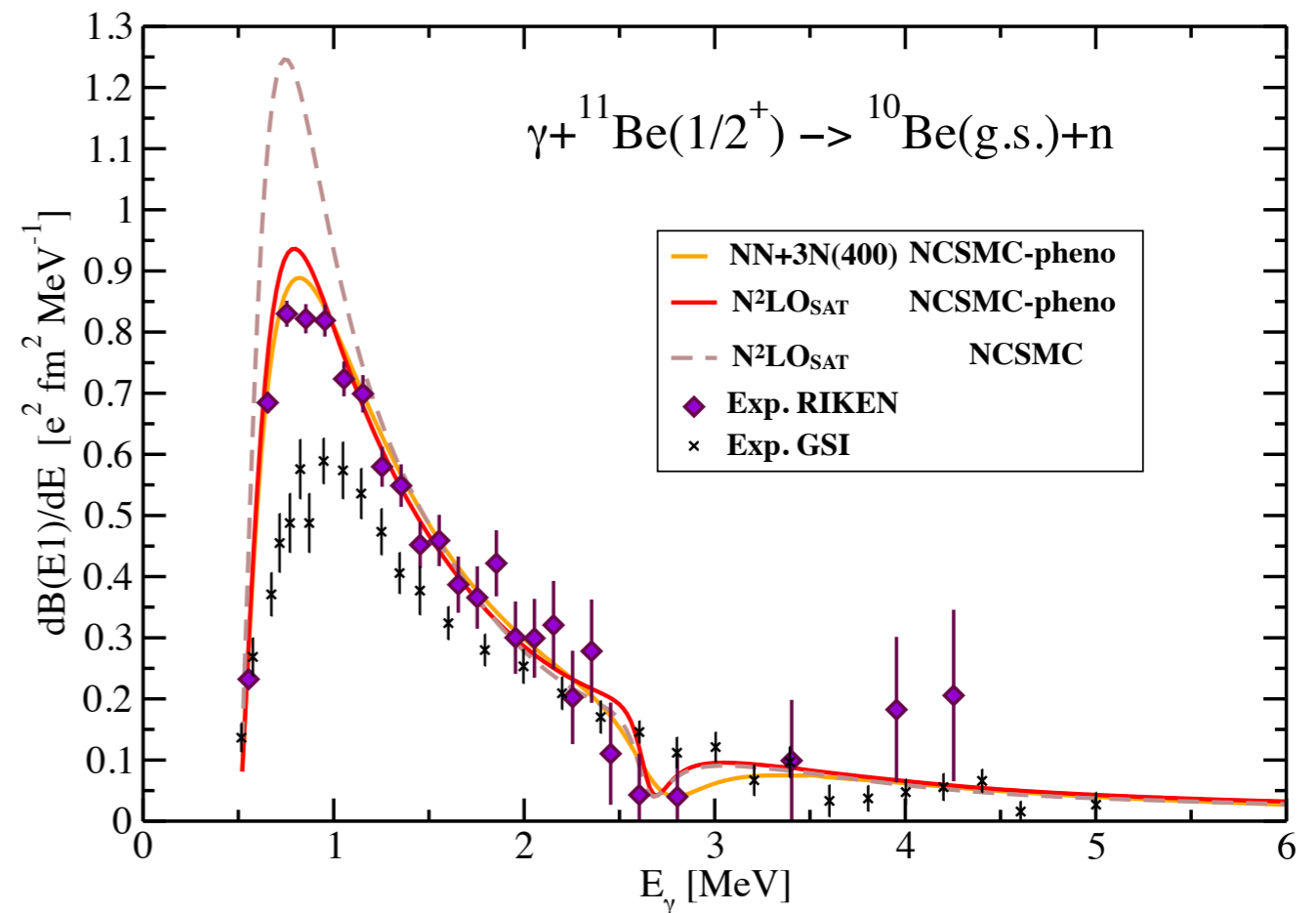
## $B(E1:1/2^- \rightarrow 1/2^+)$ [ $\text{e}^2\text{fm}^2$ ]

	NCSM	NCSMC	NCSMC-pheno	exp.
NN+3N(400)	0.0005	-	0.146	0.102(2)*
$\text{N}^2\text{LO}_{\text{SAT}}$	0.0005	0.127	0.117	

\*Kwan et al. Phys. Lett. B 732, 210 (2014)

- **strongest known E1** transition between low-lying states (attributed to halo structure)

- reproduced **only** with **continuum effects**



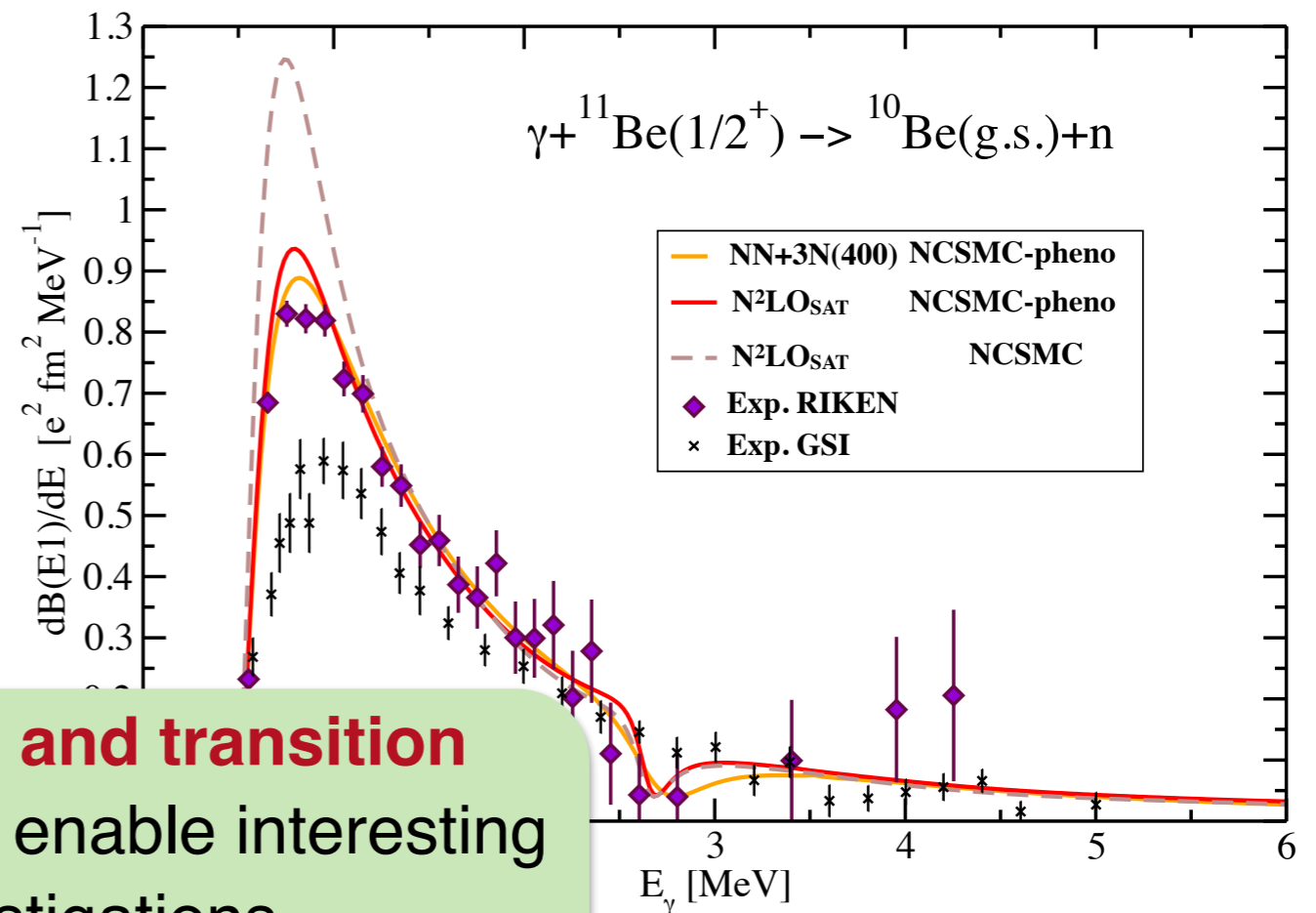
- **conflicting** experimental **measurements**
- ab initio results:
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# $^{11}\text{Be}$ : Photodisintegration process & E1 transition

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**scattering and transition observables** enable interesting investigations

- **strongest known E1 transition** between low-lying states (attributed to halo structure)
- reproduced **only** with **continuum effects**
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# NCSMC with multi-reference normal-ordered (MR-NO) 3N forces

with  
P. Navrátil, R. Roth, E. Gebrerufael

# NCSM with Continuum (NCSMC)

- representing  $H |\psi^{J\pi T}\rangle = E |\psi^{J\pi T}\rangle$  using the **over-complete basis**

$$|\psi^{J\pi T}\rangle = \sum_{\lambda} c_{\lambda} |\Psi_{A E_{\lambda} J^{\pi T}}\rangle + \sum_{\nu} \int dr r^2 \frac{\chi_{\nu}(r)}{r} |\xi_{\nu r}^{J\pi T}\rangle$$

expansion in A-body  
NCSM eigenstates

relative motion of clusters  
NCSM/RGM expansion

- leads to NCSMC equation

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# Derive NCSM/RGM Kernels

## 0B kernel

$$\begin{aligned}
 & -_{SD} \langle \epsilon_{\nu'n'}^{\mathcal{J}\pi T} | \mathbf{V}_{0N} \mathbf{T}_{A-1,A} | \epsilon_{\nu n}^{\mathcal{J}\pi T} \rangle_{SD} = \\
 = & -\frac{1}{A-1} \mathbf{V}_{0N} \sum_{M_1 m_j} \sum_{M_{T_1} m_t} \sum_{M'_1 m'_j} \sum_{M'_{T_1} m'_t} \left( \begin{array}{c} I_1 \quad j \\ M_1 \quad m_j \end{array} \middle| \begin{array}{c} \mathcal{J} \\ \mathcal{M} \end{array} \right) \left( \begin{array}{c} T_1 \quad \frac{1}{2} \\ M_{T_1} \quad m_t \end{array} \middle| \begin{array}{c} T \\ M_T \end{array} \right) \left( \begin{array}{c} I'_1 \quad j' \\ M'_1 \quad m'_j \end{array} \middle| \begin{array}{c} \mathcal{J} \\ \mathcal{M} \end{array} \right) \left( \begin{array}{c} T'_1 \quad \frac{1}{2} \\ M'_{T_1} \quad m'_t \end{array} \middle| \begin{array}{c} T \\ M_T \end{array} \right) \\
 \times & \quad_{SD} \langle \psi'_{A-1} E'_1 I'_1 \pi'_1 M'_1 T'_1 M'_{T_1} | \mathbf{a}_{nljm_j m_t}^\dagger \mathbf{a}_{n'l'j'm'_j m'_t} | \psi_{A-1} E_1 I_1 \pi_1 M_1 T_1 M_{T_1} \rangle_{SD}
 \end{aligned}$$

## 1B kernel

$$\begin{aligned}
 & \quad_{SD} \langle \epsilon_{\nu'n'}^{\mathcal{J}\pi T} | \mathbf{V}_A | \epsilon_{\nu n}^{\mathcal{J}\pi T} \rangle_{SD} \\
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 \times & \quad_{SD} \langle \psi'_{A-1} E'_1 I'_1 \pi'_1 M'_1 T'_1 M'_{T_1} | \psi_{A-1} E_1 I_1 \pi_1 M_1 T_1 M_{T_1} \rangle_{SD} \\
 \times & \langle n'l'j'm'_j \frac{1}{2} m'_t | \mathbf{V}_A | nljm_j \frac{1}{2} m_t \rangle \\
 & -_{SD} \langle \epsilon_{\nu'n'}^{\mathcal{J}\pi T} | \mathbf{V}_A \mathbf{T}_{A-1,A} | \epsilon_{\nu n}^{\mathcal{J}\pi T} \rangle_{SD} \\
 = & -\frac{1}{A-1} \sum_{M_1 m_j} \sum_{M_{T_1} m_t} \sum_{M'_1 m'_j} \sum_{M'_{T_1} m'_t} \left( \begin{array}{c} I_1 \quad j \\ M_1 \quad m_j \end{array} \middle| \begin{array}{c} \mathcal{J} \\ \mathcal{M} \end{array} \right) \left( \begin{array}{c} T_1 \quad \frac{1}{2} \\ M_{T_1} \quad m_t \end{array} \middle| \begin{array}{c} T \\ M_T \end{array} \right) \left( \begin{array}{c} I'_1 \quad j' \\ M'_1 \quad m'_j \end{array} \middle| \begin{array}{c} \mathcal{J} \\ \mathcal{M} \end{array} \right) \left( \begin{array}{c} T'_1 \quad \frac{1}{2} \\ M'_{T_1} \quad m'_t \end{array} \middle| \begin{array}{c} T \\ M_T \end{array} \right) \\
 \times & \sum_{\alpha_{A-1}} \quad_{SD} \langle \psi'_{A-1} E'_1 I'_1 \pi'_1 M'_1 T'_1 M'_{T_1} | \mathbf{a}_{nljm_j m_t}^\dagger \mathbf{a}_{\alpha_{A-1}} | \psi_{A-1} E_1 I_1 \pi_1 M_1 T_1 M_{T_1} \rangle_{SD} \\
 \times & \langle n'l'j'm'_j \frac{1}{2} m'_t | \mathbf{V}_A | \alpha_{A-1} \rangle
 \end{aligned}$$

## 2B kernel

...

# Derive NCSM/RGM Kernels

## 0B kernel

dominant 0B kernel contribution included in target eigenstates  
 $\Rightarrow$  only MR-NO 1B and 2B kernels contribute

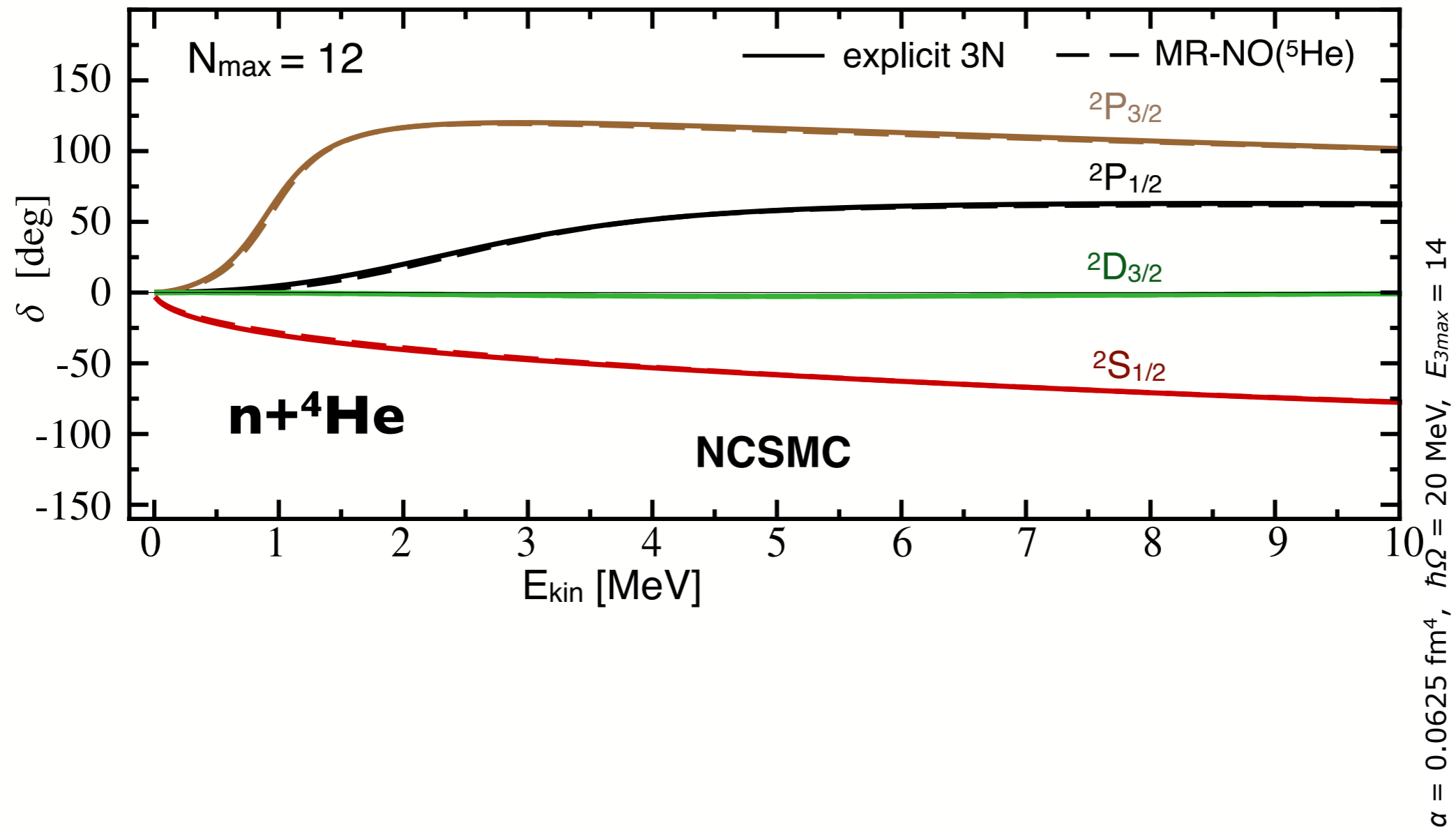
## 1B kernel

$$\begin{aligned}
 & {}_{SD} \langle \epsilon_{\nu'n'}^{\mathcal{J}\pi T} | \mathbf{V}_A | \epsilon_{\nu n}^{\mathcal{J}\pi T} \rangle_{SD} \\
 = & \sum_{M_1 m_j} \sum_{M_{T_1} m_t} \sum_{M'_1 m'_j} \sum_{M'_{T_1} m'_t} \begin{pmatrix} I_1 & j & | & \mathcal{J} \\ M_1 & m_j & | & \mathcal{M} \end{pmatrix} \begin{pmatrix} T_1 & \frac{1}{2} & | & T \\ M_{T_1} & m_t & | & M_T \end{pmatrix} \begin{pmatrix} I'_1 & j' & | & \mathcal{J} \\ M'_1 & m'_j & | & \mathcal{M} \end{pmatrix} \begin{pmatrix} T'_1 & \frac{1}{2} & | & T \\ M'_{T_1} & m'_t & | & M_T \end{pmatrix} \\
 \times & {}_{SD} \langle \psi'_{A-1} E'_1 I'_1 \pi'_1 M'_1 T'_1 M'_{T_1} | \psi_{A-1} E_1 I_1 \pi_1 M_1 T_1 M_{T_1} \rangle_{SD} \\
 \times & \langle n'l'j'm'_j \frac{1}{2} m'_t | \mathbf{V}_A | nljm_j \frac{1}{2} m_t \rangle \\
 & - {}_{SD} \langle \epsilon_{\nu'n'}^{\mathcal{J}\pi T} | \mathbf{V}_A \mathbf{T}_{A-1,A} | \epsilon_{\nu n}^{\mathcal{J}\pi T} \rangle_{SD} \\
 = & -\frac{1}{A-1} \sum_{M_1 m_j} \sum_{M_{T_1} m_t} \sum_{M'_1 m'_j} \sum_{M'_{T_1} m'_t} \begin{pmatrix} I_1 & j & | & \mathcal{J} \\ M_1 & m_j & | & \mathcal{M} \end{pmatrix} \begin{pmatrix} T_1 & \frac{1}{2} & | & T \\ M_{T_1} & m_t & | & M_T \end{pmatrix} \begin{pmatrix} I'_1 & j' & | & \mathcal{J} \\ M'_1 & m'_j & | & \mathcal{M} \end{pmatrix} \begin{pmatrix} T'_1 & \frac{1}{2} & | & T \\ M'_{T_1} & m'_t & | & M_T \end{pmatrix} \\
 \times & \sum_{\alpha_{A-1}} {}_{SD} \langle \psi'_{A-1} E'_1 I'_1 \pi'_1 M'_1 T'_1 M'_{T_1} | \mathbf{a}_{nljm_j m_t}^\dagger \mathbf{a}_{\alpha_{A-1}} | \psi_{A-1} E_1 I_1 \pi_1 M_1 T_1 M_{T_1} \rangle_{SD} \\
 \times & \langle n'l'j'm'_j \frac{1}{2} m'_t | \mathbf{V}_A | \alpha_{A-1} \rangle
 \end{aligned}$$

## 2B kernel

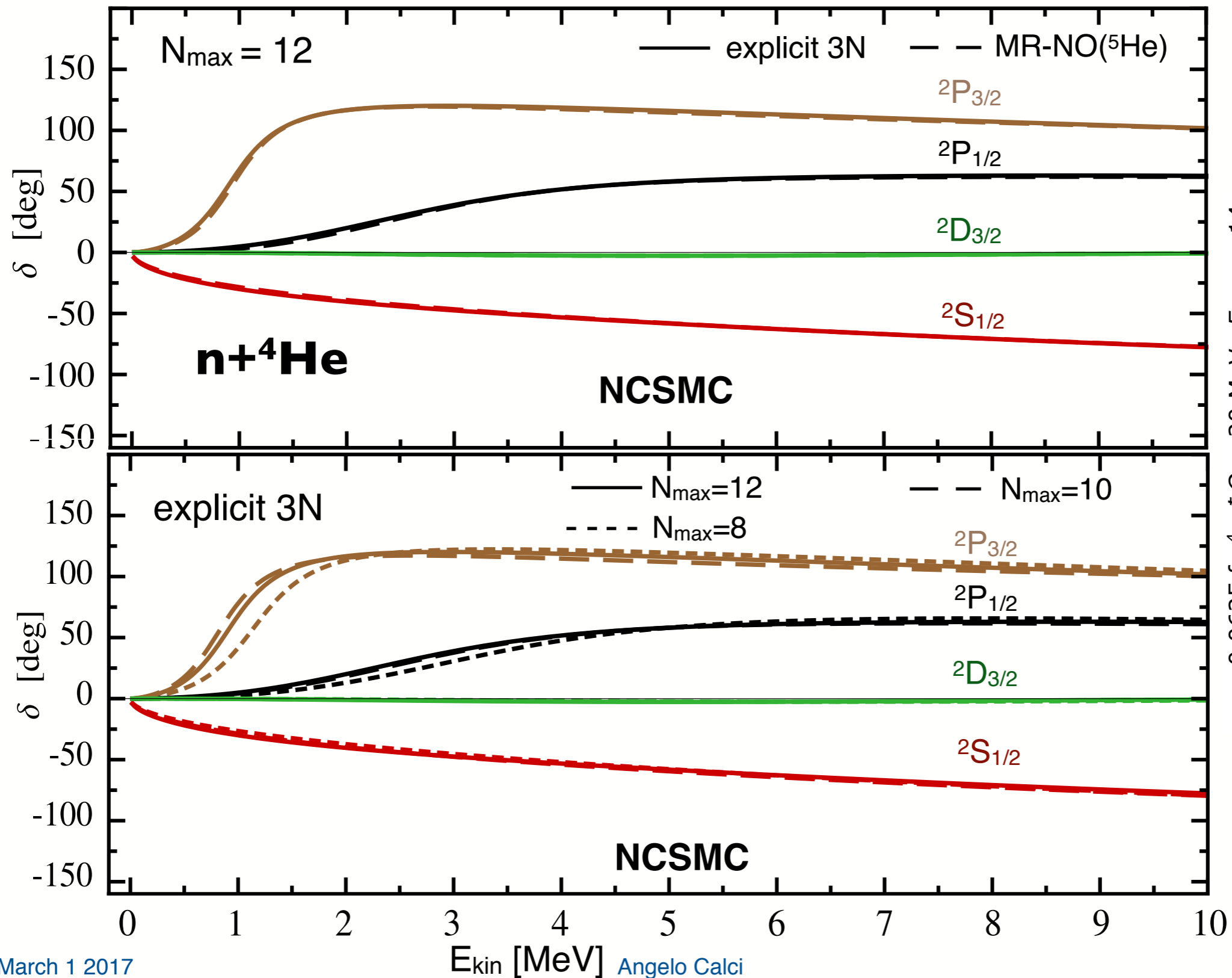
...

# NCSMC: Impact of 3N in Kernels



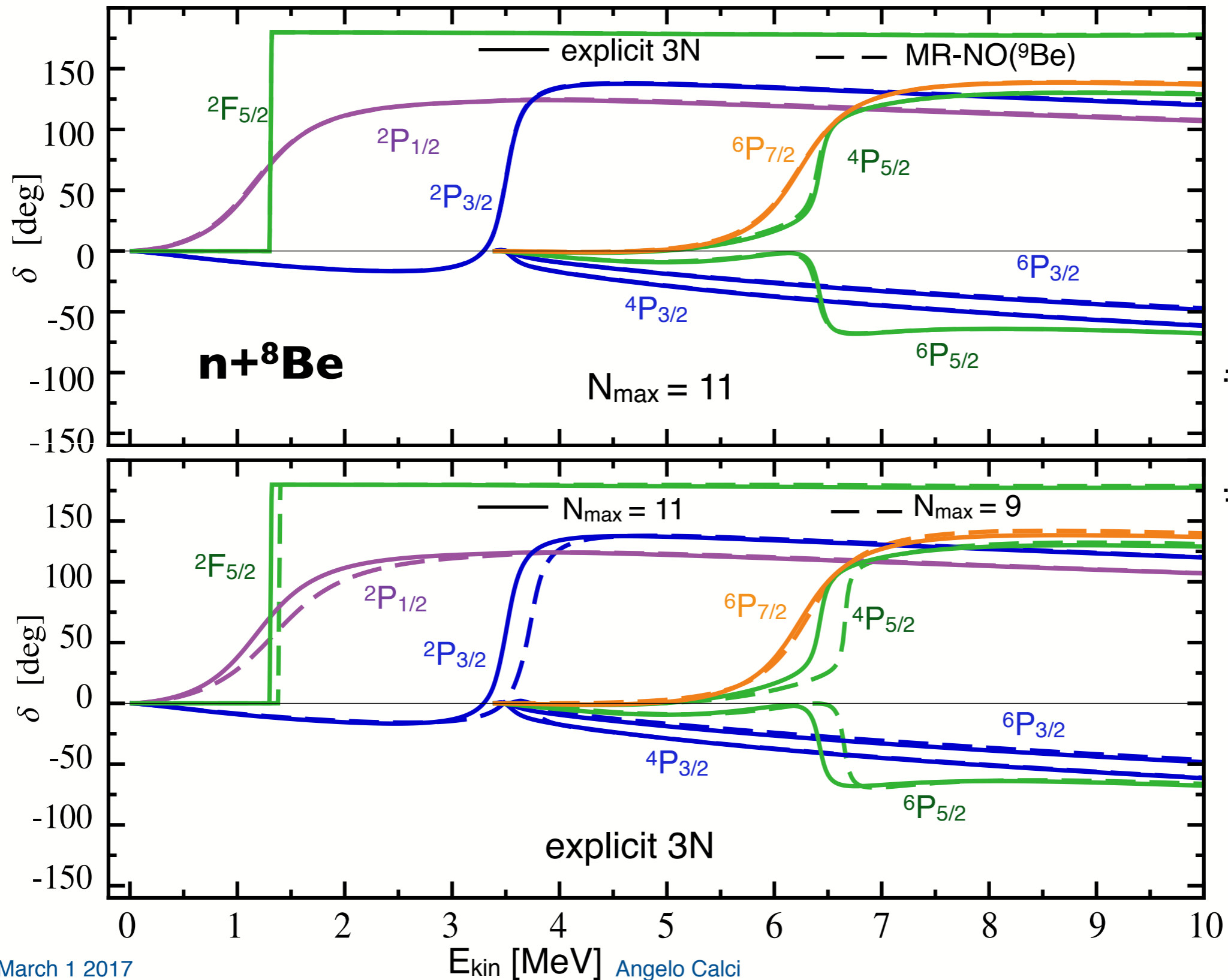
**NN+3N(500)**

# NCSMC: Impact of 3N in Kernels

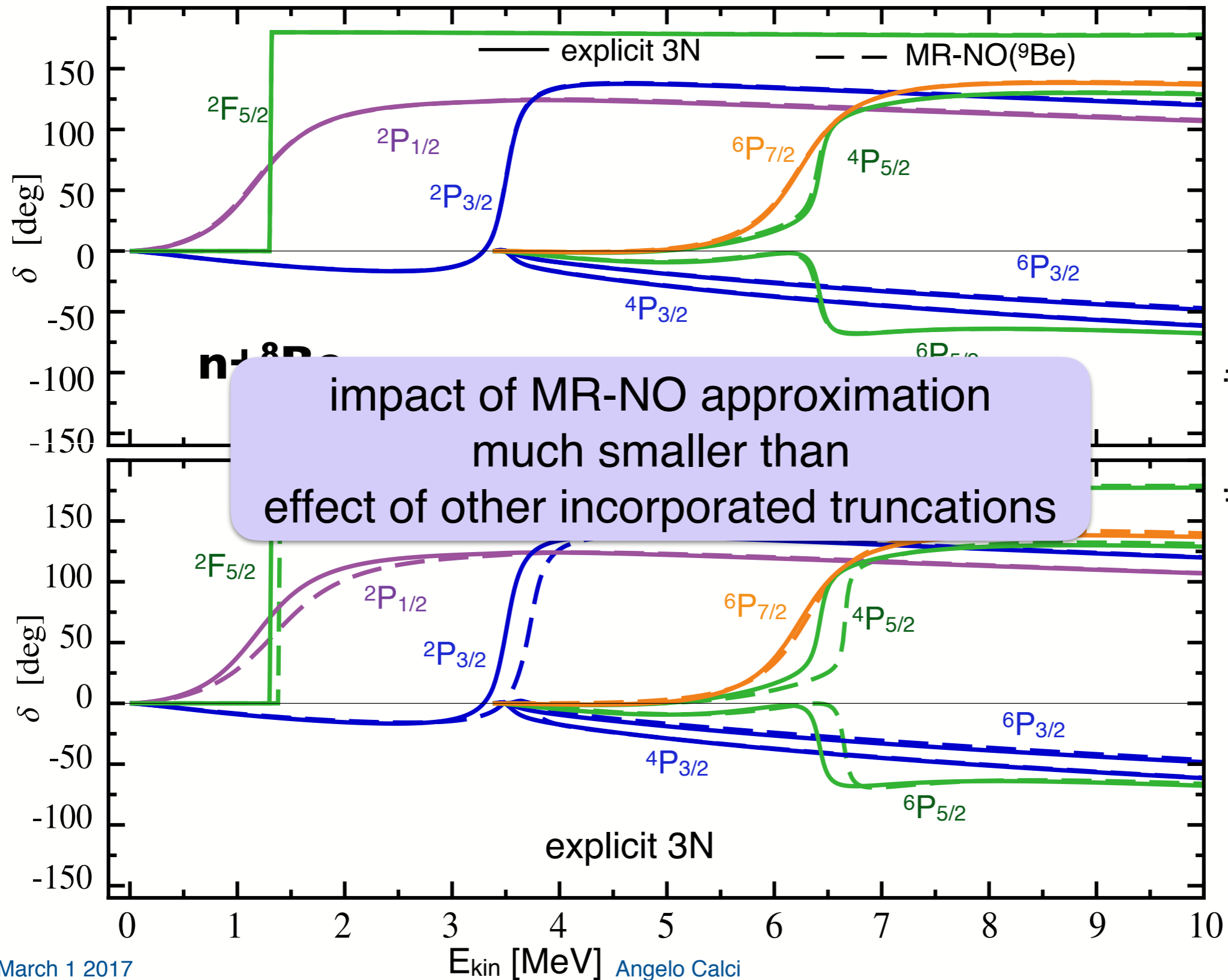


**NN+3N(500)**

# NCSMC: Impact of 3N in Kernels



# NCSMC: Impact of 3N in Kernels

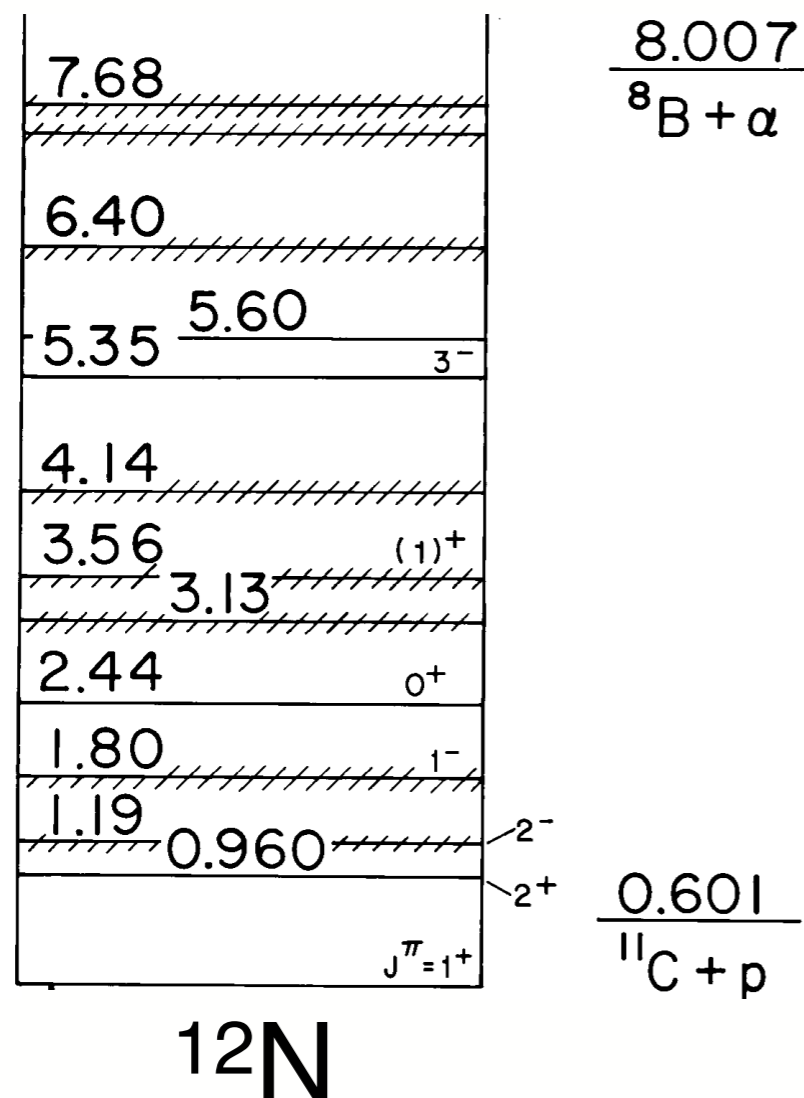


negative parity  
 $\alpha = 0.0625 \text{ fm}^4$ ,  $\hbar\Omega = 20 \text{ MeV}$ ,  $E_{3\text{max}} = 14$   
**NN+3N(400)**

# First application: $^{12}\text{N}$

- ideal candidate**

weakly bound  $J=1^+$  state  
dominated by  $p\text{-}^{11}\text{C}$

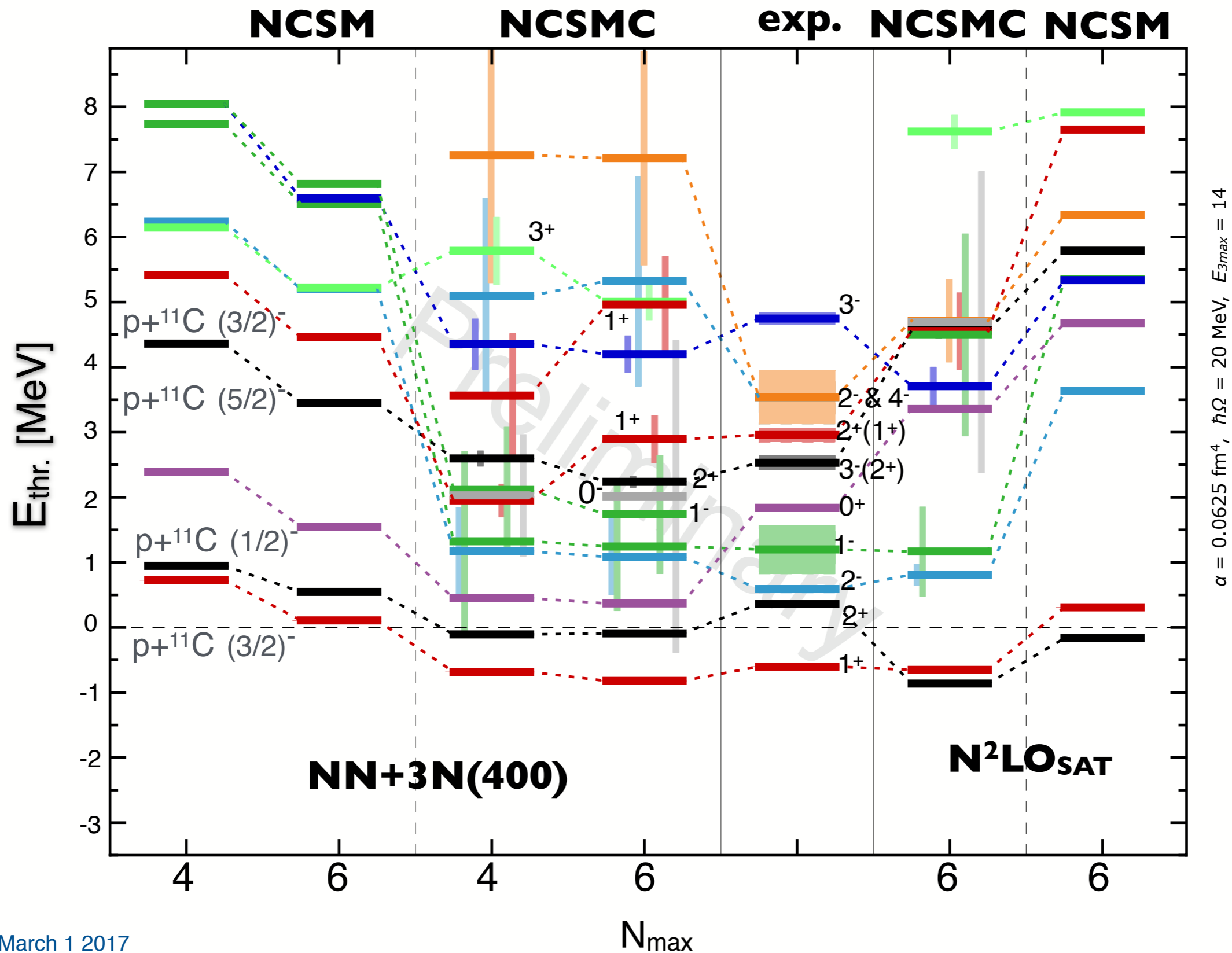


- some low lying resonances not measured precisely
- $^{11}\text{C}(p,\gamma)^{12}\text{N}$  can bypass triple-alpha process
- planned experiment at TUDA facility at TRIUMF

## ab initio NCSMC

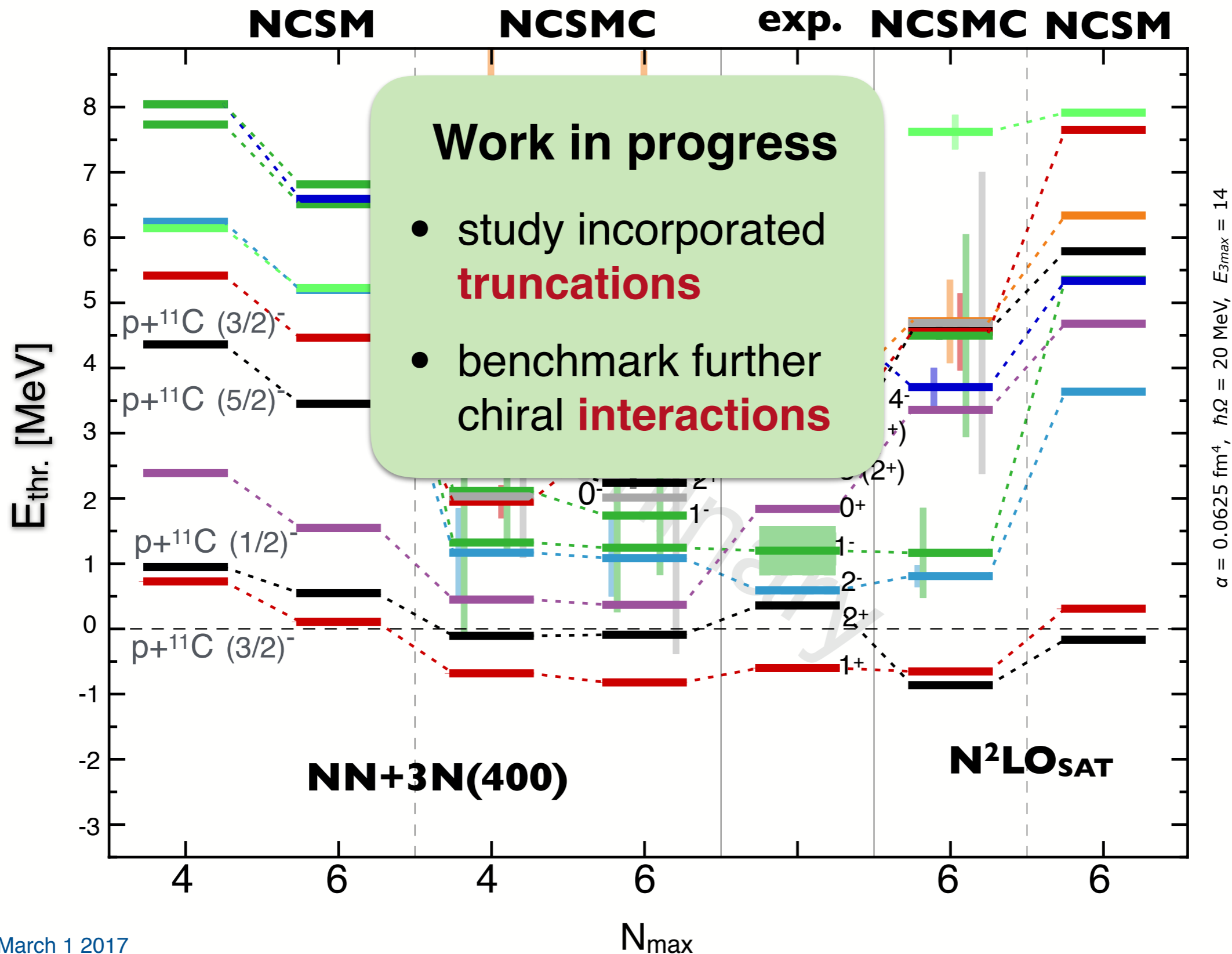
- include  $p\text{-}^{11}\text{C}$  continuum ( $3/2^-, 1/2^-, 5/2^-, 3/2^-$  states of  $^{11}\text{C}$ )
- include 4 negative and 6 positive parity states of  $^{12}\text{N}$
- MR-NO with respect to  $N_{\text{max}}=0$  eigenstate of  $^{12}\text{N}$

# $^{12}\text{N}$ spectrum with continuum effects

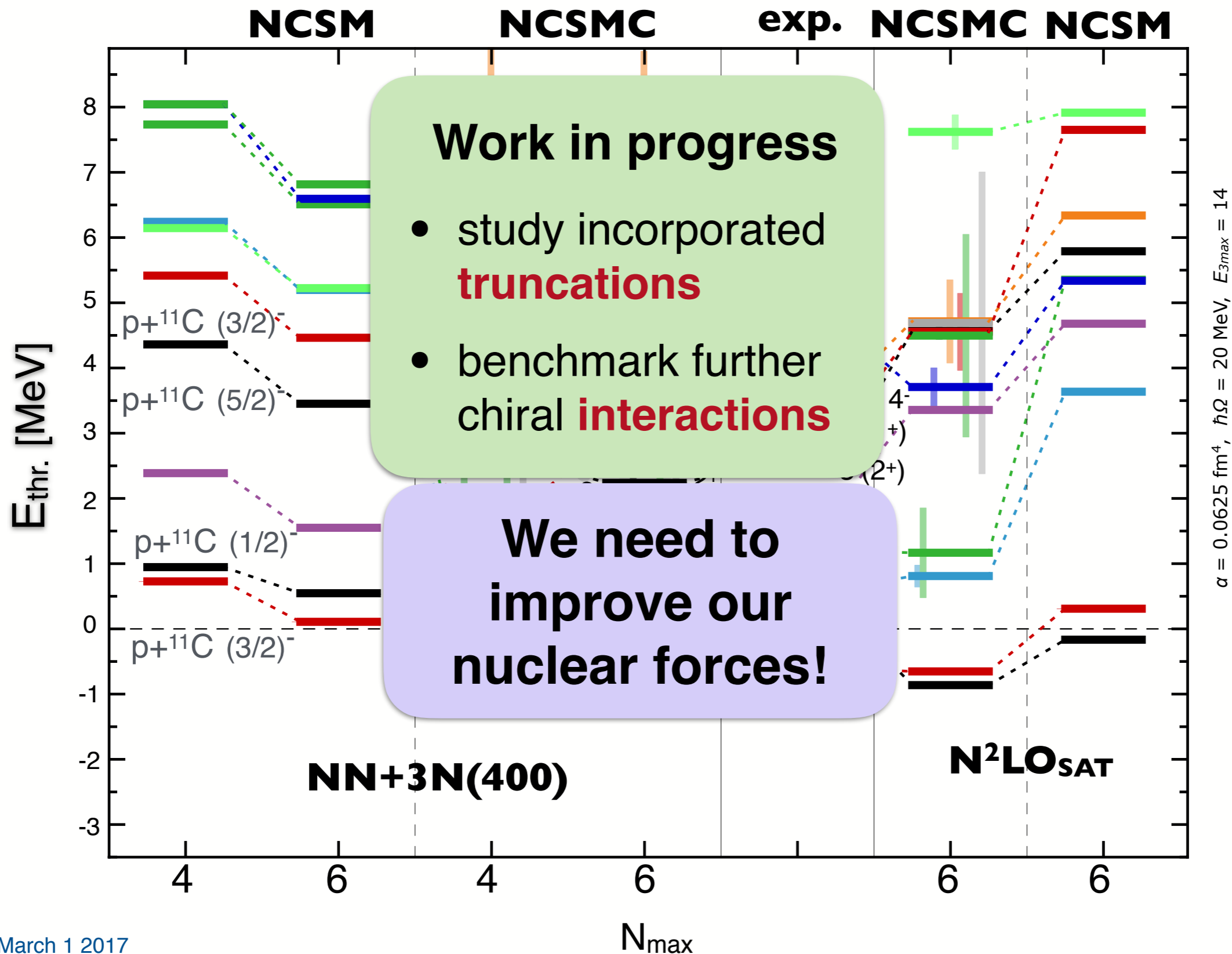




# $^{12}\text{N}$ spectrum with continuum effects

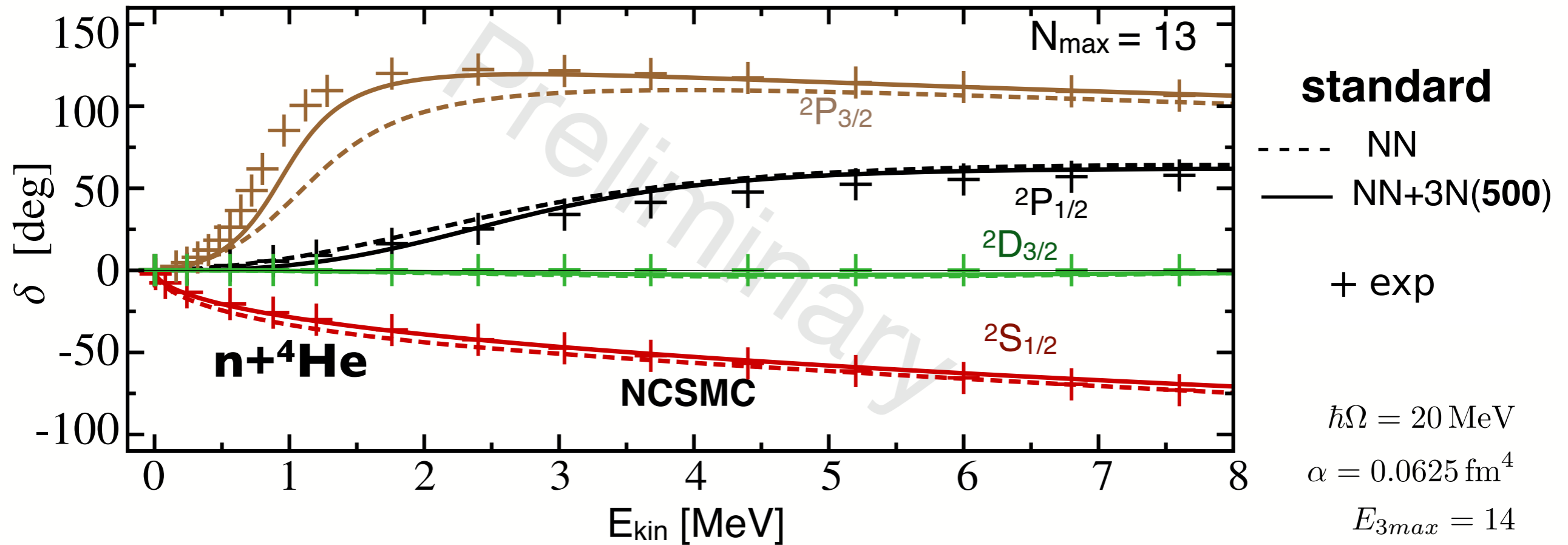


# $^{12}\text{N}$ spectrum with continuum effects

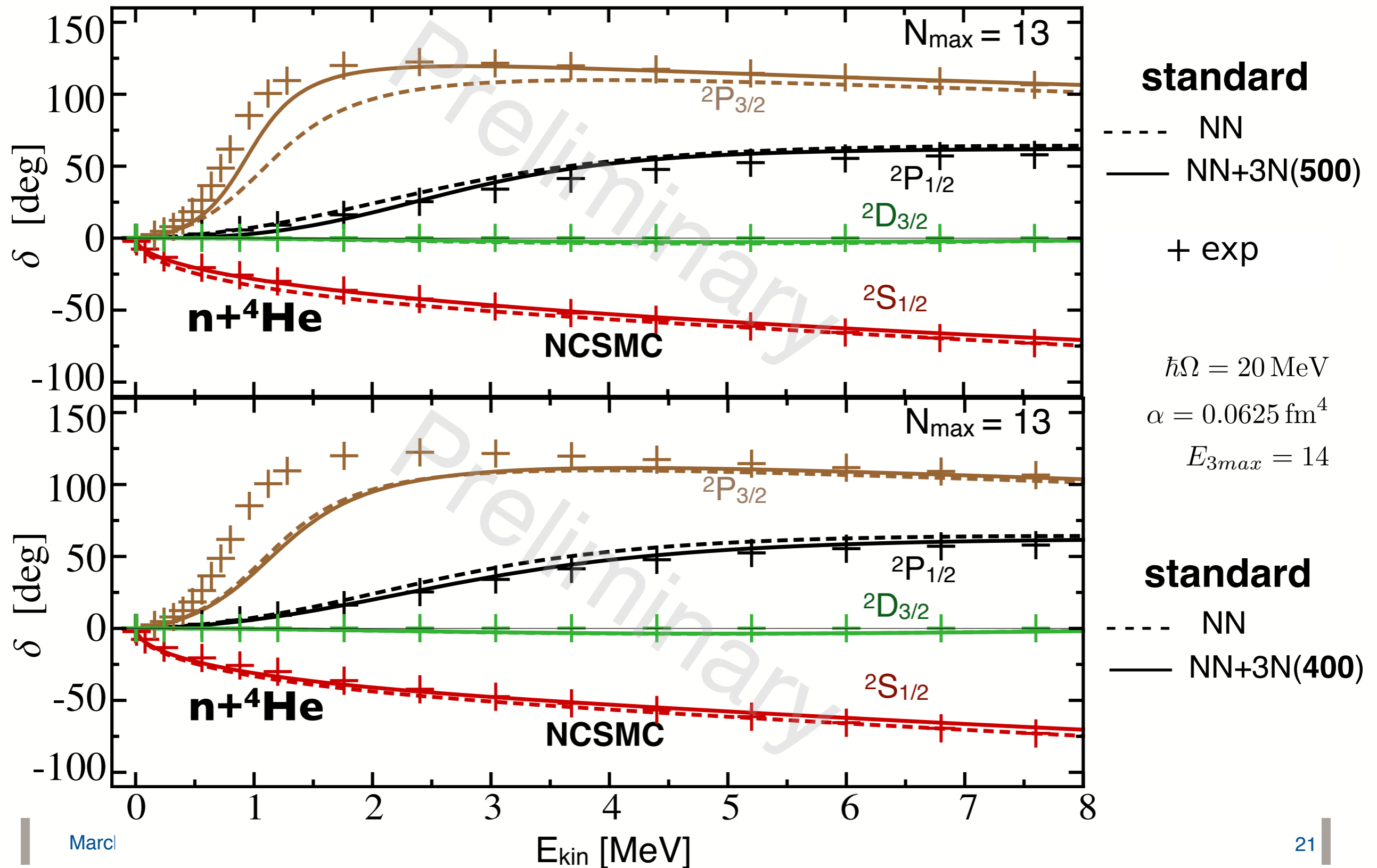


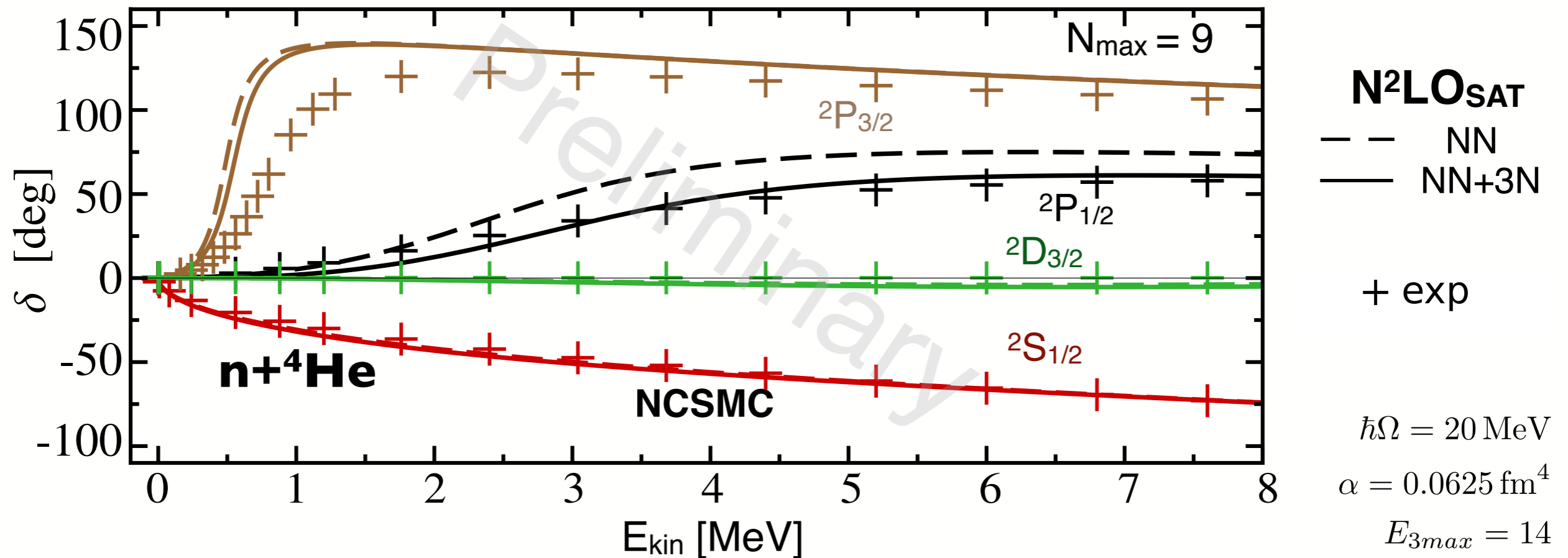
# Probe chiral interaction in light nuclear scattering

# n-<sup>4</sup>He: Standard interaction



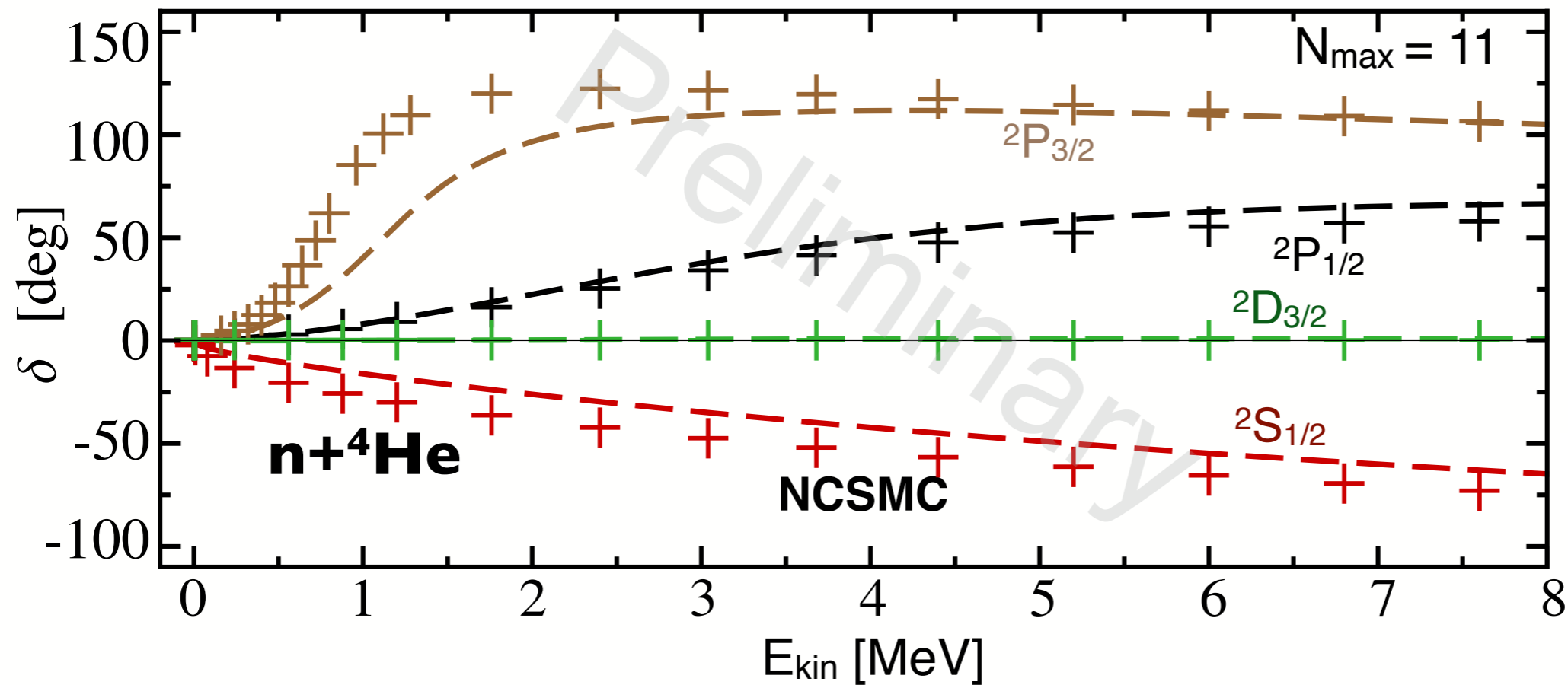
# n-<sup>4</sup>He: Standard interaction





- $P_{3/2} - P_{1/2}$  splitting sensitive to details of nuclear force
- under- or overestimated by NN+3N(400) or N<sup>2</sup>LO<sub>SAT</sub> interaction

# n-<sup>4</sup>He with LENPIC interaction



**LENPIC**  
interaction  
N<sup>2</sup>LO  
 $R = 1.0$  fm

--- NN

+ exp

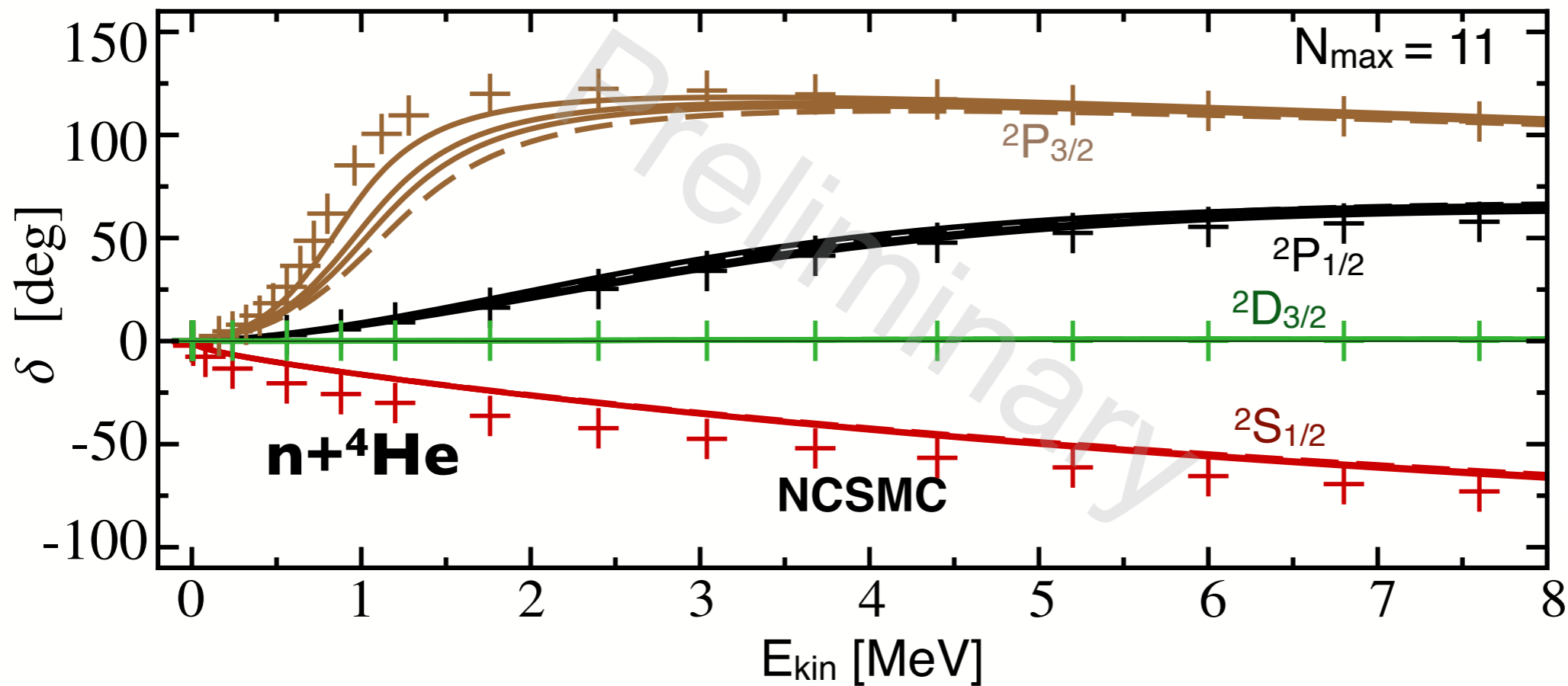
- splitting underestimated without 3N interaction

$$\hbar\Omega = 24 \text{ MeV}$$

$$\alpha = 0.08 \text{ fm}^4$$

$$E_{3max} = 14$$

# n-<sup>4</sup>He with LENPIC interaction



**LENPIC**  
interaction  
N<sup>2</sup>LO  
 $R = 1.0$  fm

--- NN  
— NN+3N

+ exp

$C_D = 2, 4, 6$

$C_E$  fitted  
to Triton g.s.

$\hbar\Omega = 24$  MeV

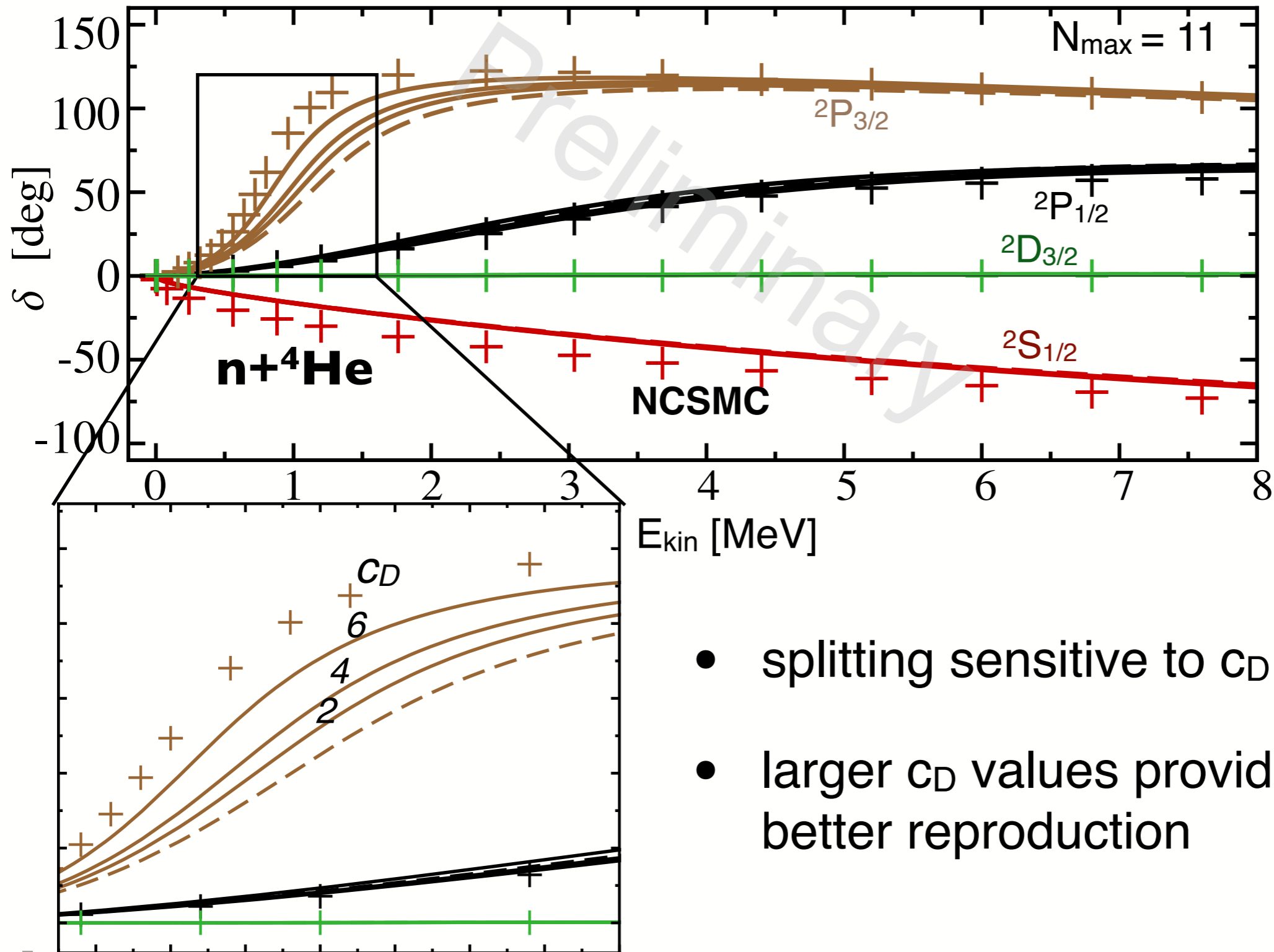
$\alpha = 0.08$  fm<sup>4</sup>

$E_{3\text{max}} = 14$

- splitting underestimated without 3N interaction



# n-<sup>4</sup>He with LENPIC interaction



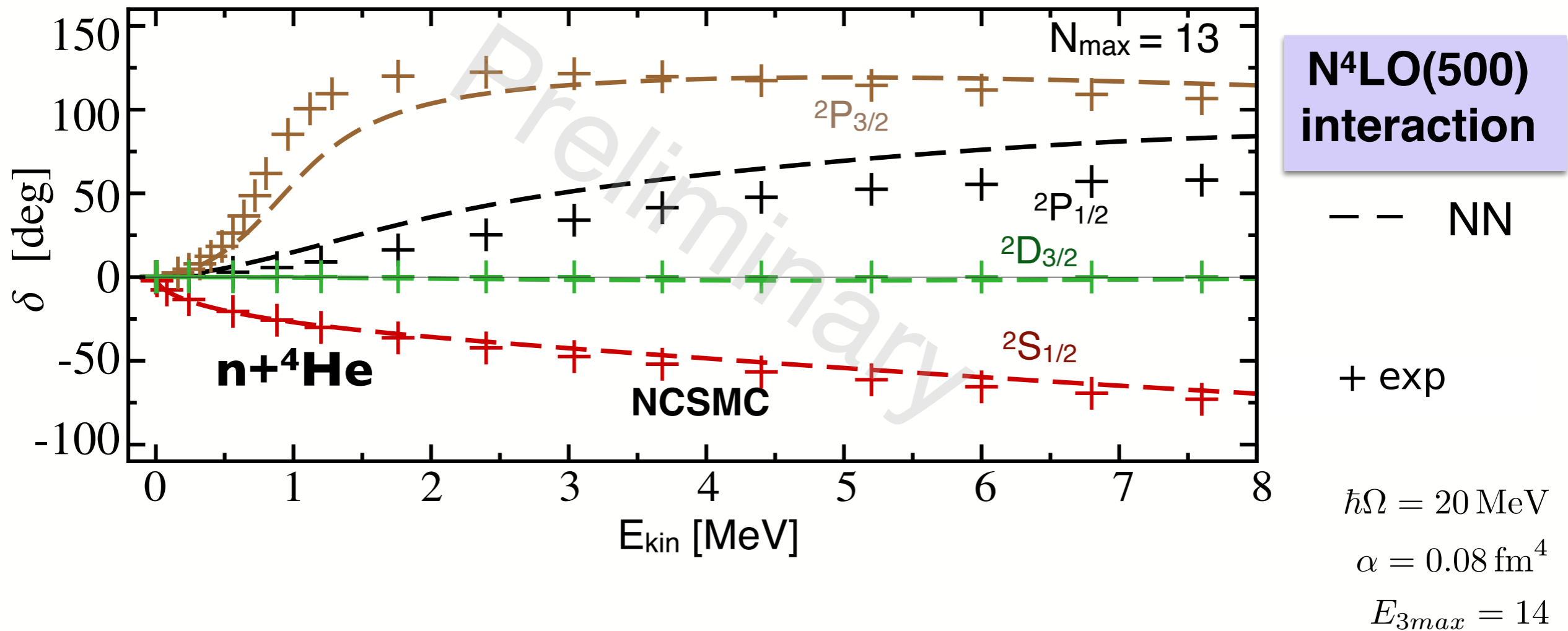
**LENPIC interaction**  
 N<sup>2</sup>LO  
 R = 1.0 fm

-- NN  
 — NN+3N  
 + exp  
 c<sub>D</sub> = 2, 4, 6  
 c<sub>E</sub> fitted to Triton g.s.

- splitting sensitive to c<sub>D</sub>
- larger c<sub>D</sub> values provide better reproduction

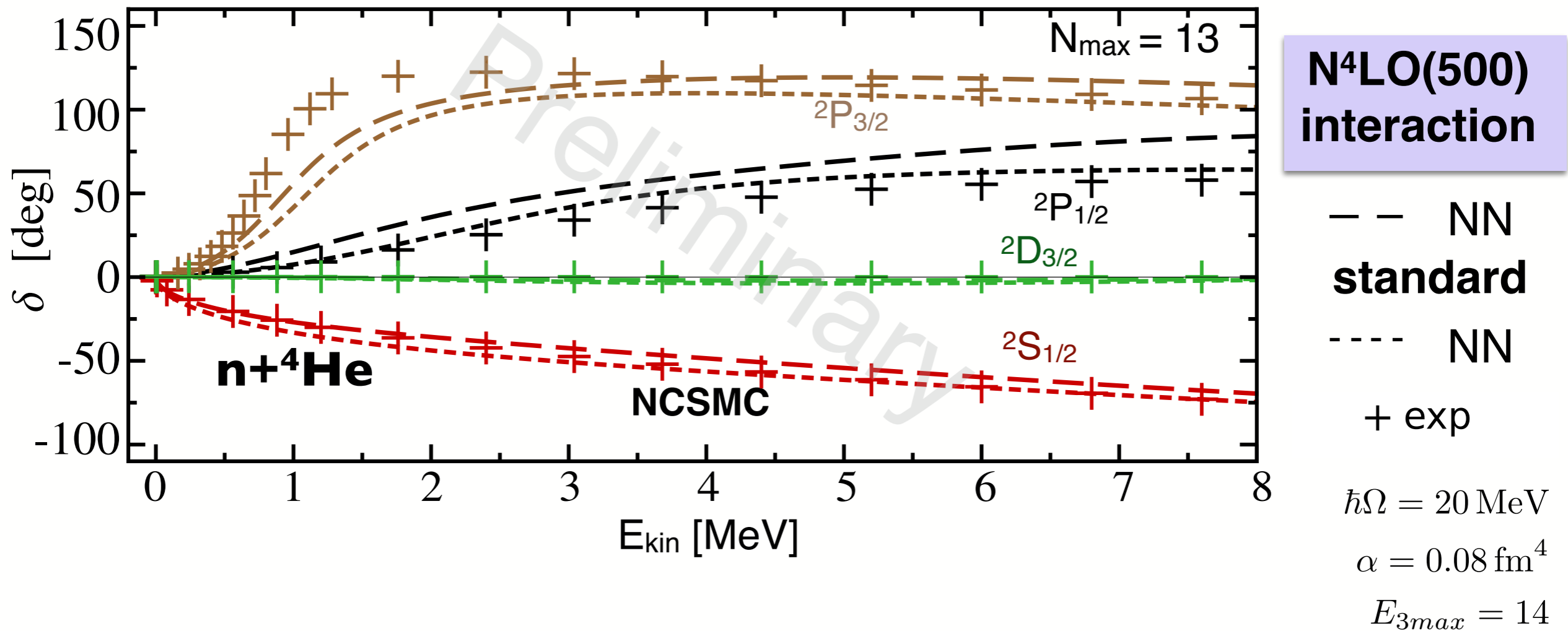
$\hbar\Omega = 24 \text{ MeV}$   
 $\alpha = 0.08 \text{ fm}^4$   
 $E_{3max} = 14$

# n-<sup>4</sup>He with N<sup>4</sup>LO(500) interaction



- promising splitting properties of N<sup>4</sup>LO NN interaction

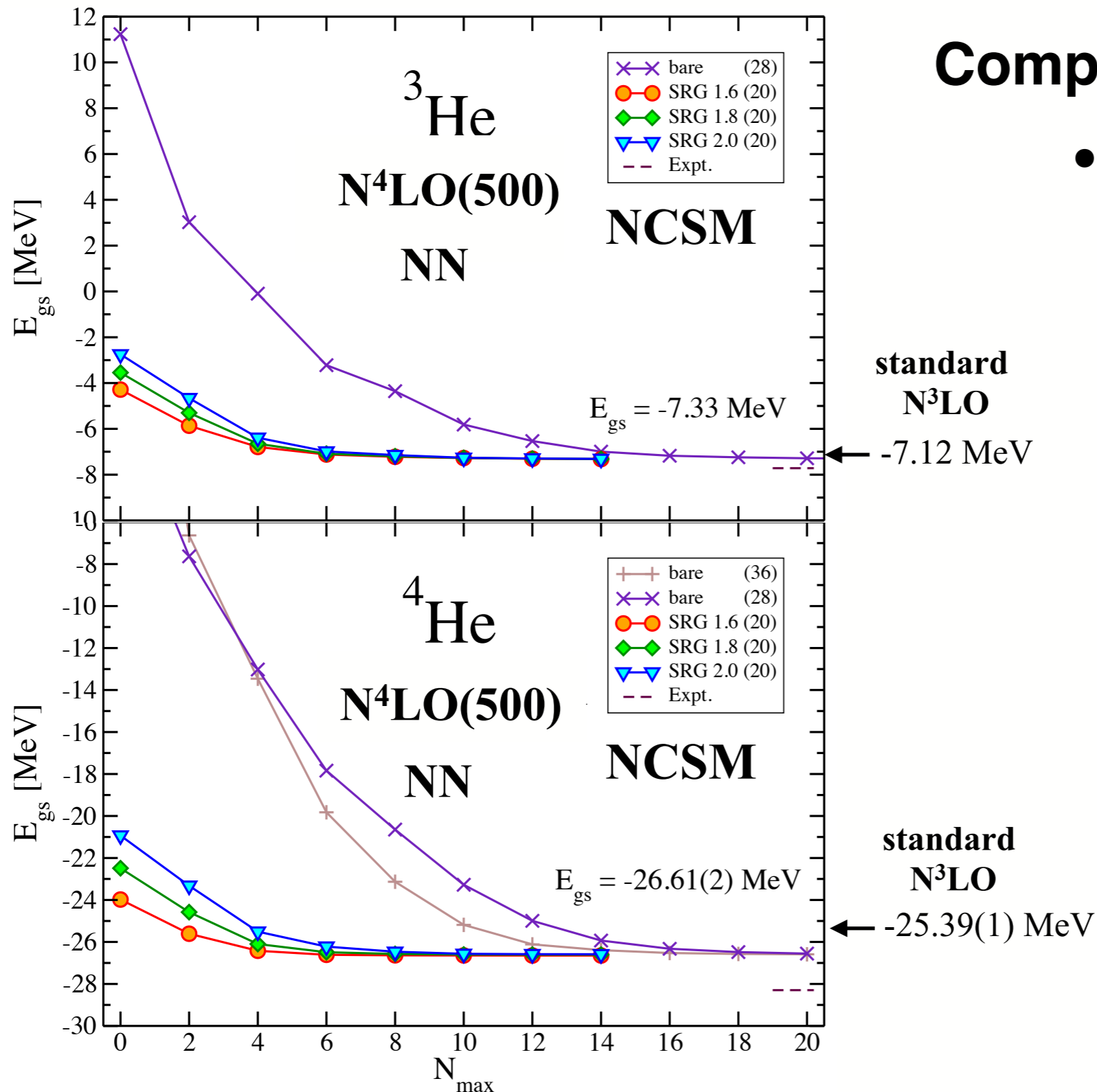
# n-<sup>4</sup>He with N<sup>4</sup>LO(500) interaction



- promising splitting properties of N<sup>4</sup>LO(500) NN interaction

# $N^4\text{LO}(500)$ NN interaction for other observables?

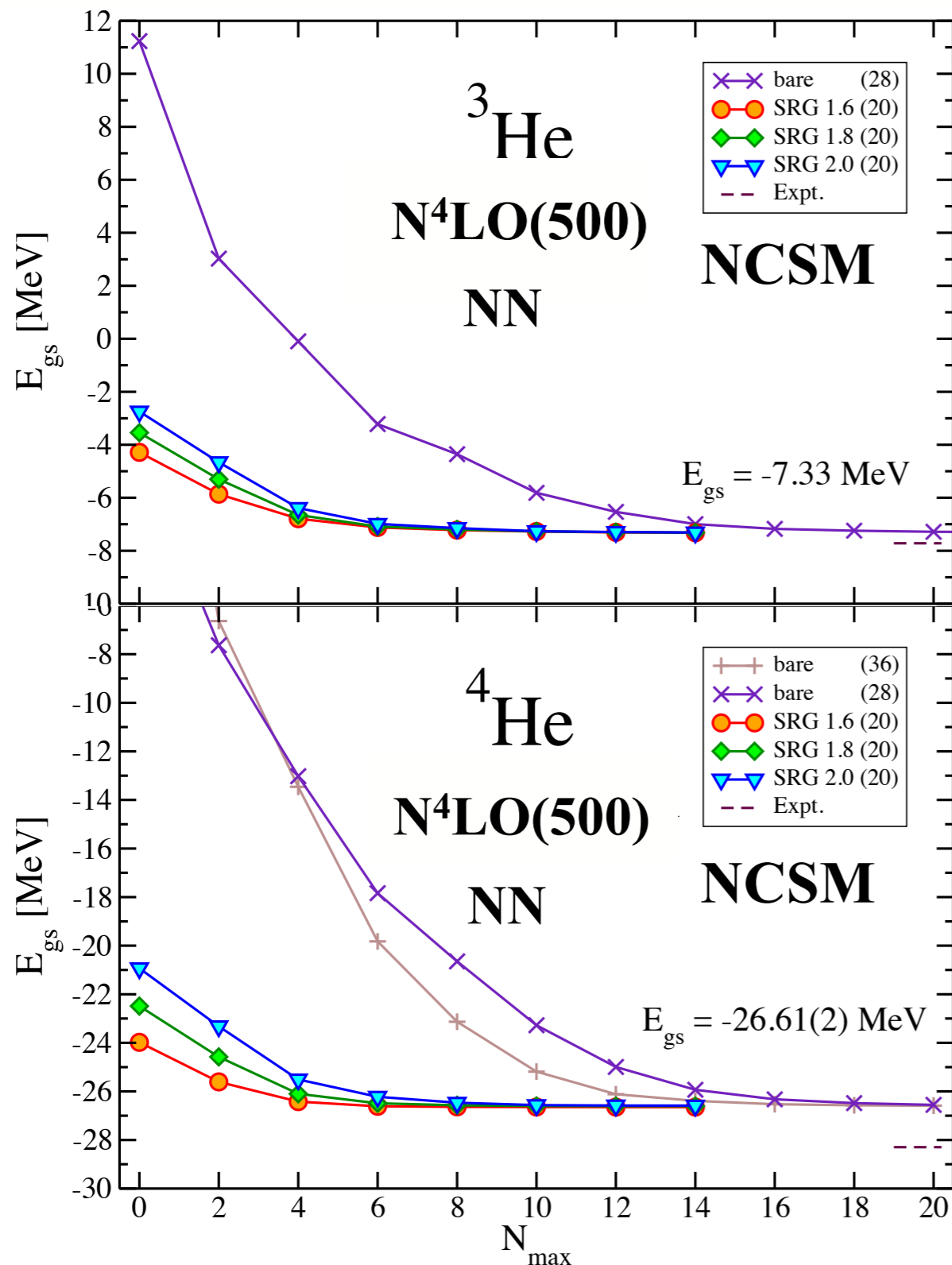
# Ground-State Energies in s-Shell



## Compared to standard $\text{N}^3\text{LO}$ :

- 3N force needs to be less attractive

# Ground-State Energies in s-Shell

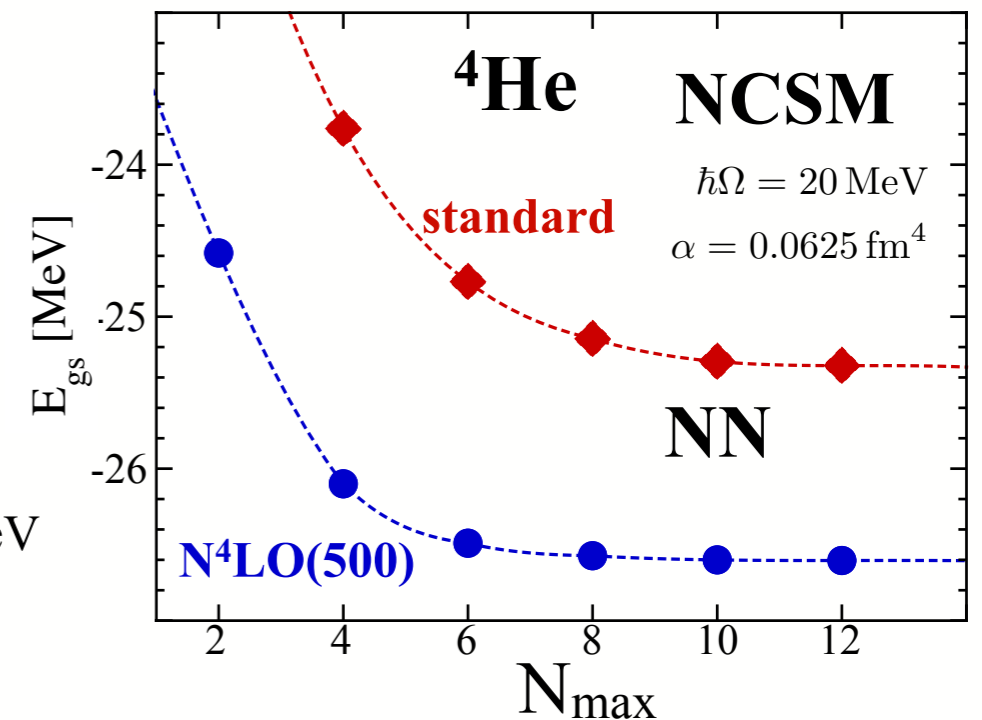


## Compared to standard $\text{N}^3\text{LO}$ :

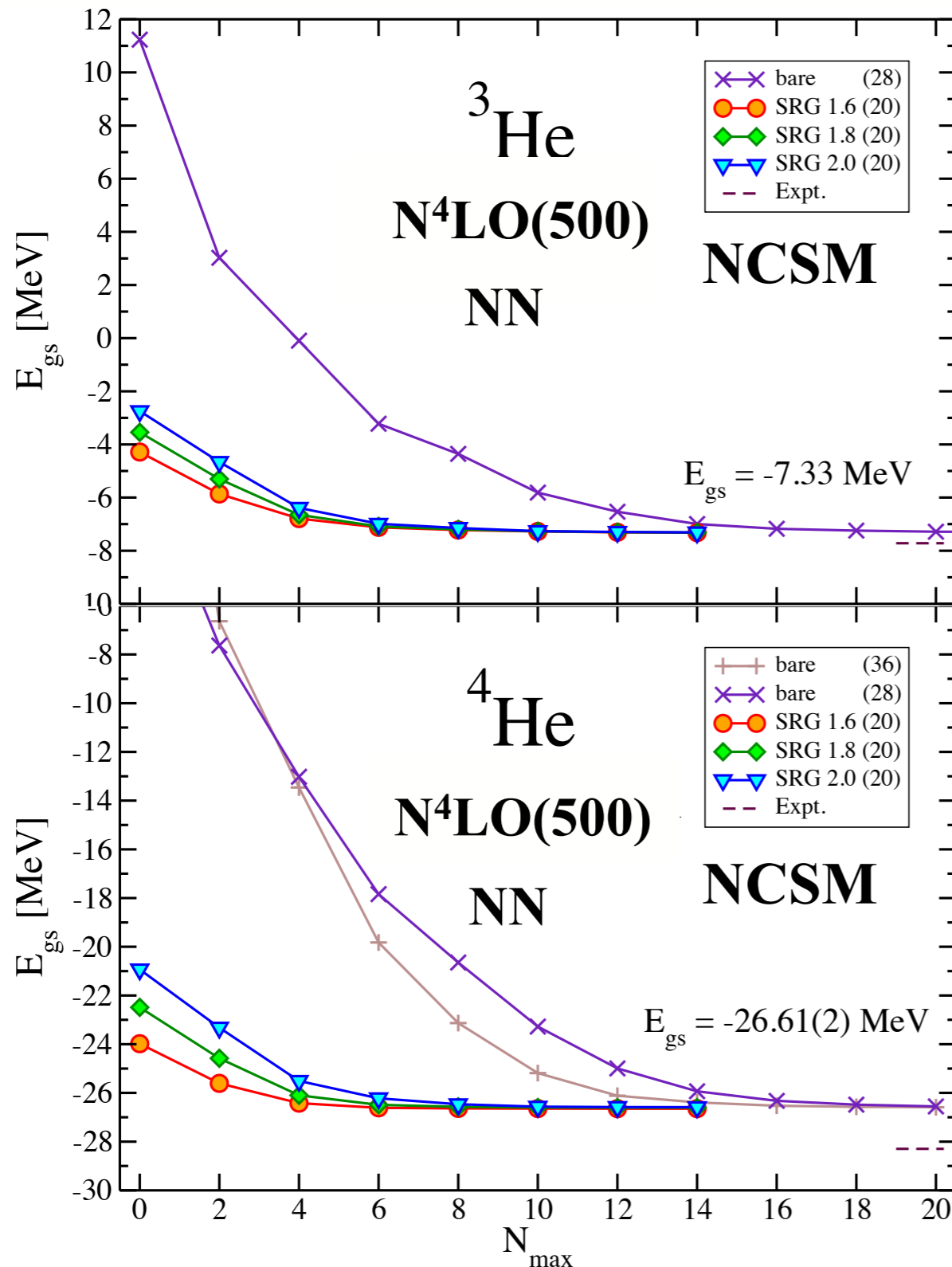
- 3N force needs to be less attractive

standard  $\text{N}^3\text{LO}$   
 $-7.12 \text{ MeV}$

standard  $\text{N}^3\text{LO}$   
 $-25.39(1) \text{ MeV}$



# Ground-State Energies in s-Shell

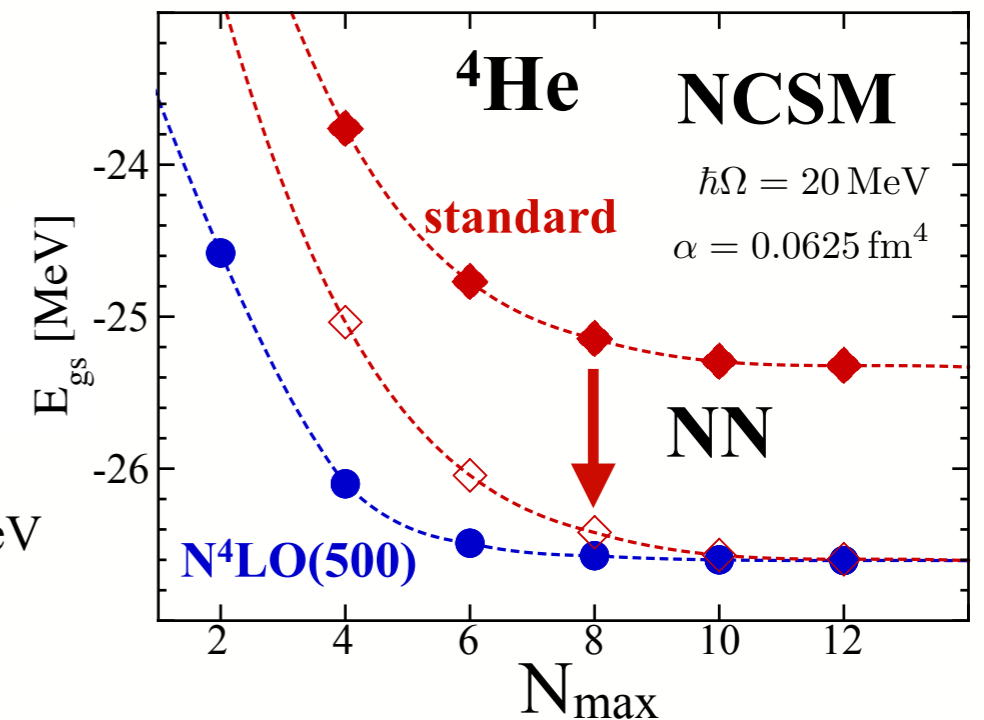


## Compared to standard $\text{N}^3\text{LO}$ :

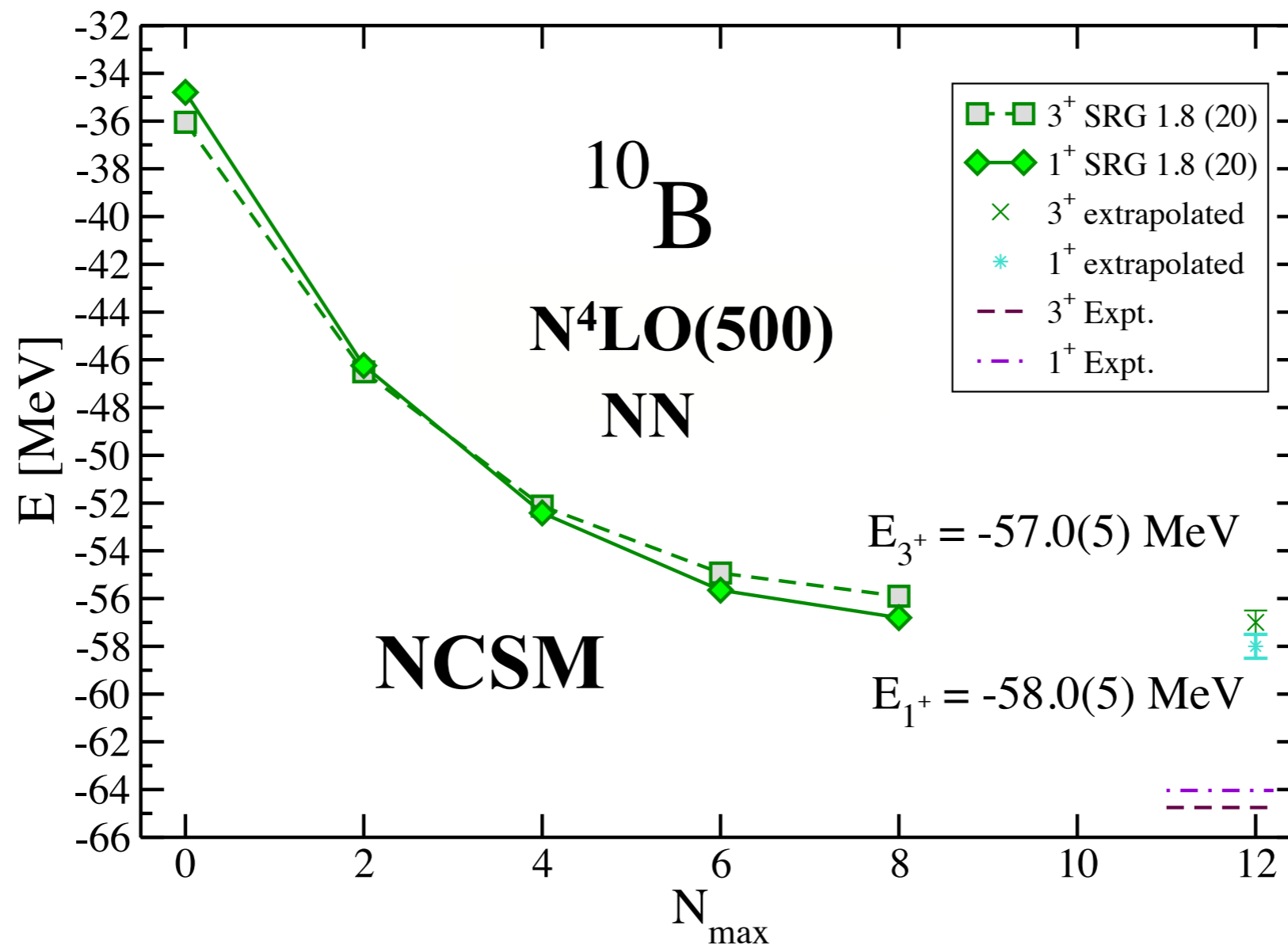
- 3N force needs to be less attractive
- $\text{N}^4\text{LO}(500)$  NN seems to be softer!

standard  $\text{N}^3\text{LO}$   
 $-7.12 \text{ MeV}$

standard  $\text{N}^3\text{LO}$   
 $-25.39(1) \text{ MeV}$



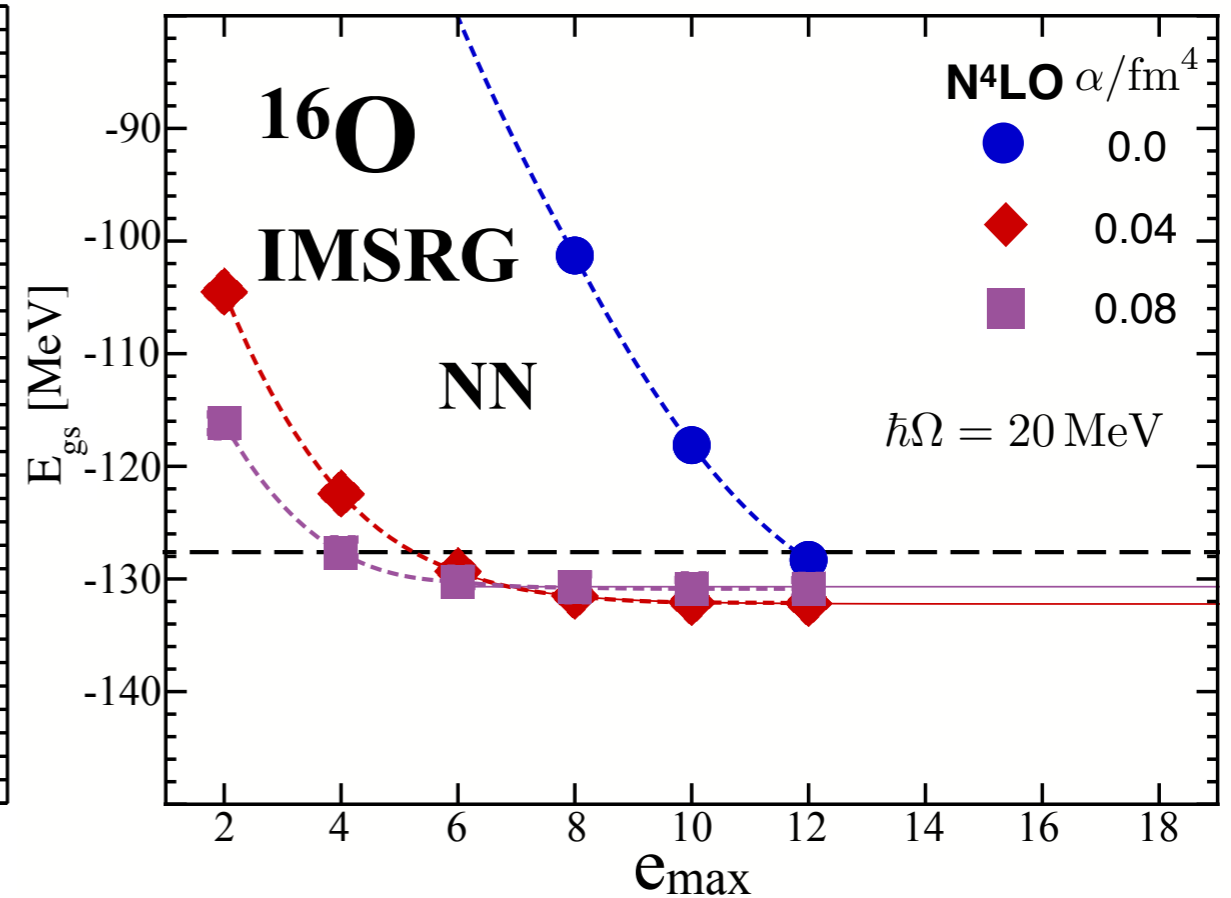
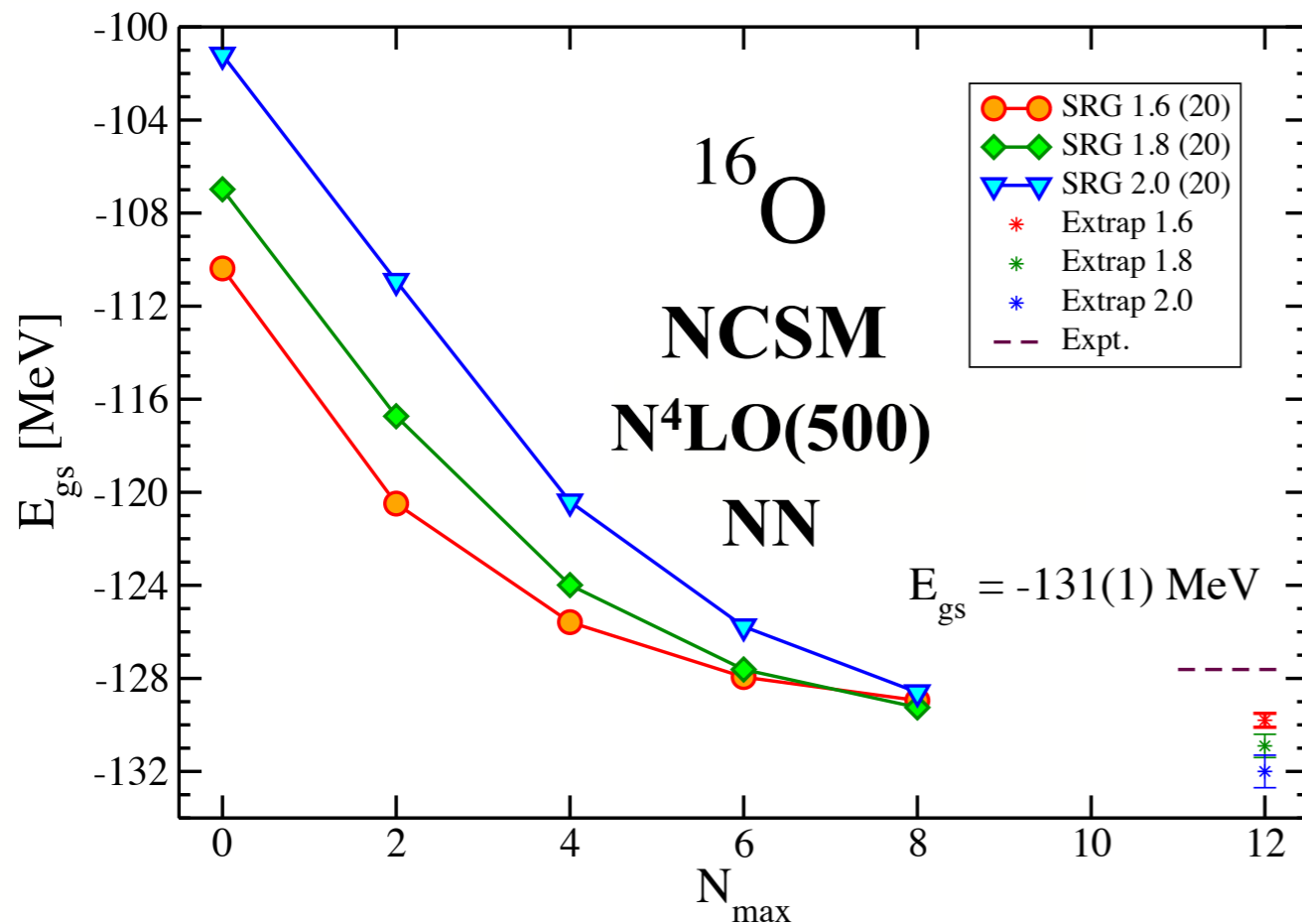
# $^{10}\text{B}$ Ground-State



- requires 3N force to describe right ground-state similar to standard N<sup>3</sup>LO

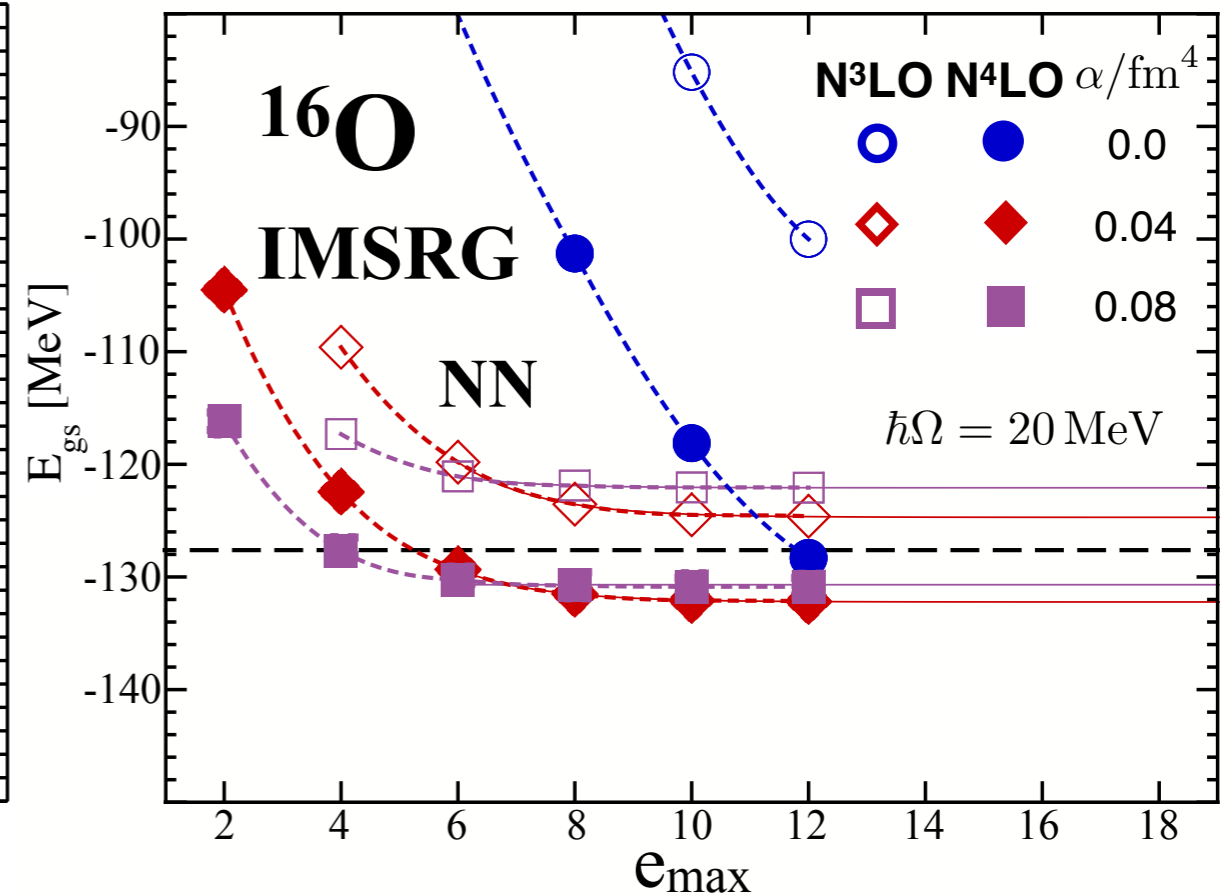
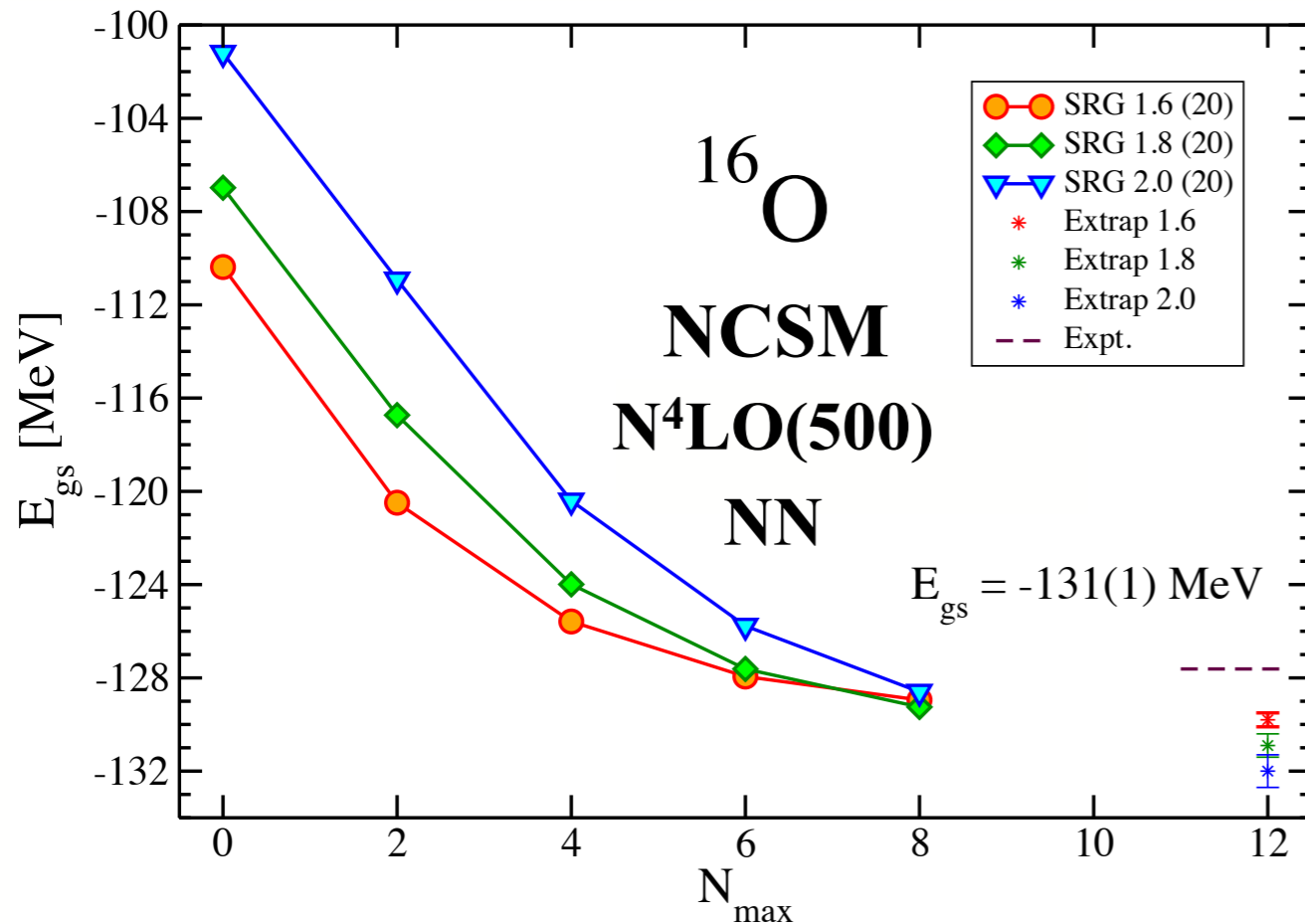


# $^{16}\text{O}$ : Ground-State Energy



- requires repulsive 3N force
- no significant SRG induced 4N contributions

# $^{16}\text{O}$ : Ground-State Energy



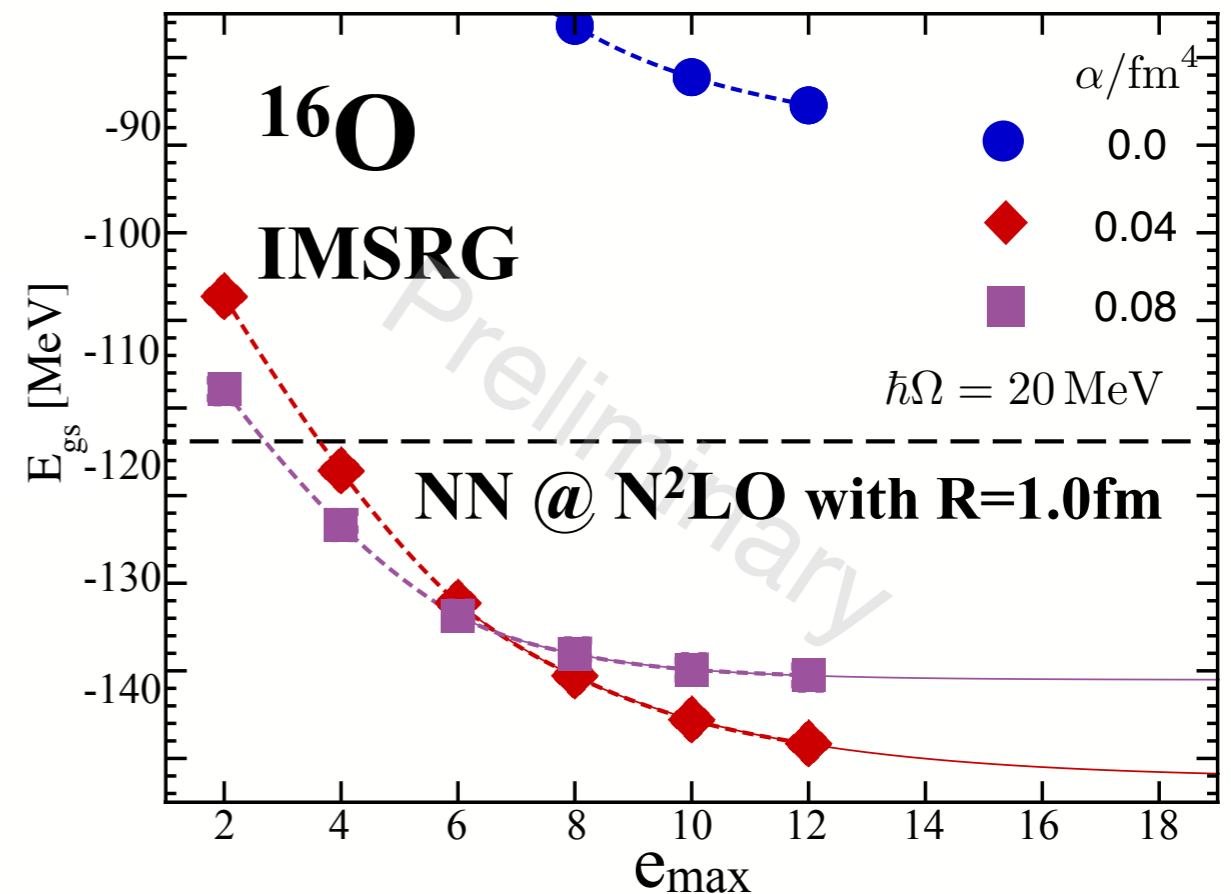
- requires repulsive 3N force
- no significant SRG induced 4N contributions

- no surprises in IMSRG calculations
- $\text{N}^4\text{LO}(500)$  is sufficiently soft

# $^{16}\text{O}$ : Ground-State Energy

## counter-example:

- fully-local  $\text{N}^2\text{LO}$  interaction used in Quantum Monte Carlo  
Gezerlis, Tews, Epelbaum et al.  
Phys. Rev. C 90, 054323 (2014)
- difficult to handle in harmonic oscillator basis



interaction for this cutoff is significantly harder

- Normal-Ordering approximation
- induced (IM)SRG many-body contributions

- **insufficient knowledge of nuclear force** provides largest uncertainties in ab initio calculations
- **combination of NCSMC with MR-NO** allows to include continuum effects at strongly reduced cost
  - enables heavier targets and **complexer projectiles**
  - splitting of  $P_{3/2}$  -  $P_{1/2}$  phase shifts in  ${}^5\text{He}$  can be used to **constrain 3N** parameters
- novel LENPIC and  $N^4\text{LO}(500)$  NN interactions have promising properties
  - 3N needs to be added

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fit  $c_D$  from  ${}^3\text{H}$   $\beta$ -decay  
poster by Peter Gysbers

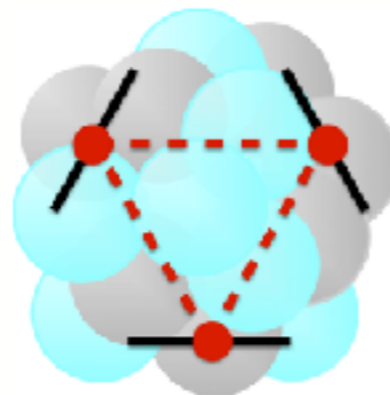
# Thank you! Merci!

- **thanks to my collaborators**

- P. Navrátil, R. Stroberg, J. Holt, R. Kanungo, G. Hackman, A. Kumar, A. Lennarz  
TRIUMF Vancouver, Canada
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TU Darmstadt, Germany
- J. Vary, P. Maris  
Iowa State University, USA
- J. Dohet-Eraly  
Istituto Nazionale di Fisica Nucleare, Pisa, Italy



**LENPIC**



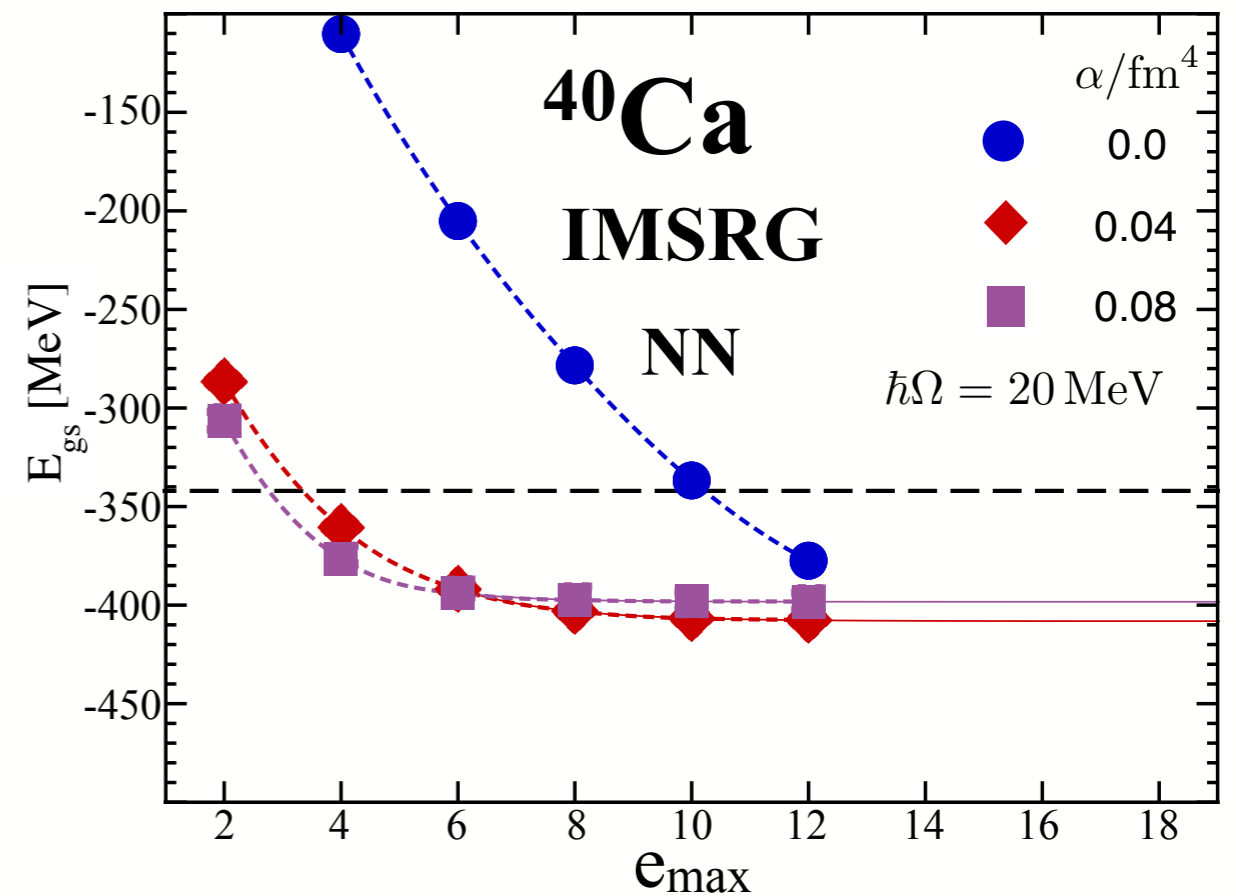
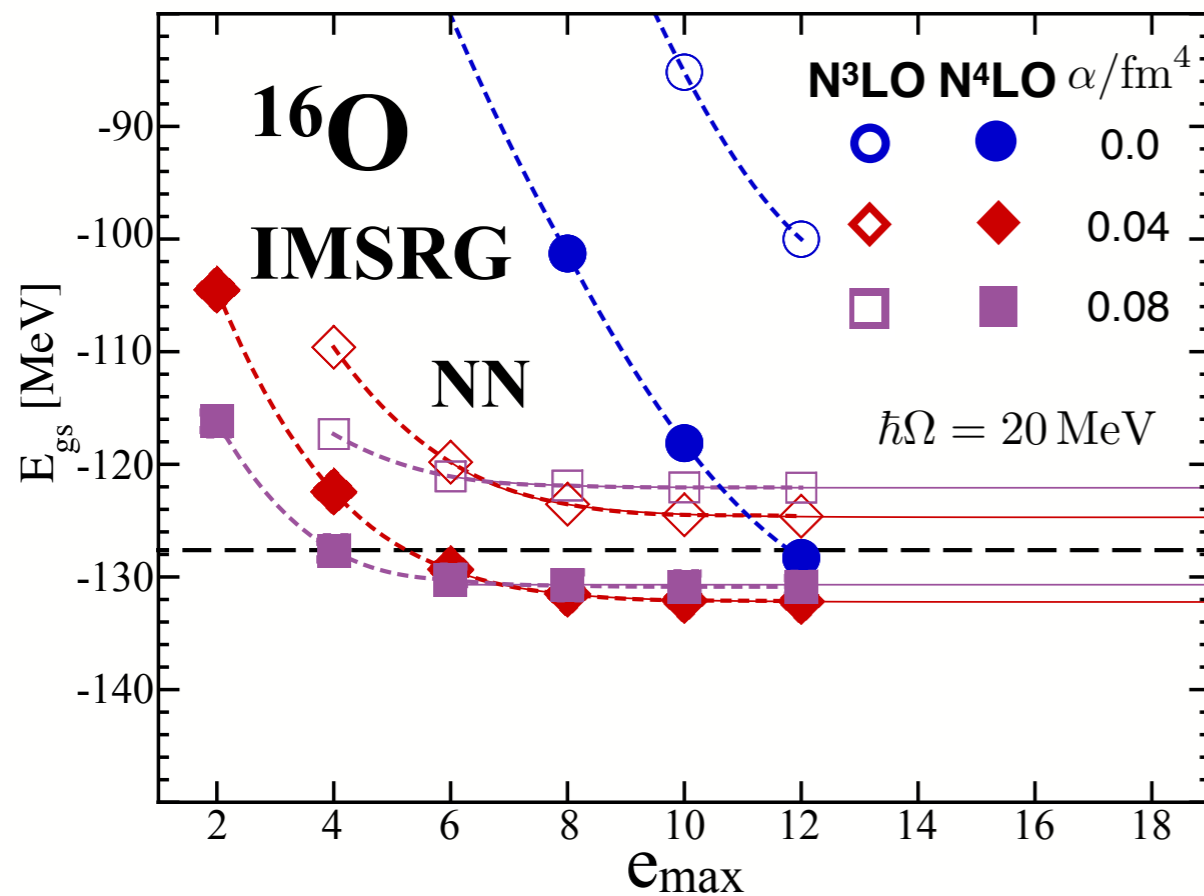
- S. Quaglioni, C. Romero-Redondo  
LLNL Livermore, USA
- G. Hupin  
Université Paris-Sud, France
- H. Hergert, S. Bogner  
MSU, USA



**COMPUTING TIME**

# Supplements

# $^{16}\text{O}$ & $^{40}\text{Ca}$ : Ground-State



- requires repulsive 3N force
- no surprises in IMSRG calculations