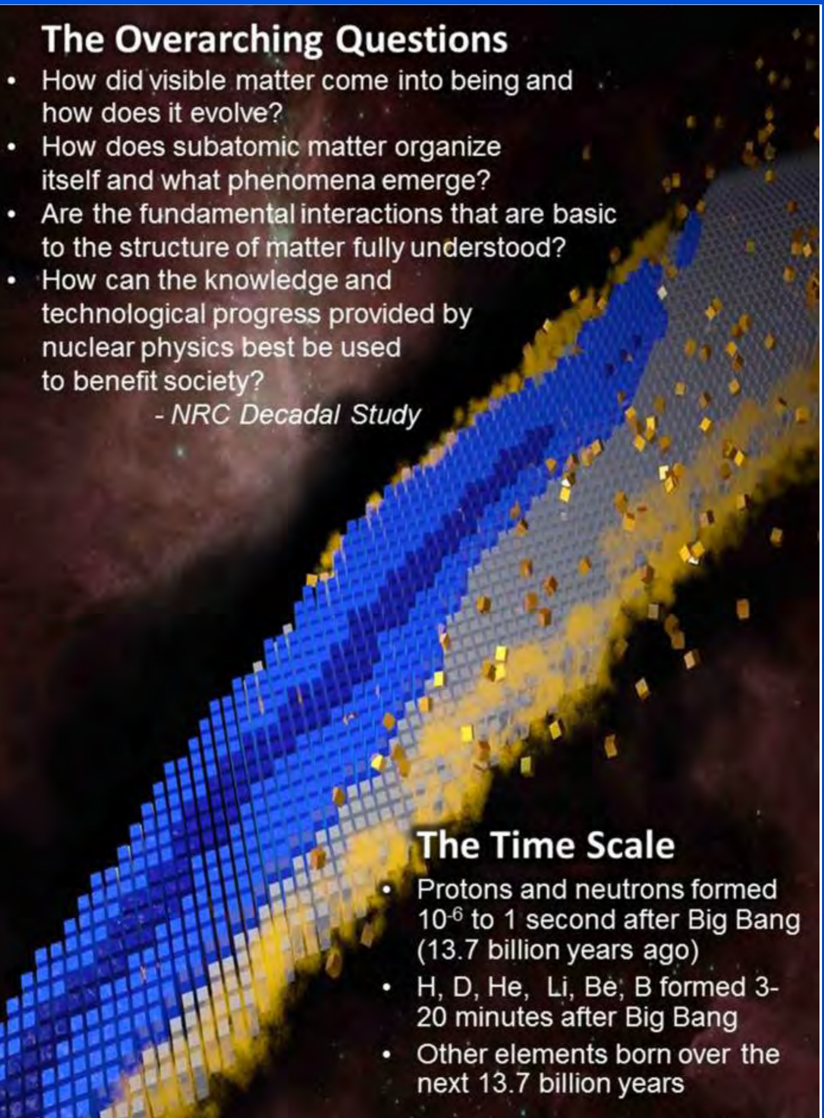
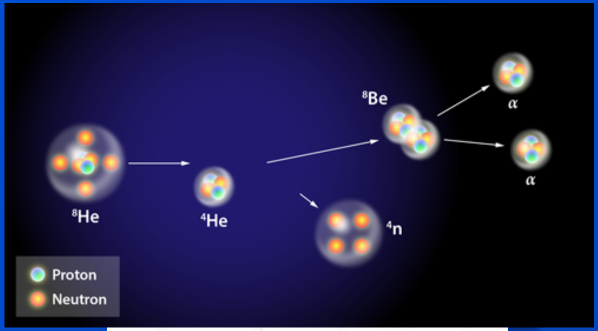
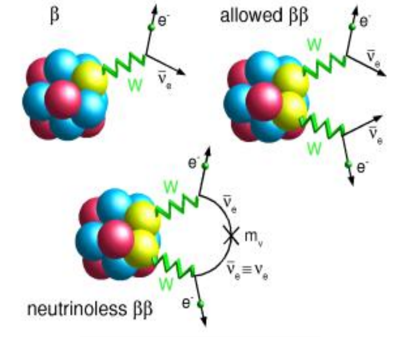
Scatterting in the NCSM: tetraneutron application

James P. Vary, Iowa State University

Progress in Ab Initio Techniques in Nuclear Physics
TRIUMF, Vancouver, Canada, February 28-March 3, 2017







No-Core Configuration Interaction calculations

Barrett, Navrátil, Vary, *Ab initio no-core shell model*, PPNP69, 131 (2013)

Given a Hamiltonian operator

$$\hat{\mathbf{H}} \;=\; \sum_{i<j} \frac{(\vec{p}_i - \vec{p}_j)^2}{2\,m\,A} + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

solve the eigenvalue problem for wavefunction of A nucleons

$$\hat{\mathbf{H}}\,\Psi(r_1,\ldots,r_A) \;=\; \lambda\,\Psi(r_1,\ldots,r_A)$$

- Expand eigenstates in basis states $|\Psi\rangle = \sum a_i |\Phi_i\rangle$
- Diagonalize Hamiltonian matrix $H_{ij} = \langle\Phi_j|\hat{\mathbf{H}}|\Phi_i\rangle$
- No Core Full Configuration (NCFC) – All A nucleons treated equally
- Complete basis $\longrightarrow$ exact result
- In practice, for bound states:
  - truncate basis
  - study behavior of observables as function of truncation

How does one use the no-core results for unbound states to extract scattering information?

That is, we want to employ results within a finite and real harmonic oscillator (HO) basis to evaluate the scattering phase shifts.

From the scattering phase shifts one then extracts resonance energies and widths.

Two examples here:
(1) neutron – alpha scattering
(2) tetraneutron

<u>Other no-core approaches:</u>
NCSMC, NCSM/RGM
Gamow Shell Model
Level density (Lifshits, Rubtsova, . . .)
Complex Scaling of the HO basis
ACCC (Lazauskas & Carbonell, . . .)

Phenomeological NN interaction: JISP16

JISP16 tuned up to ^{16}O

- Constructed to reproduce np scattering data
- Finite rank seperable potential in H.O. representation
- Nonlocal NN-only potential
- Use Phase-Equivalent Transformations (PET) to tune off-shell interaction to
  - binding energy of ^{3}H and ^{4}He
  - low-lying states of ^{6}Li (JISP6, precursor to JISP16)
  - binding energy of ^{16}O

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Realistic nuclear Hamiltonian: Ab exitu approach

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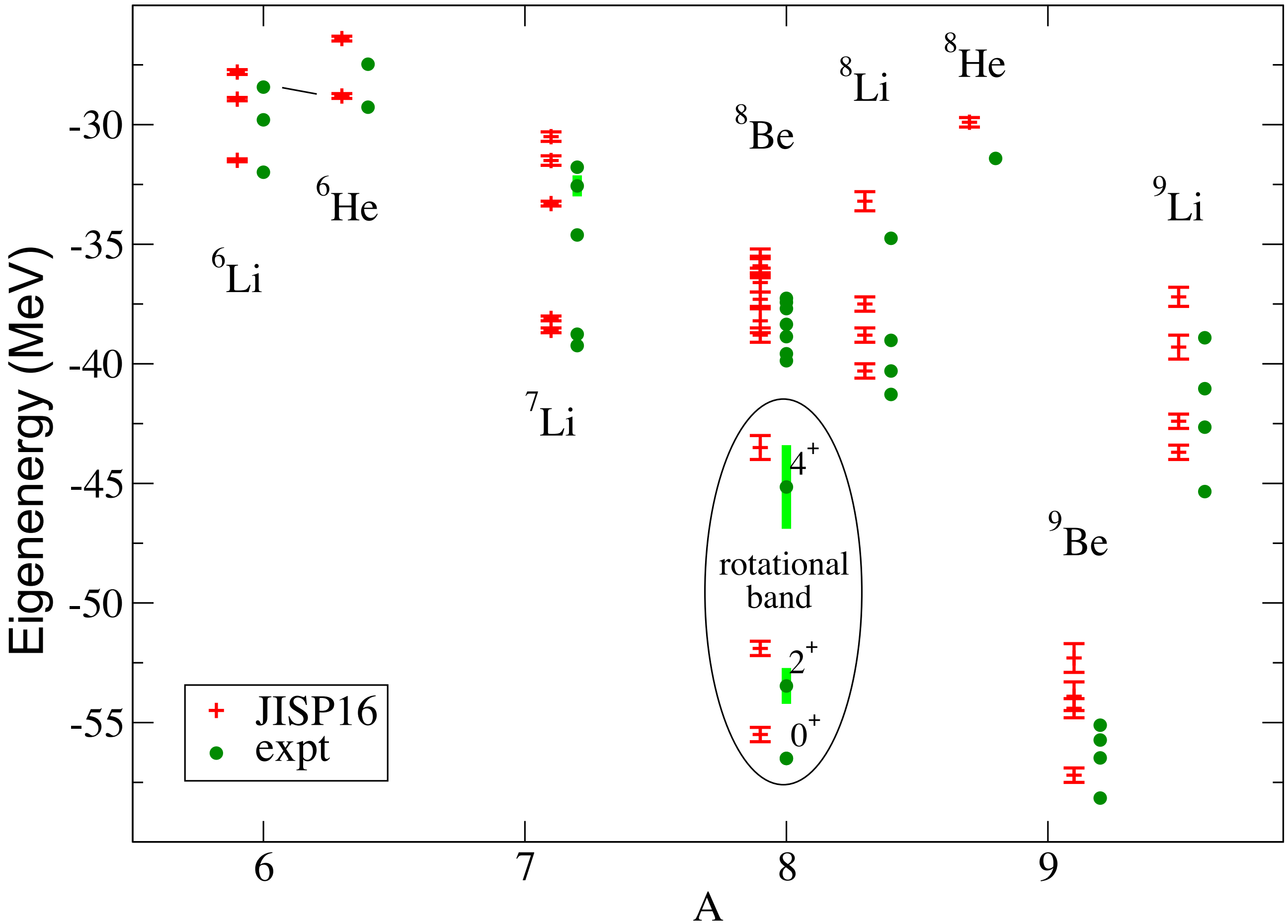
Energies of narrow A=6 to A=9 states with JISP16

Compare theory and experiment for 33 states Maris, Vary, IJMPE22, 1330016 (2013)

Many of these states are cluster-states

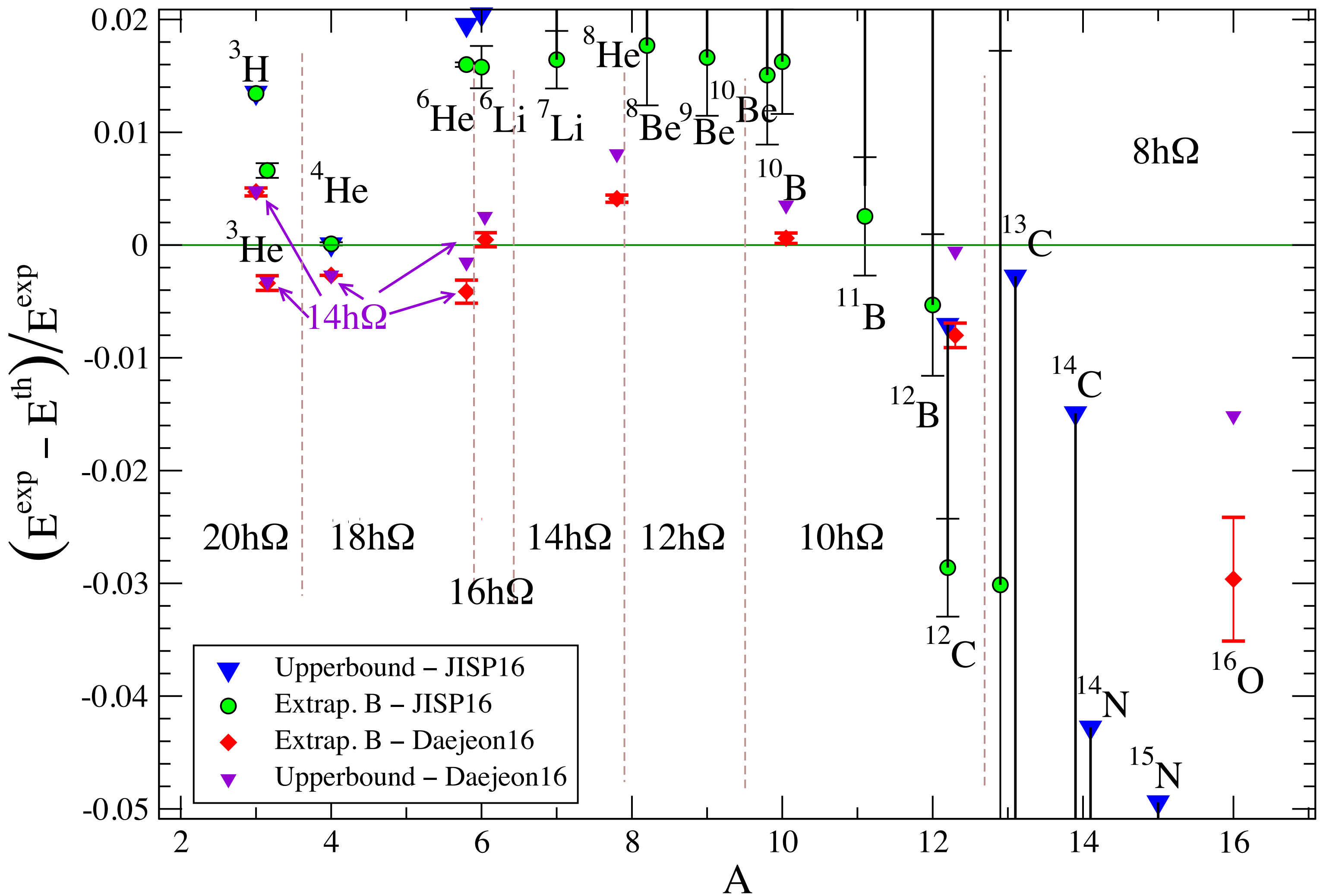


- Excitation spectrum narrow states in good agreement with data

Daejeon16 NN Interaction

Shirokov, Shin, Kim, Sosonkina, Maris, Vary, Phys. Letts. B 761, 87 (2016)

Chiral N3LO (Entem and Machleidt)
SRG (Hjorth-Jensen & Hagen, $\lambda = 1.5\ \mathrm{fm}^{-1}$)
PETs + ab exitu fits (Daejeon)

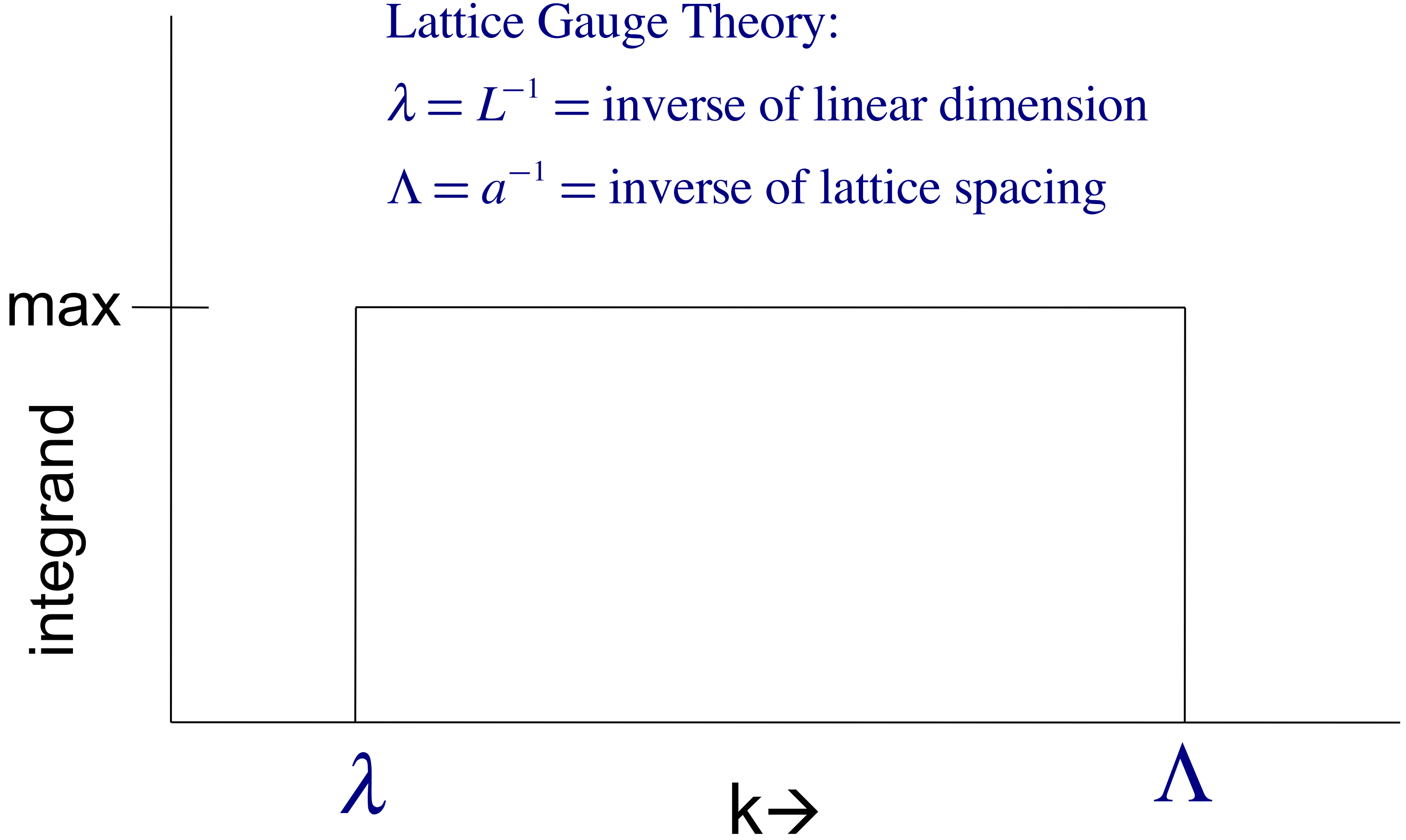


Simple example of IR and UV regulators

Lattice Gauge Theory:

$\lambda = L^{-1} =$ inverse of linear dimension

$\Lambda = a^{-1} =$ inverse of lattice spacing



=> What are the IR and UV regulators of an HO basis?

PHYSICAL REVIEW C **94**, 064320 (2016)

Shell model states in the continuum

A. M. Shirokov,[1,2,3] A. I. Mazur,[3] I. A. Mazur,[3] and J. P. Vary[2]

Synopsis of the SS-HORSE method:
accounts for the continuum limit in an infinite HO basis

Guidelines for application of the SS-HORSE method:

1. Select HO basis regulators:

$$\lambda \approx \sqrt{\hbar\Omega / N_{\max}} \qquad \lambda \leq \text{ all IR scales in H except } T_{rel}$$

$$\Lambda \approx \sqrt{\hbar\Omega \, N_{\max}} \qquad \Lambda \geq \text{ all UV scales in H except } T_{rel}$$

2. Since T_{rel} has analytic IR and UV asymptotics, scattering domain (continuum physics) is accessible – e.g. scattering phase shifts:

✧ J-matrix for scattering
✧ SS-HORSE method developed for evaluating phase shifts

General idea of the HORSE formalism

$$T + V$$

NCSM with:
$\lambda \approx \lambda_{NN}$ & $\Lambda \approx \Lambda_{NN}$

"think outside the box"=> T

Infinite set of algebraic equations:

$$\sum_{n'=0}^{N} \left(T^{l}_{nn'} + V^{l}_{nn'} - \delta_{nn'} E\right) a_{n'l}(E) = 0. \qquad n \le N-1$$

Matching condition at $n = N$

$$\sum_{n'=0}^{N} \left(T^{l}_{Nn'} + V^{l}_{Nn'} - \delta_{Nn'} E\right) a_{n'l}(E) + T^{l}_{N,N+1} a_{N+1,l}(E) = 0. \quad n \le N-1$$

Then for $n \ge N+1$

$$\sum_{n'=0}^{\infty} \left(T^{l}_{nn'} - \delta_{nn'} E\right) a_{n'l}(E) = 0, \text{ which produces:}$$

$$T^{l}_{n,n-1} a_{n-1,l}(E) + \left(T^{l}_{nn} - E\right) a_{nl}(E) + T^{l}_{n,n+1} a_{n+1,l}(E) = 0.$$

Arises as a natural extension of NCSM where both potential and kinetic energies are truncated

This is an exactly solvable algebraic problem!

Single-State HORSE (SS-HORSE)

$$\sum_{n'=0}^{N} H^{l}_{nn'} \langle n' | \lambda \rangle = E_\lambda \langle n | \lambda \rangle, \qquad n \le N$$

$$(H-E)^{-1}_{nn'} \equiv -G_{nn'} = \sum_{\lambda'=0}^{N} \frac{\langle n | \lambda' \rangle \langle \lambda' | n' \rangle}{E_{\lambda'} - E}$$

$$\tan\delta(E) = -\frac{S_{Nl}(E) - G_{NN} T^{l}_{N,N+1} S_{N+1,l}(E)}{C_{Nl}(E) - G_{NN} T^{l}_{N,N+1} C_{N+1,l}(E)}$$

Suppose $E = E_{\lambda'}$

$$\tan\delta(E_\lambda) = \frac{S_{N+1,l}(E_\lambda)}{C_{N+1,l}(E_\lambda)} = (-1)^l q^{2l+1} \Gamma(-l+\tfrac{1}{2}) \frac{L^{l+1/2}_{(N-l)/2}(q^2)}{\Phi(-\tfrac{N}{2}-\tfrac{1}{2}-\tfrac{1}{2}, -l+\tfrac{1}{2}; q^2)}$$

where $L^{l+1/2}_{(N-l)/2}$ are associated Laguerre polynomials, Φ are confluent hyper-geometric functions and $q = \sqrt{\frac{2E_\lambda}{\hbar\Omega}}$

E_λ are eigenstates of the NCSM (for given $\hbar\Omega$ and N_{max}) that are extended to scattering states, from which the phase shift is derived.

A.M. Shirokov, A.I. Mazur, I.A. Mazur and J.P. Vary, Phys. Rev. C 94, 064320 (2016); arXiv:1608.05885

Example of the neutron-alpha phase shift evaluations

Solve the NCSM on mesh of N_{max} and $\hbar\Omega$-values for ^{4}He and ^{5}He.
Use set of NCSM eigenvalues satisfying IR and UV constraints for:

$$\tan\delta(E_\lambda) = \frac{S_{N+1,\,l}(E_\lambda)}{C_{N+1,\,l}(E_\lambda)}$$

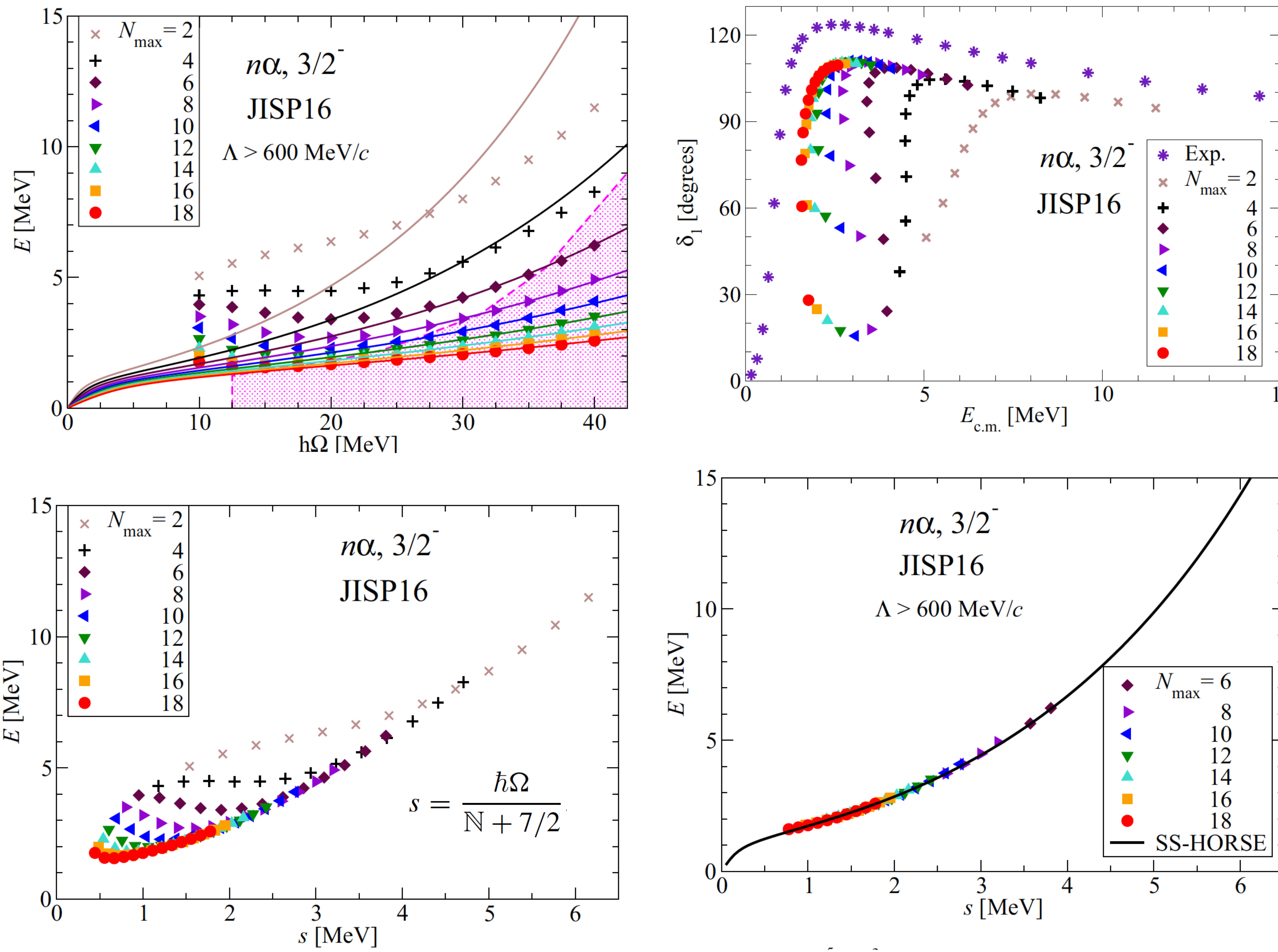
where $E_\lambda = E_\lambda({}^5\text{He}, \text{N}_{\text{max}},\, \hbar\Omega) - E_0({}^4\text{He}, \text{N}_{\text{max}},\, \hbar\Omega)$

Compare with experimental phase shifts channel-by-channel
Extract resonance parameters by fitting the phase shifts to effective range expansion that includes possible resonance contributions

Results (consistency check) should provide a single curve of $E_\lambda(s)$ with:

$$s = \frac{\hbar\Omega}{N_{\text{max}} + \ell + 7/2} = \lambda_{sc}^2$$

λ_{sc} as defined in S. A. Coon, M. I. Avetian, M. K. G. Kruse, U. van Kolck, P. Maris, and J. P. Vary, Phys. Rev. C86, 054002 (2012); arXiv: 1205.3230



A.M. Shirokov, A.I. Mazur, I.A. Mazur and J.P. Vary, Phys. Rev. C 94, 064320 (2016); arXiv:1608.05885

Single State – Harmonic Oscillator Representation of Scattering Equations
"SS-HORSE"
Application to the Tetraneutron

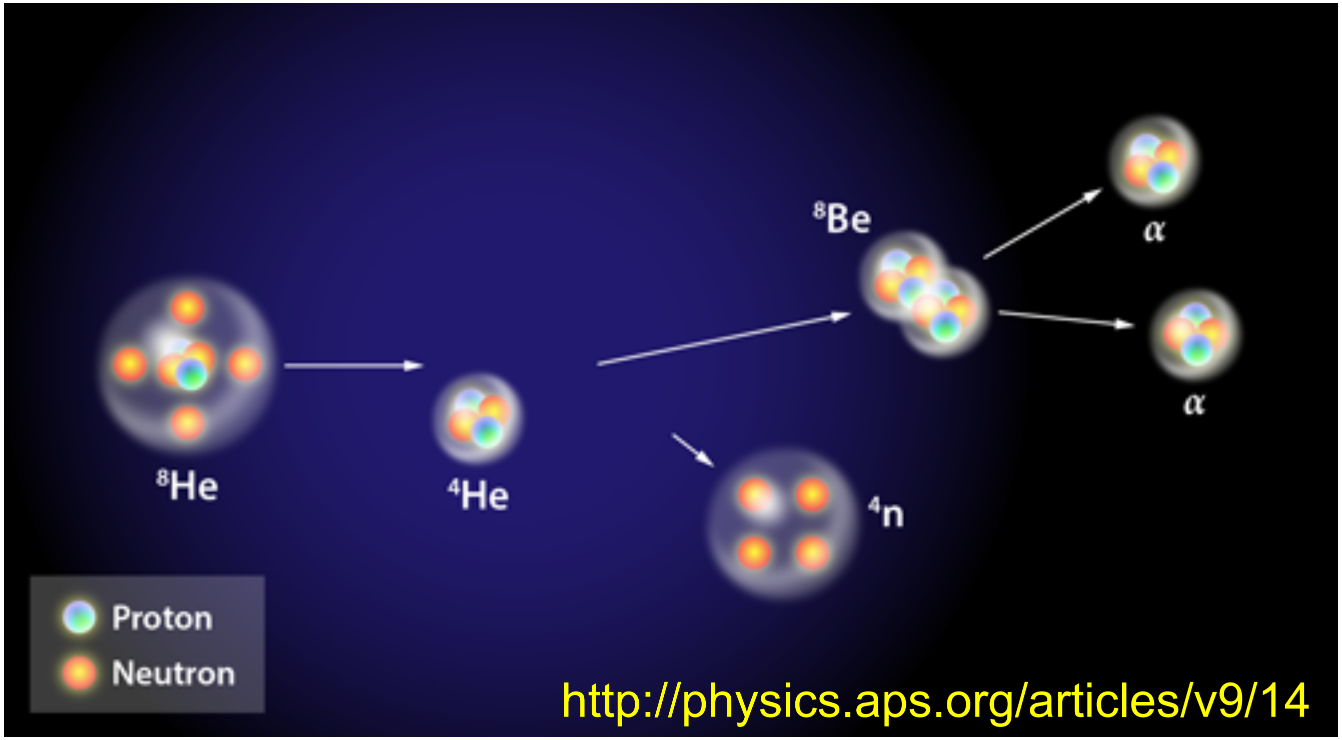
Motivations

- ❖ Better knowledge of nn interaction
- ❖ High precision access to T=3/2 nnn interacton
- ❖ Stepping stone to putative 6n, 8n, . . . systems

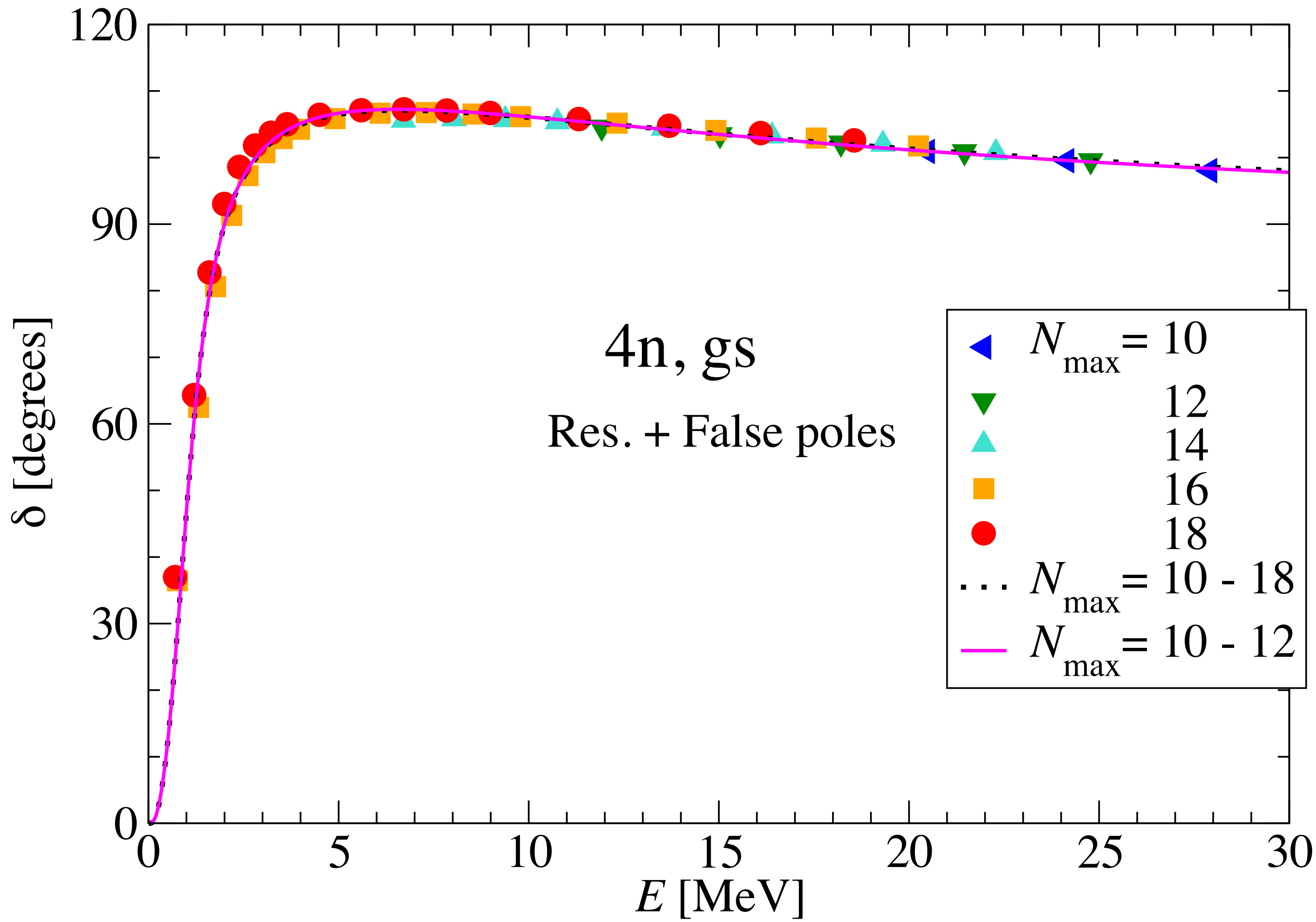
PHYSICAL REVIEW LETTERS

Candidate Resonant Tetraneutron State Populated by the ^{4}He(^{8}He, ^{8}Be) Reaction

K. Kisamori *et al.*
Phys. Rev. Lett. **116**, 052501 – Published 3 February 2016



Tetraneutron, JISP16

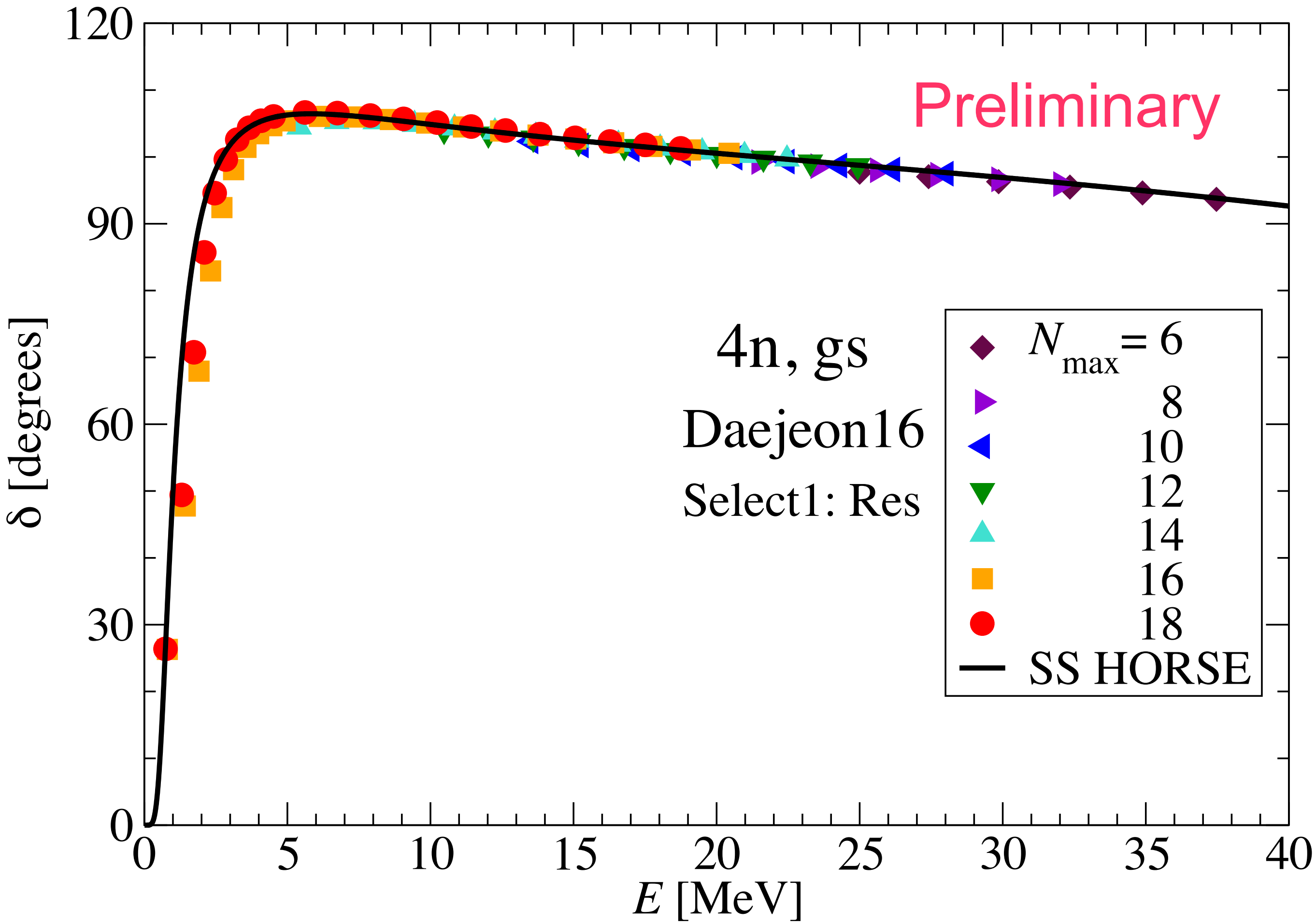


N_{max} =10-12 resonance params:
E_r = 844 keV, Γ = 1.378 MeV,
E_{false} = -55 keV.

vs

N_{max} =10-18 resonance params:
E_r = 844 keV, Γ = 1.377 MeV,
E_{false} = -55 keV.

Tetraneutron, Daejeon16

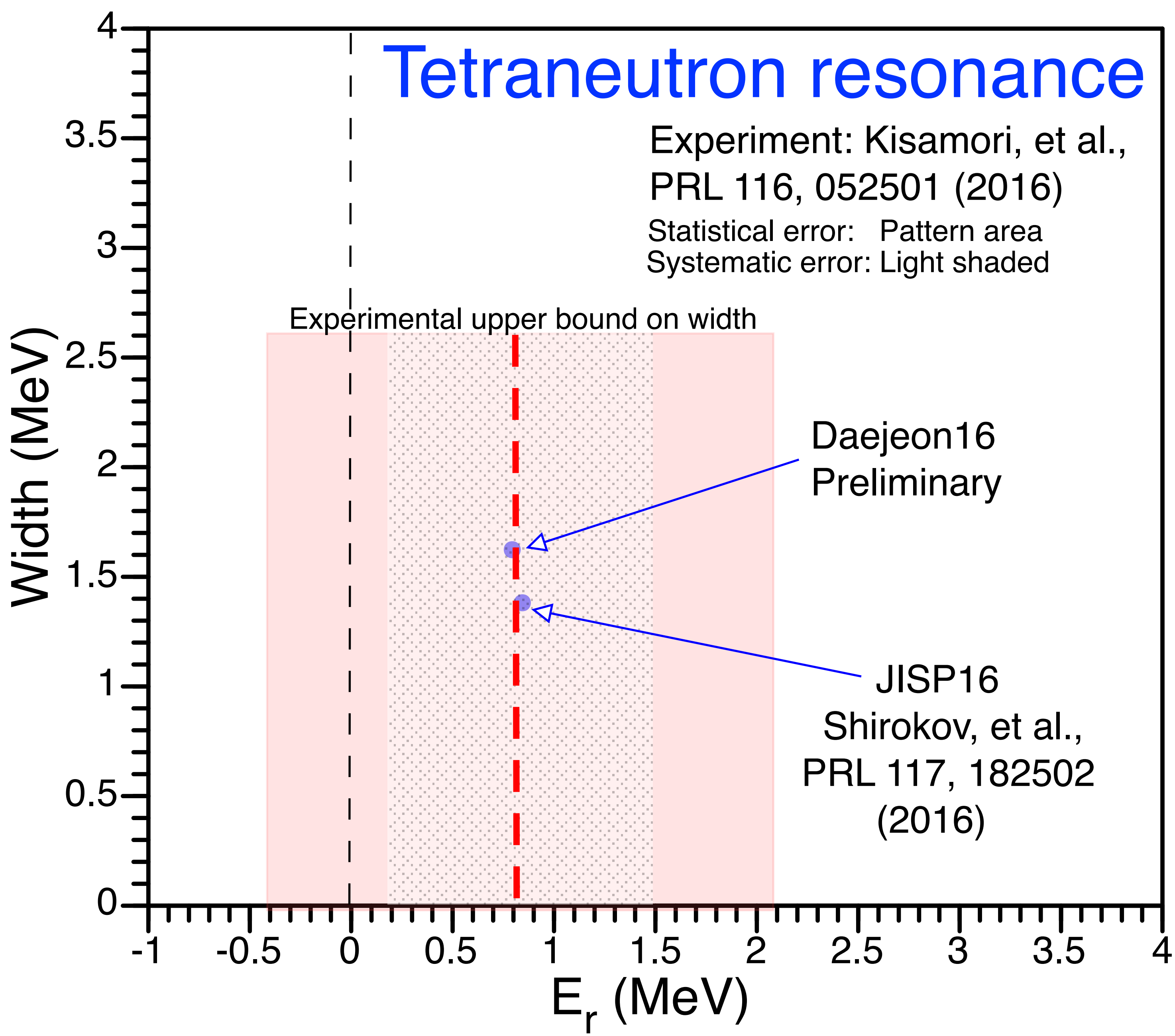


Options:

Resonance parameters:
E_r = 185 keV, Γ = 818 keV,
with large background phase

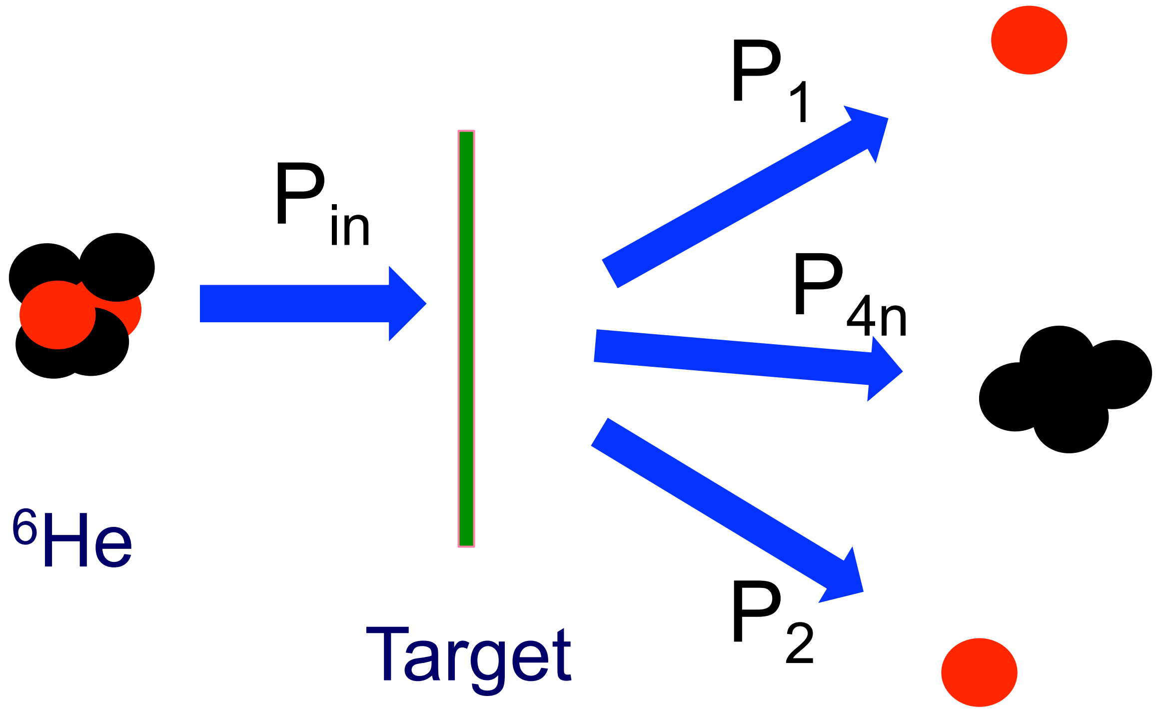
vs

Resonance parameters:
E_r = 0.794 MeV, Γ = 1.619 MeV,
E_{false} = -53.6 keV.



Tetraneutron resonance
Experiment: Kisamori, et al.,
PRL 116, 052501 (2016)
Statistical error: Pattern area
Systematic error: Light shaded
Experimental upper bound on width
Daejeon16
Preliminary
JISP16
Shirokov, et al.,
PRL 117, 182502
(2016)
Width (MeV)
E_r (MeV)
4
3.5
3
2.5
2
1.5
1
0.5
0
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4

Search for ^{6}He dissociating to 3-body final states: Tetraneutron observed as peaks in a missing mass analysis

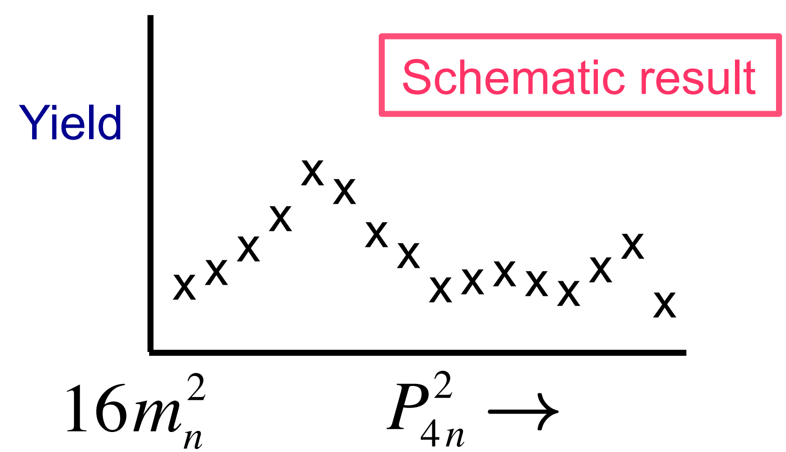


Conservation of invariant 4-momentum
with complete kinematics (“exclusive” expt)

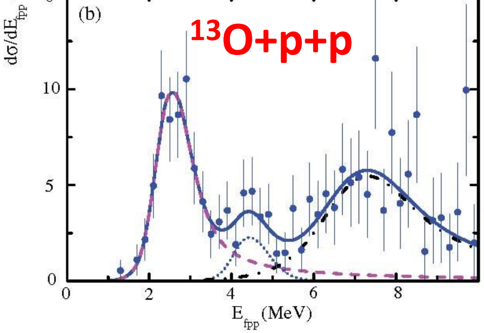
$$P_{in} + P_{\text{target}} = P_1 + P_2 + P_{4n} + P'_{\text{target}}$$

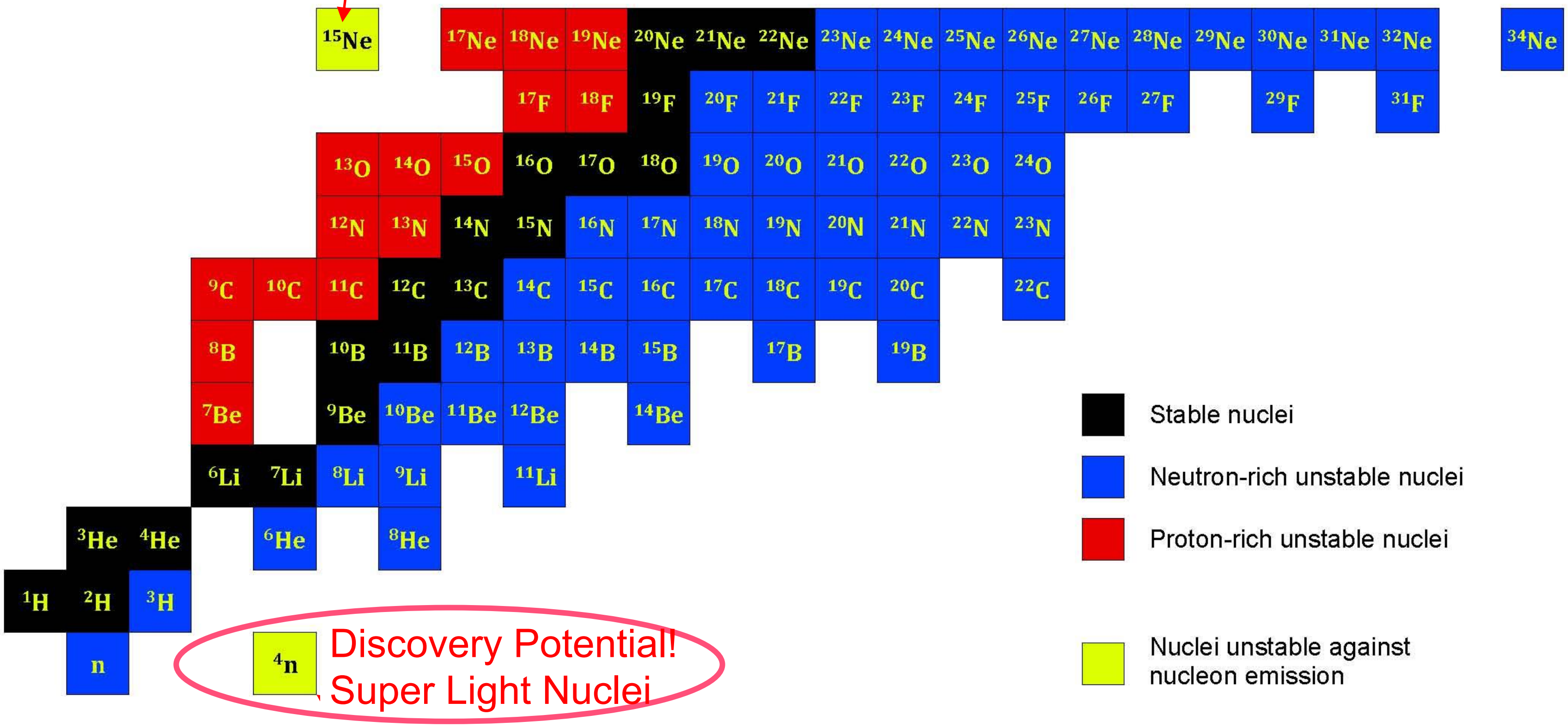
$$E_{cm} > 30 \text{ MeV}$$

$$P_{4n}^2 \geq 16 m_n^2$$



F.Wamers et al.,
PRL112(2014)132502.
C(^{17}Ne,^{16,15}Ne)





Conclusions and Outlook

Much work yet to be done on multi-neutron systems

Structure of 3n, 4n, 5n, . . .
Direct production with rare isotope beams
Transfer reactions
Production during fission

=> Better knowledge of nn, 3n and 4n interactions
=> Properties of neutron-rich nuclei
=> Properties of neutron stars
=> Extend the theory to charged particle scattering . . .

Thank you for your attention

I welcome your questions