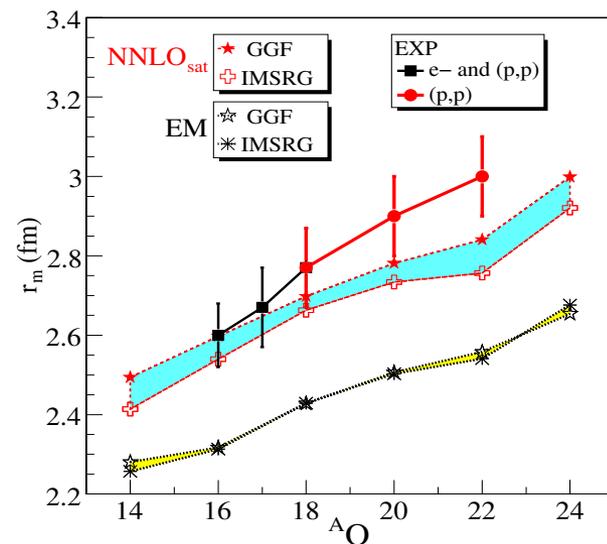
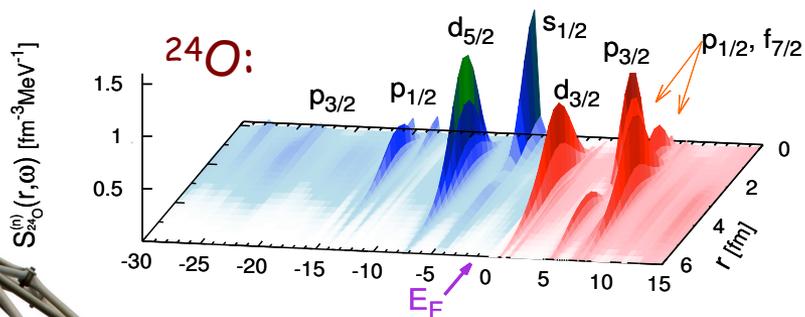
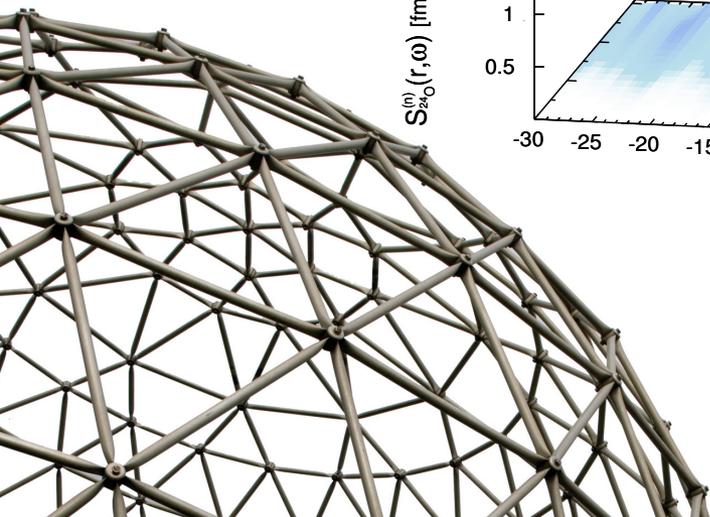


Recent Advances in SCGF Calculations

Carlo Barbieri — University of Surrey

2 March 2017



Outline

- *Some details of the current SCGF formalism (extensions for 3NFs)*
- *Application of saturating interactions and new local-nonlocal force*
- *Bubble structure of ^{34}Si*
- *occupation of valence shells*
- *Nuclei from Lattice QCD (at $m_\pi=0.47 \text{ GeV}/c^2 \dots$)*

Related calculations (see following talks):

- *Shell model effective charges*
- *Dipole response*
- *Ab-Initio optical potential from SCGF*

Current Status of low-energy nuclear physics

Composite system of interacting fermions

Binding and limits of stability

Coexistence of individual and collective behaviors

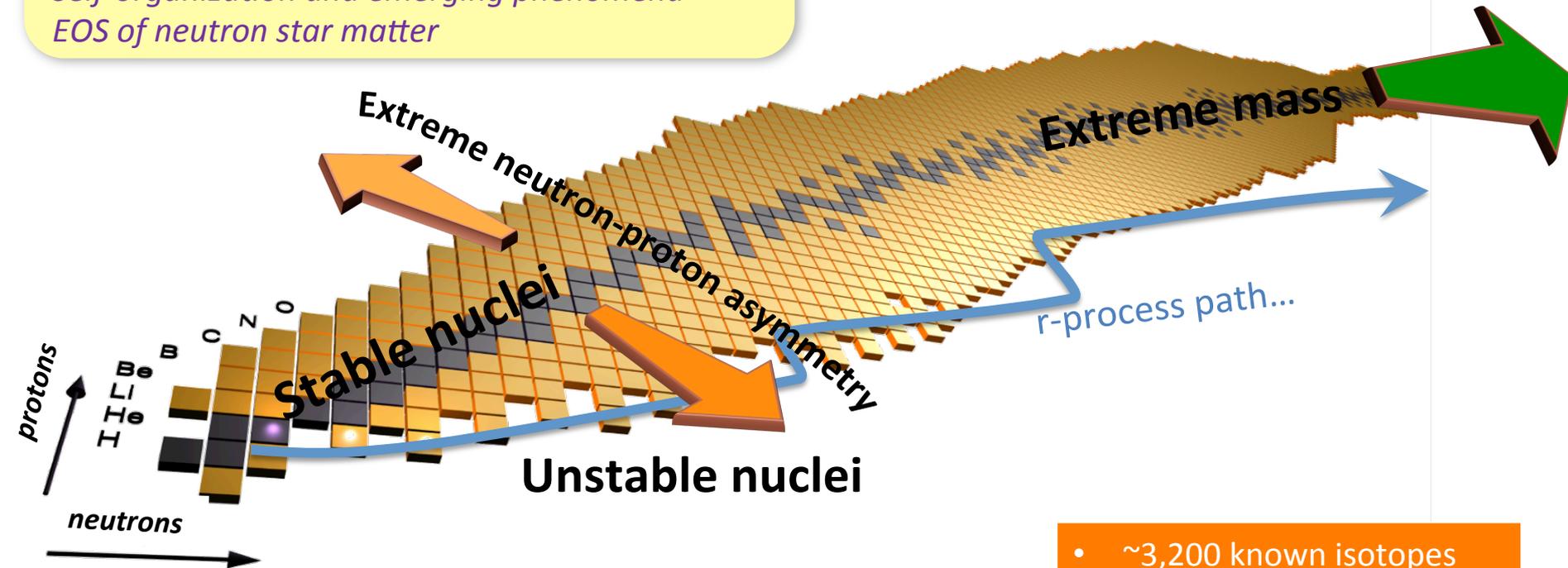
Self-organization and emerging phenomena

EOS of neutron star matter

Experimental

programs

RIKEN, FAIR, FRIB



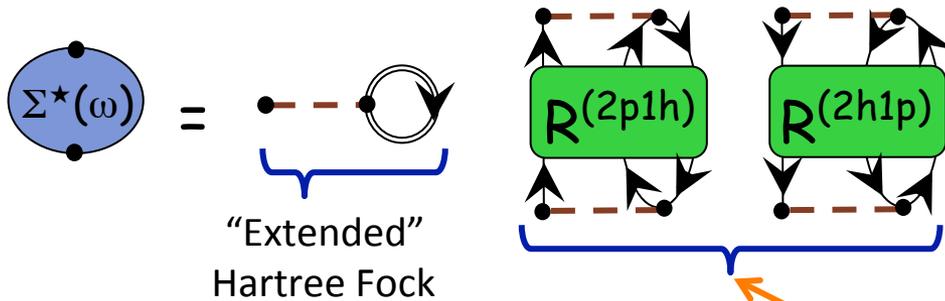
- ~3,200 known isotopes
- ~7,000 predicted to exist
- Correlation characterised in full for ~283 stable

Nature **473**, 25 (2011); **486**, 509 (2012)

The FRPA Method in Two Words

Particle vibration coupling is the main cause driving the distribution of particle strength—on both sides of the Fermi surface...

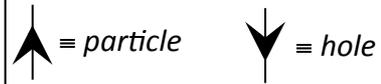
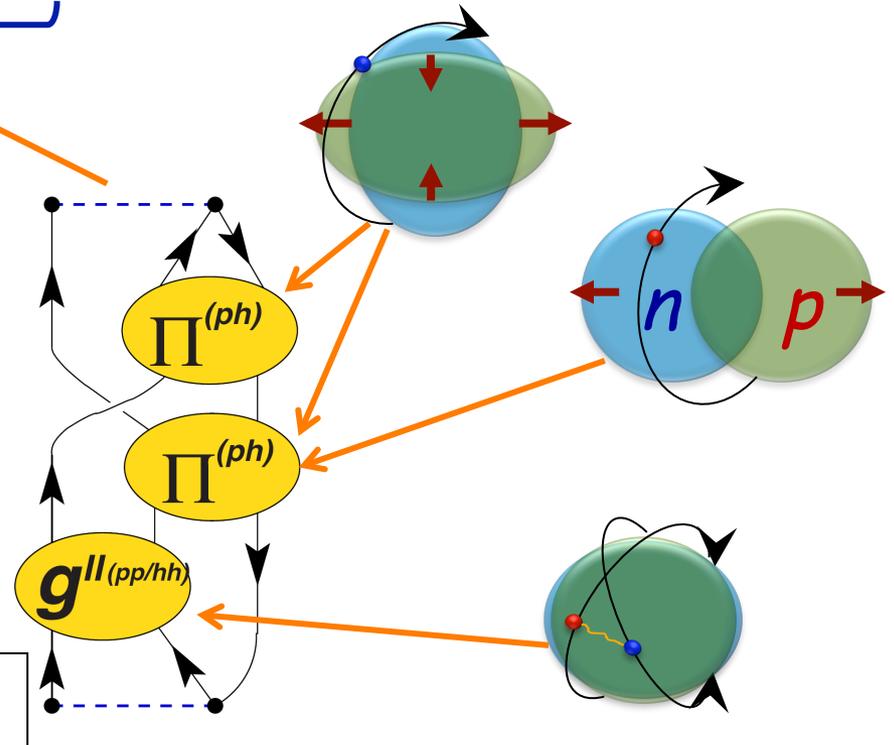
CB et al.,
 Phys. Rev. C63, 034313 (2001)
 Phys. Rev. A76, 052503 (2007)
 Phys. Rev. C79, 064313 (2009)



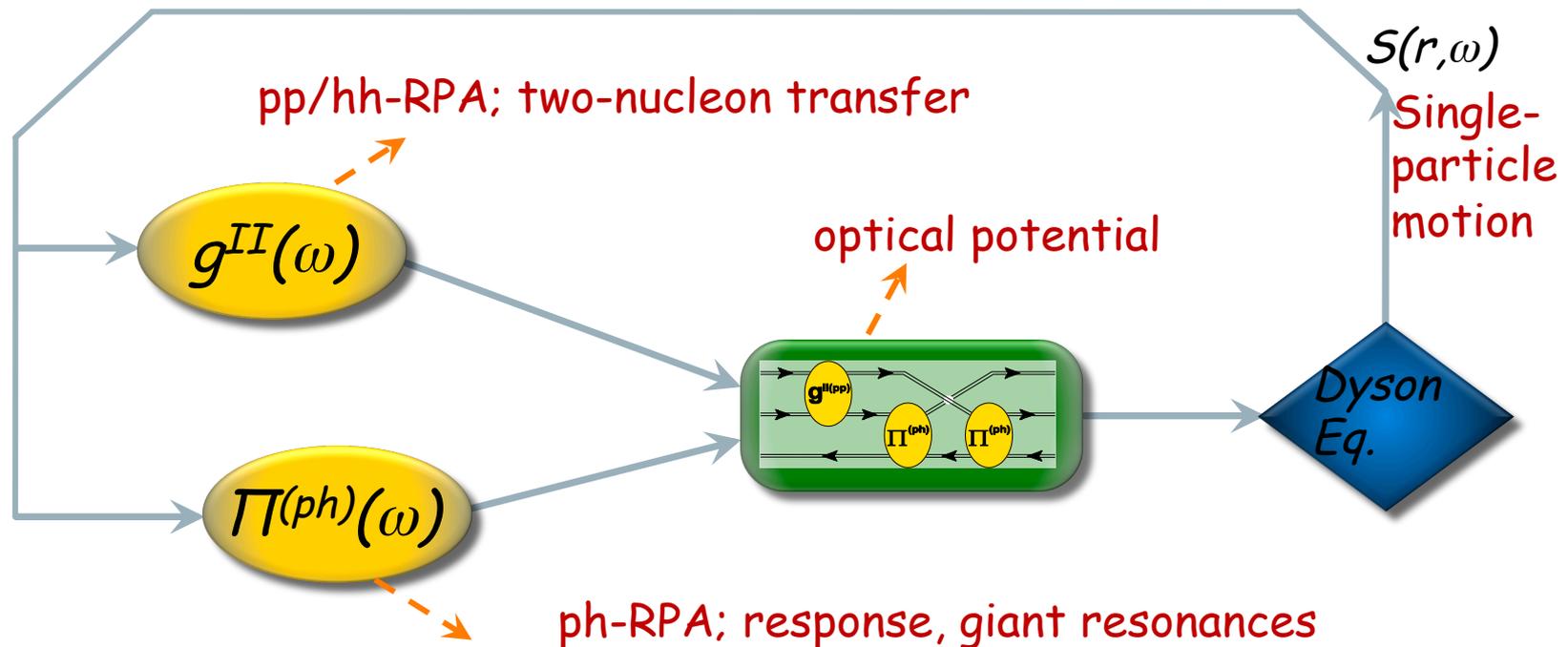
• A complete expansion requires all types of particle-vibration coupling

...these modes are all resummed exactly and to all orders in a *ab-initio* many-body expansion.

• The Self-energy $\Sigma^*(\omega)$ yields both single-particle states and scattering



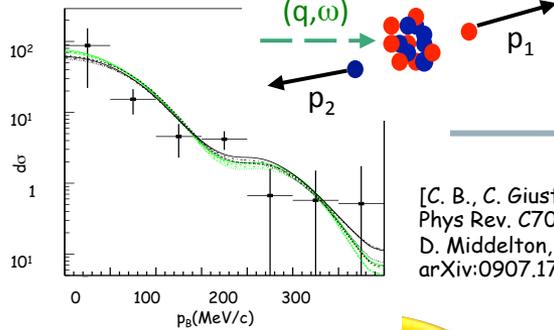
Self-Consistent Green's Function Approach



- Global picture of nuclear dynamics
- Reciprocal correlations among effective modes
- Guaranties *macroscopic conservation laws*

Self-Consistent Green's Function Approach

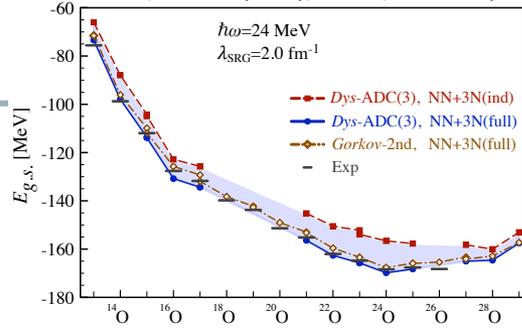
$^{16}\text{O}(e,e'pn)^{14}\text{N}$ @ MAINZ



[C. B., C. Giusti, et al. Phys Rev. C70, 014606 (2004)
D. Middleton, et al. arXiv:0907.1758; EPJA in print]

Binding energies

[PRL. 111, 062501 (2013),
PRC 92, 014306 (2015), PRC89, 061301R (2014)]



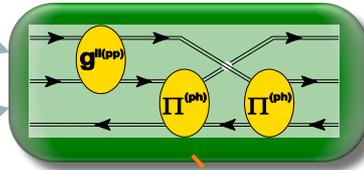
Ionization energies/
affinities, in atoms

[CB, D. Van Neck,
AIP Conf.Proc.1120,104 ('09) & in prep]

		Hartree-Fock	FRPAc	Experiment [16, 17]
He:	1s	0.918 (+14)	0.9008 (-2.9)	0.9037
Be ²⁺ :	1s	5.6672 (+116)	5.6551 (-0.5)	5.6556
Be:	2s	0.3093 (-34)	0.3224 (-20.2)	0.3426
	1s	4.733 (+200)	4.5405 (+8)	4.533
Ne:	2p	0.852 (+57)	0.8037 (+11)	0.793
	1s	1.931 (+149)	1.7967 (+15)	1.782
Mg ²⁺ :	2p	3.0068 (+56.9)	2.9537 (+3.8)	2.9499
	1s	4.4827	4.3589	
Mg:	3s	0.253 (-28)	0.280 (-1)	0.281
	2p	2.282 (+162)	2.137 (+17)	2.12
Ar:	3p	0.591 (+12)	0.579 (±0)	0.579
	3s	1.277 (+202)	1.065 (-10)	1.075
	3s		1.544	
	2p	9.571 (+411)	9.219 (+59)	9.160

$g^{II}(\omega)$

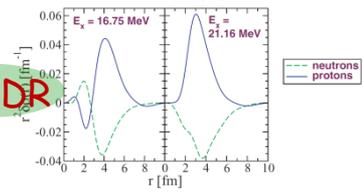
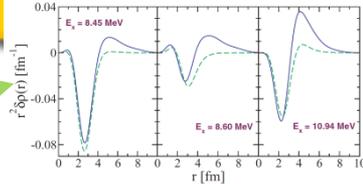
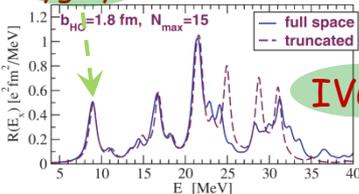
$\Pi(ph)(\omega)$



Dyson Eq.

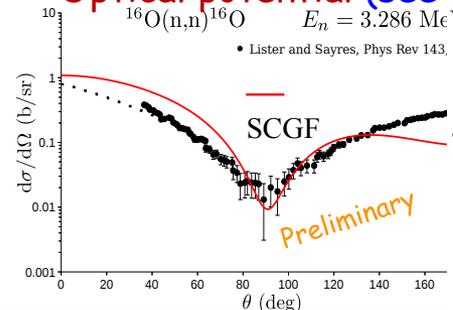
Isvector response
for ^{32}Ar , ^{34}Ar

Proton
Pygmy

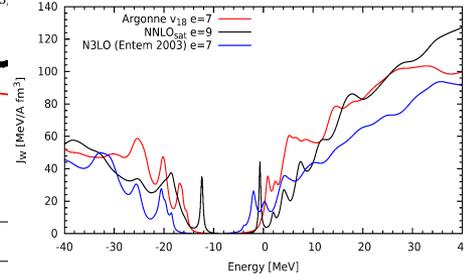


IVGDR

Optical potential (see talk by A. Idini)



arXiv:1612.01478 [nucl-th]



Gorkov and 3-nucleon forces

Approaches in GF theory

Truncation scheme:

Dyson formulation
(closed shells)

Gorkov formulation
(semi-/doubly-magic)

1st order:

Hartree-Fock

HF-Bogoliubov

2nd order:

2nd order

2nd order (w/ pairing)

...

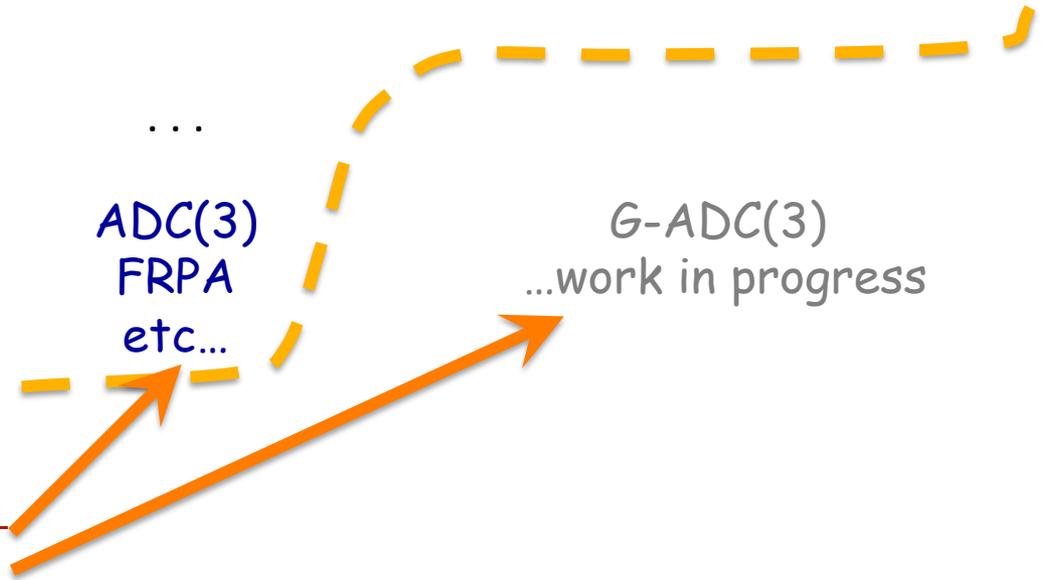
...

3rd and all-orders
sums,
P-V coupling:

ADC(3)
FRPA
etc...

G-ADC(3)
...work in progress

This is a **non-perturbative all-orders resummation**
— NOT a PT truncation!



Approaches in GF theory

Truncation scheme:

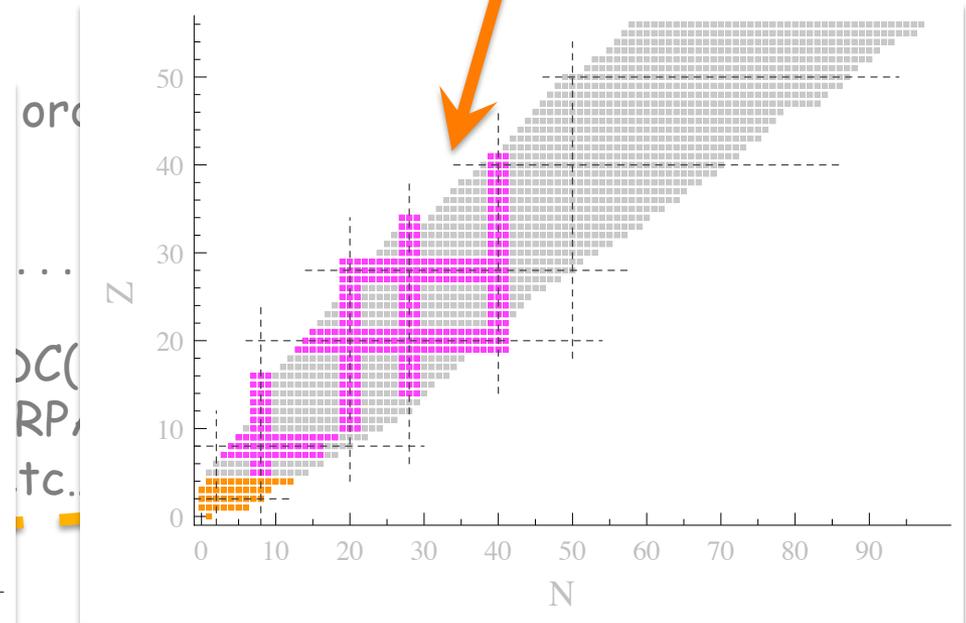
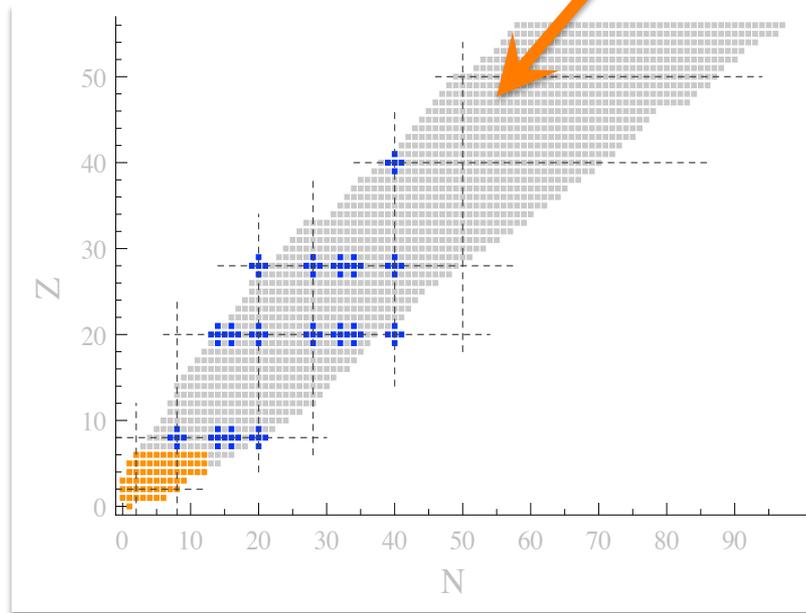
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Hartree-Fock

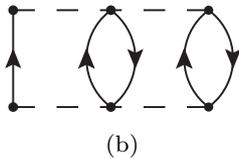
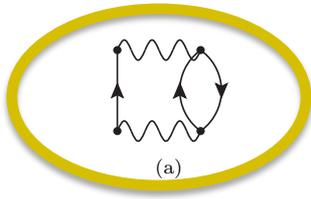
HF-Bogoliubov



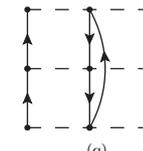
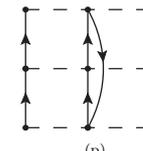
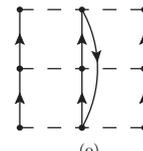
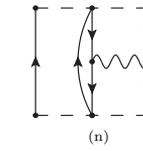
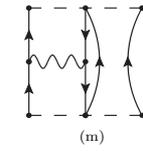
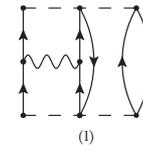
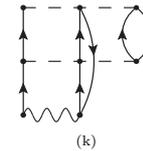
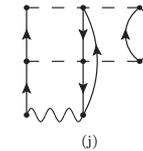
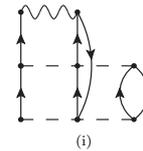
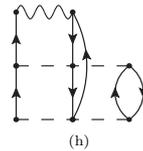
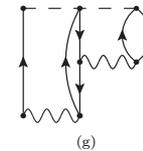
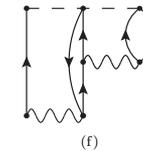
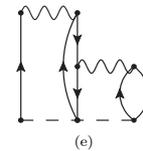
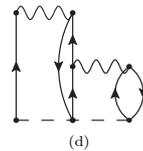
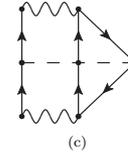
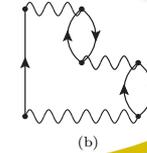
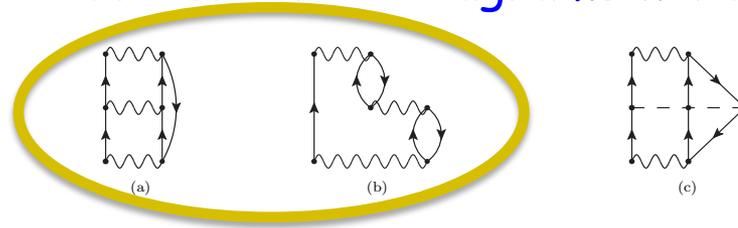
Inclusion of NNN forces

A. Carbone, CB, et al., *Phys. Rev. C* **88**, 054326 (2013)
and F. Raimondi, CB, in preparation (2017).

- Second order PT diagrams with 3BFs:



- Third order PT diagrams with 3BFs:



→ Use of irreducible 2-body interactions

→ Need to correct the Koltun sum rule (for energy)

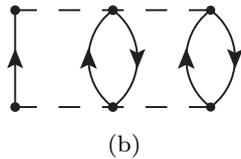
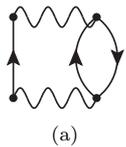
→ 3p2h/3h2p terms relevant to next-generation high-precision methods.

FIG. 5. 1PI, skeleton and interaction irreducible self-energy diagrams appearing at 3rd-order in perturbative expansion (7), making use of the effective hamiltonian of Eq. (9).

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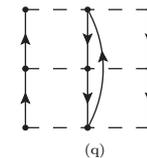
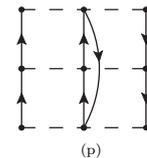
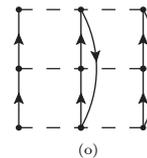
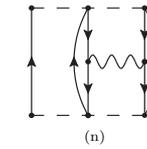
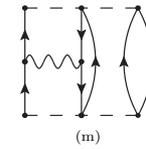
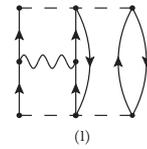
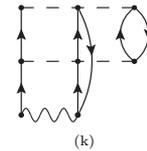
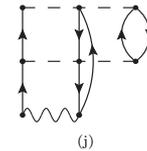
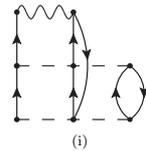
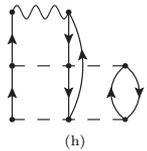
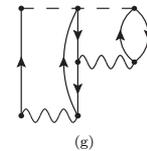
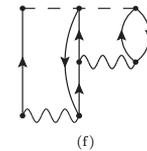
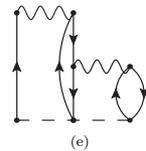
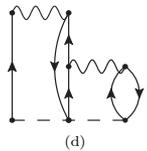
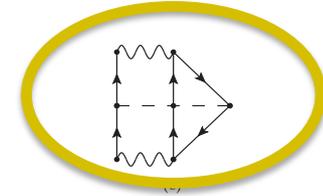
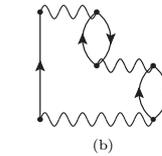
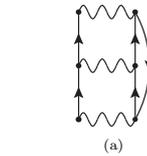
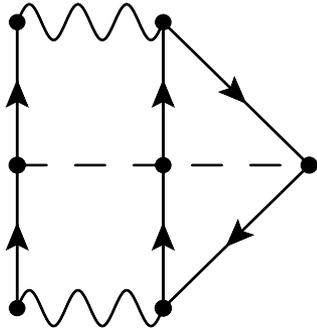


FIG. 5. 1PI, skeleton and interaction irreducible self-energy diagrams appearing at 3rd-order in perturbative expansion (7), making use of the effective hamiltonian of Eq. (9).

Inclusion of NNN forces

First interactions irreducible
3NF term:



Including 3N forces in the many-body diagrammatic with the ADC formalism 13

while the one connecting through a hole-particle (hp) interaction gives

$$\mathbf{D}_{(k_1 k_2 n_3), (k_4 k_5 n_6)}^{hp} = \frac{1}{2} \left((\mathcal{Y}_\mu^{k_2} \mathcal{X}_\rho^{n_3})^* \tilde{V}_{\mu\nu, \lambda\rho} \mathcal{Y}_\lambda^{k_4} \mathcal{X}_\nu^{n_6} \delta_{k_1 k_5} \right. \\ - (\mathcal{Y}_\mu^{k_2} \mathcal{X}_\rho^{n_3})^* \tilde{V}_{\mu\nu, \lambda\rho} \mathcal{Y}_\lambda^{k_4} \mathcal{X}_\nu^{n_6} \delta_{k_1 k_5} \\ - (\mathcal{Y}_\mu^{k_1} \mathcal{X}_\rho^{n_3})^* \tilde{V}_{\mu\nu, \lambda\rho} \mathcal{Y}_\lambda^{k_5} \mathcal{X}_\nu^{n_6} \delta_{k_2 k_4} \\ \left. + (\mathcal{Y}_\mu^{k_1} \mathcal{X}_\rho^{n_3})^* \tilde{V}_{\mu\nu, \lambda\rho} \mathcal{Y}_\lambda^{k_4} \mathcal{X}_\nu^{n_6} \delta_{k_2 k_5} \right)$$

We now turn to the Feynman diagram of Fig. 3c, which is the focus work. To our knowledge the ADC formulas arising from this term is presented before. The Feynman rules gives the following expression for

$$\Sigma_{\alpha\beta}^{(3c)}(\omega) = -\frac{(h)^4}{4} \int \frac{d\omega_1}{2\pi i} \int \frac{d\omega_2}{2\pi i} \int \frac{d\omega_3}{2\pi i} \int \frac{d\omega_4}{2\pi i} \sum_{\gamma\delta\mu\epsilon\lambda} \tilde{V}_{\alpha\gamma, \delta\nu} g_{\epsilon\gamma}(\omega_3) g_{\nu\lambda}(\omega_4) W_{\mu\lambda, \xi\eta\theta} g_{\theta\tau}(\omega - \omega_2 + \omega_4) g_{\eta\rho}(\omega_2) g_{\chi\mu}(\omega_4) \tilde{V}_{\sigma\tau, \chi\mu}$$

By performing the four integrals in the complex plane, we find six terms to the different time orderings of the three interactions. Altogether

$$\Sigma_{\alpha\beta}^{(3c)}(\omega) = \frac{1}{4} \sum_{\gamma\delta\mu\epsilon\lambda} \tilde{V}_{\alpha\gamma, \delta\nu} W_{\epsilon\lambda\mu, \eta\theta} \tilde{V}_{\sigma\tau, \beta\chi} \times \\ \left(- \sum_{\substack{n_1 n_2 k_3 \\ n_4 n_5 k_6}} \frac{(\mathcal{X}_\mu^{n_1} \mathcal{X}_\nu^{n_2} \mathcal{Y}_\gamma^{k_3})^* \mathcal{X}_\mu^{n_4} \mathcal{X}_\nu^{n_5} \mathcal{Y}_\lambda^{k_6} (\mathcal{X}_\mu^{n_4} \mathcal{X}_\nu^{n_5} \mathcal{Y}_\mu^{k_6})^* \mathcal{X}_\mu^{n_1} \mathcal{X}_\nu^{n_2} \mathcal{Y}_\gamma^{k_3}}{(h\omega - (\epsilon_{n_1}^+ + \epsilon_{n_2}^+ - \epsilon_{k_3}^- + i\eta)) (h\omega - (\epsilon_{n_4}^+ + \epsilon_{n_5}^+ - \epsilon_{k_6}^- + i\eta))} \right. \\ + \sum_{\substack{k_1 k_2 n_3 \\ n_4 n_5 k_6}} \frac{\mathcal{Y}_\delta^{k_1} \mathcal{Y}_\nu^{k_2} \mathcal{X}_\gamma^{n_3} (\mathcal{Y}_\epsilon^{k_1} \mathcal{Y}_\lambda^{k_2} \mathcal{X}_\mu^{n_4} \mathcal{X}_\nu^{n_5} \mathcal{X}_\xi^{n_3})^* \mathcal{X}_\mu^{n_4} \mathcal{X}_\nu^{n_5} \mathcal{Y}_\delta^{k_1}}{(\epsilon_{k_1}^- + \epsilon_{k_2}^- + \epsilon_{k_6}^- - \epsilon_{n_3}^+ - \epsilon_{n_4}^+ - \epsilon_{n_5}^+) (h\omega - (\epsilon_{n_4}^+ + \epsilon_{n_5}^+ - \epsilon_{k_6}^- + i\eta))} \\ + \sum_{\substack{k_1 k_2 n_3 \\ n_4 n_5 k_6}} \frac{(\mathcal{X}_\mu^{n_4} \mathcal{X}_\nu^{n_5} \mathcal{Y}_\mu^{k_6})^* \mathcal{X}_\mu^{n_4} \mathcal{X}_\nu^{n_5} \mathcal{Y}_\mu^{k_6} \mathcal{Y}_\nu^{k_2} \mathcal{X}_\gamma^{n_3} (\mathcal{Y}_\delta^{k_1} \mathcal{Y}_\nu^{k_2} \mathcal{X}_\gamma^{n_3})^* \mathcal{X}_\mu^{n_4} \mathcal{X}_\nu^{n_5} \mathcal{Y}_\mu^{k_6}}{(h\omega - (\epsilon_{n_4}^+ + \epsilon_{n_5}^+ - \epsilon_{k_6}^- + i\eta)) (\epsilon_{k_1}^- + \epsilon_{k_2}^- + \epsilon_{k_6}^- - \epsilon_{n_3}^+ - \epsilon_{n_4}^+ - \epsilon_{n_5}^+)} \\ - \sum_{\substack{k_1 k_2 n_3 \\ k_4 k_5 n_6}} \frac{\mathcal{Y}_\delta^{k_1} \mathcal{Y}_\nu^{k_2} \mathcal{X}_\gamma^{n_3} (\mathcal{Y}_\epsilon^{k_1} \mathcal{Y}_\lambda^{k_2} \mathcal{X}_\mu^{n_4} \mathcal{X}_\nu^{n_5} \mathcal{X}_\xi^{n_3})^* \mathcal{X}_\mu^{n_4} \mathcal{X}_\nu^{n_5} \mathcal{Y}_\delta^{k_1}}{(h\omega - (\epsilon_{k_1}^- + \epsilon_{k_2}^- - \epsilon_{n_3}^+ - i\eta)) (h\omega - (\epsilon_{k_4}^- + \epsilon_{k_5}^- - \epsilon_{n_6}^+ - i\eta))} \\ - \sum_{\substack{k_1 k_2 n_3 \\ n_4 n_5 k_6}} \frac{\mathcal{Y}_\delta^{k_1} \mathcal{Y}_\nu^{k_2} \mathcal{X}_\gamma^{n_3} (\mathcal{Y}_\epsilon^{k_1} \mathcal{Y}_\lambda^{k_2} \mathcal{X}_\mu^{n_4} \mathcal{X}_\nu^{n_5} \mathcal{X}_\xi^{n_3})^* \mathcal{X}_\mu^{n_4} \mathcal{X}_\nu^{n_5} \mathcal{Y}_\delta^{k_1}}{(h\omega - (\epsilon_{k_1}^- + \epsilon_{k_2}^- - \epsilon_{n_3}^+ - i\eta)) (\epsilon_{k_1}^- + \epsilon_{k_2}^- + \epsilon_{k_6}^- - \epsilon_{n_3}^+ - \epsilon_{n_4}^+ - \epsilon_{n_5}^+)} \\ - \sum_{\substack{k_1 k_2 n_3 \\ n_4 n_5 k_6}} \frac{(\mathcal{X}_\mu^{n_4} \mathcal{X}_\nu^{n_5} \mathcal{Y}_\mu^{k_6})^* \mathcal{X}_\mu^{n_4} \mathcal{X}_\nu^{n_5} \mathcal{Y}_\mu^{k_6} \mathcal{Y}_\nu^{k_2} \mathcal{X}_\gamma^{n_3} (\mathcal{Y}_\delta^{k_1} \mathcal{Y}_\nu^{k_2} \mathcal{X}_\gamma^{n_3})^* \mathcal{X}_\mu^{n_4} \mathcal{X}_\nu^{n_5} \mathcal{Y}_\mu^{k_6}}{(\epsilon_{k_1}^- + \epsilon_{k_2}^- + \epsilon_{k_6}^- - \epsilon_{n_3}^+ - \epsilon_{n_4}^+ - \epsilon_{n_5}^+) (h\omega - (\epsilon_{k_1}^- + \epsilon_{k_2}^- - \epsilon_{n_3}^+ - i\eta))} \left. \right)$$

where the first (last) three terms correspond to forward-in-time (backward-in-time) Goldstone diagrams.

14

F. Raimondi and C. Barbieri

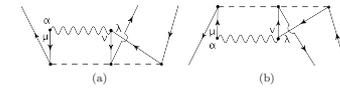


Figure 5: Diagrams of the ADC(3) coupling matrices with one effective 2NF \tilde{V} and one interaction-irreducible 3NF \tilde{W} . The coupling matrix (a) is linked to $2p1h$ ISCs and corresponds to Eq. (45), while (b) is linked to $2h1p$ ISCs and corresponds to Eq. (46).

By comparing to the third order terms in Eq. (24), one see that the new contributions to the coupling matrices contain one effective 2NF and one interaction-irreducible 3NF. The following forward-in-time matrix can be singled out from either the second or third line of Eq. (44).

$$\mathbf{M}_{(n_1 n_2 k_3)\alpha}^{(2N\ 3N\ a)} \equiv \frac{1}{2\sqrt{2}} \frac{\mathcal{X}_\xi^{n_4} \mathcal{X}_\mu^{n_5} \mathcal{X}_\nu^{n_6} W_{\xi\rho\sigma, \zeta\eta\theta} \mathcal{Y}_\zeta^{k_3} \mathcal{Y}_\theta^{k_6}}{\epsilon_{k_3}^- + \epsilon_{k_6}^- + \epsilon_{n_5}^- - \epsilon_{n_1}^+ - \epsilon_{n_2}^+ - \epsilon_{n_4}^+} (\mathcal{Y}_\mu^{k_4} \mathcal{X}_\nu^{n_6} \mathcal{X}_\lambda^{n_4})^* \tilde{V}_{\mu\nu, \alpha\lambda} \quad (45)$$

while in the last two lines of Eq. (44) we read the backward-in-time coupling matrix:

$$\mathbf{N}_{\alpha(k_1 k_2 n_3)}^{(2N\ 3N\ a)} \equiv -\frac{1}{2\sqrt{2}} \tilde{V}_{\alpha\lambda, \mu\nu} (\mathcal{Y}_\lambda^{k_4} \mathcal{X}_\mu^{n_5} \mathcal{X}_\nu^{n_6})^* \frac{\mathcal{X}_\mu^{n_4} \mathcal{X}_\nu^{n_5} \mathcal{X}_\xi^{n_6} W_{\rho\sigma\epsilon, \theta\zeta\eta} \mathcal{Y}_\theta^{k_1} \mathcal{Y}_\zeta^{k_2} \mathcal{Y}_\eta^{k_2}}{\epsilon_{k_1}^- + \epsilon_{k_2}^- + \epsilon_{k_4}^- - \epsilon_{n_3}^+ - \epsilon_{n_5}^- - \epsilon_{n_6}^-} \quad (46)$$

The diagrammatic representations of Eqs. (45) and (46) are displayed in Fig. 5.

The only interaction matrix that connects $2p1h$ ISCs through a 3NF is found from the first term of Eq. (44),

$$\mathbf{C}_{(n_1 n_2 k_3), (n_4 n_5 k_6)}^{3N} \equiv -\frac{1}{2} \mathcal{X}_\mu^{n_1} \mathcal{X}_\nu^{n_2} \mathcal{Y}_\delta^{k_3} W_{\nu\mu\lambda, \epsilon\eta\rho} (\mathcal{X}_\mu^{n_4} \mathcal{X}_\nu^{n_5} \mathcal{Y}_\lambda^{k_6})^* \quad (47)$$

which is explicitly antisymmetric in the particle indexes. With Eqs. (47) and (27) we can rewrite the first term of Eq. (44) as,

$$\mathbf{M}_{\alpha\tau}^{(1-2N)} \frac{1}{h\omega - E_\tau} \mathbf{C}_{\tau\rho}^{3N} \frac{1}{h\omega - E_{\rho'}} \mathbf{M}_{\rho'\beta}^{(1-2N)} \quad (48)$$

The expression (48) contains only the first order contribution in the interaction matrix expansion, corresponding to the second term in the r.h.s. of Eq. (23), for $B = \mathbf{C}^{3N}$. This is resummed to all order by diagonalizing the Dyson matrix (19), which will automatically include all the higher order terms in the expansion.

From the fourth term of Eq. (44), we single out the only backward-in-time interaction matrix connecting two $2h1p$ configurations through a 3N interaction, that is

$$\mathbf{D}_{(k_1 k_2 n_3), (k_4 k_5 n_6)}^{3N} \equiv \frac{1}{2} (\mathcal{Y}_\mu^{k_1} \mathcal{Y}_\nu^{k_2} \mathcal{X}_\rho^{n_3})^* W_{\nu\mu\lambda, \epsilon\eta\rho} \mathcal{Y}_\epsilon^{k_4} \mathcal{X}_\nu^{n_6} \quad (49)$$

which is also explicitly antisymmetric in the hole indexes. With Eqs. (49) and (28) we associate the fourth term of Eq. (44) to

$$\mathbf{N}_{\alpha\sigma}^{(1-2N)} \frac{1}{h\omega - E_\sigma} \mathbf{D}_{\sigma\sigma'}^{3N} \frac{1}{h\omega - E_{\sigma'}} \mathbf{N}_{\sigma'\beta}^{(1-2N)} \quad (50)$$

F. Raimondi, CB,
arXiv:1701.08127v1 [nucl-th]
and
PhysRevC
in preparation (2017).

Ab-initio Nuclear Computation & BcDor code

BoccaDorata code:

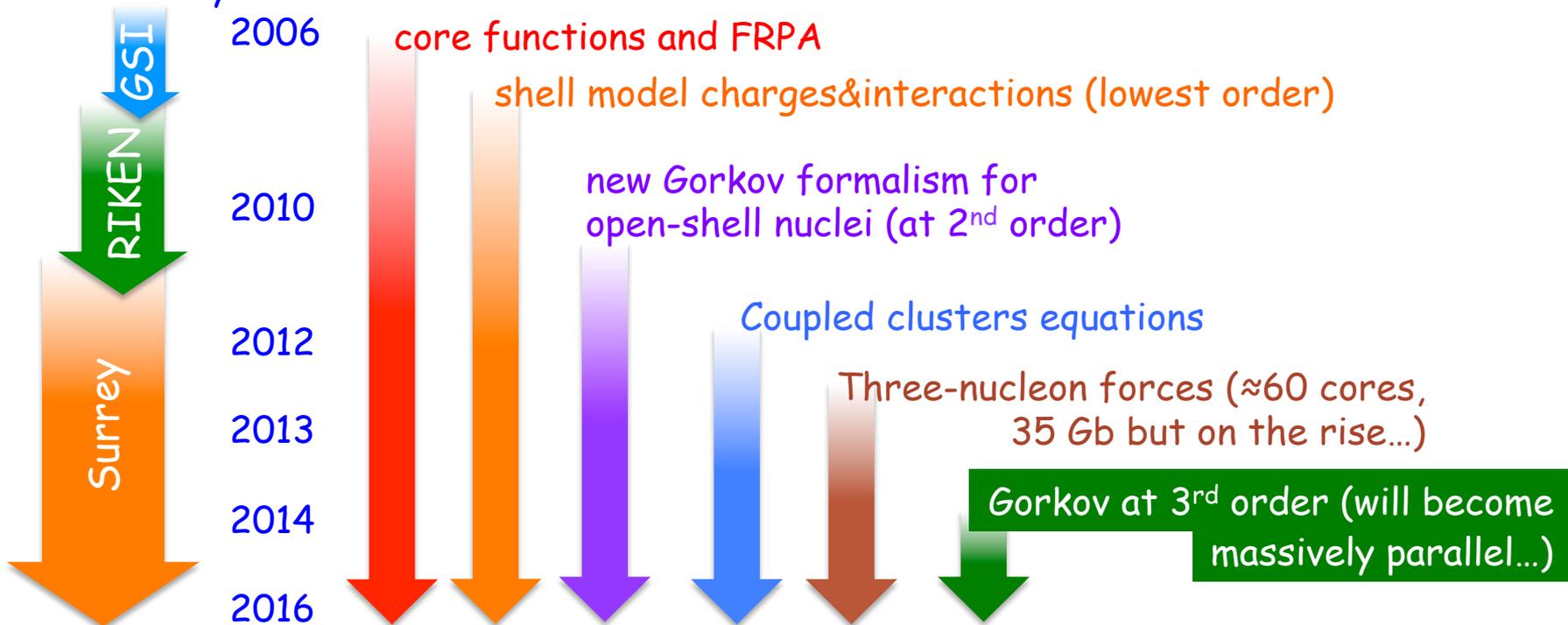
(C. Barbieri 2006-16

V. Somà 2010-15

A. Cipollone 2011-14)

- Provides a *C++ class library* for handling many-body propagators ($\approx 40,000$ lines, MPI&OpenMP based).
- Allows to solve for nuclear spectral functions, many-body propagators, RPA responses, coupled cluster equations and effective interaction/charges for the shell model.

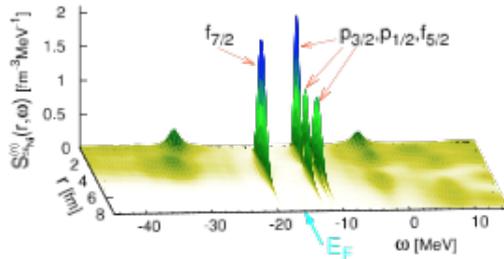
Code history:



Ab-initio Nuclear Computation & BcDor code

<http://personal.ph.surrey.ac.uk/~cb0023/bcdor/>

Computational Many-Body Physics



Welcome

From here you can download a public version of my self-consistent Green's function (SCGF) code for nuclear physics. This is a code in J-coupled scheme that allows the calculation of the single particle propagators (a.k.a. one-body Green's functions) and other many-body properties of spherical nuclei.

This version allows to:

- Perform Hartree-Fock calculations.
- Calculate the correlation energy at second order in perturbation theory (MBPT2).
- Solve the Dyson equation for propagators (self consistently) up to second order in the self-energy.
- Solve coupled cluster CCD (doubles only!) equations.

When using this code you are kindly invited to follow the creative commons license agreement, as detailed at the weblinks below. In particular, we kindly ask you to refer to the publications that led the development of this software.

Relevant references (which can also help in using this code) are:

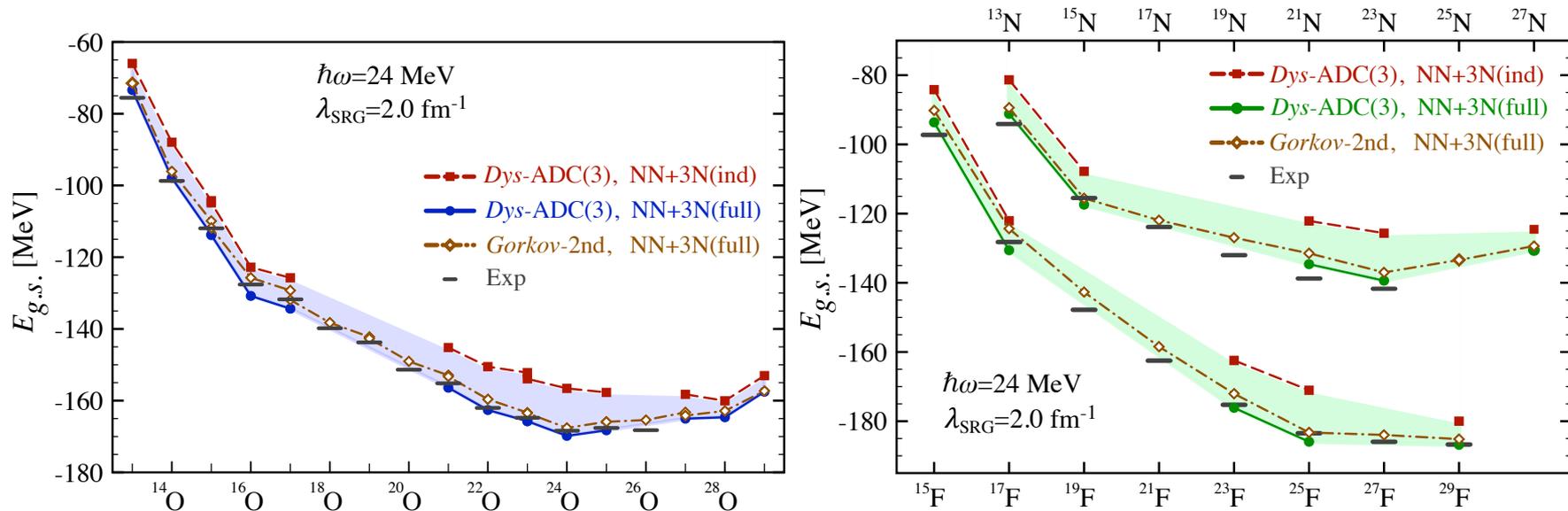
- Prog. Part. Nucl. Phys. 52, p. 377 (2004),
- Phys. Rev. A76, 052503 (2007),
- Phys. Rev. C79, 064313 (2009),
- Phys. Rev. C89, 024323 (2014)

Download

Documentation

Results for the N-O-F chains

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
and Phys. Rev. C **92**, 014306 (2015)



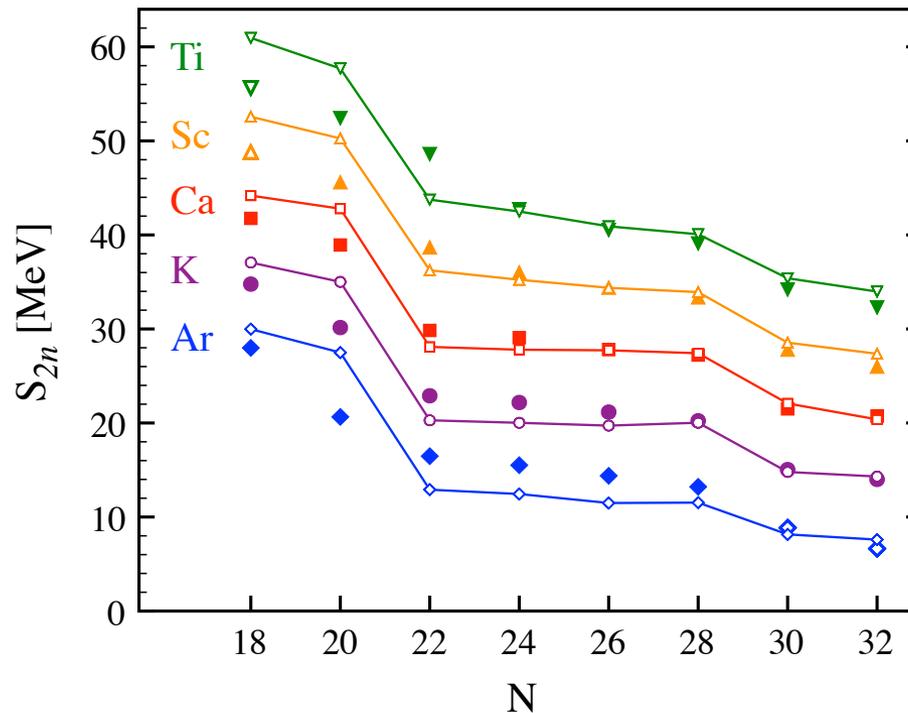
→ 3NF crucial for reproducing binding energies and driplines around oxygen

→ cf. microscopic shell model [Otsuka et al, PRL**105**, 032501 (2010).]

Neighbouring Ar, K, Ca, Sc, and Ti chains

V. Somà, CB *et al.* Phys. Rev. C89, 061301R (2014)

Two-neutron separation energies predicted by chiral $NN_{[EM(500)]}+3NF_{[N2LO(400)]}$:



→ First *ab-initio* calculation over a contiguous portion of the nuclear chart—open shells are now possible through the Gorkov-GF formalism



Radii and Binding Energies in Oxygen Isotopes: A Challenge for Nuclear Forces

V. Lapoux,^{1,*} V. Somà,¹ C. Barbieri,² H. Hergert,³ J.D. Holt,⁴ and S.R. Stroberg⁴

- New fits of chiral interactions (NNLO_{sat}) highly improve comparison to data

- Deficiencies remain for neutron rich isotopes

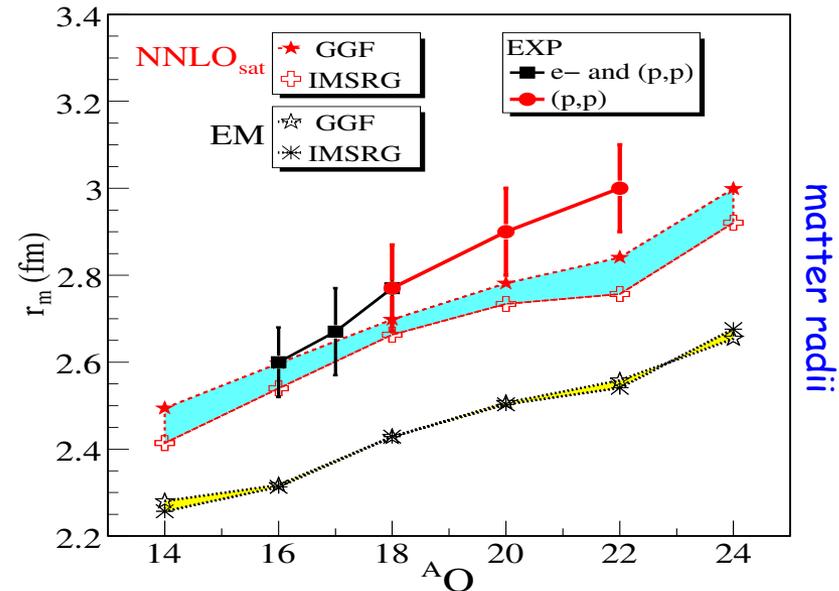
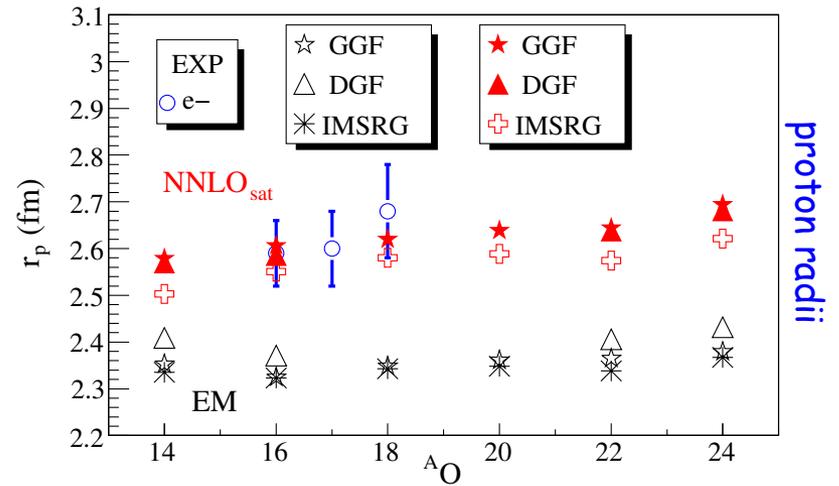
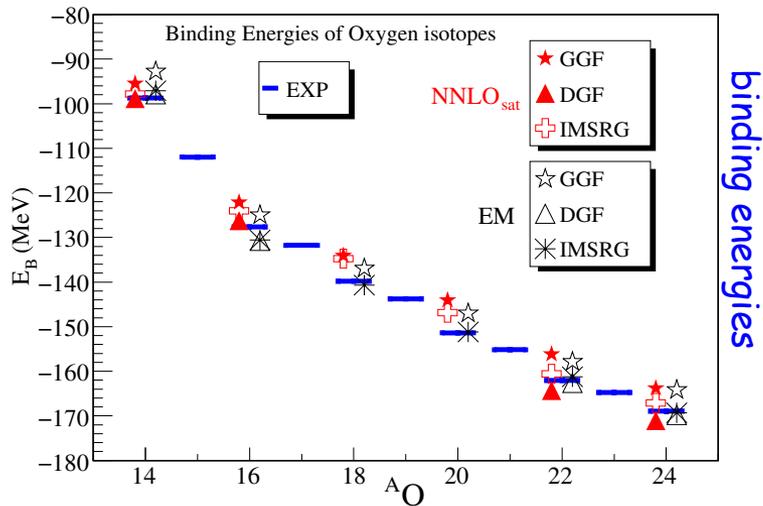


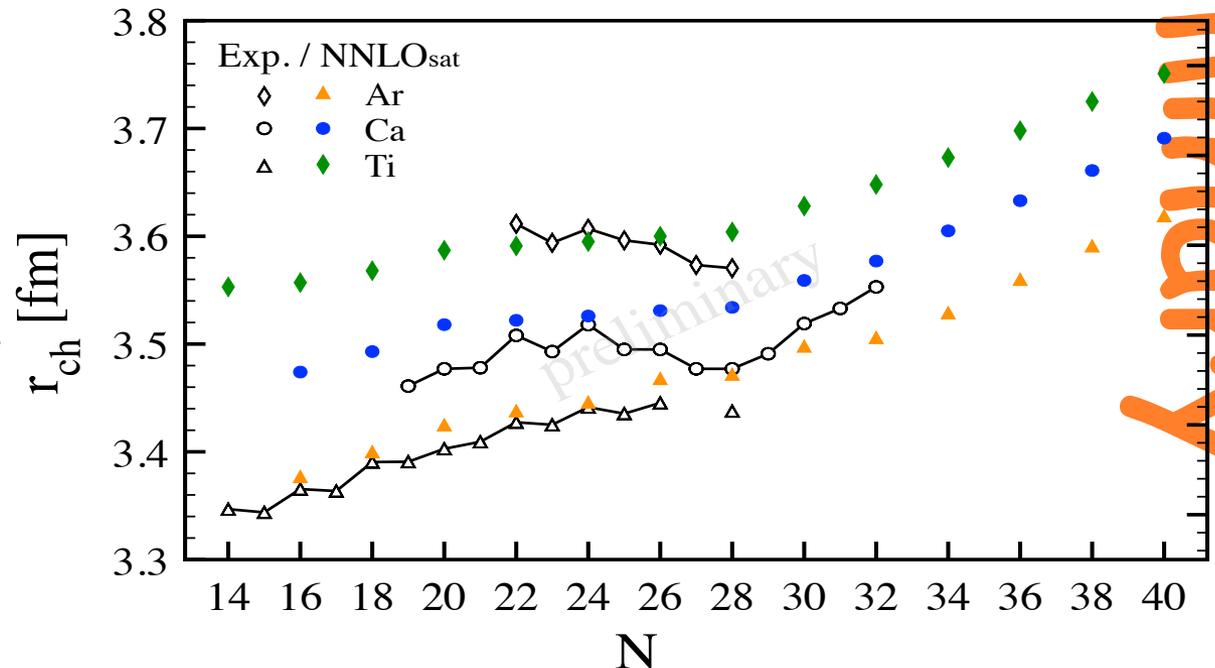
FIG. 1. Oxygen binding energies. Results from SCGF and IMSRG calculations performed with EM [20–22] and NNLO_{sat} [26] interactions are displayed along with available experimental data.

charge radii in the *pf* shell

Size of radii not perfect but remains overall correct throughout the *pf* shell with NNLO-sat.

This suggests that saturation is indeed under control.

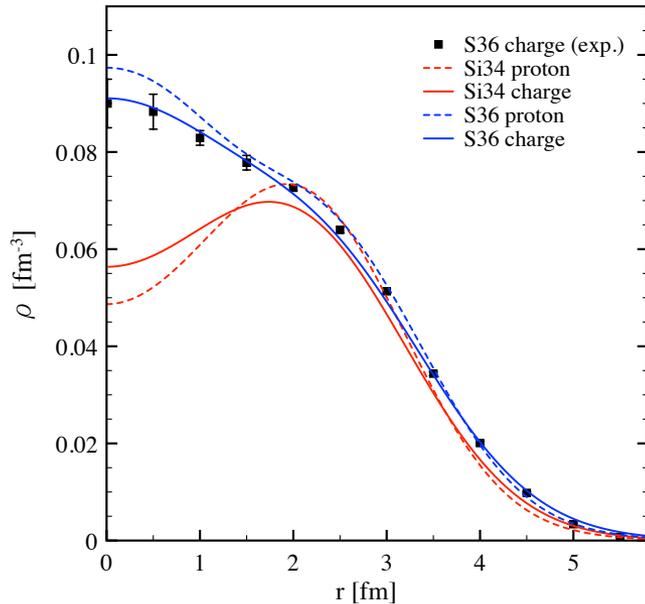
→ Improvements of many-body truncations beyond 2nd order Gorkov will also be relevant. (work in progress!)



Preliminary

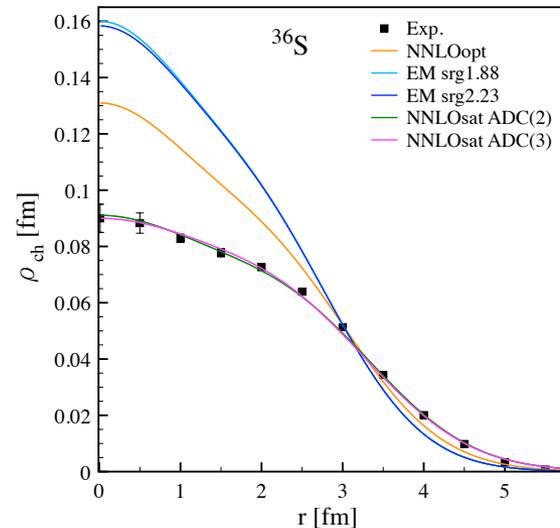
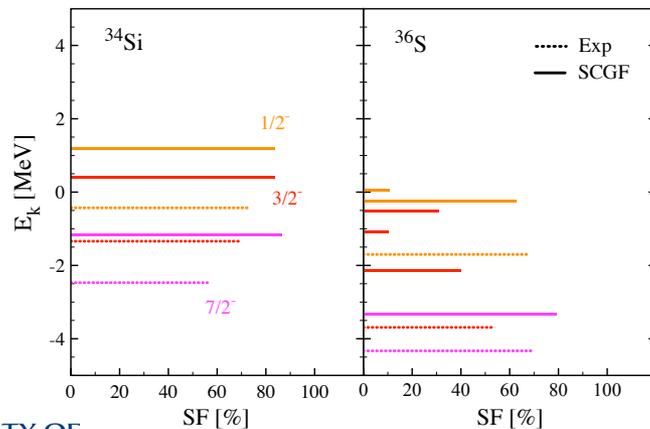
Bubble nuclei... ^{34}Si prediction

Duguet, Somà, CB, et al. arXiv:1611.08570 [nucl-th]



- ^{34}Si is unstable, charge distribution still unknown
- Suggested central depletion from mean-field simulations
- *Ab-initio* theory confirms predictions

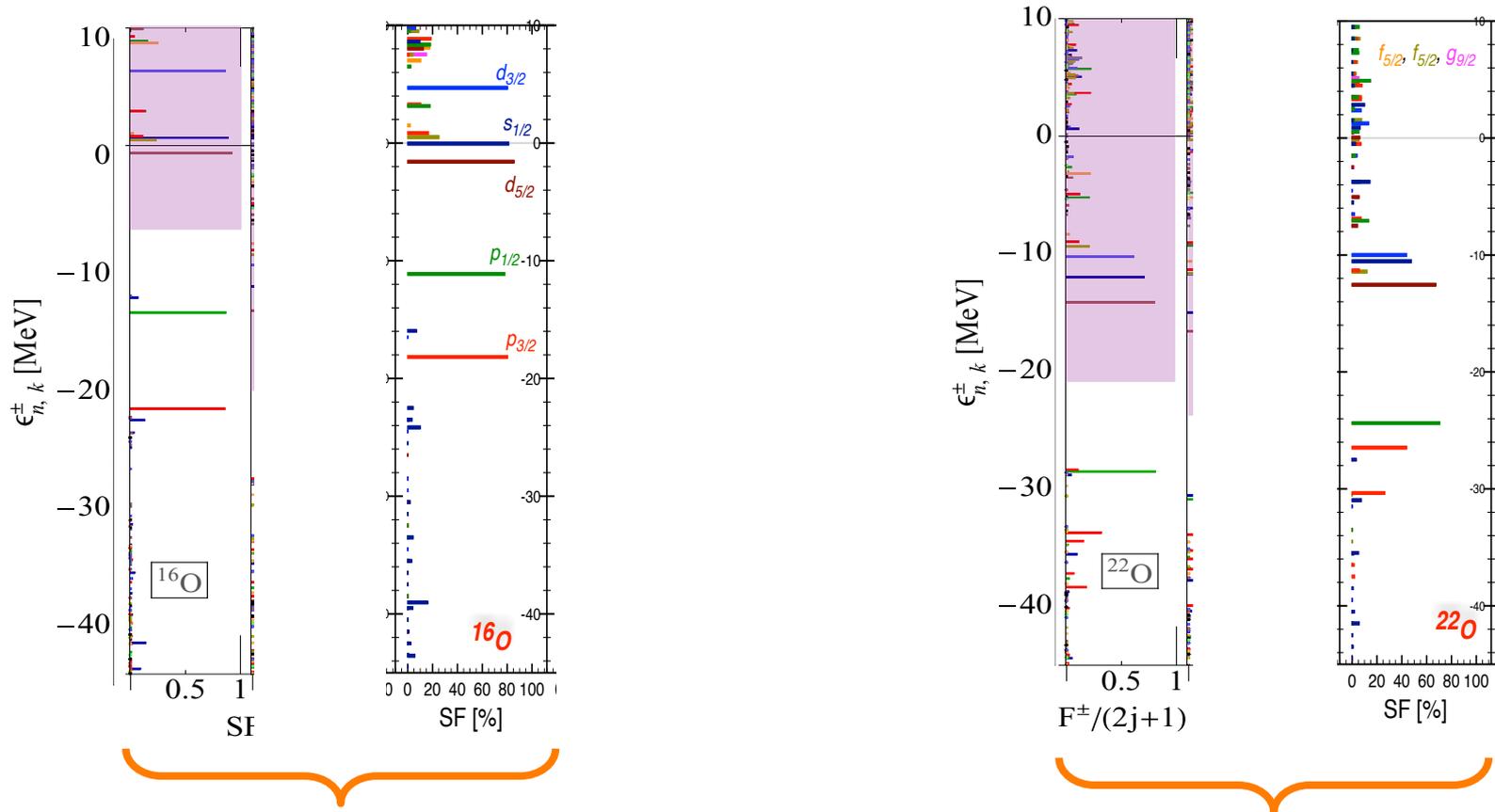
Validated by charge distributions and neutron quasiparticle spectra:



Proton spectral strength in Oxygen

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
and Phys. Rev. C **92**, 014306 (2015)
and *in preparation*

More in detail:



Quenching of absolute spectroscopic factors

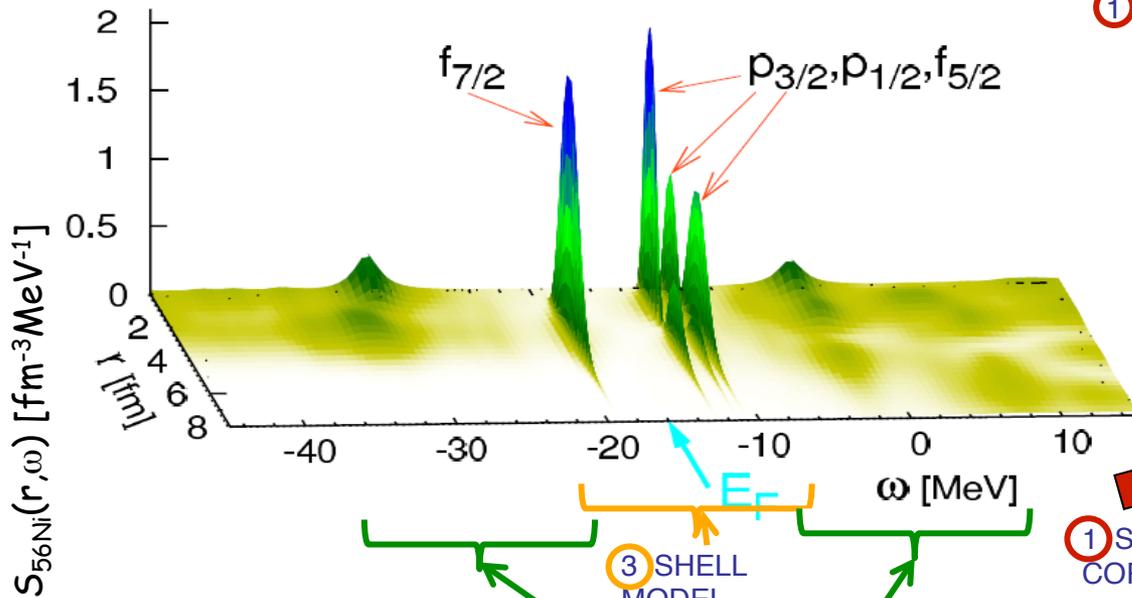
[CB, Phys. Rev. Lett. **103**, 202520 (2009)]

...with analogous conclusions for ^{48}Ca

Overall quenching of *spectroscopic factors* is driven by:

- SRC* → ~10%
- part-vibr. coupling* → dominant
- "shell-model"* → in open shell

	10 osc. shells		Exp. [30]	1p0f space			
	FRPA (SRC)	full FRPA		FRPA	SM	ΔZ_α	
^{57}Ni :							
$\nu 1p_{1/2}$	0.96	0.63	0.61	0.79	0.77	-0.02	
$\nu 0f_{5/2}$	0.95	0.59	0.55	0.79	0.75	-0.04	
$\nu 1p_{3/2}$	0.95	0.65	0.62	0.58(11)	0.82	0.79	-0.03
^{55}Ni :							
$\nu 0f_{7/2}$	0.95	0.72	0.69	0.89	0.86	-0.03	



$$Z_\alpha = \int d^3r |\psi_\alpha^{overlap}(\mathbf{r})|^2 = \frac{1}{1 - \left. \frac{\partial \Sigma_{\hat{\alpha}\hat{\alpha}}(\omega)}{\partial \omega} \right|_{\omega=\epsilon_\alpha}}$$

① SHORT RANGE CORRELATIONS

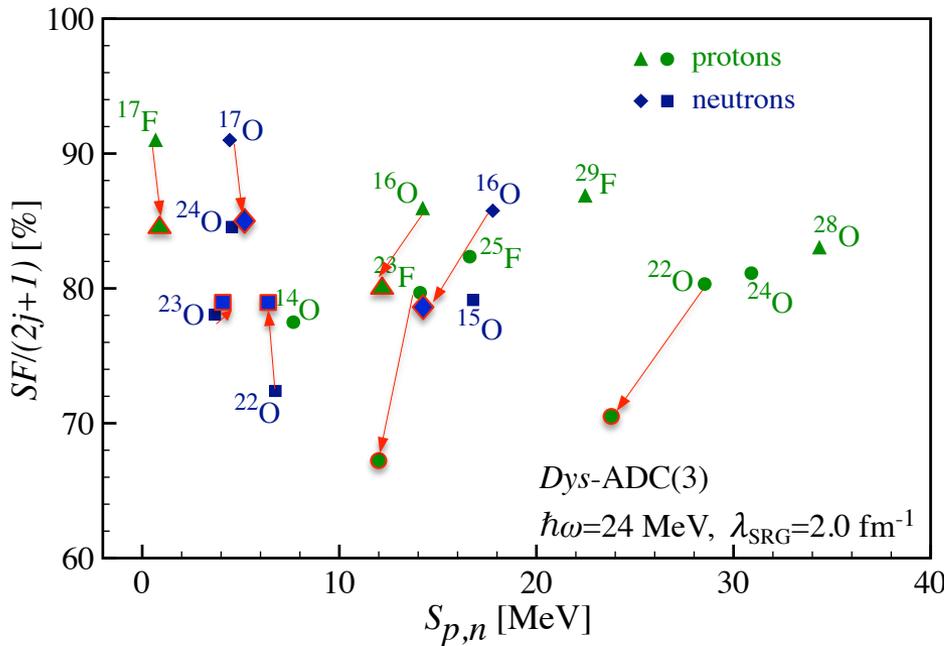
② PARTICLE-VIBRATION COUPLING

③ SHELL MODEL

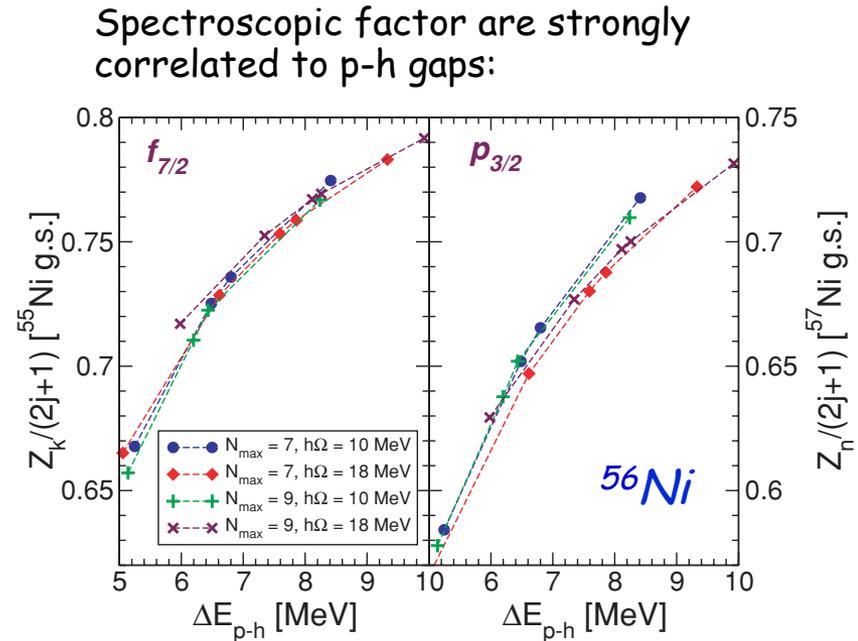
Z/N asymmetry dependence of SFs - Theory

Ab-initio calculations explain (a very weak) the Z/N dependence but the effect is much lower than suggested by direct knockout

Rather the quenching is high correlated to the gap at the Femi surface.



A. Cipollone, CB, P Navrátil
 Phys. Rev. C **92**, 014306 (2015)

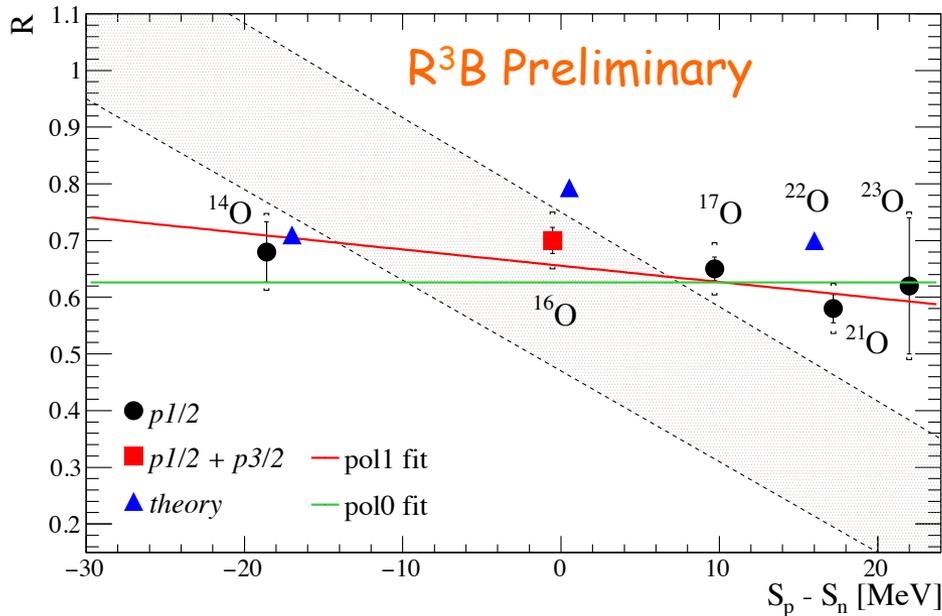


CB, M. Hjorth-Jensen,
 Phys. Rev. C **79**, 064313 (2009)

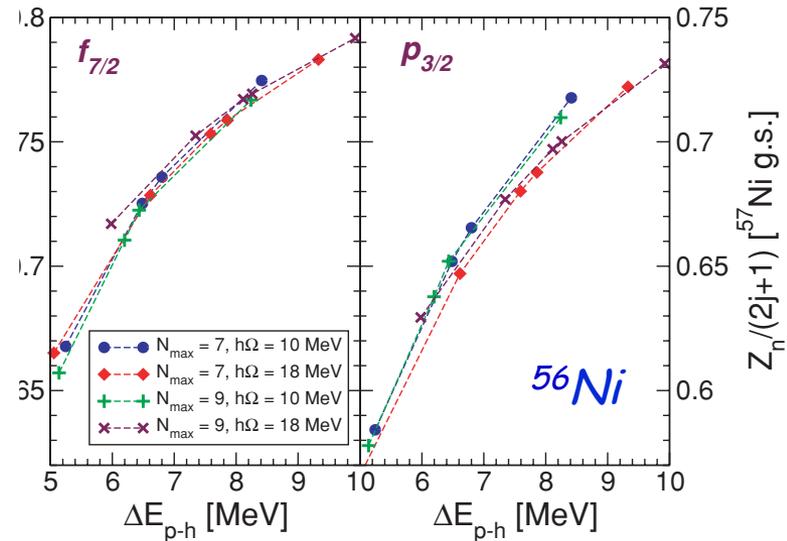
Z/N asymmetry dependence of SFs - Theory

Ab-initio calculations explain (a very weak) the Z/N dependence but the effect is much lower than suggested by direct knockout

Rather the quenching is high correlated to the gap at the Femi surface.



Spectroscopic factor are strongly correlated to p-h gaps:

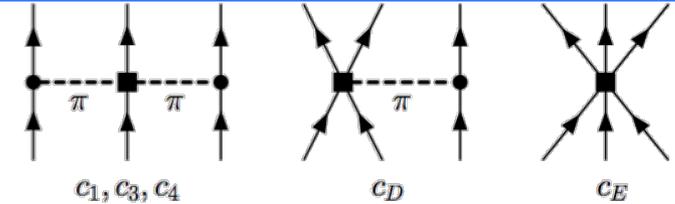


A. Cipollone, CB, P Navrátil
Phys. Rev. C **92**, 014306 (2015)

CB, M. Hjorth-Jensen,
Phys. Rev. C **79**, 064313 (2009)

Local vs. non-local chiral N²LO NNN interaction — by P. Navrátil

- Local: chiral N³LO NN+ N²LO 3N500
 - $c_D = -0.2$ $c_E = -0.205$ (${}^3\text{H } E_{\text{gs}} = -8.48$ MeV)
 - ${}^4\text{He}$



$$\langle H \rangle = -28.4939 \quad \langle V_{3b_2\pi} \rangle = -5.8819 \quad \langle V_{3b_D} \rangle = -0.2206 \quad \langle V_{3b_E} \rangle = 1.2665$$

- Non-local: chiral N²LO_{sat} NN+3N
 - $c_D = +0.8168$ $c_E = -0.0396$ (${}^3\text{H } E_{\text{gs}} = -8.53$ MeV)
 - ${}^4\text{He}$

$$\langle H \rangle = -28.4596 \quad \langle V_{3b_2\pi} \rangle = -4.7260 \quad \langle V_{3b_D} \rangle = 1.3897 \quad \langle V_{3b_E} \rangle = 0.4174$$

- Local/Non-local: chiral N³LO NN+ N²LO

$$F\left(\frac{1}{2}(\pi_1^2 + \pi_2^2); \Lambda_{\text{nonloc}}\right) W_1^Q(\Lambda_{\text{loc}}) F\left(\frac{1}{2}(\pi_1^2 + \pi_2^2); \Lambda_{\text{nonloc}}\right)$$

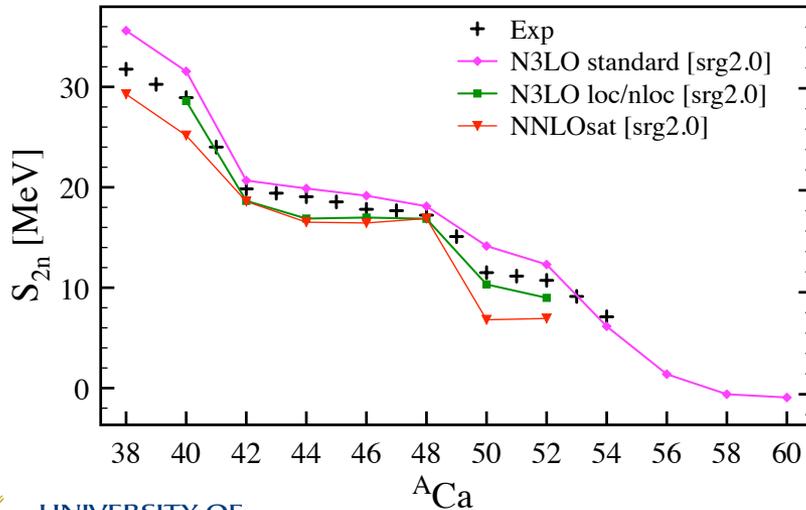
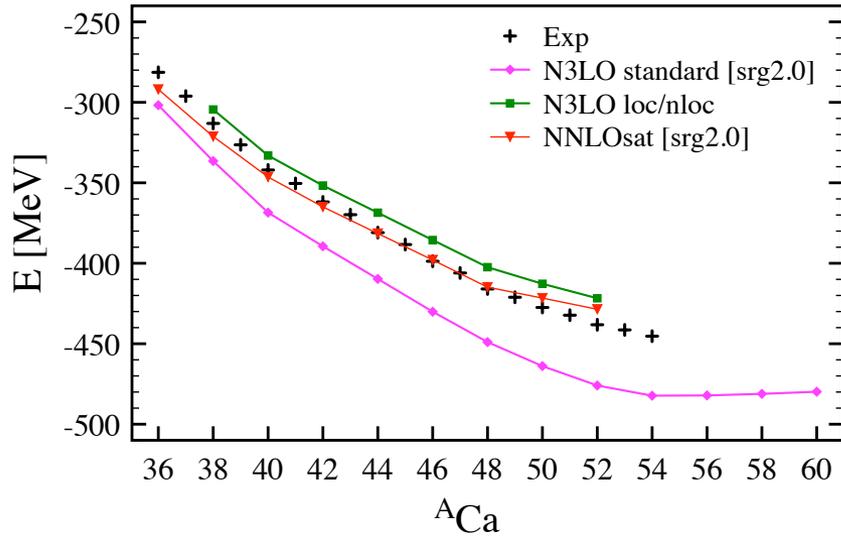
Use completeness in HO basis to calculate products of $F W F$

- $c_D = +0.7$ $c_E = -0.06$ (${}^3\text{H } E_{\text{gs}} = -8.44$ MeV)
- ${}^4\text{He}$

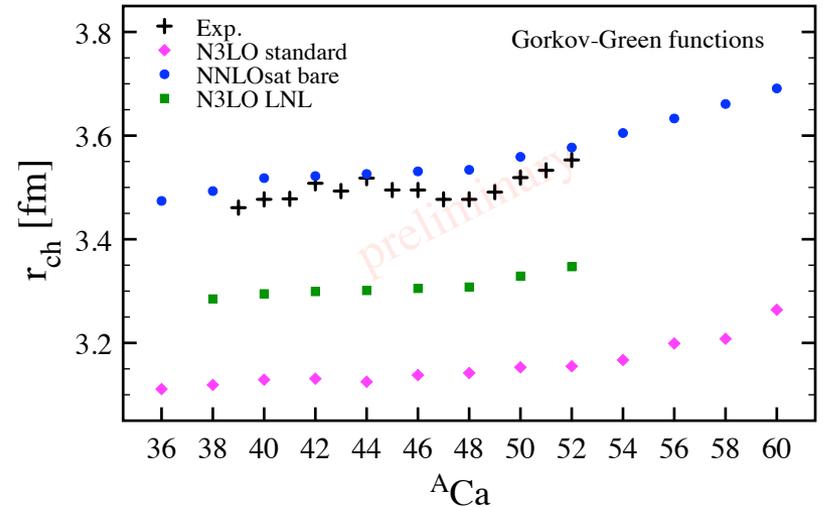
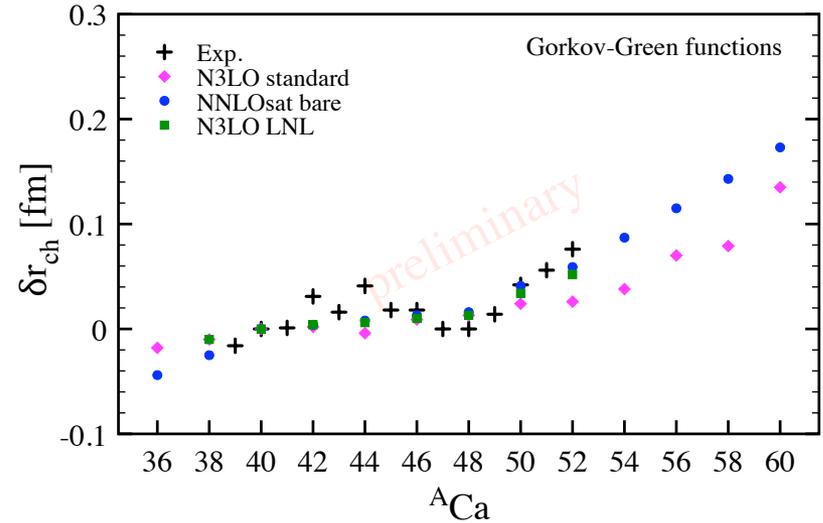
$$\langle H \rangle = -28.2530 \quad \langle V_{3b_2\pi} \rangle = -4.8124 \quad \langle V_{3b_D} \rangle = 0.7414 \quad \langle V_{3b_E} \rangle = 0.4255$$

N3LO(500) + $n \ln$ 3NF

SCGF – Gorkov-ADC(2)

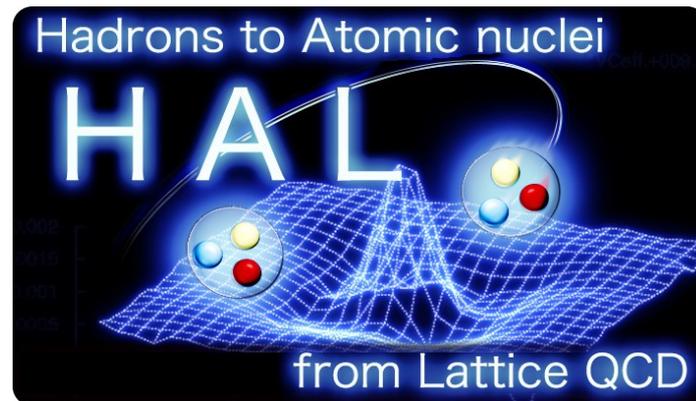


PRELIMINARY



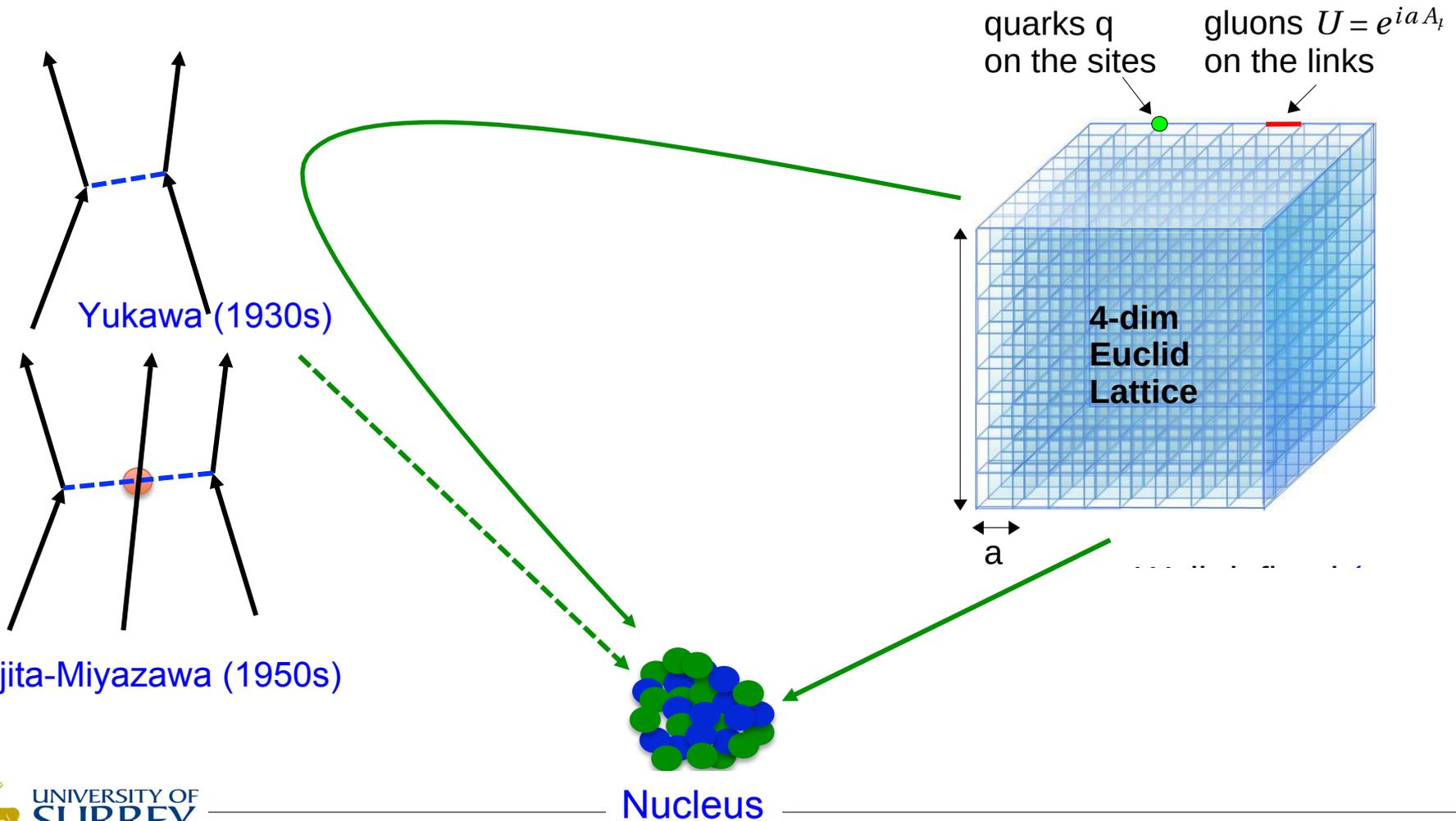
Study of nuclear interactions from Lattice QCD

In collaboration with:

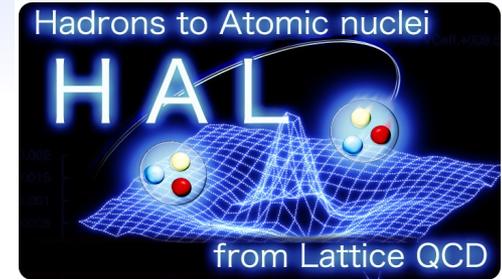


Approaches to nuclei from LQCD

$$L = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} + \bar{q} \gamma^\mu (i \partial_\mu - g t^a A_\mu^a) q - m \bar{q} q$$



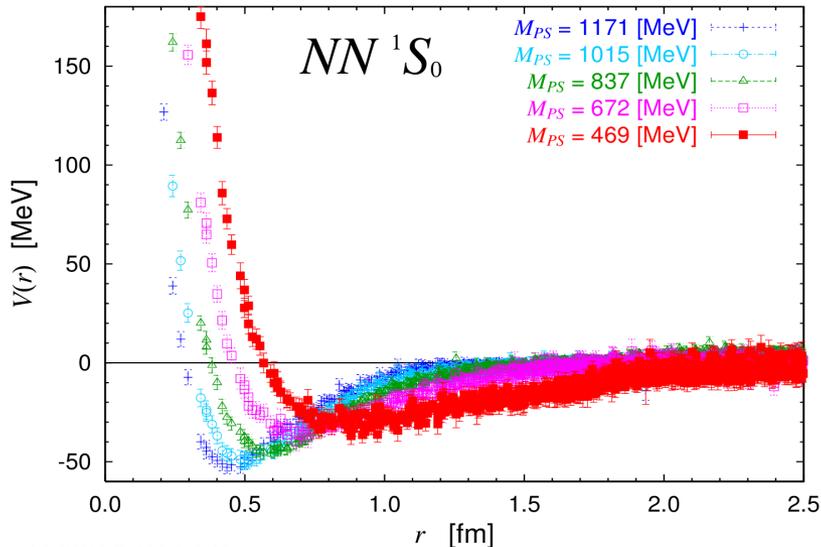
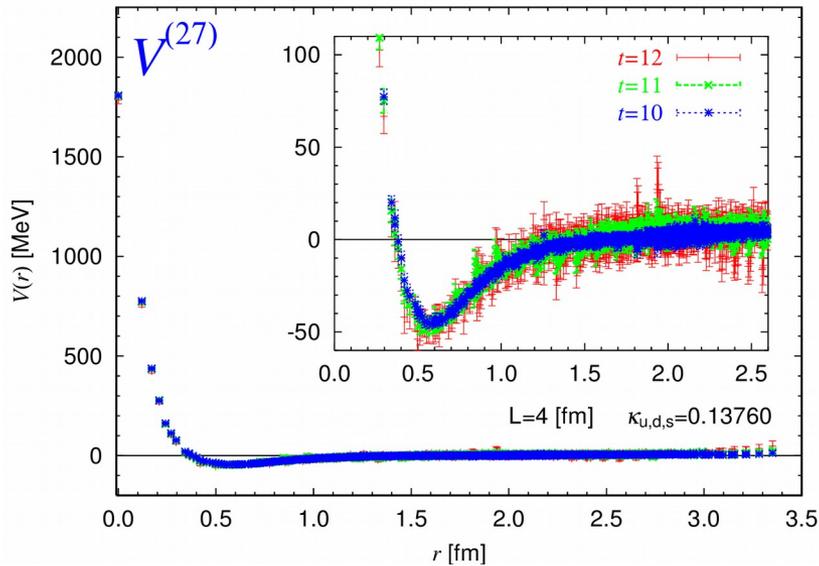
Why nuclear interactions on the Lattice??



- *Extend LQCD beyond few-bodies*
- *Not based on a specific EFT momentum scale*
→ *exploitable to high densities (e.g. Neutron stars)*
- *No LECs to worry about ...but:*
- *Variation in potentials from variation in sink operators (→ estimation of theoretical uncertainties)*
- *Direct derivation of hyperon-nucleon interactions*
- *3NF can be derived consistently with NN interactions*

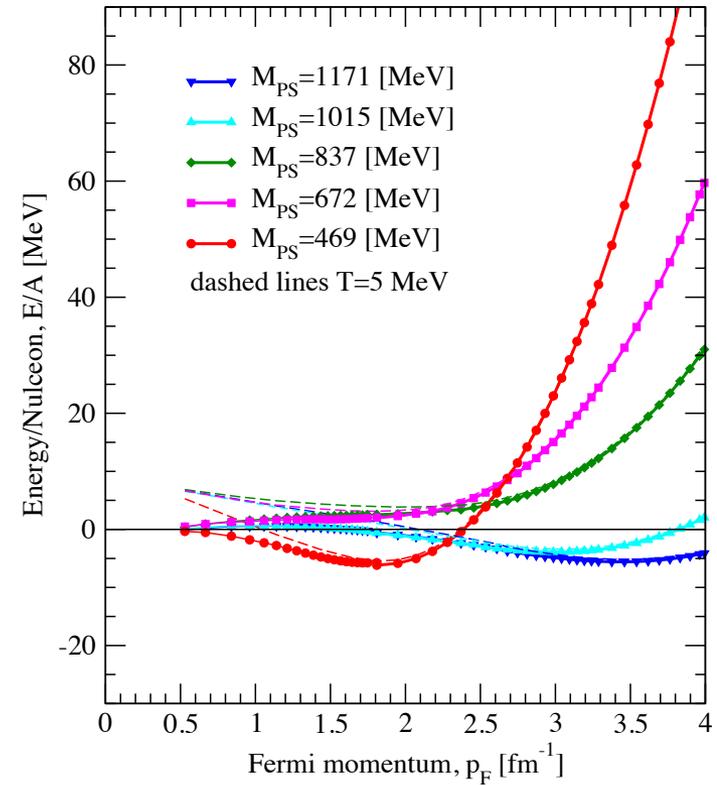
→ *Need to develop appropriate many-body methods*

Two-Nucleon HAL potentials



Quark mass dependence of $V(r)$ for NN partial wave ($^1S_0, ^3S_1, ^3S_1-^3D_1$)

→ Potentials become stronger m_{π} as decreases.

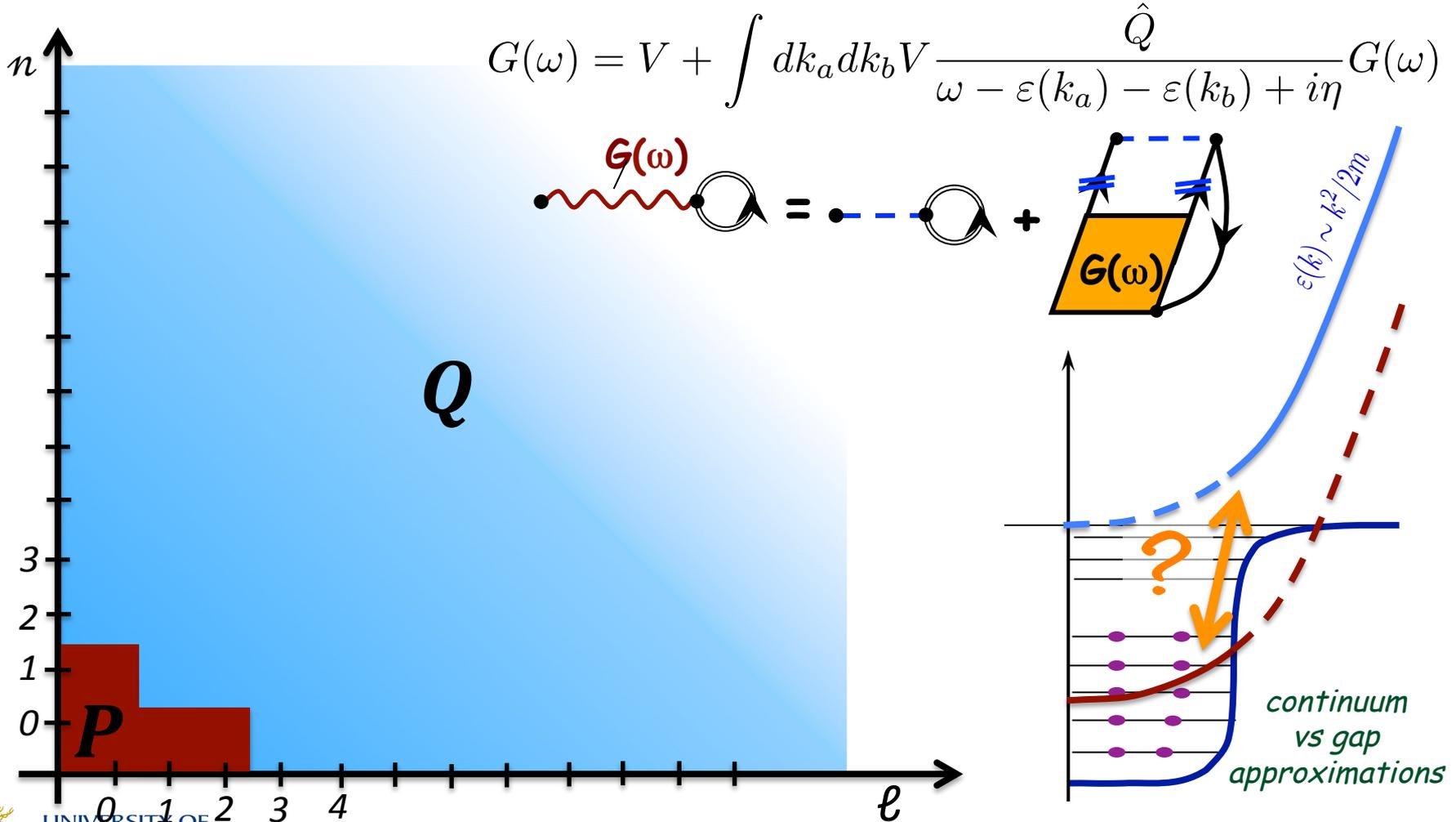


T. Inoue *et al.*,
Phys. Rev. Lett. **111** 112503 (2013).

(Finite- T results by A. Carbone, priv. comm.)

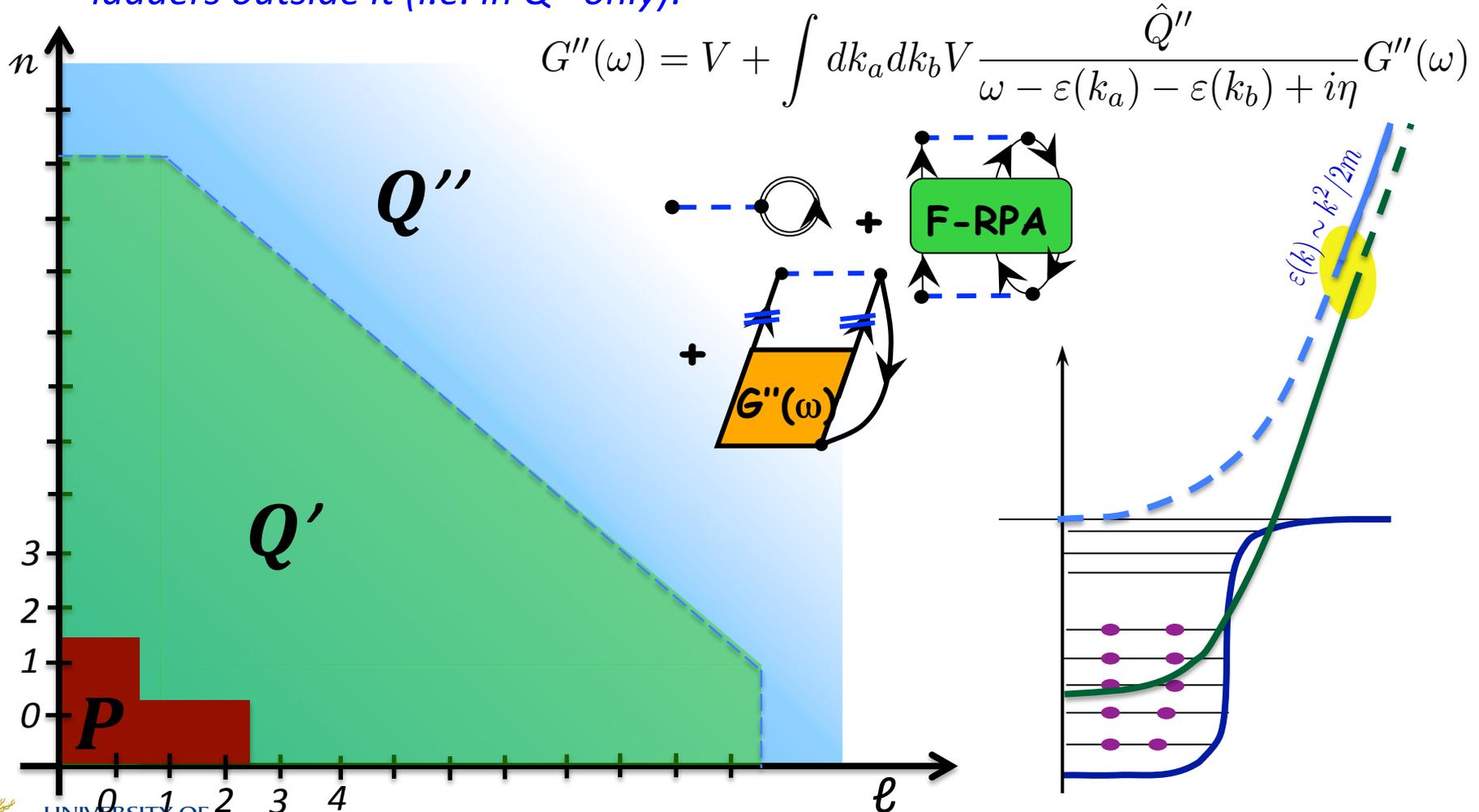
Analysis of Brueckner HF

Scattering of two nucleons outside the Fermi sea (\rightarrow BHF):



Mixed SCGF-Brueckner approach

Solve full many-body dynamics in model space ($P+Q'$) and the Goldstone's ladders outside it (i.e. in Q'' only):



Treating short-range corr. with a G-matrix

- The short-range core can be treated by summing ladders outside the model space:

$$\Sigma_{\alpha\beta}^{MF}(\omega) = i \sum_{\gamma\delta} \int \frac{d\omega'}{2\pi} G_{\alpha\gamma, \delta\beta}(\omega + \omega') g_{\delta\gamma}(\omega') = \text{Diagram}$$

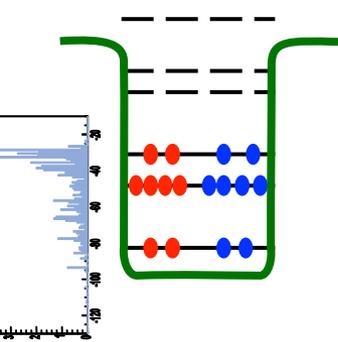
The diagram shows a red wavy line labeled $G(\omega)$ connecting two black dots. The right dot is part of a circular loop with an arrow, representing a self-energy correction.

$$\Sigma^*(\mathbf{r}, \mathbf{r}'; \omega) = \Sigma^{MF}(\mathbf{r}, \mathbf{r}'; \omega) + \tilde{\Sigma}(\mathbf{r}, \mathbf{r}'; \omega).$$

$$Z_\alpha = \int d\mathbf{r} |\psi_\alpha^{A\pm 1}(\mathbf{r})|^2 = \frac{1}{1 - \left. \frac{\partial \Sigma_{\hat{a}\hat{a}}^*(\omega)}{\partial \omega} \right|_{\omega = \pm(E_\alpha^{A\pm 1} - E_0^A)}}$$

Two contributions to the derivative:

- $\Sigma_{\alpha\beta}^{MF}(\omega)$ is due to scattering to (high-k) states in the Q space
- $\Sigma(\mathbf{r}, \mathbf{r}'; \omega)$ accounts for low-energy (long range) correlations



(Galitskii-Migdal-Boffi-) Koltun sumrule

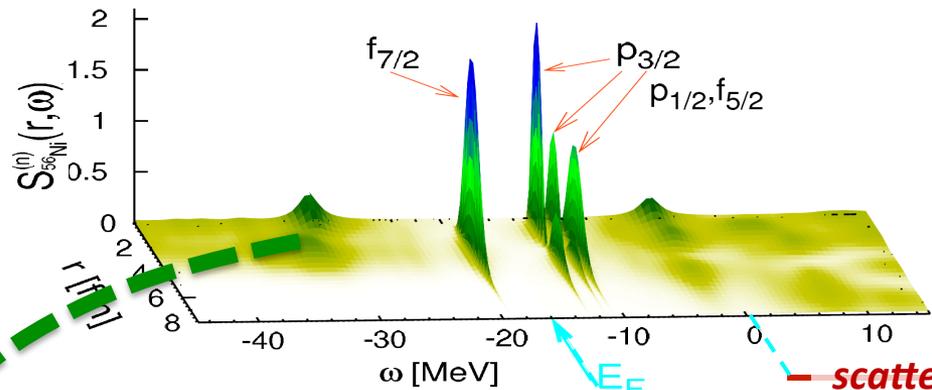
✱ Koltun sum rule (with NNN interactions):

$$\sum_{\alpha} \frac{1}{\pi} \int_{-\infty}^{\epsilon_F^-} d\omega \omega \text{Im} G_{\alpha\alpha}(\omega) = \langle \Psi_0^N | \hat{T} | \Psi_0^N \rangle + 2 \langle \Psi_0^N | \hat{V} | \Psi_0^N \rangle + 3 \langle \Psi_0^N | \hat{W} | \Psi_0^N \rangle$$

two-body

three-body

$$E_0^N = \frac{1}{2\pi} \int_{-\infty}^{\epsilon_F^-} d\omega \sum_{\alpha\beta} (T_{\alpha\beta} + \omega \delta_{\alpha\beta}) \text{Im} G_{\beta\alpha}(\omega) - \frac{1}{2} \langle \Psi_0^N | \hat{W} | \Psi_0^N \rangle$$

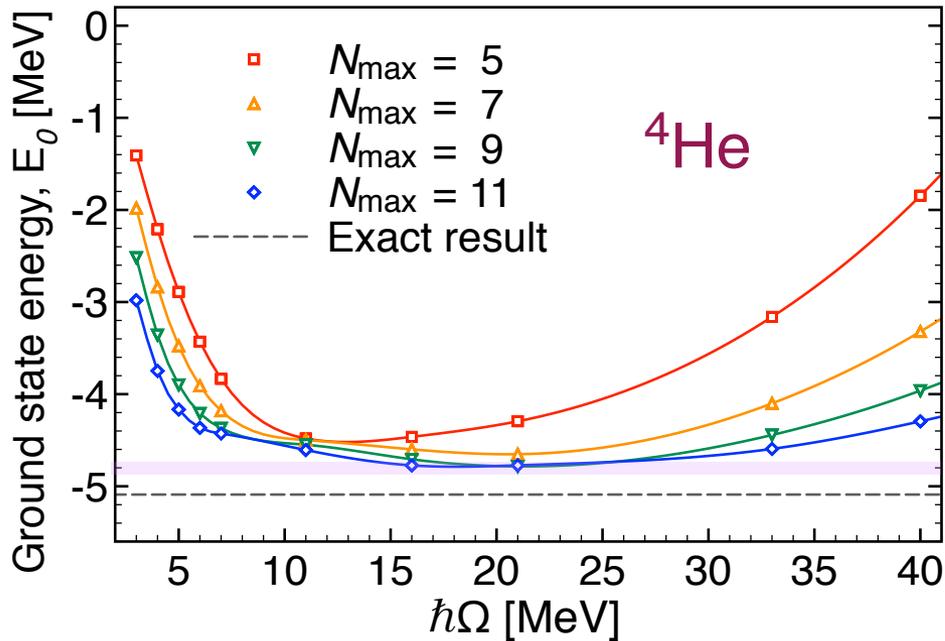


High-k and missing energy tail from SRC... (currently neglected in calculating Koltun SR)

← neutro n removal → scattering → neutro n addition

Benchmark on ${}^4\text{He}$

C. McIlroy, CB, et al., arXiv:1701.02607 [nucl-th]

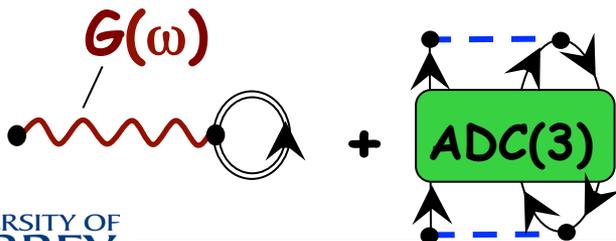


Can benchmark the $G_{\text{mtx}} + \text{ADC}(3)$ method on light ${}^4\text{He}$, where exact solutions are possible:

	$G(\omega) + \text{ADC}(3)$	Exact
HALQCD @ $m_\pi \approx 470 \text{ MeV}$	4.79(3) MeV	5.09 MeV ¹

¹H. Nemura *et al.*, Int. J. Mod. Phys. E **23**, 1461006 (2014)

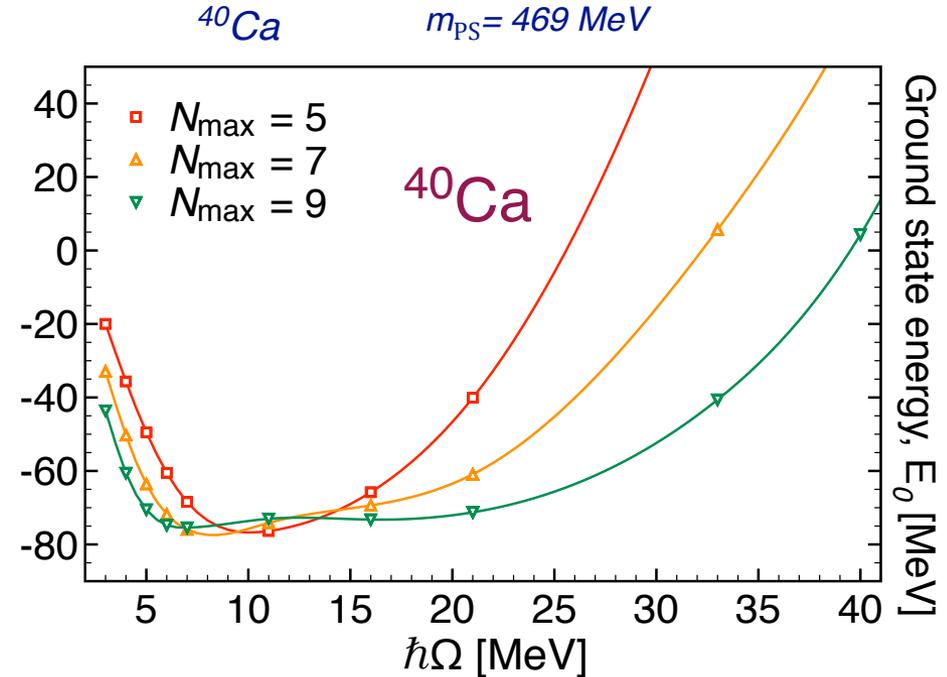
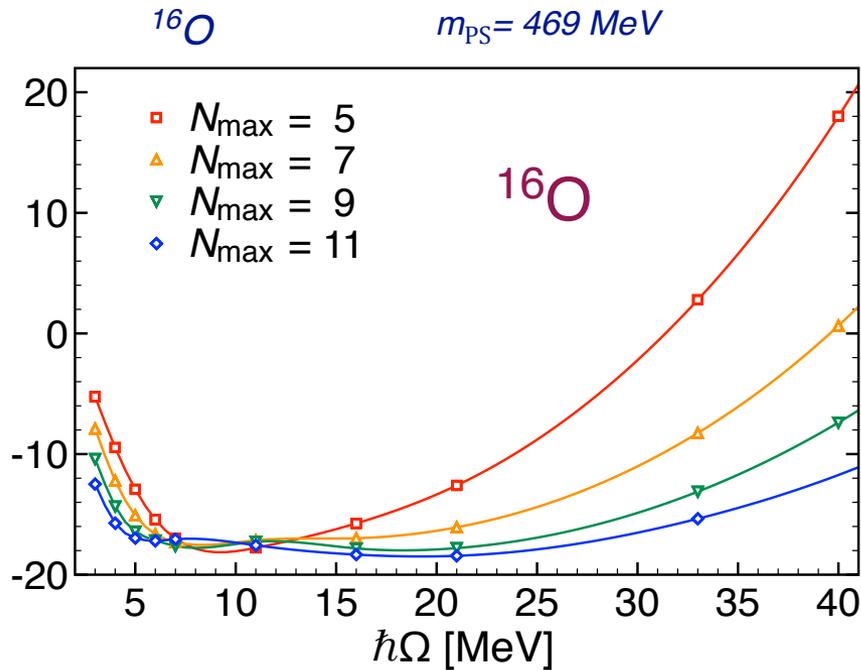
→ Can expect accuracy on binding energies at about 10%



$$G''(\omega) = V + \int dk_a dk_b V \frac{\hat{Q}''}{\omega - \varepsilon(k_a) - \varepsilon(k_b) + i\eta} G''(\omega)$$

Binding of ^{16}O and ^{40}Ca :

C. McIlroy, CB, et al., arXiv:1701.02607 [nucl-th]



Binding energies are $\sim 17 \text{ MeV}$ ^{16}O and $70\text{-}75\text{MeV}$ for ^{40}Ca . Possibly being underestimated by 10%

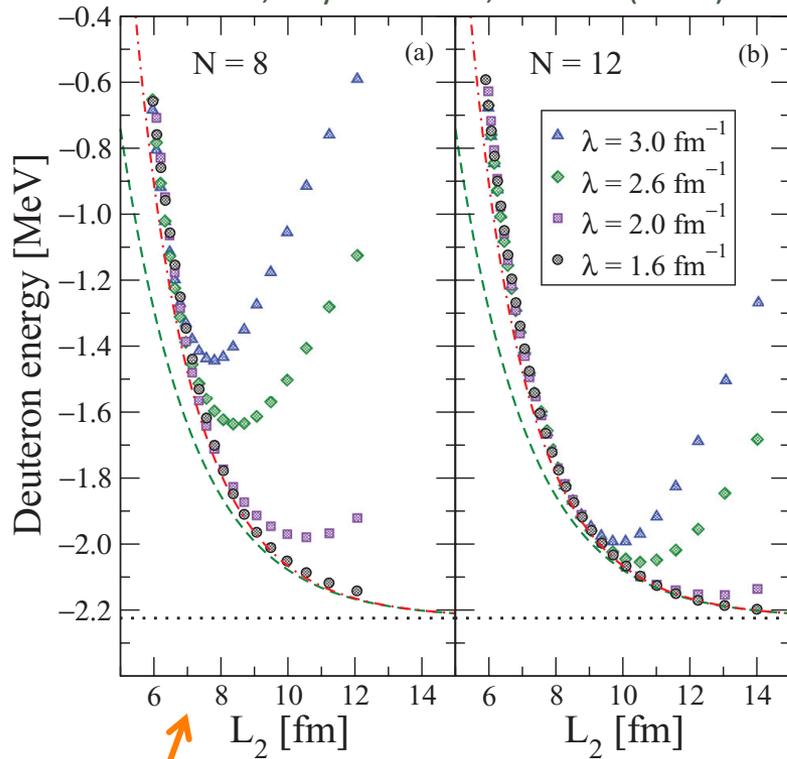
\rightarrow ^{16}O at $m_{\pi} \approx 470 \text{ MeV}$ is unstable toward $4\text{-}\alpha$ breakup!

E_0^A [MeV]	^4He	^{16}O	^{40}Ca
BHF [22]	-8.1	-34.7	-112.7
$G(\omega) + \text{ADC}(3)$	-4.80(0.03)	-17.9 (0.3) (1.8)	-75.4 (6.7) (7.5)
Exact Result [51]	-5.09	-	-
Separation into ^4He clusters:		-2.46 (0.3) (1.8)	24.5 (6.7) (7.5)

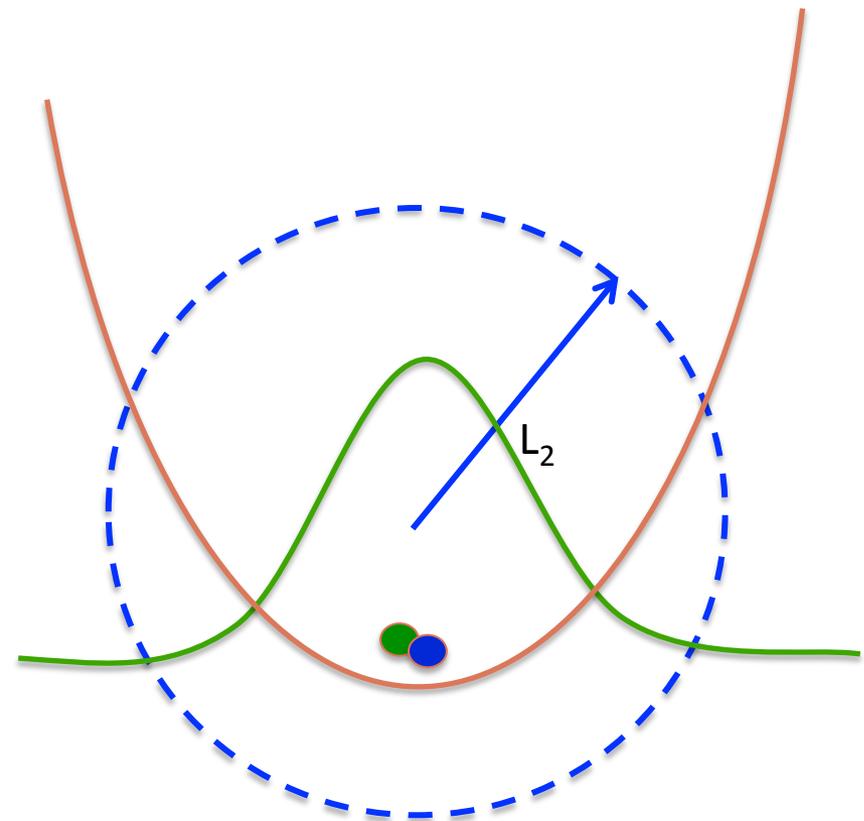
[C.S.McIlroy, CB, HAL coll., in prep]

Infrared convergence

Moore et al., Phys. Rev. C87, 044326 (2013)



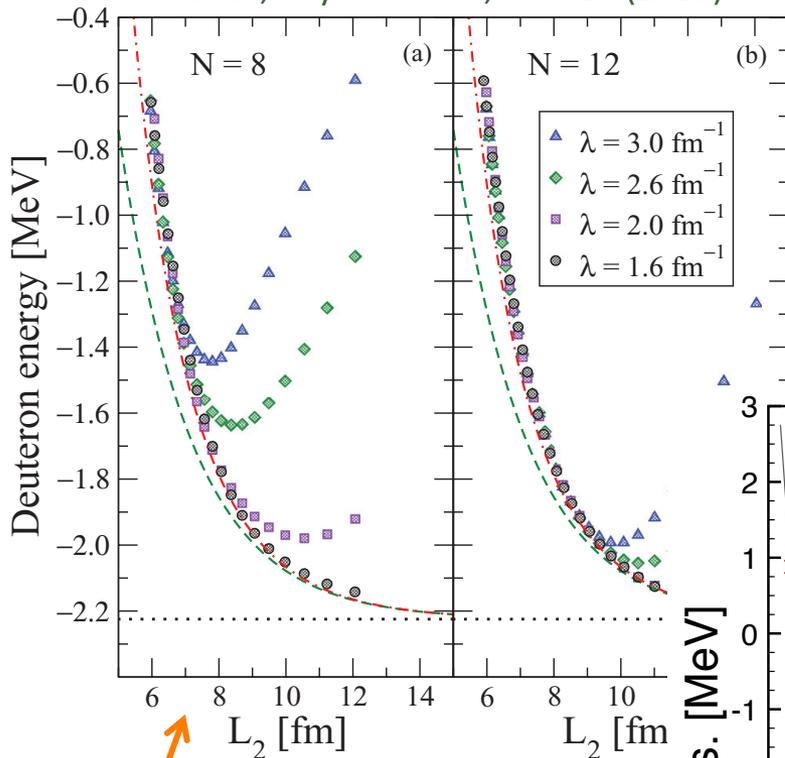
Deuteron g.s. Energy
EM(500) – N3LO two-nucleon force



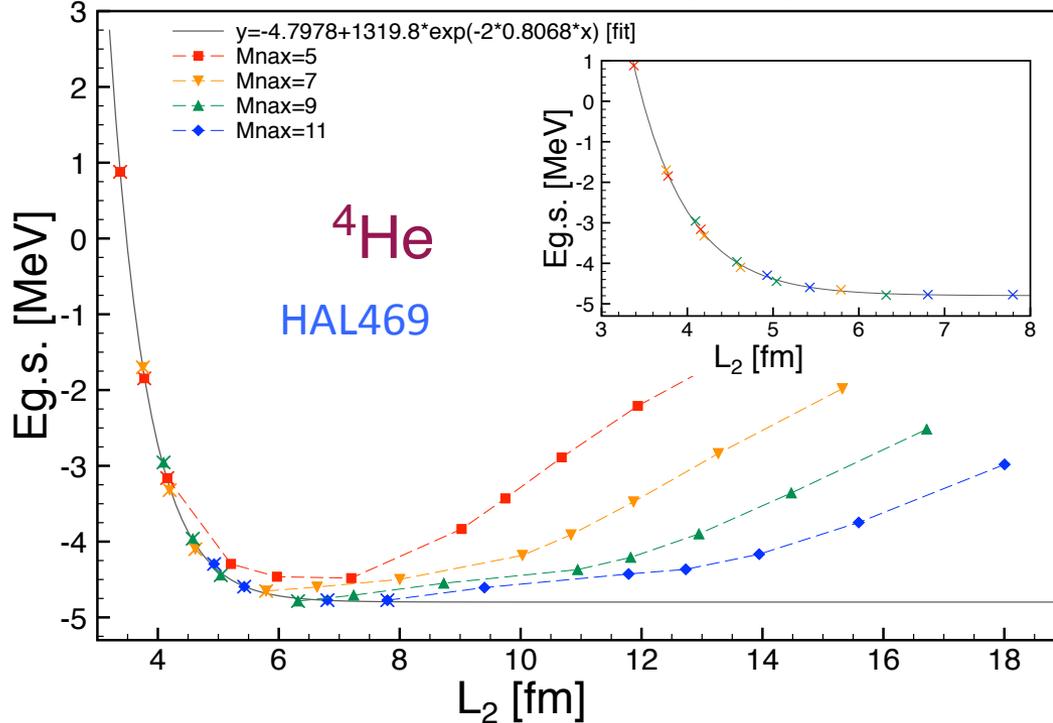
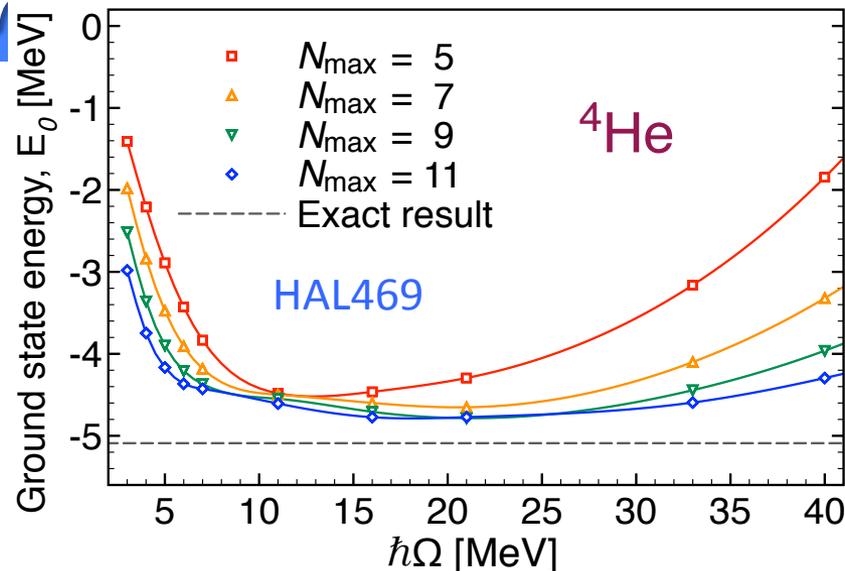
$$L_2 = \sqrt{2(N + 3/2 + 2)}b$$

Infrared convergence

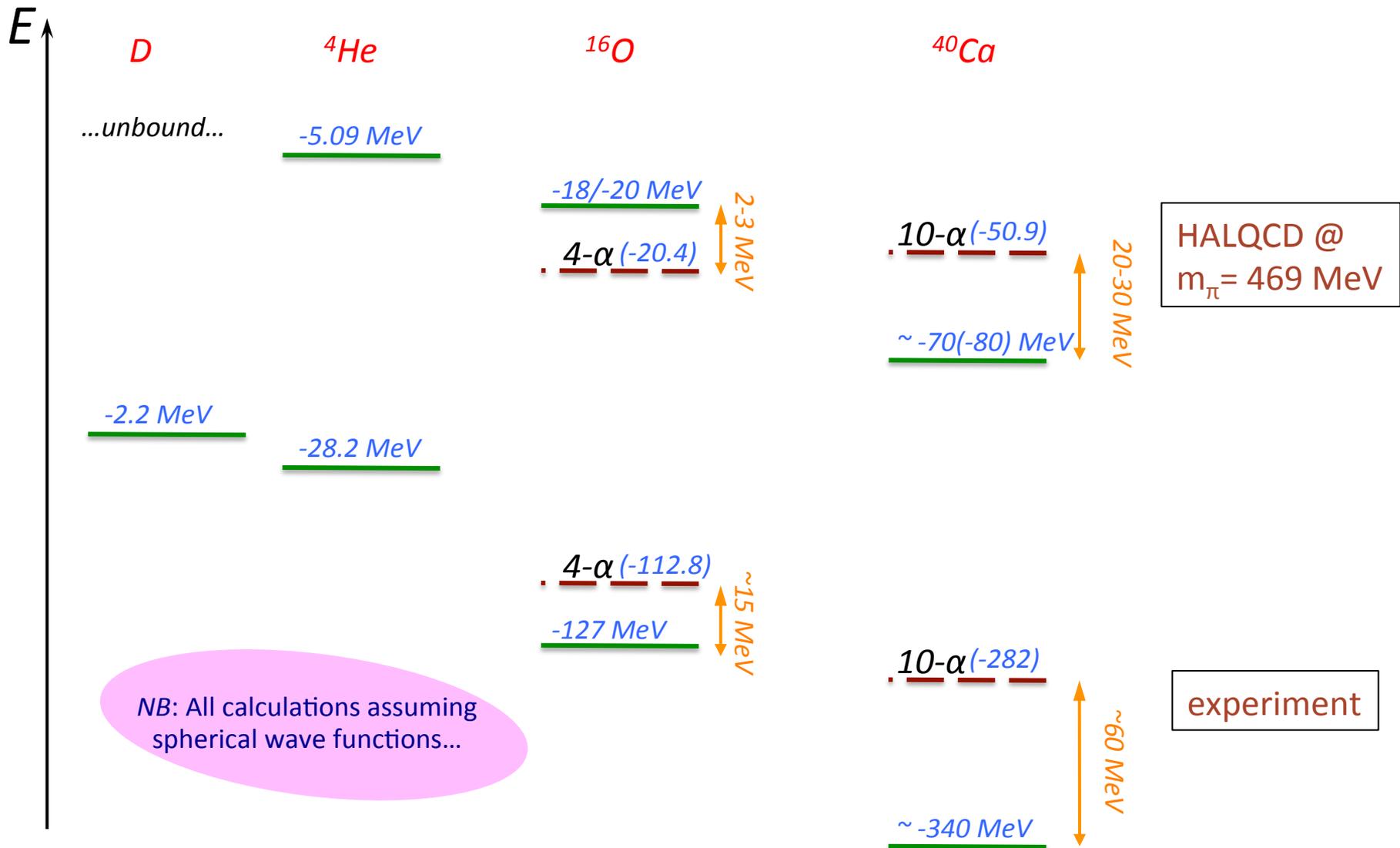
Moore et al., Phys. Rev. C87, 044326 (2013)



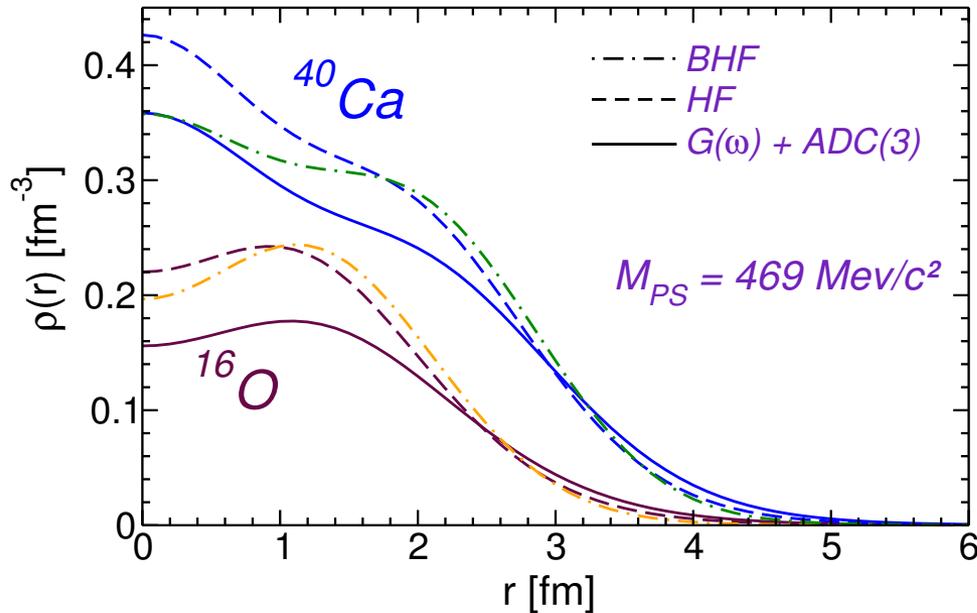
Deuteron g.s. Energy
EM(500) – N3LO two-nuc



Results for binding



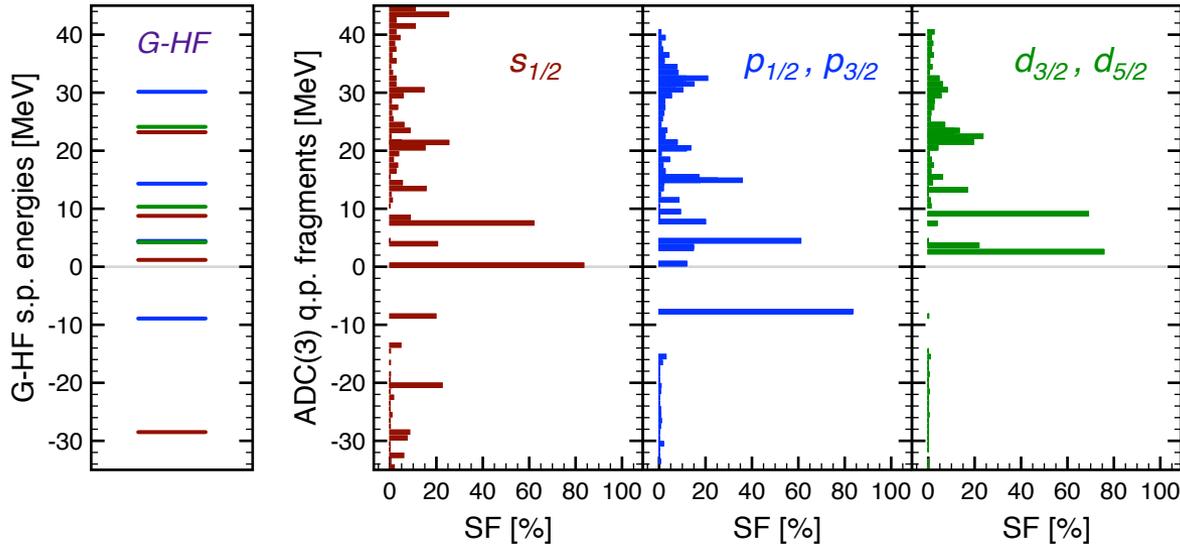
Matter distribution of ^{16}O and ^{40}Ca :



Calculated matter radii at $m_\pi \approx 470 \text{ MeV}$:

		^{16}O	^{40}Ca
$r_{pt\text{-matter}}$:	BHF [22]	2.35 fm	2.78 fm
	HF	2.39 fm	2.78 fm
	$G(\omega) + \text{ADC}(3)$	2.64 fm	2.97 fm
r_{charge} :	$G(\omega) + \text{ADC}(3)$	2.77 fm	3.08 fm
	Experiment [54, 55]	2.73 fm	3.48 fm

Spectral strength in ^{16}O and ^{40}Ca :

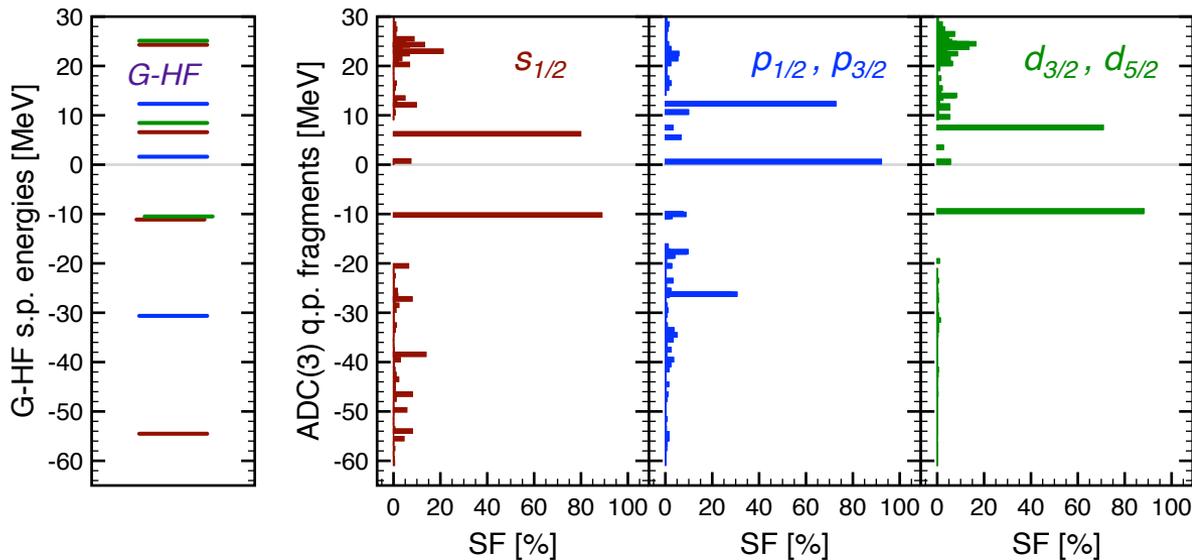


Particle-hole gaps:

^{16}O

$m_\pi = 469$ MeV: ~ 8 MeV

Expt (phys m_π): 11.5 MeV

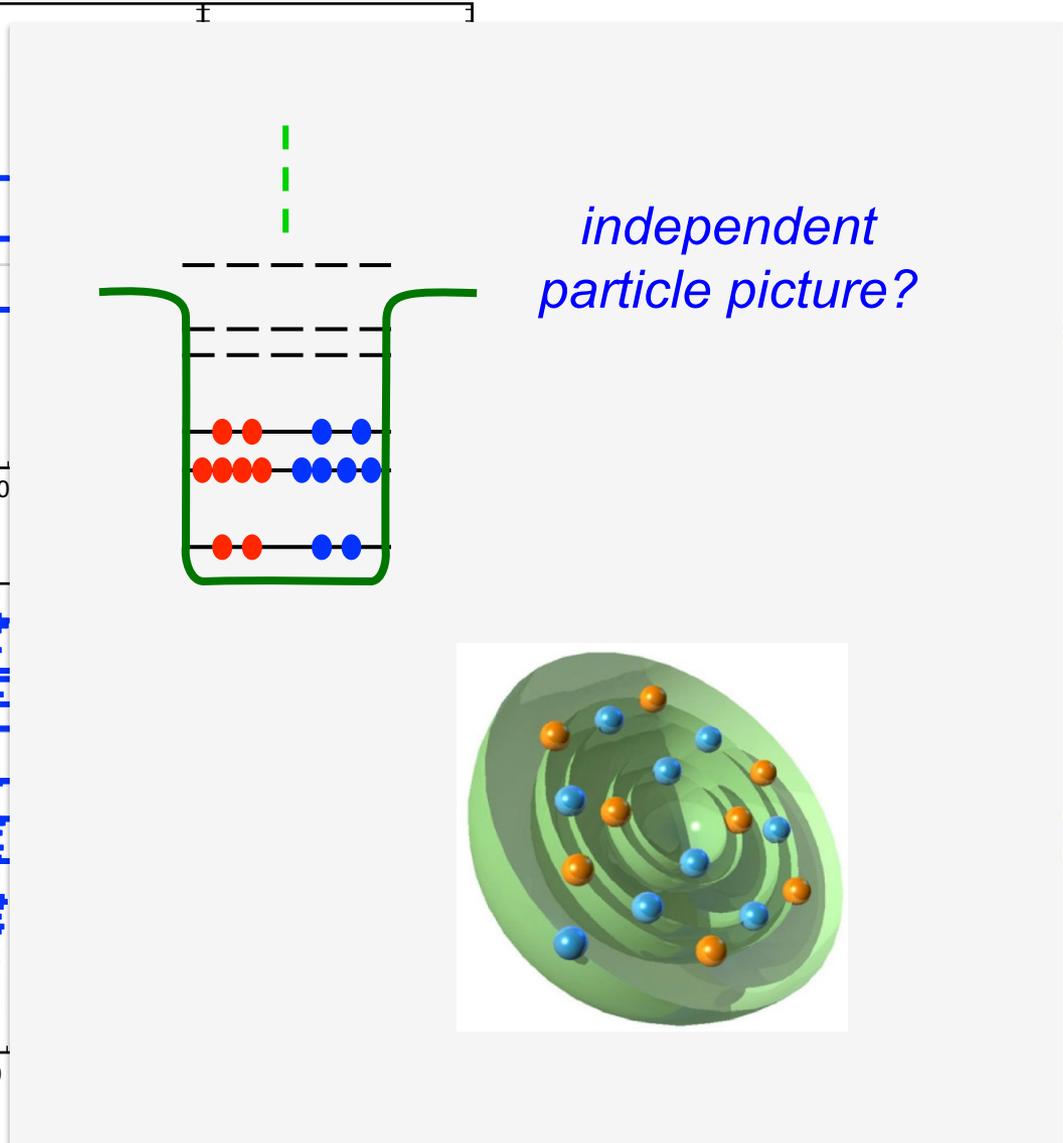
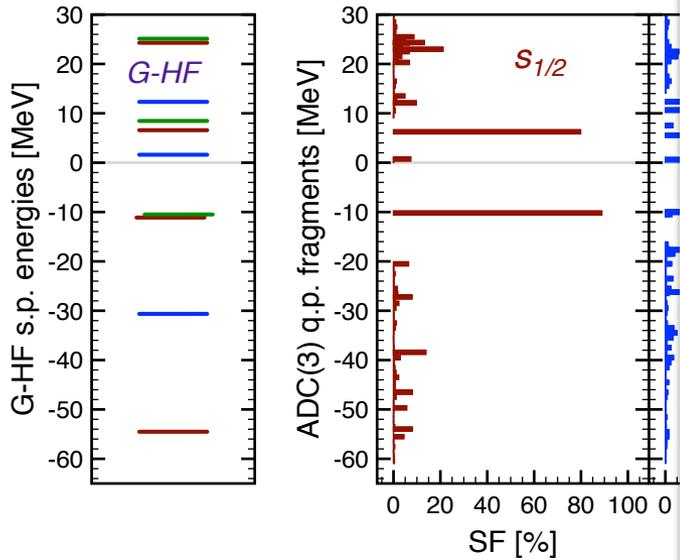
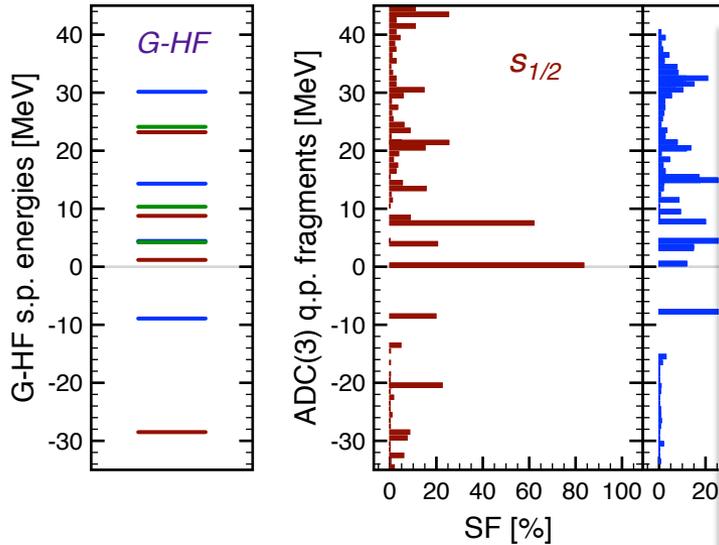


^{40}Ca

$m_\pi = 469$ MeV: ~ 10 MeV

Expt (phys m_π): 7.5 MeV

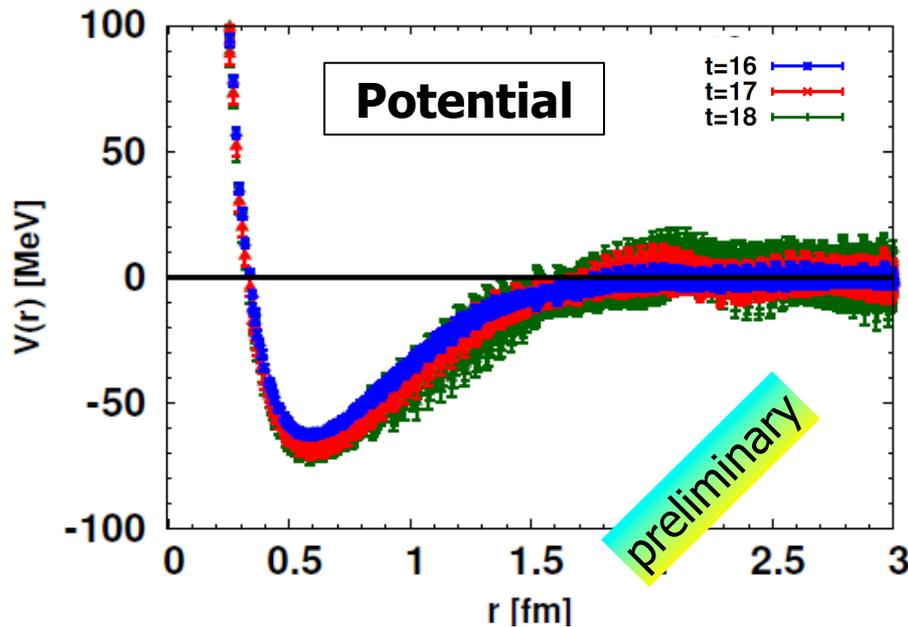
Spectral strength in ^{16}O and ^{40}Ca :



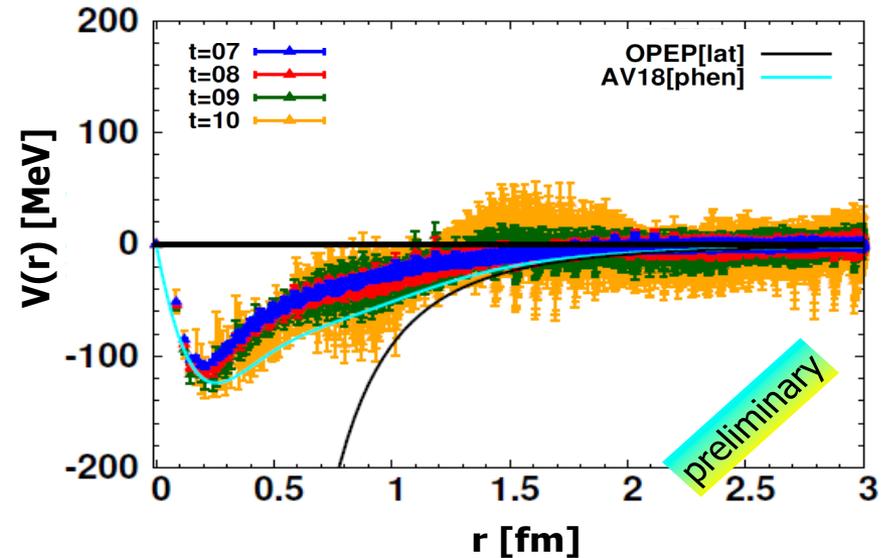
Future application for Y s in nuclei now possible

- Physical mass now under reach ($m_\pi \approx 145$ MeV) for hyperons
- Need to improve on statistic for the NN sector

$\Omega\Omega$ potential



$NN(^3S_1)$ tensor potential



HALQCD coll. -- Talk of **S. Aoki** at Kavli institute, Oct. 2016

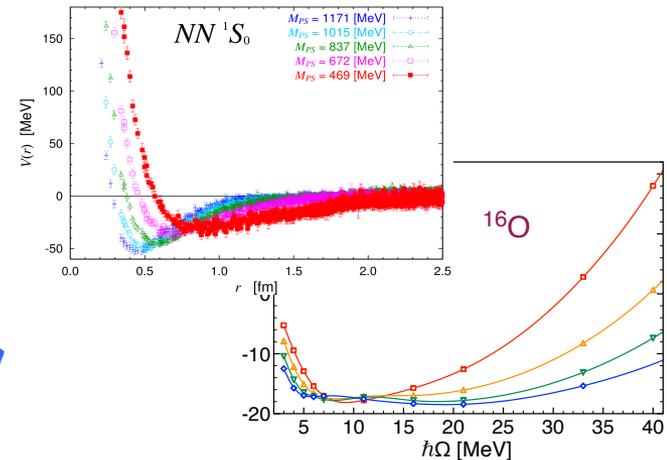
Summary

Mid-masses and chiral interactions:

- Leading order 3NF are crucial to predict many important features that are observed experimentally (drip lines, saturation, orbit evolution, etc...)
- Experimental binding is predicted accurately up to the lower *sd* shell ($A \approx 30$) but deteriorates for medium mass isotopes (Ca and above) with roughly 1 MeV/A over binding.
- New fits of chiral interaction are promising for low-energy observables and for scattering (see A. Idini, next).

HALQCD Nuclear forces:

- Strong short range behavior calls for new ideas in *ab-initio* many-body methods. Diagram resummation through *G*-matrix is good starting point (to be extended)
- At $m_\pi = 469$ MeV, closed shell 4He, 16O and 40Ca are bound. But oxygen is unstable toward 4- α break up, calcium stays bound. Underestimation of radii increases with *A* do to large saturation density (as for EM(500)+NLO3NF).



Thank you for your attention!!!

Collaborators



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energie atomique • energies alternatives



A. Carbone



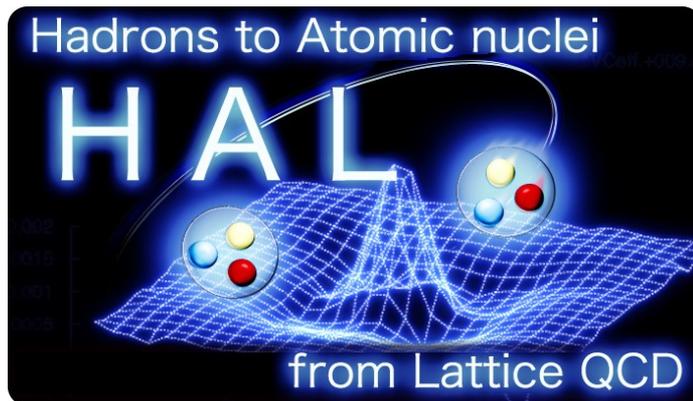
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YITP Kyoto Univ.