

Two-body densities from NCSM/SRG: What can they tell us?

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TRIUMF, Vancouver, Canada

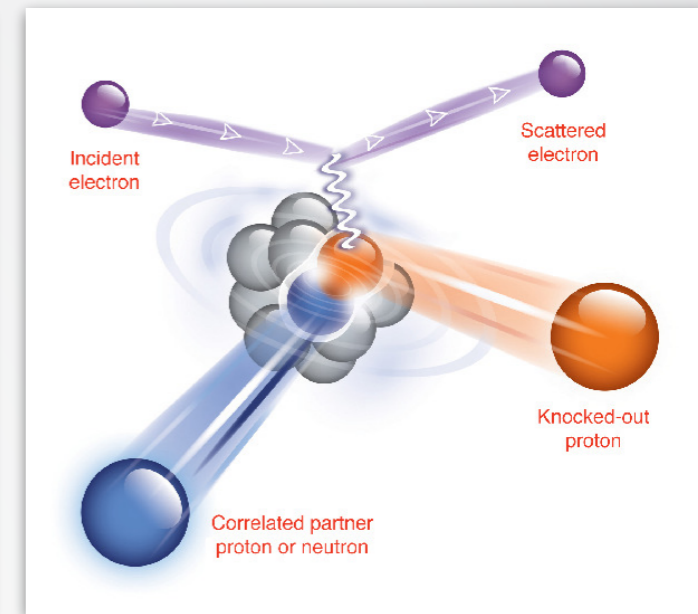
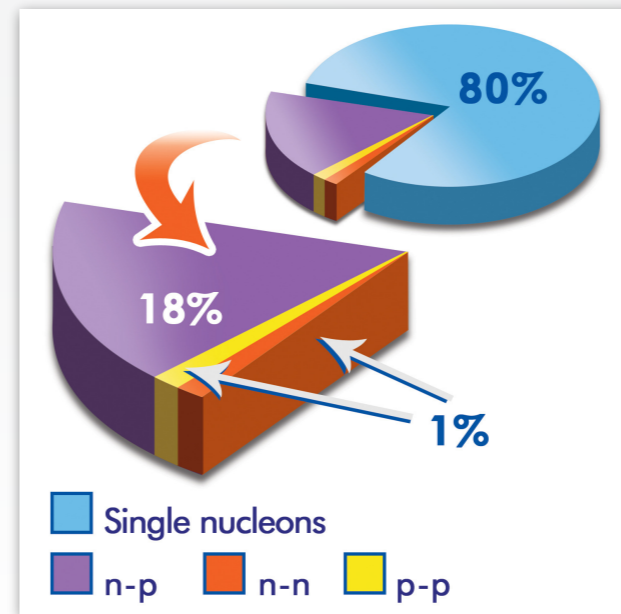


Short-range Correlations/Tensor Correlations

Observations

- JLab experiments found that a knocked out high-momentum proton is accompanied by a second nucleon with opposite momentum
- Cross sections for $(e, e'pn)$ and $(e, e'pp)$ reactions show strong dominance of pn -over pp -pairs

Subedi *et al.*, *Science* **320**, 1476 (2008), ...

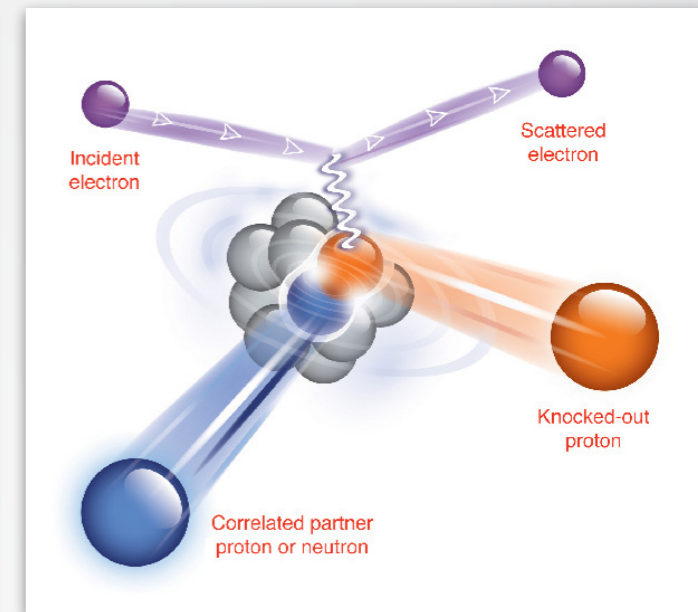
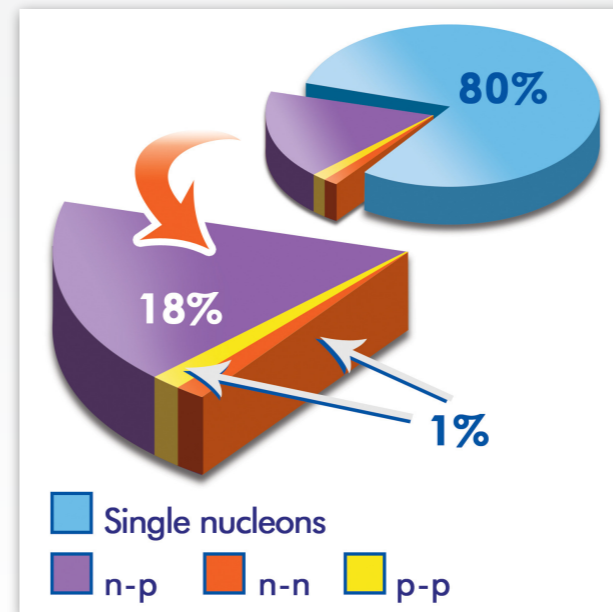


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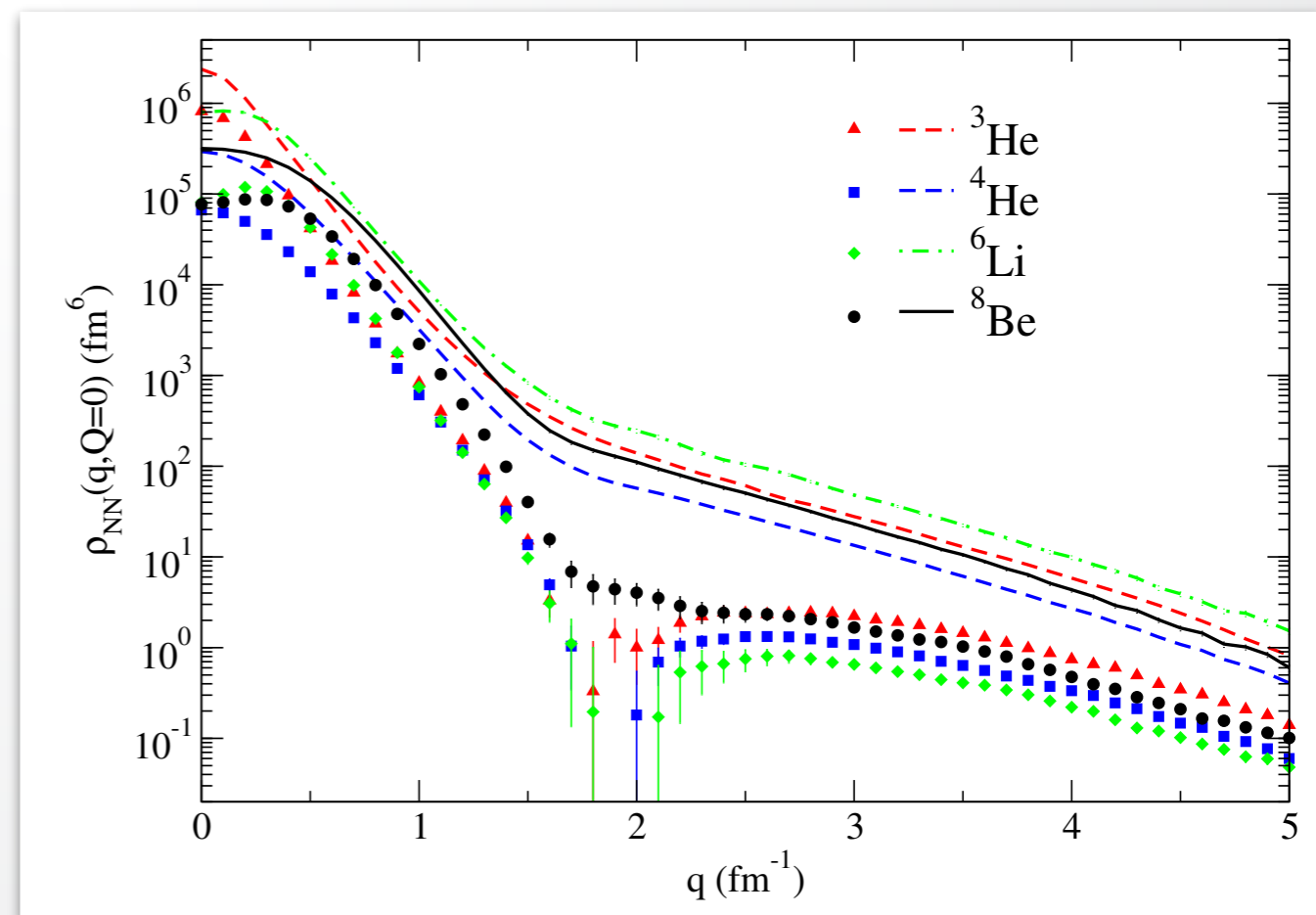


Theoretical interpretation

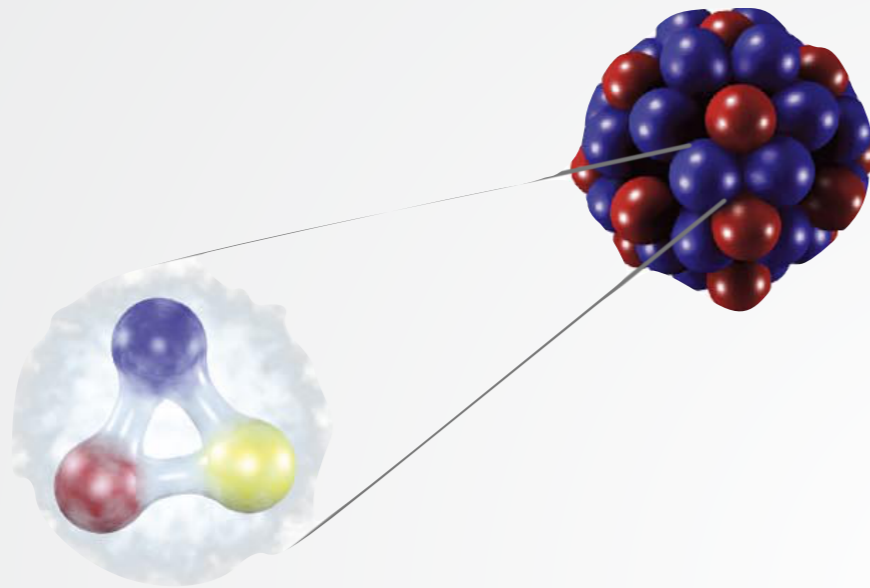
- *ab initio* calculations with Argonne interactions show high-momentum components
- dominance of pn - over pp -pairs due to the tensor force

Wiringa, Schiavilla, Pieper, Carlson, *PRC* **85**, 021001(R) (2008)

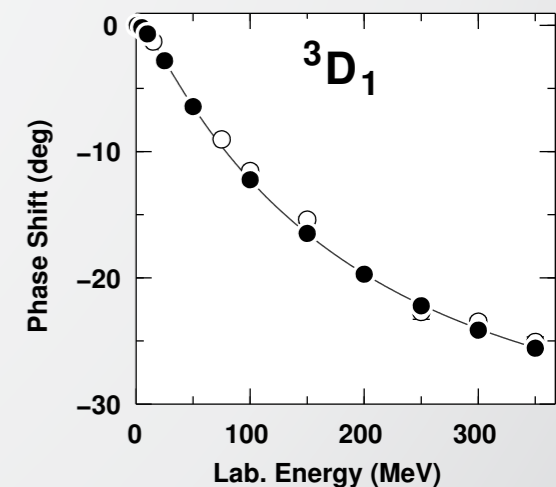
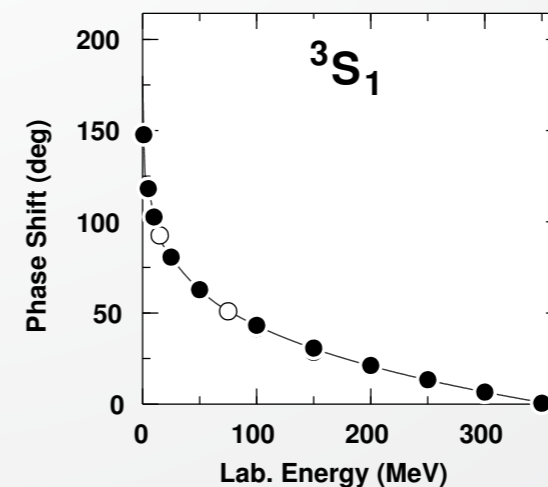
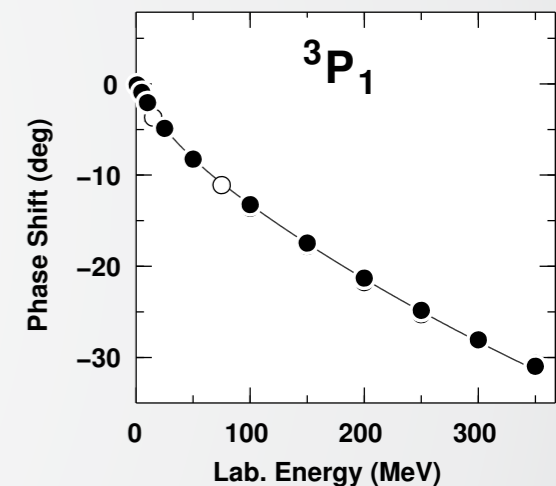
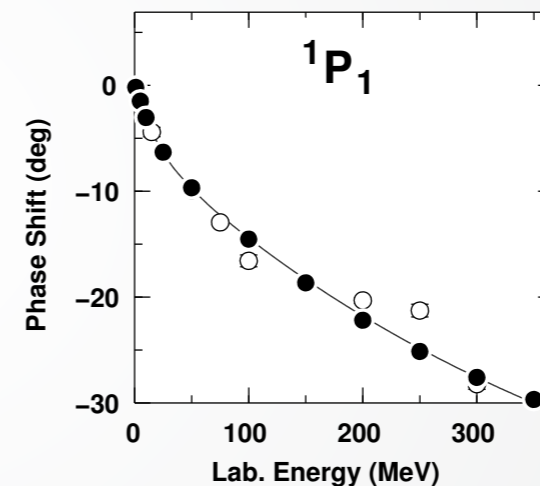
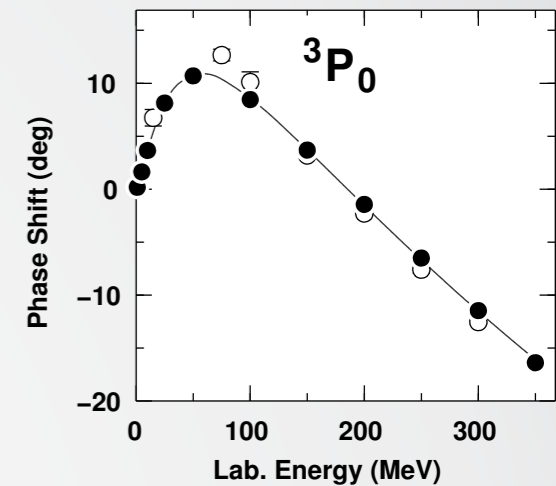
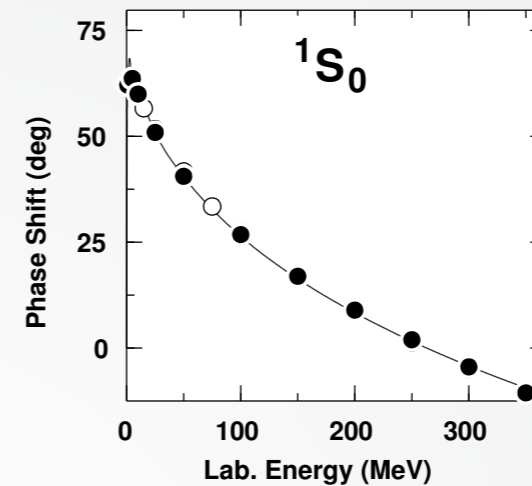
Alvioli *et al.*, *Int. J. Mod. Phys. E* **22**, 1330021 (2013), ...



Nucleon-Nucleon Interaction



- Nucleons are not point-like, complicated quark and gluon sub-structure
- Nucleon-nucleon (NN) interaction: residual interaction
- Calculation within QCD not possible yet
→ construct **realistic NN potentials** ...
- describe two-nucleon properties (scattering, deuteron) with high accuracy
- high-momentum and off-shell behavior not constrained by scattering data



Nucleon-Nucleon Interaction

Argonne V18/V8'

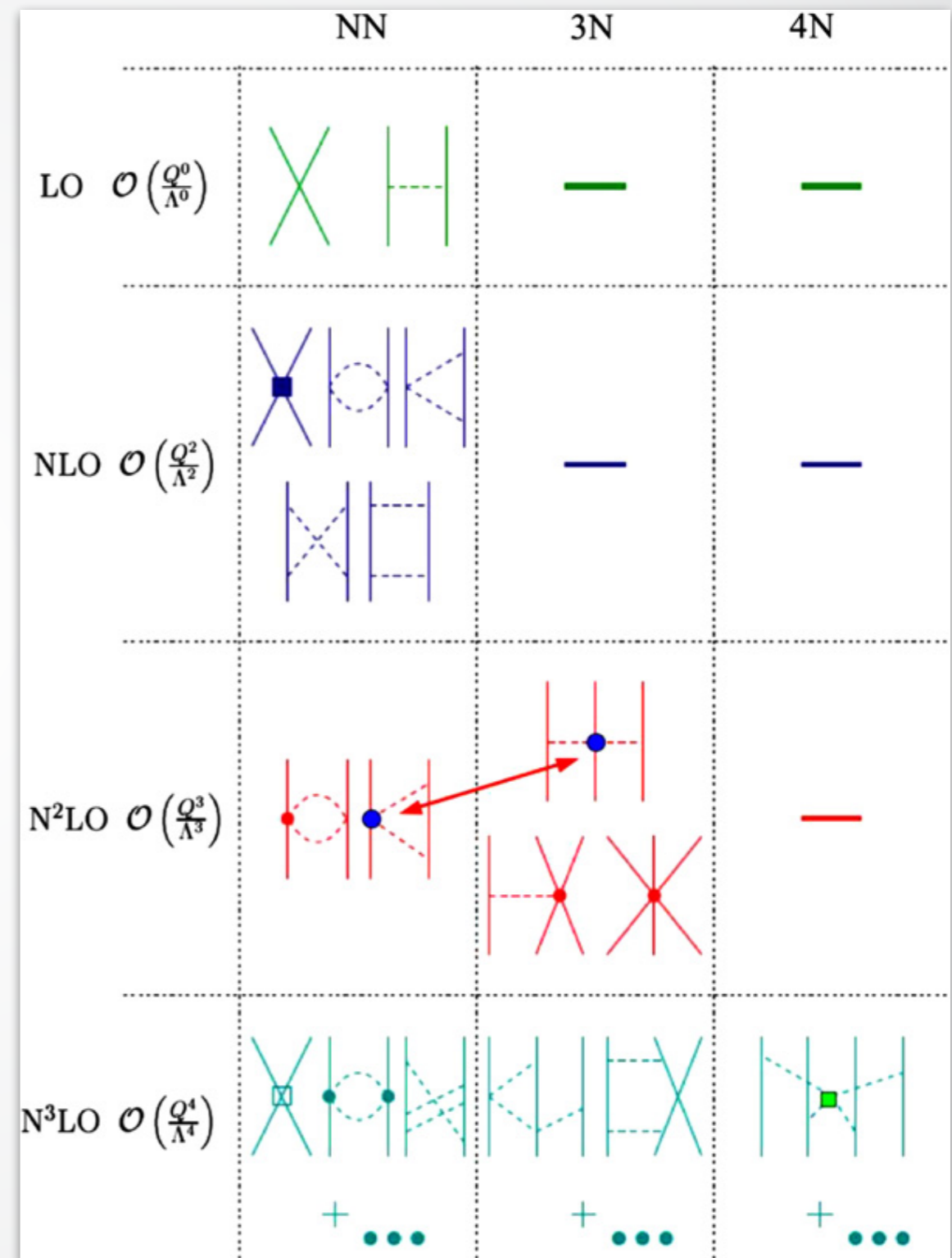
- π -exchange, phenomenological short-range
- "as local as possible"
- fitted to phase shifts up to 350 MeV, but describes elastic phase shifts up to 1 GeV

Wiringa, Stoks, Schiavilla, Phys. Rev. C **51**, 38 (1995)

N³LO

- potential derived using chiral EFT
- includes full π dynamics
- short-range behavior given by contact-terms
- power counting
- regulated by non-local cut-off (500 MeV)

Entem, Machleidt, Phys. Rev. C **68**, 041001 (2003)



Bogner, Furnstahl, Schwenk, Prog. Part. Nucl. Phys. **65**, 94 (2010)

Nucleon-Nucleon Interaction

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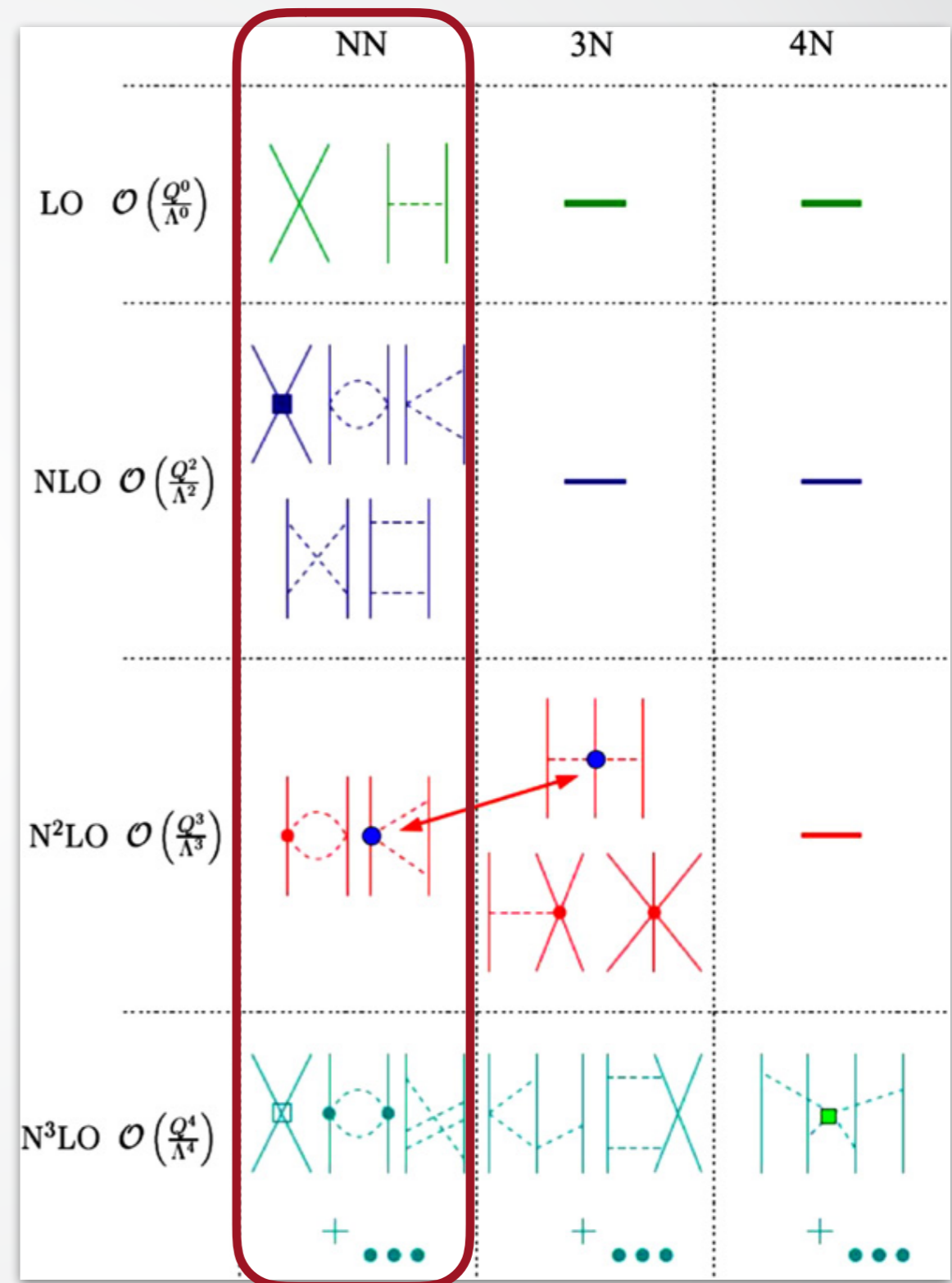
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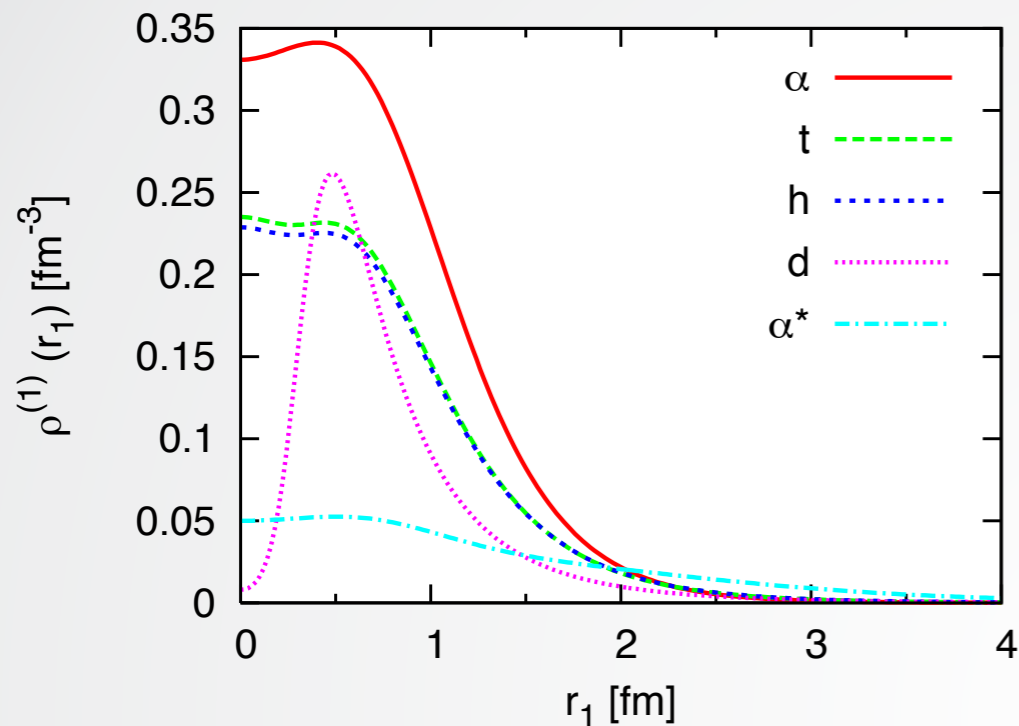
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Universality of short-range correlations

Exact solutions for light nuclei with AV8' interaction

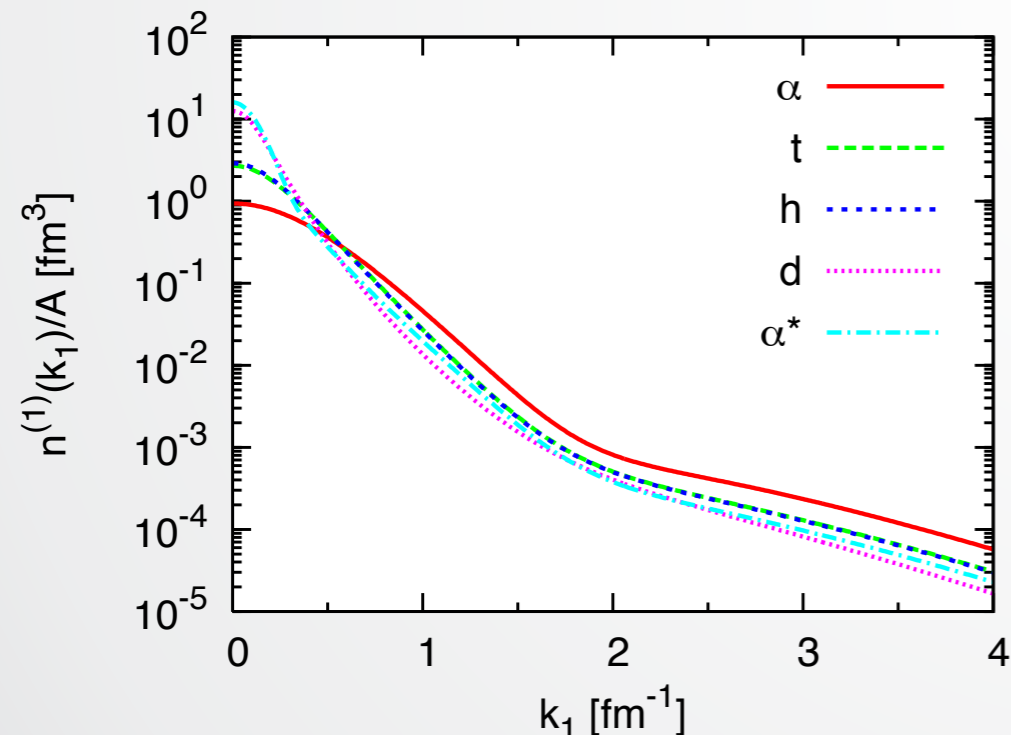
Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

One-body densities for $A=2,3,4$ nuclei



$$\rho^{(1)}(\mathbf{r}_1) = \langle \Psi | \sum_{i=1}^A \delta^3(\hat{\mathbf{r}}_i - \mathbf{r}_1) | \Psi \rangle$$

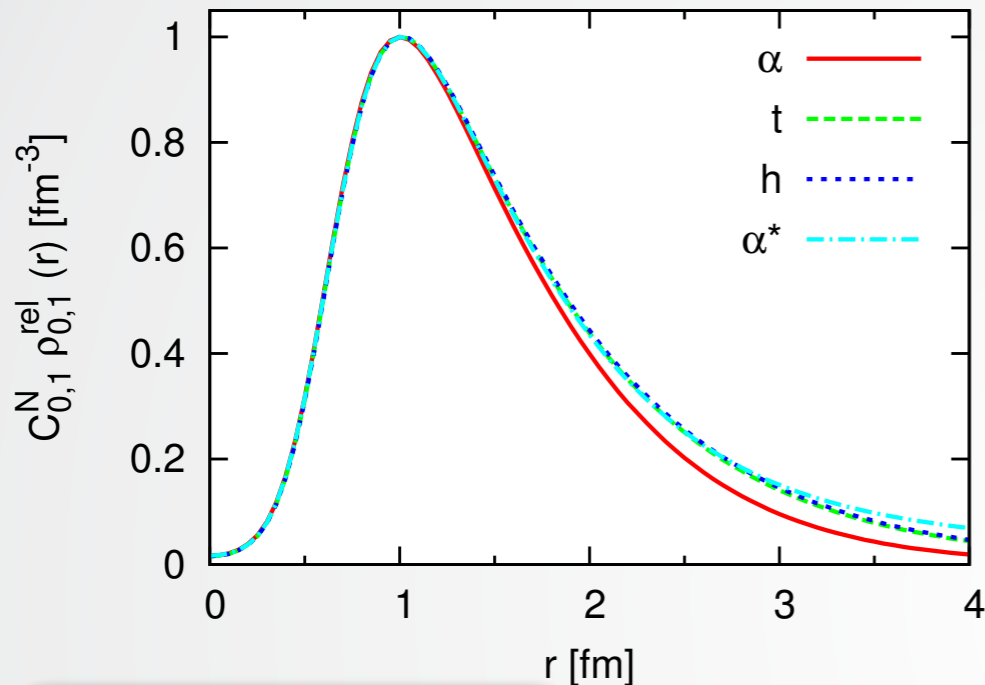
$$n^{(1)}(\mathbf{k}_1) = \langle \Psi | \sum_{i=1}^A \delta^3(\hat{\mathbf{k}}_i - \mathbf{k}_1) | \Psi \rangle$$



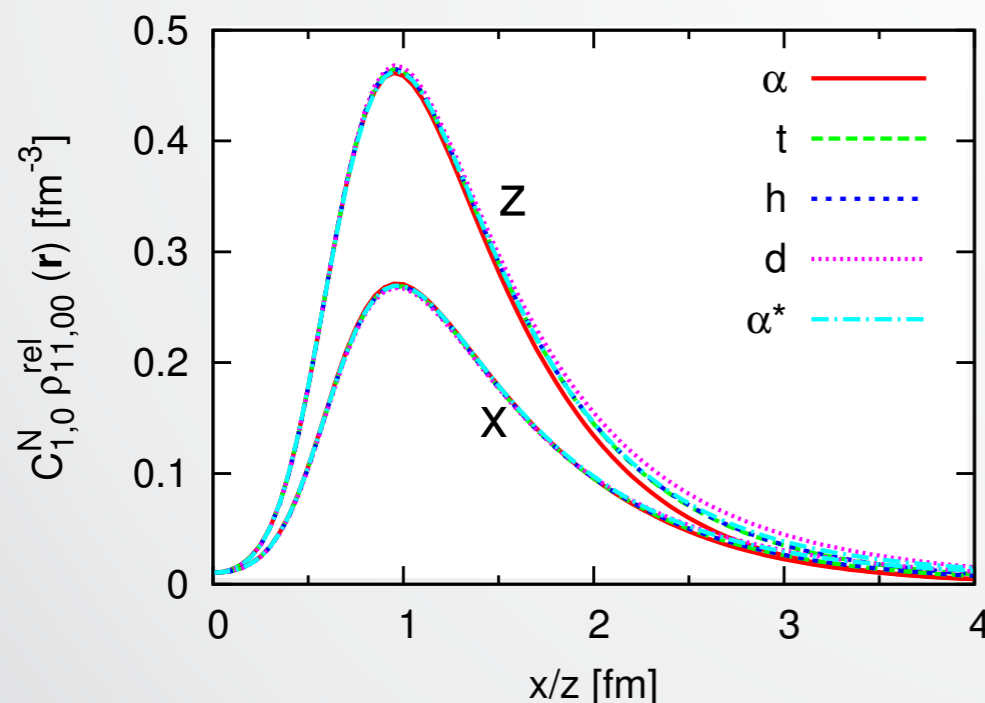
- One-body densities calculated from **exact wave functions** (Correlated Gaussian method) for AV8' interaction
- coordinate space densities reflect different sizes and densities of ${}^2\text{H}$, ${}^3\text{H}$, ${}^3\text{He}$, ${}^4\text{He}$ and the 0_2^+ state in ${}^4\text{He}$
- similar high-momentum tails in the one-body momentum distributions

Two-body Coordinate Space Densities

$S=0, T=1$



$S=1, M_S=+1, T=0$



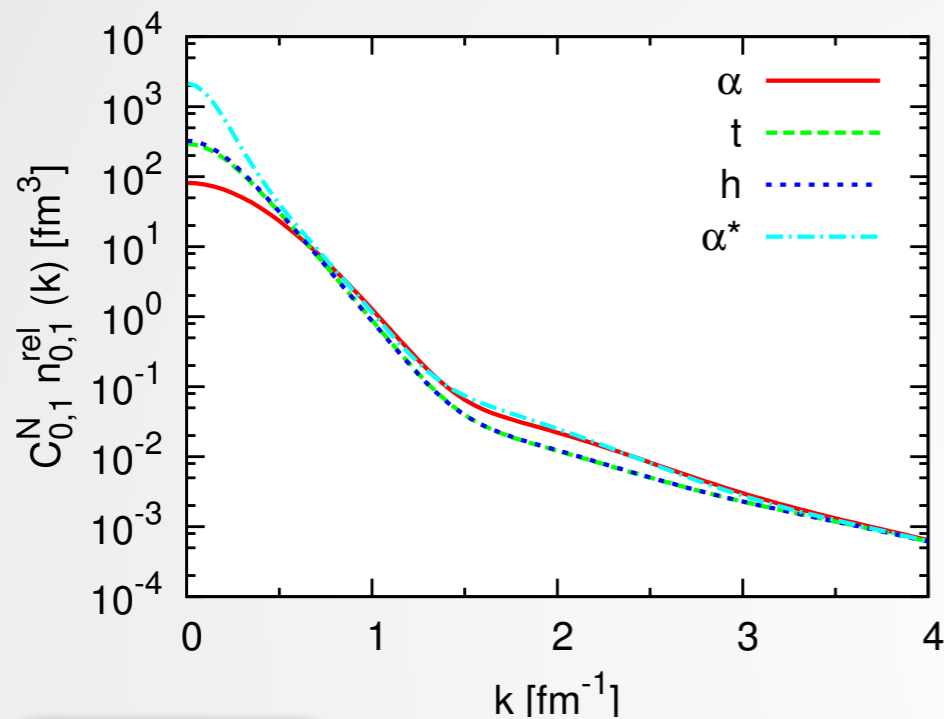
$$\rho_{SM_S, TM_T}^{\text{rel}}(\mathbf{r}) = \langle \Psi | \sum_{i < j}^A \hat{p}_{ij}^{SM_S} \hat{p}_{ij}^{TM_T} \delta^3(\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_j - \mathbf{r}) | \Psi \rangle$$

- two-body densities calculated from **exact wave functions** (Correlated Gaussian Method) for AV8' interaction
- coordinate space two-body densities reflect correlation hole and tensor correlations
- → normalize two-body density in coordinate space at $r=1.0$ fm
- → normalized two-body densities in coordinate space are **identical at short distances** for all nuclei
- also true for angular dependence in the deuteron channel

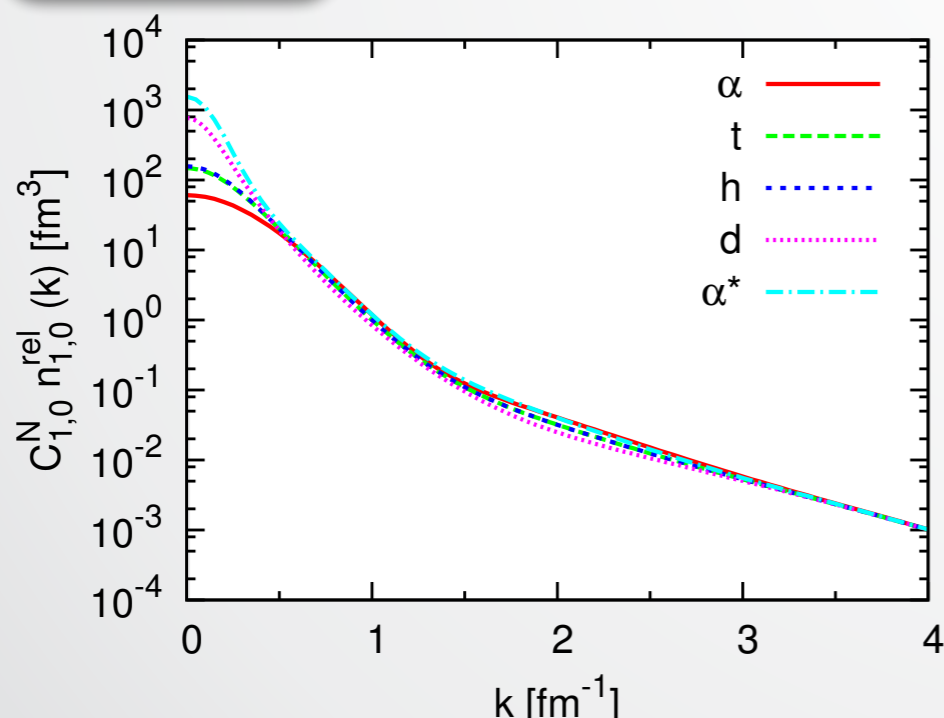
Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

Two-body Momentum Space Densities

$S=0, T=1$



$S=1, T=0$



$$n_{SM_S, TM_T}^{\text{rel}}(\mathbf{k}) = \langle \Psi | \sum_{i < j} \hat{p}_{ij}^{SM_S} \hat{p}_{ij}^{TM_T} \delta^3\left(\frac{1}{2}(\hat{\mathbf{k}}_i - \hat{\mathbf{k}}_j) - \mathbf{k}\right) | \Psi \rangle$$

- use normalization factors fixed in coordinate space
- two-body densities in momentum space agree for momenta $k > 3 \text{ fm}^{-1}$
- moderate nucleus dependence in momentum region $1.5 \text{ fm}^{-1} < k < 3 \text{ fm}^{-1}$

Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

Two-body Densities and Many-body Correlations

number of pairs in ST channels

	(00)	(01)	(10)	(11)
d	-	-	1	-
t	0.010	1.361	1.490	0.139
h	0.011	1.361	1.489	0.139
α	0.008	2.572	2.992	0.428
α^*	0.034	2.714	2.966	0.286

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Thomas Neff | Theory Workshop | Feb. 26, 2016 | TRIUMF, Vancouver

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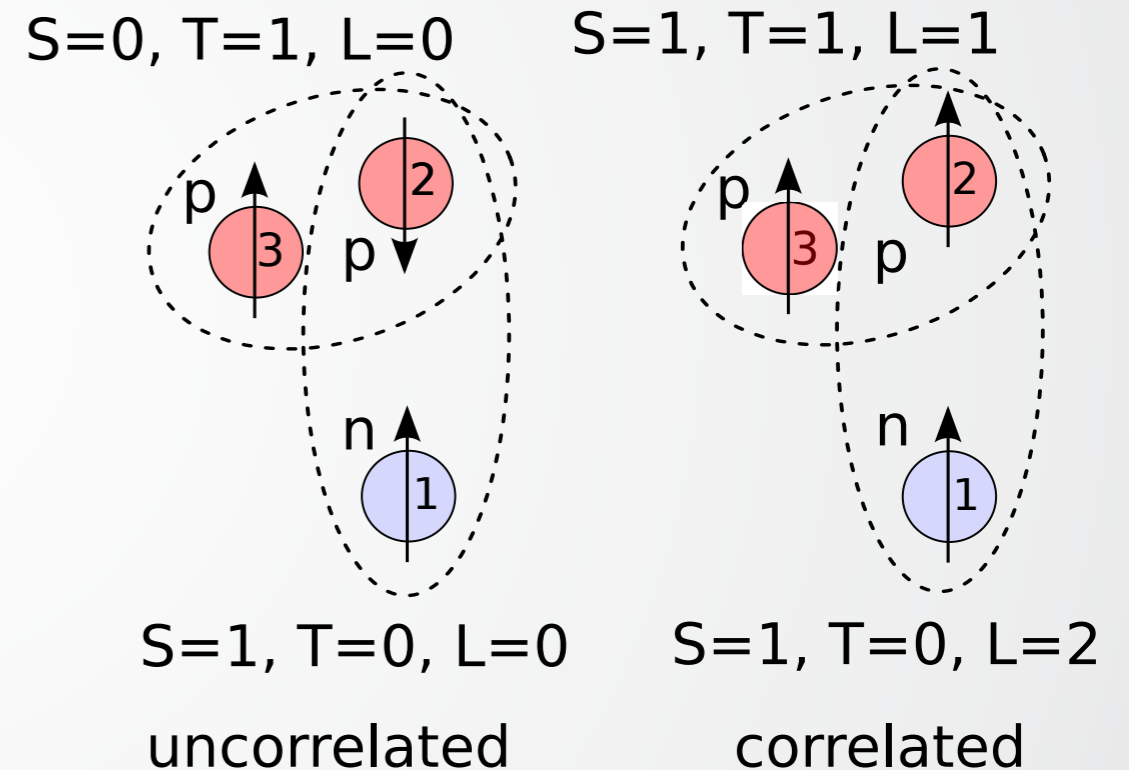
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- occupation in $(ST)=(10)$ channel almost exactly as in IPM
- $(ST)=(01)$ significantly depopulated in favor of $(ST)=(11)$ channel
- three-body correlations induced by the two-body tensor force: depopulation of $(ST)=(01)$ channel is the price one has to pay for getting the full binding from the tensor force in the $(ST)=(10)$ channel

Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

Short-range correlations studied with NCSM and SRG

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

Neff, Feldmeier, *in preparation*

Unitary Transformations

- Many-body problem very hard to solve with bare interaction
- Universality of SRC suggests to use unitary transformations to obtain a “soft” realistic interaction

$$\hat{H}_{eff} = \hat{U}^\dagger \hat{H} \hat{U}$$

- **The transformation is done in N -body approximation**

$$\hat{H}_{eff} = \hat{T} + \hat{V}_{eff}^{[2]} + \dots + \hat{V}_{eff}^{[N]}$$

and is therefore unitary only up to the N -body level

- Deuteron binding energy and NN phase shifts are conserved
- **Not only the Hamiltonian, all operators including density operators have to be transformed**

$$\hat{B}_{eff} = \hat{U}^\dagger \hat{B} \hat{U}$$

- SRG operator evolution studied for Deuteron

Anderson, Bogner, Furnstahl, Perry, Phys. Rev. C **82**, 054001 (2010)

- SRG operator evolution for radius and Gaussian two-body operator on 3-body level

Schuster, Quaglioni, Johnson, Jurgenson, Navrátil, Phys. Rev. C **90**, 011301 (2014)

Similarity Renormalization Group

- SRG provides a family of similarity transformations depending on a flow parameter α
- Evolve Hamiltonian and unitary transformation matrix (momentum space)

$$\frac{d\hat{H}_\alpha}{d\alpha} = [\hat{\eta}_\alpha, \hat{H}_\alpha]_-, \quad \frac{d\hat{U}_\alpha}{d\alpha} = -\hat{U}_\alpha \hat{\eta}_\alpha$$

- Intrinsic kinetic energy as metagenerator

$$\hat{\eta}_\alpha = (2\mu)^2 [\hat{T}_{\text{int}}, \hat{H}_\alpha]_-$$

- Evolution is done here on the **2-body level** – α -dependence can be used to investigate the role of missing higher-order contributions
- Hamiltonian evolution can now be done on the 3-body level

(Jurgenson, Roth, Hebeler, . . .)

Bogner, Furnstahl, Perry, Phys. Rev. C, **75**, 061001 (2007)

Roth, Neff, Feldmeier, Prog. Part. Nucl. Phys. **65**, 50 (2010)

$\alpha=0$:
fully correlated wave function



α large:
mean-field like wave function
with **pairwise correlations**

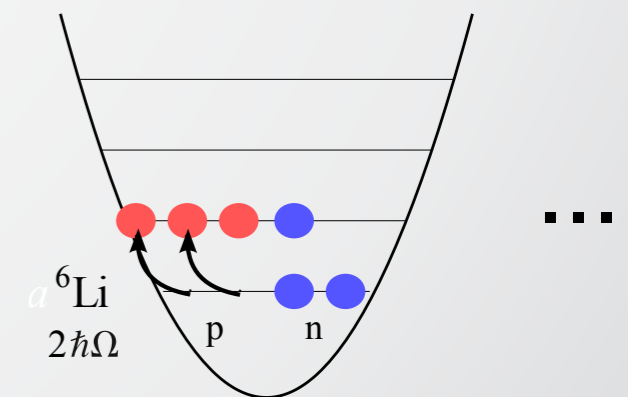
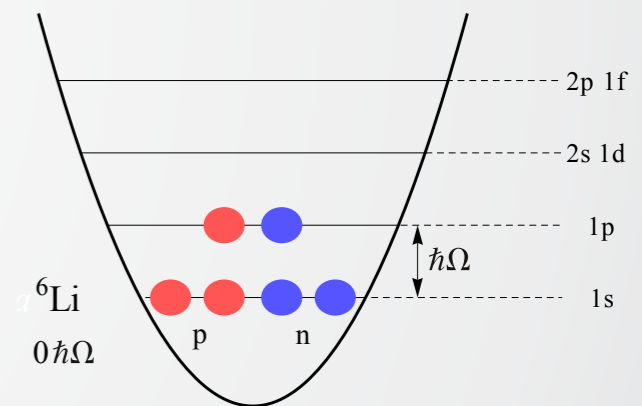
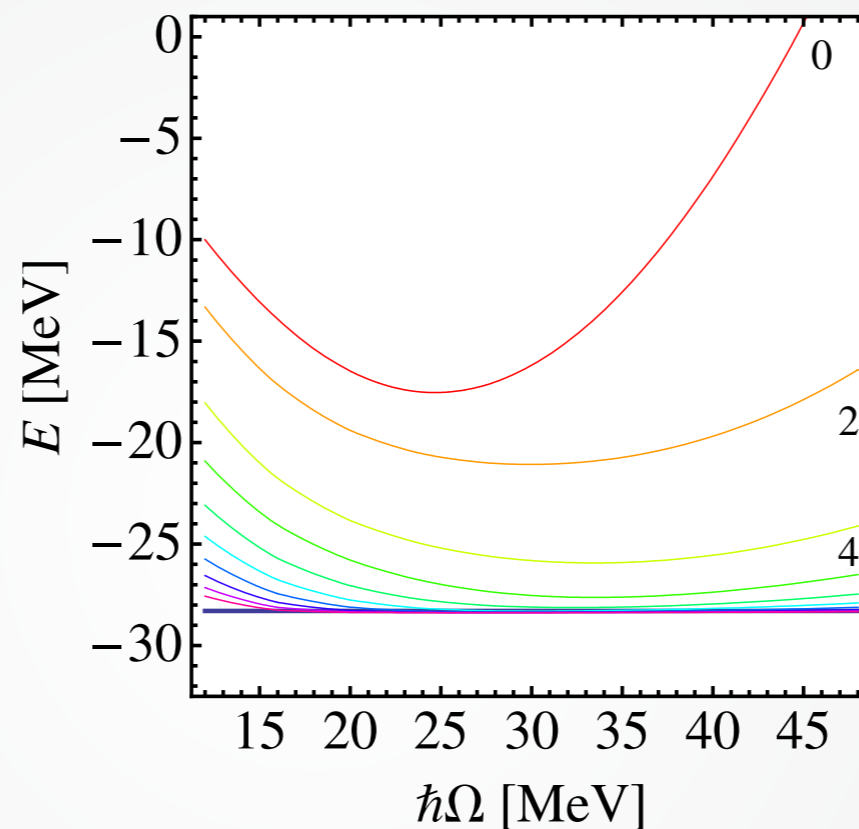
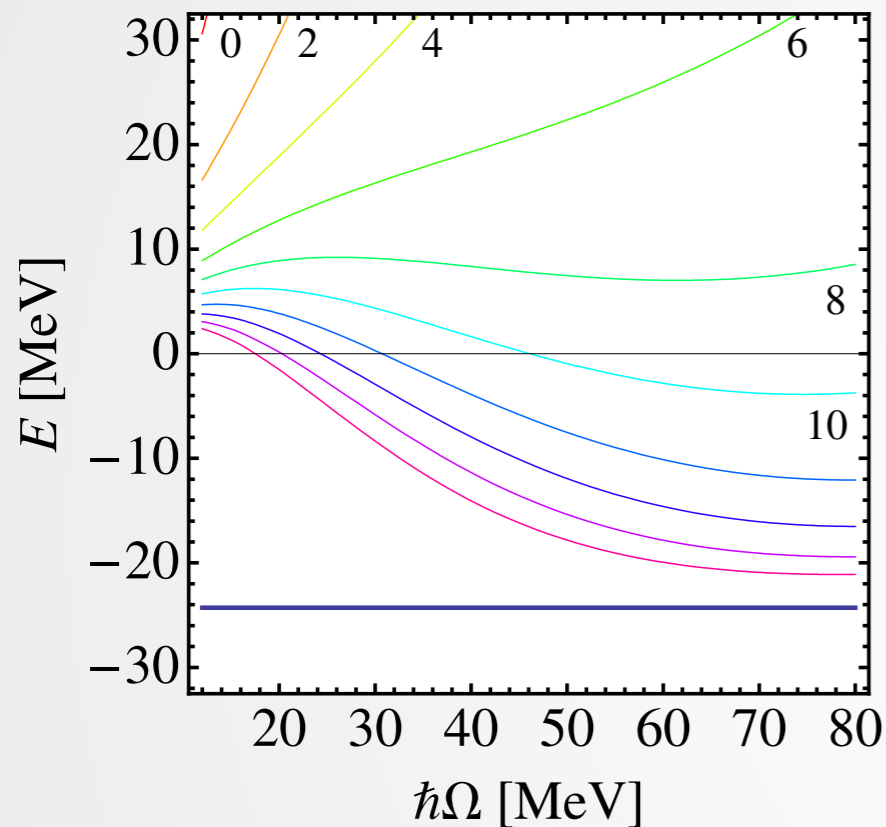
Convergence in No-Core Shell Model

bare interaction

SRG ($\alpha=0.03 \text{ fm}^4$)

${}^4\text{He} - \text{AV18}$

${}^4\text{He} - \text{SRG}$

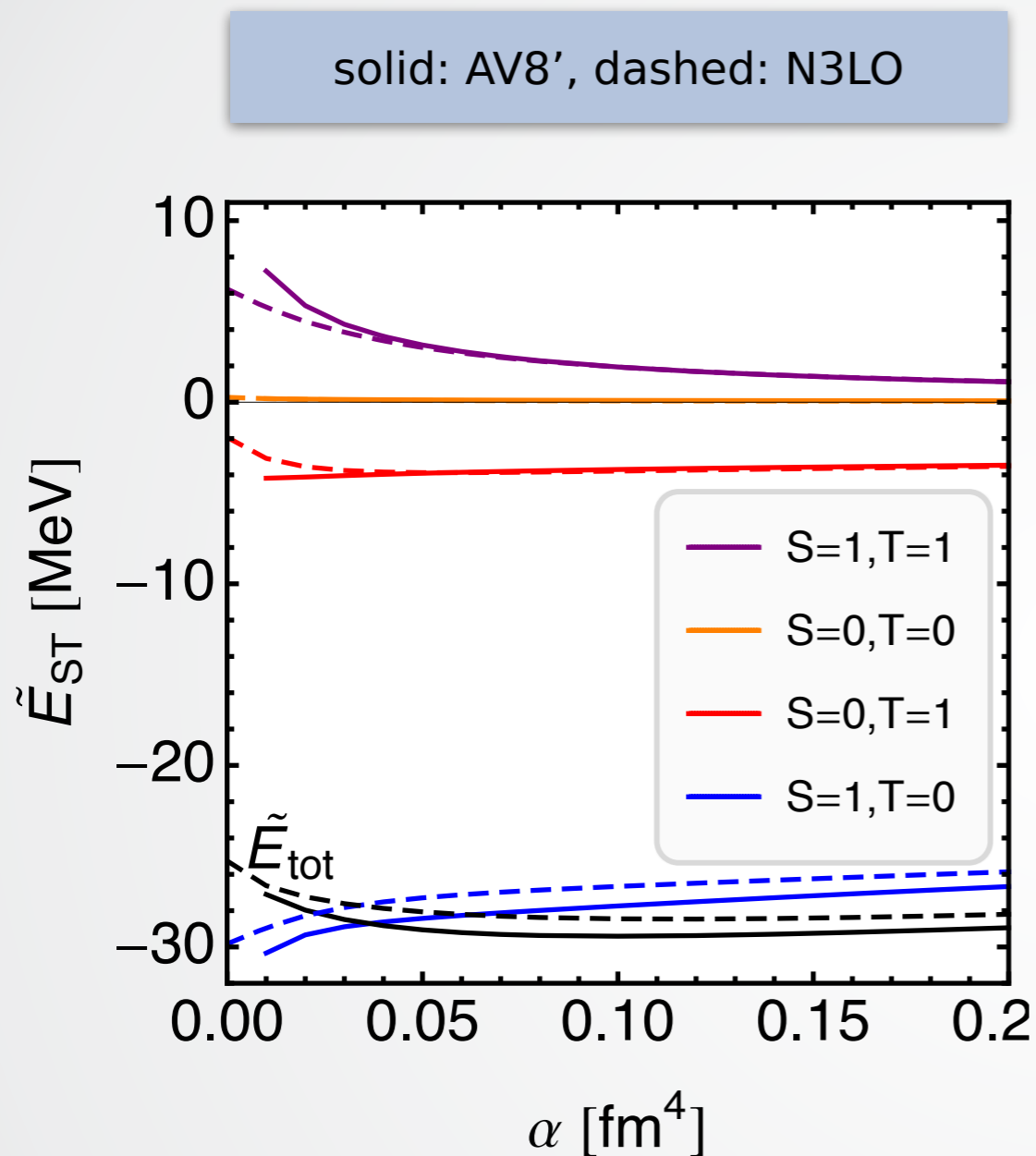


No-Core Shell Model (NCSM)

- Diagonalization of Hamiltonian in harmonic oscillator basis
- $N \hbar\Omega$ configuration: N oscillator quanta above $0 \hbar\Omega$ configuration
- Model space sizes grow rapidly with A and N_{max}

Roth, Neff, Feldmeier, Prog. Part. Nucl. Phys. **65**, 50 (2010)

Contributions to the binding energy



- Energy depends slightly on flow parameter — indicates missing three-body terms in effective Hamiltonian
- Binding energy dominated by (ST)=(10) channel, contribution from tensor part of effective Hamiltonian decreases with flow parameter
- Sizeable repulsive contribution from odd (ST)=(11) channel related to many-body correlations — decreases with flow parameter

Translational Invariant Two-body Densities

Coordinates in the lab system

$$\mathbf{x}_1, \dots, \mathbf{x}_{A-2}, \mathbf{x}_2 = \sqrt{\frac{1}{2}} [\mathbf{x}_{A-1} + \mathbf{x}_A], \mathbf{g} = \sqrt{\frac{1}{2}} [\mathbf{x}_{A-1} - \mathbf{x}_A]$$

Coordinates in the intrinsic system

$$\boldsymbol{\xi}_1 = \sqrt{\frac{1}{2}} (\mathbf{x}_1 - \mathbf{x}_2), \boldsymbol{\xi}_2 = \sqrt{\frac{2}{3}} \left[\frac{1}{2} (\mathbf{x}_1 + \mathbf{x}_2) - \mathbf{x}_3 \right], \dots, \boldsymbol{\xi}_{A-3},$$

$$\boldsymbol{\eta} = \sqrt{\frac{2(A-2)}{A}} \times \left[\frac{1}{A-2} (\mathbf{x}_1 + \dots + \mathbf{x}_{A-2}) - \frac{1}{2} (\mathbf{x}_{A-1} + \mathbf{x}_A) \right], \mathbf{g} = \sqrt{\frac{1}{2}} [\mathbf{x}_{A-1} - \mathbf{x}_A]$$

Transform two-body densities in the lab into the intrinsic system

$$\sum_{M_2} \rho_{q'N'_2L_2M_2;qN_2L_2M_2}^{\text{lab}} = \sum_{NN'L} M_{N'_2L_2,N_2L_2;N'L,NL}^{(0)} \sum_M \rho_{q'N'LM;qNLM}$$

$$M_{N'_2L_2,N_2L_2;N'L,NL}^{(0)} =$$

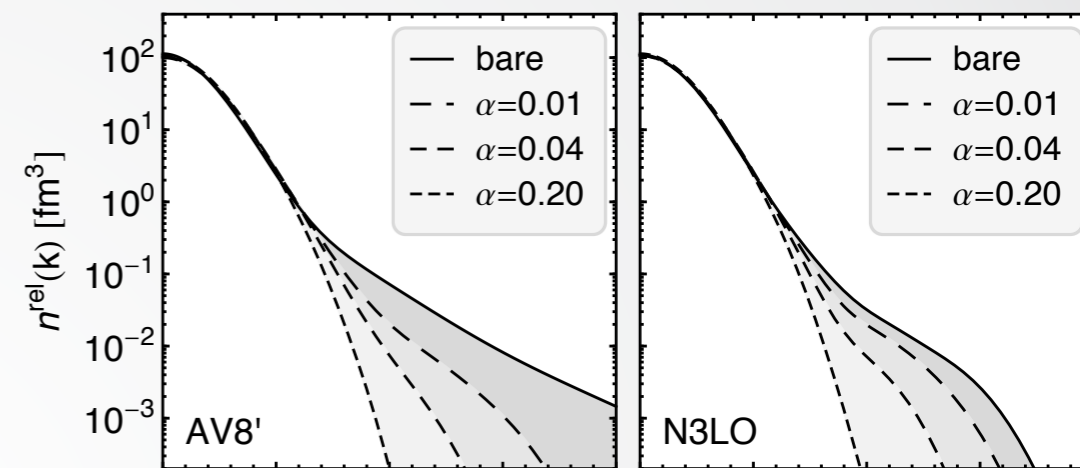
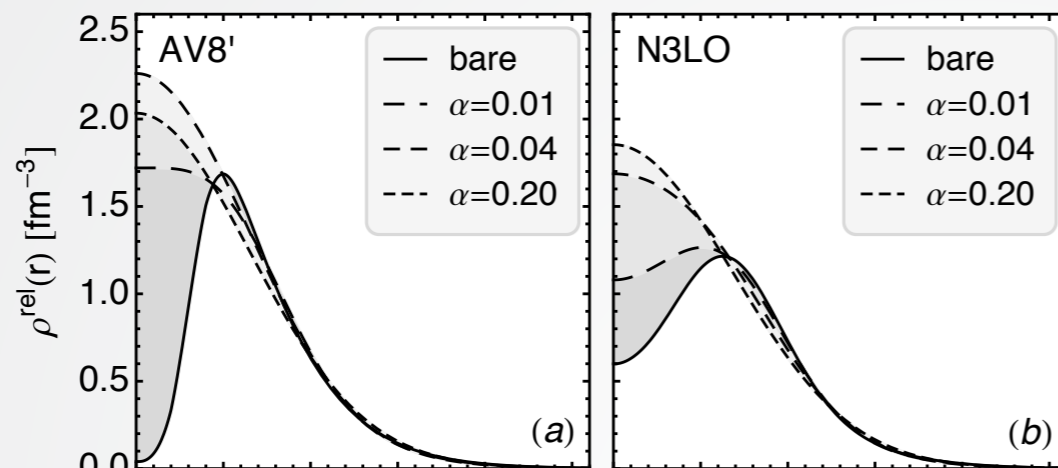
$$\sum_{N_{A-2}L_{A-2}} \langle 00, NL : L | N_{A-2}L_{A-2}, N_2L_2 : L \rangle_{(A-2)/2} \langle 00, N'L : L | N_{A-2}L_{A-2}, N'_2L_2 : L \rangle_{(A-2)/2}$$

^4He : $\rho^{\text{rel}}(r)$ and $n^{\text{rel}}(k)$

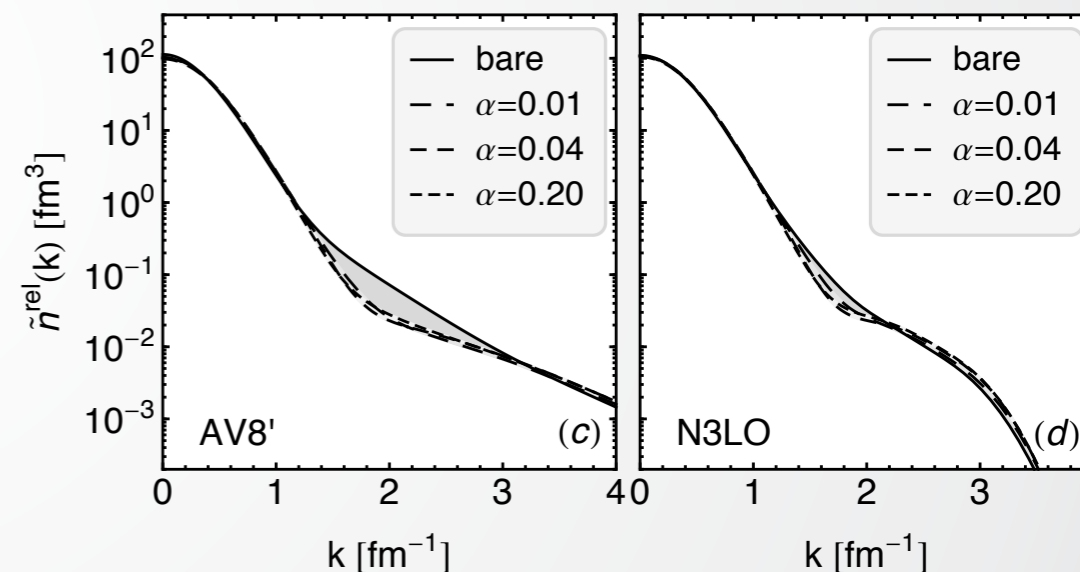
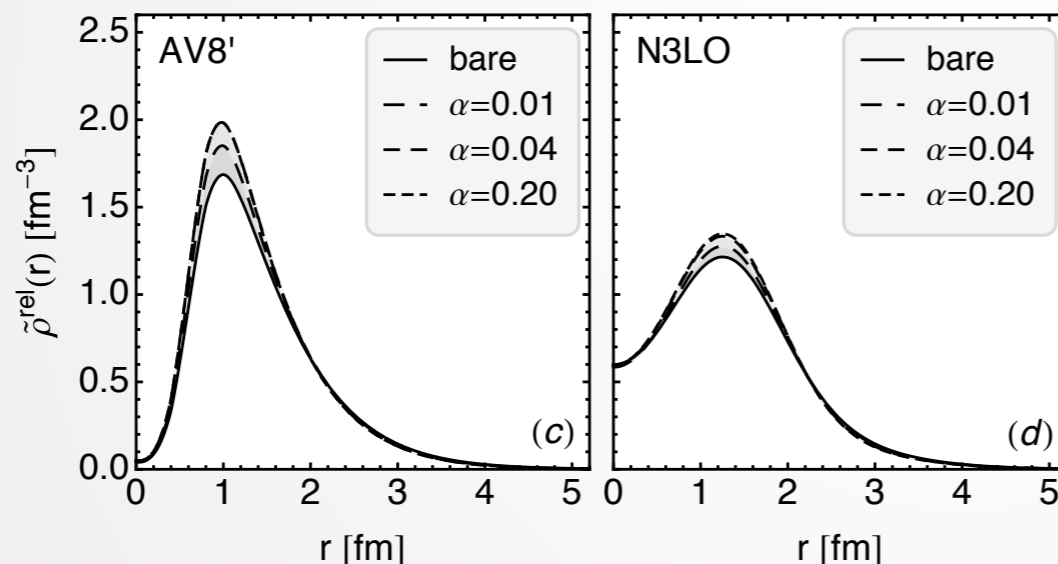
Coordinate Space

Momentum Space

bare
density operators



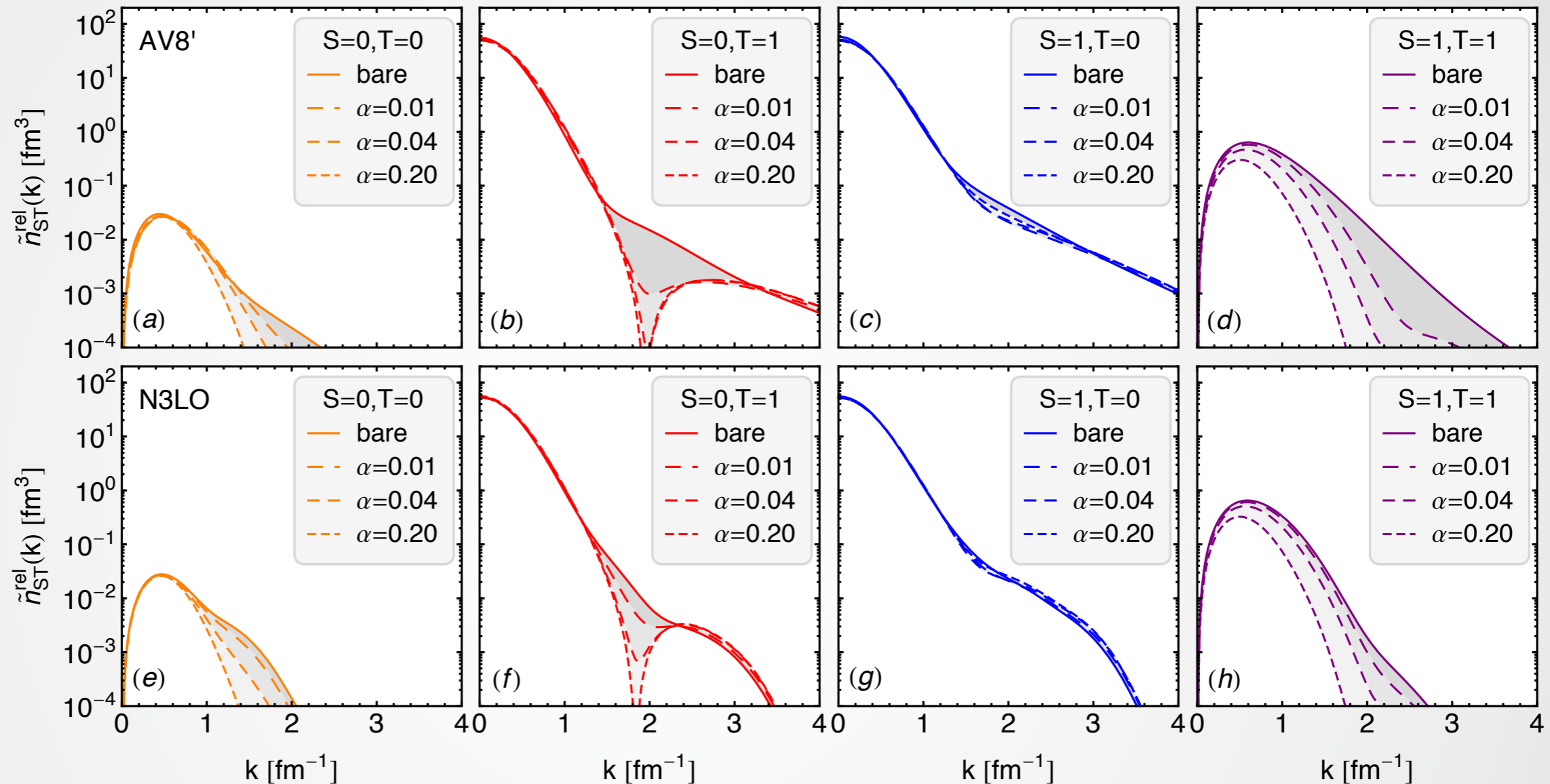
transformed
density operators



- SRG softens interaction - suppression at short distances and high-momentum components removed in wave function
- these features are recovered with SRG transformed density operators
- small but noticeable dependence on flow parameter α

${}^4\text{He}$: $n_{\text{ST}}^{\text{rel}}(k)$

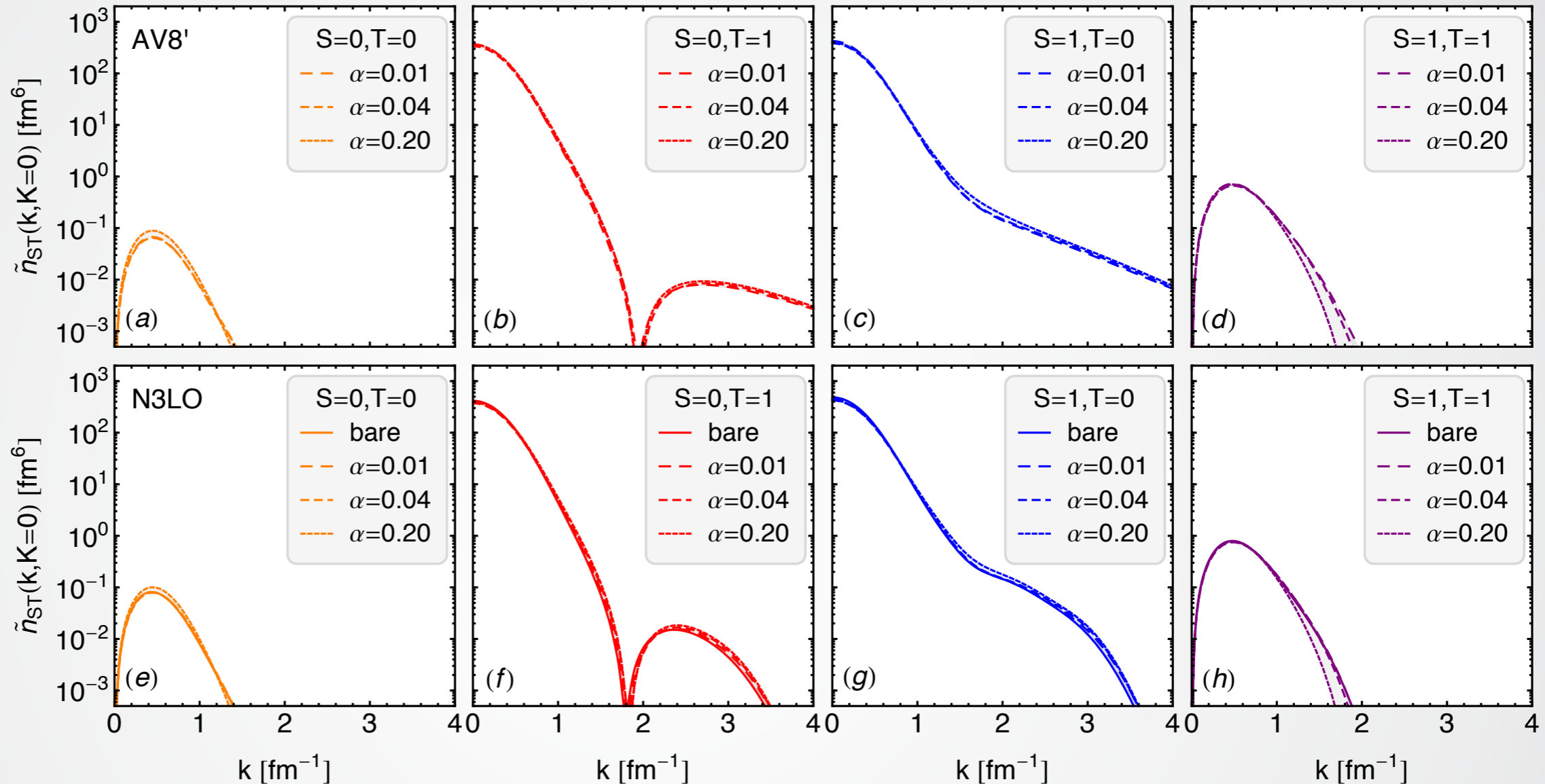
transformed
density operators



- high-momentum components much stronger in $(ST)=(10)$ channel
- flow dependence is weak in $(ST)=(10)$ channel
- flow dependence is strong in $(ST)=(01)$ and (11) channels, especially for momenta above Fermi momentum — signal of many-body correlations

^4He : $n_{ST}(k, K=0)$

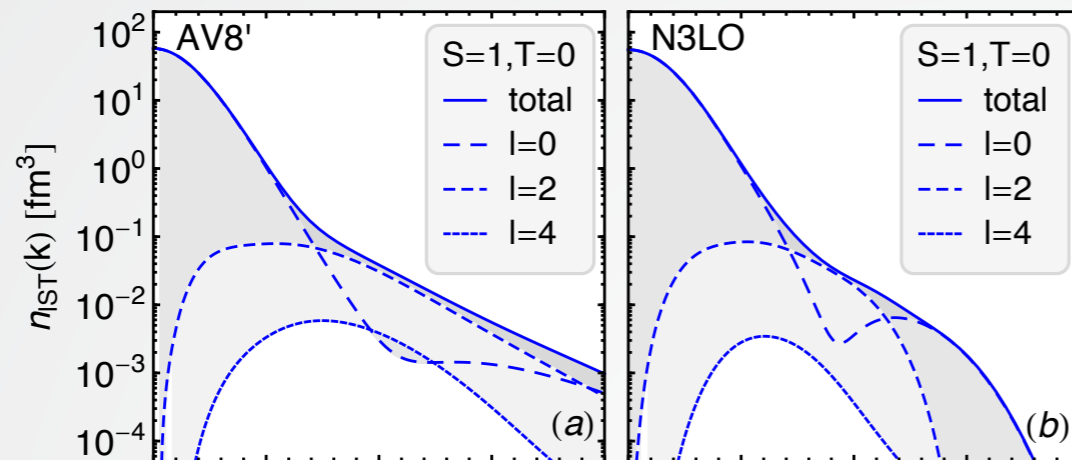
transformed
density operators



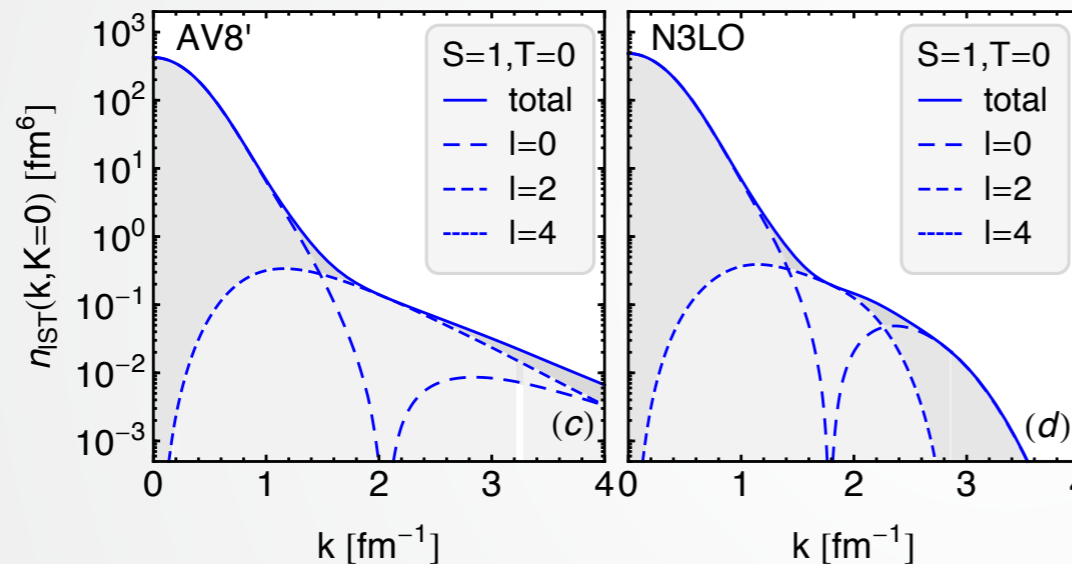
- Relative momentum distributions for $K=0$ pairs show a very weak dependence on flow parameter and therefore on many-body correlations — ideal to study two-body correlations
- Momentum distribution vanishes for relative momenta around 1.8 fm^{-1} in the $(ST)=(01)$ channel

^4He : Tensor Correlations

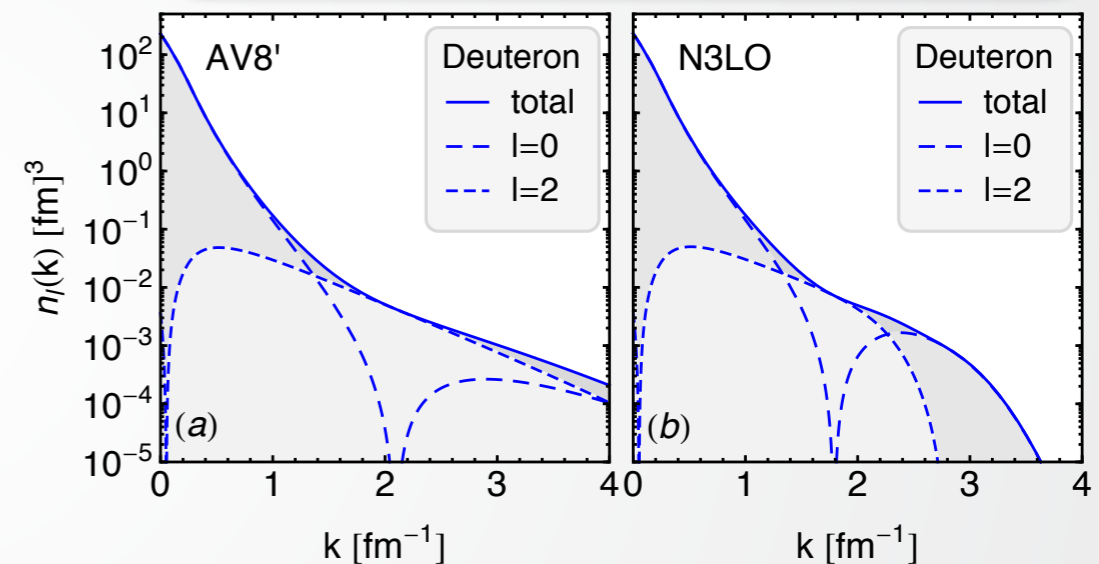
all pairs



$K=0$ pairs



Deuteron

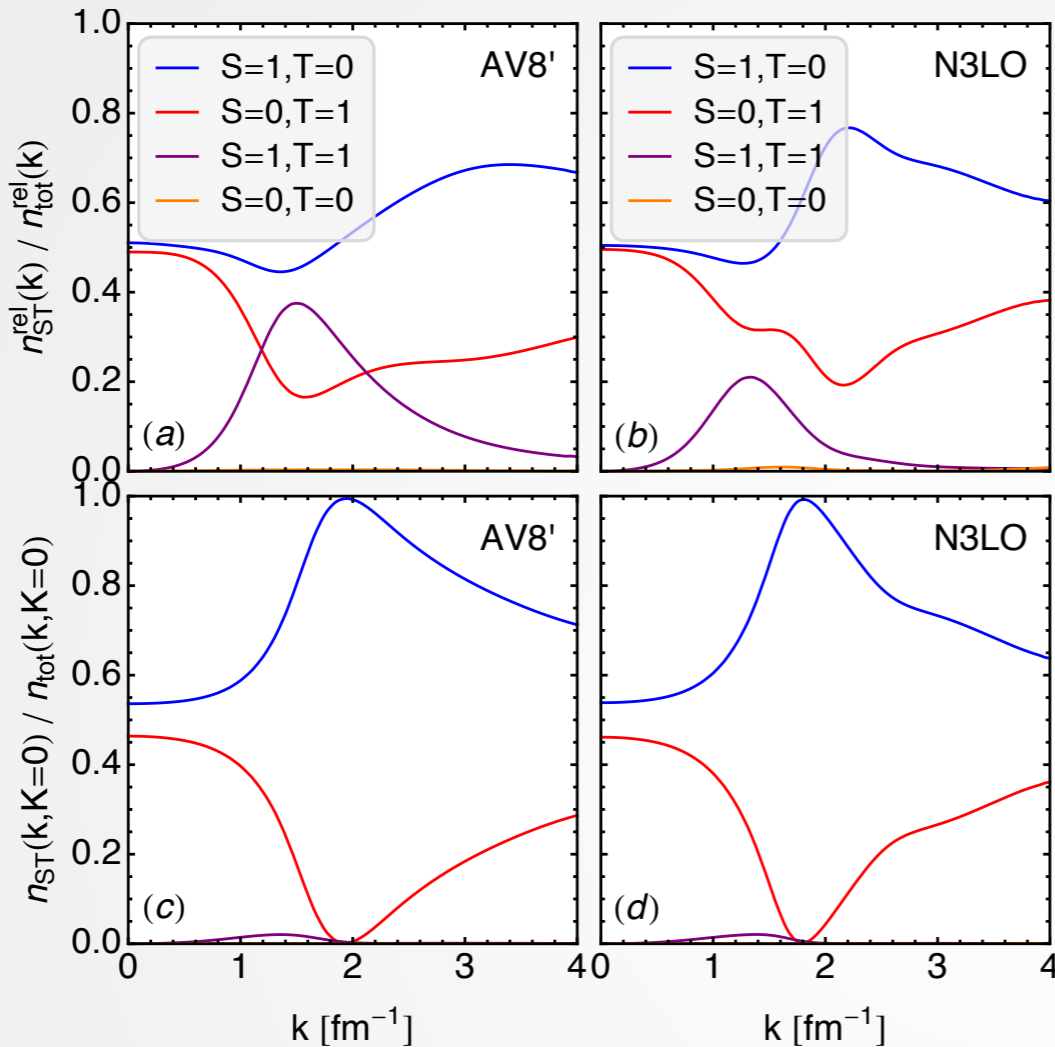


- In $(ST)=(10)$ channel momentum distributions above Fermi momentum dominated by pairs with orbital angular momentum $L=2$
- For $K=0$ pairs only $L=0,2$ relevant, for all pairs also higher orbital angular momenta contribute
- The ^4He $K=0$ momentum distributions above 1.5 fm^{-1} look like Deuteron momentum distributions

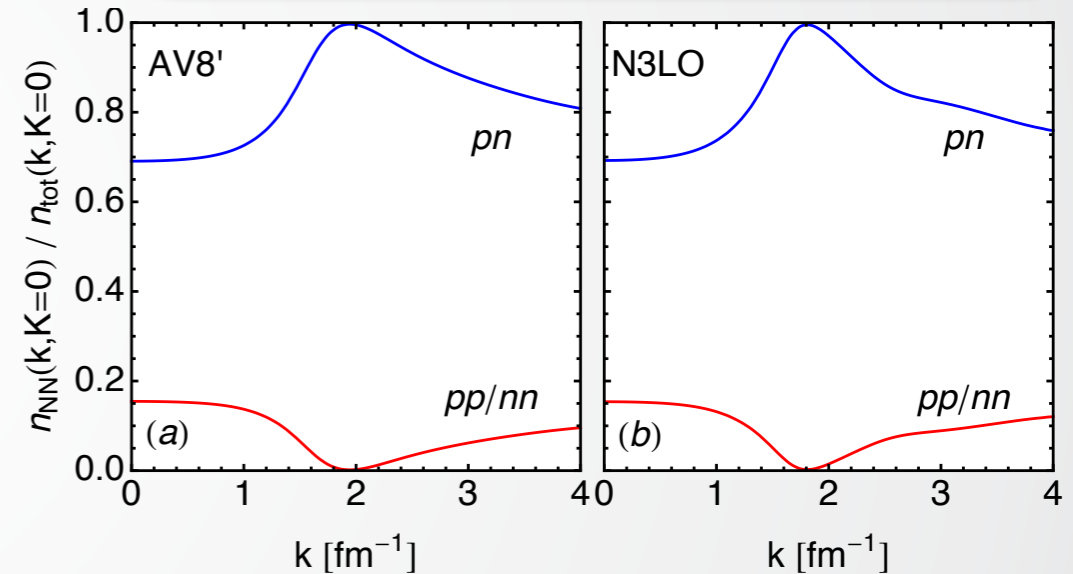
⁴He: Relative Probabilities

all pairs

K=0 pairs



pp/nn/pn pairs

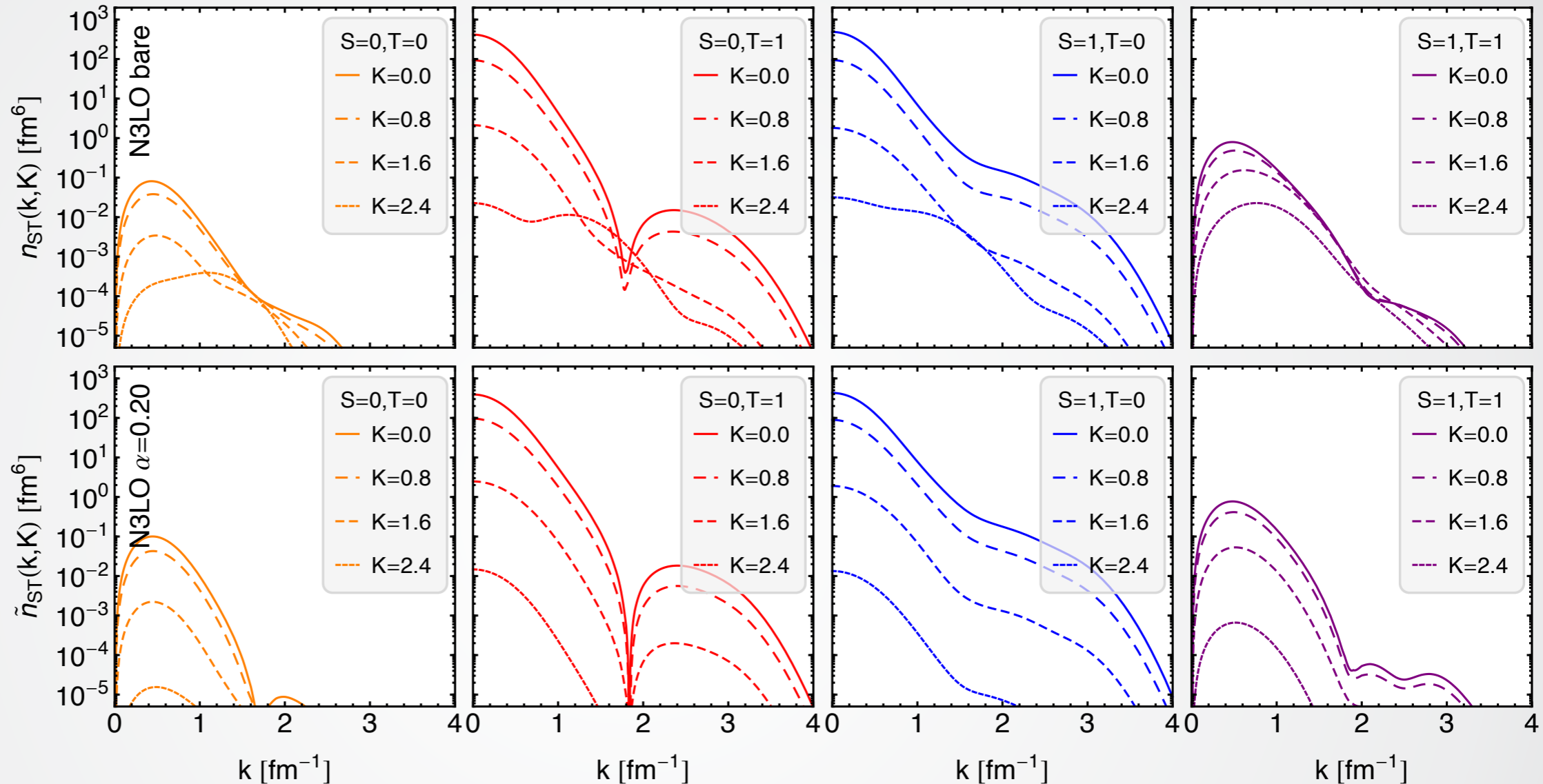


- Relative probabilities for K=0 pairs very similar for AV8' and N3LO interactions
- For K=0 pairs ratio of pn/pp pairs goes to infinity for relative momenta of about 1.8 fm⁻¹
- This is not the case if we look at all pairs, here many-body correlations generate many pairs in the (ST)=(11) channel

^4He : $n_{\text{ST}}(k, K)$

fully correlated

only pair correlations

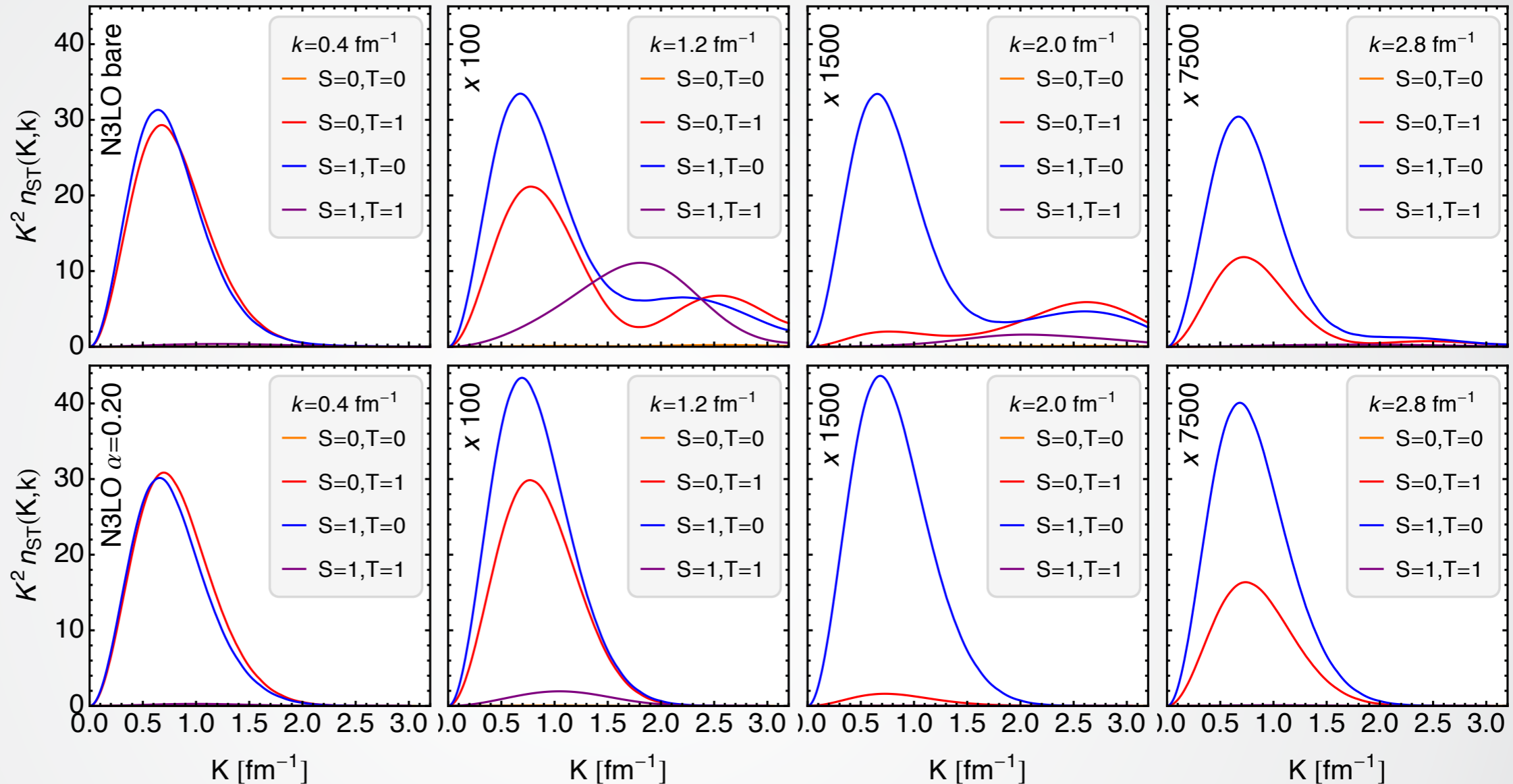


- For the bare interaction relative momentum distributions similar up to $K \approx 1.0 \text{ fm}^{-1}$
- many-body correlations significantly influence momentum distributions for larger K
- For $\alpha = 0.20$ relative momentum distributions similar for all K

^4He : Which K contribute to $n_{\text{ST}}(k)$?

fully correlated

only pair correlations

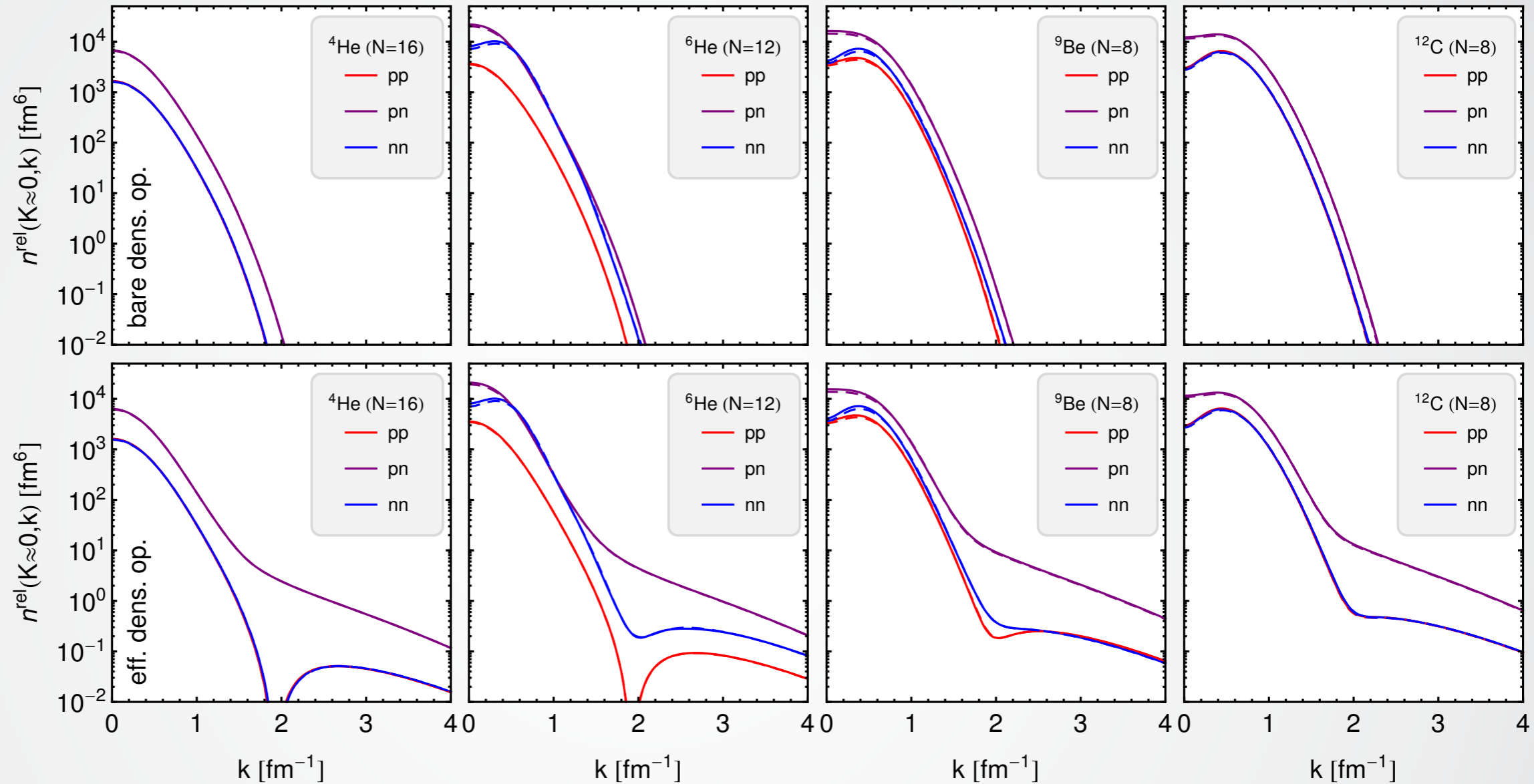


- many-body correlations responsible for pairs with pair momenta $K \gtrsim 2.0$ fm $^{-1}$
- these play a significant role for relative momenta 1.0 fm $^{-1} \lesssim k \lesssim 2.5$ fm $^{-1}$
- pairs with high relative momenta are only mildly affected

${}^4\text{He}$, ${}^6\text{He}$, ${}^9\text{Be}$, ${}^{12}\text{C}$: $n(k, K=0)$

bare
density operators

transformed
density operators



- Momentum distributions obtained in NCSM are well converged for larger flow parameters
- high-momentum pn (and total) momentum distributions very similar for all nuclei
- p -shell nucleons fill up the node around 1.8 fm^{-1} for pp/nn pairs

The Wigner Function of the Deuteron

A phase-space picture of short-range correlations

Neff, Feldmeier, *in preparation*

$$W(\mathbf{x}, \mathbf{p}) = \frac{1}{(2\pi)^3} \int d^3s \Psi(\mathbf{x} + \frac{1}{2}\mathbf{s}) \Psi(\mathbf{x} - \frac{1}{2}\mathbf{s})^* e^{-i\mathbf{p}\cdot\mathbf{s}}$$

$$\rho(\mathbf{x}) = \int d^3p W(\mathbf{x}, \mathbf{p}), \quad n(\mathbf{k}) = \int d^3x W(\mathbf{x}, \mathbf{p})$$

Wigner Function of the Deuteron

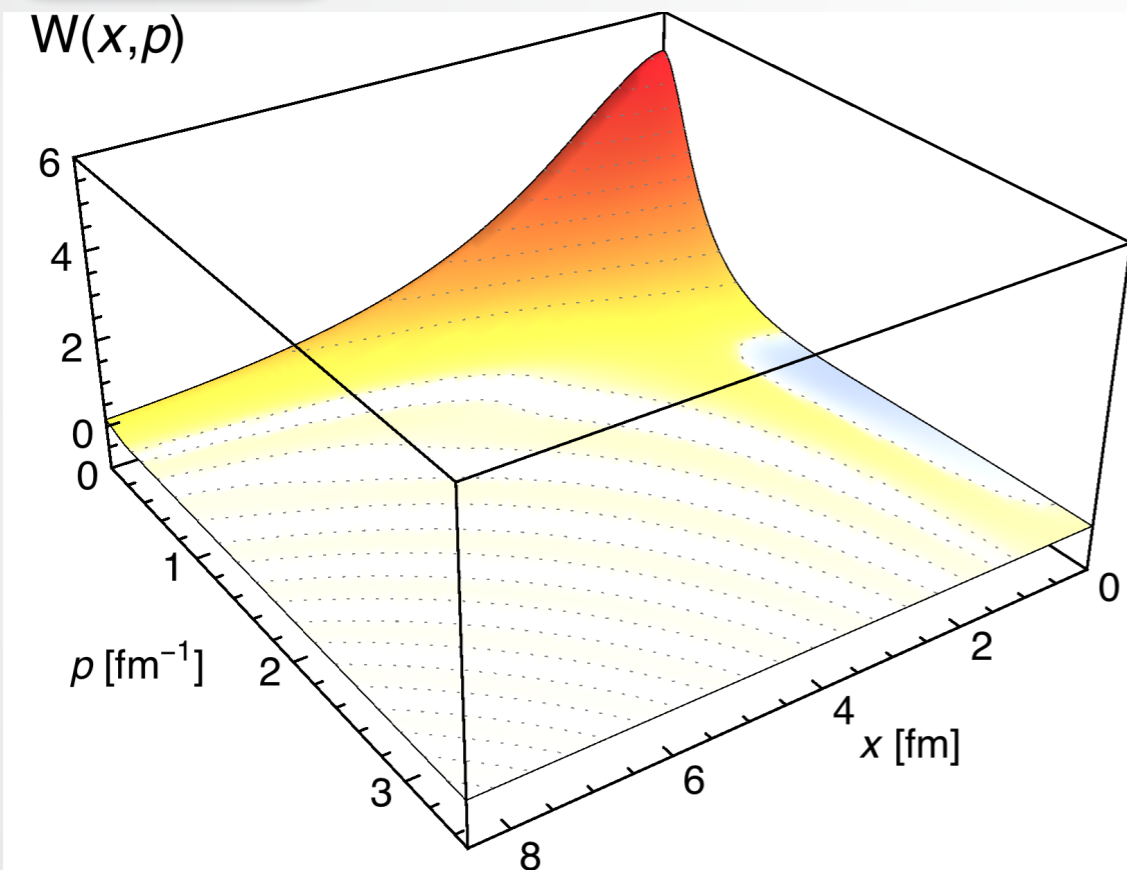
$$W(\mathbf{x}, \mathbf{p}) = \frac{1}{(2\pi)^3} \int d^3s \langle \mathbf{x} + \frac{1}{2}\mathbf{s} | \hat{\rho} | \mathbf{x} - \frac{1}{2}\mathbf{s} \rangle e^{-i\mathbf{p}\cdot\mathbf{s}}$$

$$= \frac{1}{(2\pi)^3} \int d^3s \Psi(\mathbf{x} + \frac{1}{2}\mathbf{s}) \Psi(\mathbf{x} - \frac{1}{2}\mathbf{s})^* e^{-i\mathbf{p}\cdot\mathbf{s}}$$

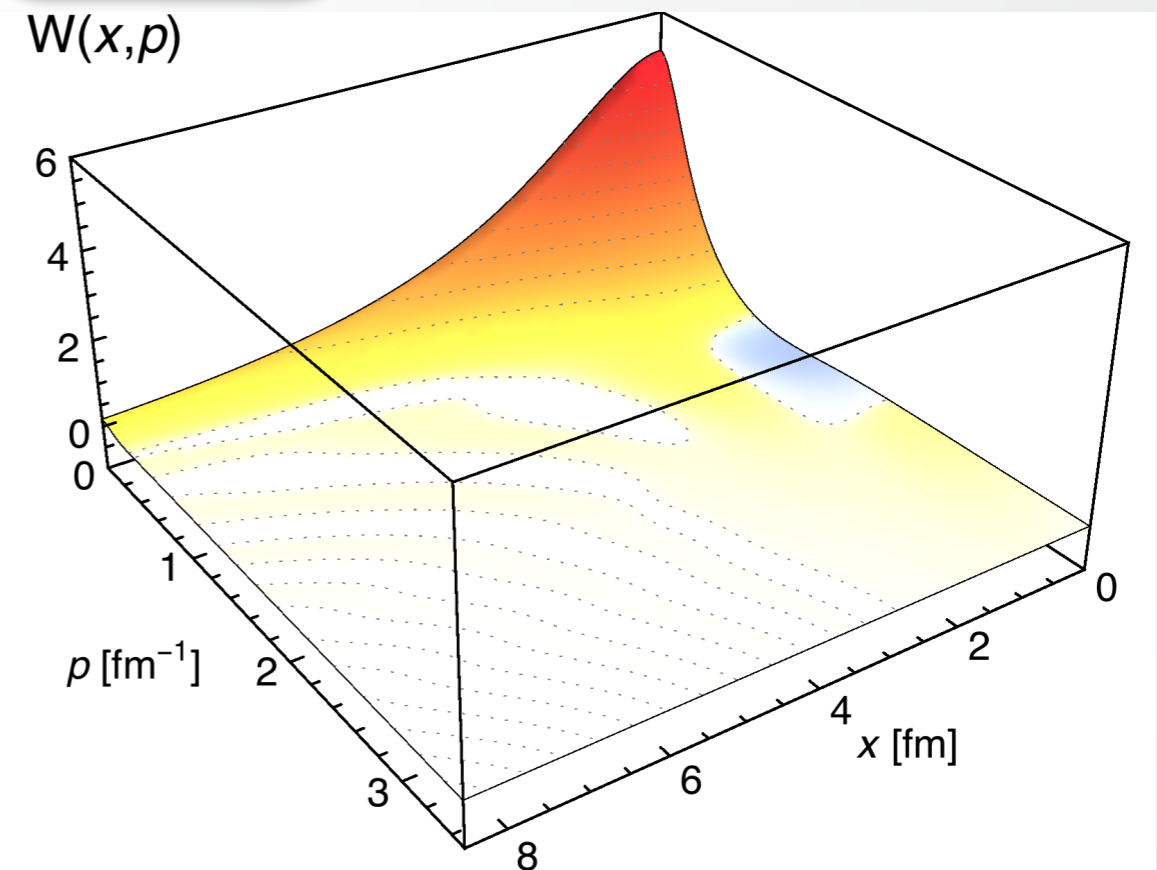
- Integrate over angles

$$W(x, p) = \int d\Omega_x \int d\Omega_p W(\mathbf{x}, \mathbf{p})$$

AV8'



N3LO



- Wigner function not suppressed at small distances x
- High-momentum components hard to spot

Wigner Function of the Deuteron

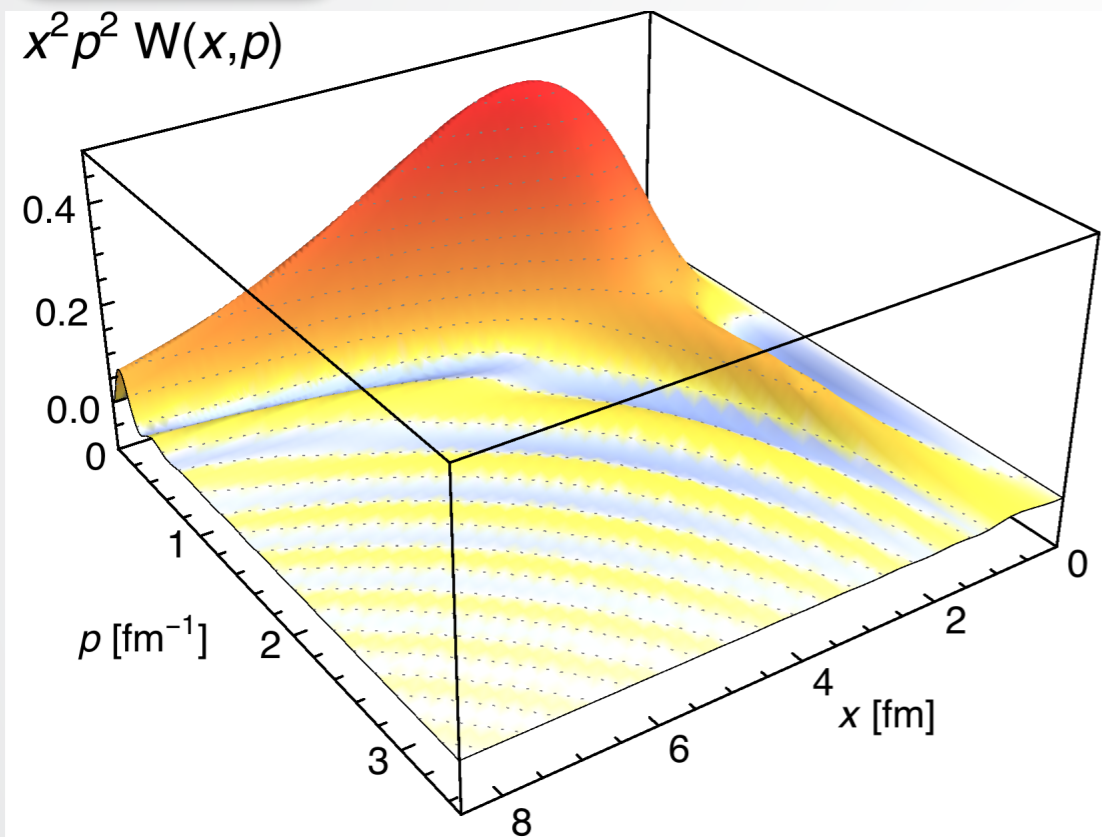
$$W(\mathbf{x}, \mathbf{p}) = \frac{1}{(2\pi)^3} \int d^3s \langle \mathbf{x} + \frac{1}{2}\mathbf{s} | \hat{\rho} | \mathbf{x} - \frac{1}{2}\mathbf{s} \rangle e^{-i\mathbf{p}\cdot\mathbf{s}}$$

$$= \frac{1}{(2\pi)^3} \int d^3s \Psi(\mathbf{x} + \frac{1}{2}\mathbf{s}) \Psi(\mathbf{x} - \frac{1}{2}\mathbf{s})^* e^{-i\mathbf{p}\cdot\mathbf{s}}$$

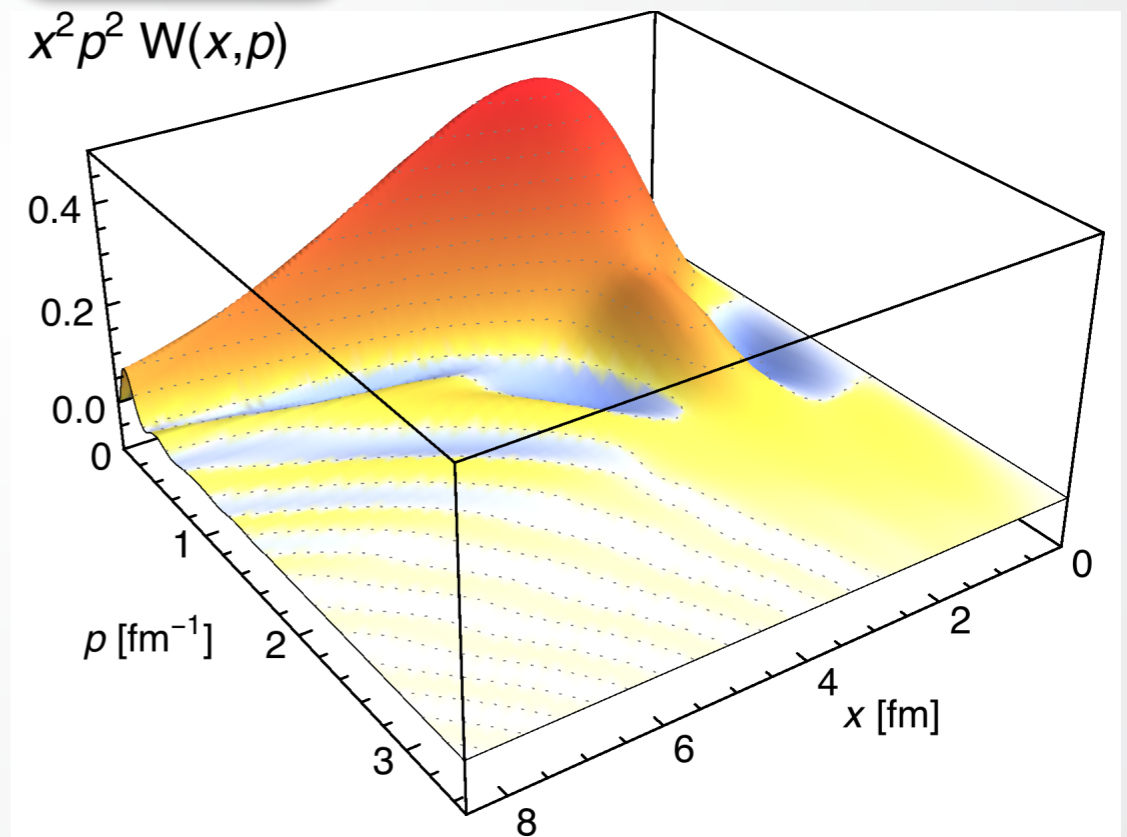
- Integrate over angles

$$W(x, p) = \int d\Omega_x \int d\Omega_p W(\mathbf{x}, \mathbf{p})$$

AV8'



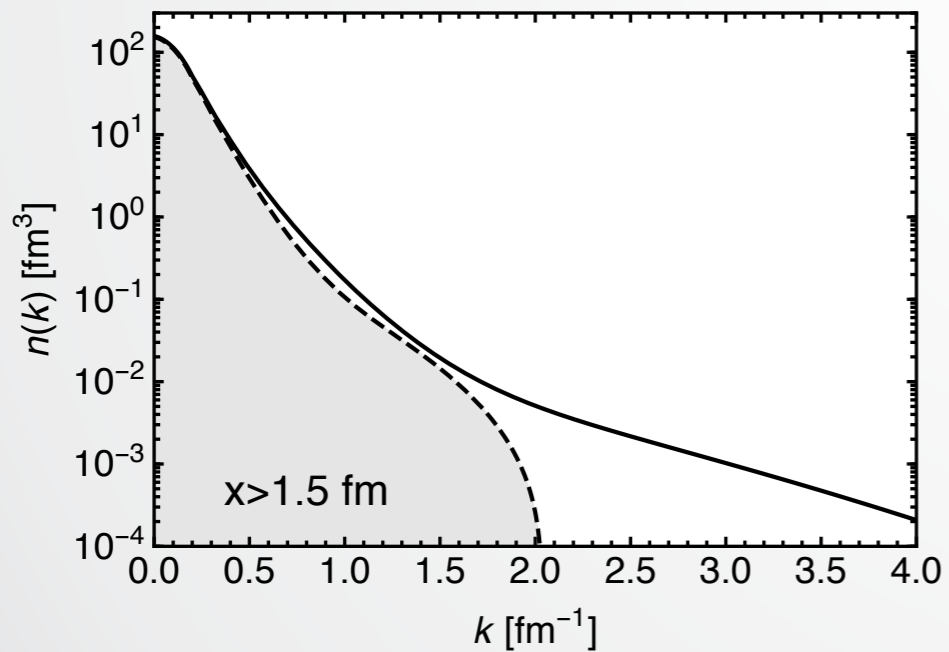
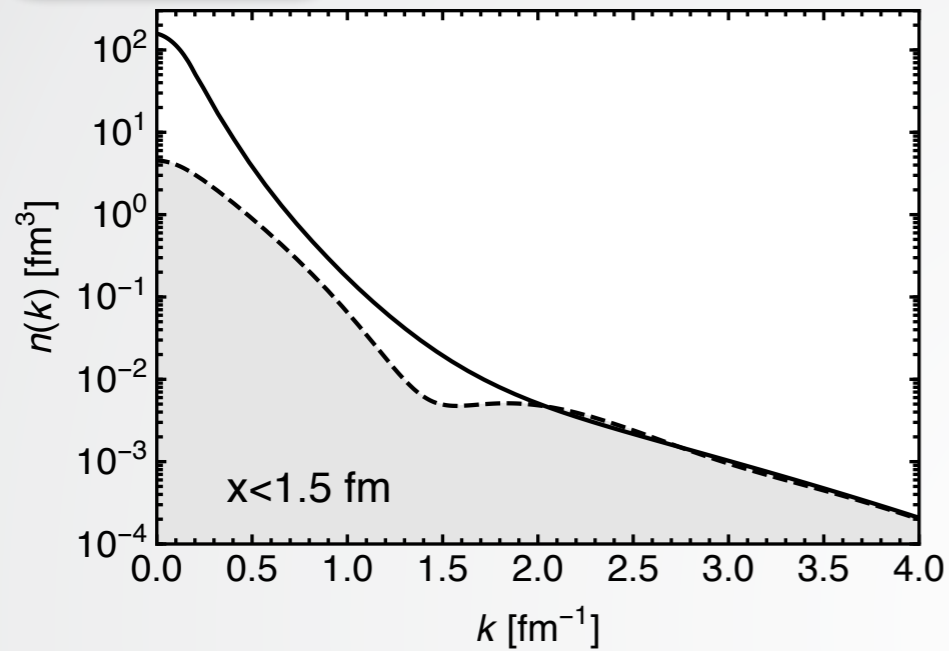
N3LO



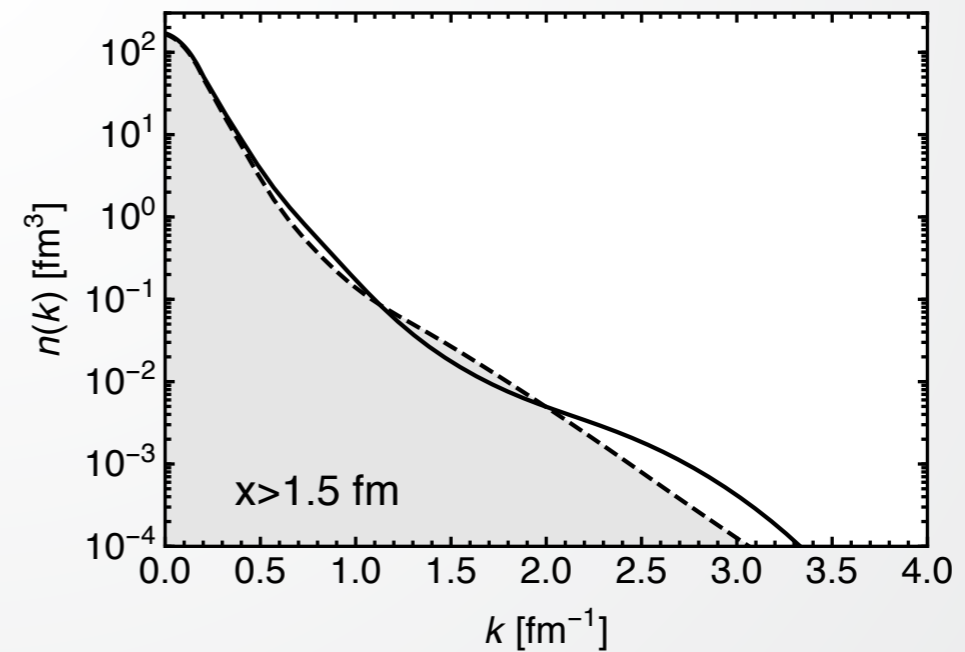
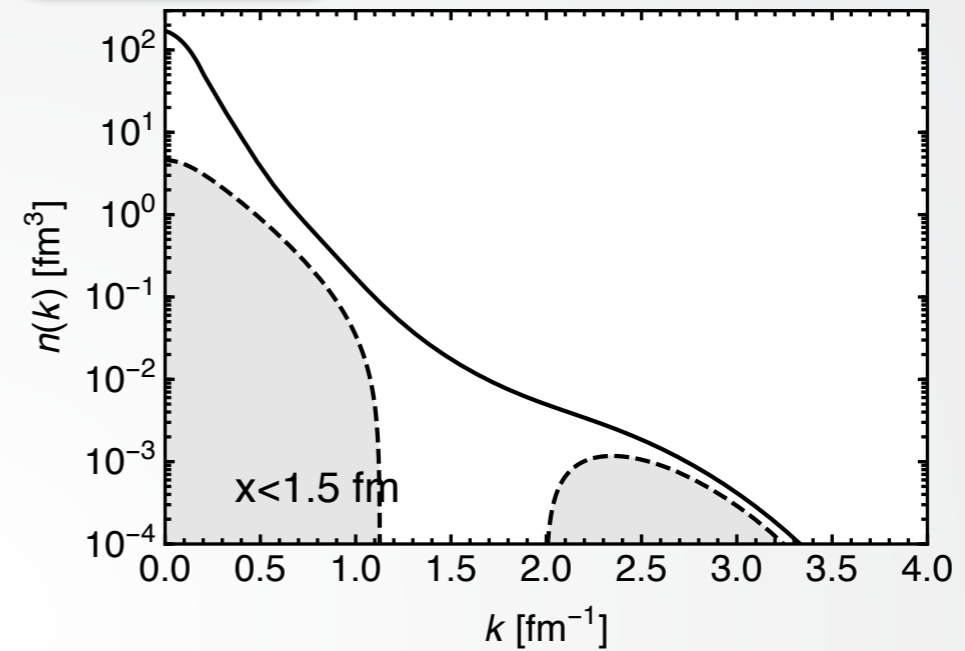
- Wigner function multiplied with phase-space volume element
- High-momentum components are seen as a shoulder at small distances
- Oscillations reflect the quantum nature (uncertainty principle)

(Partial) Momentum Distributions

AV8'



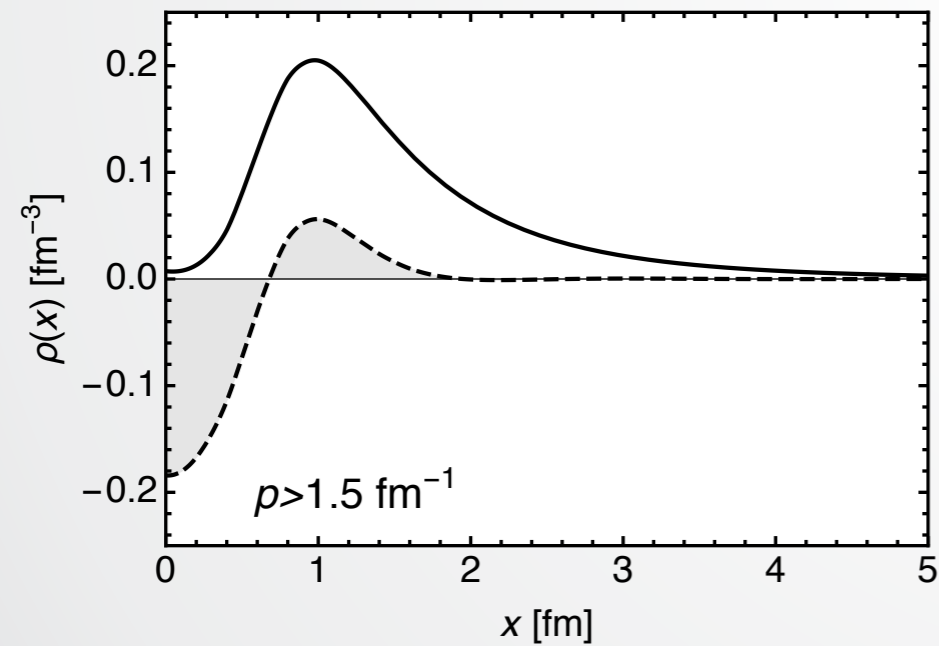
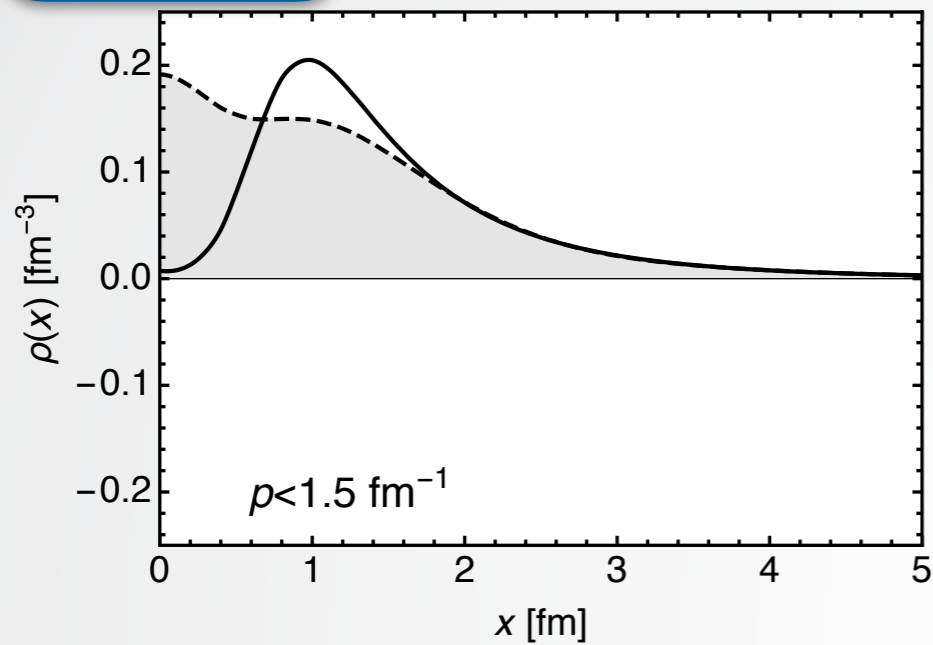
N3LO



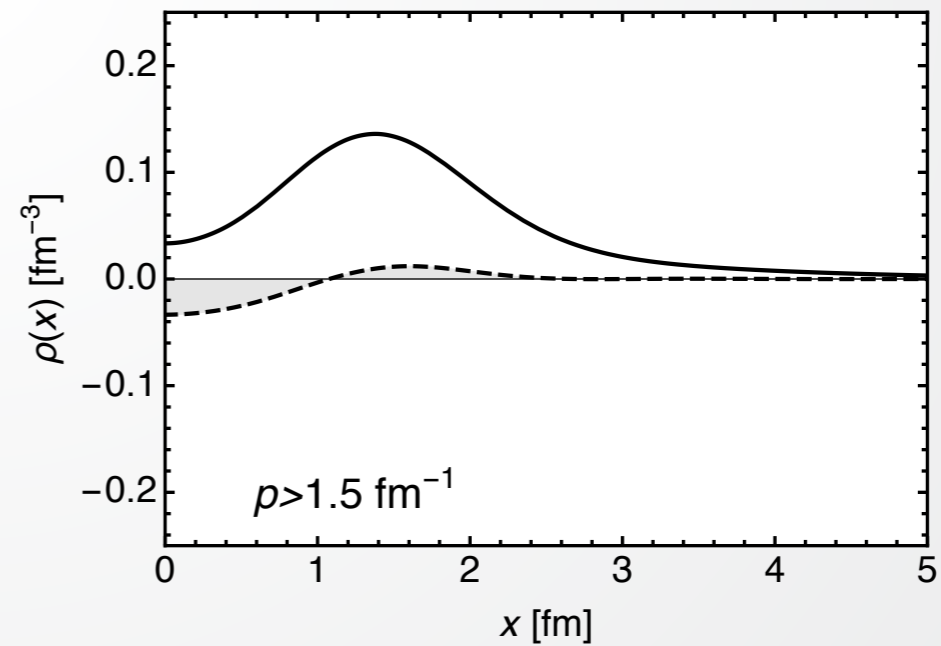
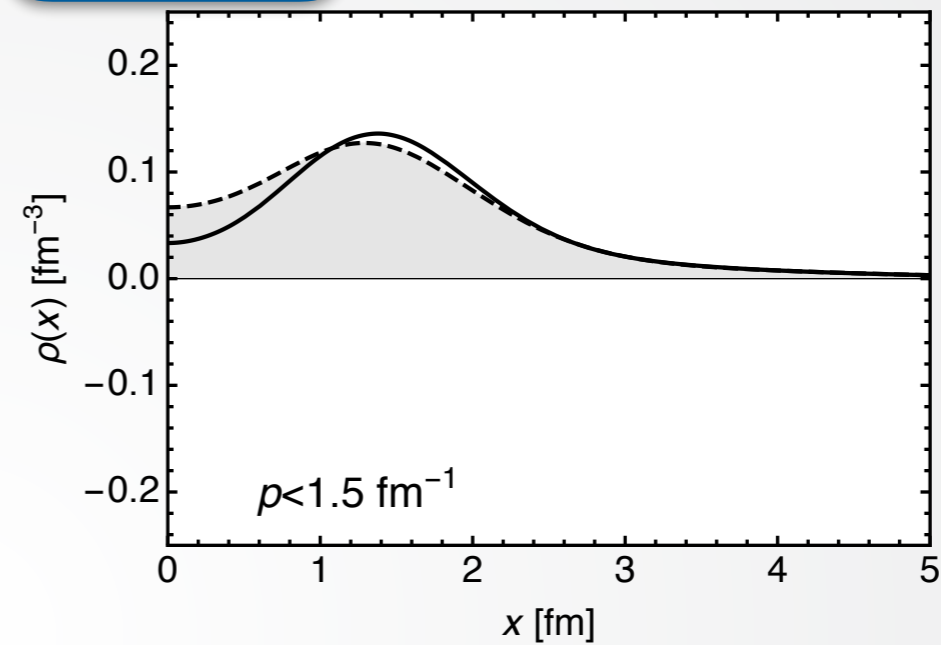
- Integrate Wigner function over small or large distance regions
- large distance pairs give momentum distributions up to Fermi momentum

(Partial) Coordinate Space Distributions

AV8'



N3LO



- Integrate Wigner function over small or large momentum regions
- correlation hole is created by negative contribution of high-momentum pairs

Summary

Universality of Short-Range correlations

- exact calculations for *s*-shell nuclei show universal behavior for two-body densities at short distances and large relative momenta

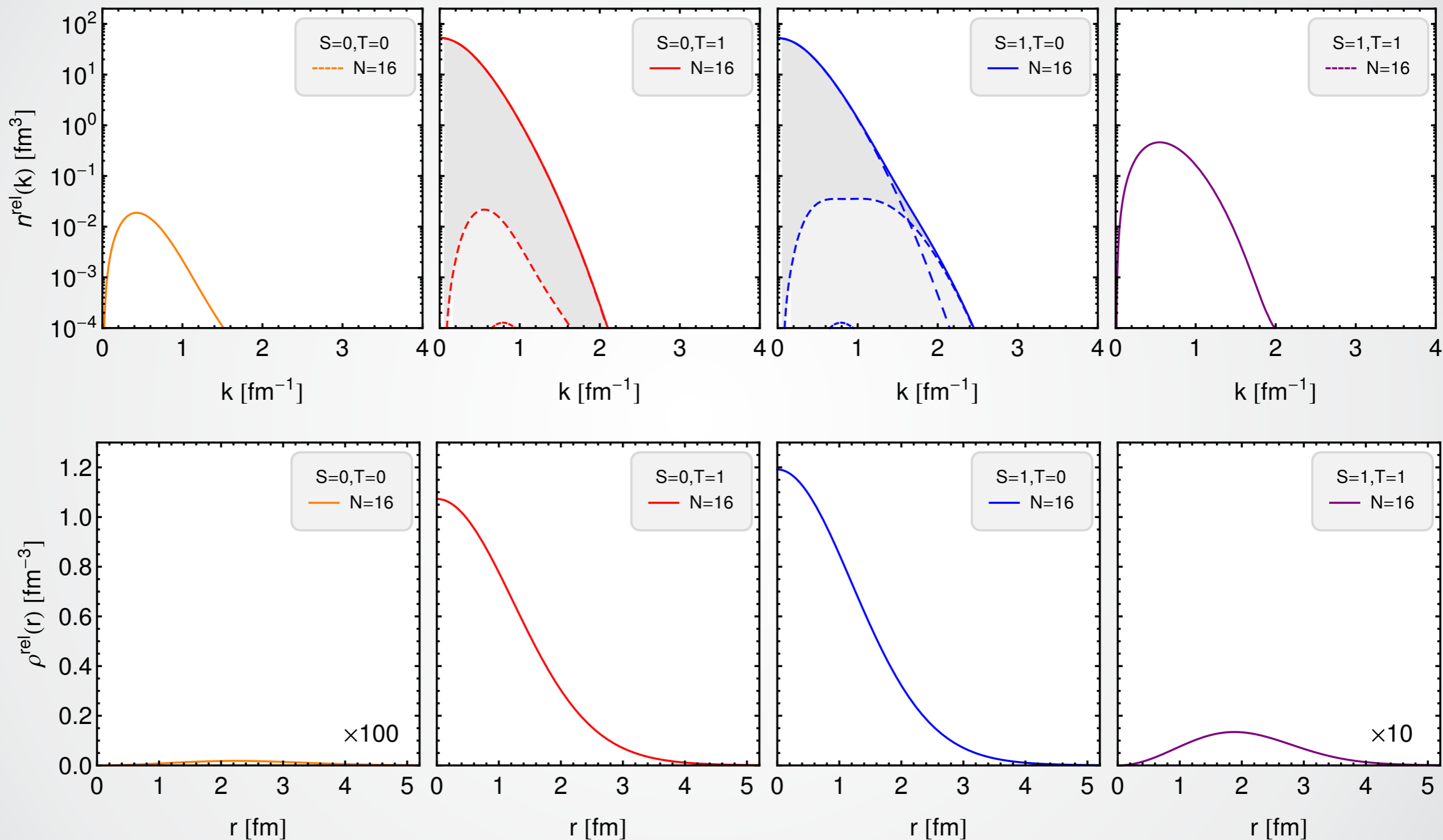
Two-body Densities with NCSM and SRG transformed Operators

- SRG eliminates high-momentum components from wave function
- use flow dependence to study many-body correlations and learn about induced three-body forces
- back-to-back pairs only weakly, pairs with large pair momentum strongly affected by many-body correlations
- high-momentum components for AV18 and N3LO interactions quite similar for momenta up to 2.5 fm^{-1} , dominated by tensor correlations, show dominance of *pn* over *pp* pairs
- two-body densities are not observables, how big is the role of MEC and FSI (see talk by Sebastian König) ?

Wigner Function of the Deuteron

- Intuitive (?) phase-space picture of short-range correlations
- large-distances contribute to high-momentum components for N3LO - artefact of regulator ?

^4He JISP16



- JISP16 is really soft !