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- C. Romero-Redondo (LLNL)
- R. Roth (TU Darmstadt)
- S. Quaglioni (LLNL)

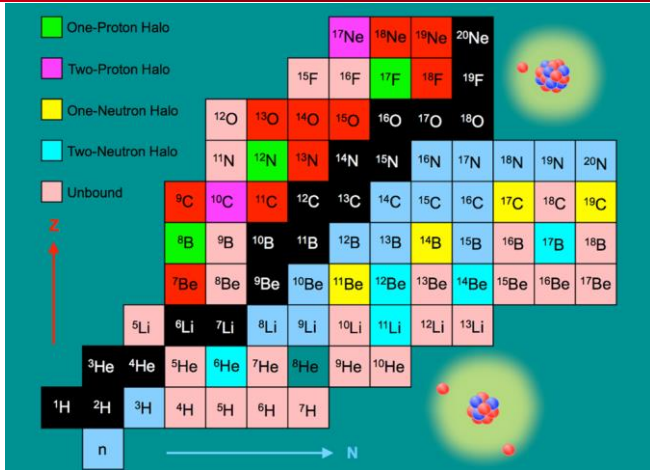
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NUCLEAR STRUCTURE AND REACTIONS FROM CHIRAL INTERACTIONS

Guillaume Hupin

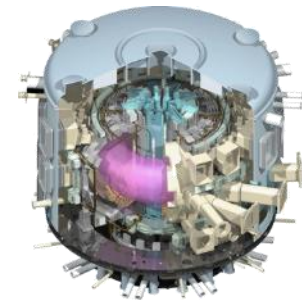
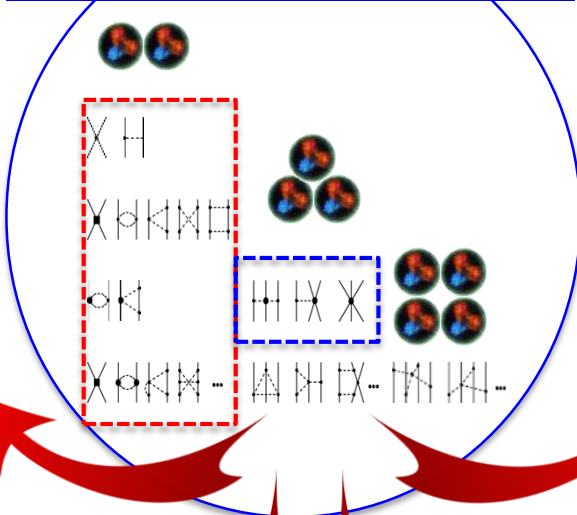
Progress in *Ab Initio* Techniques in Nuclear Physics, TRIUMF BC
Canada, February 23th 2016.

INTRODUCTION

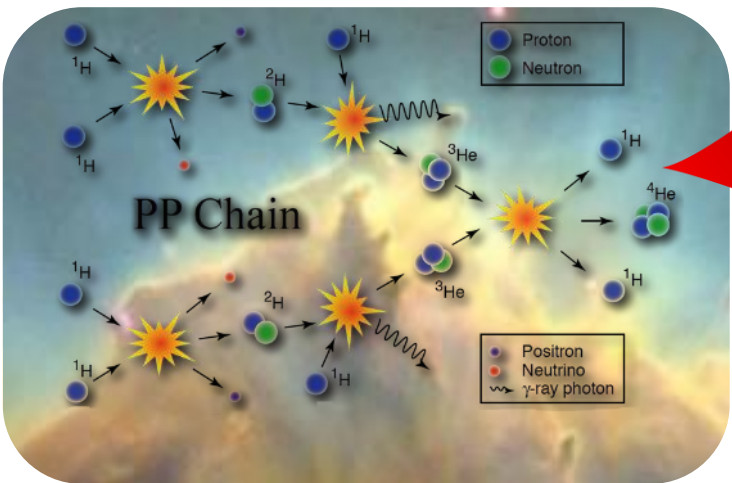


Structure of exotic nuclei

Ab initio structure and reaction



Fusion based energy generation



Nuclear astrophysics



Materials science

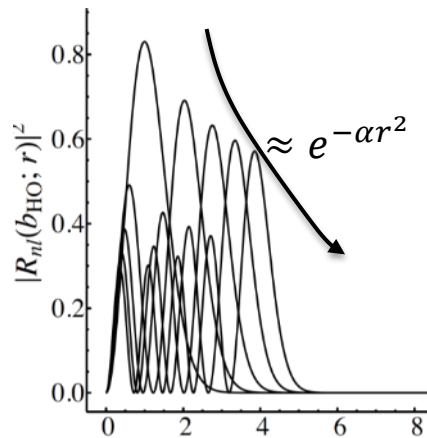
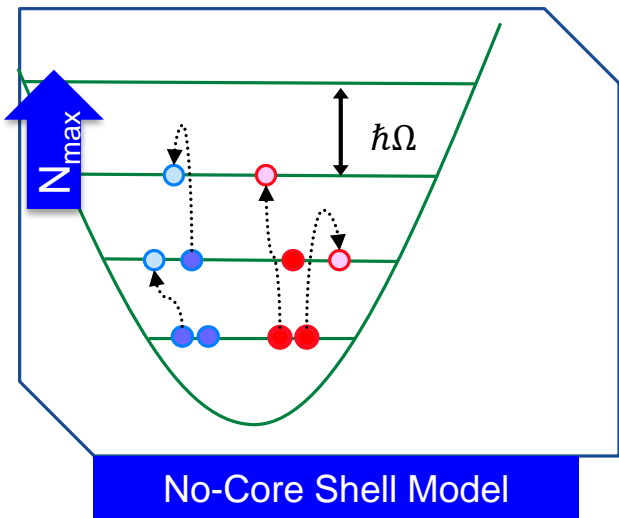
EQUAL TREATMENT OF BOUND AND RESONANT STATES: COUPLE NCSM AND NCSM/RGM (NCSMC)

- Methods develop in this presentation to solve the many body problem

$$\Psi_{NCSM}^{(A)} = |A\lambda J^\pi T\rangle = \sum_{\alpha} c_{\alpha} |A\alpha j_z^\pi t_z\rangle \leftrightarrow |A\lambda J^\pi T\rangle_{SD} \phi_{00}(\vec{R}_{c.m.}^A)$$

Mixing coefficients(unknown) A-body harmonic oscillator states Second quantization

Can address bound and low-lying resonances (short range correlations)



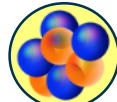
$$\Psi_{NCSMC}^{(A)} = \sum_{\lambda} c_{\lambda} |A\lambda J^\pi T\rangle$$

EQUAL TREATMENT OF BOUND AND RESONANT STATES: COUPLE NCSM AND NCSM/RGM (NCSMC)

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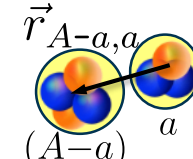
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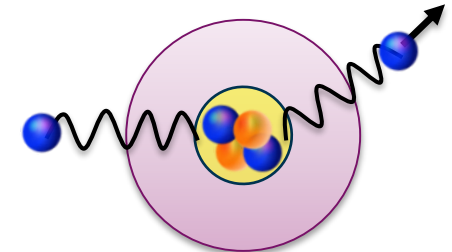


Can address bound and low-lying resonances (short range correlations)

$$\Psi_{RGM}^{(A)} = \sum_v \int d\vec{r} g_v(\vec{r}) \hat{A}_v |\Phi_{v\vec{r}}^{(A-a,a)}\rangle \leftrightarrow \psi_{\alpha_1}^{(A-a)} \psi_{\alpha_2}^{(a)} \delta(\vec{r} - \vec{r}_{A-a,a})$$

Relative wave function (unknown) Antisymmetrizer Channel basis Cluster expansion technique





NCSM/RGM
Cluster formalism for elastic/inelastic

$$\Psi_{NCSMC}^{(A)} = \sum_{\lambda} c_{\lambda} |A\lambda J^\pi T\rangle + \sum_v \int d\vec{r} g_v(\vec{r}) \hat{A}_v |\Phi_{v\vec{r}}^{(A-a,a)}\rangle$$

EQUAL TREATMENT OF BOUND AND RESONANT STATES: COUPLE NCSM AND NCSM/RGM (NCSMC)

S. Baroni, P. Navrátil and S. Quaglioni PRL110 (2013); PRC93 (2013)

- Methods develop in this presentation to solve the many body problem

$$\Psi_{NCSM}^{(A)} = |A\lambda J^\pi T\rangle = \sum_{\alpha} c_{\alpha} |A\alpha j_z^\pi t_z\rangle \leftrightarrow |A\lambda J^\pi T\rangle_{SD} \phi_{00}(\vec{R}_{c.m.}^A)$$

Mixing coefficients (unknown) A-body harmonic oscillator states Second quantization

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$$\Psi_{RGM}^{(A)} = \sum_v \int d\vec{r} g_v(\vec{r}) \hat{A}_v |\Phi_{v\vec{r}}^{(A-a,a)}\rangle \leftrightarrow \psi_{\alpha_1}^{(A-a)} \psi_{\alpha_2}^{(a)} \delta(\vec{r} - \vec{r}_{A-a,a})$$

Relative wave function (unknown) Antisymmetrizer Channel basis Cluster expansion technique

Design to account for scattering states (best for long range correlations)

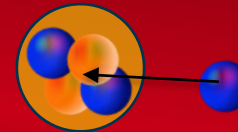
- The many body quantum problem is best described by the superposition of both type of wave functions

$$\Psi_{NCSMC}^{(A)} = \sum_{\lambda} c_{\lambda} |A\lambda J^\pi T\rangle + \sum_v \int d\vec{r} g_v(\vec{r}) \hat{A}_v |\Phi_{v\vec{r}}^{(A-a,a)}\rangle$$

NCSMC

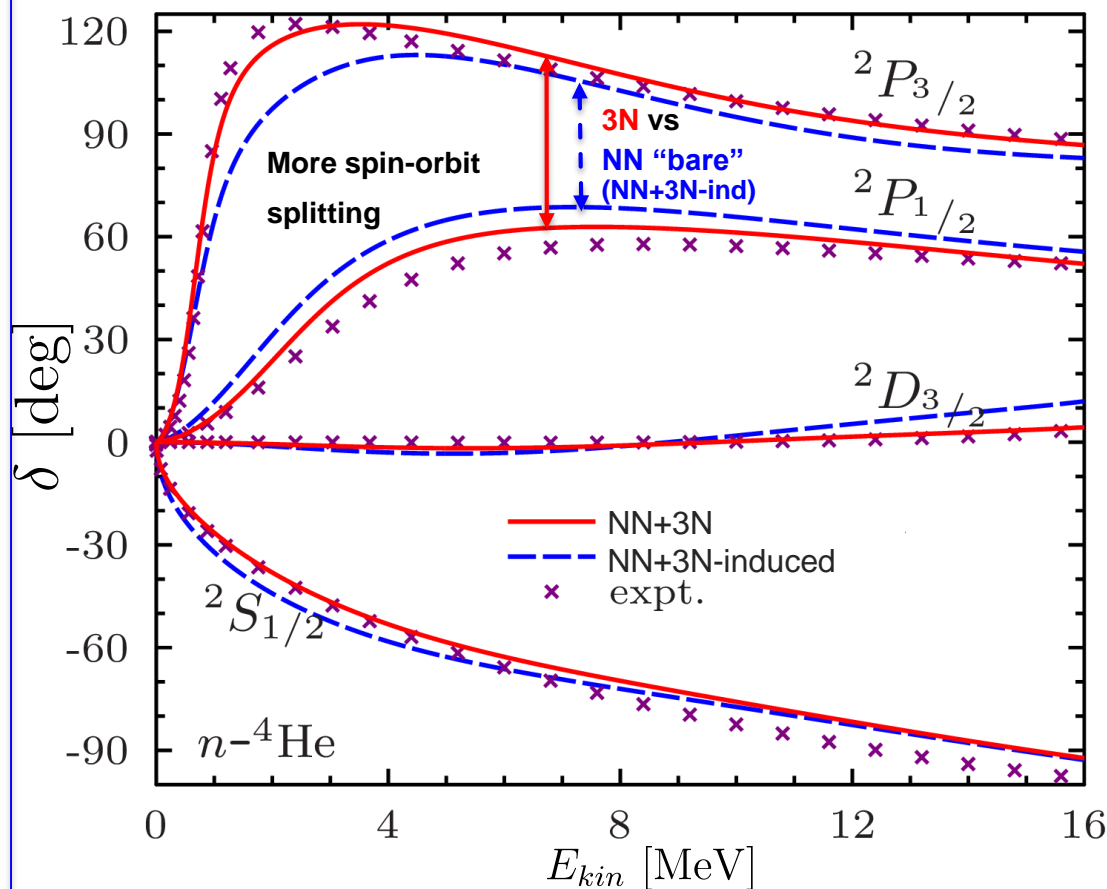
$$\begin{pmatrix} H_{NCSM} & h \\ h & H_{RGM} \end{pmatrix} \begin{pmatrix} c \\ \gamma \end{pmatrix} = E \begin{pmatrix} 1_{NCSM} & g \\ g & N_{RGM} \end{pmatrix}$$

Scattering matrix (and observables) from matching solutions to known asymptotic with microscopic R-matrix on Lagrange mesh



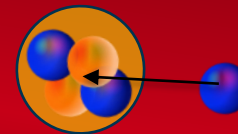
n - ^4He scattering

Two scenarii of nuclear Hamiltonians

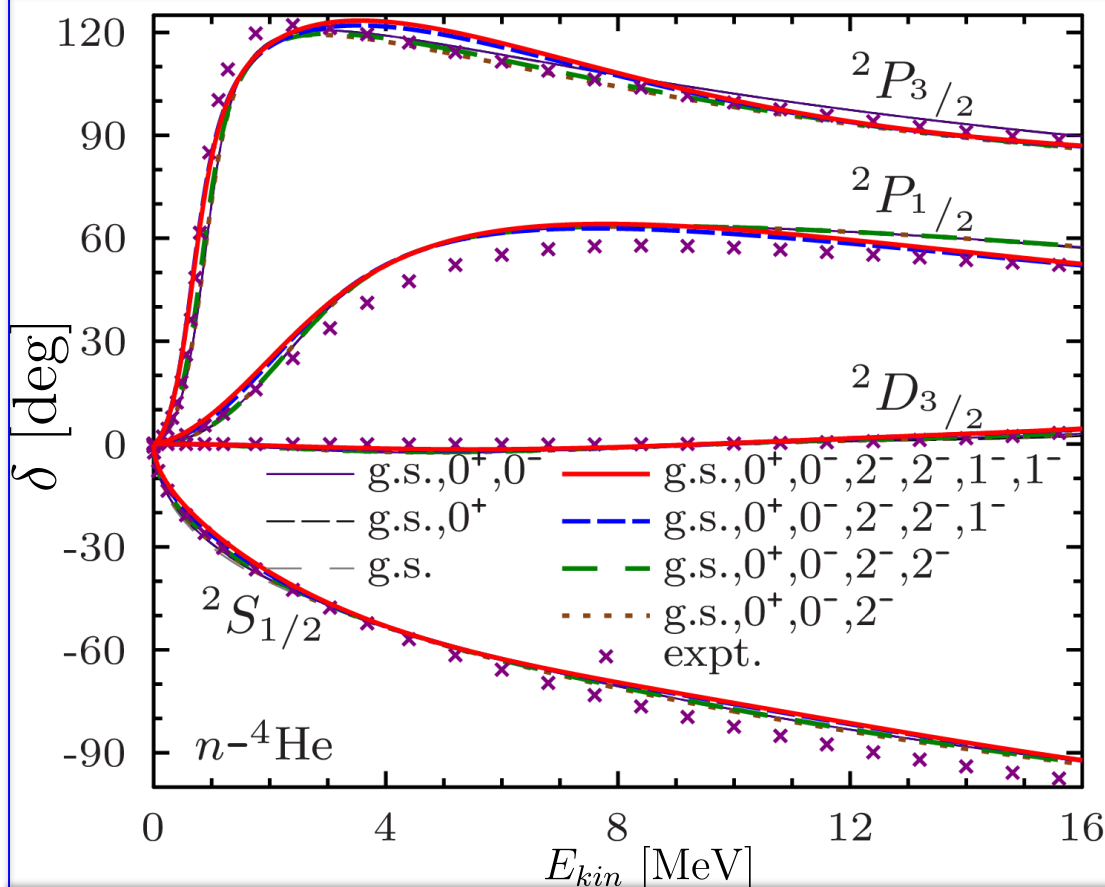


- The 3N interactions influence mostly the P waves.
- The largest splitting between P waves is obtained with NN+3N.

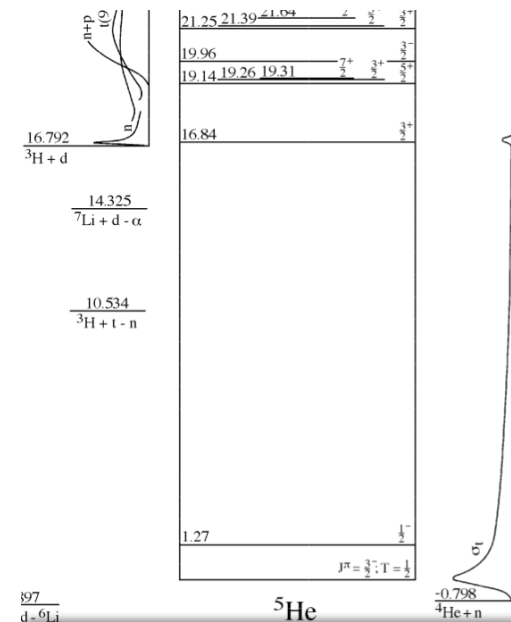
Comparison between NN+3N-ind and NN+3N at $N_{\max}=13$ with six ^4He states and 14 ^5He states.



Study of the convergence with respect to the # of ^4He low-lying states

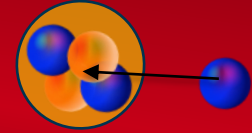


n - ^4He scattering phase-shifts for NN+3N potential with $\lambda=2.0 \text{ fm}^{-1}$ and first 14th low-lying state of ^5He .

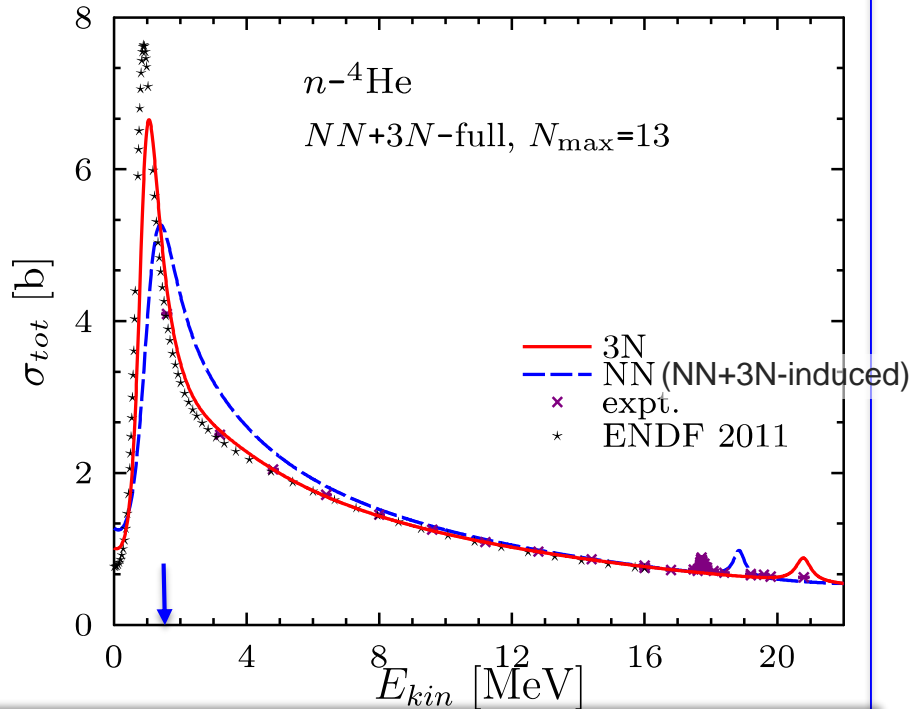


Experimental low-lying states of the $A=5$ nucleon systems.

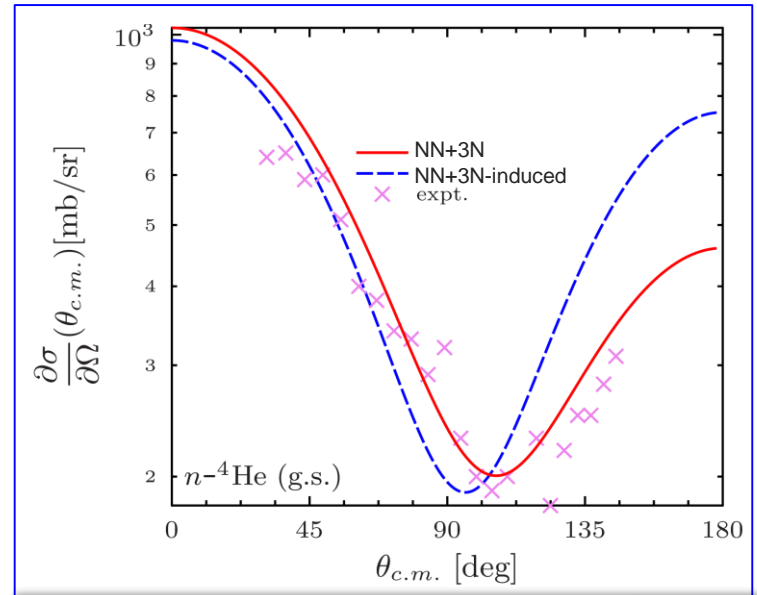
- The convergence pattern looks good.
- The experimental phase-shifts are well reproduced.



Comparison of the elastic cross-section between NN and NN+3N with ^4He (g.s.)

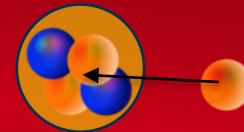


n - ^4He elastic cross-section for NN+3N-induced, NN+3N potentials compared to expt. and ENDF evaluation.



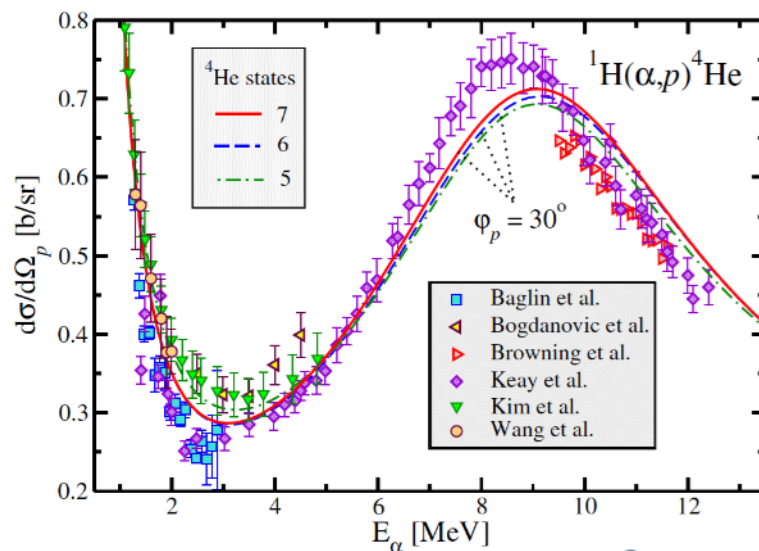
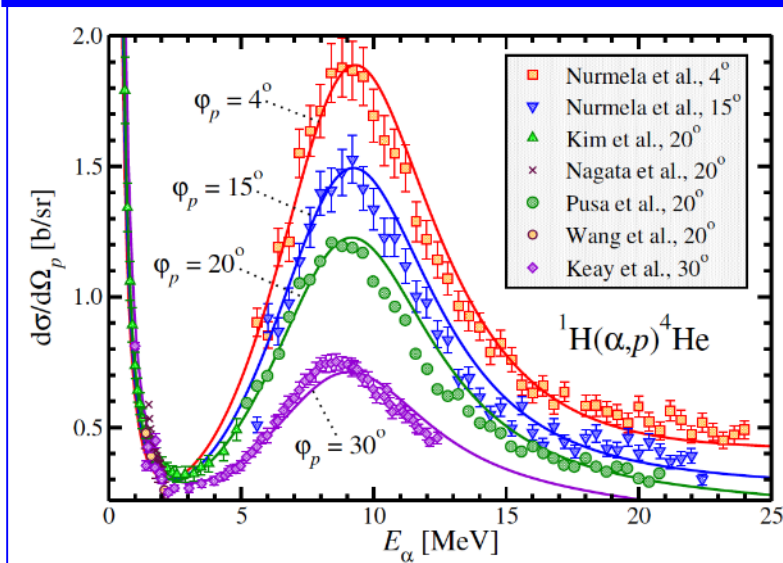
Differential cross-section at $E_{\text{neutron}} = 1.79$ MeV between NN+3N-ind and NN+3N.

- We obtained a better agreement with data when using NN+3N.
- The 3N force is constitutive to the reproduction of the $3/2^+$ resonance.



ρ - ^4He scattering

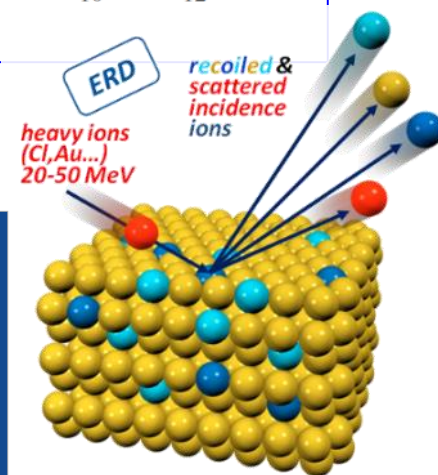
Cross-section compared to experiments focused on proton recoil analysis



ρ - ^4He differential cross-section for NN+3N at ^4He incident energies close to the Rutherford scattering.

The widths predicted by our model (the only ingredient being the interaction) is too large compared to experiment. We can however discriminate

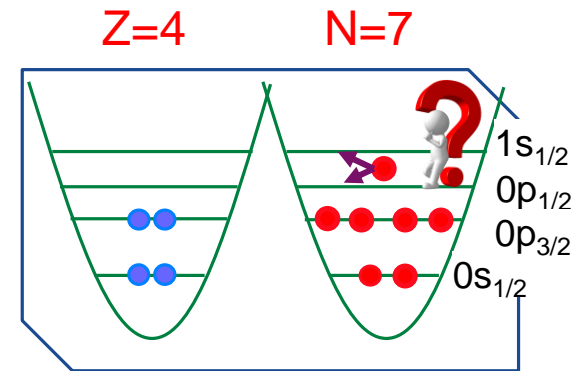
between experimental data and provide accurate predictions for all angles at small and high energies.



NEUTRON-RICH HALO NUCLEUS ^{11}Be



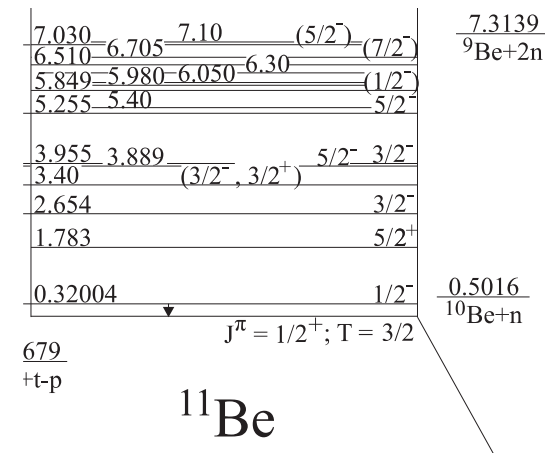
- In the shell model picture g.s. expected to be $J^\pi=1/2^-$ ($Z=6, N=7$) ^{13}C and ($Z=8, N=7$) ^{15}O have $J^\pi=1/2^-$ g.s.
- In reality, ^{11}Be g.s. is $J^\pi=1/2^+$ -- **parity inversion**
- Very weakly bound: $E_{\text{th}}=-0.5$ MeV **Halo state** -- dominated by $^{10}\text{Be}-n$ in the S-wave
- The $1/2^-$ state also bound -- **only by 180 keV**

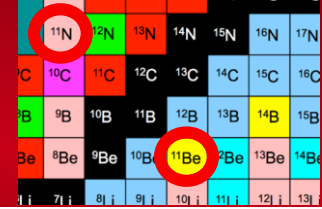


Single particle interpretation using nuclear shell model

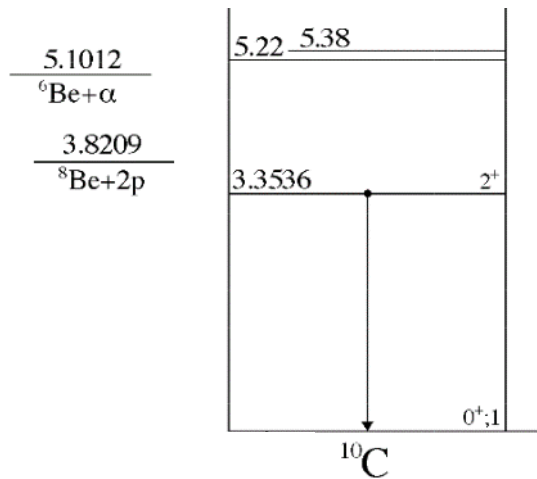
Can we describe ^{11}Be in *ab initio* calculations?

- Continuum must be included
- Does the 3N interaction play a role in the parity inversion?

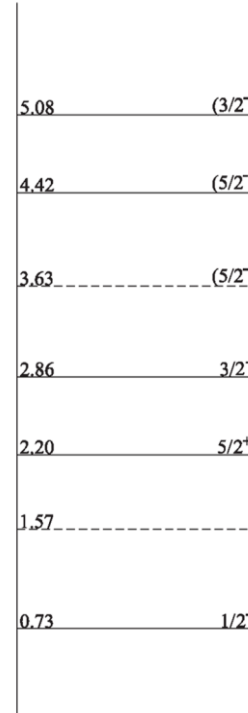




- Limited information about the structure of proton rich ^{11}N – mirror nucleus of ^{11}Be halo nucleus
- Incomplete knowledge of ^{10}C unbound excited states
- Importance of 3N force effects and continuum
- Can structure of exotic nuclei discriminate among different nuclear force models?

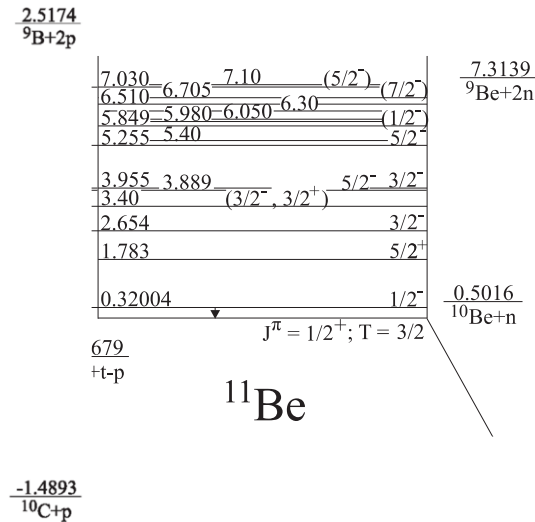


$$\frac{4.0060}{^9\text{B}+p}$$



$$J^\pi = 1/2^+; T = 3/2$$

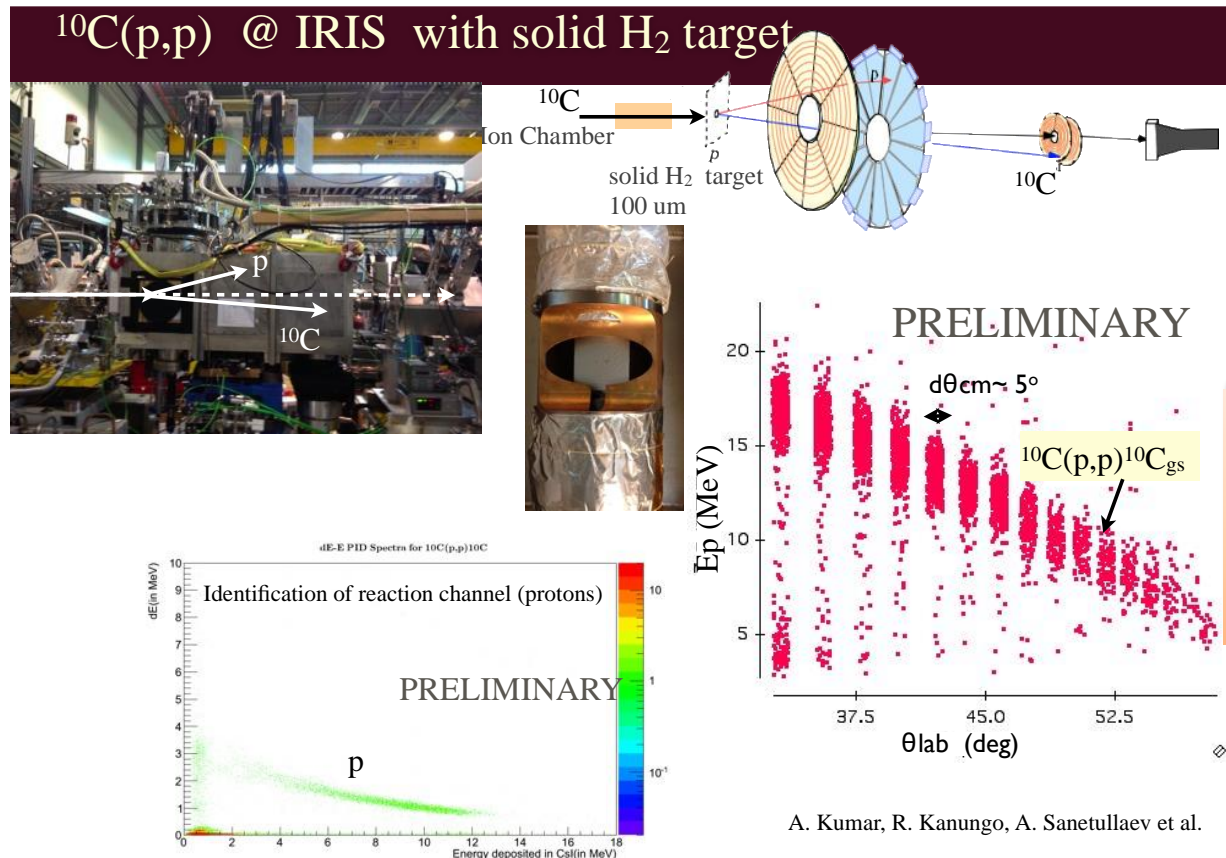
^{11}N



^{11}Be

New experiment at ISAC TRIUMF with reaccelerated ^{10}C

- The first ever ^{10}C beam at TRIUMF
- Angular distributions measured at $E_{\text{CM}} \sim 4.16$ MeV and 4.4 MeV

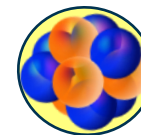
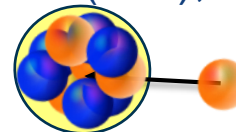


IRIS collaboration:
**A. Kumar, R. Kanungo,
A. Sanetullaev *et al.***

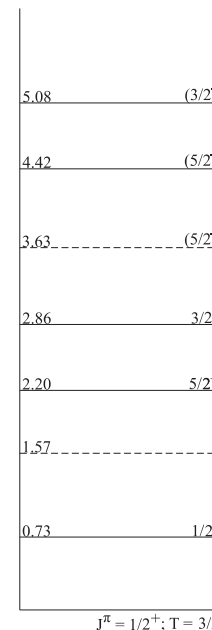
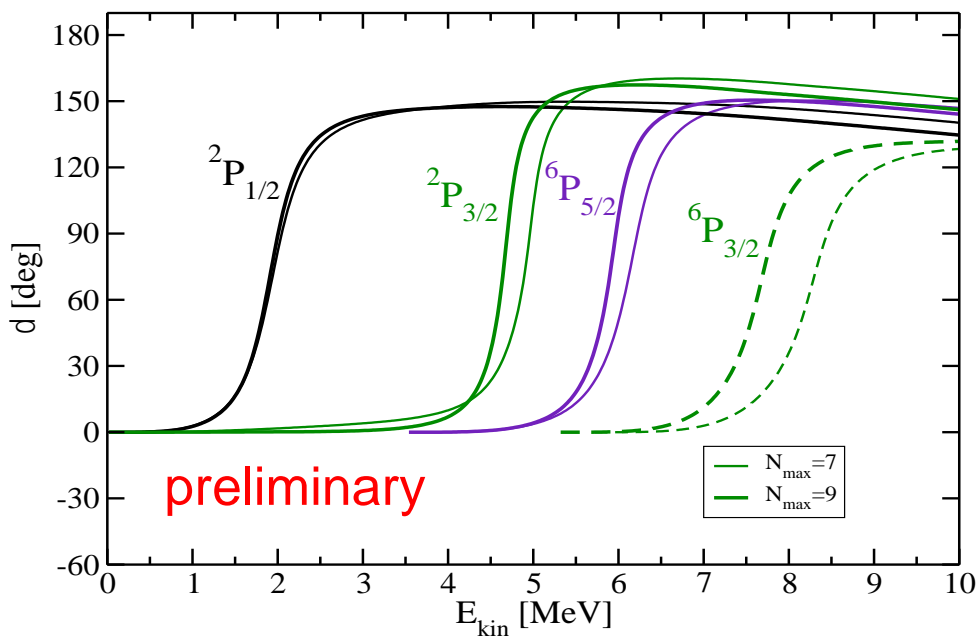
NCSMC calculations with **chiral NN+3N** [N^3 LO NN+N²LO 3NF(400), NNLOsat]

■ ¹⁰C: 0⁺, 2⁺, 2⁺ NCSM eigenstates

■ ¹¹N: ≥4 ($\pi=-1$) and ≥3 ($\pi=+1$) NCSM eigenstates

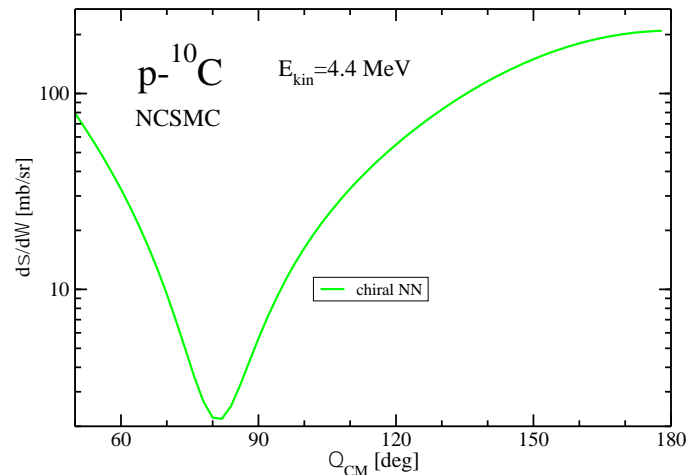
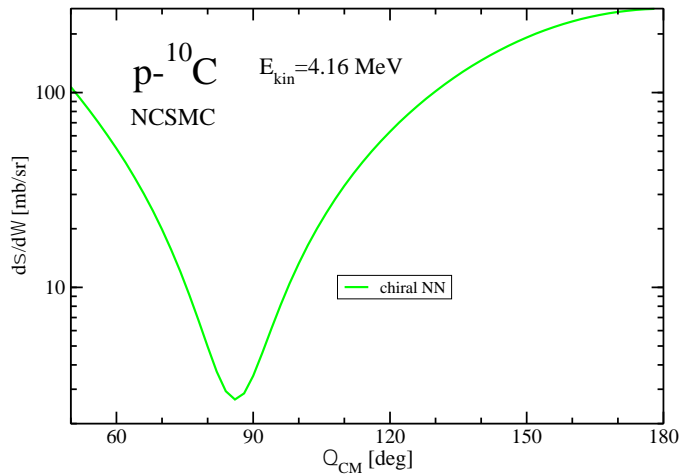
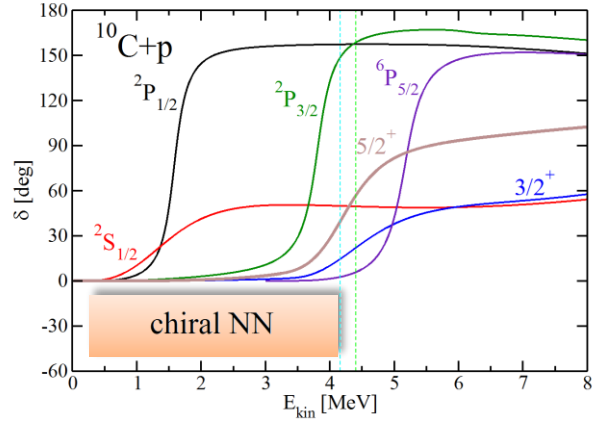


p-¹⁰C phase shifts with NN+3N ($\Lambda=400$)



$\frac{2.5174}{{}^9\text{B}+2\text{p}}$

$\frac{-1.4893}{{}^{10}\text{C}+p}$



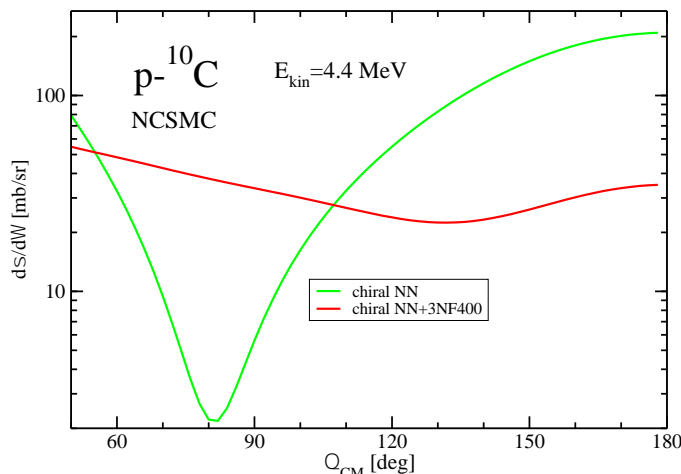
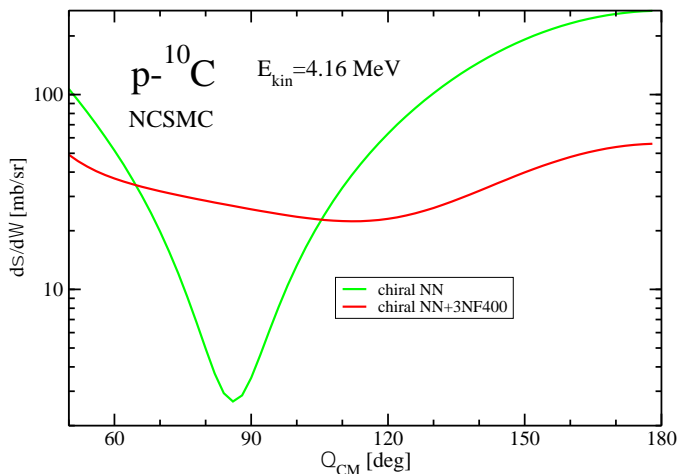
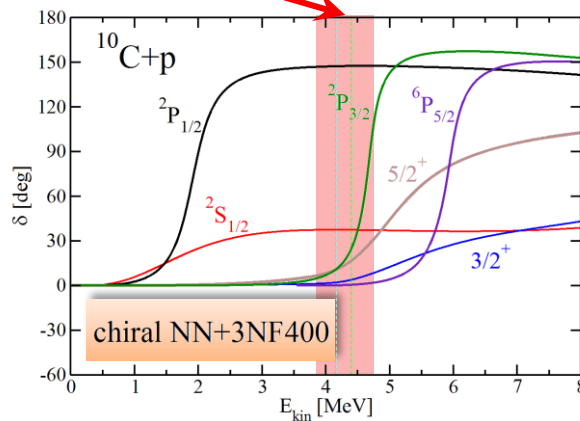
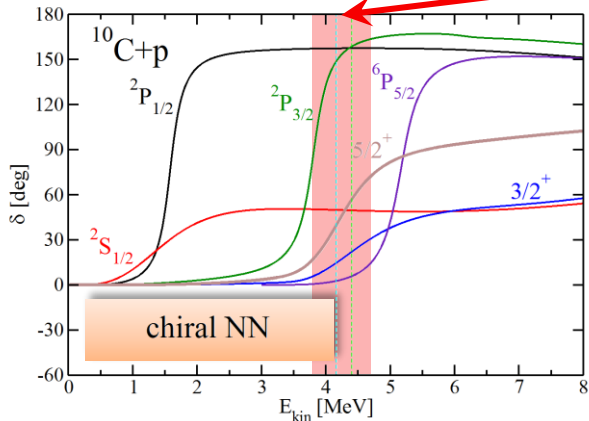
p-¹⁰C SCATTERING: STRUCTURE OF ¹¹N RESONANCES

A. Calci, P. Navratil, G. Hupin, S. Quaglioni, R. Roth *et al.* with IRIS collaboration, in preparation



Area where 3N force effect can be observed

IRIS collaboration:
A. Kumar, R. Kanungo, A. Sanetullaev *et al.*



STRUCTURE OF ^{11}Be FROM CHIRAL NN+3N FORCES

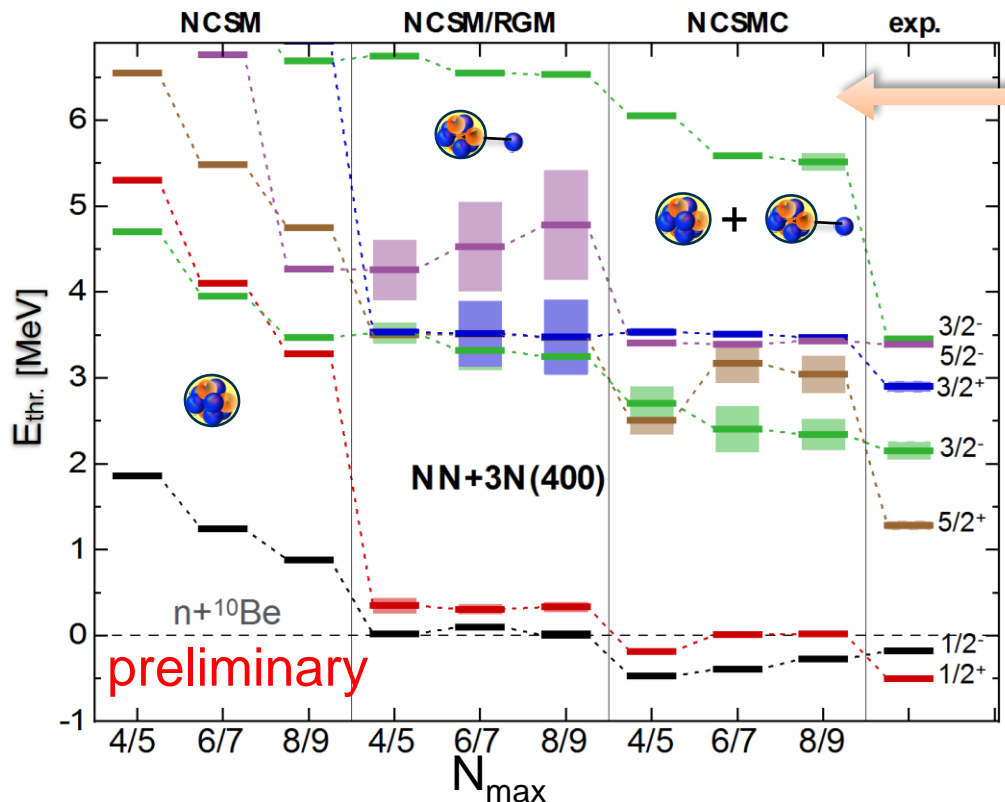
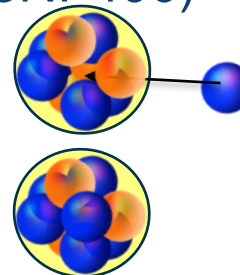
A. Calci, P. Navratil, G. Hupin, S. Quaglioni, R. Roth *et al.* with IRIS collaboration, in preparation



NCSMC calculations including chiral 3N ($\text{N}^3\text{LO NN}+\text{N}^2\text{LO 3NF400}$)

■ ^{10}Be : 0^+ , 2^+ , 2^+ NCSM eigenstates

■ ^{11}Be : ≥ 6 ($\pi=-1$) and ≥ 3 ($\pi=+1$) NCSM eigenstates

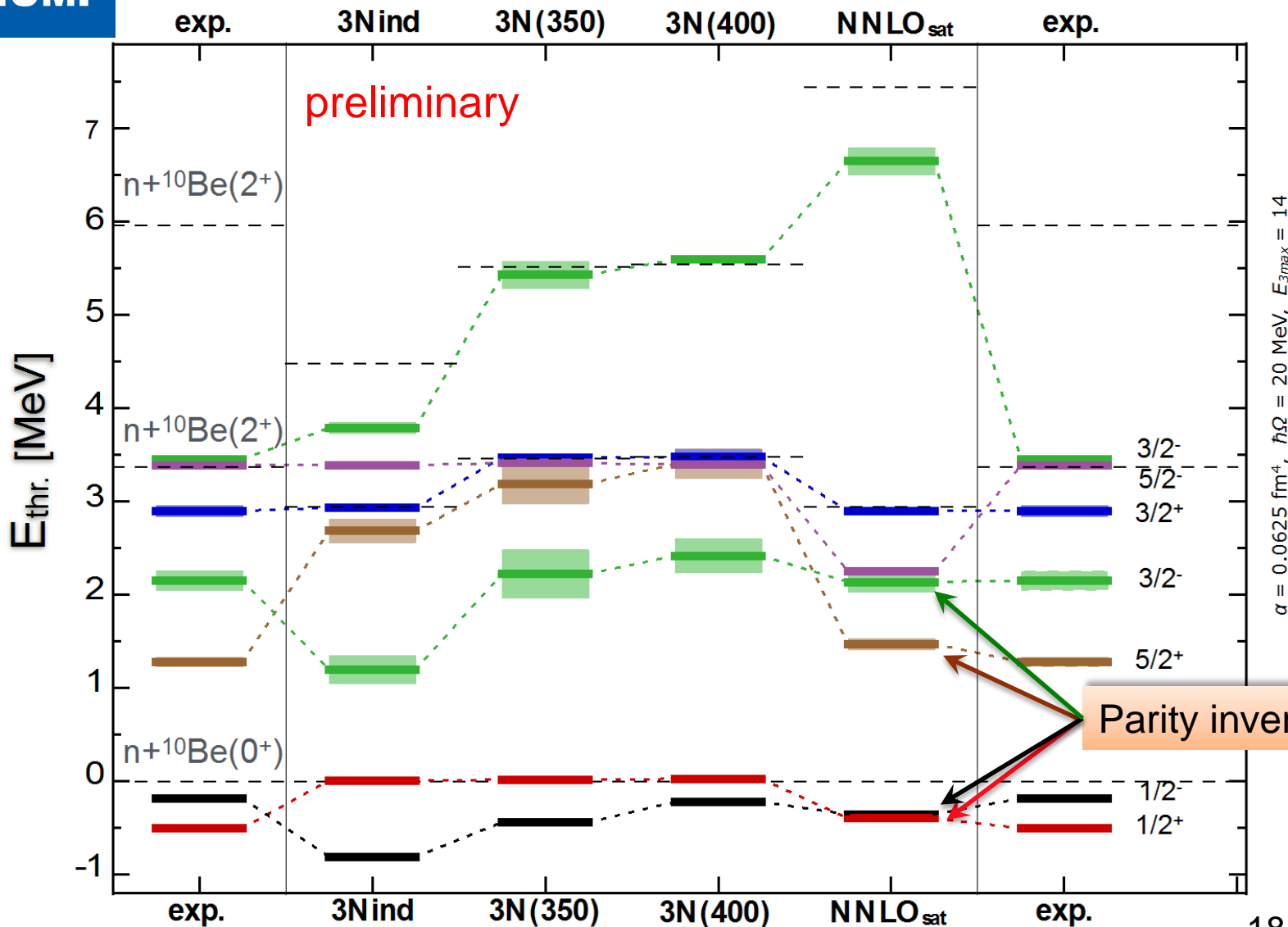


Continuum effects

7.030	6.705	7.10	(5/2 ⁻)	(7/2 ⁻)	7.3139
6.510	6.705	6.30	(1/2 ⁻)		$^9\text{Be}+2n$
5.849	5.980	6.050			
5.255	5.40		5/2 ⁻		
3.955	3.889		5/2 ⁻	3/2 ⁻	
3.40			(3/2 ⁻ , 3/2 ⁺)		
2.654				3/2 ⁻	
1.783				5/2 ⁺	
0.32004				1/2 ⁻	0.5016
					$^{10}\text{Be}+n$
679					
+t-p					
^{11}Be					

^{11}Be WITHIN NCSMC: DISCRIMINATION AMONG CHIRAL NUCLEAR FORCES

A. Calci, P. Navratil, G. Hupin, S. Quaglioni, R. Roth *et al.* with IRIS collaboration, in preparation

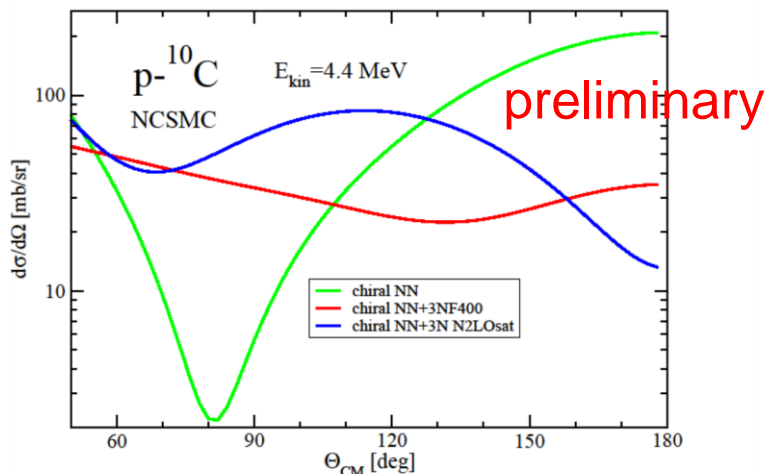
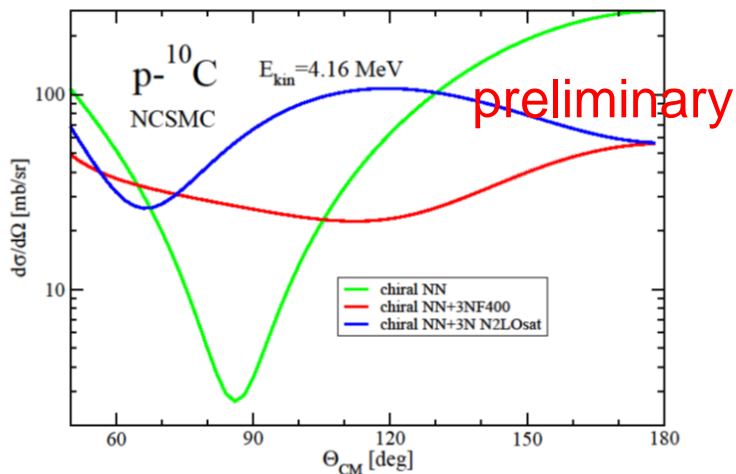
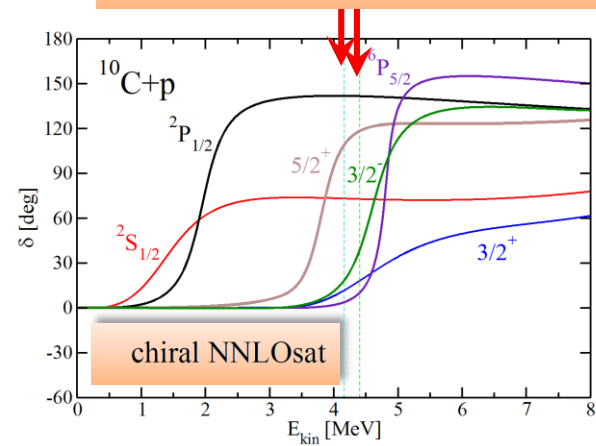
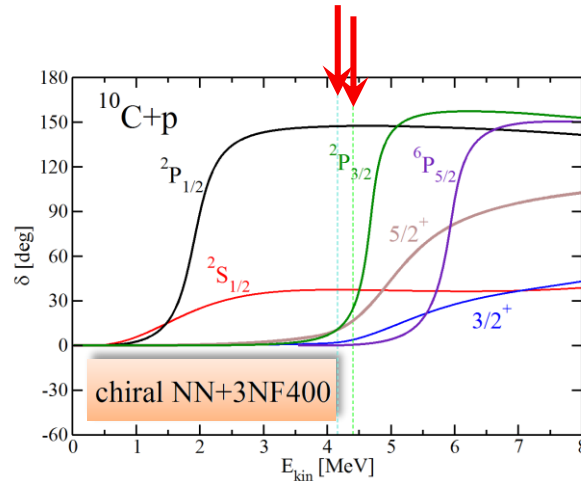
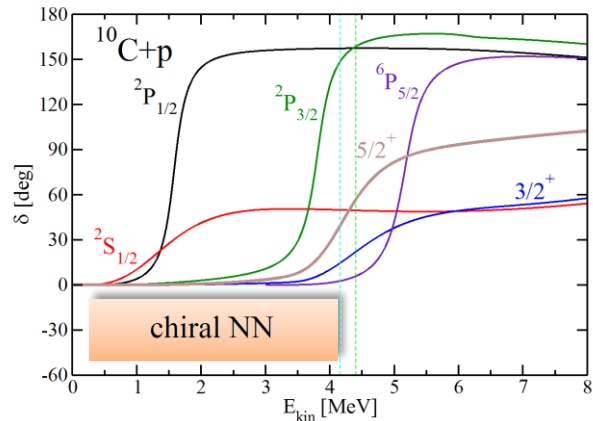


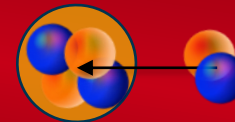
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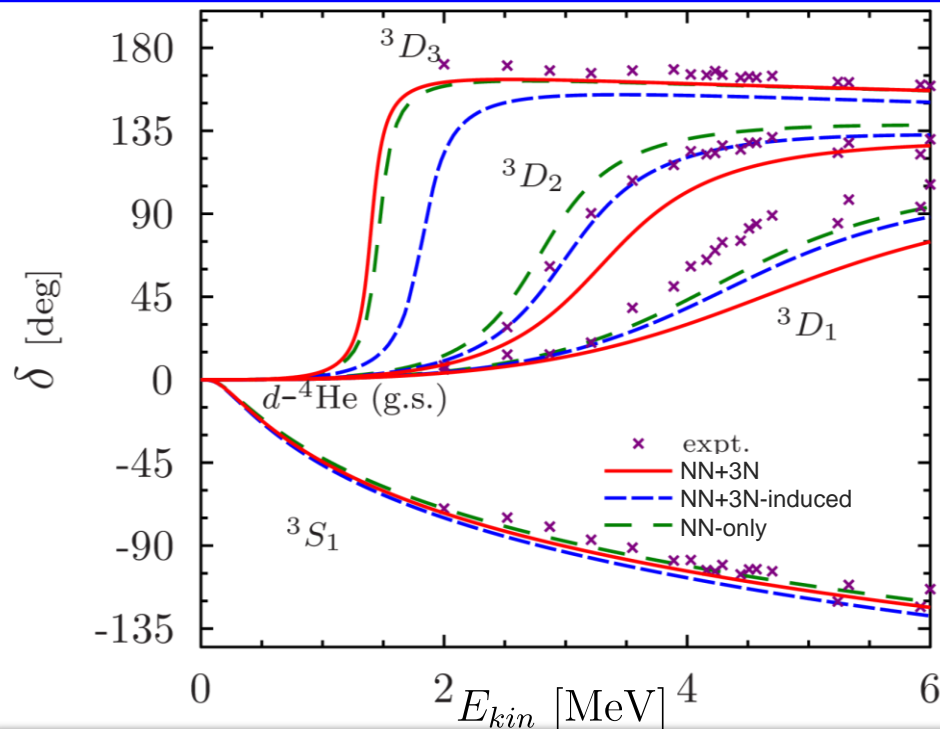
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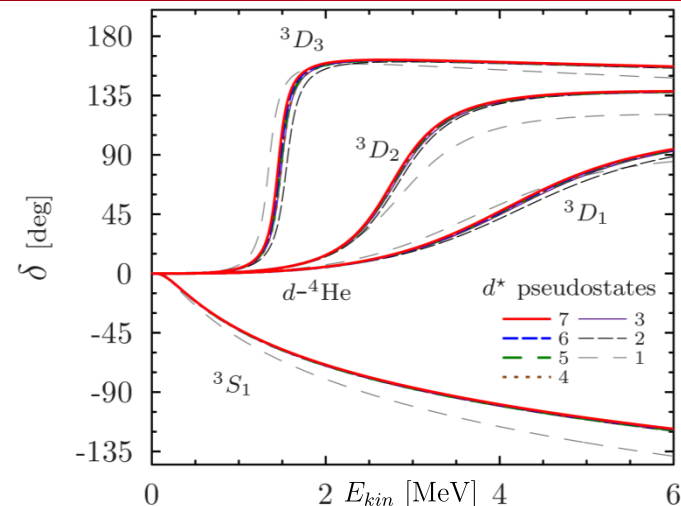


d - ^4He
scattering

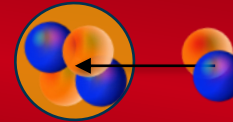
Comparison of the d - α phase-shifts with different interactions ($N_{\text{max}}=11$)



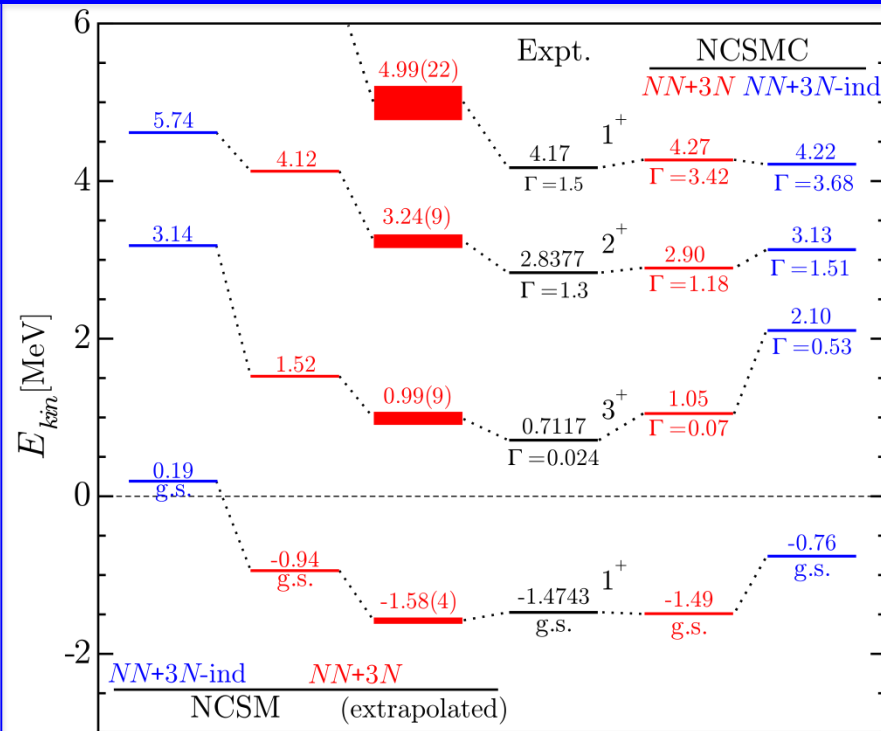
d - ^4He (g.s.) scattering phase-shifts for NN-only, NN+3N-induced, NN+3N-full potential with $\lambda=2.0 \text{ fm}^{-1}$.



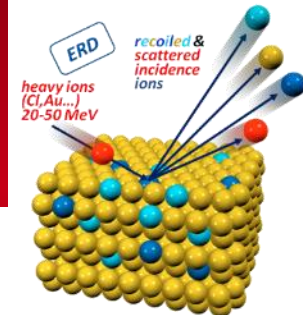
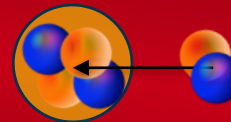
- Best results in a decent model space ($N_{\text{max}}=11$).
- The 3D_3 resonance is reproduced but the 3D_2 and 3D_1 resonance positions are underestimated.
- The 3N force corrects the D -wave resonance positions by increasing the spin-orbit splitting.
- There is room for improvements.



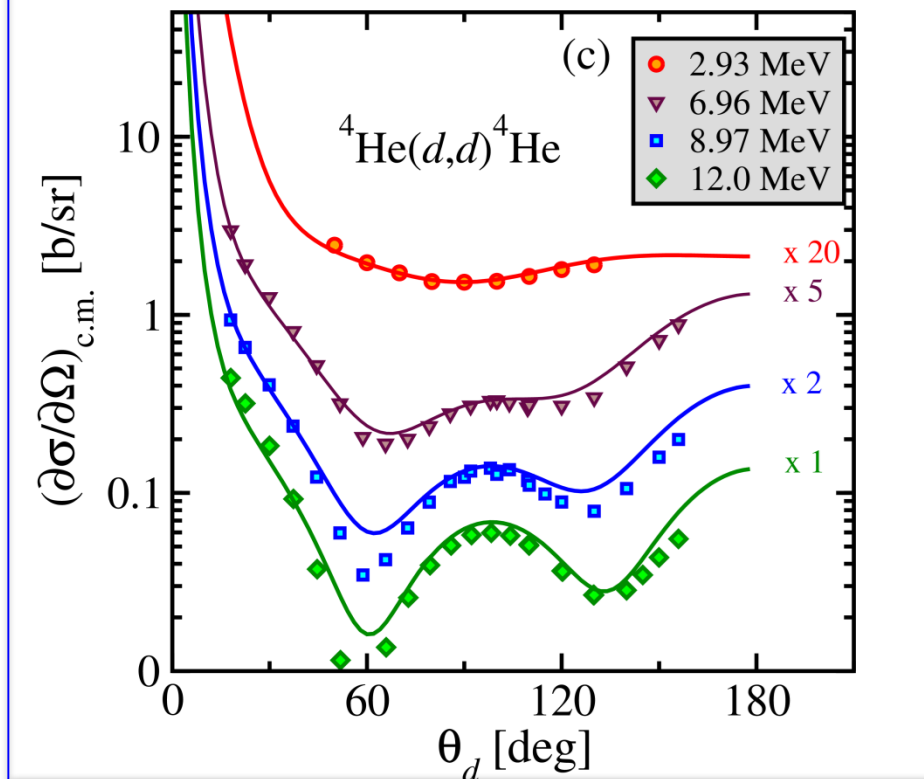
Comparison between NCSMC vs NCSM



- The 3N force is essential to get the correct ${}^6\text{Li}$ g.s. energy and splitting between the 3^+ and 2^+ states.
- The ${}^6\text{Li}$ g.s. is well reproduced.
- There is room for improvements, in particular regarding the 3^+ state.



$^4\text{He}(d,d)^4\text{He}$ angular distribution for various E_d

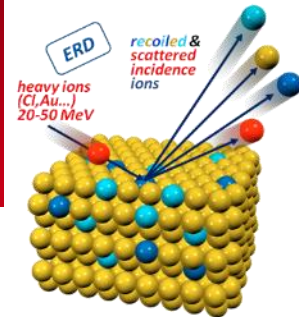
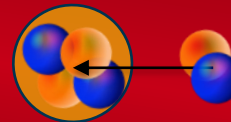


Comparison to experiment of the d - ^4He elastic angular distribution of NCSMC with NN+3N potential at $\lambda=2.0 \text{ fm}^{-1}$.

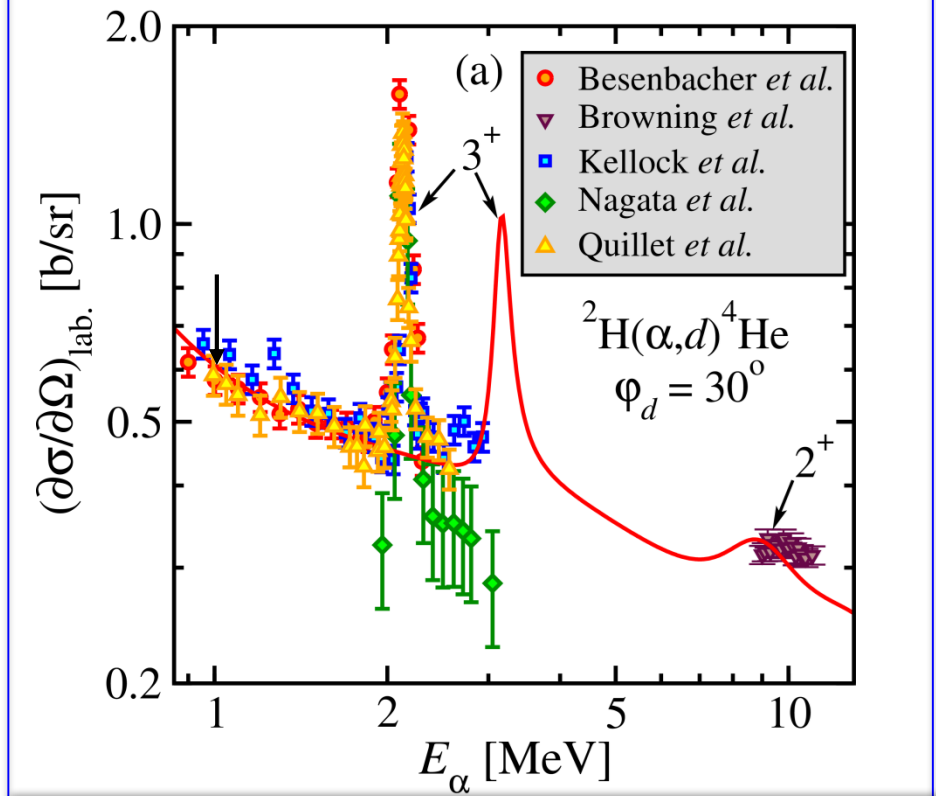
The bulk of the cross section is well reproduced for a large set of kinetic energy and scattering angle.

$^4\text{He}(d,^4\text{He})d$ CROSS-SECTION

G. Hupin, S. Quaglioni and P. Navrátil, PRL114 (2015)

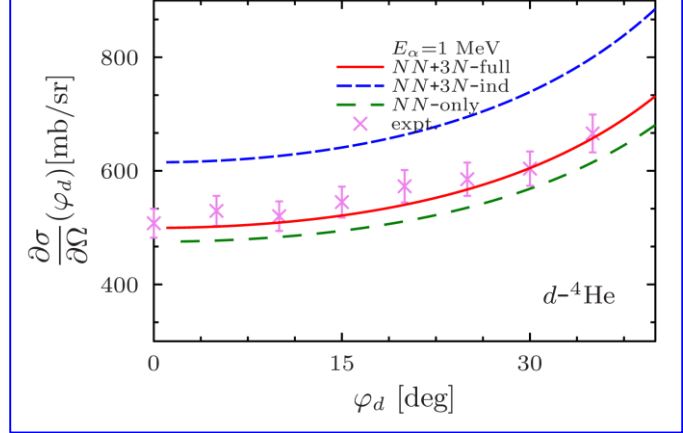


$^4\text{He}(d,^4\text{He})d$ differential cross section at $\varphi=30^\circ$



Comparison to experiment of the $d\text{-}^4\text{He}$ elastic recoil differential cross section of NCSMC with NN+3N potential at $\lambda=2.0 \text{ fm}^{-1}$.

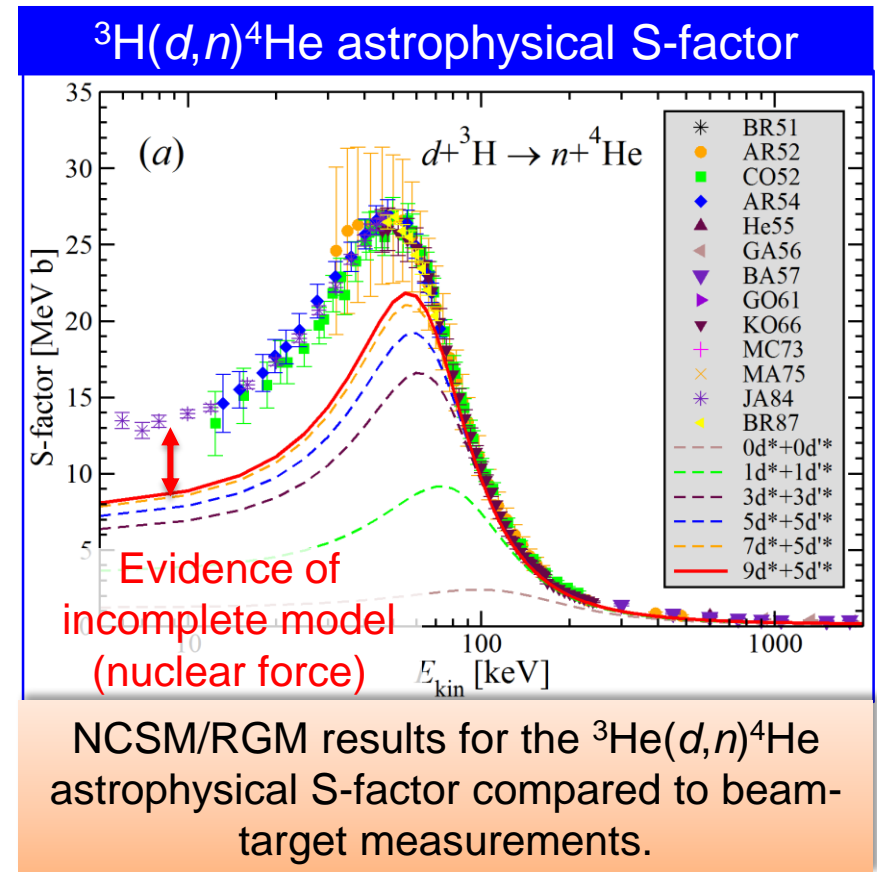
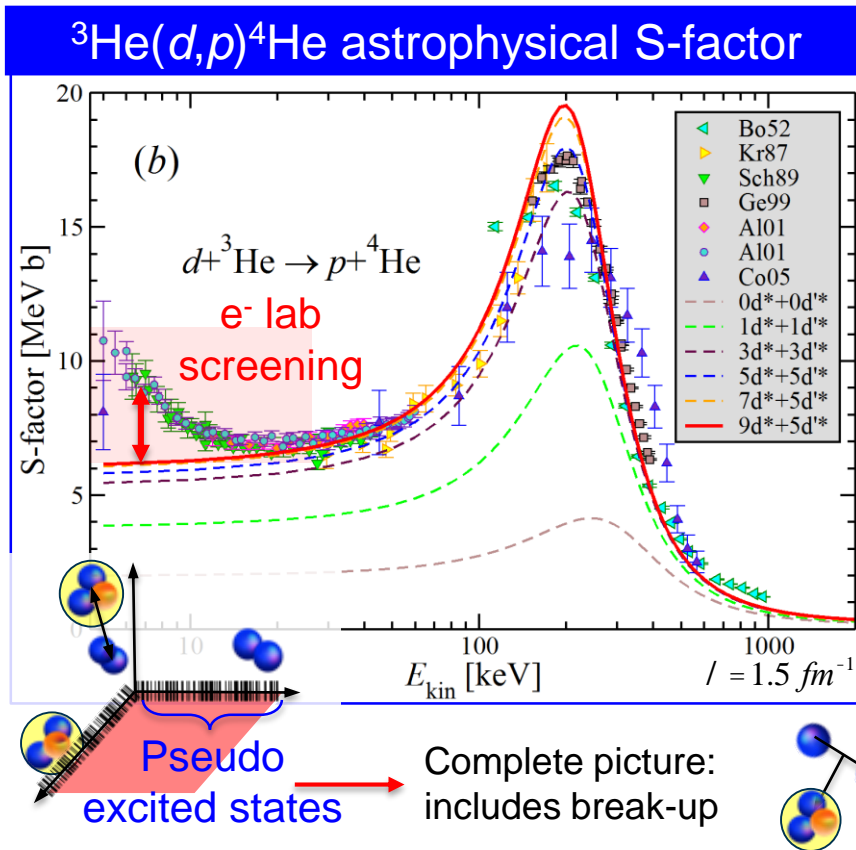
Comparison between potentials



The 3^+ resonance is missed. As its width is very narrow, it has little impact and the bulk of the cross-section.

FIRST STEPS TOWARDS *AB INITIO* CALCULATIONS OF FUSION WITH NCSM/RGM

P. Navrátil, S. Quaglioni, PRL108 (2012)



NCSM/RGM results for the $^3\text{He}(d,n)^4\text{He}$ astrophysical S-factor compared to beam-target measurements.

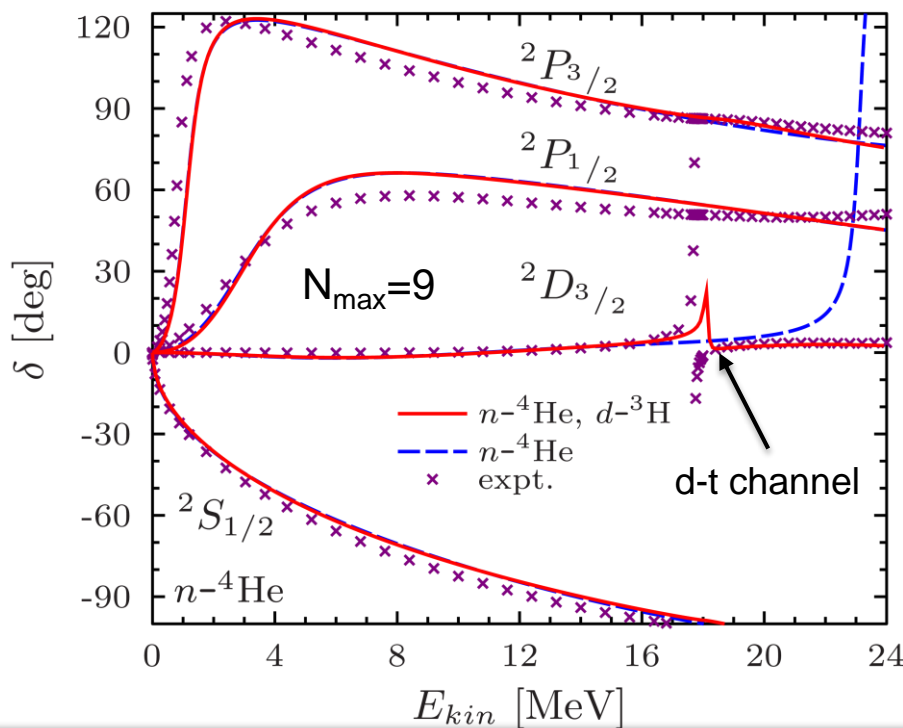
Calculated S-factors converge with the inclusion of the virtual breakup of the deuterium, obtained by means of excited 3S_1 - 3D_1 (d^*) and 3D_2 (d^{**}) pseudo-states.

Incomplete nuclear interaction: requires 3N force (SRG-induced + "real")



d-t fusion

n - ^4He phaseshifts with NCSMC and the chiral two- and three-nucleon force

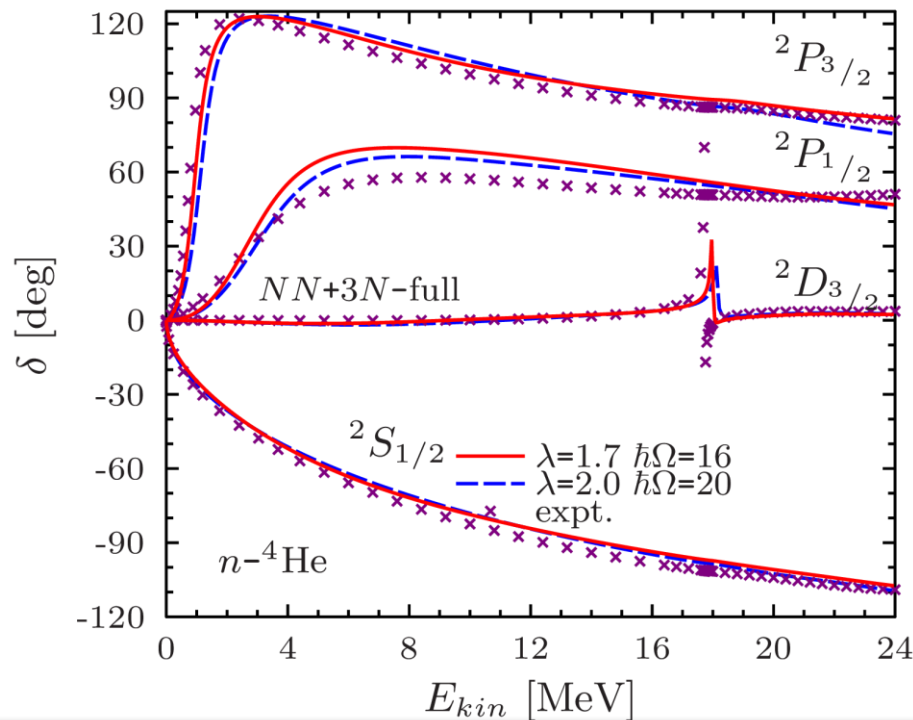


n - ^4He (g.s.) phase shifts with NN+3N potential, $\lambda=2.0 \text{ fm}^{-1}$, with eigenstates of ^5He at $N_{\text{max}}=9$.

- Perspective to provide accurate $t(d,n)^4\text{He}$ fusion cross-section for the effort toward earth-based fusion energy generation.
- The d - t fusion is known to be very sensitive to the spin-orbit and isospin part of the nuclear interaction.



Towards d-t fusion with NCSMC: comparison between effective interactions



$n+{}^4\text{He}(\text{g.s.})$ phase shifts with NN+3N potential, with eigenstates of ${}^5\text{He}$.

$\lambda = 1.7 \text{ fm}^{-1}$ and $\hbar\Omega = 16 \text{ MeV}$

N_{max}	${}^5\text{He} (\frac{3}{2}^+)$	${}^4\text{He}$	${}^3\text{H}$	d
6	-5.1574	-28.1739	-8.2909	-2.0404
8	-7.2529	-28.3677	-8.3893	-2.092
10	-8.1861	-28.4348	-8.4455	-2.1654
12			-8.4546	-2.1781
∞	-8.9373	-28.4693	-8.4565	-2.224

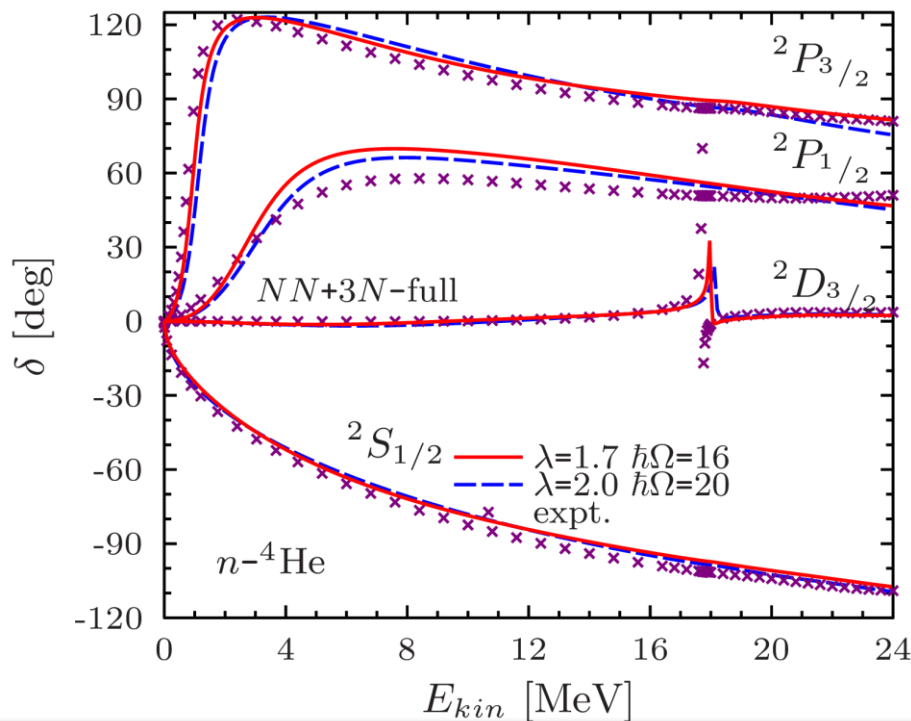
$\lambda = 2.0 \text{ fm}^{-1}$ and $\hbar\Omega = 20 \text{ MeV}$

N_{max}	${}^5\text{He} (\frac{3}{2}^+)$	${}^4\text{He}$	${}^3\text{H}$	d
6	-1.9037	-27.7923	-8.0971	-1.9199
8	-4.9122	-28.2341	-8.2721	-1.9633
10	-6.6422	-28.4078	-8.4099	-2.1172
12	-7.7062	-28.4438	-8.4387	-2.1351
∞	-9.1813	-28.4796	-8.4469	-2.224

NCSM convergence of compound and cluster states.



Towards d-t fusion with NCSMC: comparison between effective interactions



$n+{}^4\text{He}(\text{g.s.})$ phase shifts with NN+3N potential, with eigenstates of ${}^5\text{He}$.

$\lambda = 1.7 \text{ fm}^{-1}$ and $\hbar\Omega = 16 \text{ MeV}$

N_{max}	${}^5\text{He} \left(\frac{3}{2}^+\right)$	${}^4\text{He}$	${}^3\text{H}$	d
6	42.29	1.04	1.96	8.26
8	18.85	0.36	0.80	5.94
10	8.41	0.12	0.13	2.63
12	-	-	0.02	2.06

$\lambda = 2.0 \text{ fm}^{-1}$ and $\hbar\Omega = 20 \text{ MeV}$

N_{max}	${}^5\text{He} \left(\frac{3}{2}^+\right)$	${}^4\text{He}$	${}^3\text{H}$	d
6	78.70	2.38	4.25	13.67
8	45.04	0.83	2.18	11.72
10	25.68	0.22	0.55	4.80
12	13.78	0.09	0.21	4.00

Relative error (%) with respect to converged value

A smaller frequency allows us to capture the dilute nature of the $\frac{3}{2}^+$ resonance.

CONCLUSIONS AND OUTLOOK



Evolution of stars, birth, main sequence, death

We are extending the *ab initio* NCSM/RGM approach to describe low-energy reactions with two- and three-nucleon interactions.

We are able to describe:

- Nucleon-nucleus collisions with NN+3N interaction
- Deuterium-nucleus collisions with NN+3N interaction as the n-n
- NCSMC for single- and two-nucleon projectile

Work in progress:

- Fusion reactions with our best complete *ab initio* approach
- The present NNN force is "incomplete", need to go to N³LO