

# Recent advances in nuclear theory

Thomas Papenbrock

THE UNIVERSITY of TENNESSEE  KNOXVILLE

and

OAK RIDGE NATIONAL LABORATORY

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# Collaborators

@ ORNL / UTK: **E. A. Coello Pérez**, **A. Ekström**, G. Hagen, G. R. Jansen, **K. Wendt**

@ Chalmers: **B. Carlsson**, C. Forssén, **D. Sääf**

@ Hebrew U: N. Barnea

@ Michigan State U: M. Hjorth-Jensen, W. Nazarewicz

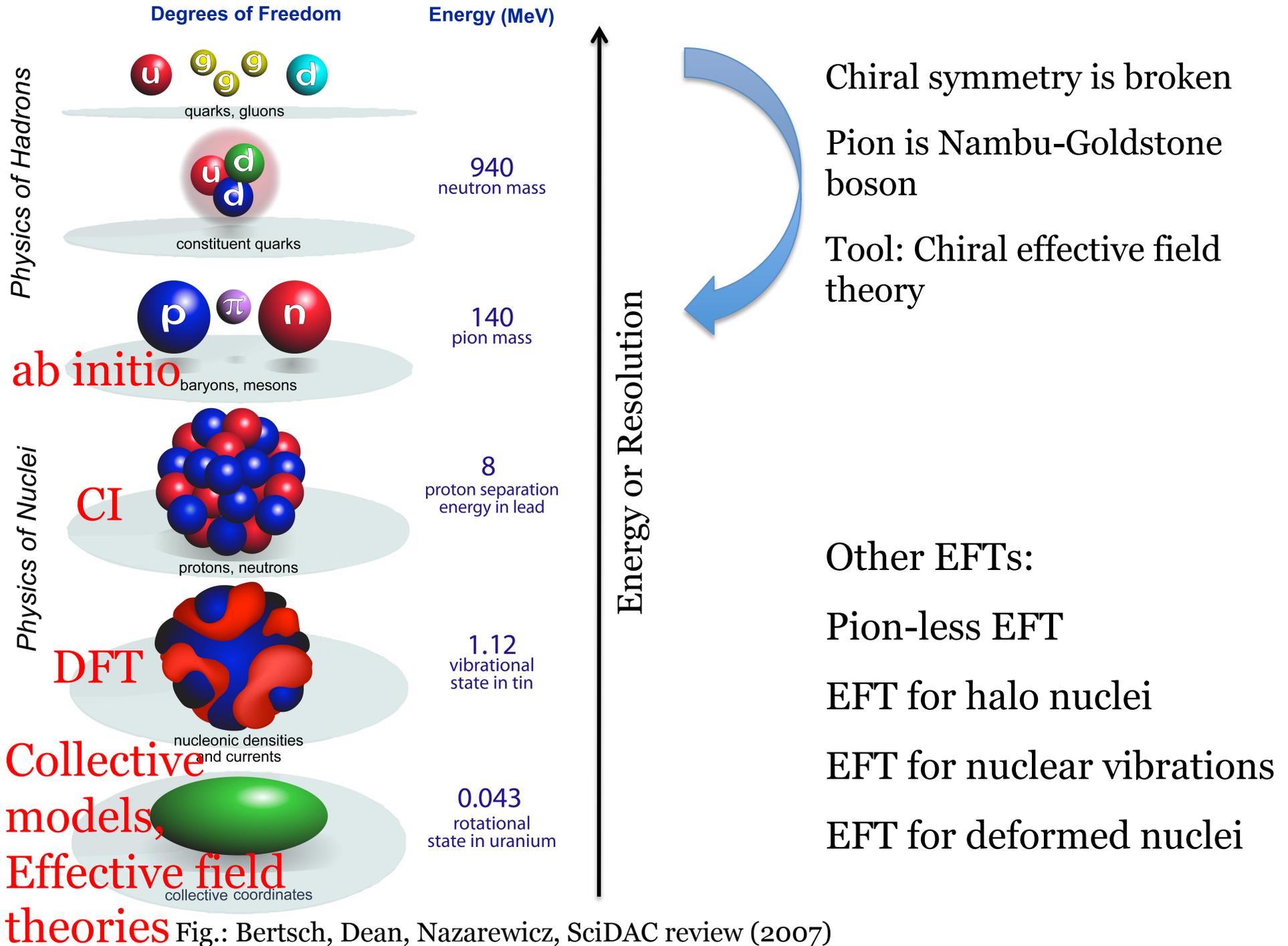
@ Trento: G. Orlandini

@ TRIUMF: S. Bacca, J. D. Holt, **M. Miorelli**, P. Navrátil, **T. Xu**

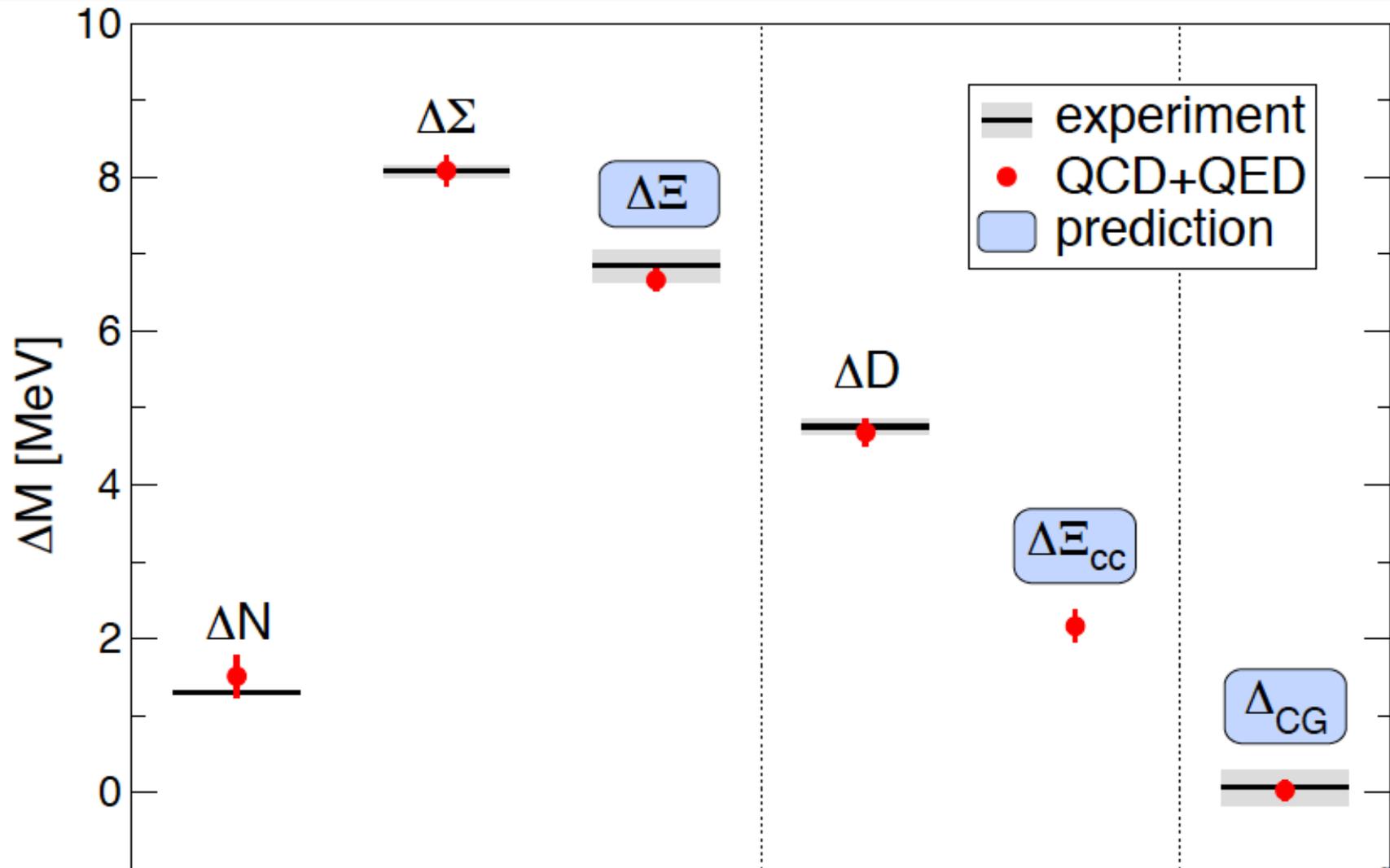
@ TU Darmstadt: **C. Drischler**, K. Hebeler, A. Schwenk, **J. Simonis**

@ CERN: COLLAPS Collaboration (**R. Garcia Ruiz** et al.)

# Energy scales and relevant degrees of freedom



# Lattice QCD describes the nucleon



Mass splittings from lattice QCD & QED  
Borsanyi et al., Science (2015)

# Toward bridging QCD and nuclei

$m_\pi$	140	510	805	805
Nucleus	[Nature]	[5]	[6]	[This work]
n	939.6	1320.0	1634.0	1634.0
p	938.3	1320.0	1634.0	1634.0
nn	-	$7.4 \pm 1.4$	$15.9 \pm 3.8$	$15.9 \pm 3.8$ *
D	2.224	$11.5 \pm 1.3$	$19.5 \pm 4.8$	$19.5 \pm 4.8$ *
$^3\text{n}$	-			-
$^3\text{H}$	8.482	$20.3 \pm 4.5$	$53.9 \pm 10.7$	$53.9 \pm 10.7$ *
$^3\text{He}$	7.718	$20.3 \pm 4.5$	$53.9 \pm 10.7$	$53.9 \pm 10.7$
$^4\text{He}$	28.30	$43.0 \pm 14.4$	$107.0 \pm 24.2$	$89 \pm 36$
$^5\text{He}$	27.50			$98 \pm 39$
$^5\text{Li}$	26.61			$98 \pm 39$
$^6\text{Li}$	32.00			$122 \pm 50$

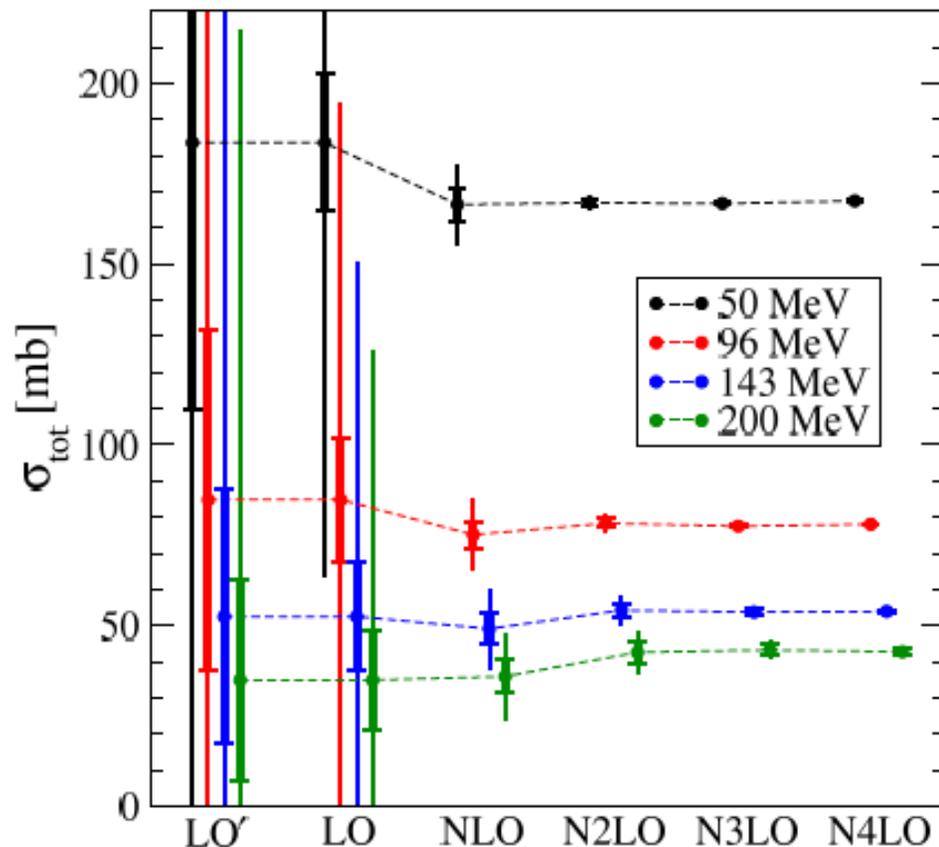
Match pion-less EFT to lattice QCD at large pion masses.  
Not yet in the phase of spontaneously broken chiral symmetry.

Barnea et al., PRL (2015)

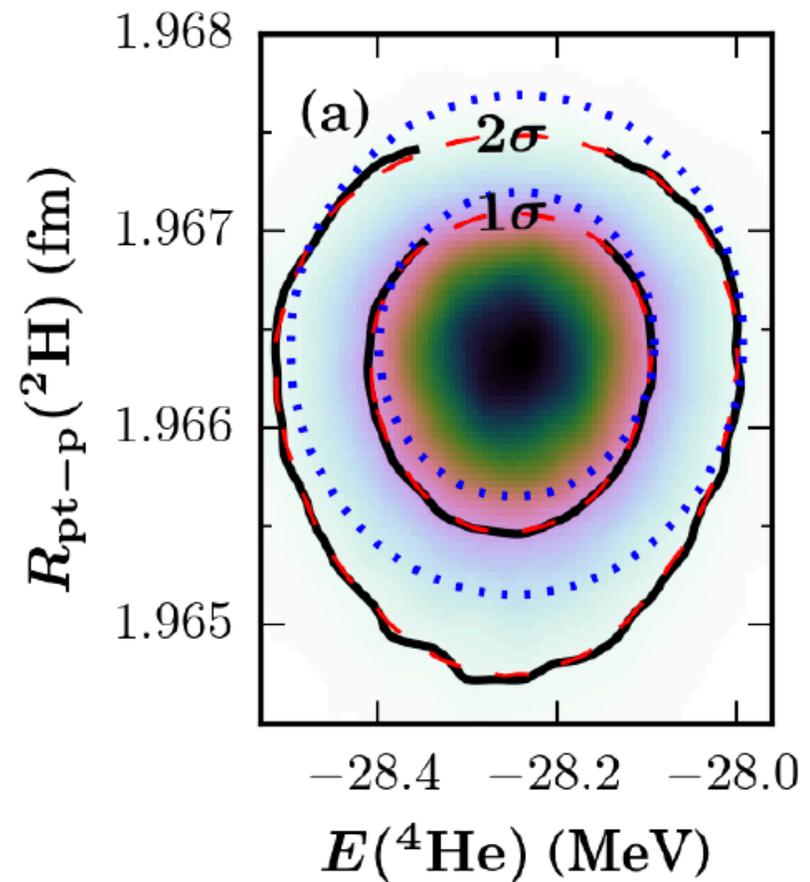
# Quantified theoretical uncertainties

EFTs provide us with advantages over models:

- Uncertainty estimates readily available (based on power counting)
- Quantified uncertainties (based on Bayesian statistics *and* testable assumptions)



Furnstahl et al., PRC (2015)



Carlsson et al., PRX (2016)

# Computation of emergent phenomena

## Emergent phenomena

- Nuclear saturation
- Nuclear deformation and vibrations
- Clustering ( $\alpha$  particles, halos, ...)

## Really hard to compute from first principles

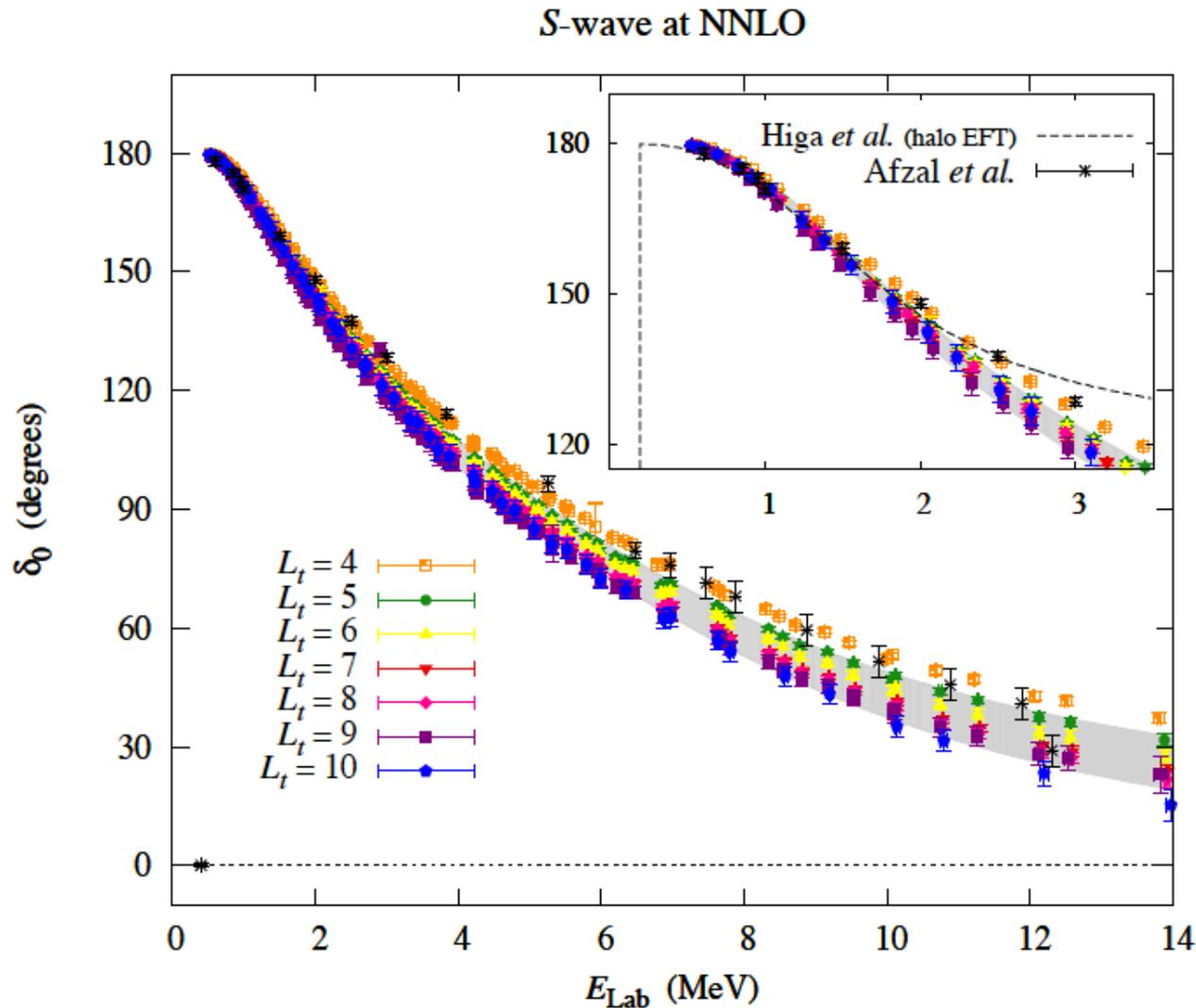
- Finely tuned
- Emergent low-energy scales / multi-scale problem
- Complex and collective in nature

## Usually fixed in models

- $\hbar\omega$  sets nuclear saturation & radii in shell model
- Deformed shell model, collective & algebraic models
- $\alpha$ -particle cluster models of the nucleus

Opportunities for EFTs & challenges for ab initio approaches

# $\alpha$ - $\alpha$ scattering from lattice EFT



Recent reviews on ab initio reactions:

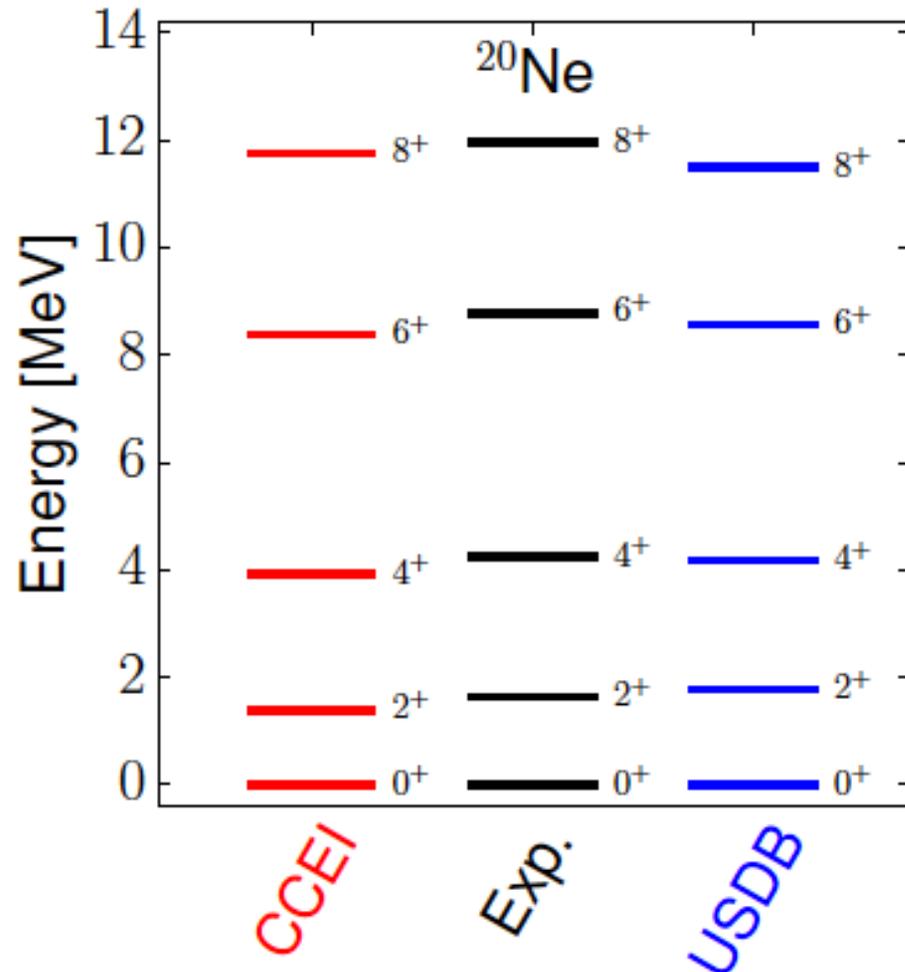
Bacca & Pastore (2014);  
Navrátil, Quaglioni, Hupin,  
Romero-Redondo, Calci (2016).

Electroweak processes:

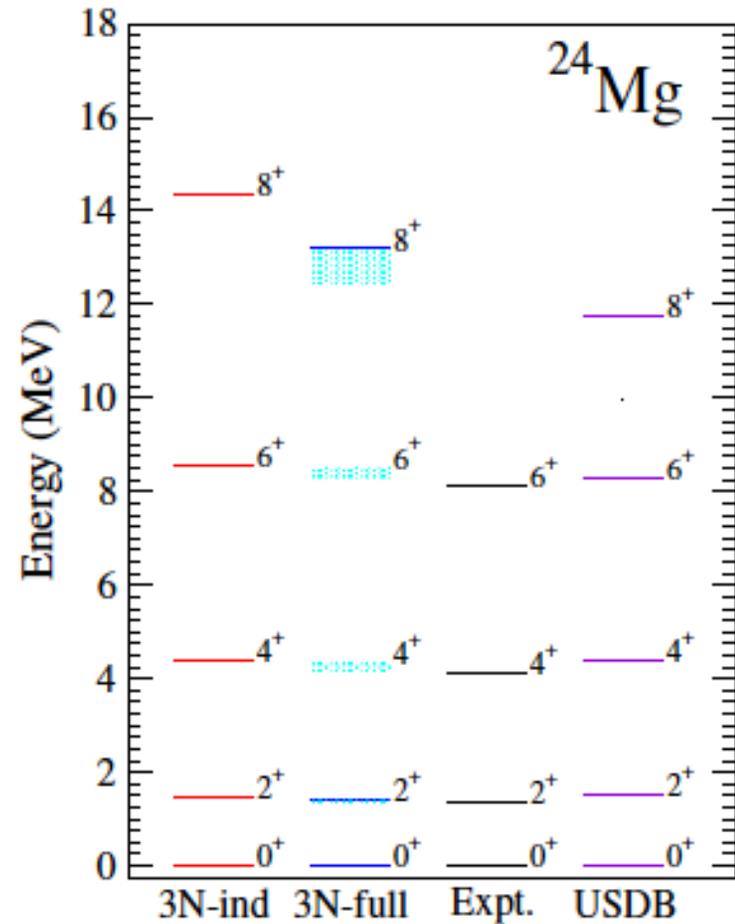
Pastore et al. (2013);  
Lovato et al. (2013);  
Carlson et al. (2015).

S. Elhatisari et al., Nature 528, 111 (2015)

# Nuclear deformation from first principles



Jansen et al., 1511.00757



Stroberg et al., 1511.02802

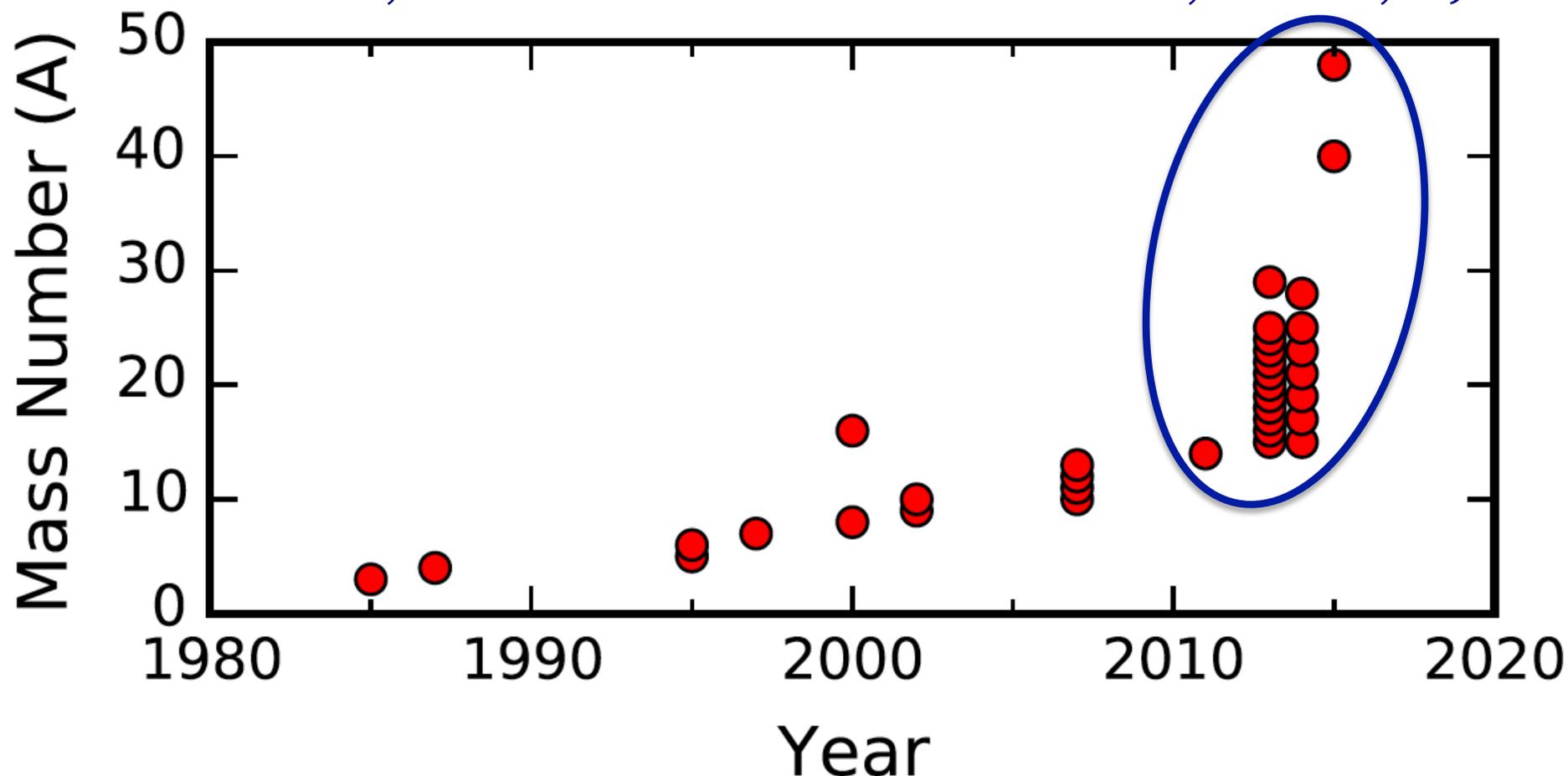
Deformation in *p*-shell nuclei:

Caprio, Maris & Vary, PLB (2013); Caprio et al., IJMPE (2015); Dytrych et al., PRL (2013)

# Trend in realistic *ab initio* calculations

## Explosion of many-body methods

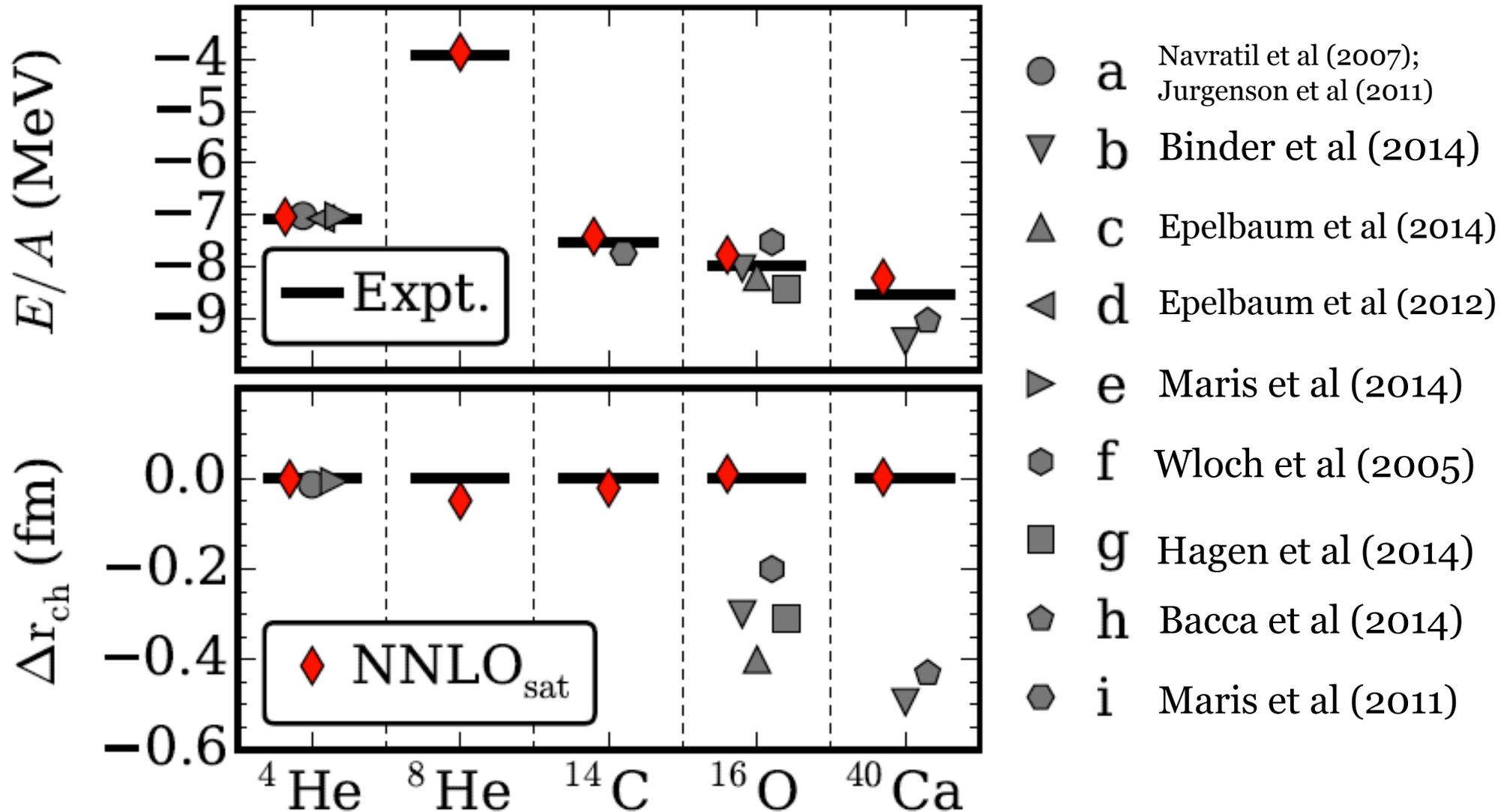
(Coupled clusters, Green's function Monte Carlo, In-Medium SRG, Lattice EFT, MCSM, No-Core Shell Model, Self-Consistent Green's Function, UMOA, ...)



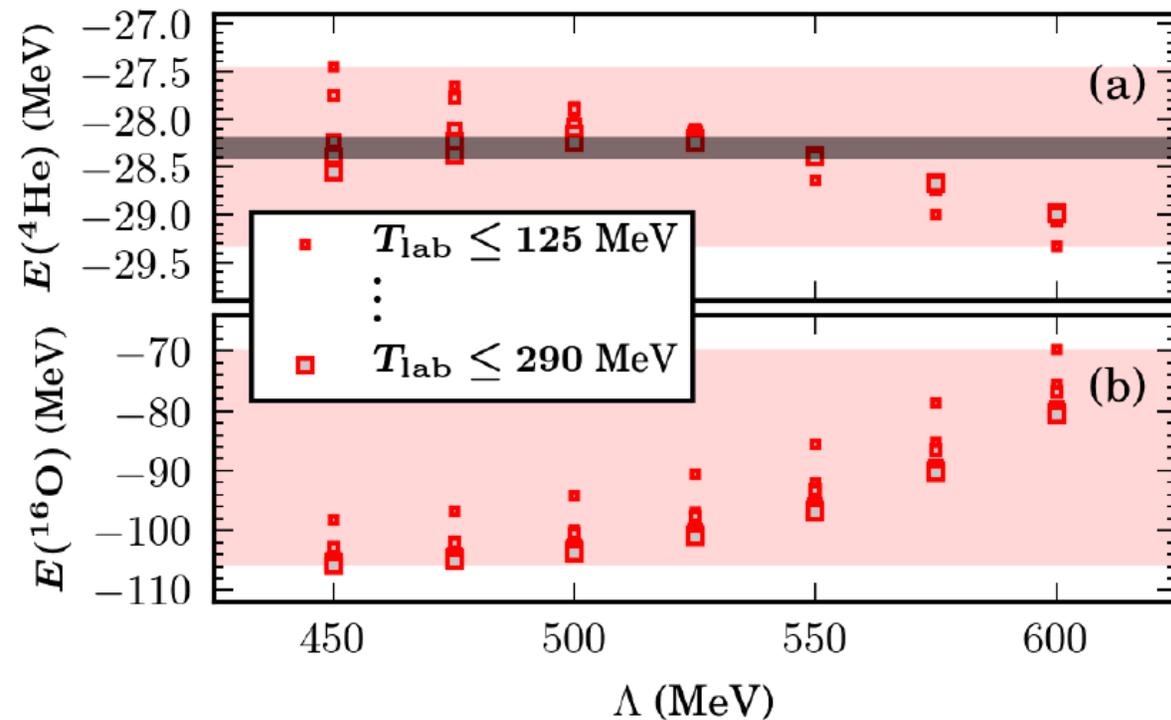
Computational capabilities exceed accuracy of available interactions  
[Binder *et al*, Phys. Lett. B 736 (2014) 119]



# NNLO<sub>sat</sub> – improved binding and radii by construction



# Nuclear saturation is finely tuned

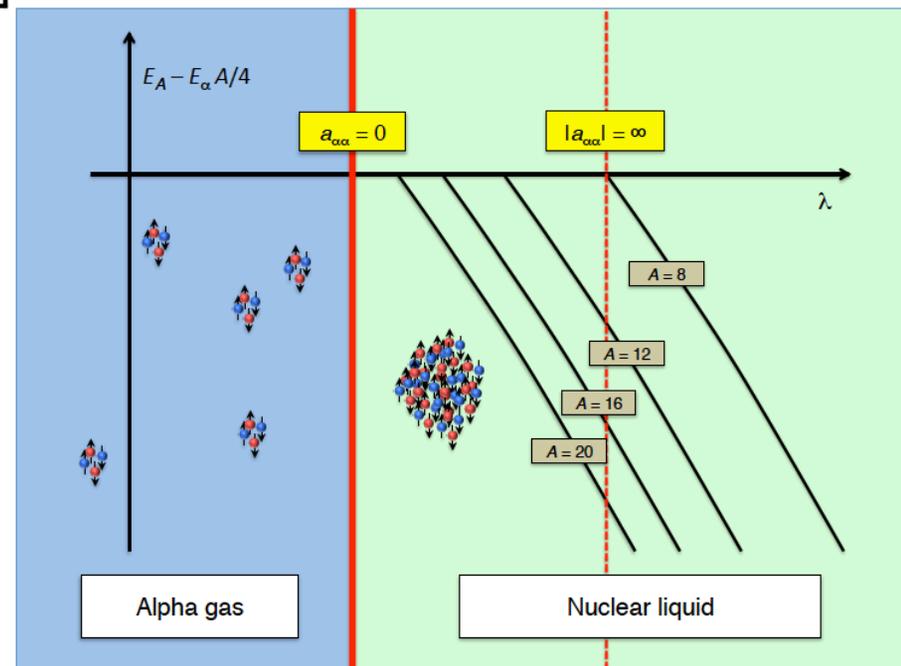


A 4% change in the binding energy of  ${}^4\text{He}$  yields a 15% change in  ${}^{16}\text{O}$  [B. Carlsson, A. Ekström, C. Forssén et al., PRX **6**, 011019 (2016)].

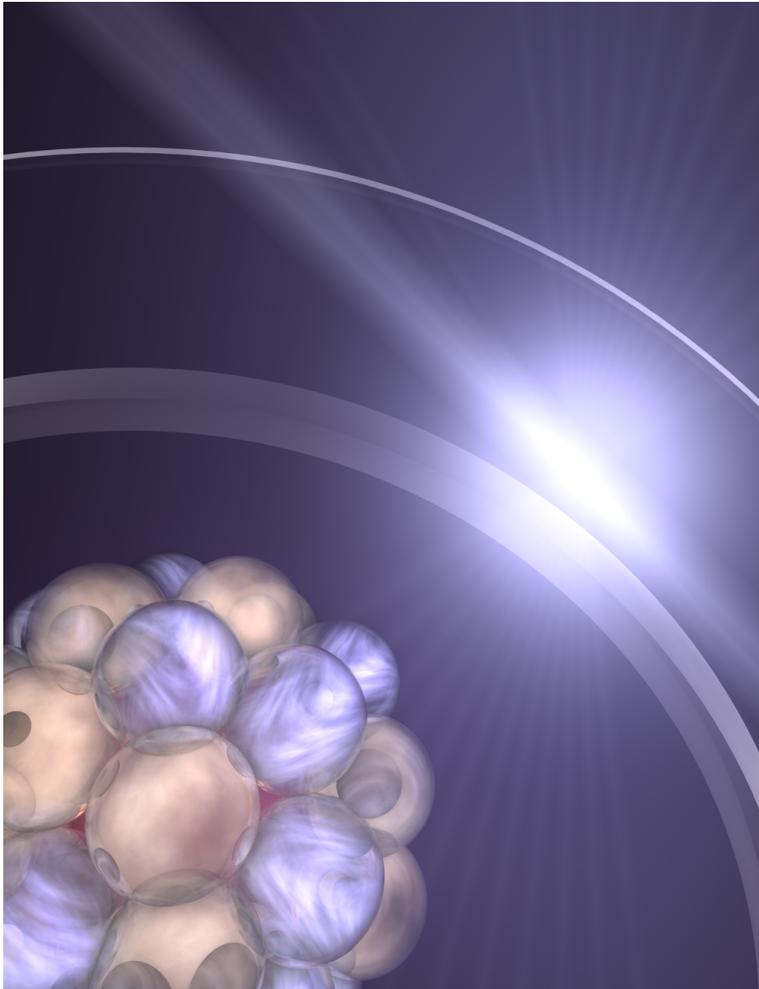
Light nuclei:

Illinois 3NF fitted to 7 states in  $A \leq 8$  nuclei [Pieper et al. (2001)].

Lattice EFT suggests that nuclei are close to a quantum phase transition [Elhatisari et al., (2016)]



# What is the neutron skin in $^{48}\text{Ca}$ ?



**Neutron skin** = Difference between radii of neutron and proton distributions

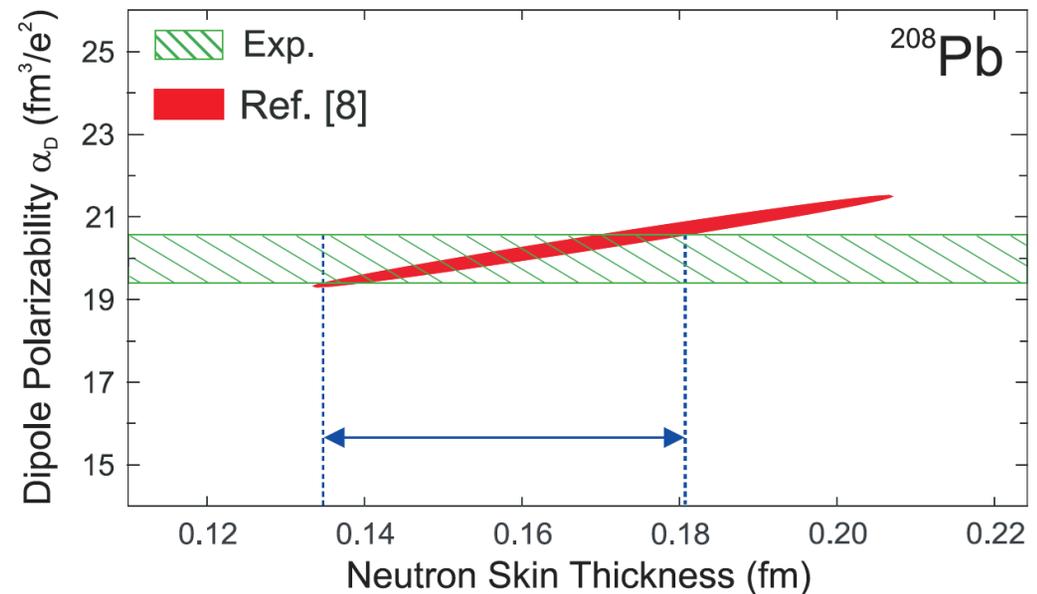
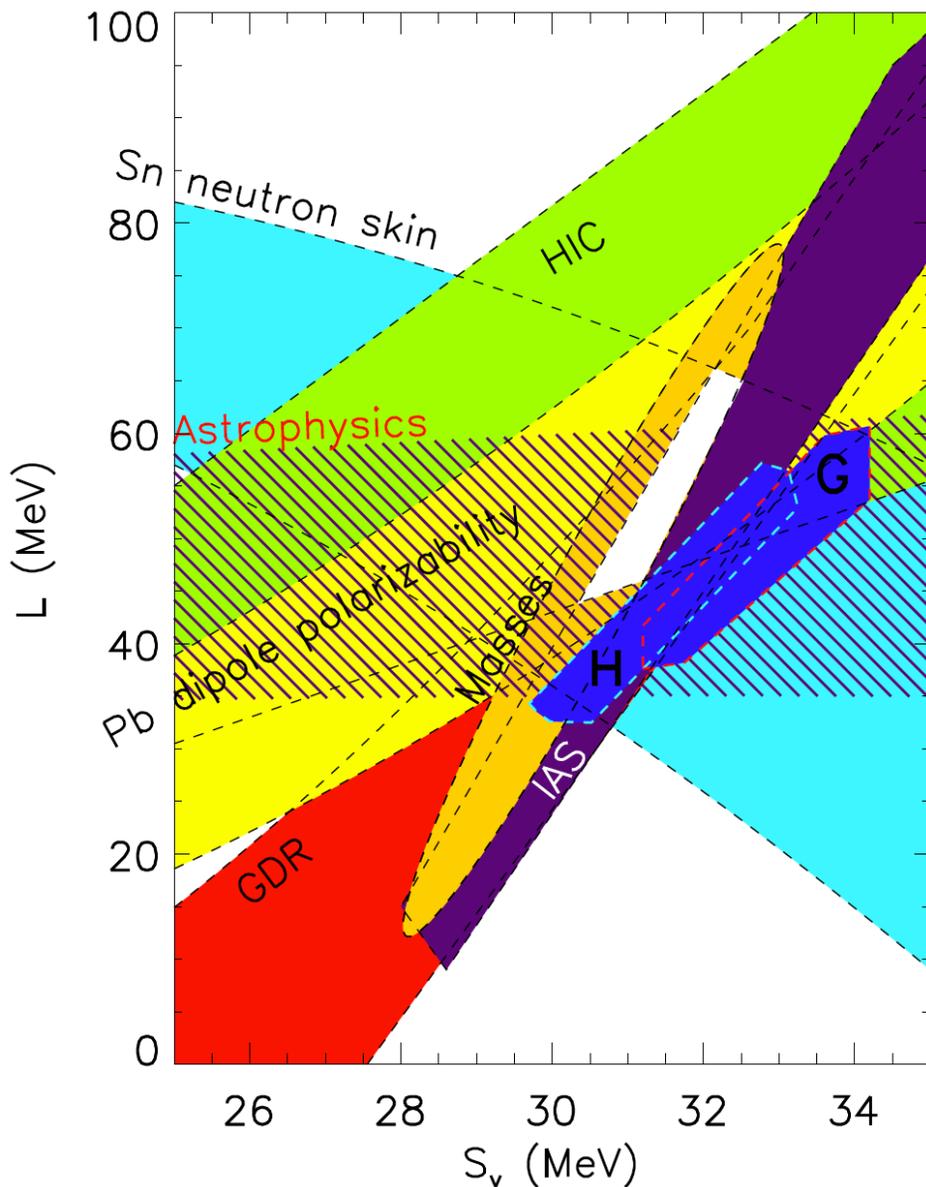
Relates atomic nuclei to neutron stars via neutron EOS

Correlated quantity: dipole polarizability

Model-independent measurement possible via parity-violating electron scattering

# Neutron radii and dipole polarizabilities

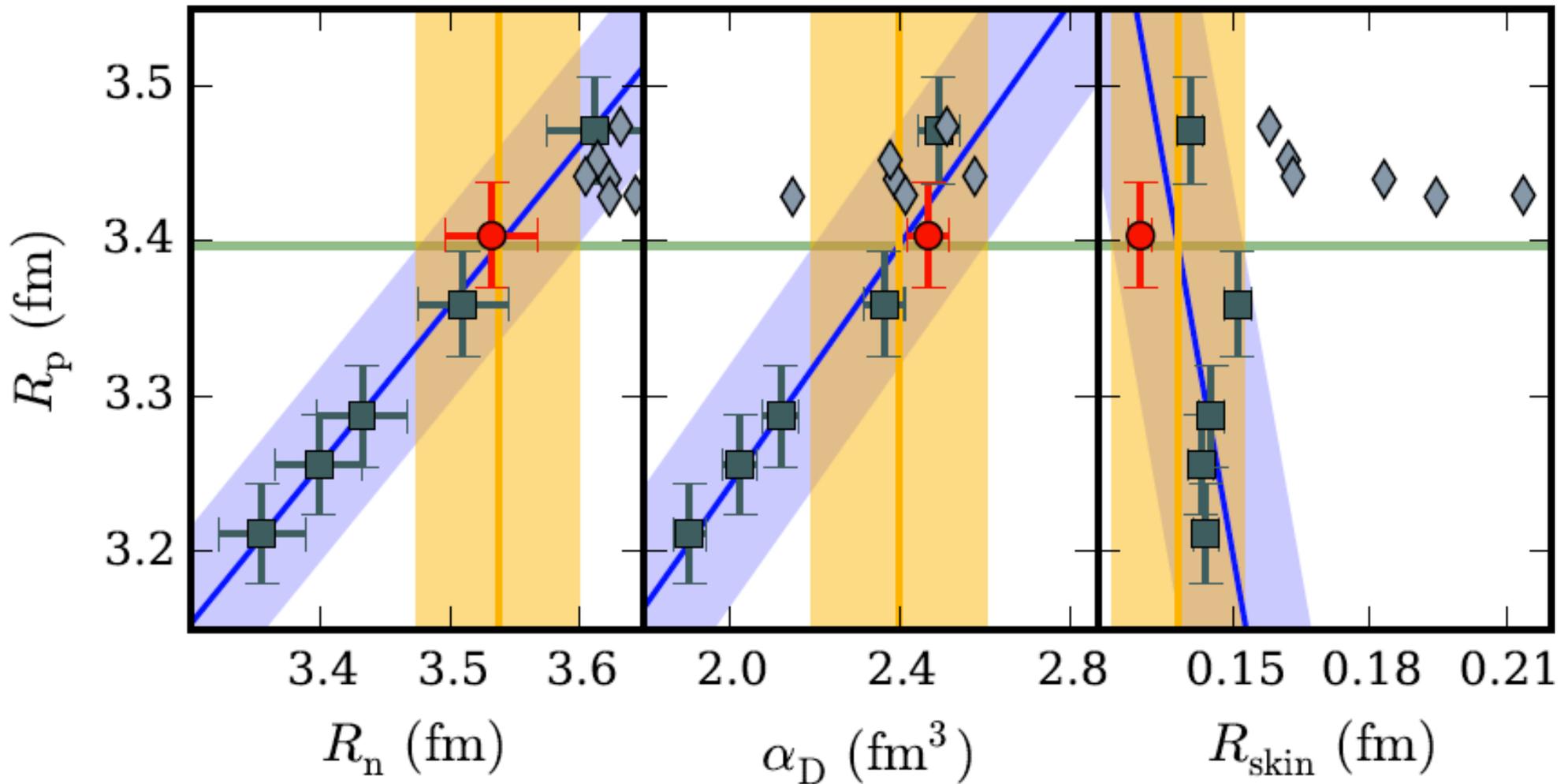
Brown, PRL 2000, Piekarewicz & Horowitz, PRL 2001; Furnstahl, NPA 2002; Reinhard & Nazarewicz, PRC 2010; Piekarewicz et al., PRC 2012; Horowitz et al, PRC 2012; ...



$\alpha_D$ :  $^{208}\text{Pb}$  by Tamii et al, PRL 2011;  $^{68}\text{Ni}$  by Rossi et al, PRL 2013;  $^{120}\text{Sn}$  by Hashimoto et al. (2015);  $^{48}\text{Ca}$  coming soon ...

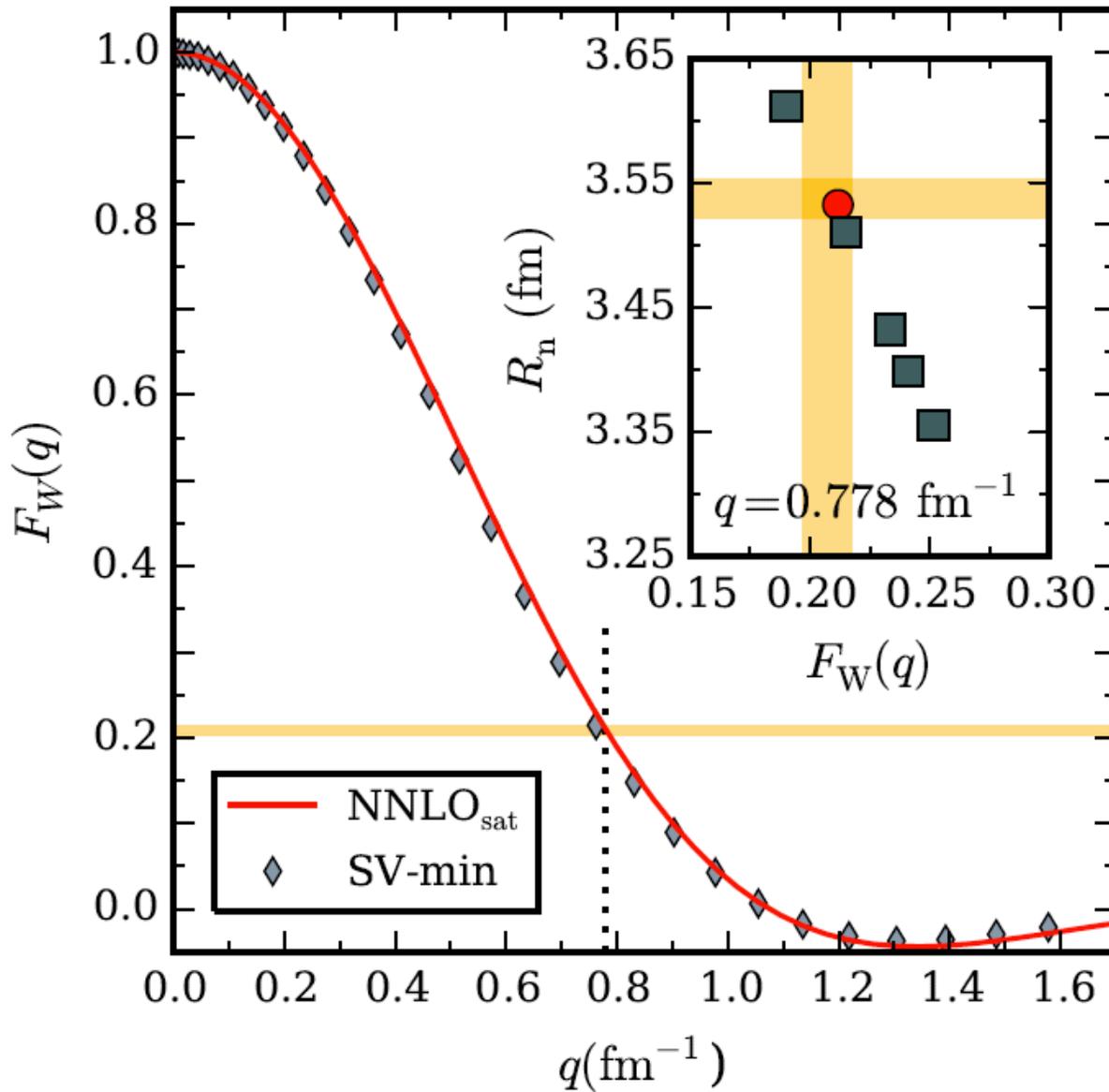
$R_n$ :  $^{208}\text{Pb}$  by Abrahamyan et al, PRL 2012;  $^{48}\text{Ca} \rightarrow \text{CREX}$

# Correlations of critical observables

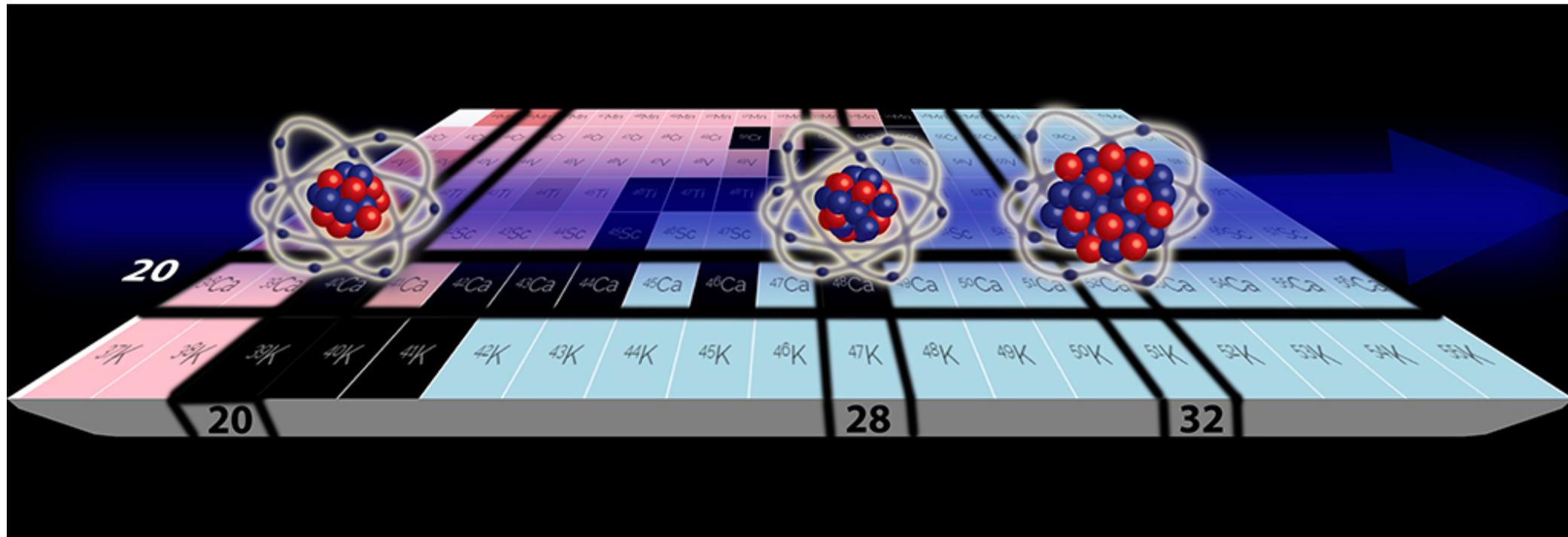


Uncertainty estimates from family of chiral interactions [NNLO<sub>sat</sub>, other potentials from Hebeler (2011), and DFT].

# Weak form factor



# Magicity in calcium isotopes

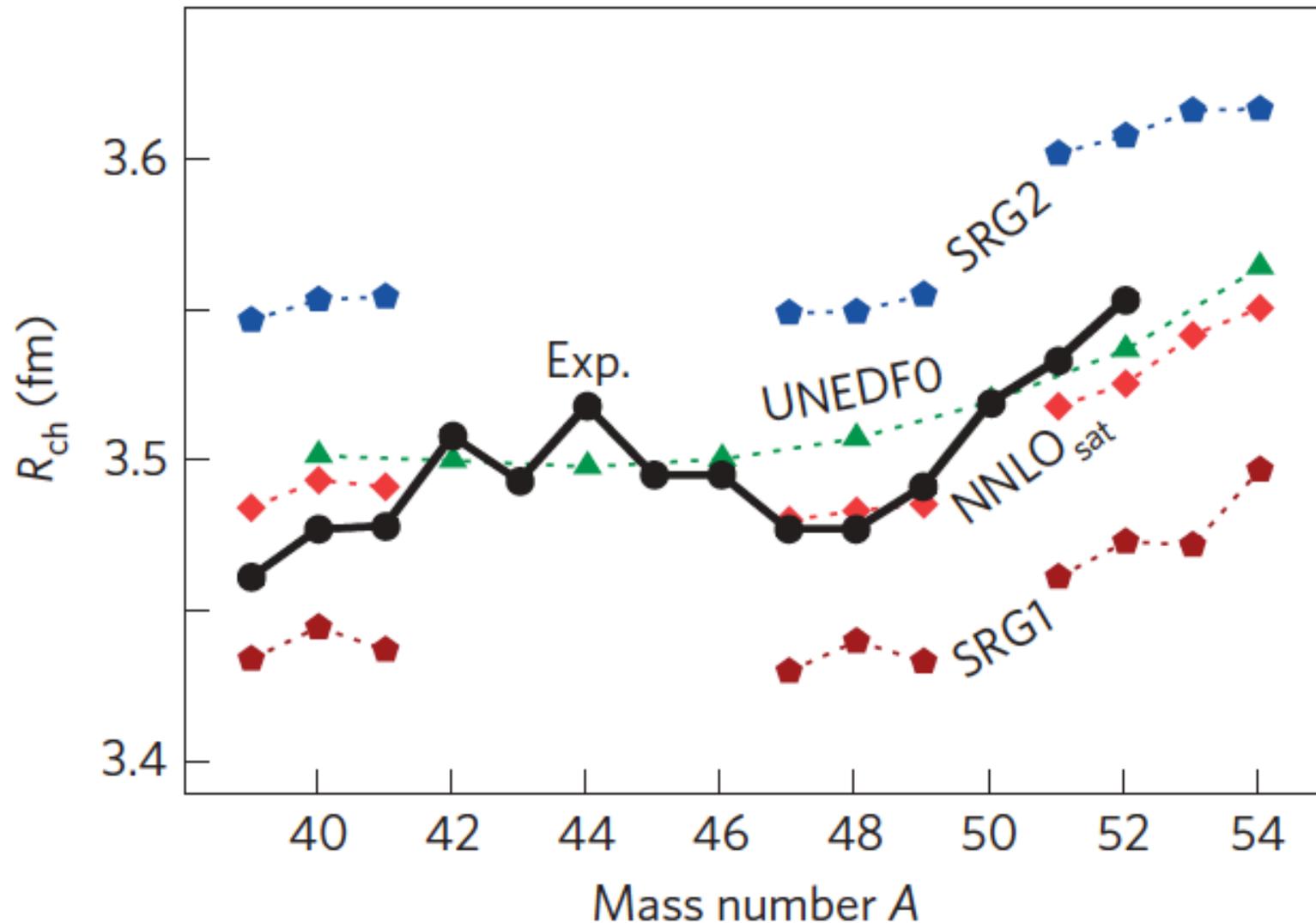


Magicity manifests itself through many observables:

- Separation energies
- Energy of  $2^+$  excited state
- Charge radii
- ...

Figure: R. Garcia Ruiz and COLLAPS collaboration

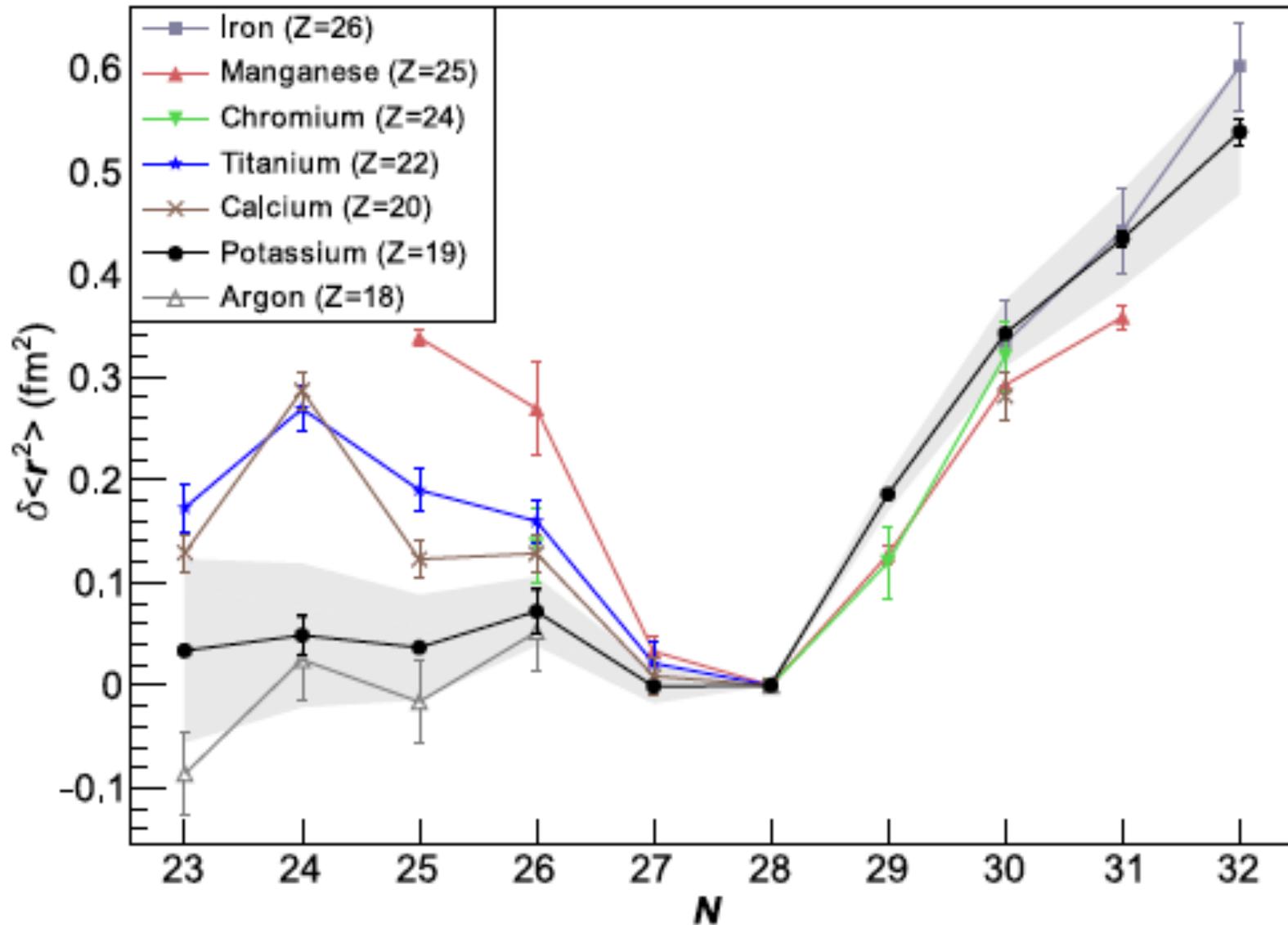
# Charge radii in calcium isotopes



... question the magicity at  $N=32$ .

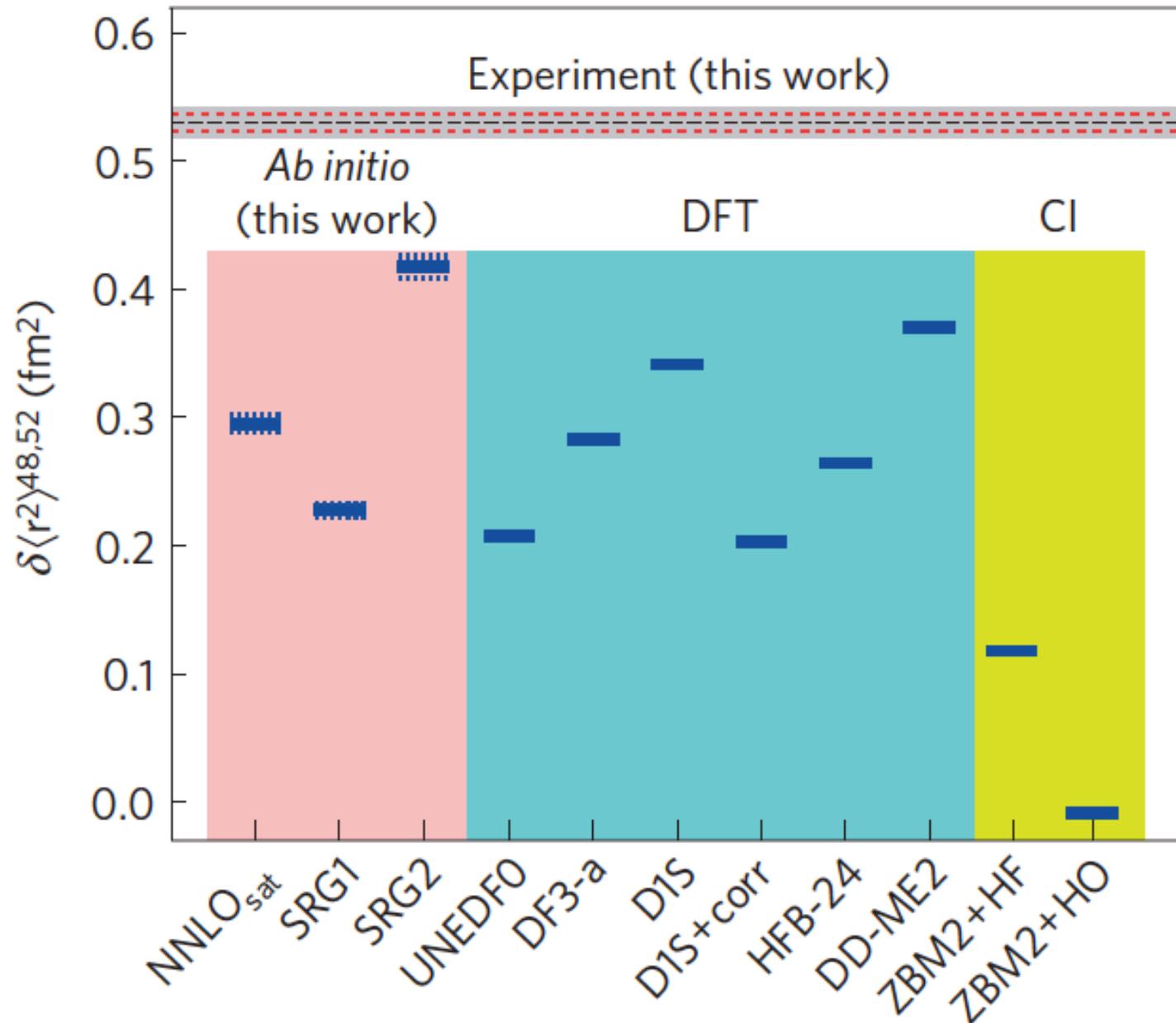
R. Garcia Ruiz et al., Nature Physics (advance online, 2016)

# Isotope shifts around N=28



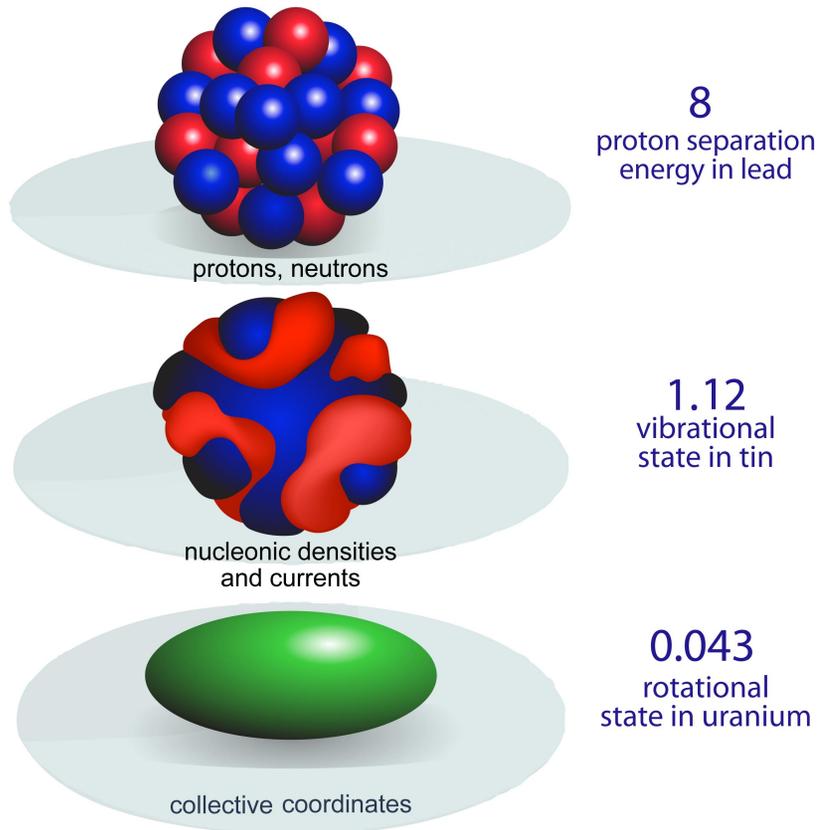
Kreim et al., PLB (2014)

# Theory challenge: Charge radius in $^{52}\text{Ca}$



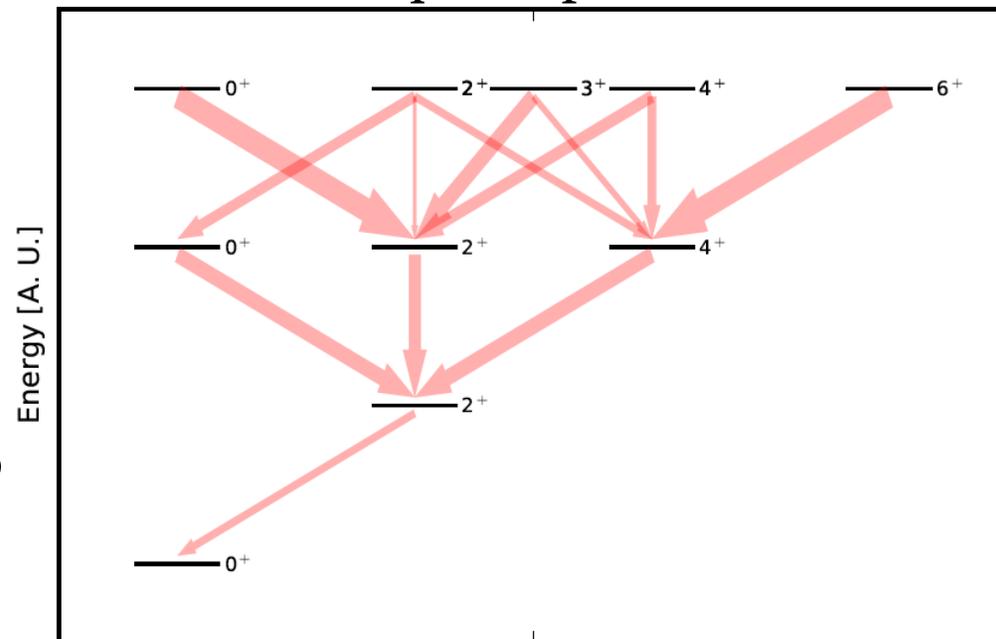
# EFT for nuclear vibrations

[with E. A. Coello Pérez, PRC 92, 064309 (2015)]



EFT for nuclear vibrations

*Harmonic quadrupole oscillator*

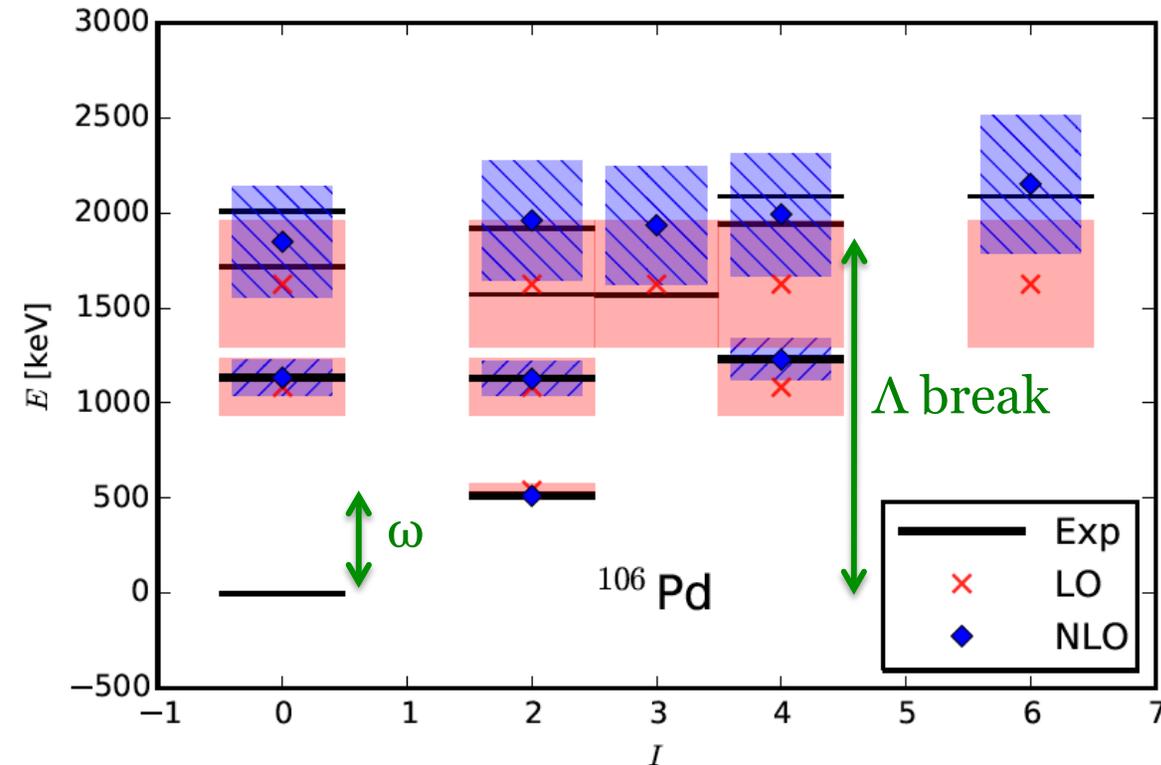


Spectrum and B(E2) transitions

While spectra of certain nuclei appear to be harmonic, B(E2) transitions do not.

Garrett & Wood (2010): “Where are the quadrupole vibrations in atomic nuclei?”

# EFT for nuclear vibrations



EFT ingredients:

- quadrupole degrees of freedom
- breakdown scale around three-phonon levels
- “small” expansion parameter: ratio of vibrational energy to breakdown scale:  $\omega/\Lambda \approx 1/3$

- Uncertainties show 68% DOB intervals from Bayesian analysis of EFT truncation effects, following [Cacciari & Houdeau (2011); Bagnaschi et al (2015); Furnstahl, Klco, Phillips & Wesolowski (2015)]
  - Expand observables according to power counting
  - Employ “naturalness” assumptions as log-normal priors in Bayes’ theorem
  - Compute distribution function of uncertainties due to EFT truncation
  - Compute degree-of-believe (DOB) intervals.

# Hamiltonian

LO Hamiltonian  $\hat{H}_{\text{LO}} = \omega \hat{N}$

NLO correction  $\hat{h}_{\text{NLO}} = g_N \hat{N}^2 + g_v \hat{\Lambda}^2 + g_I \hat{I}^2$

with  $\hat{N}^2 = (d^\dagger \cdot \tilde{d})^2,$

$$\hat{\Lambda}^2 = -(d^\dagger \cdot d^\dagger)(\tilde{d} \cdot \tilde{d}) + \hat{N}^2 - 3\hat{N},$$

$$\hat{I}^2 = 10(d^\dagger \otimes \tilde{d})^{(1)} \cdot (d^\dagger \otimes \tilde{d})^{(1)}.$$

Small expansion parameter  $\varepsilon \equiv (N\omega/\Lambda)$

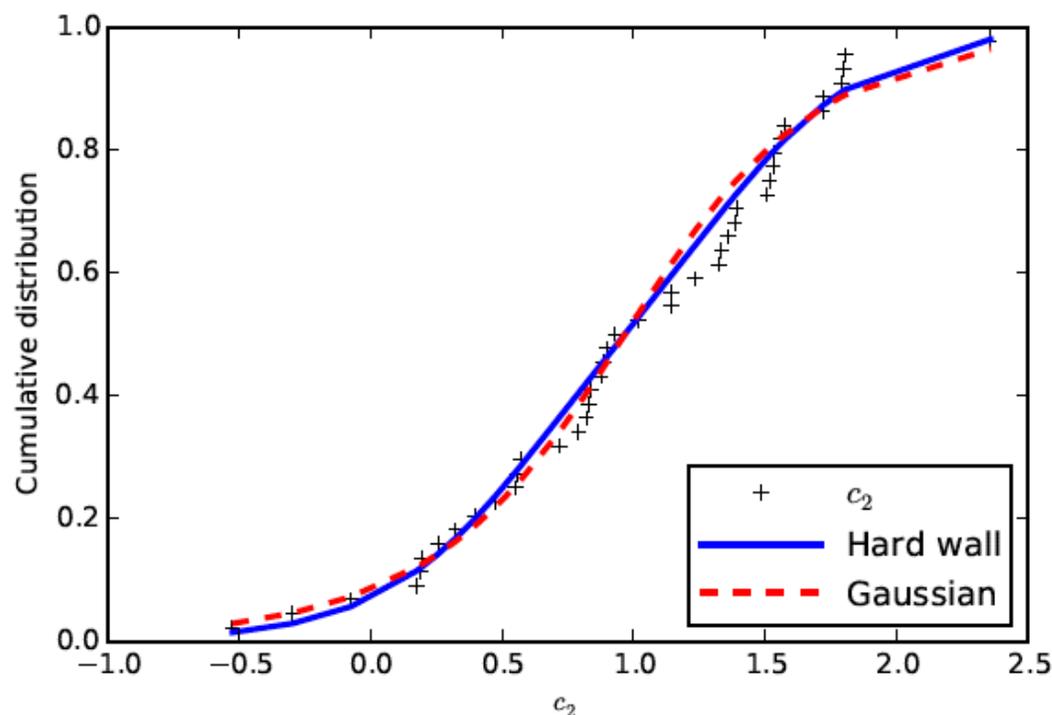
# Uncertainty quantification

$$E_{\text{NLO}} = \omega N + g_\omega N + g_N N^2 + g_v v(v+3) + g_I I(I+1)$$

$$c_2 \equiv c_2(N, v, I)$$

$$= \frac{g_\omega N + g_N N^2 + g_v v(v+3) + g_I I(I+1)}{\varepsilon^2 \omega}$$

$$X = X_0 \sum_{n=0}^{\infty} c_n \varepsilon^n$$



Linear combinations of LECs enter observables. LECs are random, but with EFT expectations, i.e. log-normal distributed. Making assumptions about these distributions then allows one to quantify uncertainties. The assumptions can be tested.

$$\Delta_k^{(M)} = \sum_{n=k+1}^{k+M} c_n \varepsilon^n$$

$$p_M(\Delta|c_0, \dots, c_k) = \frac{\int_0^\infty dc \text{pr}(c) p_M(\Delta|c) \prod_{m=0}^k \text{pr}(c_m|c)}{\int_0^\infty dc \text{pr}(c) \prod_{m=0}^k \text{pr}(c_m|c)}$$

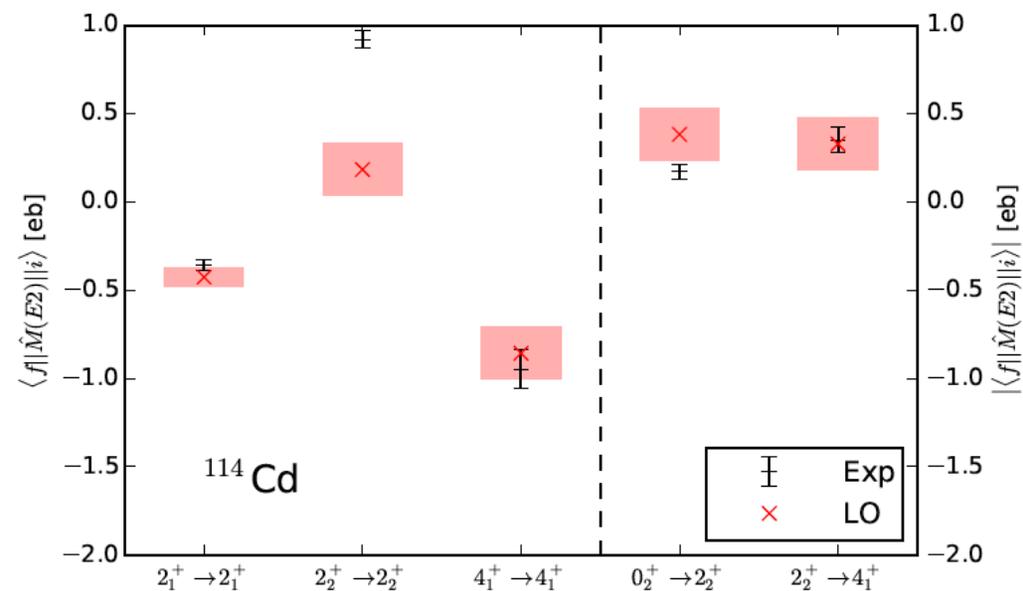
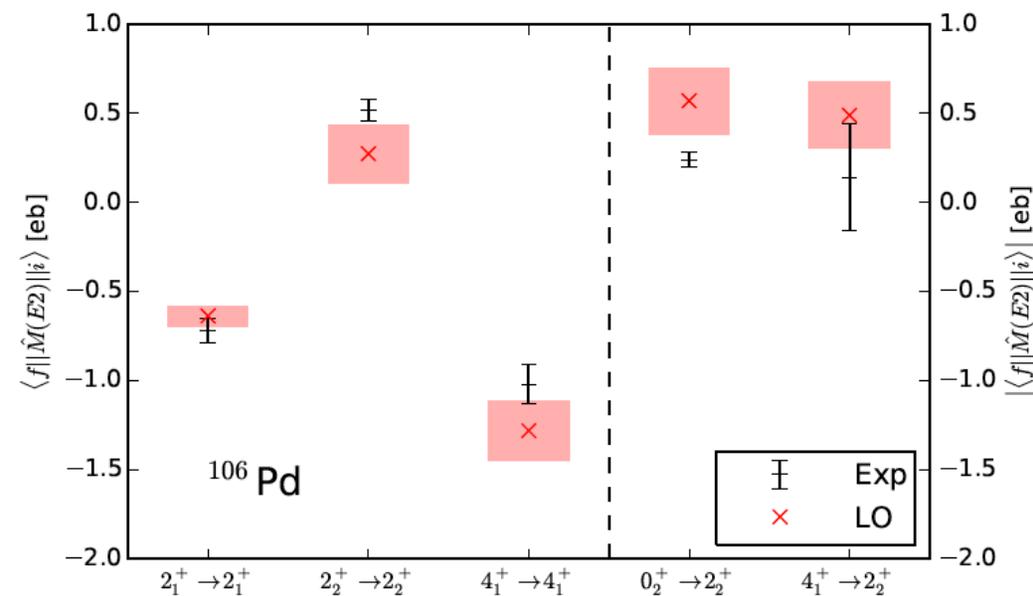
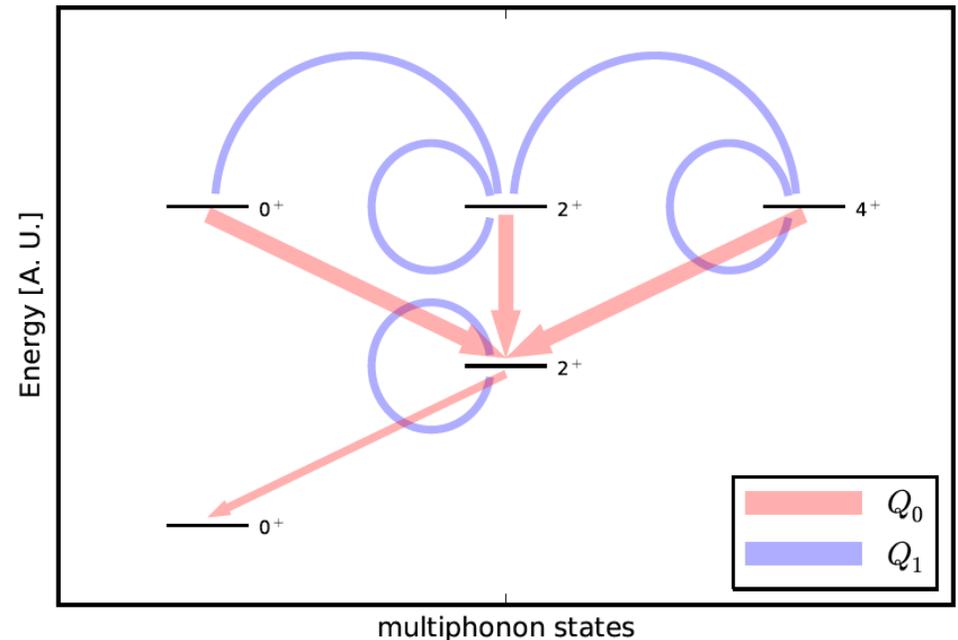
# EFT result: sizeable quadrupole matrix elements are natural

In the EFT, the quadrupole operator is also expanded:

$$\hat{Q}_\mu = Q_0 (d_\mu^\dagger + \tilde{d}_\mu) + Q_1 (d^\dagger \times d^\dagger + \tilde{d} \times \tilde{d} + 2d^\dagger \times \tilde{d})_\mu^{(2)}$$

Subleading corrections are sizable:

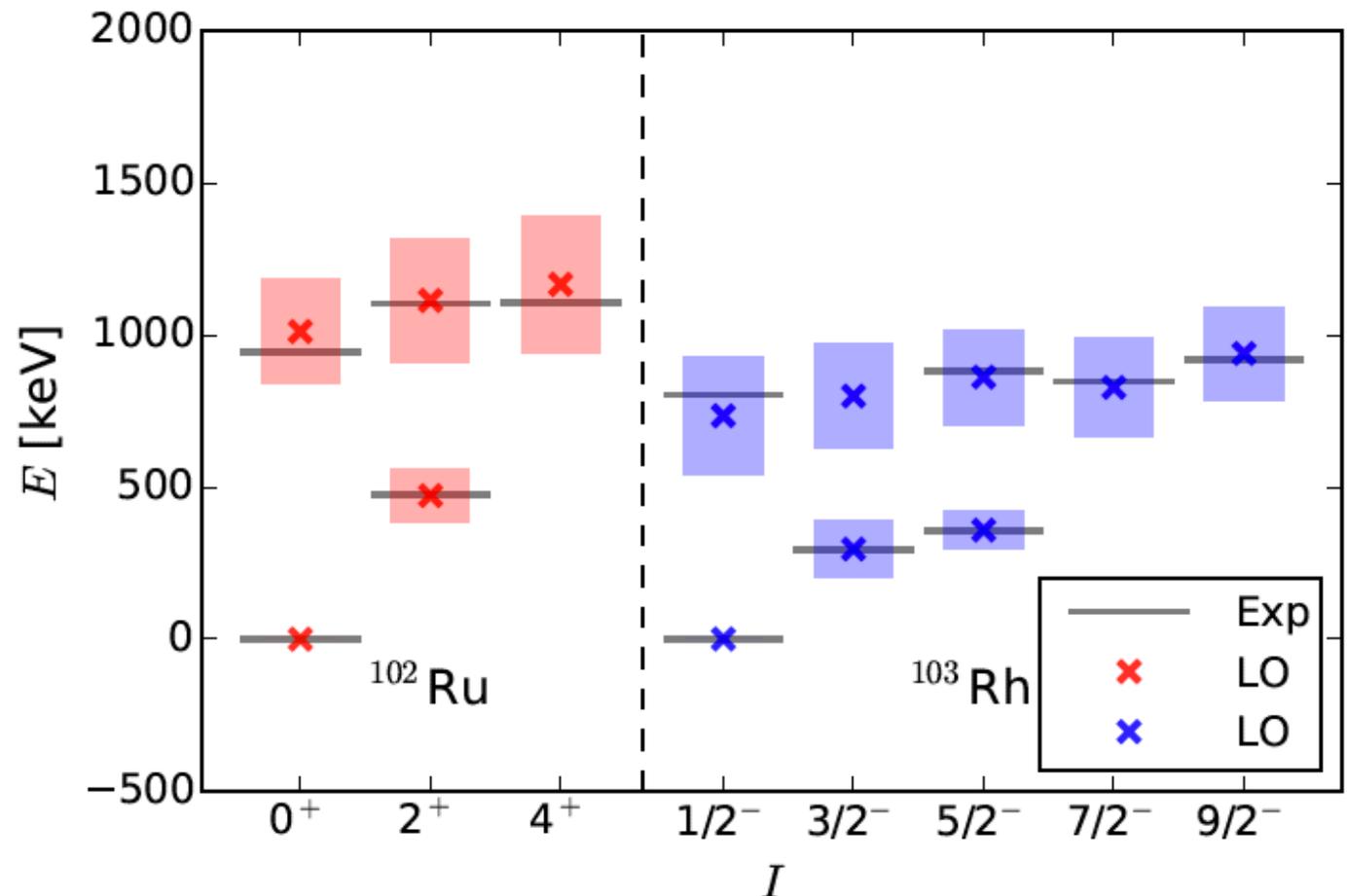
$$Q_1 \sim \left(\frac{\omega}{\Lambda}\right)^{1/2} Q_0$$



# Work in progress: Fermion coupled to vibrating nucleus

Idea: In the spirit of Halo EFT [Bertulani, Hammer, van Kolck (2002); Higa, Hammer, van Kolck (2008); Hammer & Philipps (2011); Ryberg et al. (2014)], add a fermion to describe odd-mass neighbors

Two new LECs enter at LO



# Magnetic moments: Relations between even-even and even-odd nuclei

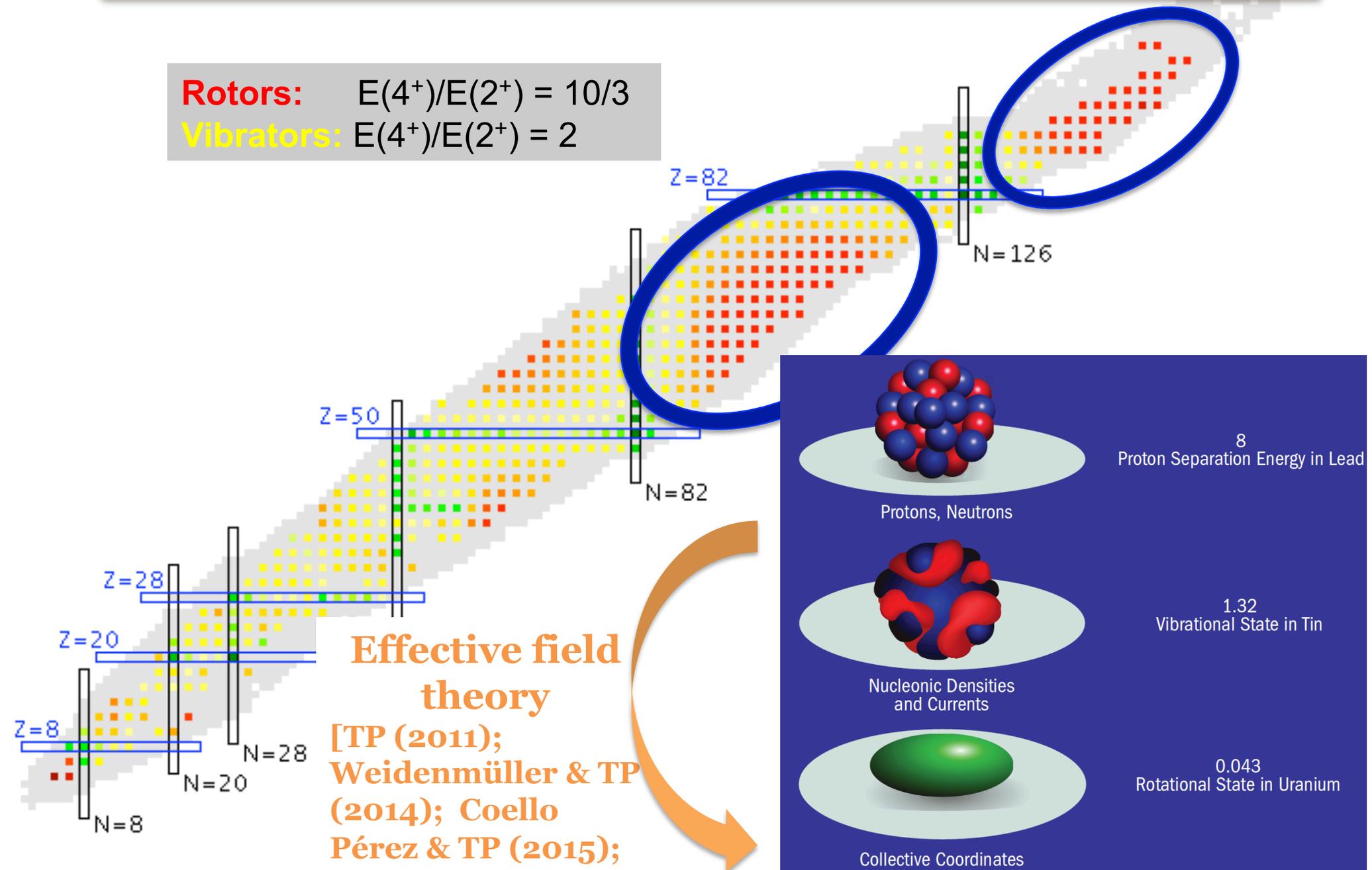
Nucleus	$I_i^\pi$	$\mu_{\text{exp}}(I_i^\pi)$	$\mu_{\text{EFT}}(I_i^\pi)$	Nucleus	$I_i^\pi$	$\mu_{\text{exp}}(I_i^\pi)$	$\mu_{\text{EFT}}(I_i^\pi)$
$^{102}\text{Ru}$	$2_1^+$	$0.85(3)^*$	$0.85(16)$	$^{106}\text{Pd}$	$2_1^+$	$0.79(2)^*$	$0.79(15)$
	$2_2^+$		$0.85(33)$		$2_2^+$	$0.71(10)$	$0.79(30)$
	$4_1^+$		$2.08(33)$		$4_1^+$	$1.8(4)$	$1.93(30)$
$^{103}\text{Rh}$	$\frac{1}{2}_1$	$-0.088^*$	$-0.088(16)$	$^{107}\text{Ag}$	$\frac{1}{2}_1$	$-0.11^*$	$-0.11(15)$
	$\frac{3}{2}_1$	$0.77(7)$	$0.78(16)$		$\frac{3}{2}_1$	$0.98(9)$	$0.74(15)$
	$\frac{5}{2}_1$	$1.08(4)$	$0.79(16)$		$\frac{5}{2}_1$	$1.02(9)$	$0.71(15)$
	$\frac{7}{2}_1$	$2.0(6)$	$2.0(3)$		$\frac{7}{2}_1$		$1.9(3)$
	$\frac{9}{2}_1$	$2.8(5)$	$2.0(3)$		$\frac{9}{2}_1$		$1.9(3)$
	$\frac{11}{2}_1$				$\frac{11}{2}_1$		

At LO, one new LEC enters to describe odd-mass neighbor

# EFT for deformed nuclei

**Rotors:**  $E(4^+)/E(2^+) = 10/3$

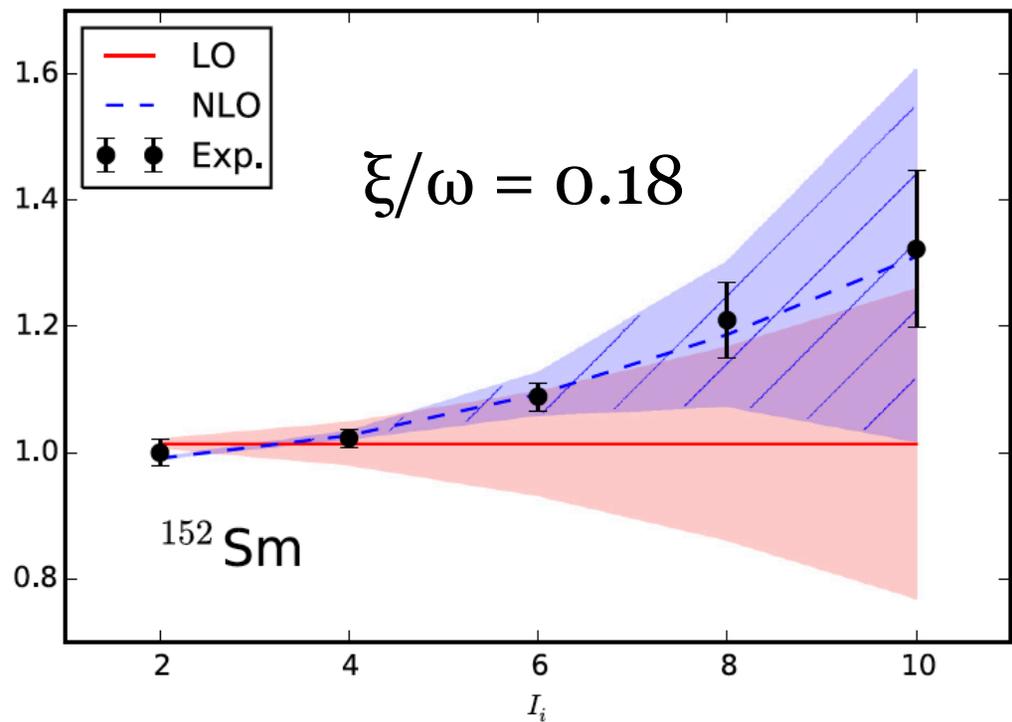
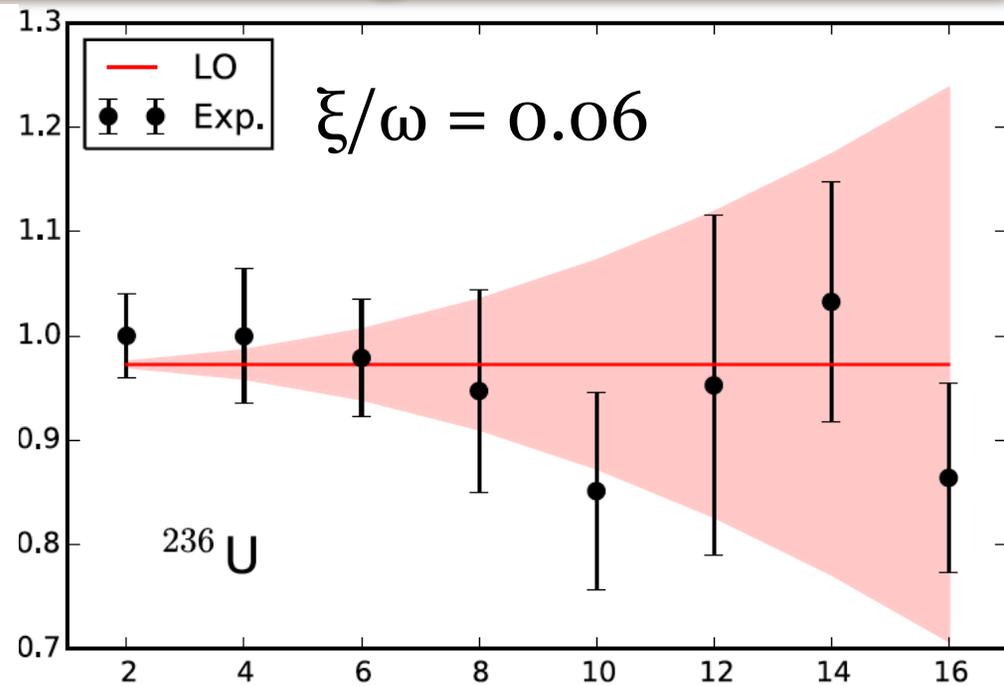
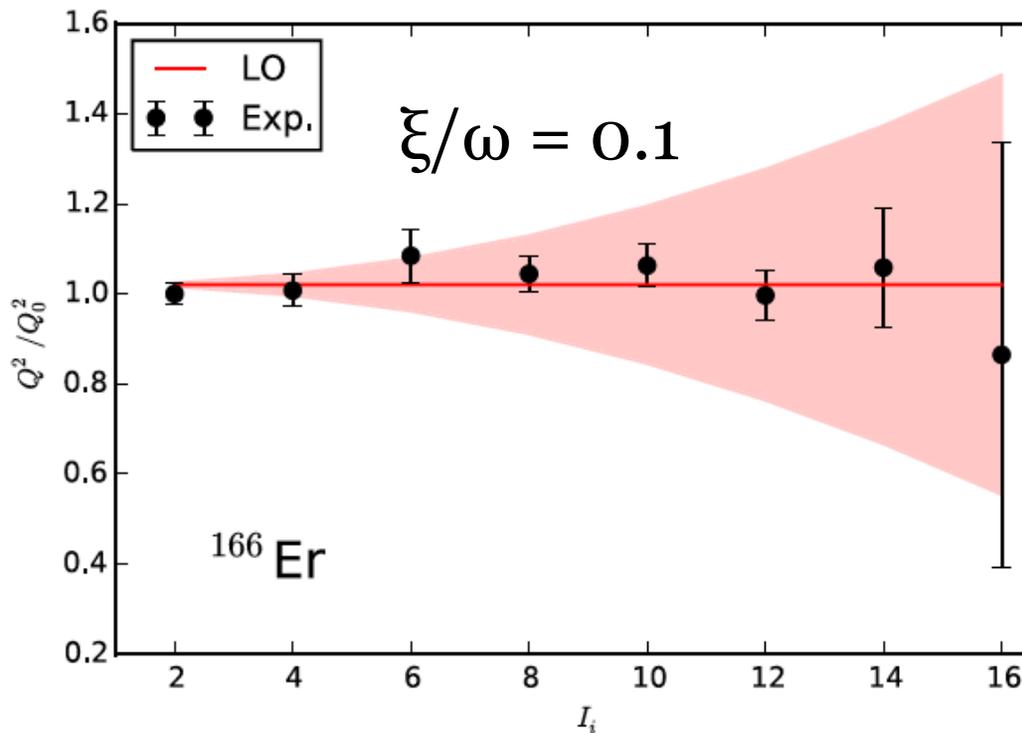
**Vibrators:**  $E(4^+)/E(2^+) = 2$



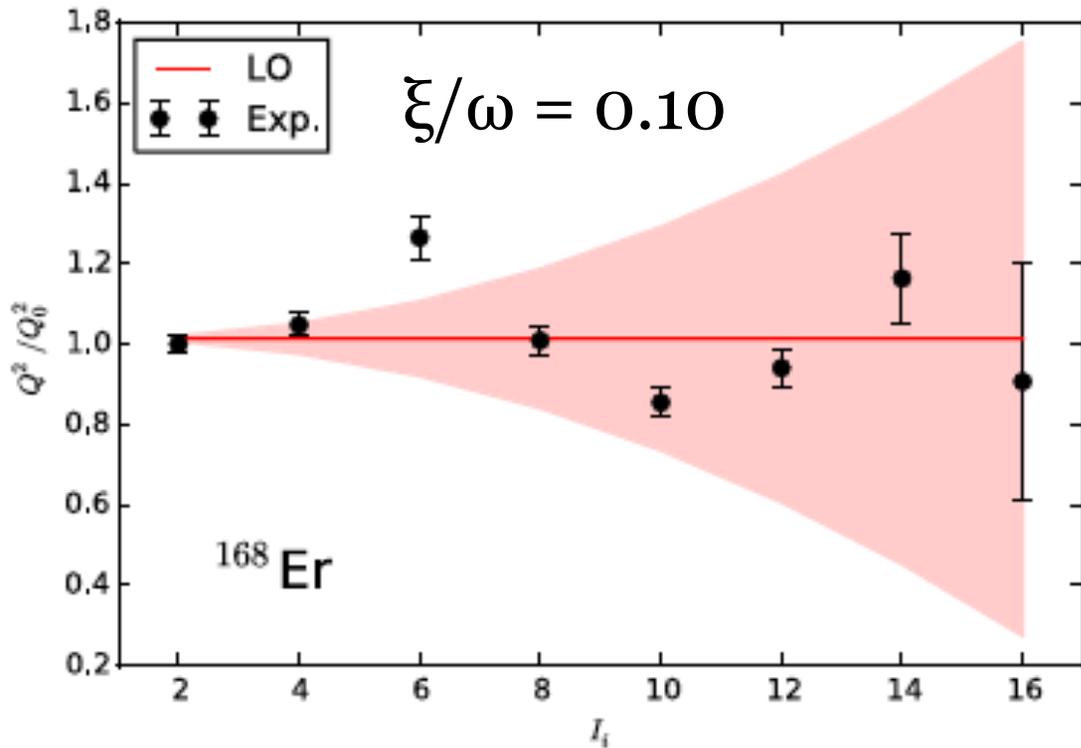
# EFT works well for a wide range of rotors

Bohr & Mottelson (1975):

“The accuracy of the present measurements of E2-matrix elements in the ground-state bands of even even nuclei is in most cases barely sufficient to detect deviations from the leading-order intensity relations.”



# EFT can not explain oscillatory patterns in supposedly “good” rotors $^{168}\text{Er}$ , $^{174}\text{Yb}$

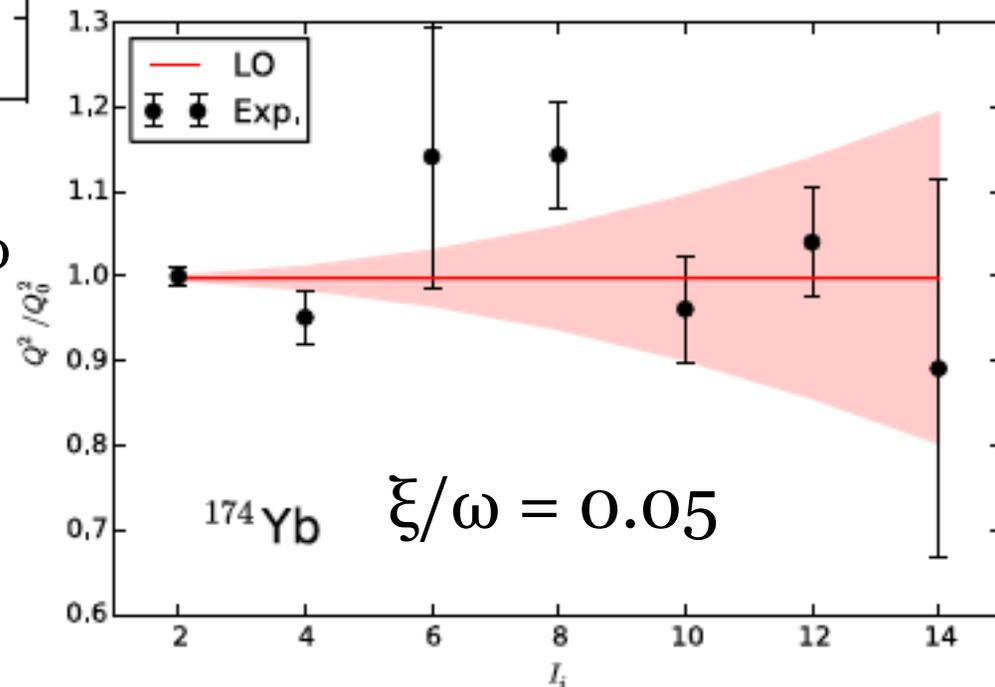


$^{168}\text{Er}$ :  $B(E2)$  for  $6^+ \rightarrow 4^+$  very difficult to understand.

$^{174}\text{Yb}$ :  $B(E2)$  for  $8^+ \rightarrow 6^+$  difficult to reconcile with  $4^+ \rightarrow 2^+$ .

Theoretical uncertainty estimates relevant.

Based on results for molecules, well-deformed nuclei, and transitional nuclei, EFT suggests that a few transitions in text-book rotors could merit re-measurement.



# EFT and weak interband transitions ( $^{154}\text{Sm}$ )

$i \rightarrow f$	$B(E2)_{\text{exp}}$	$B(E2)_{\text{ET}}$	$B(E2)_{\text{CBS}}$	$B(E2)_{\text{BH}}$
$2_g^+ \rightarrow 0_g^+$	0.863 (5)	0.863 <sup>a</sup>	0.853	0.863
$4_g^+ \rightarrow 2_g^+$	1.201 (29)	1.233 (9)	1.231	1.234
$6_g^+ \rightarrow 4_g^+$	1.417 (39)	1.358 (23)	1.378	1.355
$8_g^+ \rightarrow 6_g^+$	1.564 (83)	1.421 (43)	1.471	1.424
$2_\gamma^+ \rightarrow 0_g^+$	0.0093 (10)	0.0110 (28)		0.0492
$2_\gamma^+ \rightarrow 2_g^+$	0.0157 (15)	0.0157 <sup>a</sup>		0.0703
$2_\gamma^+ \rightarrow 4_g^+$	0.0018 (2)	0.0008 (2)		0.0050
$2_\beta^+ \rightarrow 0_g^+$	0.0016 (2)	0.0025 (6)	0.0024	0.0319
$2_\beta^+ \rightarrow 2_g^+$	0.0035 (4)	0.0035 <sup>a</sup>	0.0069	0.0456
$2_\beta^+ \rightarrow 4_g^+$	0.0065 (7)	0.0063 (16)	0.0348	0.0821

<sup>a</sup>Values employed to adjust the LECs of the effective theory.

In-band transitions [in  $e^2b^2$ ] are LO, inter-band transitions are NLO. Effective theory is more complicated than Bohr Hamiltonian both in Hamiltonian and E2 transition operator. EFT correctly predicts strengths of inter-band transitions with natural LECs.

[E. A. Coello Pérez and TP, Phys. Rev. C 92, 014323 (2015)]

# Summary

- Exciting times in nuclear theory
  - explosion of many-body solvers
  - many new developments regarding interactions and currents
- Optimization of chiral interaction  $\text{NNLO}_{\text{sat}}$  : improved radii and binding
- Weak charge, neutron radius, and dipole polarizability in  $^{48}\text{Ca}$ 
  - predictions for soon-to-be measured quantities
  - charge radii in neutron-rich calcium isotopes not well understood
- EFT for nuclear vibrations
  - Quadrupole moments are of natural size (and sizeable) due to NLO corrections
  - anharmonic vibrations
- EFT for deformed nuclei
  - interband transitions correctly described due to new terms in operator

# Outlook

We have fast cars (CCM, GFMC, IMSRG, MCSM, NCSM, UMOA, QMC) ...



but the roads have potholes



New interactions are being worked on ...

