Understanding the proton radius puzzle: the role of nuclear structure corrections

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Outline

- **Introduction**
  - The proton radius puzzle
  - Lamb shift, charge radius & polarization
- **Methods**
  - *Ab-initio* calculation of nuclear polarization
  - The LSR method
- **Results**
  - $\mu\,^4$He$^+$
  - μD
  - $\mu\,^3$He$^+$ & $\mu\,^3$H
- **Uncertainty estimates**
- **Summary & Outlook**

How large is the proton?

- **r_p from electron-proton interaction**
  1. e-p scattering: $r_p = 0.875(10)$ fm
  2. Hydrogen spectroscopy: $r_p = 0.8768(69)$ fm
  3. $\Longrightarrow$ CODATA-2010: $\boldsymbol{r_p = 0.8775(51)}$ fm

     Mohr *et al.*, **Rev. Mod. Phys. (2012)**

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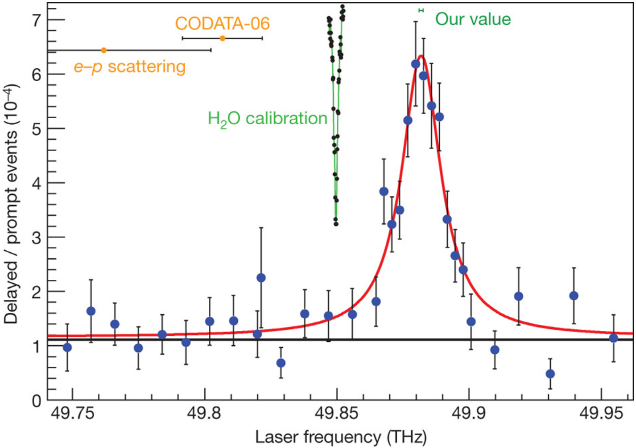
- r_p **from μH Lamb shift (2S-2P)**
  1. μH 2S$^{F=1}_{1/2}$ - 2P$^{F=2}_{3/2}$: $r_p = 0.84184(67)$ fm (5σ)

     Pohl *et al.*, **Nature (2010)**
  2. Combined with

     μH 2S$^{F=0}_{1/2}$ - 2P$^{F=1}_{3/2}$: $\boldsymbol{r_p = 0.84087(39)}$ fm (7σ)

     Antognini *et al.*, **Science (2013)**


CODATA-06
e–p scattering
H2O calibration
Our value
Delayed / prompt events (10−4)
Laser frequency (THz)
0
1
2
3
4
5
6
7
49.75
49.8
49.85
49.9
49.95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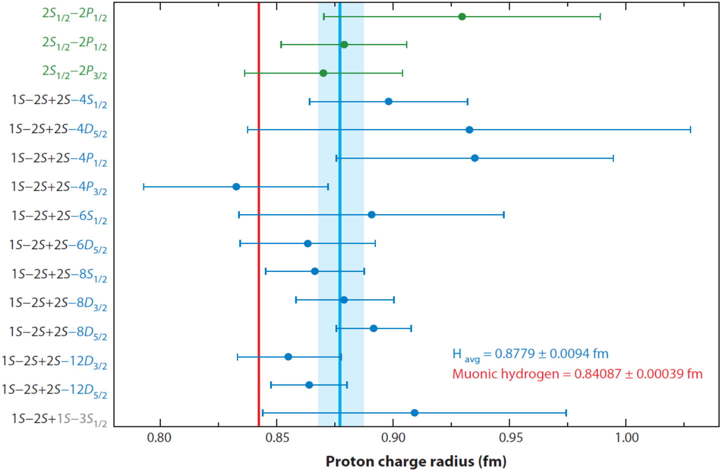
New value for charge radius of key subatomic particle

Origins of the discrepancy? — old data

Hydrogen spectroscopy



Pohl *et al.*, **Annu. Rev. Nucl. Part. Sci. (2013)**

Origins of the discrepancy? — old data

ep scattering measurements / fits?

(Q^2 not small enough / floating normalization)

$G_E^p(Q^2) = 1 - \frac{1}{6}\, \boldsymbol{r_p^2}\, Q^2 + \ldots$

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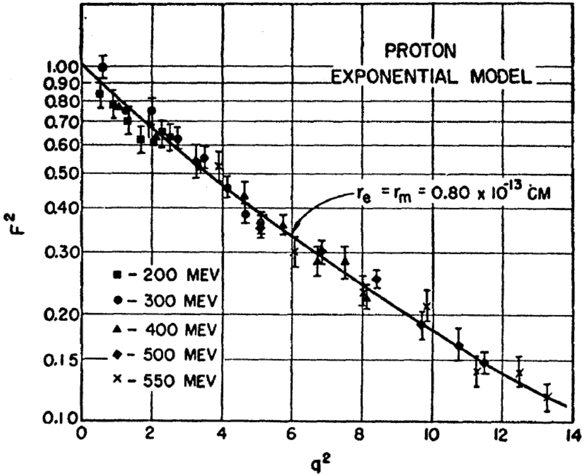

Chambers and Hofstadter, **Phys. Rev. (1956)**

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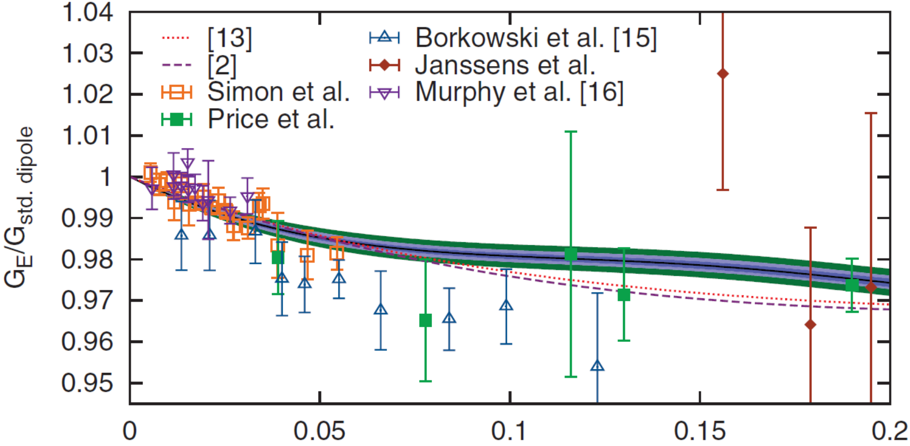

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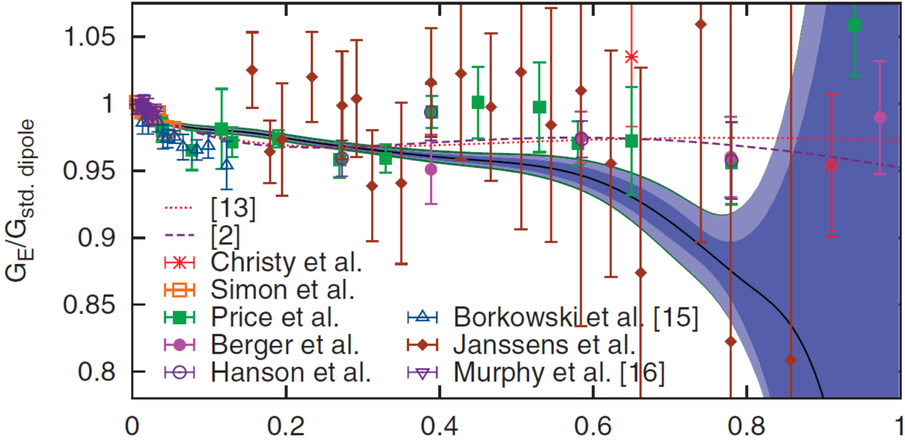

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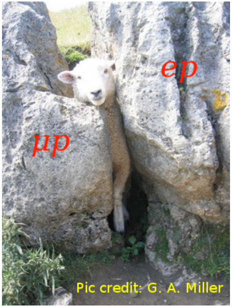
Origins of the discrepancy? — new ideas

Study the discrepancy between r_p from ep and μp experiments

- Problems with electronic measurements ?
  - eH spectroscopy
  - ep scattering measurements / fits?



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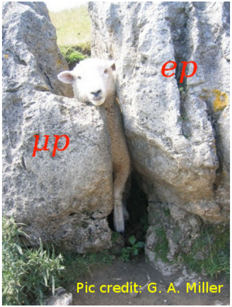
      Lorenz *et al.*, **EPJA '12**



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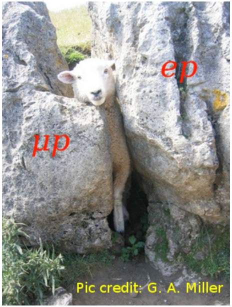

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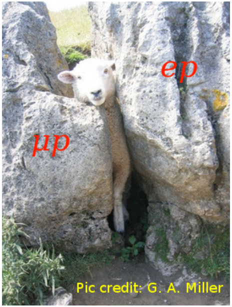

Parametrization	MAMI (1422 data points)	World data incl. MAMI (1922 data points)
Unconstrained z expansion	$r_E = 0.64, r_M = 1.97, (\chi_r^2 = 1.12)$	$r_E = 0.85, r_M = 0.98, (\chi_r^2 = 1.17)$
z expansion, $\lvert e_k \rvert < 10$	$r_E = 0.91, r_M = 0.79, (\chi_r^2 = 1.17)$	$r_E = 0.89, r_M = 0.77, (\chi_r^2 = 1.23)$
DR approach	$r_E = 0.84, r_M = 0.85, (\chi_r^2 = 1.41)$	$r_E = 0.84, r_M = 0.85, (\chi_r^2 = 1.32)$
Combination of the above	$r_E = 0.84, r_M = 0.85, (\chi_r^2 = 1.38)$	$r_E = 0.84, r_M = 0.85, (\chi_r^2 = 1.30)$

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    - non-dipole fit: $r_p = 0.849(7)$
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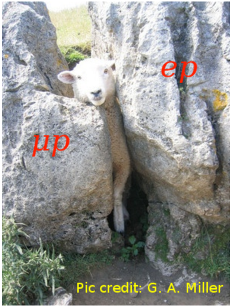

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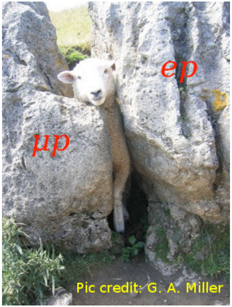
- exotic hadronic structure?
  Birse & McGovern, **EPJA '12** *vs.* Miller, **PLB '13**
  Hill & Paz, **PRD '10**; **PRL '11** & Jentschura **PRA '13**



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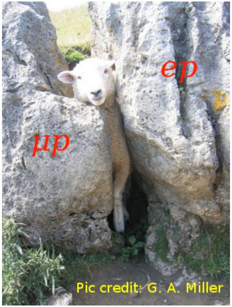

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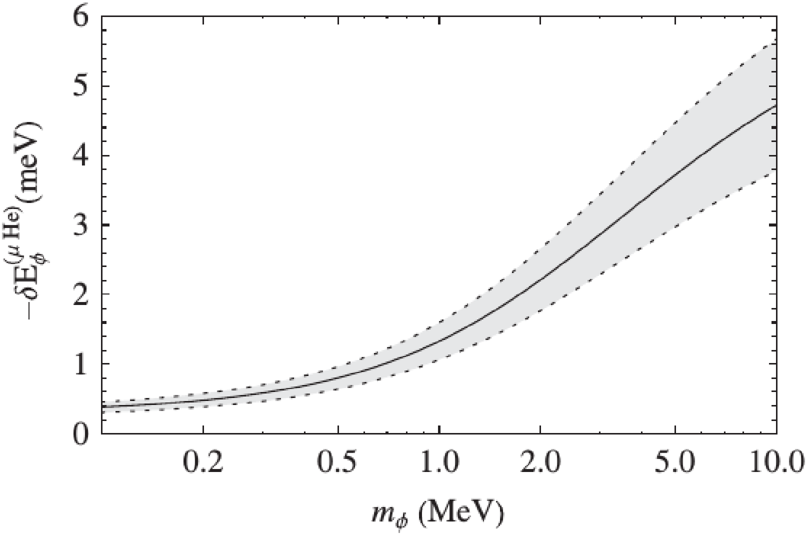
- beyond-standard-model physics?
  - new force carriers, e.g., dark photon: interact differently with e and μ
  - address both r_p puzzle & $(g_\mu - 2)$ puzzle
    Tucker-Smith & Yavin, **PRD '11**; Batell, McKeen & Pospelov, **PRL '11**;
    Carlson & Rislow, **PRD '12**; **PRD '14**

Tucker-Smith & Yavin's prediction for $\mu\,^4$He$^+$



Origins of the discrepancy? - more data!

New experiments to shed light on the puzzle

- **Jefferson Lab**
  - ep scattering for Q^2 from 10^{-4} GeV^2 to 10^{-2} GeV^2

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    - measure $e^{\pm}p$ and $\mu^{\pm}p$: can study 2γ exchange

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  - and other electronic systems — Rydberg const. ...

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high-precision measurements $\Longleftrightarrow$ accurate theoretical inputs

Lamb shift, charge radius & polarization

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$$\Delta E_{LS} = \delta_{QED} + \delta_{size}(r) + \delta_{\text{pol}}$$

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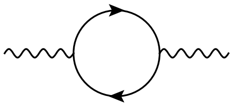
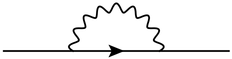
- QED corrections:
  - vacuum polarization
  - lepton self energy
  - relativistic recoil effects





- Theory of μ-p, D, ^{3,4}He$^+$ reexamined

  Martynenko *et al.* **'07**, Borie **'12**, Krutov *et al.* **'15**

  Karshenboim *et al.* **'15**, Krauth *et al.* **'15** ...

Lamb shift, charge radius & polarization

Extract $r \equiv \sqrt{\langle r^2 \rangle}$ from Lamb shift measurement

$$\Delta E_{LS} = \delta_{QED} + \delta_{size}(r) + \delta_{\rm pol}$$

- Nuclear finite-size corrections (elastic):
  - leading term: $\frac{m_r^3}{12}(Z\alpha)^4 \times r^2$
  - Zemach/Friar term: $-\frac{m_r^4}{24}(Z\alpha)^5 \times \langle r^3 \rangle_{(2)} \quad \propto r^3$

$$\langle r^3 \rangle_{(2)} \equiv \iint d\boldsymbol{R} d\boldsymbol{R}' |\boldsymbol{R} - \boldsymbol{R}'|^3 \rho_0(\boldsymbol{R}) \rho_0(\boldsymbol{R}')$$

Friar **'79**, Pohl *et al.* **'10**, Borie **'12('14)**, Krutov *et al.* **'15**

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    $\Longrightarrow \boldsymbol{\delta}_{\rm TPE} \equiv |\delta_{\rm Zem} + \delta_{\rm pol}|$
    c.f. Pachucki '11 & Friar '13 (μD)

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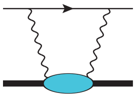
Extract $r \equiv \sqrt{\langle r^2 \rangle}$ from Lamb shift measurement

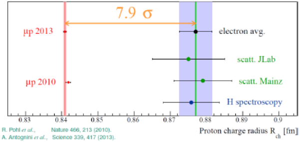
$$\Delta E_{LS} = \delta_{QED} + \delta_{size}(r) + \delta_{\rm pol}$$

- Nuclear (A) polarization corrections (inelastic):
  - exchange of two virtual photons
  - dominant contribution $\sim (Z\alpha)^5$



- Nucleon (p/n) polarization corrections (inelastic)

Update from CREMA

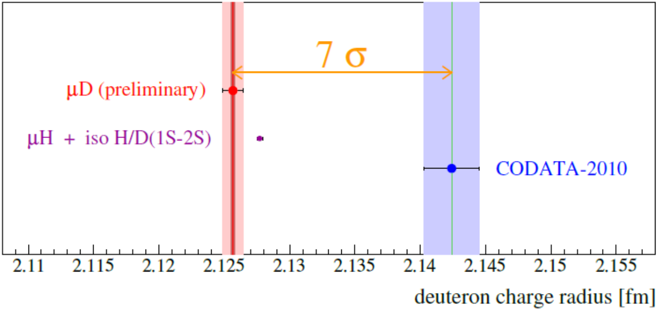


- μp:
  - is published, incl. theory summary
  - raised the "proton radius puzzle"!
- μd:
  - frequency measurement is thoroughly analysed, incl. sytematics
  - theory summary is being being peer-reviewed
- $\mu^4\text{He}^+$:
  - frequency measurement is thoroughly analysed, incl. sytematics
  - theory summary is being reviewed internally (CREMA)
- $\mu^3\text{He}^+$:
  - Work in progress! Measurement analysis & theory investigations ongoing

Courtesy of Beatrice Frank @ CREMA

Update from CREMA

- Deuteron charge radius $r_{\mathrm{d}} = 2.12\mathrm{XX}(8)$ fm
- Close to extraction from μH & isotope shift (1S-2S)
- Not in agreement with 2010 CODATA value



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Update from CREMA

$$\Delta E^{\rm LS}_{\rm QED} = 228.77356(75)\,{\rm meV}$$

$$\Delta E^{\rm LS}_{\rm rad.-dep.} = -6.11025(28)\,r_{\rm d}^2\,{\rm meV/fm^2} + 0.00300(60)\,{\rm meV}$$

$$\Delta E^{\rm LS}_{\rm TPE} = 1.70910(2000)\,{\rm meV}$$

Courtesy of Beatrice Frank @ CREMA

Uncertainty in nuclear polarization

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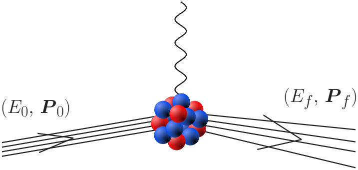
- Nuclear polarization $\Longrightarrow$ inputs from nuclear response functions

$$S_O(\omega) = \sum\!\!\!\!\!\!\int_f |\langle \psi_f | \hat{O} | \psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$$



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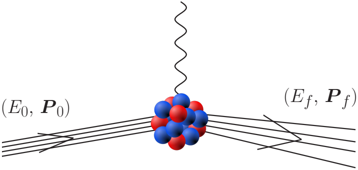

- Early calculations of δ_{pol} in muonic atoms:
  $\Rightarrow S_O(\omega)$ inputs were not accurate enough

Previous calculations of $S_O(\omega)$ & $\delta_{\rm pol}$

- μD (^{2}H)
  - Modern forces: AV14 - Leidemann & Rosenfelder **'95**
  - State-of-the-art: AV18 - Pachucki **'11**
    - underestimates nuclear physics uncertainty
    - includes nucleon polarizability $\delta^p_{\rm pol}$ (incorrect?)
  - $\not\pi$EFT: zero-range expansion - Friar **'13**
    - estimated uncertainty 1–2%
    - includes nucleon-size corrections (correct?)
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- $\mu\,^{3,4}$He$^+$
  - From photoabsorption: Bernabeu & Jarlskog **'74**; Rinker **'76**; Friar **'77**
  - $\delta_{\rm pol}^{A=3/4} = -4.9/-3.1$ meV $\pm 20\%$ (c.f. experimental requirement $\sim \pm 5\%$)

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  - **experimental input** for S_O is unsatisfactory

Previous calculations of $S_O(\omega)$ & $\delta_{\rm pol}$

- **μD (^{2}H)**
  - Modern forces: AV14 - Leidemann & Rosenfelder **'95**
  - State-of-the-art: AV18 - Pachucki **'11**
    - underestimates nuclear physics uncertainty
    - includes nucleon polarizability $\delta^p_{\rm pol}$ (incorrect?)
  - $\not\pi$EFT: zero-range expansion - Friar **'13**
    - estimated uncertainty 1–2%
    - includes nucleon-size corrections (correct?)
  - From e^-D scattering: Dispersion relations - Carlson *et al.* **'14**
    - estimate nuclear uncertainty $\sim 40\%$
- **$\mu\,^{3,4}$He$^+$**
  - From photoabsorption: Bernabeu & Jarlskog **'74**; Rinker **'76**; Friar **'77**
  - $\delta^{A=3/4}_{\rm pol} = -4.9/-3.1$ meV $\pm 20\%$ (c.f. experimental requirement $\sim \pm 5\%$)
- **μT (^{3}H)**
  - Very crude theoretical estimation: C. Joachain **'61** (in French)
- **Status of $\delta_{\rm pol}$ in light muonic atoms**
  - **experimental input** for S_O is unsatisfactory
  - need to calculate $\delta_{\rm pol}$ using **modern potentials and *ab-initio* methods**

Ab initio nuclear structure corrections in light muonic atoms

We performed the first ab-initio calculation of nuclear polarization and Zemach corrections with state-of-the-art forces — AV18+UIX and χEFT

in:

$\mu\,^4\mathrm{He}^+$ Ji, NND, Bacca & Barnea, **PRL '13**; **FBS '14**

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- **Few-body methods**
  - **EIHH**: Effective Interaction Hyperspherical Harmonics
  - **LIT**: Lorentz Integral Transform
  - **LSR**: A new method we developed based on the Lanczos algorithm

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$$H_{nucl} \Longrightarrow S_O \Longrightarrow \delta_{\rm pol}$$

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  - $\delta_{\rm pol}$ from different $H_{nucl} \Longrightarrow$ nuclear physics uncertainty
  - also estimate many other sources of uncertainty

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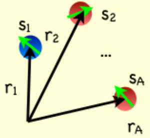
  - also estimate many other sources of uncertainty
- **Our Goal**

  provide $\boldsymbol{\delta_{\rm pol}}$ with accuracy comparable to the $\pm 5\%$ experimental needs

Nuclear potentials: two approaches



$$H_N|\psi_i\rangle = E_i|\psi_i\rangle$$

$$H_N = T + V_{NN} + V_{3N} + \ldots$$

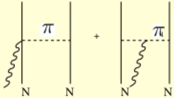
High precision two-nucleon potentials:
well constraint on NN phase shifts

Three nucleon forces:
less known, constraint on A>2 observables

Traditional Nuclear Physics
AV18+UIX, ..., J_2

Effective Field Theory
N^2LO, N^3LO ...

$$J^\mu = J^\mu_N + J^\mu_{NN} + \ldots$$



two-body currents (or MEC)
subnuclear d.o.f.

J^μ consistent with V

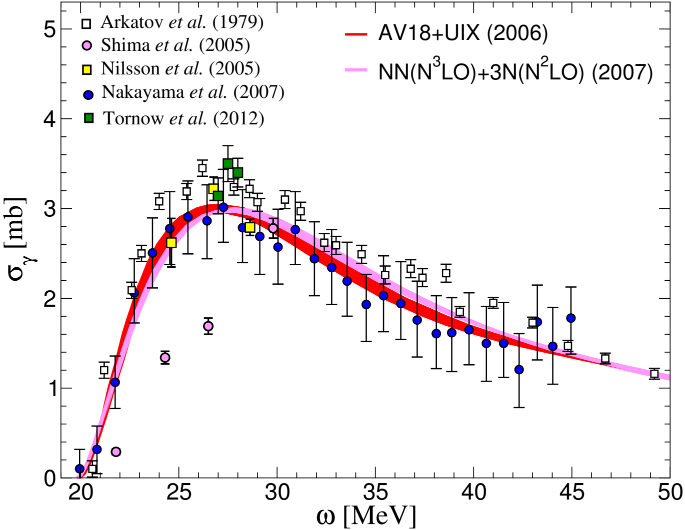
$$\nabla \cdot J = -i[V, \rho]$$

$$S(\omega) \propto |\langle\psi_f|J^\mu|\psi_0\rangle|^2$$

Exact Initial state &
Final state in the continuum at
different energies and for different A

Nuclear potentials: ^{4}He Photoabsorption

electric dipole photoabsorption cross section $\sigma_\gamma(\omega) = 4\pi^2\alpha\omega S_{D1}(\omega)$

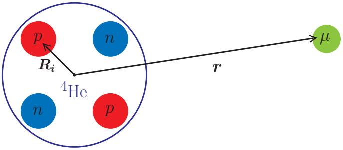


Nuclear polarization: basic idea

- Hamiltonian for muonic atoms

$$H = H_{nucl} + H_\mu + \Delta H$$

$$H_\mu = \frac{p^2}{2m_r} - \frac{Z\alpha}{r}$$



- Corrections to the point Coulomb from protons

$$\Delta H = \alpha \sum_i^Z \Delta V(\boldsymbol{r}, \boldsymbol{R}_i) \equiv \alpha \sum_i^Z \left(\frac{1}{r} - \frac{1}{|\boldsymbol{r} - \boldsymbol{R}_i|} \right)$$

- Evaluate inelastic effects of ΔH on muonic spectrum in 2^{nd}-order perturbation theory

$$\delta_{\rm pol} = \sum_{N \neq N_0, \mu} \langle N_0 \mu_0 | \Delta H | N \mu \rangle \frac{1}{E_{N_0} - E_N + \epsilon_{\mu_0} - \epsilon_\mu} \langle N \mu | \Delta H | \mu_0 N_0 \rangle$$

$|\mu_0\rangle$: muon wave function for $2S/2P$ state

Nuclear polarization: contributions

Systematic contributions to nuclear polarization

δ_{NR}	**Non-Relativistic** limit
$\delta_L + \delta_T$	Longitudinal and Transverse **relativistic** corrections
δ_C	**Coulomb** distortions
δ_{NS}	Corrections from **finite Nucleon Size**

Test: $\delta^{(0)}$ in μD

- **Test Run: electric-dipole polarization effects in μD**
  - previous $\delta_{\rm pol}$ in $\boldsymbol{\mu}$D (AV18): Pachucki '11
  - we calculate $\delta^{(0)}$ from dipole response of D (AV18)
    [$S_{D_1}(\omega)$ from Bampa, Leidemann & Arenhövel '11]

$\delta^{(0)}$ [meV]		Pachucki '11	Our work
non-rel dipole	$\delta^{(0)}_{D1}$	-1.910	-1.907
relativistic	$\delta^{(0)}_{L}$	0.035	0.029
	$\delta^{(0)}_{T}$	–	-0.012
Coulomb	$\delta^{(0)}_{C}$	0.261	0.259

  - The difference in $\delta^{(0)}_{L}$ is due to small energy expansion used in Pachucki '11

LSR: Lanczos sum rule method

- Nuclear polarization $\Longrightarrow$ energy-dependent sum rules of the response functions

$$\delta_{\rm pol} \propto I_O = \int_0^\infty d\omega\, S_O\left(\omega\right) g\left(\omega\right)$$

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  It can be extended to calculate the sum rules.

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- With the Lanczos sum rule (LSR) method, we directly calculate I_O,
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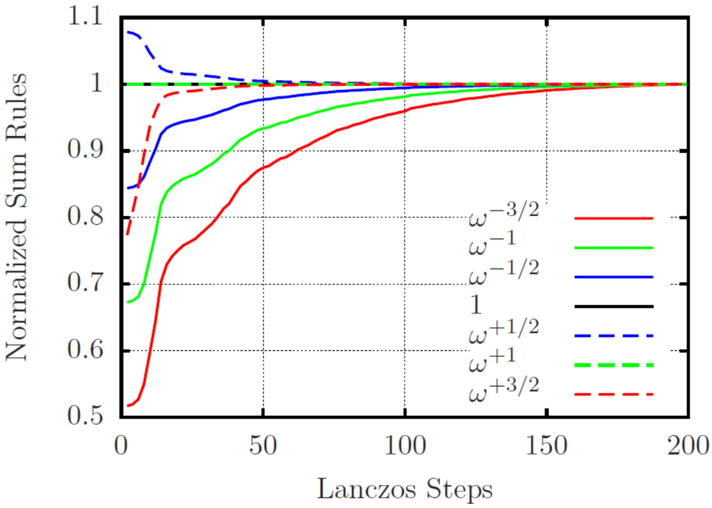
- With the Lanczos sum rule (LSR) method, we directly calculate I_O,
  without explicitly solving S_O.

- The calculated I_O converges as the LIT of S_O, if $g\left(\omega\right)$ is smooth.

NND, Ji, Bacca, Barnea, Phys. Rev. C **89**, 064317 (2014)

LSR: Example — ^{4}He dipole response integrals



(Model space size $M \sim 10^5$)

NND, Barnea, Ji, and Bacca, PRC (2014)

Nuclear polarization in $\mu\,^4$He$^+$

[meV]		AV18+UIX	χEFT★
$\delta^{(0)}$	$\delta^{(0)}_{D1}$	-4.418	-4.701
	$\delta^{(0)}_{L}$	0.289	0.308
	$\delta^{(0)}_{T}$	-0.126	-0.134
	$\delta^{(0)}_{C}$	0.512	0.546

★ NN: N^{3}LO-EM
$3N$: N^{2}LO (c_D=1, c_E=-0.029)

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δ_{NS}	$\delta^{(1)}_{R1pp}$	-1.036	-1.071
	$\delta^{(1)}_{Z1}$	1.753	1.811
	$\delta^{(2)}_{NS}$	-0.200	-0.210

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δ_{pol}		-2.408	-2.542

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Nuclear polarization in $\mu\,^4\text{He}^+$

[meV]	AV18+UIX	χEFT★
$\delta^{(0)}$	-3.743	-3.981
$\delta^{(1)}$	0.741	0.809
$\delta^{(2)}$	0.077	0.101
δ_{NS}	0.517	0.530
δ_{pol}	-2.408	-2.542

- Convergence from $\delta^{(0)}$ to $\delta^{(2)}$ from an expansion in $\eta \sim \sqrt{m_r/M_N} \approx 0.3$

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- Convergence from $\delta^{(0)}$ to $\delta^{(2)}$ from an expansion in $\eta \sim \sqrt{m_r/M_N} \approx 0.3$
- $\delta_{\rm pol}$ with AV18+UIX & χEFT differ: $\sim 5.5\%$ (0.134 meV)

★ NN: N^{3}LO-EM
$3N$: N^{2}LO (c_D=1, c_E=-0.029)

Nuclear physics uncertainty

^{4}He obesrvable		AV18+UIX	χEFT-EM	Difference
$\mu\,^4\mathrm{He}^+$ nuclear polarization	δ_{pol} [meV]	-2.408	-2.542	5.5%

Nuclear physics uncertainty

^{4}He obesrvable		AV18+UIX	χEFT-EM	Difference
binding energy	B_0 [MeV]	28.422	28.343	0.28%
point-proton nuclear radius	R_{pp} [fm]	1.432	1.475	3.0%
electric-dipole polarizability	α_E [fm^3]	0.0651	0.0694	6.4%
$\boldsymbol{\mu}\,^4$He$^+$ nuclear polarization	$\delta_{\rm pol}$ [meV]	-2.408	-2.542	5.5%

- B_0, R_{pp} & α_E in good agreement with previous calculations
  Kievsky *et al.* '08, Gazit *et al.* '06 & Stetcu *et al.* '09

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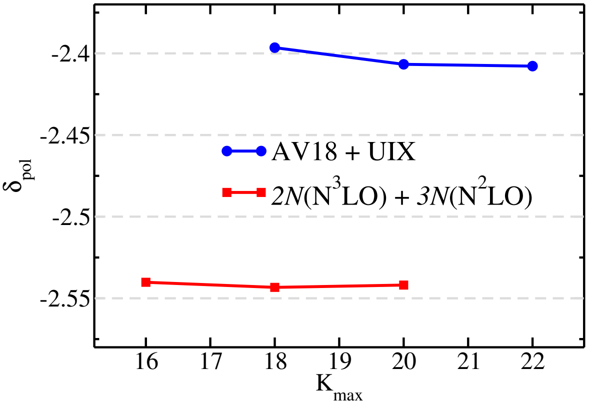
- systematic uncertainty in $\delta_{\rm pol}$ from nuclear physics:
  $\frac{5.5\%}{\sqrt{2}} \implies \pm 4\% \; (1\sigma)$

Numerical accuracy

- **Convergence with model space size**

  Compare $\delta_{\text{pol}}^{(K_{max})}$ with $\delta_{\text{pol}}^{(K_{max}-4)}$
  - AV18+UIX $\sim 0.4\%$
  - χEFT-EM $\sim 0.2\%$



Error budget — $\mu\,^4\text{He}^+$

- **Nuclear physics**
  4% from two potentials
- **Numerical accuracy**
  0.4% from convergence
- **Additional corrections**
  - $(Z\alpha)^6$ terms (beyond 2nd-order perturbation theory)
  - Rel. & Coulomb corrections (other than dipole)
  - higher-order nucleon-size corrections

  $\Rightarrow\ \sim 4\%$ estimated from additional corrections
- **Final result (quadratic sum)**

our prediction:	$\delta_{\text{pol}} = -2.47$ meV $\pm 6\%$
previous estimates:	$\delta_{\text{pol}} = -3.1$ meV $\pm 20\%$
experimental needs:	δ_{pol} uncertainty $\sim 5\%$

Reflections on $\mu\,^4\text{He}^+$

- The accuracy of δ_{pol} in $\mu\,^4\text{He}^+$ is limited by the nuclear physics (AV18+UIX vs. χEFT-EM)
- To study this we can further vary the nuclear potentials
  - Use χEFT at different orders to track the convergence
  - At each order vary the cutoff to estimate the theoretical error (χEFT-EGM: Epelbaum, Glöckle, & Meißner, **Nucl. Phys. A '05**)

Reflections on μD

- Pachucki only used AV18
  - $\Rightarrow$ No nuclear physics uncertainty
  - $\Rightarrow$ We can add χEFT-EM & χEFT-EGM
- 2-body problem
  - $\Rightarrow$ only NN interaction
  - $\Rightarrow$ simple numerics
- Other issues
  - Pachucki did not include nucleon-size corrections
  - Pachucki did not treat nucleon-polarization correctly
  - We already reproduced Pachucki's leading term as a check for $\mu\,^4\text{He}^+$
  - There is also a Magnetic contribution
  - Pachucki only calculated $\delta_{\text{TPE}} \equiv |\delta_{\text{Zem}} + \delta_{\text{pol}}|$

Nuclear polarization in μD

	Pachucki '11 (AV18)	our work (AV18)
$\delta^{(0)}_{D1}$	-1.910	-1.907
$\delta^{(0)}_{L}$	0.035	0.029
$\delta^{(0)}_{T}$	–	-0.012
$\delta^{(0)}_{C}$	0.261	0.262
$\delta^{(2)}_{R2}$	0.045	0.042
$\delta^{(2)}_{Q}$	0.066	0.061
$\delta^{(2)}_{D1D3}$	-0.151	-0.139
$\delta^{(1)}_{NS}$	–	0.017
$\delta^{(2)}_{NS}$	–	-0.015
δ_{M}	0.016	0.008
$\delta_{\rm TPE} \equiv \lvert\delta_{\rm pol} + \delta_{\rm Zem}\rvert$	**1.638**	**1.656**

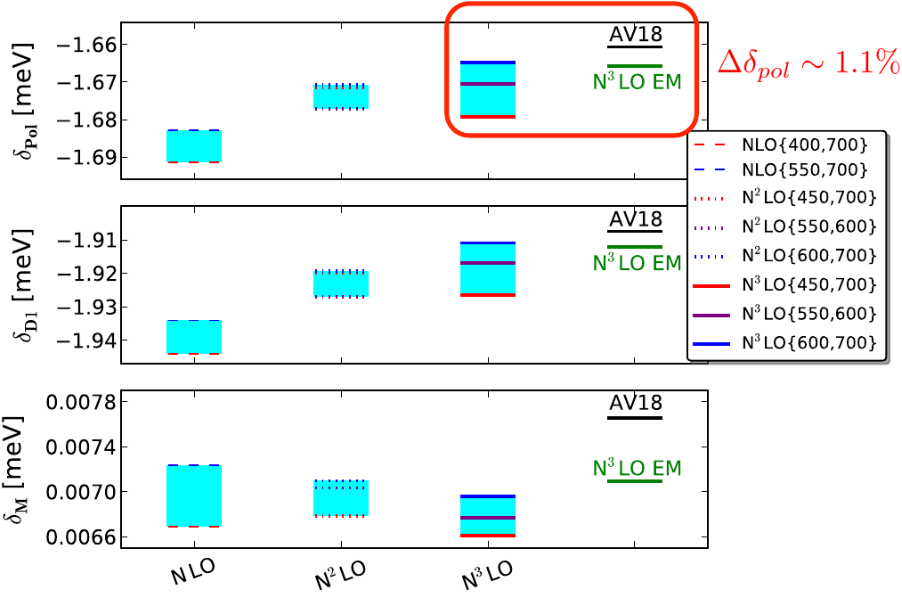
- compare: $\delta^{(0)}_{L}$ & $\delta^{(0)}_{T}$; $\delta^{(2)}$; δ_M ; δ_{NS}

Nuclear polarization in μD

	Pachucki '11	our work		
	(AV18)	(AV18)	N^3LO-EM	N^3LO-EGM
$\delta^{(0)}_{D1}$	-1.910	-1.907	-1.912	(-1.911,-1.926)
$\delta^{(0)}_{L}$	0.035	0.029	0.029	(0.029, 0.030)
$\delta^{(0)}_{T}$	–	-0.012	-0.012	-0.013
$\delta^{(0)}_{C}$	0.261	0.262	0.262	(0.262, 0.264)
$\delta^{(2)}_{R2}$	0.045	0.042	0.041	0.041
$\delta^{(2)}_{Q}$	0.066	0.061	0.061	0.061
$\delta^{(2)}_{D1D3}$	-0.151	-0.139	-0.139	(-0.139,-0.140)
$\delta^{(1)}_{NS}$	–	0.017	0.017	0.017
$\delta^{(2)}_{NS}$	–	-0.015	-0.015	-0.015
δ_M	0.016	0.008	0.007	0.007
$\delta_{\mathrm{TPE}} \equiv \lvert\delta_{\mathrm{pol}} + \delta_{\mathrm{Zem}}\rvert$	**1.638**	**1.656**	**1.661**	**(1.660,1.674)**

- compare: $\delta^{(0)}_{L}$ & $\delta^{(0)}_{T}$; $\delta^{(2)}$; δ_M ; δ_{NS}

Improved nuclear uncertainty in μD



Error budget — μD

- **Nuclear physics**
  $\sim 1.1\%$ from a range of potentials
- **Numerical accuracy**
  (negligible)
- **Additional corrections**
  0.95% (as estimated by Pachucki)
- **Final result (quadratic sum)**
  our prediction: $\delta_{\mathrm{TPE}} \equiv |\delta_{\mathrm{pol}} + \delta_{\mathrm{Zem}}| = 1.66$ meV $\pm 1.5\%$
  includes nuclear error & nucleon-size corrections
  $\Rightarrow$ improved result and error estimate

μD update: Krauth *et al.* arXiv:1506.01298v2

Item	Contribution	Pachucki [55] AV18		Friar [60] ZRA		Hernandez *et al.* [58] AV18	Hernandez *et al.* [58] N^3LO †		Pach.& Wienczek [65] AV18	
	Source	1		2		3	4		5	
p1	Dipole	1.910	$\delta_0 E$	1.925	Leading C1	1.907	1.926	$\delta_{D1}^{(0)}$	1.910	$\delta_0 E$
p2	Rel. corr. to p1, longitudinal part	−0.035	$\delta_R E$	−0.037	Subleading C1	−0.029	−0.030	$\delta_L^{(0)}$	−0.026	$\delta_R E$
p3	Rel. corr. to p1, transverse part					0.012	0.013	$\delta_T^{(0)}$		
p4	Rel. corr. to p1, higher order								0.004	$\delta_{HO} E$
sum	Total rel. corr., p2+p3+p4	−0.035		−0.037		−0.017	−0.017		−0.022	
p5	Coulomb distortion, leading	−0.255	$\delta_{C1} E$						−0.255	$\delta_{C1} E$
p6	Coul. distortion, next order	−0.006	$\delta_{C2} E$						−0.006	$\delta_{C2} E$
sum	Total Coulomb distortion, p5+p6	−0.261				−0.262	−0.264	$\delta_C^{(0)}$	−0.261	
p7	El. monopole excitation	−0.045	$\delta_{Q0} E$	−0.042	C0	−0.042	−0.041	$\delta_{R2}^{(2)}$	−0.042	$\delta_{Q0} E$
p8	El. dipole excitation	0.151	$\delta_{Q1} E$	0.137	Retarded C1	0.139	0.140	$\delta_{D1D3}^{(2)}$	0.139	$\delta_{Q1} E$
p9	El. quadrupole excitation	−0.066	$\delta_{Q2} E$	−0.061	C2	−0.061	−0.061	$\delta_Q^{(2)}$	−0.061	$\delta_{Q2} E$
sum	Tot. nuclear excitation, p7+p8+p9	0.040		0.034	C0 + ret-C1 + C2	0.036	0.038		0.036	
p10	Magnetic	−0.008 ◇[a]	$\delta_M E$	−0.011	M1	−0.008	−0.007	$\delta_M^{(0)}$	−0.008	$\delta_M E$
SUM_1	Total nuclear (corrected)	1.646		1.648 [b]		1.656	1.676		1.655	
p11	Finite nucleon size			0.021	Retarded C1 f.s.	0.020 ◇[c]	0.020 ◇[c]	$\delta_{NS}^{(2)}$	0.020	$\delta_{FS} E$
p12	n p charge correlation			−0.023	pn correl. f.s.	−0.017	−0.017	$\delta_{np}^{(1)}$	−0.018	$\delta_{FZ} E$
sum	p11+p12			−0.002		0.003	0.003		0.002	
p13	Proton elastic 3rd Zemach moment	} 0.043(3)	$\delta_P E$	0.030	$\langle r^3 \rangle_{(2)}^{pp}$				} 0.043(3)	$\delta_P E$
p14	Proton inelastic polarizab.					} 0.027(2)		δ_{pol}^N [64]		
p15	Neutron inelastic polarizab.								0.016(8)	$\delta_N E$
p16	Proton & neutron subtraction term									
sum	Nucleon TPE, p13+p14+p15+p16	0.043(3)		0.030		0.027(2)			0.059(9)	
SUM_2	Total nucleon contrib.	0.043(3)		0.028		0.030(2)			0.061(9)	
	Sum, published	1.680(16)		1.941(19)		1.690(20)			1.717(20)	
	Sum, corrected			1.697(19) [g]		1.714(20) [h]			1.707(20) [i]	

μD update: Krauth *et al.* arXiv:1506.01298v2

Item	Contribution	Pachucki '11		Friar		Our Work (Hernandez *et al.* '14)			Pachucki *et al.* '15	
		AV18		ZRA		AV18	N^3LO †		AV18	
	Source	1		2		3	4		5	
p1	Dipole	1.910	$\delta_0 E$	1.925	Leading C1	1.907	1.926	$\delta_{D1}^{(0)}$	1.910	$\delta_0 E$
p2	Rel. corr. to p1, longitudinal part	−0.035	$\delta_R E$	−0.037	Subleading C1	−0.029	−0.030	$\delta_L^{(0)}$	−0.026	$\delta_R E$
p3	Rel. corr. to p1, transverse part					0.012	0.013	$\delta_T^{(0)}$		
p4	Rel. corr. to p1, higher order								0.004	$\delta_{HO} E$
sum	Total rel. corr., p2+p3+p4	−0.035		−0.037		−0.017	−0.017		−0.022	
p5	Coulomb distortion, leading	−0.255	$\delta_{C1} E$						−0.255	$\delta_{C1} E$
p6	Coul. distortion, next order	−0.006	$\delta_{C2} E$						−0.006	$\delta_{C2} E$
sum	Total Coulomb distortion, p5+p6	−0.261				−0.262	−0.264	$\delta_C^{(0)}$	−0.261	
p7	El. monopole excitation	−0.045	$\delta_{Q0} E$	−0.042	C0	−0.042	−0.041	$\delta_{R2}^{(2)}$	−0.042	$\delta_{Q0} E$
p8	El. dipole excitation	0.151	$\delta_{Q1} E$	0.137	Retarded C1	0.139	0.140	$\delta_{D1D3}^{(2)}$	0.139	$\delta_{Q1} E$
p9	El. quadrupole excitation	−0.066	$\delta_{Q2} E$	−0.061	C2	−0.061	−0.061	$\delta_Q^{(2)}$	−0.061	$\delta_{Q2} E$
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Work in progress

The work is not completed yet ...

Hot from the press: δ_{pol} in $\mu\,^3\text{He}^+$

[meV]	AV18+UIX	χEFT★
$\delta^{(0)}$	-5.346	-5.486
$\delta^{(1)}$	-0.438	-0.384
$\delta^{(2)}$	0.806	0.830
δ_{NS}	0.783	0.792
δ_{Mag}	0.081	0.047
δ_{pol}	-4.114	-4.201

- Convergence from $\delta^{(0)}$ to $\delta^{(2)}$?

★ NN: N^{3}LO-EM

$3N$: N^{2}LO (c_D=1, c_E=-0.029)

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  $\sim 1\%$ (~ 0.04 meV)
  c.f. $\mu\,^4$He$^+$:
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  $-10.5(2)$ meV w/$r_p(\mu^-)$
  $-10.7(2)$ meV w/$r_p(e^-)$
  both agree with $-10.87(27)$ meV
  from scattering data (Sick, **PRC '14**)

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Hot from the press: $\delta_{\rm pol}$ in $\mu\,^3$H

[meV]	AV18+UIX	χEFT★
$\delta^{(0)}$	-0.680	-0.695
$\delta^{(1)}$	0.178	0.184
$\delta^{(2)}$	-0.025	-0.029
δ_{NS}	0.053	0.054
δ_{Mag}	0.010	0.006
$\delta_{\rm pol}$	-0.465	-0.480

- Convergence from $\delta^{(0)}$ to $\delta^{(2)}$?
- AV18+UIX vs. χEFT:
  $\sim 3\%$ (~ 0.02 meV)
- Probe $\delta^N_{\rm pol}$ of the nucleons ?

  $\delta^N_{\rm pol} \approx 34(16)$ μeV
- Precise triton radius

★ NN: N^{3}LO-EM
$3N$: N^{2}LO (c_D=1, c_E=-0.029)

Uncertainty estimates

	$\mu\,^3\mathbf{He}^+$			$\mu\,^3\mathbf{H}$		
Error type	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$
Numerical	0.4	0.1	0.1	0.1	0.0	0.1
Nuclear model	1.5	1.8	1.7	2.2	2.3	2.2
ISB	2.0	0.2	0.5	0.9	0.2	0.6
Nucleon size	1.6	1.5	0.6	0.6	1.3	0.0
Relativistic	0.6	-	1.5	1.4	-	0.3
Coulomb	1.2	-	0.3	0.3	-	0.2
Multipole expansion	2.0	-	0.6	2.0	-	1.4
Higher $Z\alpha$	1.5	-	0.4	0.7	-	0.5
Magnetic MEC	0.4	-	0.1	0.3	-	0.2
Total	4.1%	2.3%	2.5%	3.6%	2.7%	2.7%

Summary

- **Lamb shifts in muonic atoms**
  - raise interesting questions about lepton universality
  - probe isospin dependence of the proton radius puzzle
  - allow high precision determination of the nuclear charge radius $\langle \boldsymbol{r^2} \rangle$
  - For $A > 1$ the precision of $\langle \boldsymbol{r^2} \rangle$ is bound by the nuclear polarization $\delta^A_{\rm pol}$

- **We performed first *ab-initio* calculations of $\delta_{\rm pol}$ and $\delta_{\rm Zem}$ in $\boldsymbol{\mu}\,^{\boldsymbol{3,4}}He^+$ and μT, and improved the treatment of μD**

μD	$\delta_{\rm TPE} = 1.66(2)$	meV	[PLB **736**, 344 (2014)]
μT	$\delta_{\rm TPE} = 0.70(2)$	meV	arXiv:1512.05773; PLB online
μ^3He$^+$	$\delta_{\rm TPE} = 14.6(4)$	meV	(arXiv:1512.05773; PLB online)
μ^4He$^+$	$\delta_{\rm TPE} = 8.6(3)$	meV	[PRL **111**, 143402 (2013)]

  - more accurate than previous calculations
  - will significantly improve the precision of $\langle \boldsymbol{r^2} \rangle$ extracted from ongoing $\boldsymbol{\mu}\,^{\boldsymbol{3,4}}He^+$ Lamb shift measurements

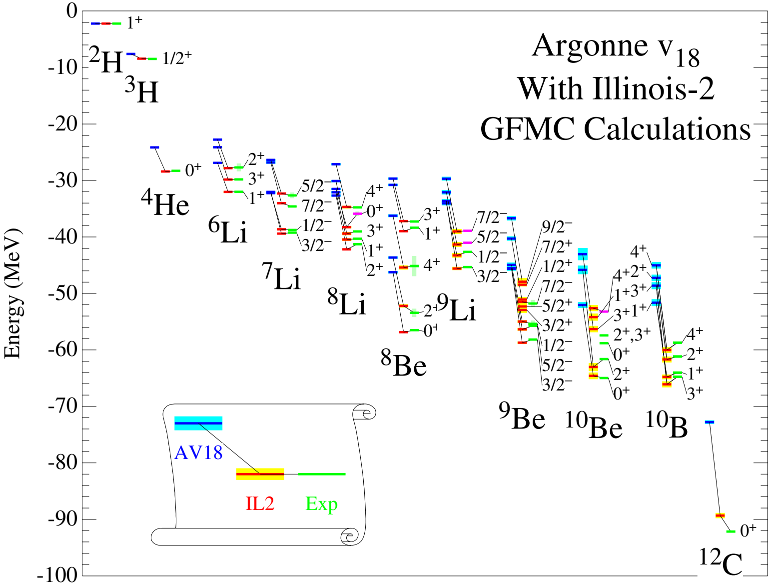
Outlook

- Study higher-order terms
- Reduce nuclear physics uncertainty
  - understand why various nuclear potentials differ
  - further explore the various parameterizations (3NF?)
  - include higher-order or otherwise improved χEFT forces
- Investigate nuclear polarization in *e.g.* $\boldsymbol{\mu\,^6\mathrm{Li}^{+2}}$, $\boldsymbol{\mu}\,^{\mathbf{6}}\mathrm{He}^{+}$, $\cdots$
- Investigate nuclear polarization in HFS of electronic and muonic atoms

The muonic Lamb shift is still puzzling !!!

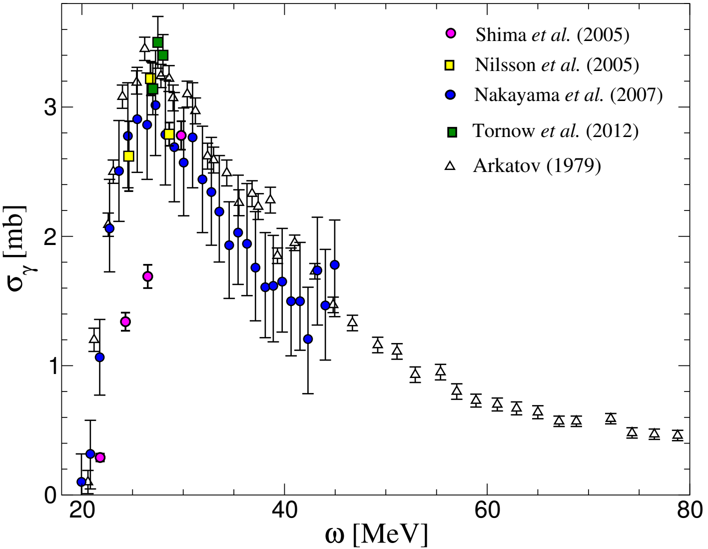
BACK UP

Phenomenological potentials



^{4}He photoabsorption cross sections

electric-dipole photoabsorption cross section $\sigma_\gamma(\omega) = 4\pi^2\alpha\omega S_{D1}(\omega)$



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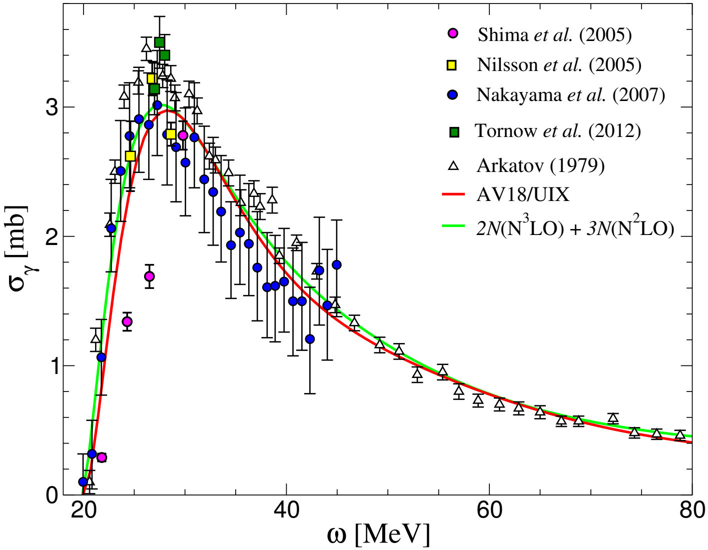

Non-Relativistic Approximation

- Coulomb interactions in the (virtual) intermediate state are neglected
- Results are expressed in an expansion of $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'|$

$\Delta E_{NR} = \Delta E_{NR}^{(2)} + \Delta E_{NR}^{(3)} + \Delta E_{NR}^{(4)}$

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1. $\sim R^2$ **term:**

- $\Delta E_{NR}^{(2)}$ **is the dominant polarizability contribution**

$$\boxed{\Delta E_{NR}^{(2)} = -\frac{2\pi m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{2m_r}{\omega}} S^{D_1}(\omega)}$$

- $S^{D_1}(\omega) = \frac{1}{2J_0+1} \sum_{N \neq N_0, J} |\langle N_0 J_0 || \hat{D}_1 || N J \rangle|^2 \delta(\omega - E_N + E_{N_0})$
- $\hat{D}_1 = \frac{1}{Z} \sum_{i}^{Z} R_i Y_1(\hat{R}_i)$

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2. $\sim R^3$ **term:**

$$\Delta E_{NR}^{(3)} = -\frac{m_r^4}{24}(Z\alpha)^5 \left[\iint d^3R d^3R' |\boldsymbol{R}-\boldsymbol{R}'|^3 \langle N_0|\hat{\rho}^\dagger(\boldsymbol{R})\hat{\rho}(\boldsymbol{R}')|N_0\rangle - \langle r^3\rangle_{(2)} \right]$$

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- 1st term: charge correlation function
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- 1st term: charge correlation function
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- 2nd term: Zemach moment
  $\langle r^3\rangle_{(2)} = \iint d^3R d^3R' |\boldsymbol{R}-\boldsymbol{R}'|^3 \rho_0(\boldsymbol{R})\,\rho_0(\boldsymbol{R}')$
  $\rho_0(\boldsymbol{R}) = \langle N_0|\hat{\rho}(\boldsymbol{R})|N_0\rangle$

  cancels exactly the **Zemach term** in (elastic) finite-size corrections
  c.f. Pachucki PRL 2011 (μD)

Non-Relativistic Approximation

3. $\sim R^4$ **term:**

- $\Delta E_{NR}^{(4)}$ **corresponds to higher-multipole corrections**

$$\Delta E_{NR}^{(4)} = \frac{m_r^5}{18}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{\omega}{2m_r}} \left[S^{Q_0}(\omega) + \frac{16\pi}{25} S^{Q_2}(\omega) + \frac{16\pi}{5} S^{D_{13}}(\omega) \right]$$

- $S^{R^2}(\omega) = \frac{1}{2J_0+1} \sum_{N \neq N_0, J} |\langle N_0 J_0 || \hat{R}^2 || N J \rangle|^2 \delta(\omega - E_N + E_{N_0})$

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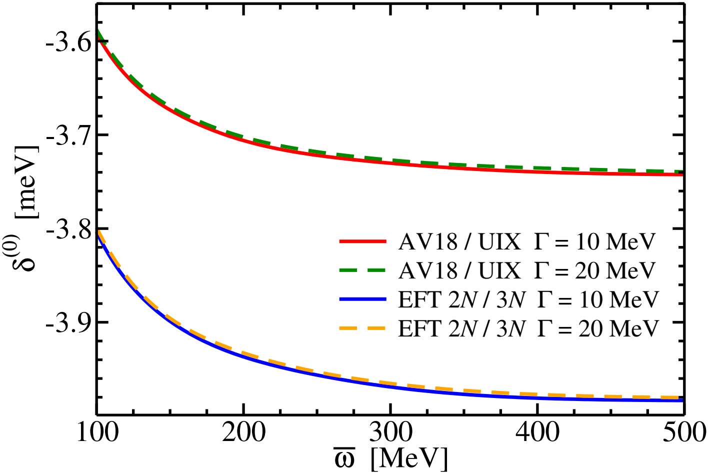
$$S^{D_{13}}(\omega) = \frac{1}{2J_0+1} \sum_{N \neq N_0, J} (-1)^{J_0 - J} \times \mathrm{Re} \left(\langle N_0 J_0 || \hat{D}_3 || N J \rangle \langle N J || \hat{D}_1 || N_0 J_0 \rangle \right) \delta(\omega - E_N + E_{N_0})$$

- $\hat{R}^2 = \frac{1}{Z} \sum_i^Z R_i^2$ $\qquad \hat{D}_1 = \frac{1}{Z} \sum_i^Z R_i Y_1(\hat{R}_i)$

$\hat{Q}_2 = \frac{1}{Z} \sum_i^Z R_i^2 Y_2(\hat{R}_i)$ $\qquad \hat{D}_3 = \frac{1}{Z} \sum_i^Z R^3 Y_1(\hat{R}_i)$

Convergence of Ab-initio calculations

Convergence of $\bar{\omega}$



Convergence of Ab-initio calculations

$\delta^{(0)}$ convergence with the largest model space K_{max}