# Deuteron electrodisintegration with unitarily evolved potentials

Sebastian König

in collaboration with S. N. More, R. J. Furnstahl, and K. Hebeler

**Nuclear Theory Workshop**

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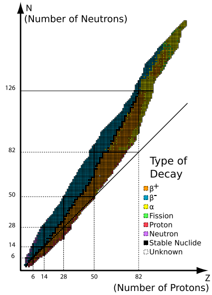
More, SK, Furnstahl, Hebeler, PRC **92** 064002 (2015)
and work in progress

# Motivation

**nuclear structure**

ground states, stability, transitions, etc.

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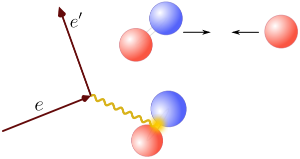


**reactions**

disintegration, scattering, ...

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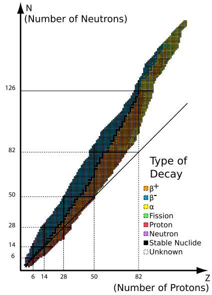


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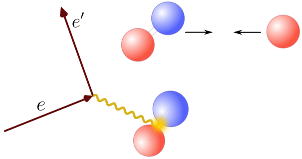


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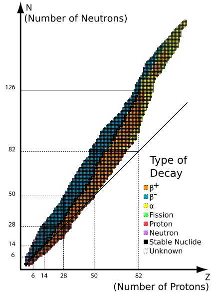


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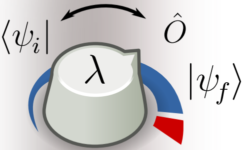
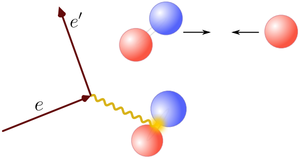



GNOME icon artists

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# Motivation

**Typical *ab initio* calculation**

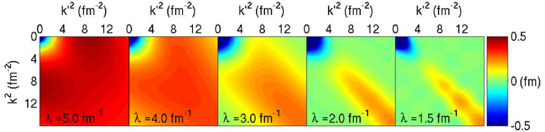
**(chiral) potential → SRG → many-body method → result**

**evolve ("soften") interaction with unitary transformations**

- transform Hamiltonian with flow equation: $\mathrm{d}H_s/\mathrm{d}s = [[G, H_s], H_s]$, $\lambda = 1/s^{1/4}$
- equivalently: unitary transformation $H \to H_\lambda = U_\lambda H U_\lambda^\dagger \rightsquigarrow V_\lambda$
- $\hookrightarrow$ **interaction becomes amenable to numerical methods**

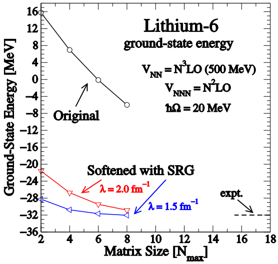
see, *e.g.*, Bogner *et al.*, PPNP **65** 94 (2010)

matrix elements $\langle k|V|k'\rangle$ evolve towards diagonal form




$\lambda$ = effective resolution scale

R. Furnstahl, HUGS 2014 lecture slides

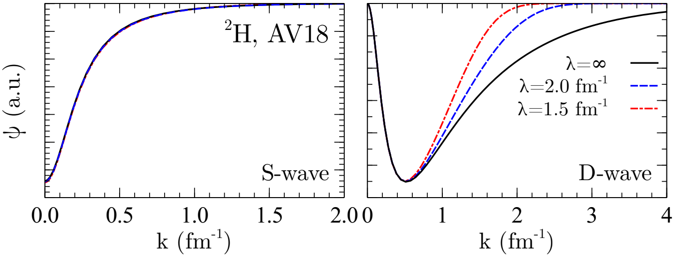


# SRG unitarity

**SRG evolution is a unitary transformation**

$\hookrightarrow$ eigenvalues stay invariant, eigenstates do not!

$$\underbrace{\langle\psi|U_\lambda^\dagger}_{\langle\psi_\lambda|}\;\underbrace{U_\lambda H\,U_\lambda^\dagger}_{H_\lambda}\;\underbrace{U_\lambda|\psi\rangle}_{|\psi_\lambda\rangle}$$




- high-momentum modes are suppressed by SRG evolution...
- ...but of course the physics does not go away!

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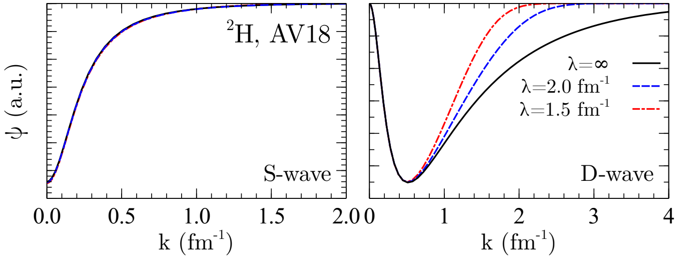


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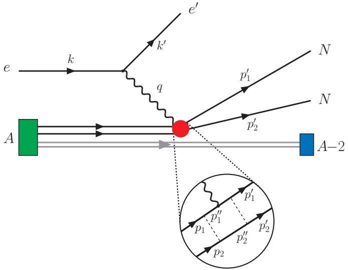
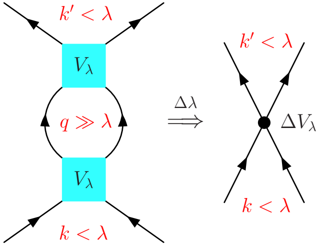
- . . . but of course the physics does not go away!

**bottom line: all operators have to be evolved consistently!**

# Decoupling and factorization

- SRG evolution can be interpreted as a change in **resolution**
- choice of potential and $\lambda$ introduce **scheme and scale dependence**
- treating nuclear structure and reactions separately assumes **factorization**




Furnstahl, 1309.5771 [nucl-th]

## Questions

- is there a net simplification for reaction calculations?
- how to understand knock-out reactions in the absence of SRCs?

# Status of SRG operator evolution

- **static deuteron properties** Anderson *et al.*, PRC **82** 054001 (2010)
  - momentum distribution, $\langle r^2 \rangle$, ...
  - no pathologies in evolved operators
  - small evolution effects for low-momentum observables

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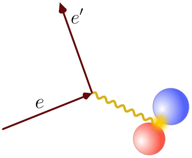
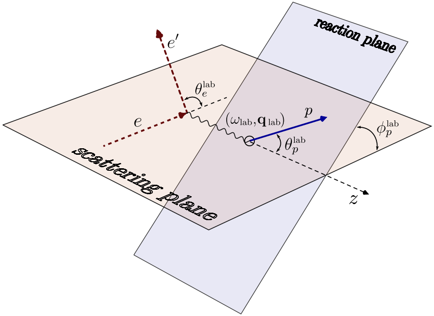
**What about nuclear knock-out reactions?**

$\hookrightarrow$ study deuteron electrodisintegration!



# Deuteron disintegration

**use deuteron electrodisintegration as controlled laboratory**




- study evolution of initial state, current operator, and FSI
  ↪ **all mixed under evolution**
- no three-body effects
- rich kinematic structure

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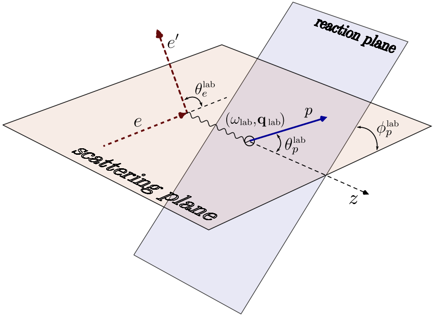


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**longitudinal structure function**

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}k'^{\rm lab}\mathrm{d}\Omega_e^{\rm lab}} \sim v_L f_L + v_T f_T + \cdots$$

- $v_L$, $v_T$, ...: kinematic factors
- $f_L$, $f_T$, ...: observables

Yang+Phillips (2013), Arenhövel *et al.* (1988), Donnelly+Raskin (1986), ...

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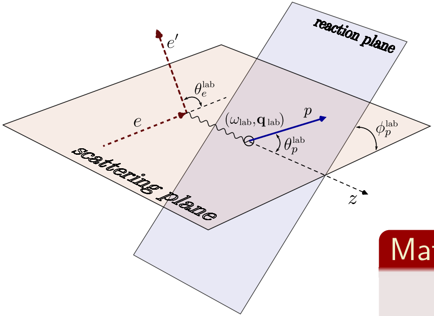


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## Matrix elements

$$f_L(E', \mathbf{q}^2; \cos\theta') \propto |\langle\psi_f|J_0|\psi_i\rangle|^2$$

$$\langle\psi_f|J_0|\psi_i\rangle = \underbrace{\langle\phi|J_0|\psi_i\rangle}_{\text{IA}} + \underbrace{\langle\phi|t\, G_0 J_0|\psi_i\rangle}_{\text{FSI}}$$

- $E'$ = energy of outgoing nucleons (c.m. frame)
- $\theta'$ = angle of outgoing nucleons (c.m. frame)
- $\mathbf{q}^2$ = momentum transfer in c.m. frame

Yang+Phillips (2013), Arenhövel *et al.* (1988), Donnelly+Raskin (1986), . . .

# Deuteron disintegration

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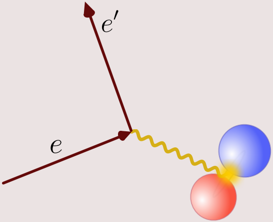
- $|\psi_i\rangle$ = deuteron wavefunction
- $|\psi_f\rangle = |\phi\rangle + G_0 t|\phi\rangle = NN$ scattering state
- $J_0$ = e.m. current from virtual photon



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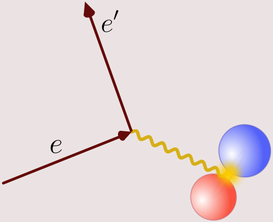


- e.m. current given in terms of nucleon formfactors ($T, T_1$ = isospin):

$$\langle\mathbf{k}_1\,T_1|J_0(\mathbf{q})|\mathbf{k}_2\,T=0\rangle$$
$$= \frac{1}{2}(G_E^p + (-1)^{T_1} G_E^n)\,\delta(\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{q}/2) + \frac{1}{2}((-1)^{T_1} G_E^p + G_E^n)\,\delta(\mathbf{k}_1 - \mathbf{k}_2 + \mathbf{q}/2)$$

- evolution of initial/final state: *just replace* $V \to V_\lambda$, for current: $U_\lambda J_0 U_\lambda^\dagger$

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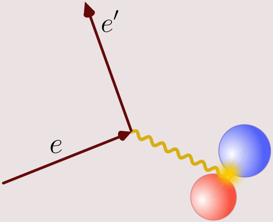


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- evolution of initial/final state: *just replace* $V \to V_\lambda$, for current: $U_\lambda J_0 U_\lambda^\dagger$
- **study evolution of individual pieces (and their interplay)!**

# SRG unitarity at work

## Invariance of matrix elements

- since $U_\lambda^\dagger U_\lambda = \mathbf{1}$, matrix elements are invariant: $\langle \psi_f | \widehat{O} | \psi_i \rangle = \langle \psi_f^\lambda | \widehat{O}^\lambda | \psi_i^\lambda \rangle$

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- individual changes add up to zero $\rightarrow$ unitarity preserved ✓
- changes in initial and final states compensated by the evolved operator
- $\hookrightarrow$ **physics “reshuffled” between structure and reaction**

# Computational issues

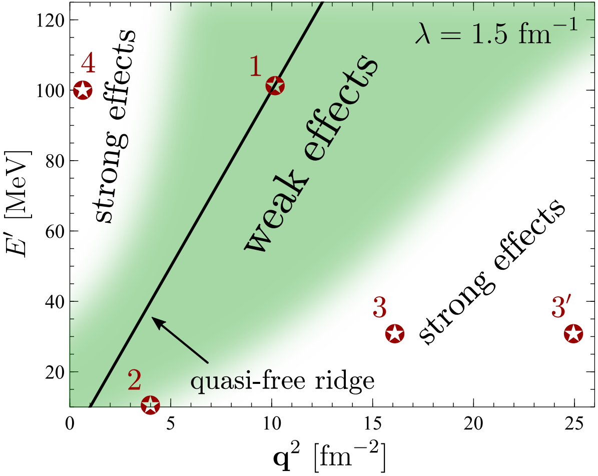
**Only a two-body system, but still computationally intensive. . .**

- need off-shell T-matrices in many (coupled) partial waves
- delta functions in current operator
- large number of intermediate sums and integrals, *e.g.* $\langle\phi|t_\lambda^\dagger G_0^\dagger \widetilde{U} J_0 \widetilde{U}^\dagger|\psi_i^\lambda\rangle$

## Solutions

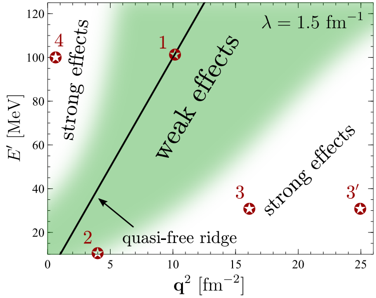
- implementation completely in modern C++11
  - object-oriented code design $\rightarrow$ easily extendable!
  - functional techniques $\rightarrow$ stay close to math on paper!
  - rigorous const-ness annotations $\rightarrow$ thread-safety easily achieved!
- use Schrödinger and LS equations for high-accuracy interpolation
- transparent caching techniques ("memoization")
- parallel implementation with Intel TBB library (scales very well)

**Importance of consistent evolution depends on kinematics!**




More, SK, Furnstahl, Hebeler, PRC **92** 064002 (2015)

# Evolution effects

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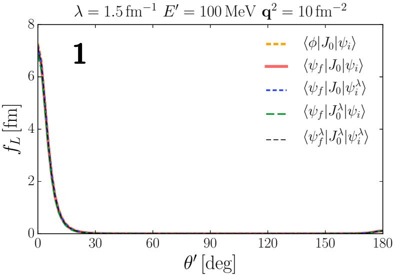

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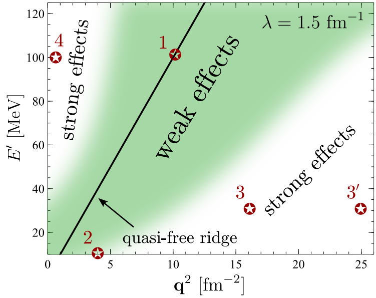


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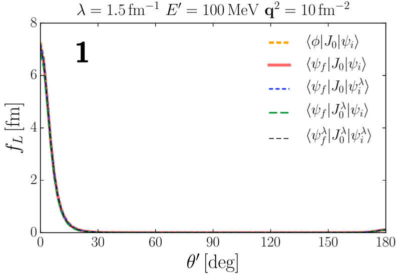
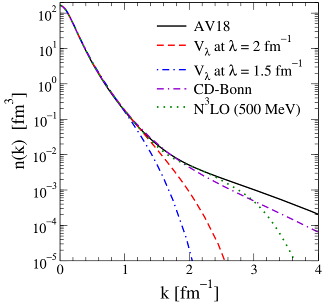



**on the quasi-free ridge:**

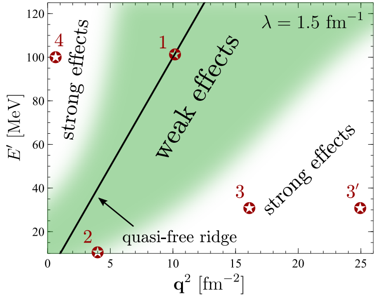
- energy transfer $\omega = 0$
- nucleons on-shell, FSI are minimal
- only low-momentum modes are probed

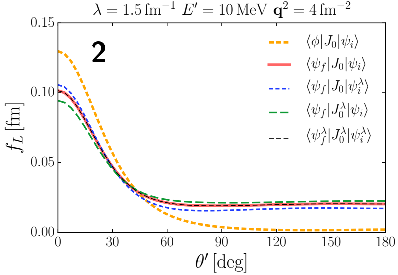
**low-momentum modes stay invariant!** $\rightarrow$

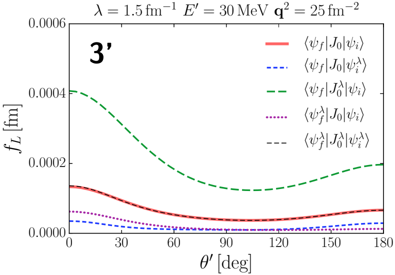


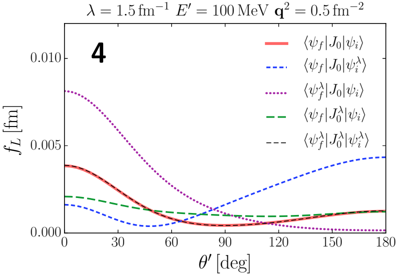
# Evolution away from the quasi-free ridge

More, SK, Furnstahl, Hebeler, PRC **92** 064002 (2015)



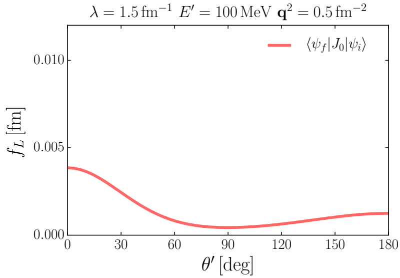



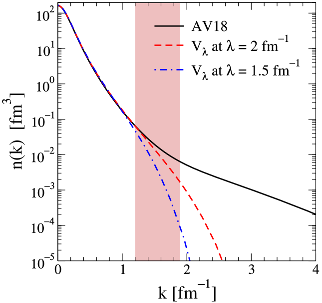



# Dissection of evolution effects

## Detailed look at evolution above the quasi-free ridge



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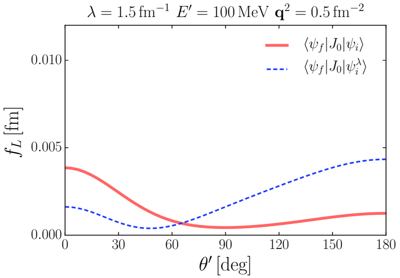
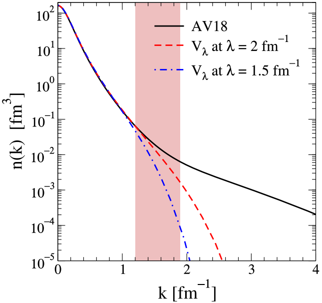
## Detailed look at evolution above the quasi-free ridge




- destructive interference between IA and FSI parts

$$\langle\psi_f|J_0|\psi_i\rangle = \underbrace{\langle\phi|J_0|\psi_i\rangle}_{\text{IA}} + \underbrace{\langle\phi|t\,G_0 J_0|\psi_i\rangle}_{\text{FSI}}$$

- small angles, IA dominates over FSI, vice versa for large angles
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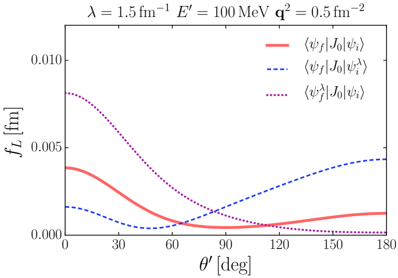


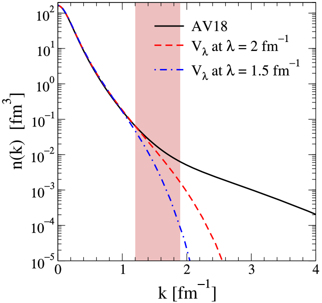


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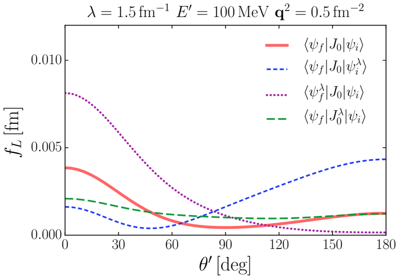


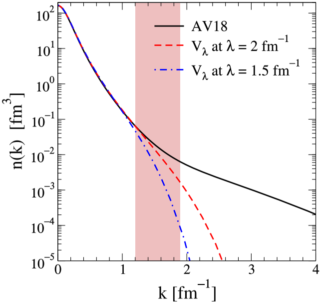


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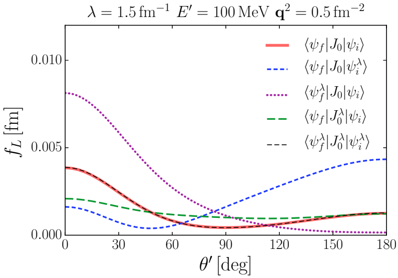


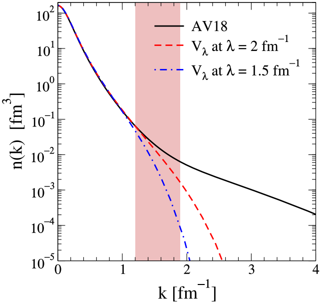


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**So what exactly happens to the current operator?**

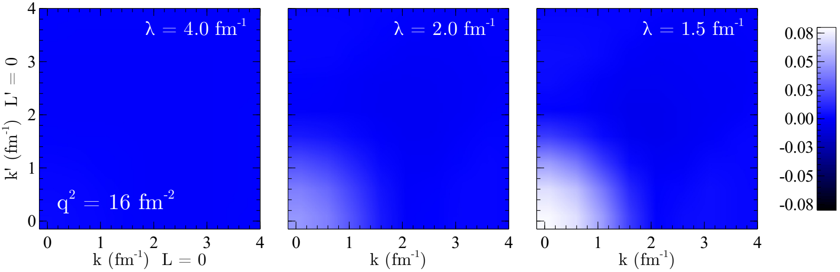
$\hookrightarrow$ work in progress...

# Current evolution status

- delta functions complicate analysis...

$$\langle \mathbf{k}_1\, T_1 | J_0(\mathbf{q}) | \mathbf{k}_2\, T=0 \rangle$$
$$= \frac{1}{2}\left(G_E^p + (-1)^{T_1} G_E^n\right) \delta(\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{q}/2) + \frac{1}{2}\left((-1)^{T_1} G_E^p + G_E^n\right) \delta(\mathbf{k}_1 - \mathbf{k}_2 + \mathbf{q}/2)$$

- look at partial-wave matrix elements of $J_0^\lambda(\mathbf{q}) - J_0(\mathbf{q})$

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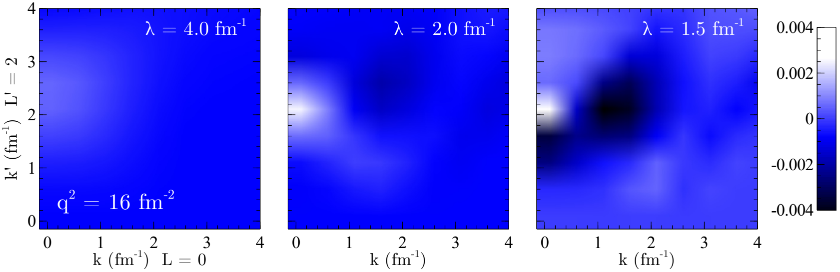


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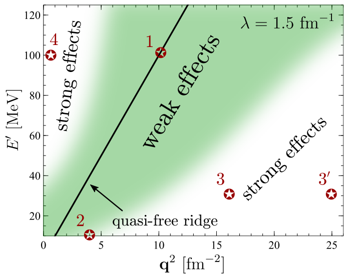
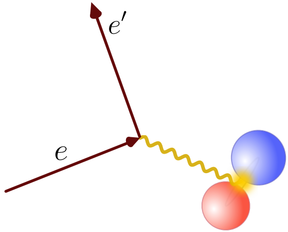
# Summary

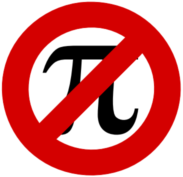


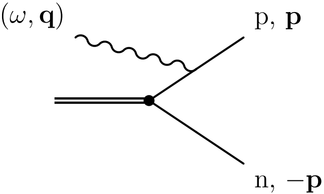



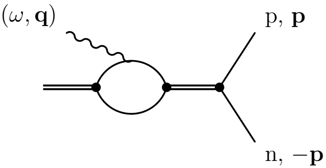

- operator – and FSI – evolution has to compensate suppression of high-momentum modes (unitarity!)
- effects of SRG evolution depend (strongly) on kinematics. . .
- . . . but in a *systematic* way
- evolution effects are minimal for quasi-free kinematics
- scale and scheme dependence is, in general, very significant
- $\hookrightarrow$ **important to use evolved operators for consistency!**

# Outlook

- extend to **larger systems**
  - inclusion of three-body forces
  - evolution of three-body currents
    $\hookrightarrow$ power counting for operator evolution?
- **understand current evolution** in more detail
  - emergence of many-body components?
  - impact on factorization assumptions?
- study electrodisintegration in **pionless EFT**
  $\hookrightarrow$ simplicity should allow analytical insights!






*cf.* Christlmeier+Grießhammer, PRC **77** 064001 (2008)