Bound and continuum properties of A=6 nuclei

Progress in Ab Initio Techniques in Nuclear Physics

February 18th, 2015

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Collaborators:
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Outline

- Introduction
- NCSM/RGM
- NCSMC
- ^{6}Li structure and d+^{4}He dynamics
- ^{6}He within a ^{4}He+n+n basis
- Preliminary calculation for ^{3}H+n+n
- Summary and outlook

Introduction

Ab initio in nuclear physics

- Assumes nucleons as the effective degrees of freedom
- Uses realistic interactions
- The goal is twofold:

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To achieve a predictive theory for light nuclear systems:

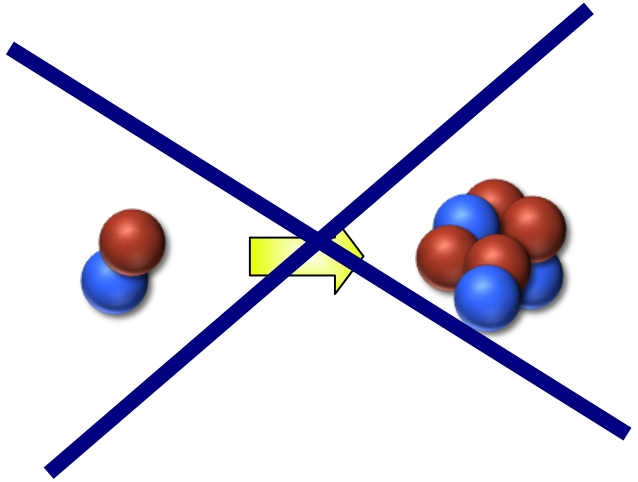
- Exotic nuclei
- Reactions important in nuclear astrophysics
- Reactions important for energy production projects

Introduction

No-core shell model (NCSM)

Is an ***ab initio*** method capable of studying light bound nuclei from an accurate Hamiltonian.

Is not able to deal with continuum states and therefore is not applicable to reactions.



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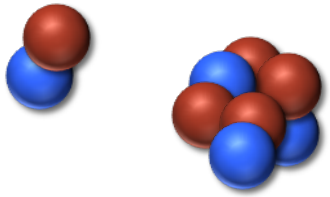
Resonating group method (RGM)

Microscopic cluster approach.

Permits studying the scattering of clusters

Traditionally uses non-realistic Hamiltonian



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NCSM/RGM

Uses NCSM wave functions within the RGM to obtain an *ab initio* formalism which uses an accurate nuclear Hamiltonian and is capable of studying both structure and scattering problems in light nuclear systems

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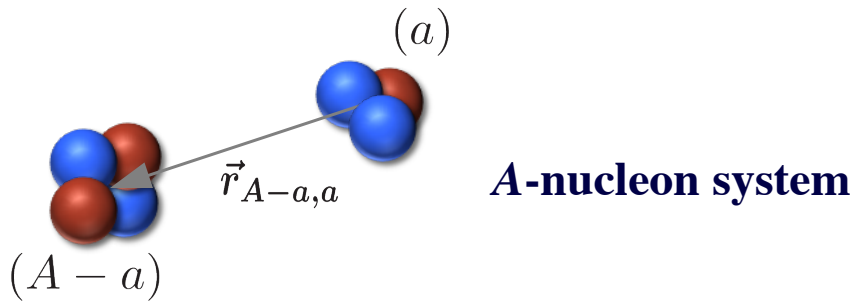
NCSM/RGM

Summary: binary clusters



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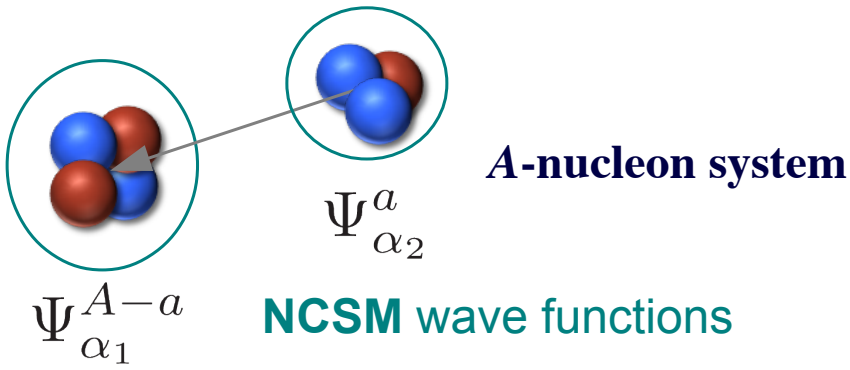

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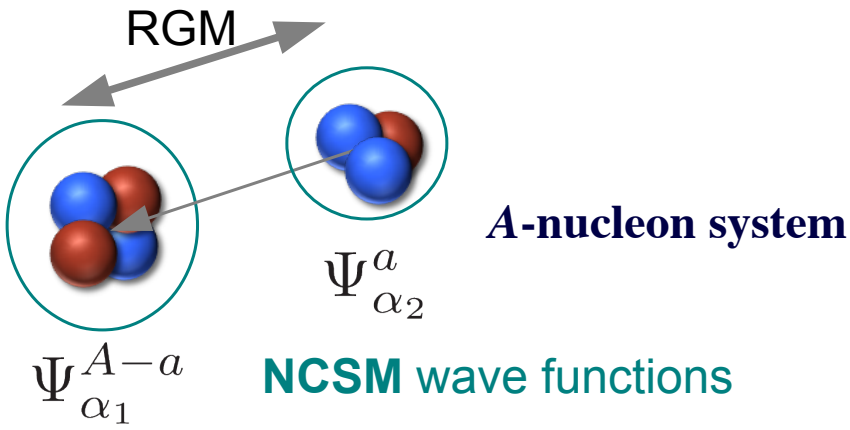

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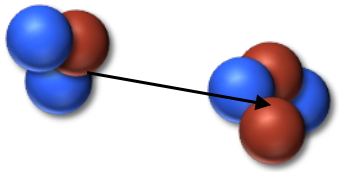
NCSMC

NCSM/RGM
Long range description



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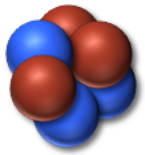

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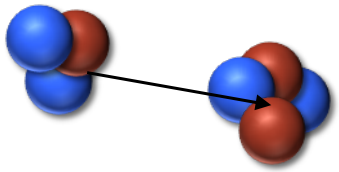

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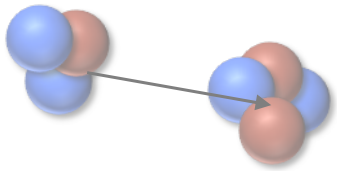

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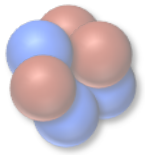


+

NCSM eigenstates

NCSM/RGM channel states

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| (A) \; , \lambda \right\rangle + \sum_{\nu} \int d\vec{r} \; \gamma_{\nu}(\vec{r}) \; \hat{A}_{\nu} \left| \; \vec{r} \; , \nu \right\rangle$$

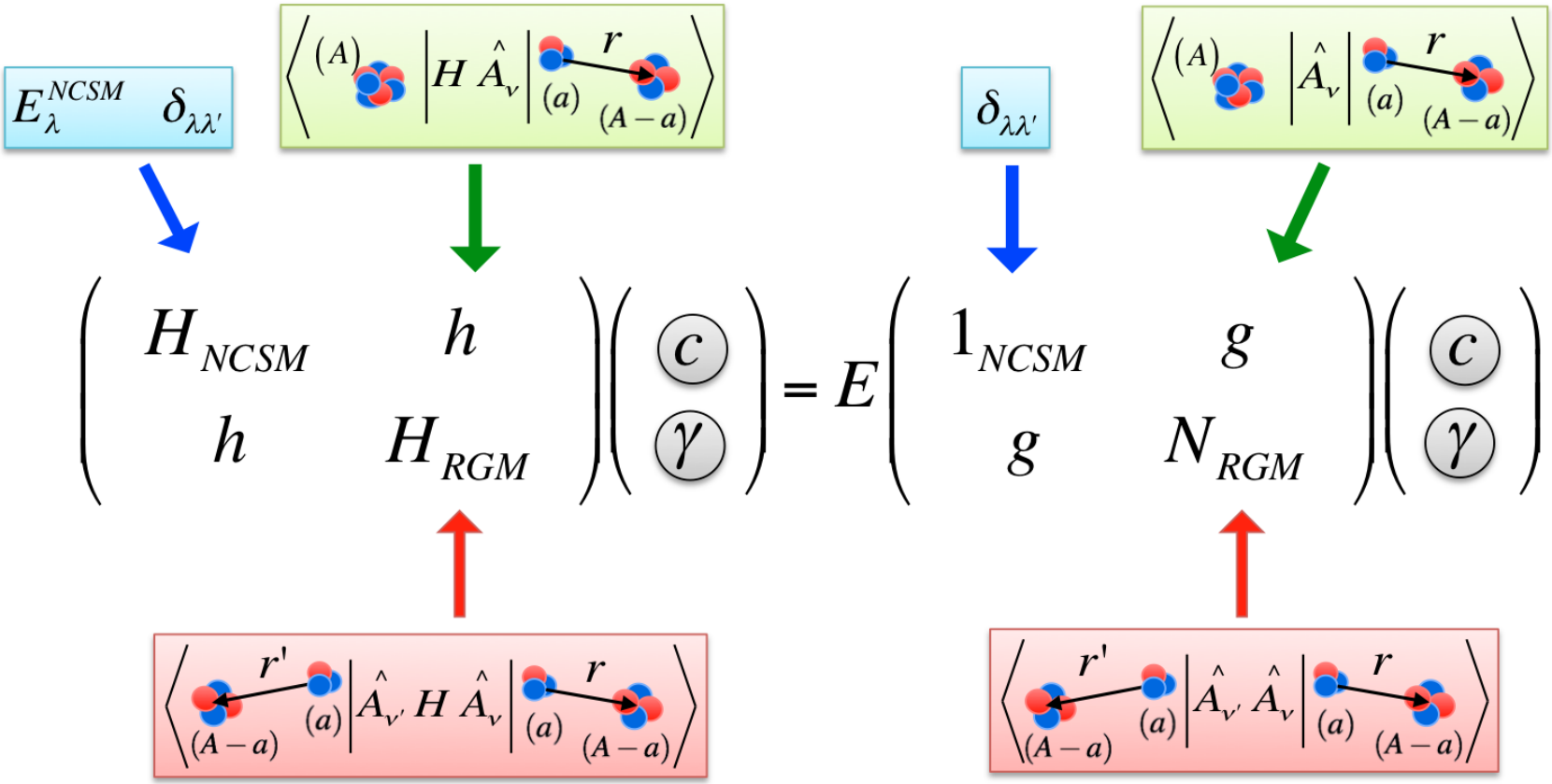
$(A-a)$ (a)

Unknowns

NCSMC

$$\begin{pmatrix} H_{NCSM} & h \\ h & H_{RGM} \end{pmatrix} \begin{pmatrix} c \\ \gamma \end{pmatrix} = E \begin{pmatrix} 1_{NCSM} & g \\ g & N_{RGM} \end{pmatrix} \begin{pmatrix} c \\ \gamma \end{pmatrix}$$

NCSMC


E_λ^{NCSM} $\delta_{\lambda\lambda'}$
$\left\langle (A) \left| H \hat{A}_\nu \right| (a)\ r\ (A-a) \right\rangle$
$\delta_{\lambda\lambda'}$
$\left\langle (A) \left| \hat{A}_\nu \right| (a)\ r\ (A-a) \right\rangle$
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$\left\langle (A-a)\ r' \ (a) \left| \hat{A}_{\nu'} H \hat{A}_\nu \right| (a)\ r\ (A-a) \right\rangle$
$\left\langle (A-a)\ r' \ (a) \left| \hat{A}_{\nu'} \hat{A}_\nu \right| (a)\ r\ (A-a) \right\rangle$

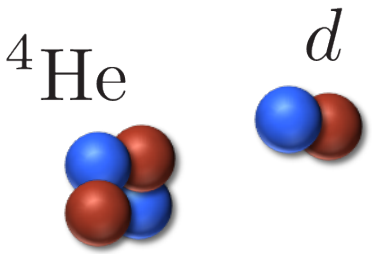
NCSMC

- One nucleon projectile: PRL **110**, 022505 (2013)
- Two-nucleon projectile: This talk
- Three-nucleon projectile: J. Dohet-Eraly, today at 11am

NCSM/RGM

- (d,p) transfer reactions with p-shell target: F. Raimondi tomorrow at 10am
- Three-cluster basis: This talk

^{6}Li structure and d+^{4}He dynamics



***6*-nucleon system**

* Big bang nucleosynthesis

* Characterization of deuterium impurities in materials

NN forces	P. Navratil and S. Quaglioni. PRC **83**, 044609 (2011)
NN+3N forces	G. Hupin, S. Quaglioni and P. Navrátil. ArXiv: 1412.4101

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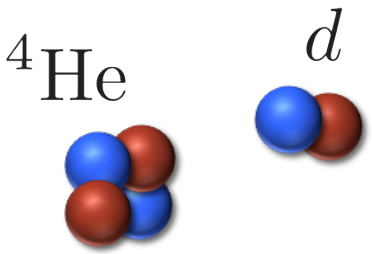

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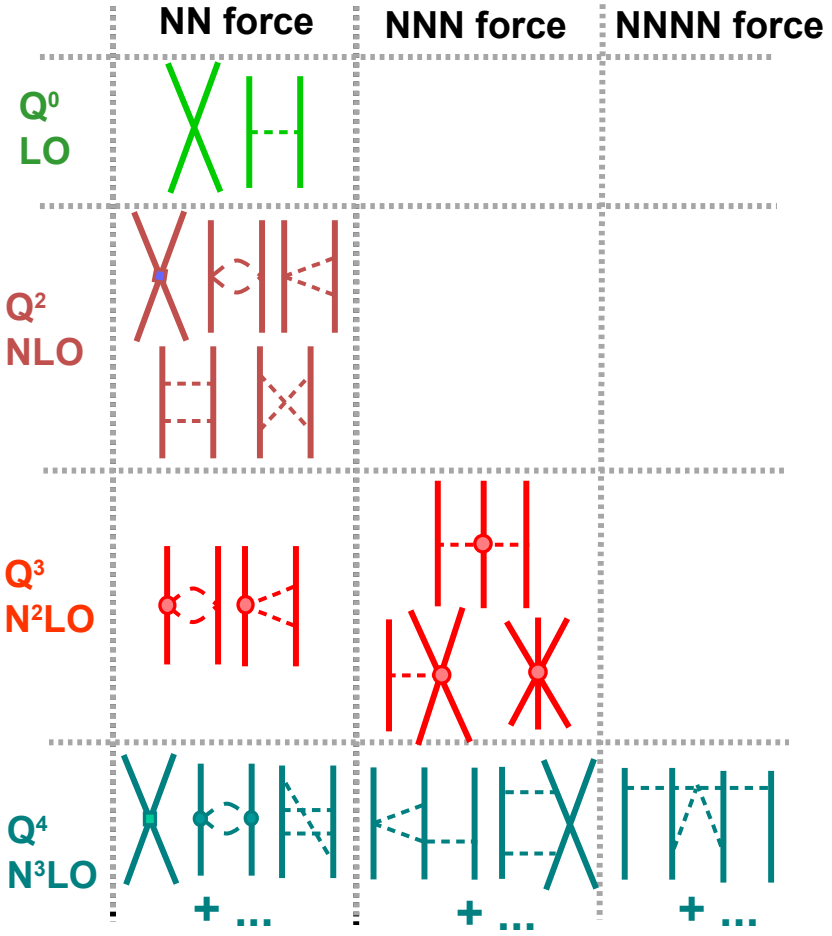
Nuclear interaction

- Two- plus three-nucleon (NN+3N) forces from chiral effective field theory (EFT):
  - NN potential at N^{3}LO (by Entem & Machleidt).
  - 3N force at N^{2}LO (in the local form by Navrátil) with: Λ_{3N} = 500 MeV, constrained to reproduce ^{3}H binding energy and b-decay half-life.



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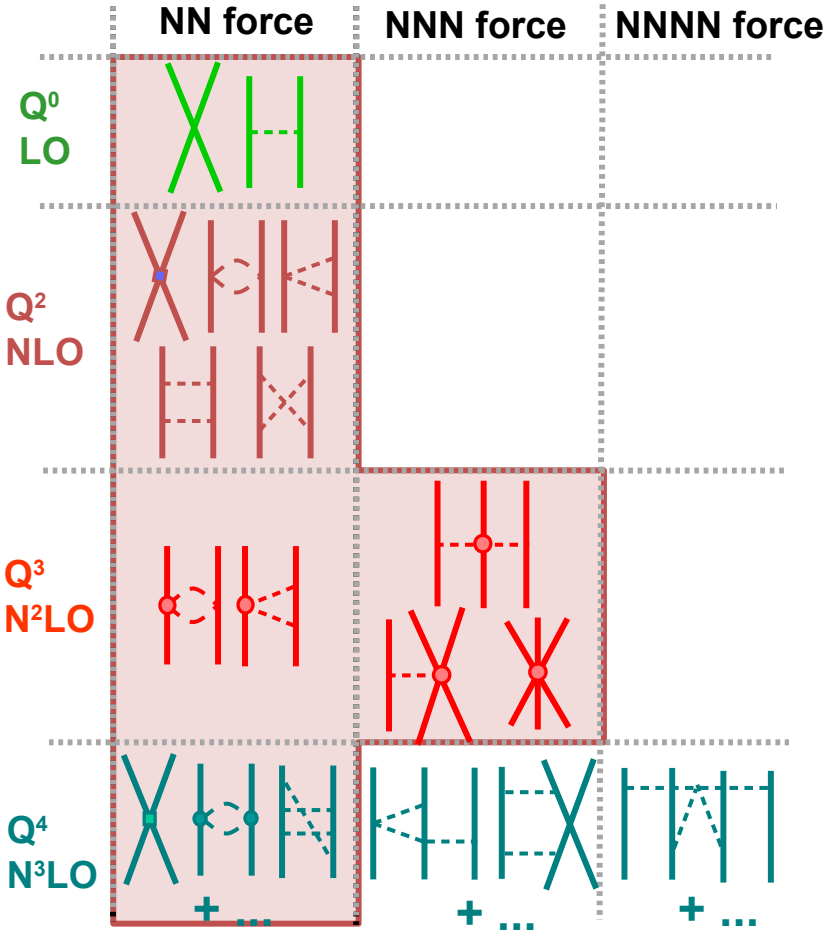

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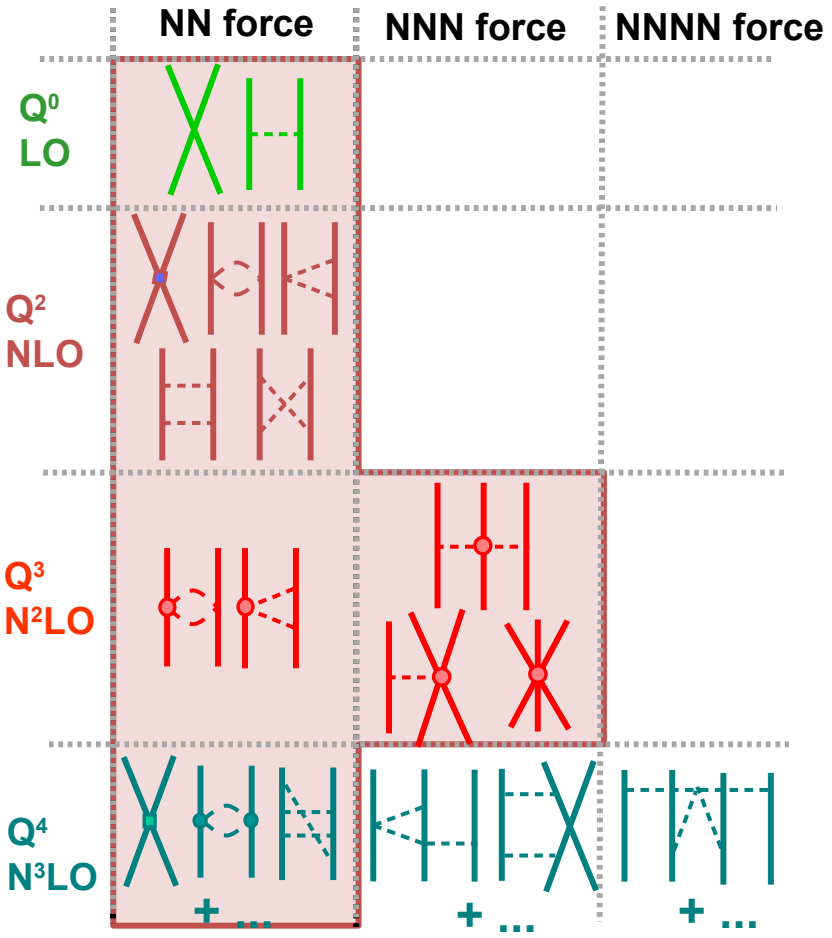
  - 'Soften' the interactions using unitary transformations:Similarity Renormalization Group (SRG) method (λ=2.0 fm^{-1}).



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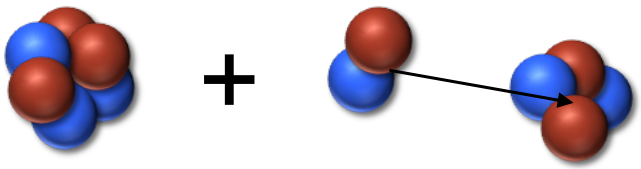
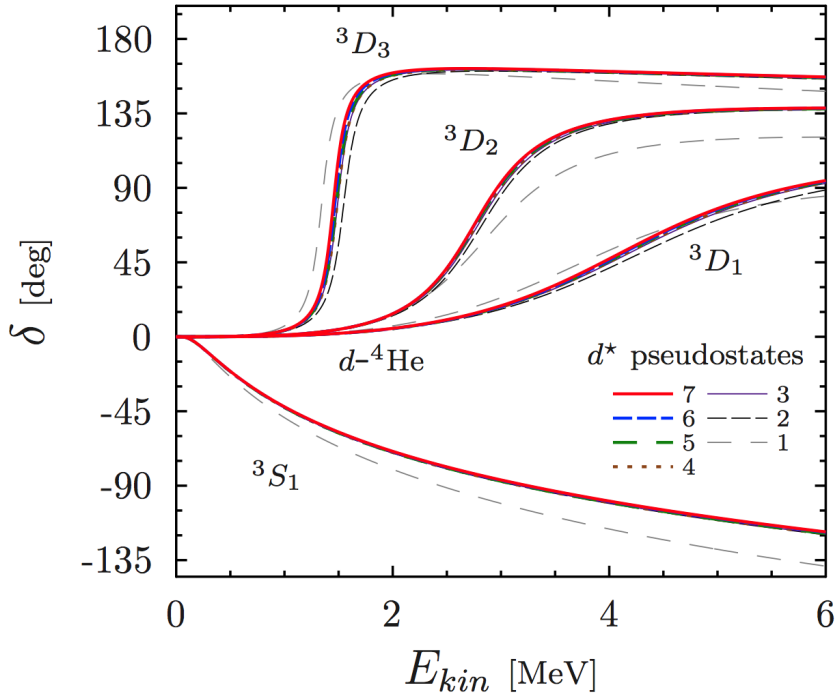
^{6}Li structure and d+^{4}He dynamics

- HO expansion at N_{max}=11, $\hbar\Omega$ = 20MeV
- Fifteen discrete eigenstates of ^{6}Li
- Seven deuteron pseudostates





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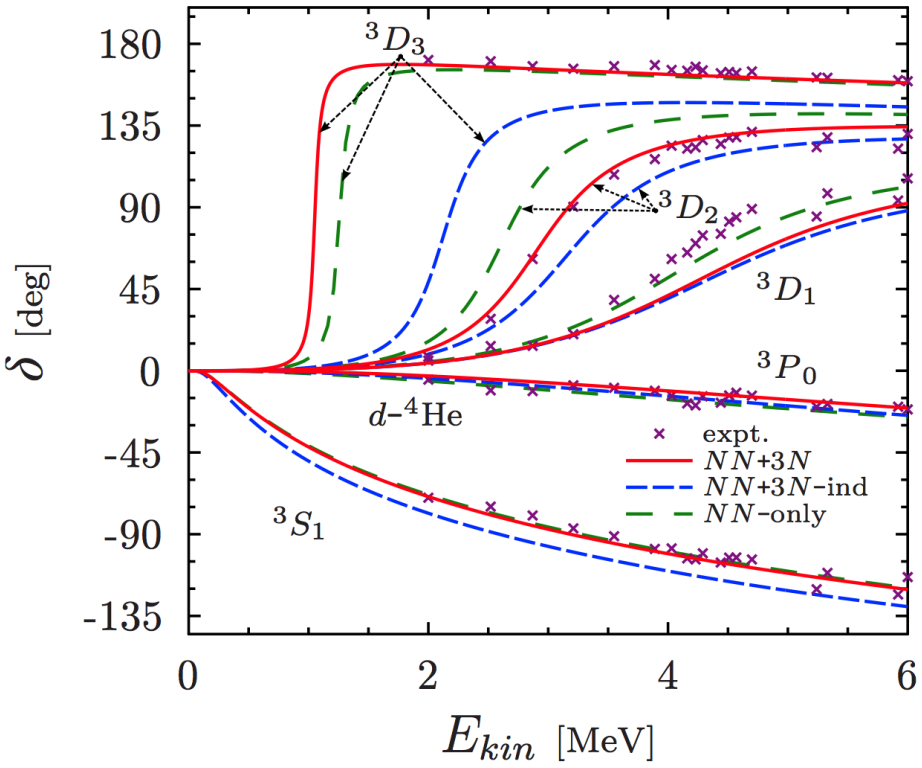
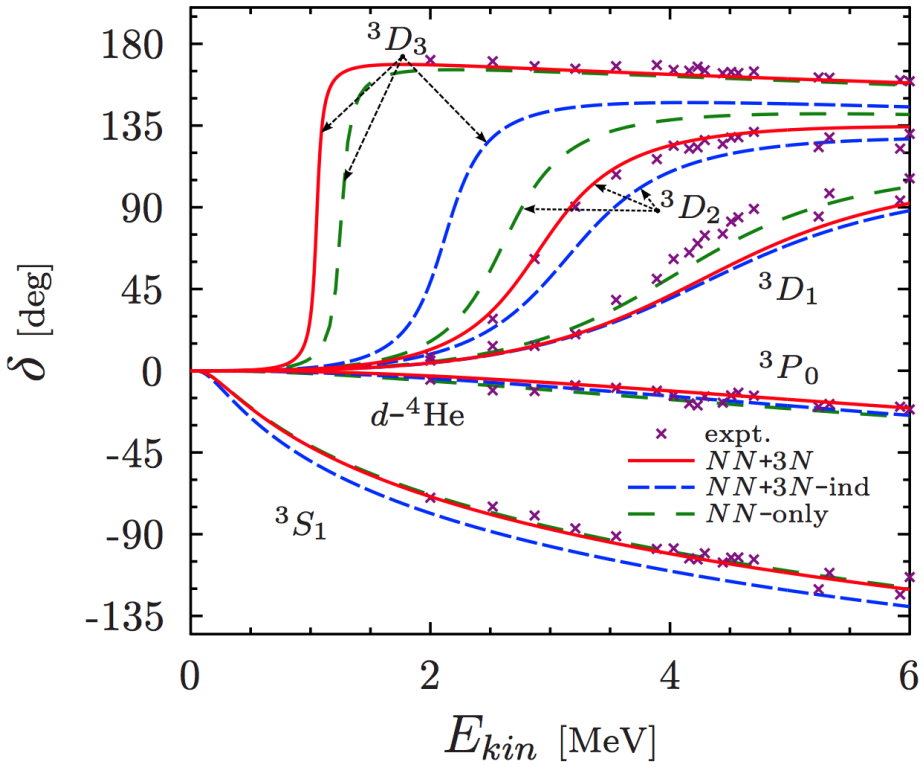


NCSMC phase shifts

^{6}Li structure and d+^{4}He dynamics



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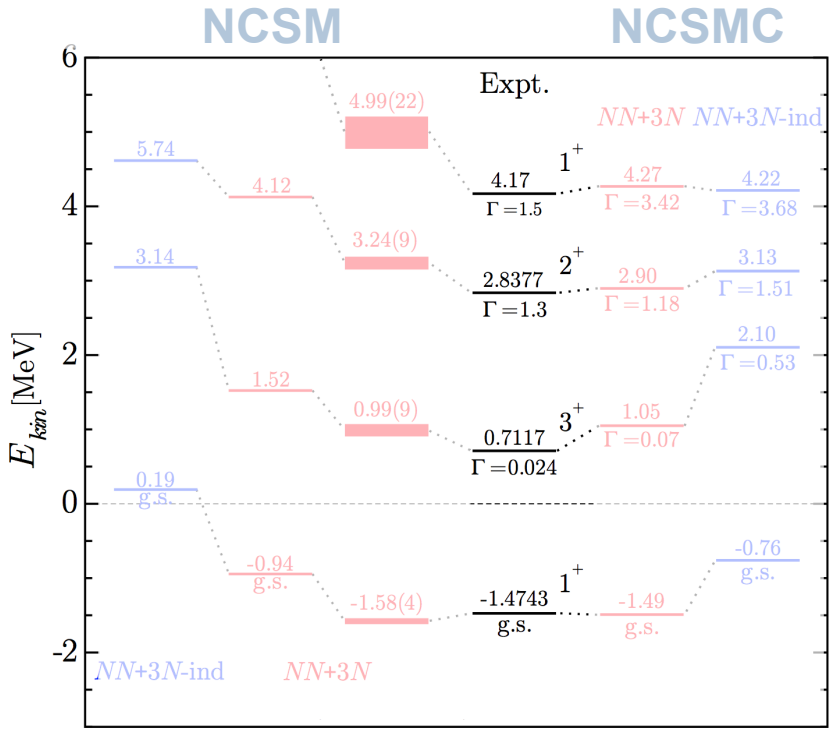
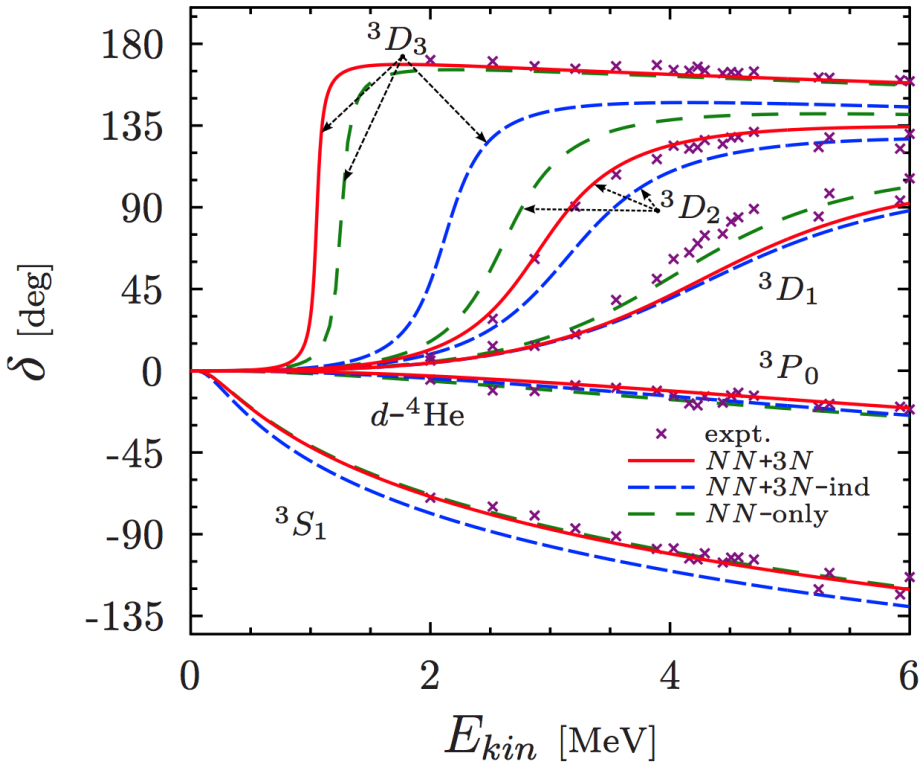


^{6}Li spectrum

^{6}Li structure and d+^{4}He dynamics



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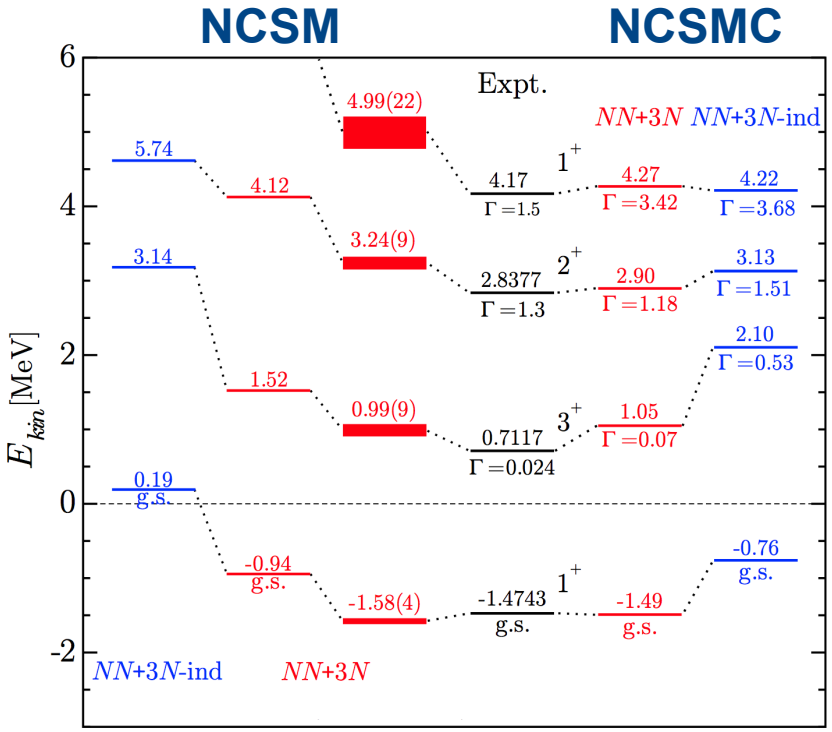


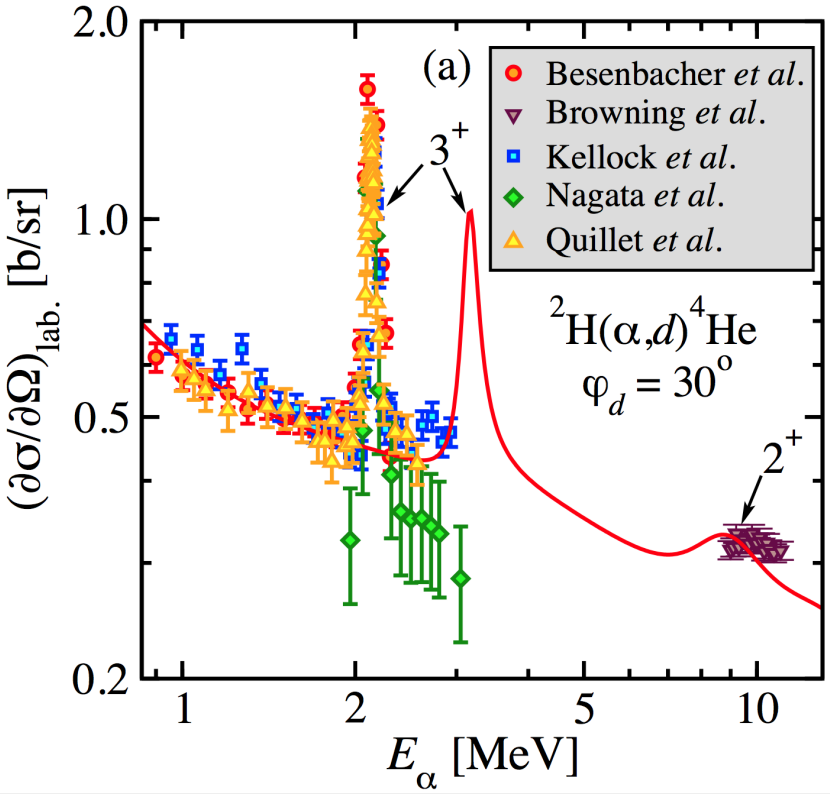
^{6}Li spectrum

^{6}Li structure and d+^{4}He dynamics

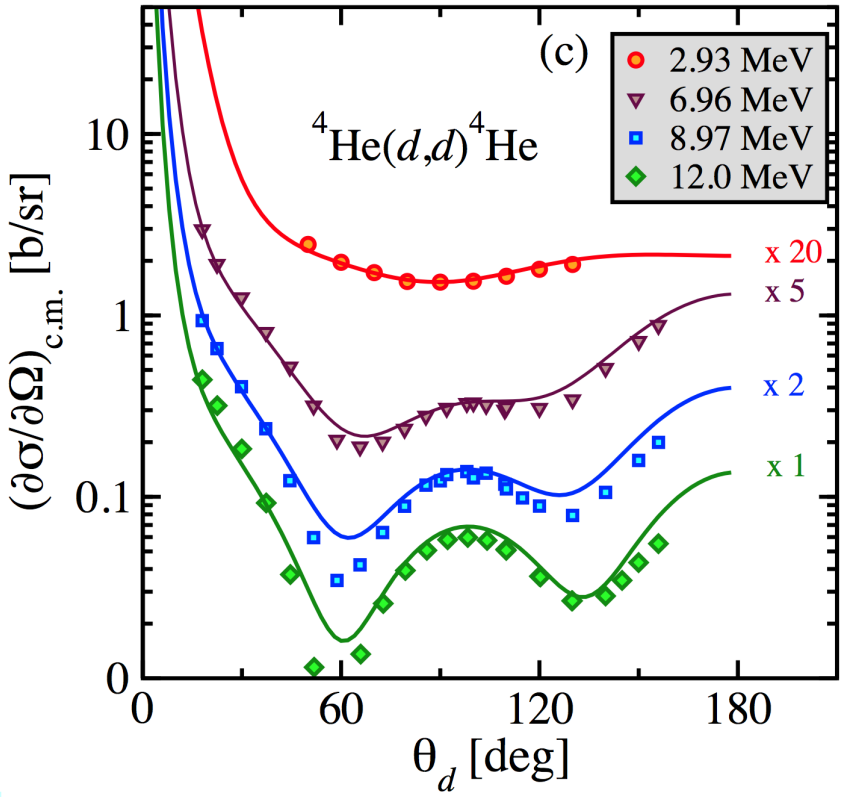
Asymptotic normalization constants ratio

	C_2(D-wave)/C_0(S-wave)
NCSMC	-0.027
Exp. PRC **59** 598 (1999)	-0.025(6)(10)
Exp. PRL **81** 1187 (1998)	0.0003(9)

^{6}Li structure and d+^{4}He dynamics



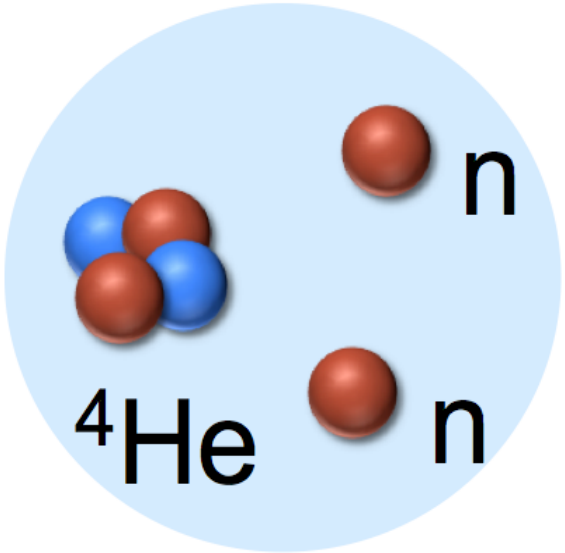
Comparison to experiment of the α-d elastic recoil differential cross section at φ=30°. NCSMC with NN+3N potential at λ=2.0 fm^{-1}



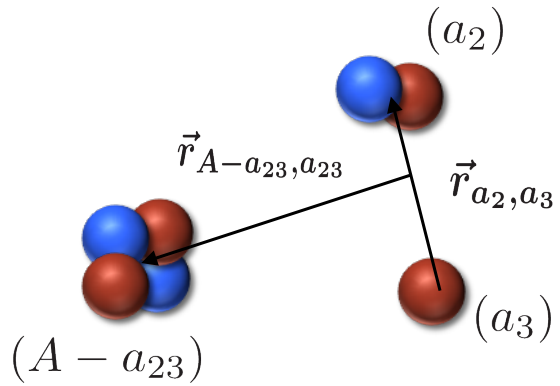
Center-of-mass angular distributions at different incident energies E_d

^{4}He+n+n

^{6}He: 2 neutron halo (^{4}He-n-n)



NCMS/RGM-3B



Extension to three-body cluster

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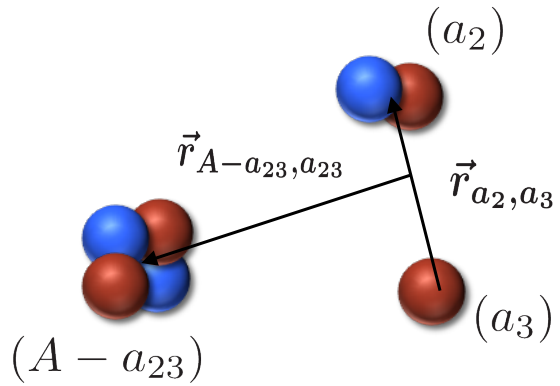
C. Romero-Redondo

NCMS/RGM-3B



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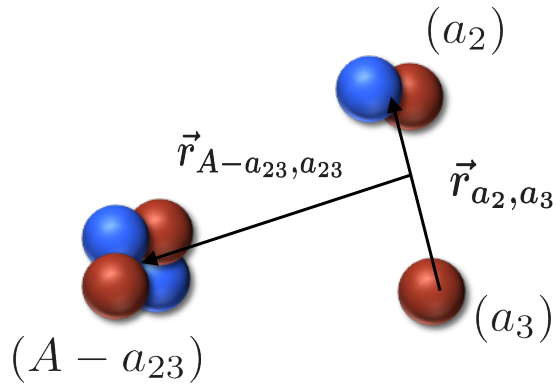
Why?

NCMS/RGM-3B



Extension to three-body cluster

Bound and resonant states:
2n Halo nuclei

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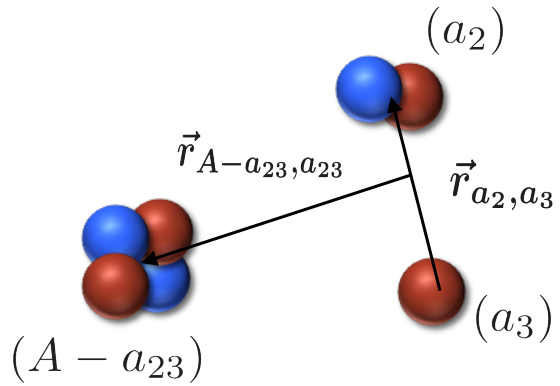

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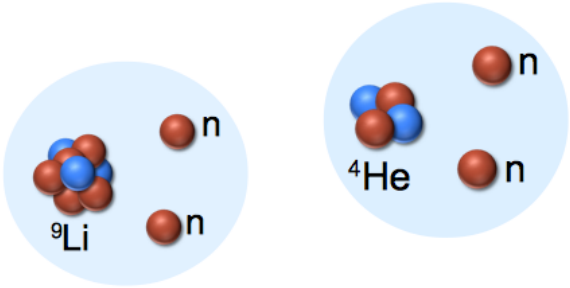

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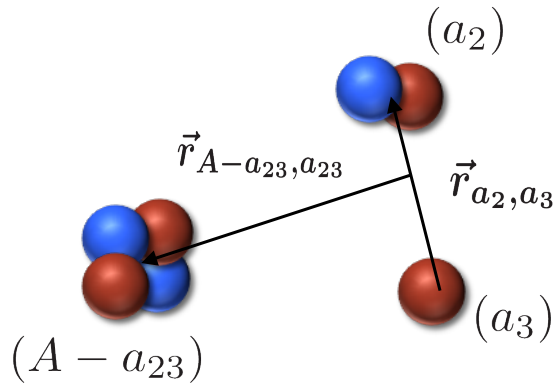

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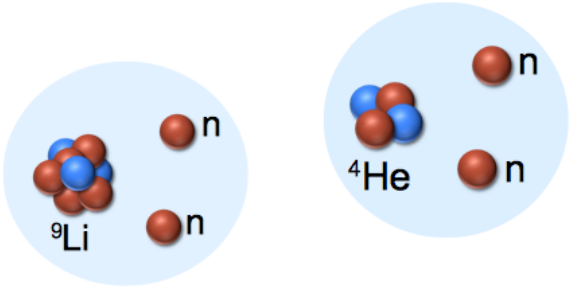
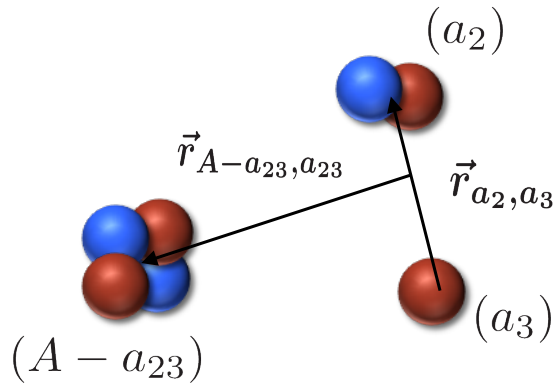


3-body continuum states:
Transfer reactions

NCMS/RGM-3B



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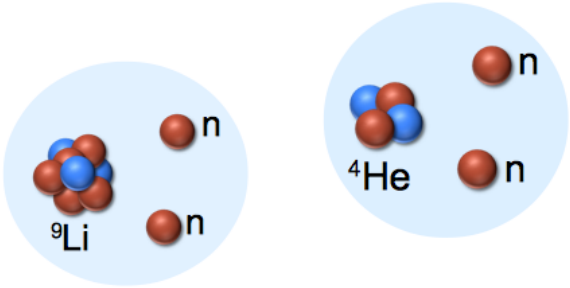

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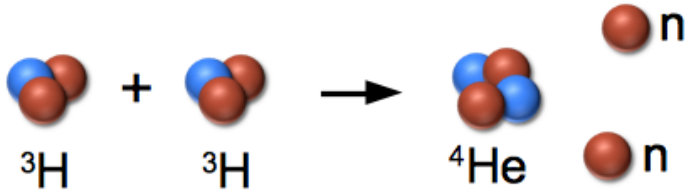

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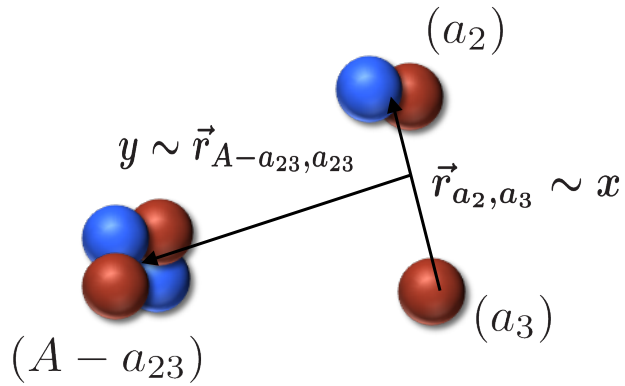
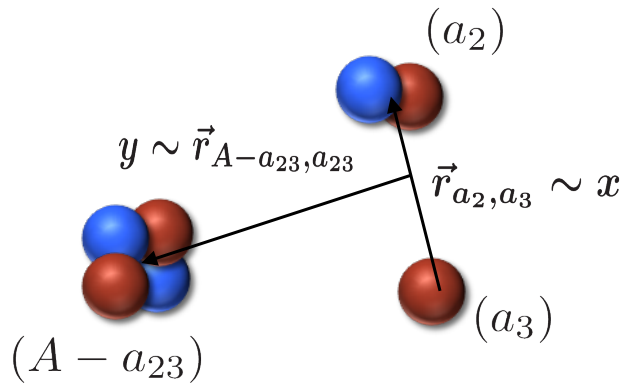


Jacobi coordinates

Basis

$$|\Psi^{J^\pi T}\rangle = \sum_\nu \int dx x^2 \int dy y^2 G_\nu^{J^\pi T}(x,y) \hat{A}_\nu |\boldsymbol{\Phi}^{\boldsymbol{J^\pi T}}_{\boldsymbol{\nu x y}}\rangle$$

NCMS/RGM-3B



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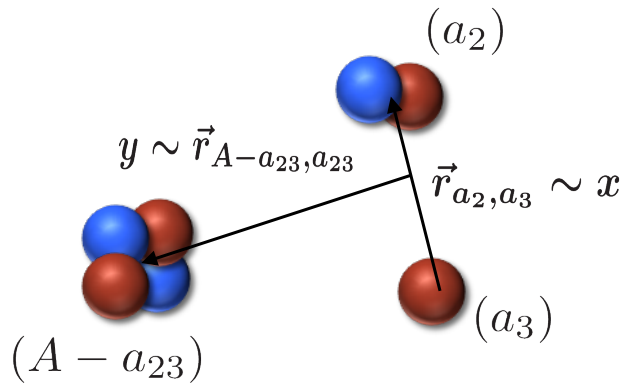
$$|\Phi^{J^\pi T}_{\nu x y}\rangle = \left\{ \left[|A - a_{23}\alpha_1 I_1^{\pi_1} T_1\rangle \left(|a_2\alpha_2 I_2^{\pi_2} T_2\rangle |a_3\alpha_3 I_3^{\pi_3} T_3\rangle \right)^{S_{23}T_{23}} \right]^{sT} \left(Y_{\ell_x}(\hat{r}_{a_2,a_3}) Y_{\ell_y}(\hat{r}_{A-a_{23},a_{23}}) \right)^L \right\}^{J^\pi T} \frac{\delta(x - r_{a_2,a_3})}{x r_{a_2,a_3}} \frac{\delta(y - r_{A-a_{23},a_{23}})}{y r_{A-a_{23},a_{23}}}$$

NCMS/RGM-3B



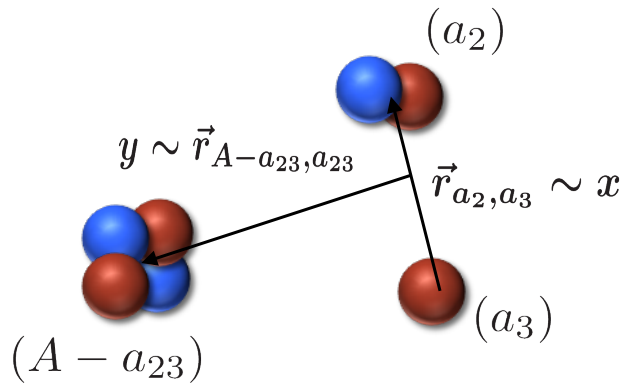
Basis

$$|\Psi^{J^\pi T}\rangle = \sum_\nu \int dx x^2 \int dy y^2 G_\nu^{J^\pi T}(x,y) \hat{A}_\nu |\boldsymbol{\Phi}^{\boldsymbol{J^\pi T}}_{\boldsymbol{\nu x y}}\rangle$$

NCSM wave functions

$$|\Phi^{J^\pi T}_{\nu x y}\rangle = \left\{ \left[|A - a_{23}\alpha_1 I_1^{\pi_1} T_1\rangle \left(|a_2\alpha_2 I_2^{\pi_2} T_2\rangle |a_3\alpha_3 I_3^{\pi_3} T_3\rangle \right)^{S_{23}T_{23}} \right]^{sT} \left(Y_{\ell_x}(\hat{r}_{a_2,a_3}) Y_{\ell_y}(\hat{r}_{A-a_{23},a_{23}}) \right)^L \right\}^{J^\pi T} \frac{\delta(x - r_{a_2,a_3})}{x r_{a_2,a_3}} \frac{\delta(y - r_{A-a_{23},a_{23}})}{y r_{A-a_{23},a_{23}}}$$

NCMS/RGM-3B



Basis

$$|\Psi^{J^\pi T}\rangle = \sum_\nu \int dx x^2 \int dy y^2 G_\nu^{J^\pi T}(x,y) \hat{A}_\nu |\mathbf{\Phi}^{J^\pi T}_{\nu x y}\rangle$$

Intercluster antisymmetrizer

$$|\Phi^{J^\pi T}_{\nu x y}\rangle = \left\{ \left[|A - a_{23}\alpha_1 I_1^{\pi_1} T_1\rangle \left(|a_2\alpha_2 I_2^{\pi_2} T_2\rangle |a_3\alpha_3 I_3^{\pi_3} T_3\rangle \right)^{S_{23}T_{23}} \right]^{sT} \right.$$
$$\left. \left(Y_{\ell_x}(\hat{r}_{a_2,a_3}) Y_{\ell_y}(\hat{r}_{A-a_{23},a_{23}}) \right)^L \right\}^{J^\pi T} \frac{\delta(x - r_{a_2,a_3})}{x r_{a_2,a_3}} \frac{\delta(y - r_{A-a_{23},a_{23}})}{y r_{A-a_{23},a_{23}}}$$

NCMS/RGM-3B

$$|\Psi^{J^\pi T}\rangle = \sum_\nu \int dx x^2 \int dy y^2 G_\nu^{J^\pi T}(x,y)\hat{A}_\nu |\boldsymbol{\Phi}^{\boldsymbol{J^\pi T}}_{\boldsymbol{\nu x y}}\rangle$$

Schrödinger equation ⇓ $(\mathcal{H} - E)\,|\Psi^{J^\pi T}\rangle = 0$

$$\sum_\nu \int dx dy x^2 y^2 \left[\mathcal{H}_{\nu'\nu}(x,y,x',y') - E\mathcal{N}_{\nu'\nu}(x,y,x',y')\right] G_\nu^{J^\pi T}(x,y) = 0$$

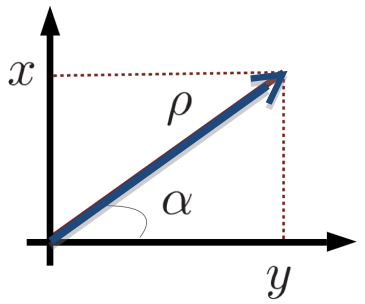
Hamiltonian Kernel $\langle \Phi^{J^\pi T}_{\nu' x' y'} | \hat{A}_{\nu'} \mathcal{H} \hat{A}_\nu | \Phi^{J^\pi T}_{\nu x y}\rangle$

Norm kernel $\langle \Phi^{J^\pi T}_{\nu' x' y'} | \hat{A}^2 | \Phi^{J^\pi T}_{\nu x y}\rangle$

NCMS/RGM-3B

Hyperspherical coordinates:

$$\rho = \sqrt{x^2 + y^2}, \quad \alpha = \arctan(x/y)$$



After changing to hyperspherical coordinates and integrating in α,α':

$$\sum_{\nu k} \int d\rho \rho^5 \left[\bar{\mathcal{H}}^{k'k}_{\nu'\nu}(\rho', \rho) - E \frac{\delta(\rho - \rho')}{\rho^5} \delta_{\nu'\nu} \delta_{k'k} \right] C^{J^\pi T}_{k\nu}(\rho) = 0$$

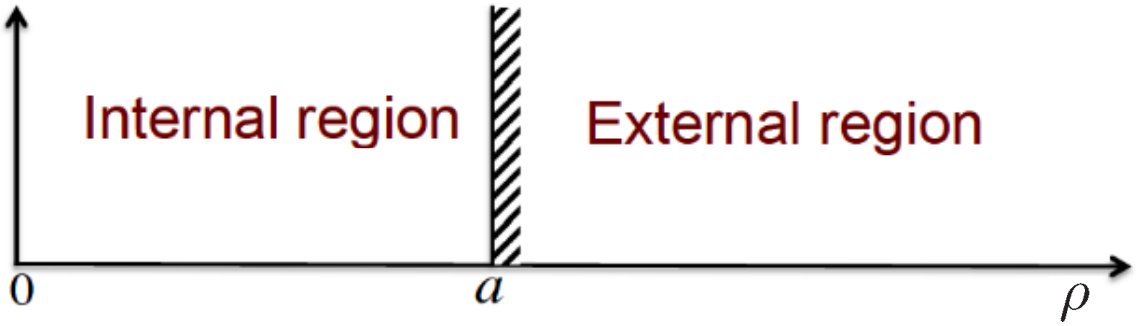
Coupled-channel microscopic R-matrix method on a Lagrange mesh*

*M. Hesse, J.-M. Sparenberg 1, E Van Raemdonck, D. Baye. Nuclear Physics A 640 (1998) 37-51

NCMS/RGM-3B

Internal region: expansion on a basis ($\rho < a$)

$$C_{k\nu}(\rho) = \sum_i \beta_{k\nu i} f_i(\rho)$$



External region: known asymptotic behaviour ($\rho > a$)

* Bound state: $C_{k\nu}(\rho) = A_{k\nu}\sqrt{\kappa\rho}K_{k+2}(\kappa\rho)$

* Continuum state: $C_{k\nu}(\rho) = A_{k\nu}\left[H_k^-(\kappa\rho)\delta_{\nu,\nu'}\delta_{k,k'} - S_{\nu k,\nu' k'}H_k^+(\kappa\rho)\right]$

^{4}He+n+n

NCSM/RGM results

- n+n+^{4}He, N_{max} = 11, ħΩ = 14 MeV

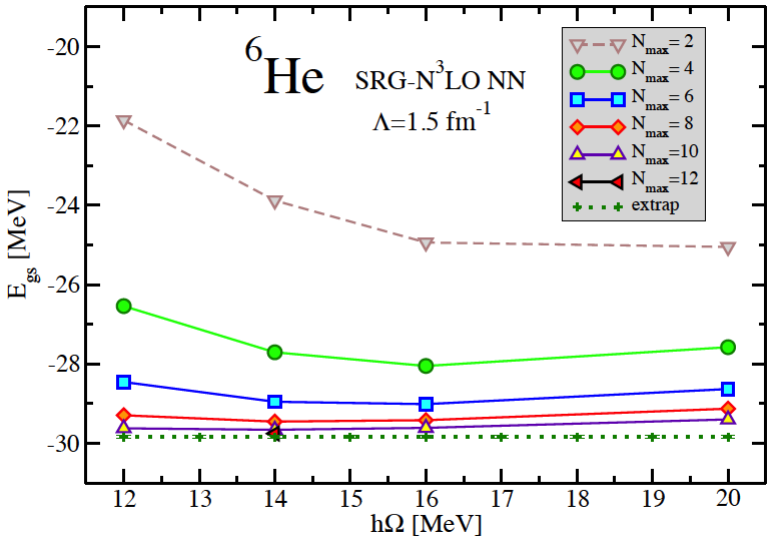
- SRG-N^{3}LO NN-only interaction with λ=1.5 fm^{-1}

Comparison with NCSM:

- ~ 1MeV difference in E_{gs} due to excitations of the ^{4}He core only included in the NCSM calculation.

-Contrary to NCSM, NCSM/RGM n+n+^{4}He w.f. Has the appropriate asymptotic behavior.

^{6}He ground state, NCSM

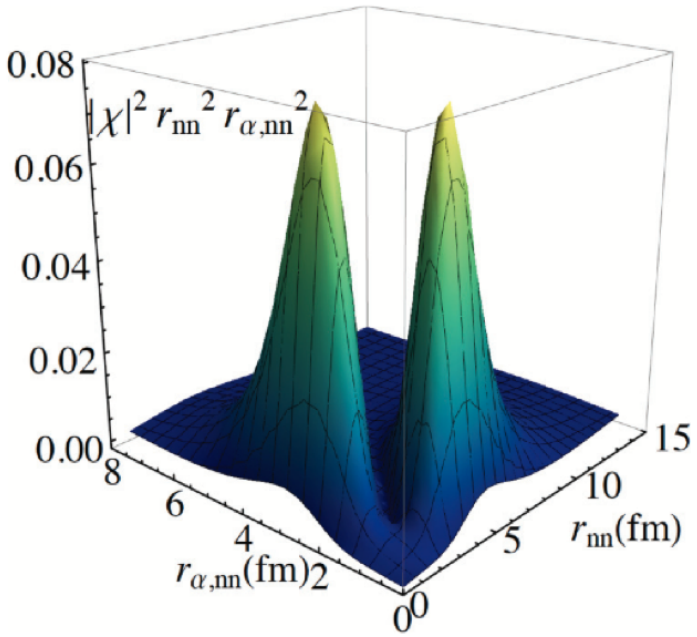


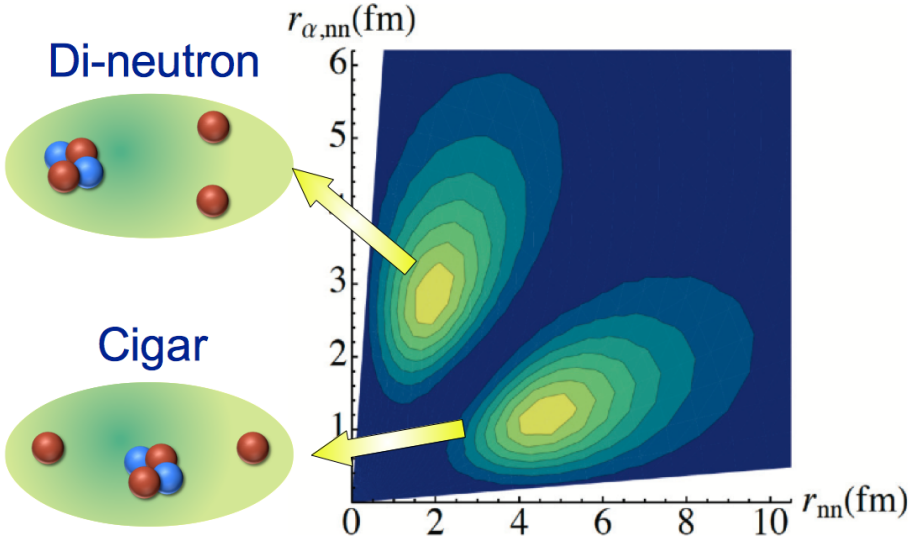
HO model space	*E*g.s. (^{4}He) [MeV] (NCSM)	*E*g.s. (^{6}He) [MeV] (NCSM)	*E*g.s. (^{6}He) [MeV] (NCSM/RGM)
*N*max = 12	-28.224	-29.658	-28.697
Extrapolation	-28.230(5)	-29.84(4)	-28.70(3)

S. Quaglioni, CRR, P. Navrátil PRC **88**, 034320 (2013)

^{4}He+n+n. Ground state

^{6}He g.s. Probability distribution





PHYSICAL REVIEW C **88**, 034320 (2013)

Three-cluster dynamics within an *ab initio* framework

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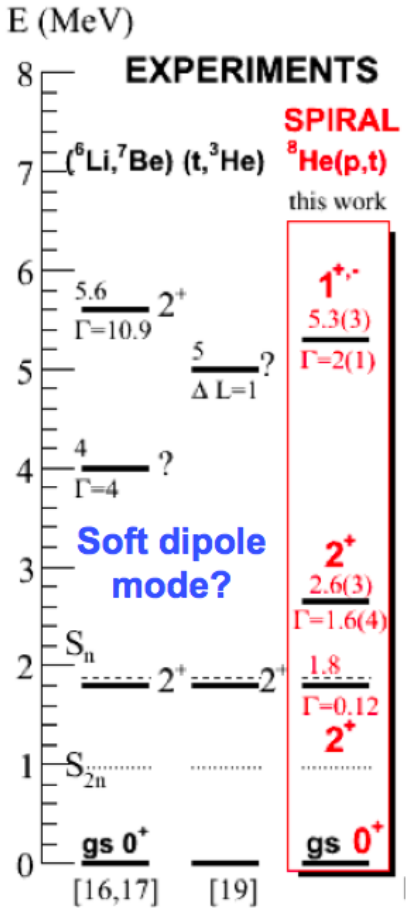
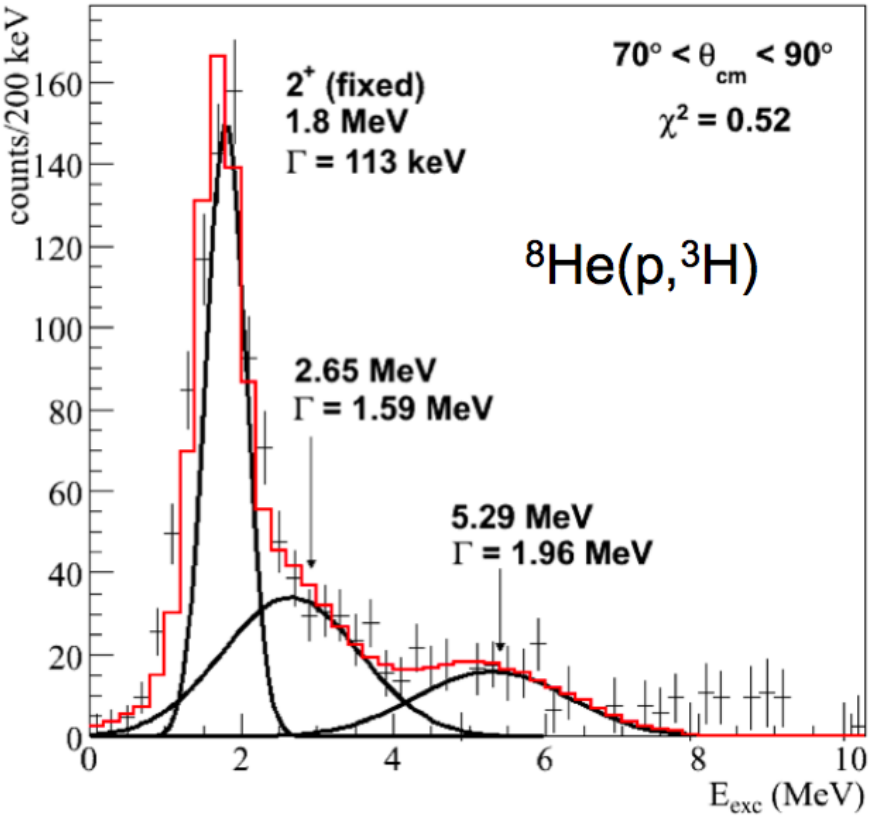
(Received 1 August 2013; published 26 September 2013)

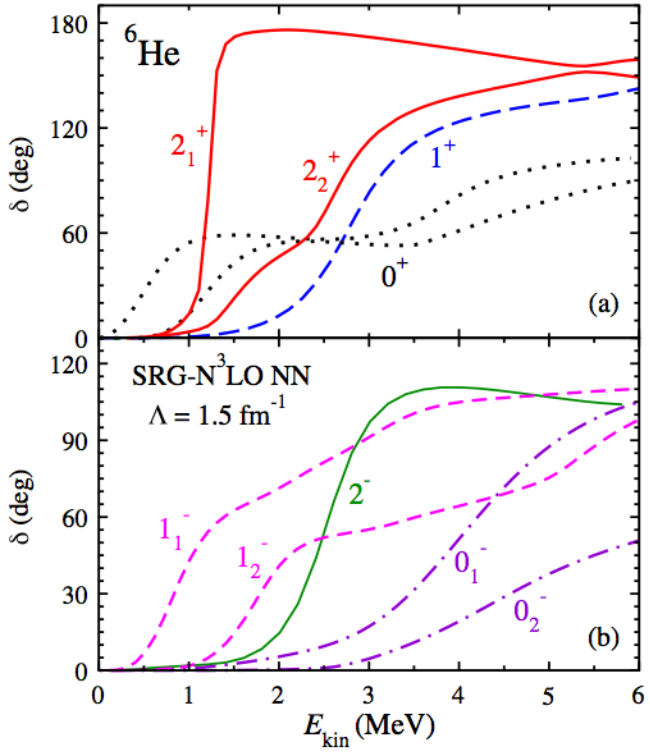
^{4}He+n+n. Experimental picture



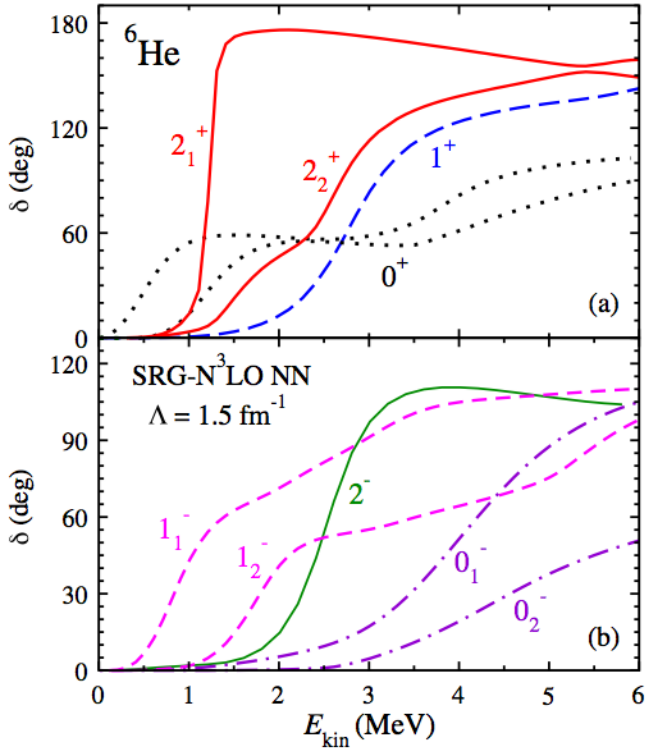
Recent expt. @SPIRAL, GANIL: PLB **718** (2012) 441

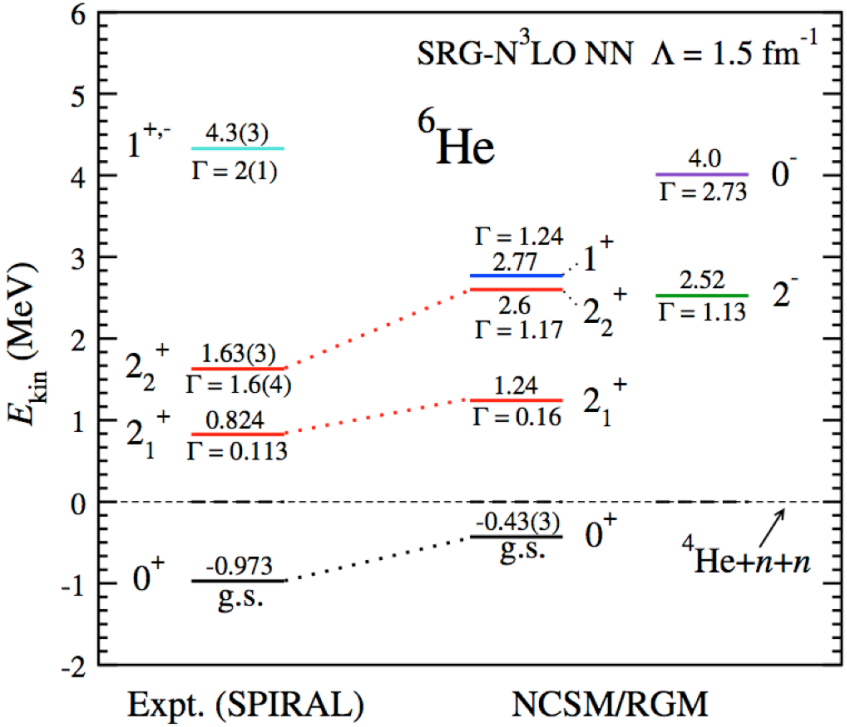


^{4}He+n+n. Spectrum



^{4}He+n+n. Spectrum





PRL 113, 032503 (2014) PHYSICAL REVIEW LETTERS week ending 18 JULY 2014

^{4}He + n + n Continuum within an *Ab initio* Framework

Carolina Romero-Redondo,[1,*] Sofia Quaglioni,[2,†] Petr Navrátil,[1,‡] and Guillaume Hupin[2,§]
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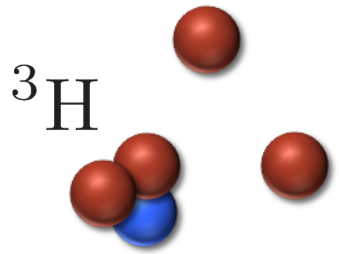
(Received 4 April 2014; published 16 July 2014)

^{3}H+n+n



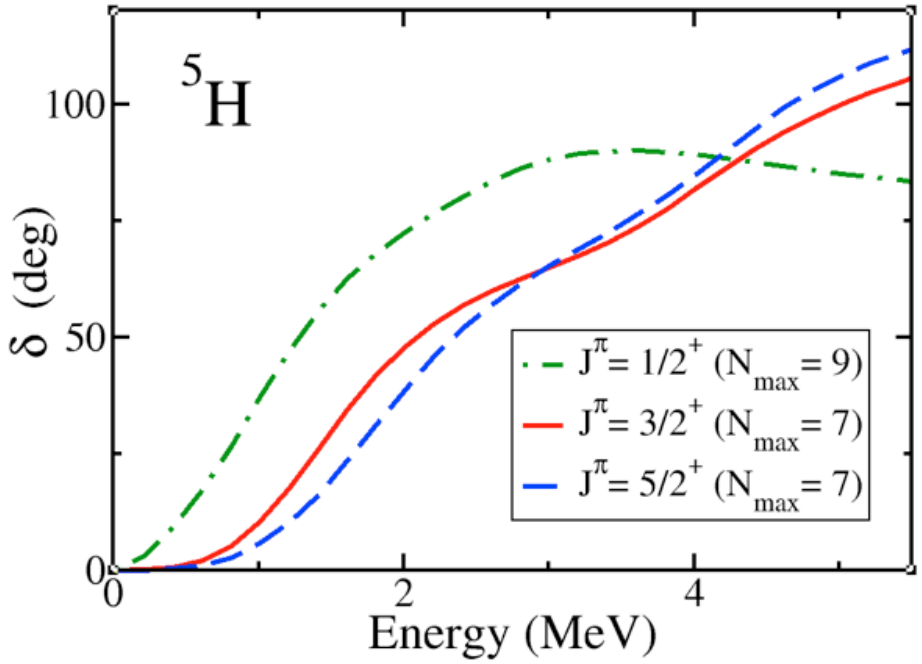
Experimental Picture:

	$1/2^+$	$3/2^+$	$5/2^+$
PRL **87** 092501 (2001)	$(1.7 \pm 0.3, 1.9 \pm 0.4)$	$(-,-)$	$(-,-)$
NPA **719** 229c (2003)	$(1.8 \pm 0.1, < 0.5)$	$(-,-)$	$(2.7 \pm 0.1, < 0.5)$
PRC **72** 064612 (2005)	$(1.8, 1.3)$	$(> 2.5, -)$	$(> 2.5, -)$
EPJ A **25** 315 (2005)	$(2, 2.5)$	$(> 2.5, -)$	$(> 2.5, -)$
PRL **91** 162504 (2003)	$(3, 6)$	$(-,-)$	$(-, -)$
EPJ A **24** 231 (2005)	$(5.5 \pm 0.2, 5.4 \pm 0.6)$	$(> 10, > 2)$	$(> 10, > 2)$

^{3}H+n+n (preliminary)

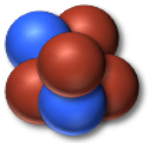
- Same accurate soft NN potential
(SRG-evolved chiral N^{3}LO with λ=1.5fm^{-1})

- NSCM ^{3}H wave function



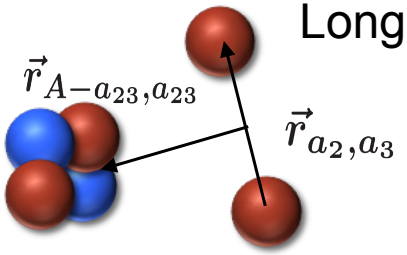
NCSMC-3B

NCSM
Short range description



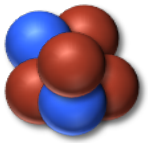
+

NCSM/RGM
Long range description



NCSMC-3B

NCSM
Short range description



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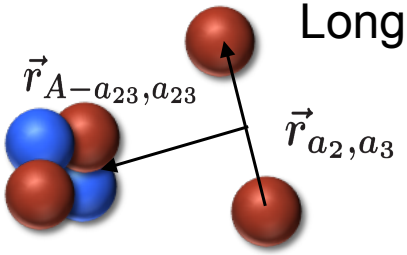
NCSM/RGM
Long range description



In progress

Summary

- We are able to study nuclear systems:
  - Bound and resonant states in structure problems
  - Continuum states for reaction problems
- Results for
  - Structure and dynamics of ^{4}He+d system

    (shown importance of 3N forces and short range correlations)
  - Ground state of ^{6}He
  - Continuum ^{4}He+n+n
- Preliminary calculations for ^{3}H+n+n basis

Outlook

- Improvement of current calculations
  - Introduce core excitations by coupling the (A-2)-n-n basis to A-body NCSM eigenstates (NCSMC)
  - Run calculations with 3N force
- Transfer reactions, i.e, $^{3}H(^{3}H,2n)^{4}He$
  - Derive and calculate couplings between two and three body clusters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