

Canada's National Laboratory for Particle and Nuclear Physics Laboratoire national canadien pour la recherche en physique nucléaire et en physique des particules

Halo Nuclei in Effective Field Theory

Chen Ji || TRIUMF

Progress in Ab Initio Techniques in Nuclear Physics TRIUMF, Feb 17-20, 2015



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Effective Theories & Resolution Scales

• We study physics at different resolution scales with different effective theories

describe nucleon structures
physics scale: Q ≥ GeV
d.o.f.: quarks & gluons
effective theory: lattice QCD



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Effective Theories & Resolution Scales



- light/medium mass nuclei
 - physics scale: $Q\sim 200~{\rm MeV}$
 - d.o.f.: nucleons & pions
 - effective theory: chiral EFT
 - use ab initio methods



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Effective Theories & Resolution Scales



• We study physics at different resolution scales with different effective theories

- very light nuclei (d, t, ³He, α)
 - physics scale: $Q \ll m_{\pi}$
 - d.o.f.: nucleons in contact
 - effective theory: pionless EFT
 - use few-body methods







- halo nuclei (core + valence N)
- separation in length scales

 $R_{\rm core} \ll R_{\rm halo}$



Effective Theories for Halo Nuclei



ab initio methods

- capture dynamics inside and outside the core
- numerically expensive for loosely bound systems



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 $R_{\rm core} \ll R_{\rm halo}$



Effective Theories for Halo Nuclei



ab initio methods

- capture dynamics inside and outside the core
- numerically expensive for loosely bound systems

halo effective field theory

- valence nucleon + core d.o.f.
- systematic expansion in $R_{\rm core}/R_{\rm halo}$
- capture only clustering mechanism
- numerically simpler
- complimentary to *ab initio* methods
- explain universal correlations in clustering physics

halo nuclei (core + valence N) separation in length scales

 $R_{\rm core} \ll R_{\rm halo}$

0



Halo Nuclei in EFT

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Halo Effective Field Theory



• We adopt EFT with contact interactions to describe clustering in halo nuclei

$$\mathcal{L} = \psi^{\dagger} \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi + \eta \, d^{\dagger} \left(i\partial_0 + \frac{\nabla^2}{4m} - \Delta \right) d - \frac{g}{\sqrt{2}} \left(d^{\dagger} \psi \psi + \text{h.c} \right) + h d^{\dagger} d\psi^{\dagger} \psi + \cdots$$

 \cdots are higher orders in $R_{
m core}/R_{
m halo}$ expansion

Halo Effective Field Theory

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m core}/R_{
m halo}$ expansion

• 2-body contact (LO) introduce a two-body field

$$= -iC_0 \qquad \xrightarrow{C_0 = g^2/\Delta} \qquad = -i\sqrt{2}g$$

g determined by a 2-body observable

• 3-body contact (LO) $= -iD_0 \qquad \underline{D_0 = -3hg^2/\Delta^2} \qquad = ih$

h determined by a 3-body observable Bedaque, Hammer, van Kolck '99

One-Neutron Halo Systems



• EFT for 1n halo



• ⁵He shallow resonance ($P_{3/2}$)

$$a = \frac{1}{4\pi^2 \mu_{n\alpha}} \frac{\vec{p} \cdot \vec{q}}{-1/a_1 + r_1 k^2/2 - ik^3} a_1 = -62.95 \text{ fm}^3, r_1 = -0.8819 \text{ fm}^{-1} \text{Ardnt et al. '73}$$

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Ardnt et al. '73

• $n\alpha$ p-wave EFT power counting

Bertulani, Hammer, van Kolck '02

- $a_1 \sim 1/(Q^3)$ $r_1 \sim Q$
- two fine tunings at LO
- shallow resonance: $k_R, \Gamma \sim Q$
- shallow bound state: $\gamma_1 \sim Q$



One-Neutron Halo Systems



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$$a = \frac{1}{4\pi^2 \mu_{n\alpha}} \frac{\vec{p} \cdot \vec{q}}{-1/a_1 + r_1 k^2/2 - ik^3} a_1 = -62.95 \text{ fm}^3, r_1 = -0.8819 \text{ fm}^{-1}$$

Ardnt et al. '73

• $n\alpha$ p-wave EFT power counting

Bedaque, Hammer, van Kolck '02

- $a_1 \sim 1/(Q^2 \Lambda_H)$ $r_1 \sim \Lambda_H$
- $Q/\Lambda_H \sim 0.15$
- one fine tuning at LO
- shallow resonance: $k_R \sim Q$, $\Gamma \sim Q^2/\Lambda_H$
- deep bound state: $\gamma_1 \sim \Lambda_H$



• EFT for 1p halo nucleus

p- α and α - α scattering [Higa '08] ¹⁷F [Ryberg, Forssén, Platter '13]

Photo-Dissociation in Halos





E1 transition

$^{11}{\rm Be}$ photo-dissociation



 $^{19}\mathrm{C}$ photo-dissociation



data: Nakamura *et al*, RIKEN '99,'03; calculation: Acharya, Phillips '13

[Hammer, Phillips '11]

Radiative Nucleon Captures

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proton captures



$$\label{eq:constraint} \begin{split} ^{7}\mathrm{Li} &+ n \rightarrow {}^{8}\mathrm{Li} + \gamma \; [\mathrm{Rupak, Higga \; '11, Zhang, Nollett, Phillips \; '13]} \\ ^{14}\mathrm{C} &+ n \rightarrow {}^{15}\mathrm{C} + \gamma \; [\mathrm{Rupak, Fernando, Vaghani \; '12]} \\ ^{7}\mathrm{Be} &+ p \rightarrow {}^{8}\mathrm{B} + \gamma \; [\mathrm{Zhang, \; Nollett, \; Phillips \; '14]} \\ ^{16}\mathrm{O} &+ p \rightarrow {}^{17}\mathrm{F}^{*}(1/2^{+}) + \gamma \; [\mathrm{Ryberg, \; Forssén, \; Platter \; '13]} \end{split}$$

Radiative Nucleon Captures

proton captures

⁷Li + $n \rightarrow {}^{8}$ Li + γ [Rupak, Higga '11, Zhang, Nollett, Phillips '13] $^{14}\mathrm{C} + n \rightarrow {}^{15}\mathrm{C} + \gamma$ [Rupak, Fernando, Vaghani '12] $^{7}\text{Be} + p \rightarrow {}^{8}\text{B} + \gamma$ [Zhang, Nollett, Phillips '14] ${}^{16}\text{O} + p \rightarrow {}^{17}\text{F}^*(1/2^+) + \gamma$ [Ryberg, Forssén, Platter '13]

- E1 S-factor for ${}^{7}\text{Be}(p,\gamma){}^{8}\text{B}$ [Zhang, Nollett, Phillips '14]
- [Navratil, Roth, Quaglioni, PLB '11]
- ---- LO EFT: fit to NSCM-GRM ANC
- LO EFT: fit to ANC from VMC VMC [Nollett, Wiringa, PRC '11]

• 2n-halo wave functions

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• Three-body Faddeev equation

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EFT For 2n Halos

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 n-core in s-wave virtual/real bound state: ¹¹Li, ¹²Be, ²⁰C [Canham, Hammer '08, '10] ²²C [Yamashita, Carvalho, Frederico, Tomio '11] ²²C Acharya, C.J., Phillips PLB 723 (2013)

• charge radius of 2n s-wave halos [Hagen, Hammer, Platter '13]

• heaviest 2n s-wave halo:

⁶²Ca [Hagen, Hagen, Hammer, Platter '13] fit n-⁶⁰Ca scattering length from coupled-cluster calculations

- ⁶He: n- α in p-wave resonance
 - EFT + Gamow shell model [Rotureau, van Kolck, Few Body Syst. '13]
 - EFT + Faddeev Equations C.J., Elster, Phillips, PRC 90, 044004 (2014)

Universality in 2n s-wave halo

• Implication of excited Efimov halo assume excited states $S_{2n} = 0$

Canham, Hammer EPJA 2008

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²²C: 2*n* Halo

	²⁰ C	²¹ C	²² C
bound/unbound	bound		
ground state	0+		
binding/virtual	S_{2n} =4.76 MeV		
energy	Ozawa et al. '11		
matter radius	2.97(5) fm		
r_m	Ozawa et al. '01		

²²C: 2n Halo

	²⁰ C	^{21}C	^{22}C
bound/unbound	bound	unbound	
ground state	0+	$S_{1/2}$	
binding/virtual	S_{2n} =4.76 MeV	$E_{nc} > 2.9 \mathrm{MeV}$	
energy	Ozawa et al. '11	Mosby et al. '13	
		??	
matter radius	2.97(5) fm		
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²²C: 2*n* Halo

	²⁰ C	²¹ C	22 C
bound/unbound	bound	unbound	bound
ground state	0^{+}	$S_{1/2}$	0^+
			$S_{2n}{=}0.42(94) {\rm ~MeV}$
binding/virtual	S_{2n} =4.76 MeV	$E_{nc} > 2.9 \mathrm{MeV}$	Audi et al. '03
energy	Ozawa et al. '11	Mosby et al. '13	$S_{2n}{=}{-}0.14(46)~{\rm MeV}$
		??	Gaudefroy et al. '12
matter radius	2.97(5) fm		5.4(9) fm
r_m	Ozawa et al. '01		Tanaka et al. '10

22 C: 2n Halo

	²⁰ C	^{21}C	²² C
bound/unbound	bound	unbound	bound
ground state	0+	$S_{1/2}$	0^+
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- Halo EFT [Acharya, C.J., Phillips, PLB **723** 196 (2013)] we fit to ²²C matter radius to constrain:
 - E_{nc} in ²¹C (a < 0)
 - S_{2n} in ${}^{22}\mathrm{C}$

Constraints On ²¹C and ²²C

Input: $r_m[^{22}C] = 5.4^{+0.9}_{-0.9}$ fm

bands: uncertainty from higher-order EFT

Acharya, C.J., Phillips, PLB 723 196 (2013)

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c.f. Yamashita et al. '11 (theo) $\rightarrow S_{2n} < 120 \text{ keV}$ Fortune & Sherr '12 (theo) $\rightarrow S_{2n} < 220 \text{ keV}$ Gaudefroy et al. '12 (expt) $\rightarrow S_{2n} < 320 \text{ keV}$ Mosby et al. '13 $E_{nc} > 2.9 \text{ MeV}$ Halo EFT $\rightarrow S_{2n} < 20 \text{ keV}$ (inconsistent with other measurements)

bands: uncertainty from higher-order EFT

Acharya, C.J., Phillips, PLB 723 196 (2013)

Efimov States In ²²C ?

 possibility of finding Efimov excited states in ²²C Mazumdar et al. '00, Frederico et al. '12, Acharya, C.J., Phillips, '13

• An Efimov excited state exists if G.S. $S_{2n} > B_{min}$

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 An Efimov excited state exists if C.S. S. > P

• An Efimov excited state exists if G.S. $S_{2n} > B_{min}$

• The Efimov excited state only occurs in ${}^{22}C$ if: \rightarrow the virtual energy of ${}^{21}C E_{nc} < 1$ keV (unlikely)

	21 N	22 N	^{23}N
S_{1n} [MeV]	4.59(11)	1.28(21)	1.79(36)
S_{2n} [MeV]	6.75(10)	5.87(20)	3.07(31)

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• We study ²³N in $n + n + {}^{21}$ N cluster model Zhang, Ren, Lyu, C.J., PRC 91, 024001 (2015)

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- We study ²³N in $n + n + {}^{21}$ N cluster model Zhang, Ren, Lyu, C.J., PRC 91, 024001 (2015)
- Adopted interactions
 - realistic nn (Gogny-Pires-De Tourreil (GPT))
 - phenomenological n-21N (Wood Saxon)

$$V_{n\text{-core}}(r) = -\frac{V_0}{1 + \exp(\frac{r-r_0}{a})} - \frac{V_{\text{so}}}{ra} \frac{\exp(\frac{r-r_0}{a})}{(1 + \exp(\frac{r-r_0}{a}))^2} \mathbf{L} \cdot \mathbf{S}$$

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 Faddeev equation in hyperspherical harmonics expansion numerical tool: FaCE [Thompson, Nunes, Danilin, Comp. Phys. Comm. '04]

²³N G.S. & Excited Halo States

- We tune $V_{n\text{-core}}$ to reproduce 21 N $S_{1n} = 4.59(11)$ MeV 22 N $S_{1n} = 1.28^{+21}_{-21}$ MeV
- We predict S_{2n} and r_m

S_{2n}	r_m	S_{2n}^*	r_m^*
MeV	fm	MeV	fm
4.13	2.969	0.315	4.272
3.64	2.985	0.185	4.358
3.13	3.004	0.069	4.476

Experiment: $S_{2n} = 3.07(31)$ MeV

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- We tune $V_{n\text{-core}}$ to reproduce ${}^{21}\text{N} \ S_{1n} = 4.59(11) \text{ MeV}$ ${}^{22}\text{N} \ S_{1n} = 1.28{}^{+21}_{-21} \text{ MeV}$
- We predict S_{2n} and r_m

• add 3BF $V_3(\rho) = W_0 e^{-\rho^2/\rho_0^2}$ to reproduce 23 N $S_{2n} = 3.07$ MeV

• Predictions in S_{2n} and r_m

3.011

3.07

S_{2n}	r_m	S_{2n}^*	r_m^*
MeV	fm	MeV	fm
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0.064

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5.011

²³N Probability Density Distributions

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• future work: Halo EFT analysis of universal correlations in ²³N

• experiment in ⁶He

- matter radius Tanihata et al. '92, Alkhazov et al. '97, Kislev et al. '05
- charge radius Wang et al. '04, Mueller et al. '07
- ⁶He mass Brodeur *et al.* '12

• ab intio calculation

- no-core shell model Navrátil et al. '01; Sääf, Forssén '14
- NCSM-RGM Romero-Redondo et al. '14
- Green's function Monte Carlo Pieper et al. '01; '08
- hyperspherical harmonics (EIHH) Bacca et al. '12
- halo EFT C.J., Elster, Phillips, PRC 90, 044004 (2014)
 - explore universal correlations in ⁶He
 - compare predictions with experiments and ab initio calculations

⁶He: P-Wave *n*-core Interactions

spin-orbit coupling for ⁶He ($J = 0^+$)

pair, spec	pair	spectator	total L , S	total J
nn, α	$\ell_{nn} = 0, \ s_{nn} = 0$	$\lambda_{\alpha-nn} = 0, \ s_{\alpha-nn} = 0$	L = 0, S = 0	
$n\alpha$, n	$\ell = 1 \ a = 1$) -1 - 1	L = 0, S = 0	$J = 0^{+}$
	$c_{n\alpha} = 1, \ s_{n\alpha} = \frac{1}{2}$	$\lambda_{n-n\alpha} = 1, \ s_{n-n\alpha} = \frac{1}{2}$	L = 1, S = 1	

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TRIUMF

• without $nn\alpha$ 3-body force:

• S_{2n} is strongly cutoff dependent: $S_{2n} \sim \Lambda^3 \quad \leftarrow$ need 3body force!

$nn\alpha$ Three-Body Force (3BF)

 p-wave 3BF: reproduce S_{2n} = 0.973 MeV

$nn\alpha$ Three-Body Force (3BF)

Renormalized Faddeev Components

 $F_{\alpha}(\alpha, nn)$ and $F_{n}(n, \alpha n)$ are cutoff independent

C.J., Elster, Phillips, PRC 90, 044004 (2014)

• 3-body form factor (with p-wave *n*-core interactions)

• 3-body form factor (with p-wave *n*-core interactions)

- The $n\alpha$ two-body current counterterm is fixed by r_1 in $n\alpha$ $3/2^-$ state
- It does not require an additional 3-body input

⁶He Radii

[Preliminary]

• He-6 point-proton radius

• He-6 matter radius

compare with NSCM: Caurier, Navratil, PRC '06 GFMC: Pieper, RNC '08 EIHH: Bacca, Barnea, Schwenk, PRC '12 Halo EFT: preliminary (_____ uncertainty)

Atomic Isotope Shift

- Nuclear polarization in muonic atoms:
 N. Nevo Dinur's talk; O.J. Hernandez's poster
- The nuclear charge radius can also be extracted from the isotope shifts in electronic atoms:

$$\delta_{AA'} = \delta^{MS}_{AA'} + K_{FS} \, \delta \langle r^2 \rangle_{AA'}$$

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Pachucki, Moro PRA '07

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• δ_{pol} is larger in atoms with unstable nuclear isotopes (lower threshold energy) halo nuclei: δ_{pol} is important for accurately extracting nuclear charge radii

ab initio methods are computationally expensive for halo systems / continuum

- halo EFT works economically at low energies
- future EFT calculations of σ_{γ} in ⁶He ; δ_{pol} in ⁶He isotope shift

Chen Ji [TRIUMF]

CRIUMF

- Halo EFT describes structure/reaction in halo nuclei in a systematic expansion of R_{core}/R_{halo}
- We studied 2n-halo nuclei
 - ²²C: *n*-core in s-wave resonance
 - ²³N: ground and excited halo
 - ⁶He: n- α p-wave resonance
- Halo EFT can be complimentary to ab initio calculations
 - adopt inputs from ab initio results
 - benchmark with ab initio calculations
 - explain universal correlations from observables in *ab initio* work