Momentum Representation Similarity Renormalization Group Evolution of Three Nucleon Forces

Kyle A. Wendt

Department of Physics and Astronomy, University of Tennessee, Knoxville
Physics Division, Oak Ridge National Laboratory

February 20, 2014

Nuclear Structure and Reactions: Experimental and Ab Initio Theoretical Perspectives, TRIUMF 2014 February 18-21

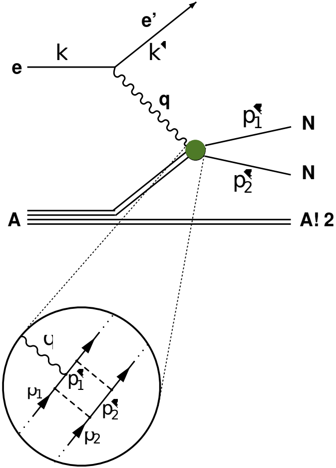
Acknowledgments:
R.J. Furnstahl, **S. Ramanan**, **K. Hebeler**, **R.J. Perry**
S.K. Bogner, H. Hergert, L. Platter, E.R. Anderson, B.S. Dainton, S. More, T. Papenbrock, G. Hagen, G. Jansen, A. Ekström C. Elster, M.R. Hadizadeh

The Similarity Renormalization Group

The Similarity Renormalization Group

[Phys. Rev. D 48, 5863 (1993)][AdP 3, 77 (1994)][PRC 75, 061001 (2007)]

- A way to control resolution
- Can decouple the long and short wavelength information
- $\boldsymbol{H}_s = \boldsymbol{U}_s \boldsymbol{H}_{s=0} \boldsymbol{U}_s^\dagger$
- $\frac{d\boldsymbol{H}_s}{ds} = \left[\frac{d\boldsymbol{U}_s}{ds} \boldsymbol{U}_s^\dagger, \boldsymbol{H}_s \right]$, define $\eta(s) = \frac{d\boldsymbol{U}_s}{ds} \boldsymbol{U}_s^\dagger$
- Any anti-hermitian $\eta(s)$ will generate a unitary flow of the Hamiltonian
- Often it is convenient to choose $\eta(s) = [\boldsymbol{G}(s), \boldsymbol{H}_s]$
  - This form will generate some form of decoupling

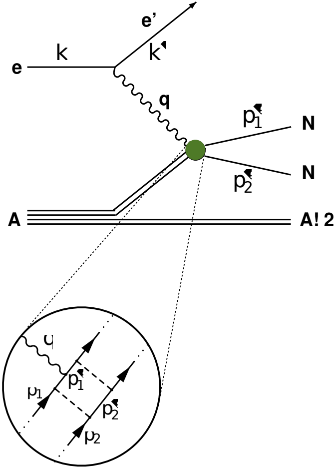


The Similarity Renormalization Group

The Similarity Renormalization Group

[Phys. Rev. D 48, 5863 (1993)][AdP 3, 77 (1994)][PRC 75, 061001 (2007)]

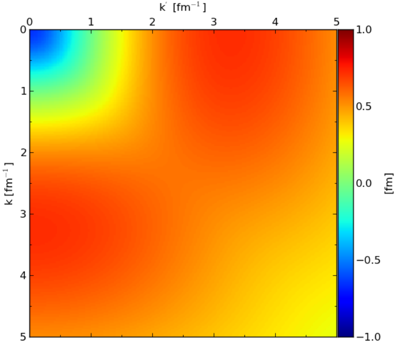
- Most work in nuclear physics has used $\eta(s) = [\boldsymbol{T}_{\mathrm{rel}}, \boldsymbol{H}_s]$
  - This leads to a decoupling between high- and low-momentum scales
- Operators can be evolved/decoupled consistently:
  - $\boldsymbol{O}_s = \boldsymbol{U}_s \boldsymbol{O}_{s=0} \boldsymbol{U}_s^\dagger$
- Few nucleon forces can be decoupled consistently

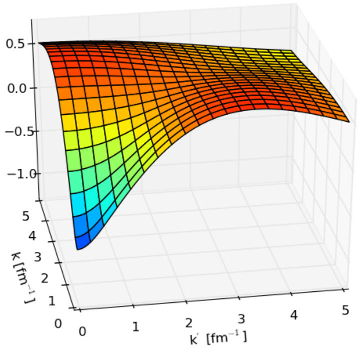


T_{rel} Similarity Renormalization Group

$$\frac{d}{ds}V_s(k,k') = -(k^2 - k'^2)^2 V_s(k,k') + \frac{2}{\pi}\int q^2 dq (k^2 + k'^2 - 2q^2) V_s(k,q) V_s(q,k')$$

Argonne V18 $^1S_0$ $\lambda = \infty$ $\quad s = \lambda^{-4}$ $\quad \lambda$ in units of fm^{-1}

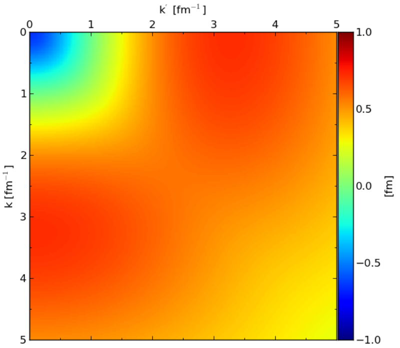


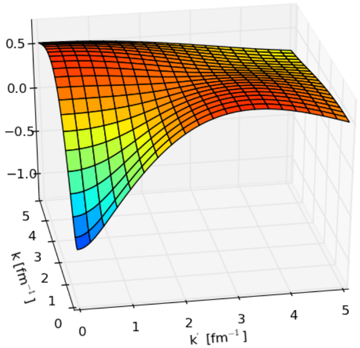


T_{rel} Similarity Renormalization Group

$$\frac{d}{ds}V_s(k,k') = -(k^2 - k'^2)^2 V_s(k,k') + \frac{2}{\pi}\int q^2 dq (k^2 + k'^2 - 2q^2) V_s(k,q) V_s(q,k')$$

Argonne V18 $^1S_0$ $\lambda = \infty$ $\quad s = \lambda^{-4}$ $\quad \lambda$ in units of fm^{-1}

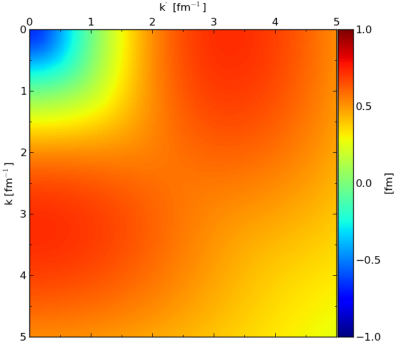


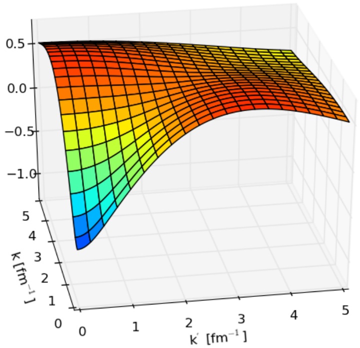


T_{rel} Similarity Renormalization Group

$$\frac{d}{ds}V_s(k,k') = -(k^2 - k'^2)^2 V_s(k,k') + \frac{2}{\pi}\int q^2 dq (k^2 + k'^2 - 2q^2) V_s(k,q) V_s(q,k')$$

Argonne V18 $^1S_0$ $\lambda = \infty$ $\quad s = \lambda^{-4}$ $\quad \lambda$ in units of fm^{-1}

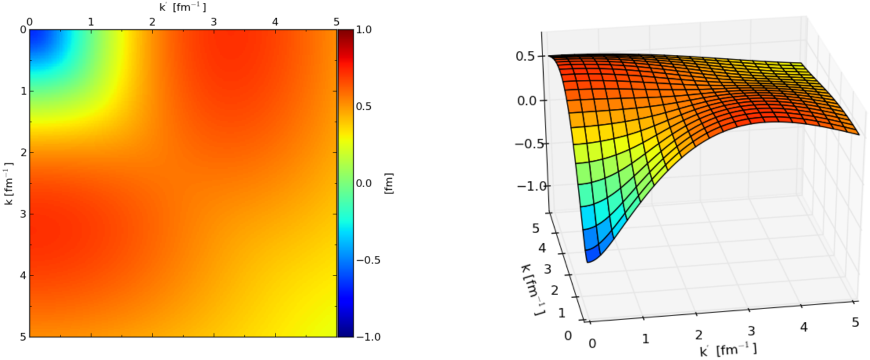




$T_{\rm rel}$ Similarity Renormalization Group

$$\frac{d}{ds}V_s(k,k') = -(k^2 - k'^2)^2 V_s(k,k') + \frac{2}{\pi}\int q^2 dq (k^2 + k'^2 - 2q^2) V_s(k,q) V_s(q,k')$$

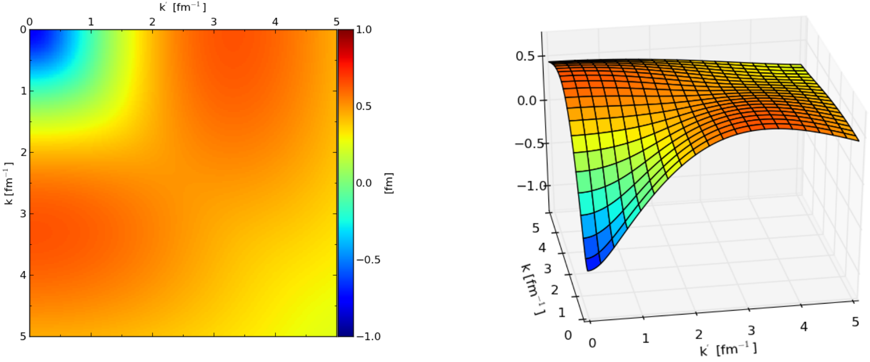
Argonne V18 $^1S_0$ $\lambda = 20.0\,\mathrm{fm}^{-1}$ $\quad s = \lambda^{-4}$ $\quad \lambda$ in units of fm^{-1}



$T_{\rm rel}$ Similarity Renormalization Group

$$\frac{d}{ds}V_s(k,k') = -(k^2-k'^2)^2 V_s(k,k') + \frac{2}{\pi}\int q^2 dq (k^2+k'^2-2q^2) V_s(k,q) V_s(q,k')$$

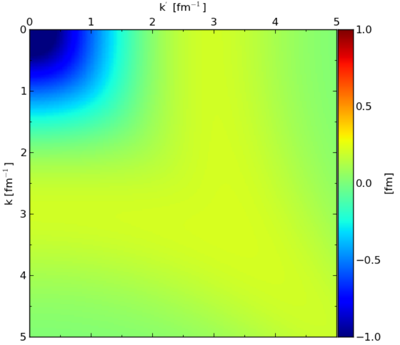
Argonne V18 $^1S_0$ $\lambda = 10.0\,\mathrm{fm}^{-1}$ $\quad s=\lambda^{-4}$ $\quad \lambda$ in units of fm^{-1}

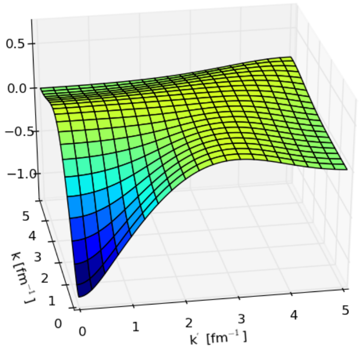


$T_{\rm rel}$ Similarity Renormalization Group

$$\frac{d}{ds}V_s(k,k') = -(k^2 - k'^2)^2 V_s(k,k') + \frac{2}{\pi}\int q^2 dq (k^2 + k'^2 - 2q^2) V_s(k,q) V_s(q,k')$$

Argonne V18 $^1S_0$ $\lambda = 4.0\,\mathrm{fm}^{-1}$ $s = \lambda^{-4}$ λ in units of fm^{-1}

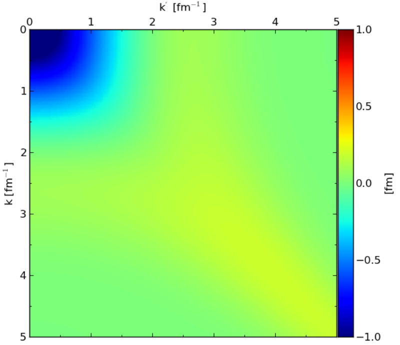


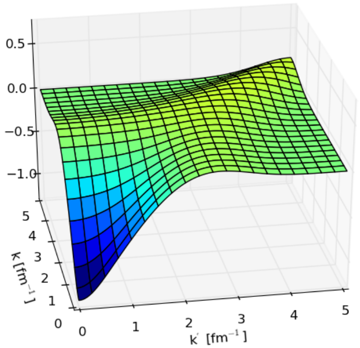


T_{rel} Similarity Renormalization Group

$$\frac{d}{ds}V_s(k,k') = -(k^2 - k'^2)^2 V_s(k,k') + \frac{2}{\pi}\int q^2 dq (k^2 + k'^2 - 2q^2) V_s(k,q) V_s(q,k')$$

Argonne V18 $^1S_0$ $\lambda = 3.0\,\mathrm{fm}^{-1}$ $s = \lambda^{-4}$ λ in units of fm^{-1}

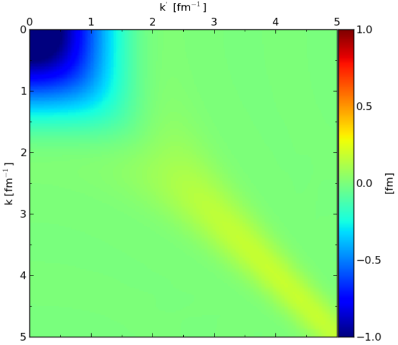


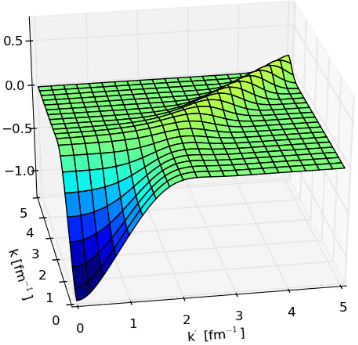


T_{rel} Similarity Renormalization Group

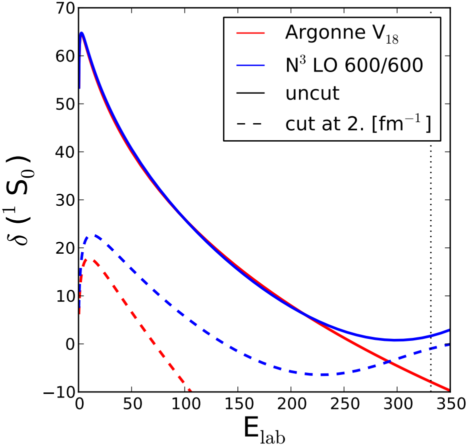
$$\frac{d}{ds}V_s(k,k') = -(k^2-k'^2)^2 V_s(k,k') + \frac{2}{\pi}\int q^2 dq (k^2+k'^2-2q^2) V_s(k,q) V_s(q,k')$$

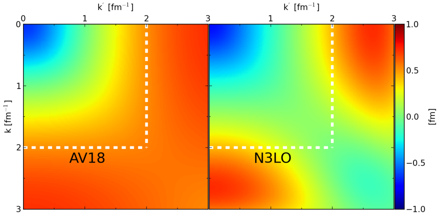
Argonne V18 $^1S_0$ $\lambda = 2.0\,\text{fm}^{-1}$ $s = \lambda^{-4}$ λ in units of fm^{-1}





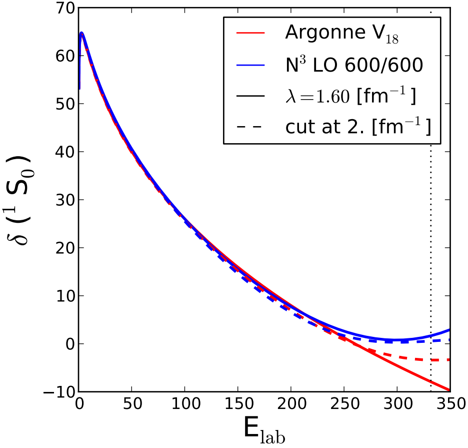
Importance of Decoupling

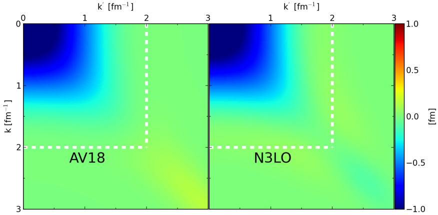




- Working in momentum space truncate potential at $\Lambda = 2\,\mathrm{fm}^{-1}$ and compute phase shifts
- A momentum space truncation is similar to the $N_{\max}$ truncation H.O. basis/NCSM.
- Lowpass filter does not work!

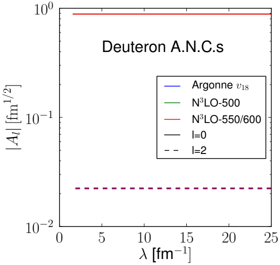
Importance of Decoupling

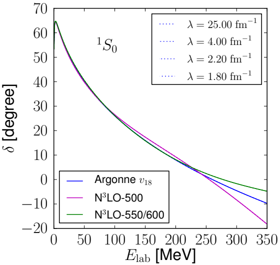




- Low pass filter works!
- Result of decoupling low- and high-momentum physics

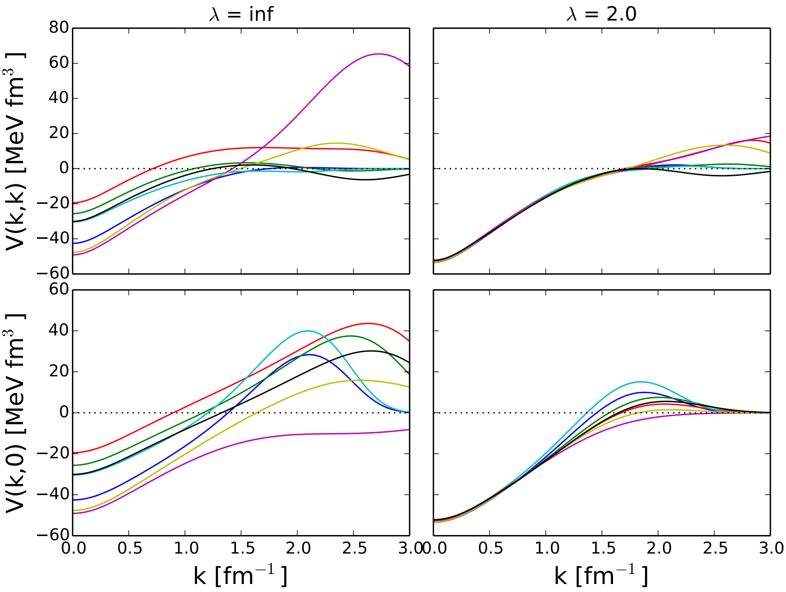
What happens as $T_{\rm rel}$-SRG softens interactions?





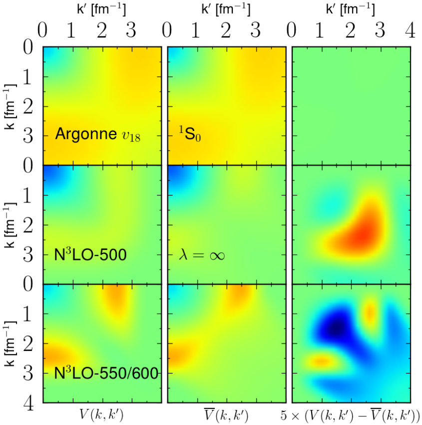
- SRG is unitary $\rightarrow$ observables are preserved
  - on-shell S-matrix is invariant
  - other $\langle \widehat{O} \rangle$ using evolved operators
- **Induced non-locality**
- **Induced many-body forces**

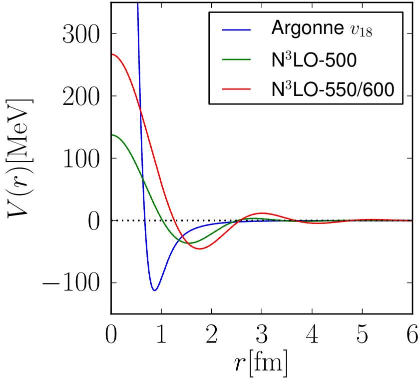
What happens as T_{rel}-SRG softens interactions?



What happens as T_{rel}-SRG softens interactions?

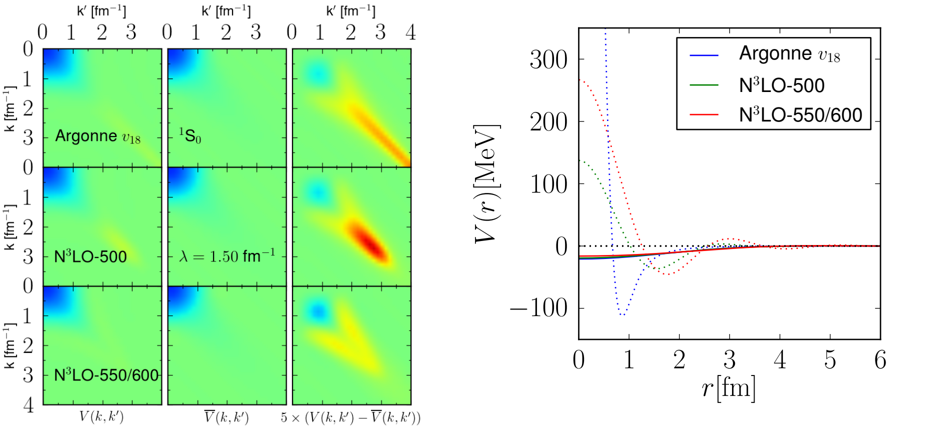
Local projection of non-local forces





What happens as T_{rel}-SRG softens interactions?

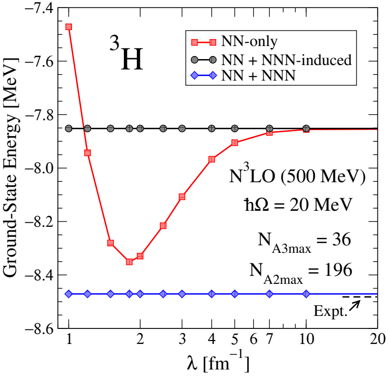
Local projection of non-local forces

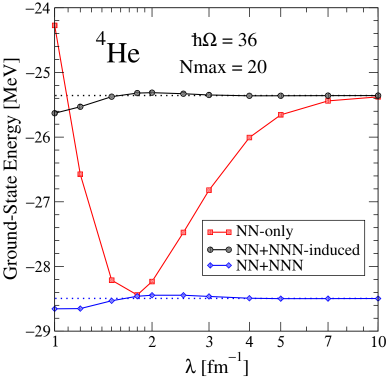


Induced many-body forces in the SRG

$$\frac{d\hat{V}}{ds} = \left[\left[G\hat{a}^\dagger\hat{a}\,, H\hat{a}^\dagger\hat{a}^\dagger\hat{a}\;\hat{a}\right], H\hat{a}^\dagger\hat{a}^\dagger\hat{a}\;\hat{a}\right]$$

$$\hat{V}_{s=0+\delta s} = V_{s=0}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\;\hat{a} + \delta s\left(\frac{dV^{(2)}}{ds}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\;\hat{a} + \frac{dV^{(3)}}{ds}\hat{a}^\dagger\hat{a}^\dagger\hat{a}^\dagger\hat{a}\;\hat{a}\;\hat{a}\right)$$





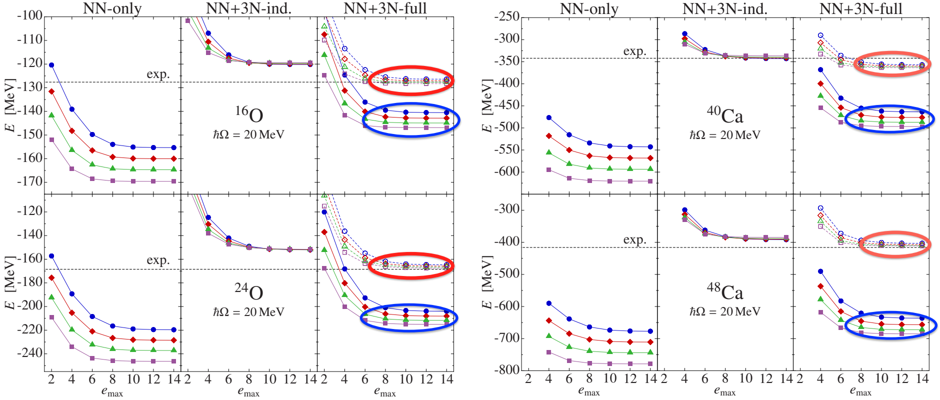
[E.D. Jurgenson et al. PRC 83, 034301 (2011)]

Growth of forces

Couple cluster calculation from R. Roth et al. PRL 109, 052501 (2012)

- Red Circle: 3NF fit with $\Lambda_3 \ll \Lambda_2$
- Blue Circle: fit with“consistent” Λ_3

- Induced 4N force (Roth et al 2013)
- What to study effect of IR and UV cutoffs and other generators.



SRG and Few-Body Evolution

- Want to study induced forces in a manner that does not add additional scales to problems
- Likewise want to study local/non-local features of the 3NF and 4NF
- Want to study operators under SRG.

H.O. basis (E.D. Jurgenson)

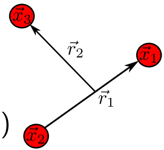
- 3- and 4- body SRG evolution is straightforward!
- Easy to get good permutational symmetry
- $\hbar\omega$ is a new scale in the calculation!
- Evolution has a IR cutoff
  - May cause issues studying asymptotic features of evolved interactions.

Jacobi plane waves (K. Hebeler)

- 3-body SRG evolution is straightforward!
- $q_{\rm max}$ can be chosen to not interfere with SRG scale
- 4-body is not!
- Hard to get good permutational symmetry for SRG evolution
- Generators other than $\left[\hat{T}_{\rm rel}, \hat{H}\right]$ are harder to express

What about Hyper-spherical Harmonics?

- Another alternative for good permutational symmetry in Jacobi coordinates is the Hyper-spherical Harmonics
- Start with P.W.E of Jacobi coordinates:



$$|\vec{k}_1\vec{k}_2\rangle \rightarrow |k_1 k_2 l_1 l_2, m_1, m_2\rangle \qquad \begin{aligned}\vec{k}_1 &= (\vec{p}_1 - \vec{p}_2)/\sqrt{2} \\ \vec{k}_2 &= \sqrt{2/3}(\vec{p}_3 - (\vec{p}_1 + \vec{p}_2)/2)\end{aligned}$$

- Expand (k_1, k_2) in polar coordinates (Q, θ) $\theta \in [0, \pi/2]$ as moments:

$$|k_1 k_2 l_1 l_2, m_1, m_2\rangle \rightarrow |Q\theta l_1 l_2, m_1, m_2\rangle \rightarrow |Q n_{12} l_1 l_2 m_1 m_2\rangle$$

- Permutation operators are now only a function of n_{12}, l_1, l_2
  - Block diagonal in $K = 2 * n_{12} + l_p + l_q$
- Completely antisymmetric representation that is closed within truncation $G_{\max}$:

$$|QKi\rangle = \sum_{l_1, l_2} c^{K,i}_{l_1, l_2} |QKl_1, l_2\rangle$$

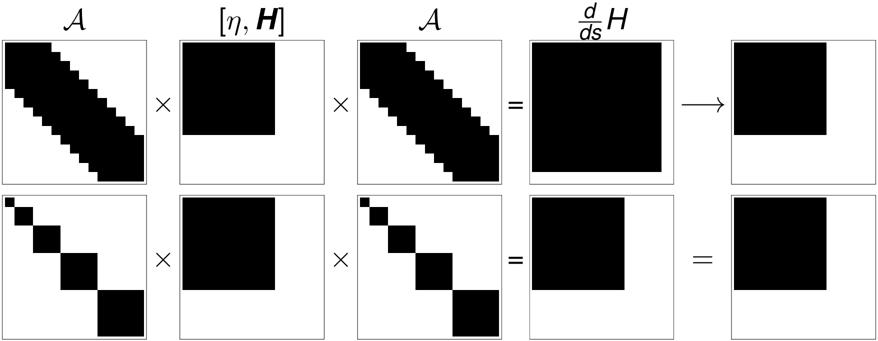
Hyper-spherical Plane Waves

- Modern H.H. methods expand potential into polynomials of Q and H.H. functions

$$\langle QG'i' \mid mGi\rangle = f_m^G(Q)\delta_{G,G'}\delta_{i,i'}$$

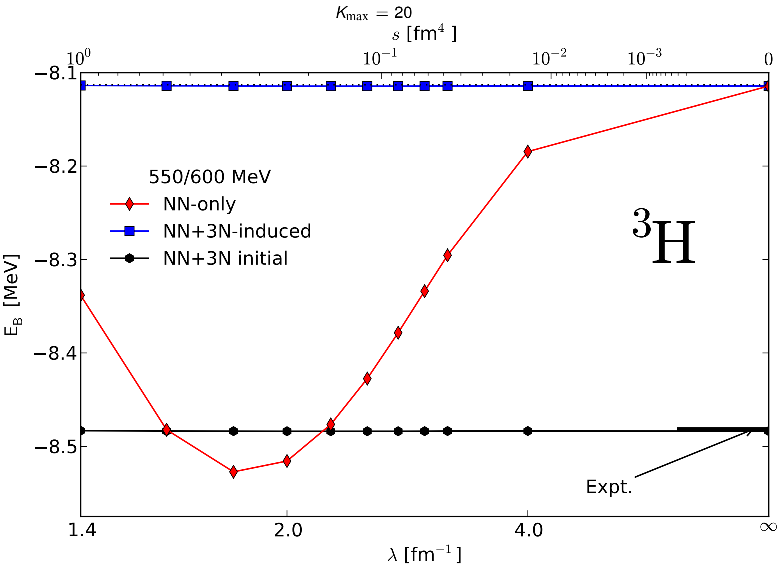
- Similar issues as H.O. basis
- Instead leave Hamiltonian in terms of hyper-spherical momentum states!
    - Has good permutational symmetry!
    - Extension from 3- to 4-body SRG is straightforward
    - Extension to other SRG generators is easy
    - Clean control of IR cutoff
    - Clean control of UV cutoff
    - Numerical convergence is relatively slow

Difference from other Momentum Rep. Evolutions

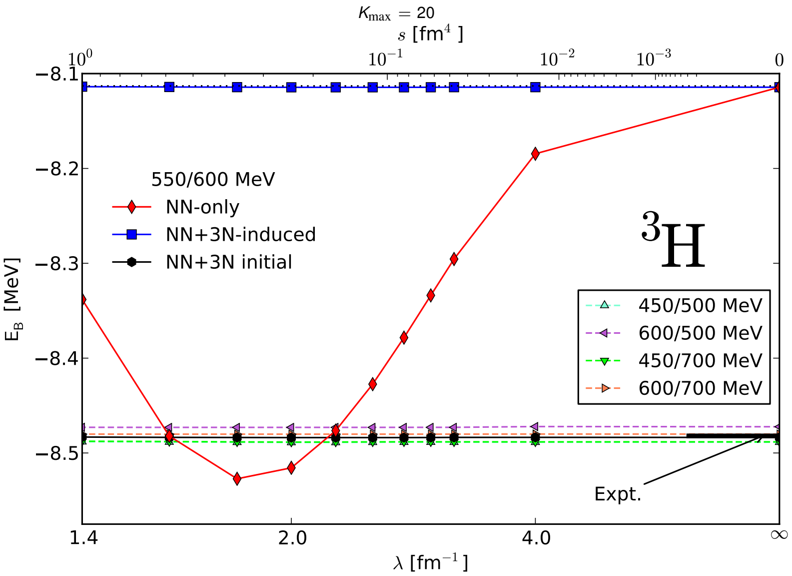


- Need to ensure antisymmetry during evolution
- Antisymmetric RHS couples outside basis truncation
  $\rightarrow$ no consistent basis truncation!
- Antisymmetric RHS does not couple outside basis truncation
  $\rightarrow$ consistent basis truncation!

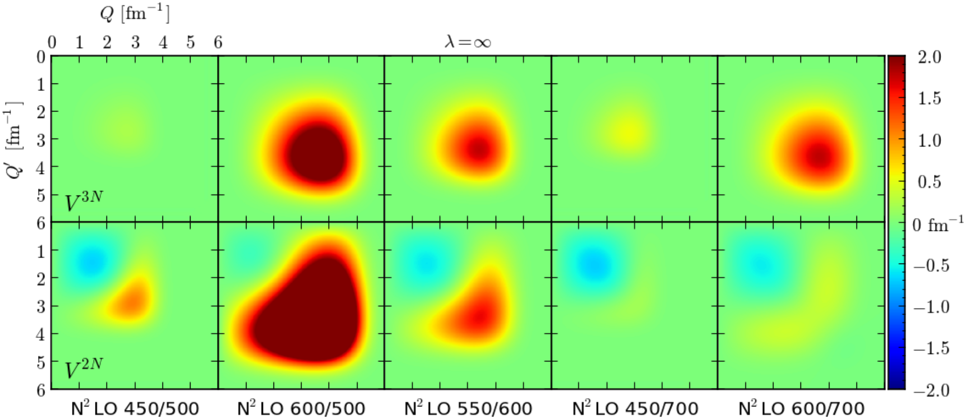
Test of SRG implementation



Test of SRG implementation

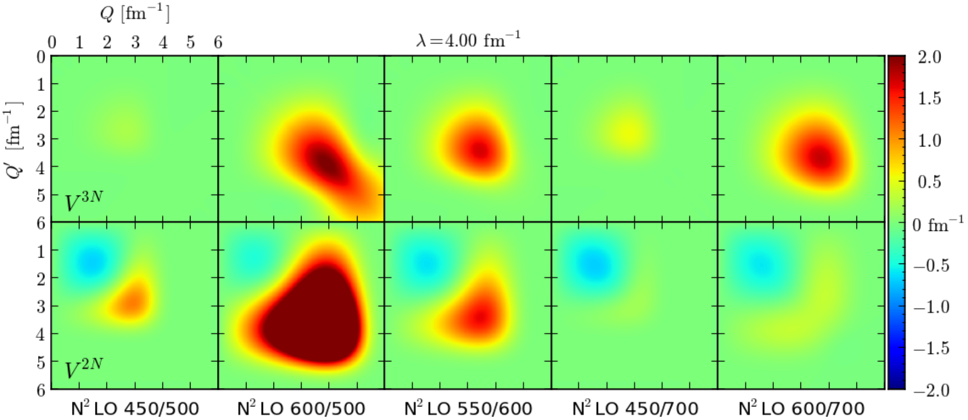


Evolution of the Induced Force



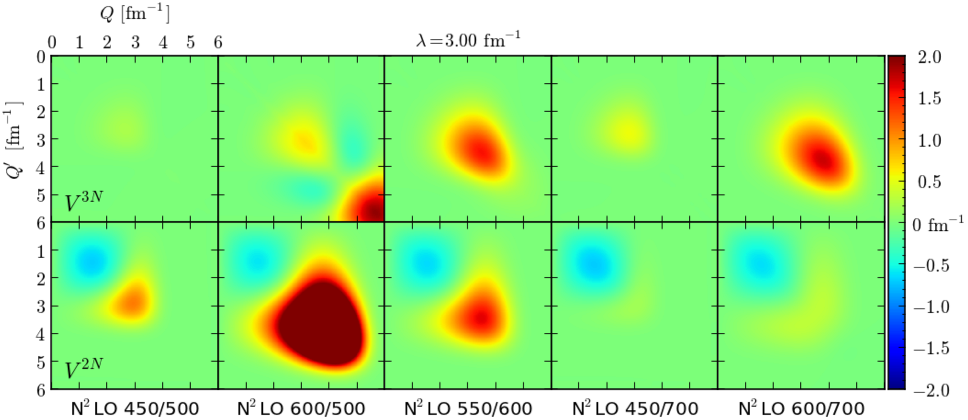
$$|\, H.H.\rangle = \frac{1}{\sqrt{2}}\left[|\ {}^1S_0 : (0\tfrac{1}{2})\rangle - |\ {}^3S_1 : (0\tfrac{1}{2})\rangle\right]$$

Evolution of the Induced Force



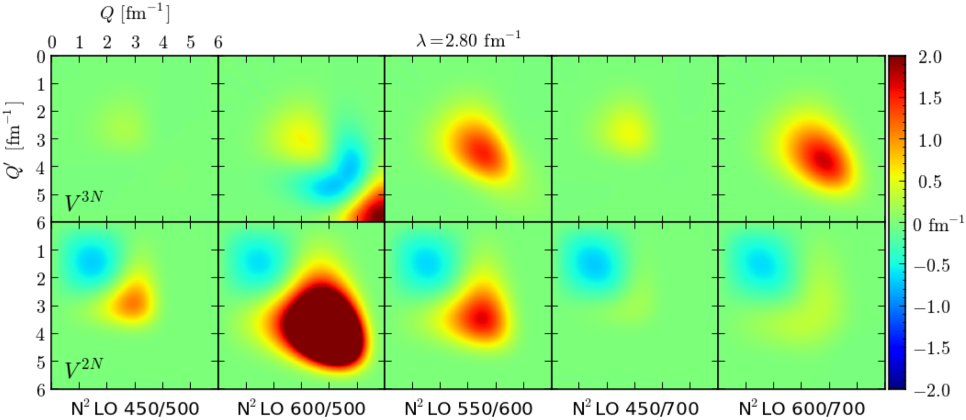
$$|\, H.H. \rangle = \frac{1}{\sqrt{2}} \left[|\, {}^1S_0 : (0\tfrac{1}{2}) \rangle - |\, {}^3S_1 : (0\tfrac{1}{2}) \rangle \right]$$

Evolution of the Induced Force



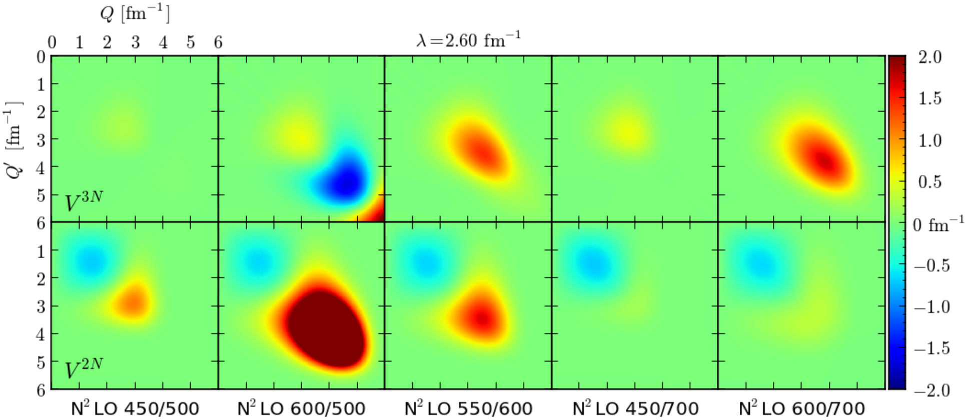
$$| H.H.\rangle = \frac{1}{\sqrt{2}}\left[| \, ^1S_0 : (0\tfrac{1}{2})\rangle - | \, ^3S_1 : (0\tfrac{1}{2})\rangle\right]$$

Evolution of the Induced Force



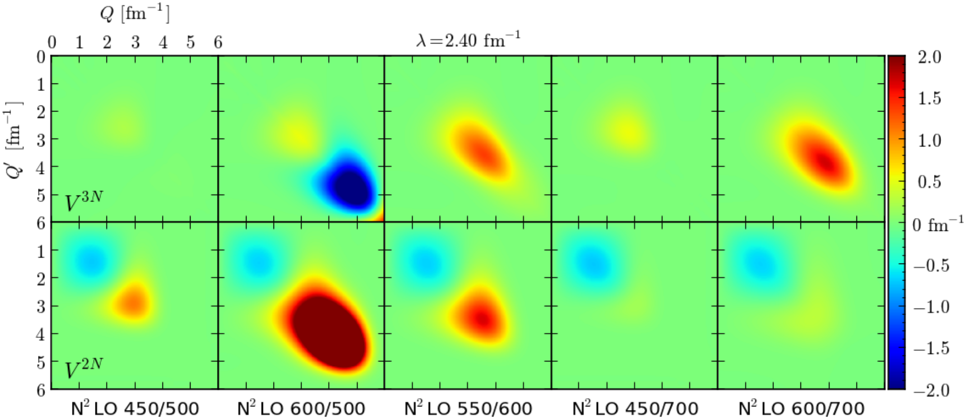
$$|\ H.H.\rangle = \frac{1}{\sqrt{2}}\left[|\ {}^1S_0 : (0\tfrac{1}{2})\rangle - |\ {}^3S_1 : (0\tfrac{1}{2})\rangle\right]$$

Evolution of the Induced Force



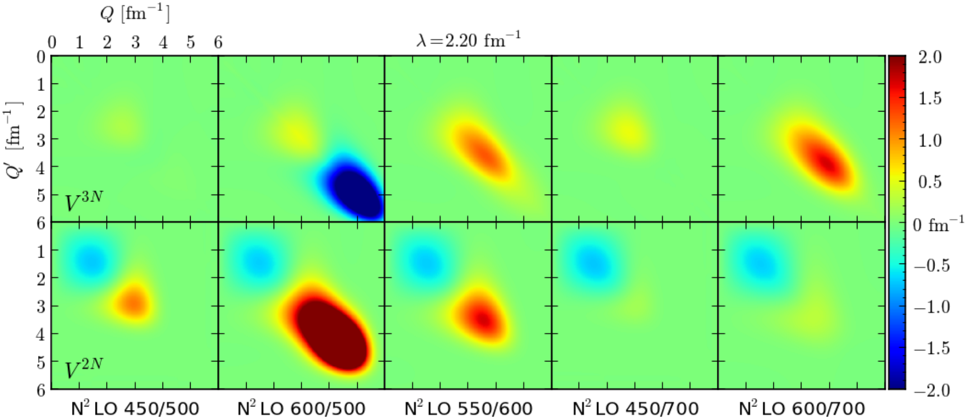
$$| H.H.\rangle = \frac{1}{\sqrt{2}} \left[| \, ^1S_0 : (0\tfrac{1}{2})\rangle - | \, ^3S_1 : (0\tfrac{1}{2})\rangle \right]$$

Evolution of the Induced Force



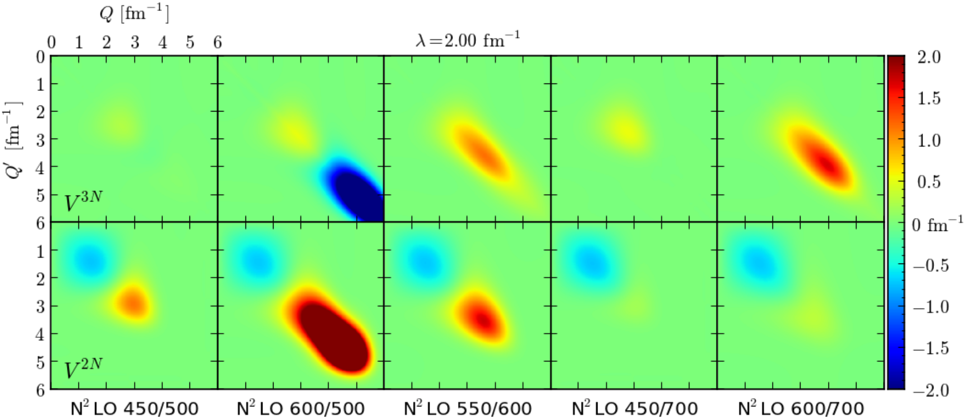
$$| H.H.\rangle = \frac{1}{\sqrt{2}} \left[| \, {}^1S_0 : (0\tfrac{1}{2})\rangle - | \, {}^3S_1 : (0\tfrac{1}{2})\rangle \right]$$

Evolution of the Induced Force



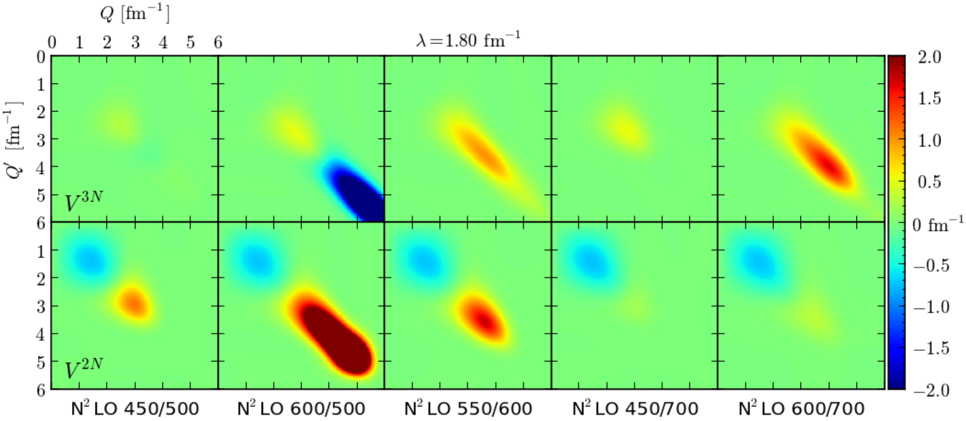
$$| H.H.\rangle = \frac{1}{\sqrt{2}} \left[| \, {}^1S_0 : (0\tfrac{1}{2})\rangle - | \, {}^3S_1 : (0\tfrac{1}{2})\rangle \right]$$

Evolution of the Induced Force



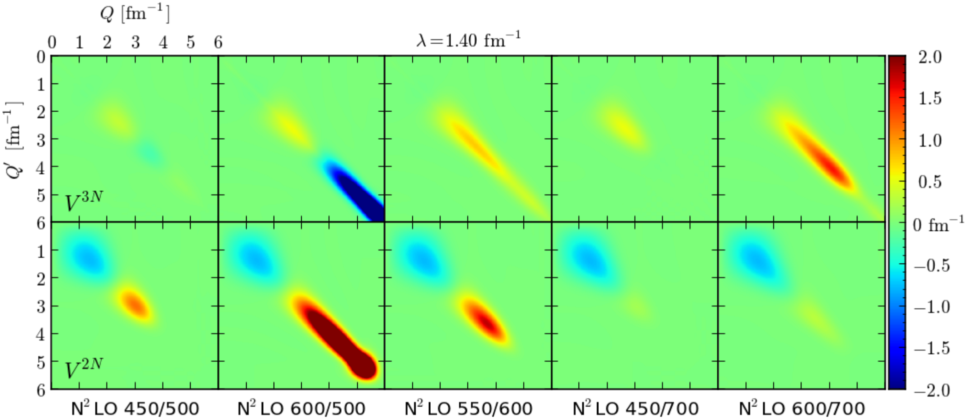
$$| H.H.\rangle = \frac{1}{\sqrt{2}} \left[| \, {}^1S_0 : (0\tfrac{1}{2})\rangle - | \, {}^3S_1 : (0\tfrac{1}{2})\rangle \right]$$

Evolution of the Induced Force



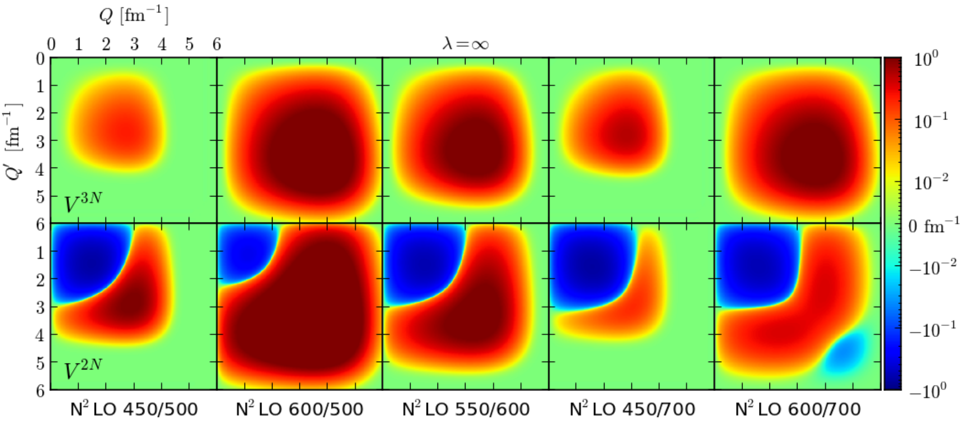
$$| H.H.\rangle = \frac{1}{\sqrt{2}} \left[| \, {}^1S_0 : (0\tfrac{1}{2})\rangle - | \, {}^3S_1 : (0\tfrac{1}{2})\rangle \right]$$

Evolution of the Induced Force



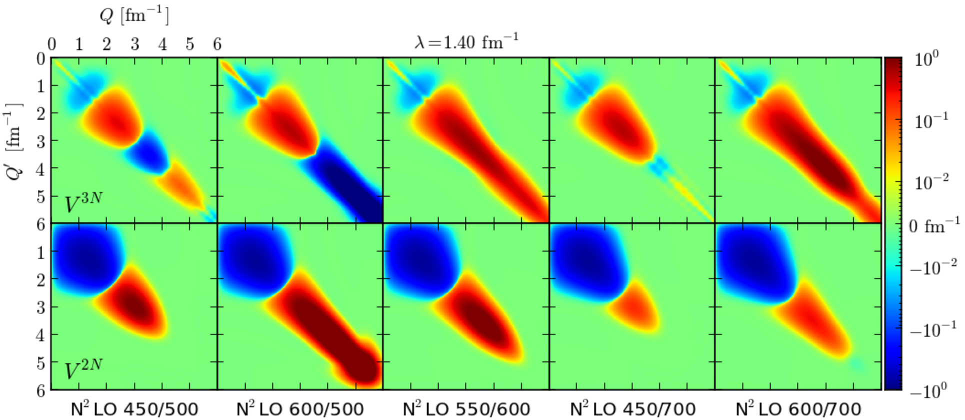
$$| H.H.\rangle = \frac{1}{\sqrt{2}} \left[| \, {}^1S_0 : (0\tfrac{1}{2}) \rangle - | \, {}^3S_1 : (0\tfrac{1}{2}) \rangle \right]$$

Evolution of the Induced Force



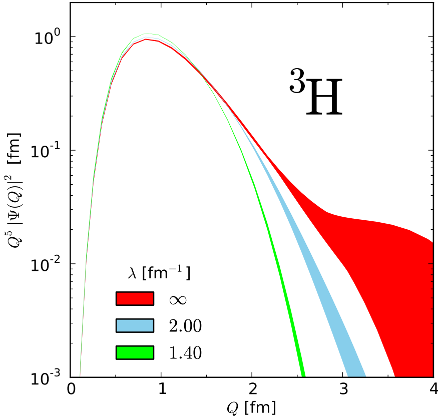
$$H.H.\rangle = \frac{1}{\sqrt{2}} \left[|\ {}^1S_0 : (0\tfrac{1}{2})\rangle - |\ {}^3S_1 : (0\tfrac{1}{2})\rangle \right]$$

Evolution of the Induced Force



$$H.H.\rangle = \frac{1}{\sqrt{2}}\left[|\ {}^1S_0 : (0\tfrac{1}{2})\rangle - |\ {}^3S_1 : (0\tfrac{1}{2})\rangle\right]$$

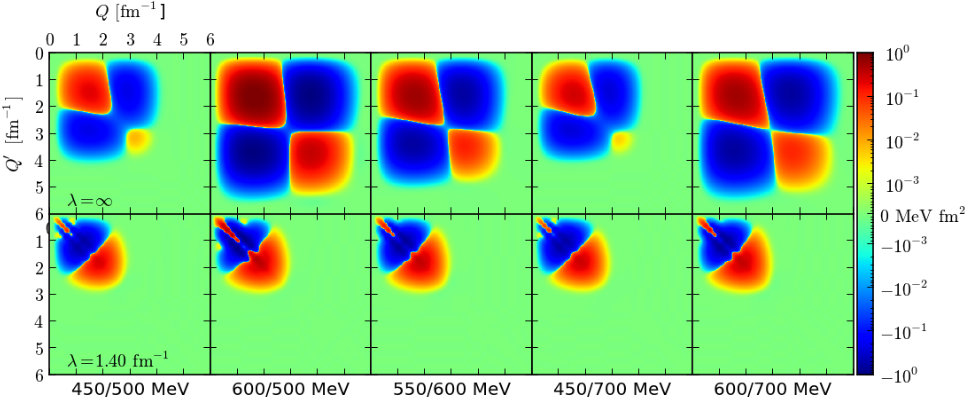
Evolution of the 3N Force



- Similar behavior to what has been demonstrated for deuteron in past works.
- SRG suppresses high momentum tail of wave functions
- SRG low and large momentum WF is universal!
- Seemingly dominant high momentum 3NF is irrelevant

Evolution of the 3N Force

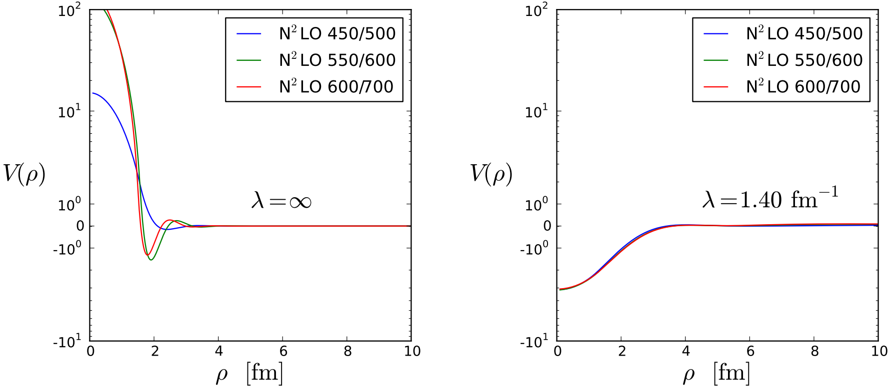
Integrand of ⟨Triton | V^{3N} | Triton⟩



Clear Signal of low momentum universality!

Local projection of 3NF

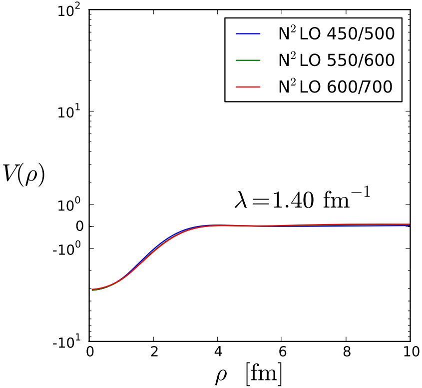
Local projection of non-local forces



Local projection of 3NF

Local projection of non-local forces

Local projection of 3NF is also universal!



Operators also have induced many-body components

$$\frac{d\hat{O}}{ds} = \left[\left[G\hat{a}^\dagger\hat{a}\ , H\hat{a}^\dagger\hat{a}^\dagger\hat{a}\ \hat{a}\ \right], O\hat{a}^\dagger\hat{a}^\dagger\hat{a}\ \hat{a}\ \right]$$

$$\hat{O}_{s=\delta s} = O_{s=0}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\ \hat{a}\ + \delta s\left(\frac{dO^{(2)}}{ds}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\ \hat{a}\ + \frac{dO^{(3)}}{ds}\hat{a}^\dagger\hat{a}^\dagger\hat{a}^\dagger\hat{a}\ \hat{a}\ \hat{a}\ \right)$$

- Operators will have induced many-body currents
- Currents can be computed via flow equation
- Or via $\hat{O}_s = \hat{U}_s\hat{O}_0\hat{U}_s^\dagger$

Two Body Momentum Distribution

- Can use two body momentum distribution to probe the evolution of operators
- Simple structure allows for easy decomposition of various induced three-body components
- Has been studied for SRG in 1D models (E.R. Anderson et al. 2010) and in UCOM for realistic interactions (H. Feldmeier et al. 2011).

$$\rho(q_0) = \sum_{l_1 s_1 j_1} |\, q_0 l_1 s_1 j_1 \rangle \langle q_0 l_1 s_1 j_1 \,|$$

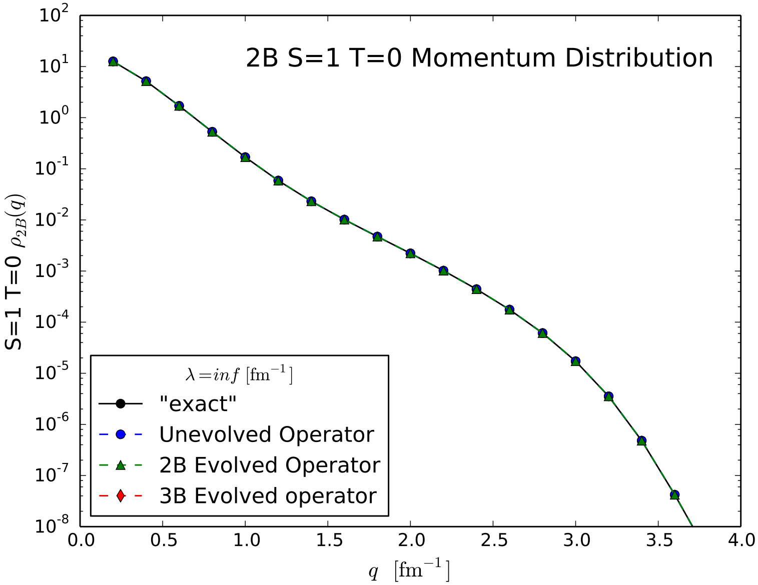
$$\boldsymbol{U}_s = 1 + \boldsymbol{U}_s^{(2)} + \boldsymbol{U}_s^{(3)} + \ldots$$

$$\rho_s(q_0) = \boldsymbol{U}_s \rho(q_0) \boldsymbol{U}_s^{\dagger} = \rho_s^{2B}(q_0) + \rho_s^{3B}(q_0) + \ldots$$

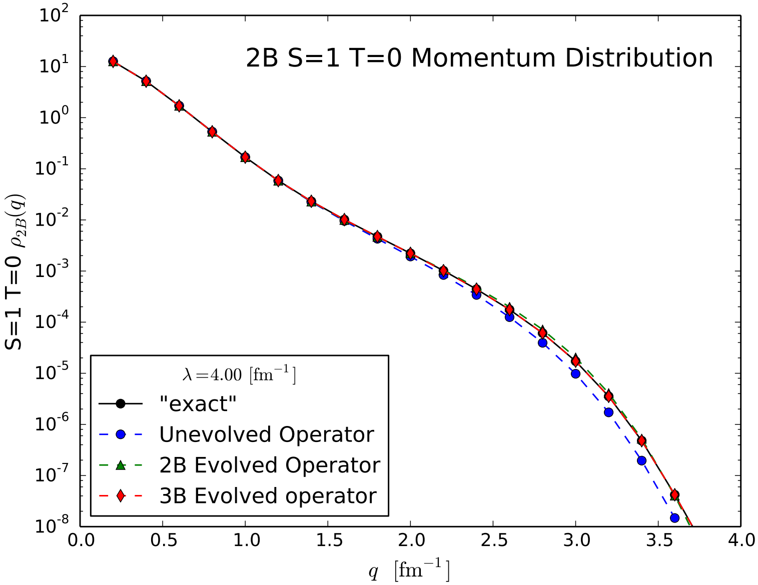
$$= \rho(q_0) + (\boldsymbol{U}_s^{(2)} \rho(q_0) + \mathrm{c.c.}) + \boldsymbol{U}_s^{(2)} \rho(q_0) \boldsymbol{U}_s^{(2)^{\mathsf{T}}})$$

$$+ (\boldsymbol{U}_s^{(3)} \rho(q_0) + \mathrm{c.c.}) + \boldsymbol{U}_s^{(3)} \rho(q_0) \boldsymbol{U}_s^{(3)^{\mathsf{T}}}) + (\boldsymbol{U}_s^{(2)} \rho(q_0) \boldsymbol{U}_s^{(3)^{\mathsf{T}}} + \mathrm{c.c.})$$

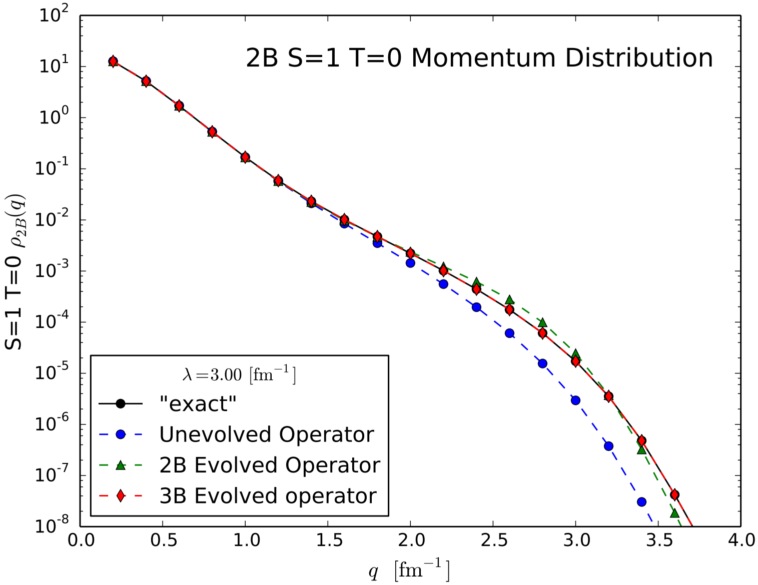
Two Body Momentum Distribution



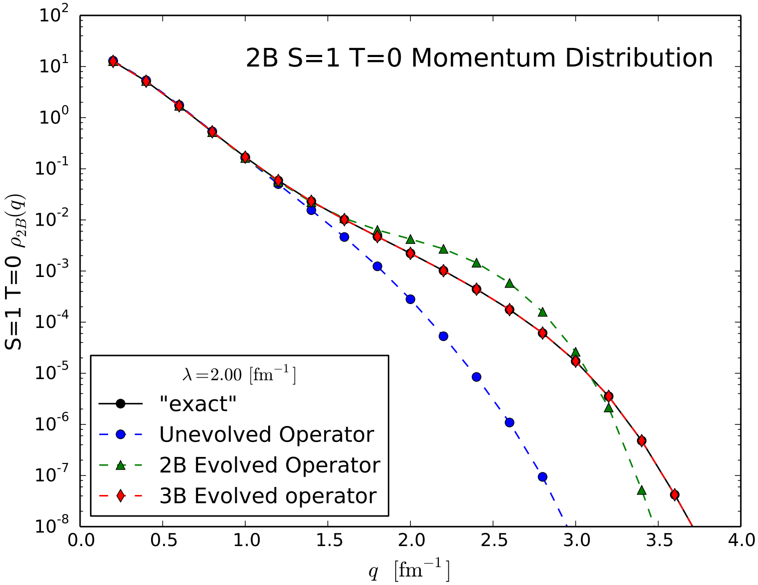
Two Body Momentum Distribution



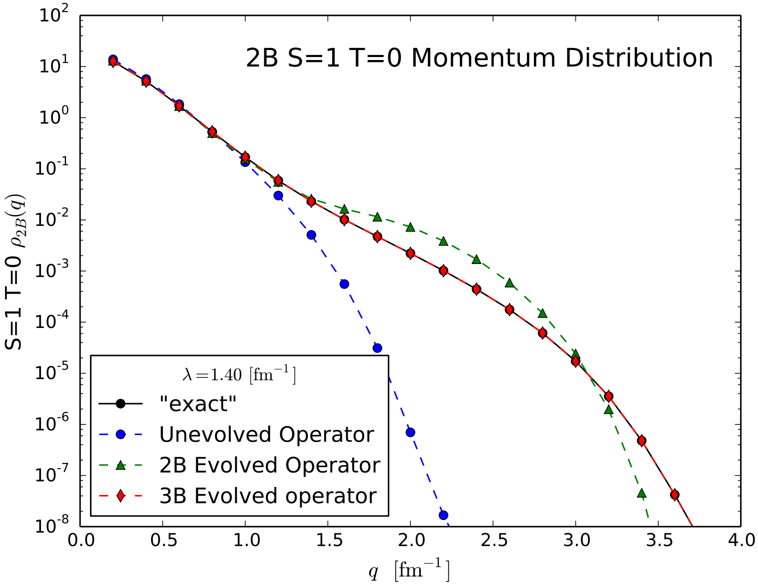
Two Body Momentum Distribution



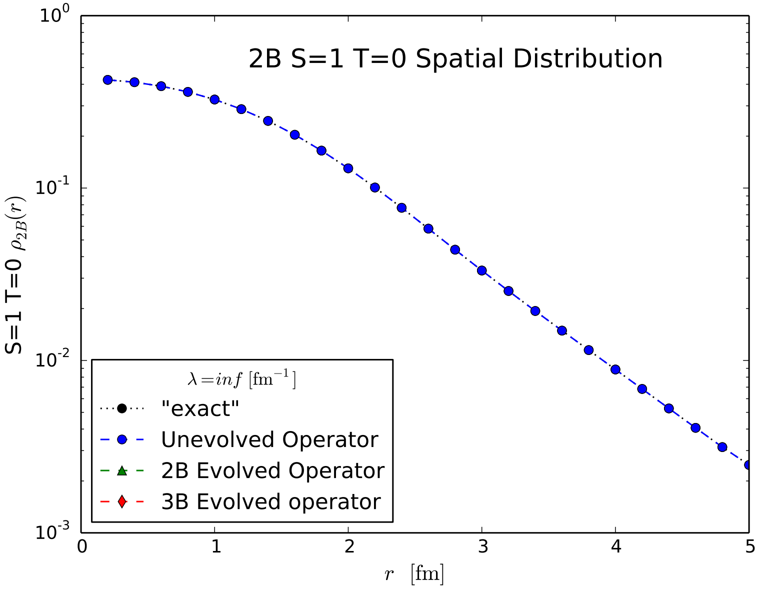
Two Body Momentum Distribution



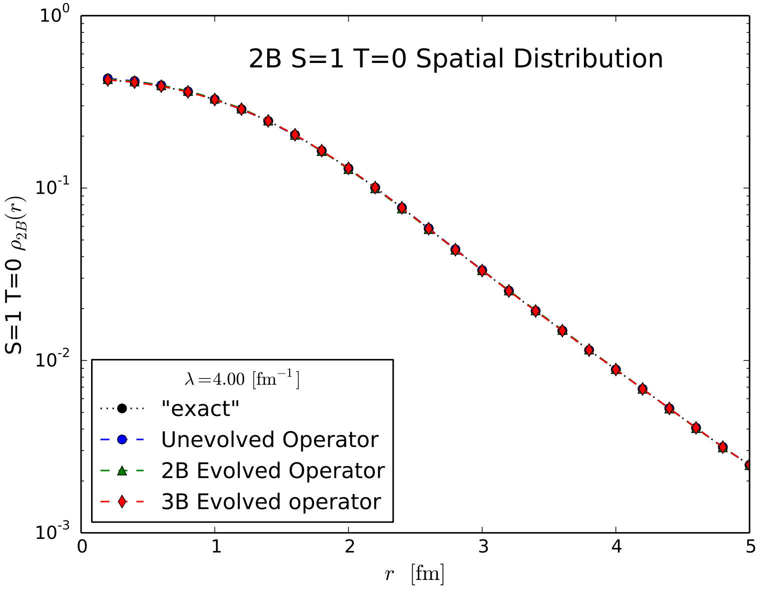
Two Body Momentum Distribution



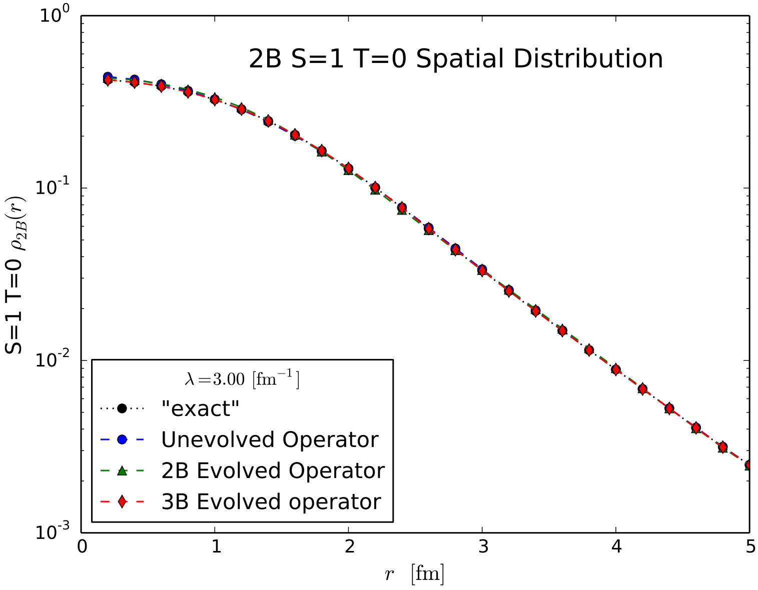
Two Body Spatial Distribution



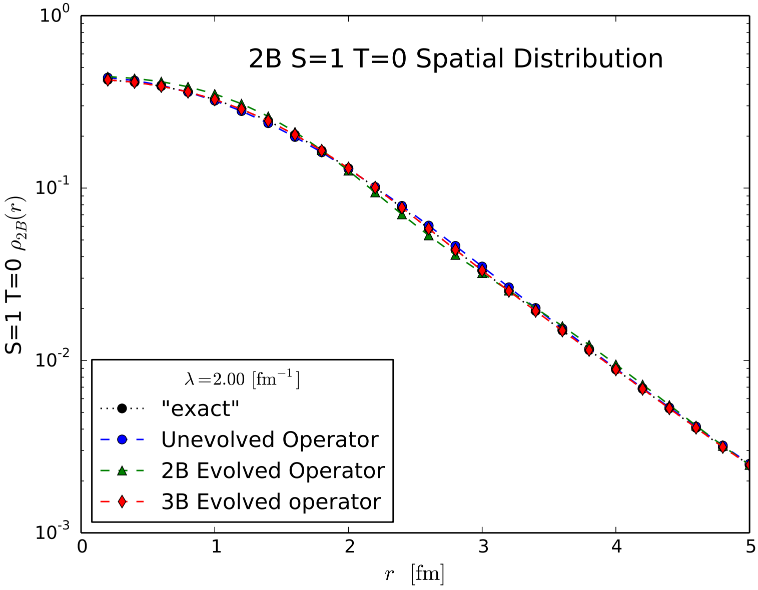
Two Body Spatial Distribution



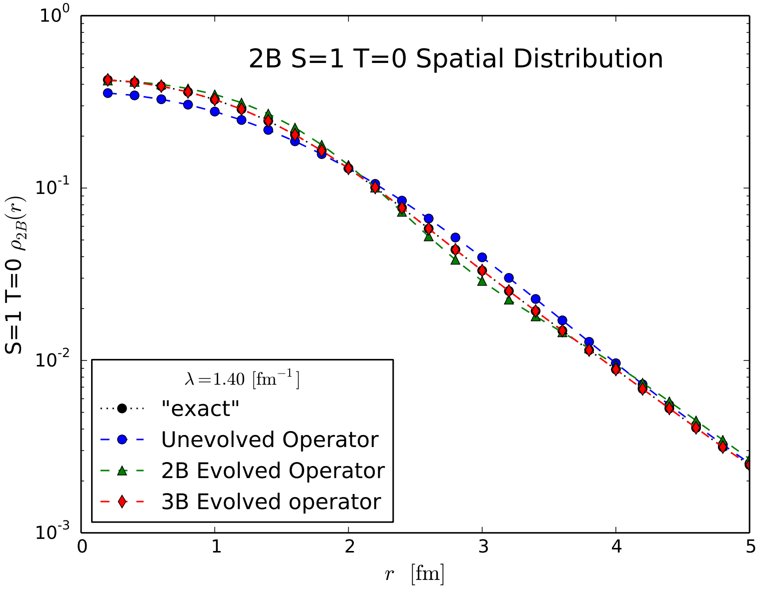
Two Body Spatial Distribution



Two Body Spatial Distribution



Two Body Spatial Distribution

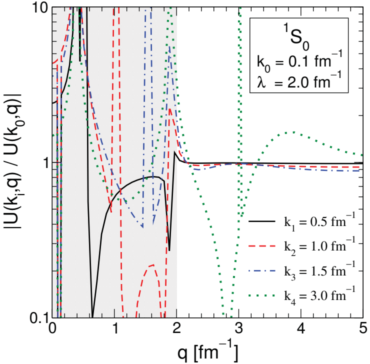


Factorization of the Unitary Operator

- SRG evolution of all operators is controlled by $\boldsymbol{U}_\lambda$
- $\boldsymbol{U}_\lambda = \mathbb{1} + \boldsymbol{U}_\lambda^{(2)} + \boldsymbol{U}_\lambda^{(3)}$
- Formal Structure of $\boldsymbol{U}_\lambda$ has implications
- For A=2
  - Will ultimately apply $\langle k \mid \boldsymbol{U}_\lambda^{(2)} \mid q\rangle$ at low momentum (small k)
  - $\langle k \mid \boldsymbol{U}_\lambda^{(2)} \mid q\rangle \approx K_\lambda(k) Q_\lambda(q)$ for $k \ll \lambda$ and $q \gg \lambda$
  - OPE allows separation of expectation values into low momentum integrals and universal high momentum factors:

$A = 2$

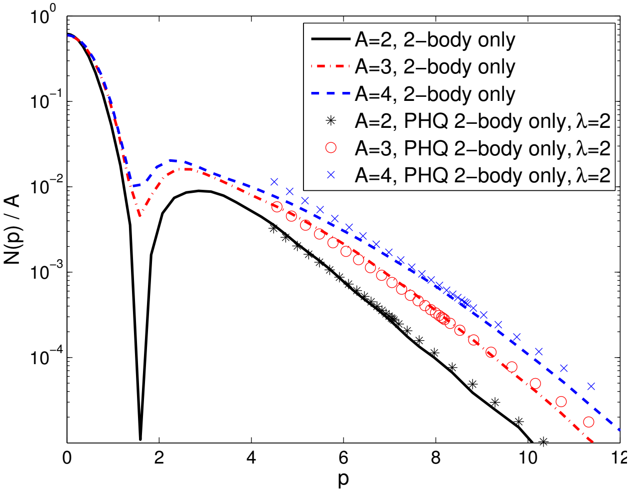
$U(k,q)/U(k_0,q) \quad k_0 \ll \lambda$



E.R. Anderson et al. Phys. Rev. C 82, 054001 (2010)

$$\langle \psi_\lambda \mid \boldsymbol{O}^{(2)} \mid \psi_\lambda \rangle = \langle \psi_\lambda \mid \boldsymbol{O}^{(2)} \mid \psi_\lambda \rangle_{\text{Truncated}} + I^{(2)}_{QOQ} \langle \psi_\lambda \mid K_\lambda \rangle \langle K_\lambda \mid \psi_\lambda \rangle_{\text{Truncated}}$$

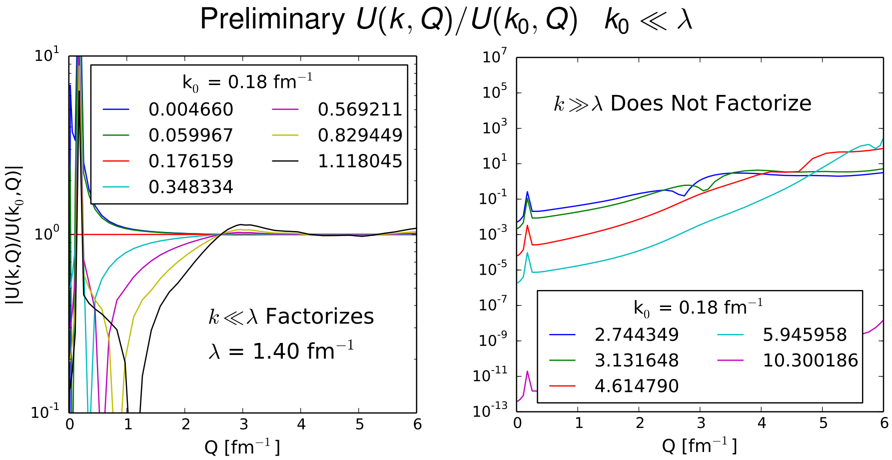
Factorization of the Unitary Operator



E.R. Anderson et al. Phys. Rev. C 82, 054001 (2010)

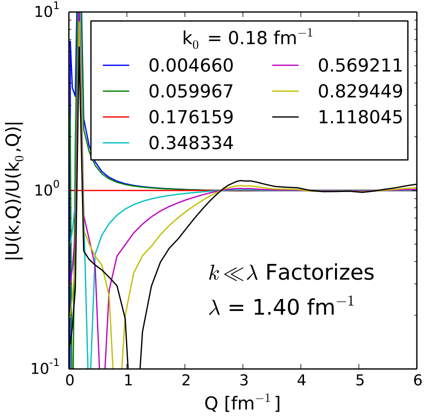
$$\langle \psi_\lambda \mid \boldsymbol{O}^{(2)} \mid \psi_\lambda \rangle = \langle \psi_\lambda \mid \boldsymbol{O}^{(2)} \mid \psi_\lambda \rangle_{\text{Truncated}} + I^{(2)}_{QOQ} \langle \psi_\lambda \mid K_\lambda \rangle \langle K_\lambda \mid \psi_\lambda \rangle_{\text{Truncated}}$$

Does the Three Body U Factorize as Well?



Does the Three Body U Factorize as Well?

Preliminary $U(k,Q)/U(k_0,Q) \quad k_0 \ll \lambda$



- Low Momentum Three Body U factorizes!
- May be able to exploit OPE in similar manner to two body:

$$\langle \psi_\lambda \mid \boldsymbol{O}^{(3)} \mid \psi_\lambda \rangle = \langle \psi_\lambda \mid \boldsymbol{O}^{(3)} \mid \psi_\lambda \rangle_{\text{Truncated}} + I^{(3)}_{QOQ} \langle \psi_\lambda \mid K^{(3)}_\lambda \rangle \langle K^{(3)}_\lambda \mid \psi_\lambda \rangle_{\text{Truncated}}$$

Summary

- H.H. momentum representation provides a permutationally consistent momentum space evolution
  - Not possible in other momentum representations due to non-closure of the basis under particle permutation
  - Generates a reduced basis for computing the evolution on
  - Provides a convenient way to visualize the three-body force
- Universality at N^2LO
  - T_{rel}-SRG in H.H. momentum rep.
  - 5 different fits / regulators
  - Common low momentum evolved form
    - Three-body Universality!
- Induced Three Body Operators
  - Three body induced components are important for high momentum operators!
  - Three body low momentum ***U*** factorizes!
  - May provide a path to computing high-momentum observables that are outside reach of low momentum model spaces!

Local Projections and the SRG

- SRG flow is often computed in momentum space using a finite quadrature or using H.O. basis
  - "Sharp" features in coordinate space are lost to truncation
    - Such as the delta function ($\delta(\mathbf{r}-\mathbf{r}')$) on local potentials
    - Want a way to partially recover this delta function
- Inhibits use of SRG (and chiral) interactions with QMC methods

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r}) + \int d\mathbf{r}'\, V(\mathbf{r},\mathbf{r}')\Psi(\mathbf{r}') = E\Psi(\mathbf{r})$$

Local Projections and the SRG

- SRG flow is often computed in momentum space using a finite quadrature or using H.O. basis
  - "Sharp" features in coordinate space are lost to truncation
    - Such as the delta function ($\delta(\mathbf{r}-\mathbf{r}')$) on local potentials
    - Want a way to partially recover this delta function
- Inhibits use of SRG (and chiral) interactions with QMC methods

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r}) + \left[\int d\mathbf{r}'\, V(\mathbf{r},\mathbf{r}')\right]\Psi(\mathbf{r}) = E\Psi(\mathbf{r})$$

- Can examine local "projections" of the interaction. Ex:

$$V_{\text{LP}}(\mathbf{r}) = \int d^3\mathbf{r}'\, V(\mathbf{r},\mathbf{r}')$$

- For a local potential, $V(\mathbf{r},\mathbf{r}') = V(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}')$, such "projections" should be identity:

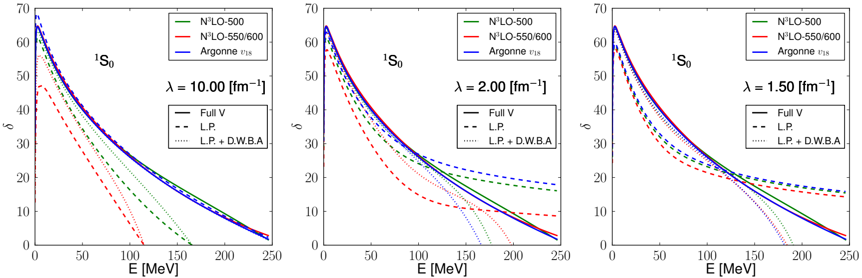
$$V_{\text{LP}}(\mathbf{r}) = \int d^3\mathbf{r}'\, V(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') = V(\mathbf{r})$$

Recent Progress In 2NF Local Projections

A more thorough look at perturbation theory for the local projection.

$$\hat{V}(\lambda) = \hat{V}_{\mathrm{L}}(\lambda) + \hat{V}_{\mathrm{NL}}(\lambda)$$

$$T^{(+)}(p, p; p^2) \approx T^{(+)}_{\mathrm{LP}}(p, p; p^2) + \langle \psi^{(-)}_{\mathrm{LP}}(p) \mid V_{\mathrm{NL}} \mid \psi^{(+)}_{\mathrm{LP}}(p) \rangle$$



KW, R.J. Furnstahl, S. Ramanan (2012)

Recent Progress In 2NF Local Projections

A more thorough look at perturbation theory for the local projection.

$$\hat{V}(\lambda) = \hat{V}_{\text{L}}(\lambda) + \hat{V}_{\text{NL}}(\lambda)$$

- Can generalize tools used for analyzing how perturbative RG interactions are to understand non-local residual.
  S.K. Bogner, R.J. Furnstahl, S. Ramanan, A. Schwenk (2006)
- Weinberg Eigenvalues:

$$\hat{T}(p^2) = \hat{V} + \hat{V}\hat{G}_0(p^2)\hat{T}(p^2)$$
$$= \left(\mathbb{1} + \hat{V}\hat{G}_0(p^2) + \left(\hat{V}\hat{G}_0(p^2)\right)^2 + \left(\hat{V}\hat{G}_0(p^2)\right)^3 + \ldots\right)\hat{V}$$

$$\hat{V}\hat{G}_0(p^2) \mid \Gamma_\nu(p^2)\rangle = \eta_\nu(p^2) \mid \Gamma_\nu(p^2)\rangle$$

$$\hat{T} = \sum_\nu \left(1 + \eta_\nu + \eta_\nu^2 + \eta_\nu^3 + \ldots\right) \mid \Gamma_\nu\rangle\langle\Gamma_\nu \mid \hat{V}$$

Recent Progress In 2NF Local Projections

Can use a two potential formula to generalize WeVs:

$$\hat{T}(p^2) = \hat{T}_L(p^2) + \left(\mathbb{1} + \hat{T}_L(p^2)\hat{G}_0(p^2)\right) \hat{\tilde{T}}_{NL}(p^2) \left(\hat{G}_0(p^2)\hat{T}_L(p^2) + \mathbb{1}\right)$$

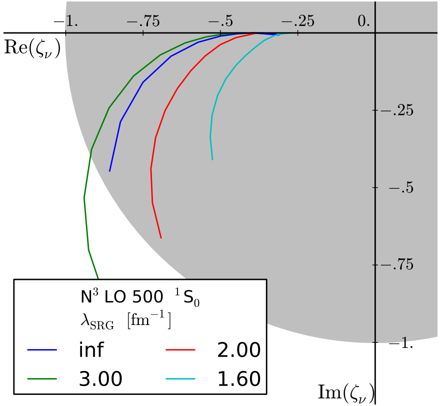
$$\begin{aligned}\hat{\tilde{T}}_{NL}(p^2) &= \hat{V}_{NL} + \hat{V}_{NL}\hat{G}_L(p^2)\hat{\tilde{T}}_{NL}(p^2) \\ &= \left(\mathbb{1} + \hat{V}_{NL}\hat{G}_L(p^2) + \hat{V}_{NL}\hat{G}_L(p^2)\hat{V}_{NL}\hat{G}_L(p^2) + \ldots\right)\hat{V}_{NL}\end{aligned}$$

$$\hat{G}_L(p^2) = \hat{G}_0(p^2) + \hat{G}_0(p^2)\hat{T}_L(p^2)\hat{G}_0(p^2)$$

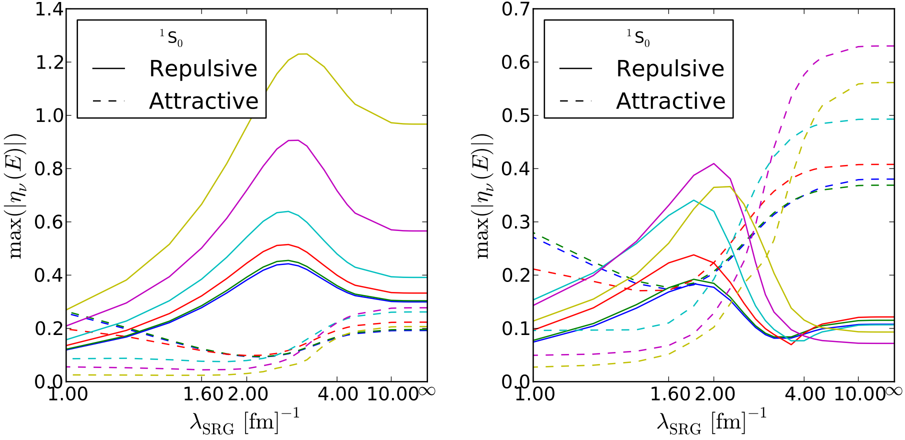
$$\hat{V}_{NL}\hat{G}_L(p^2) \mid \Sigma\nu(p^2)\rangle = \zeta_\nu(p^2) \mid \Sigma\nu(p^2)\rangle$$

$$\hat{\tilde{T}}_{NL} = \sum_\nu \left(1 + \zeta_\nu + {\zeta_\nu}^2 + {\zeta_\nu}^3 + \ldots\right) \mid \Sigma_\nu\rangle\langle\Sigma_\nu \mid \hat{V}_{NL}$$

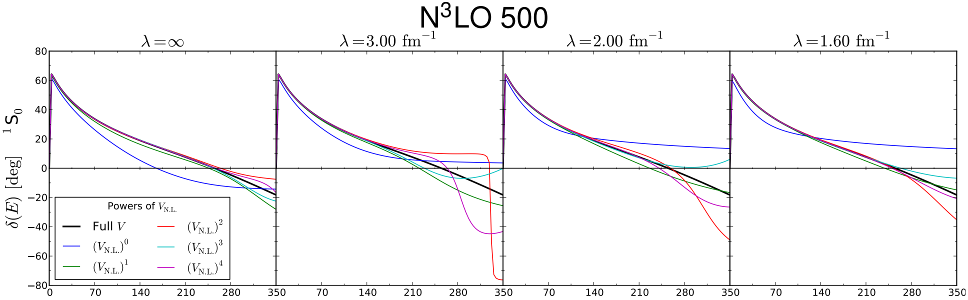
Recent Progress In 2NF Local Projections



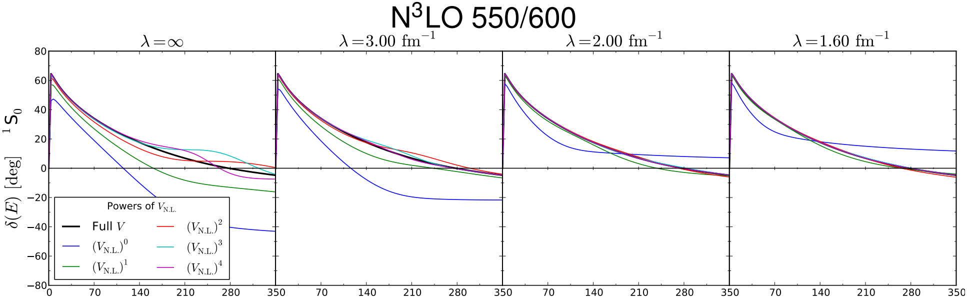
Recent Progress In 2NF Local Projections



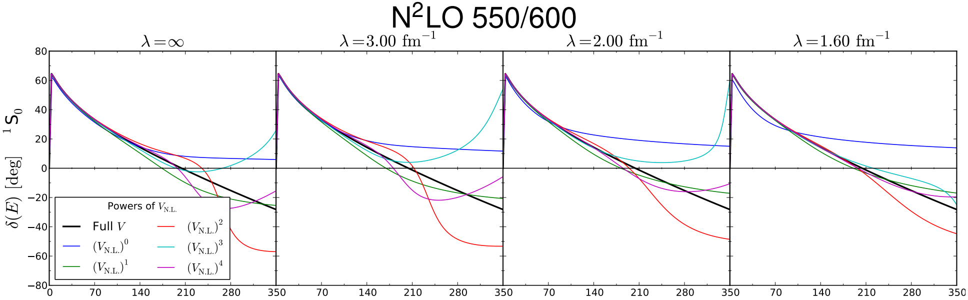
Recent Progress In 2NF Local Projections



Recent Progress In 2NF Local Projections



Recent Progress In 2NF Local Projections



Summary 2

- Local projection residuals are perturbative for $\hat{T}_{\mathrm{rel}}$-SRG interactions
  - This was demonstrated using DWBA
  - Now done with DW Weinberg eigenvalues
    - More robust, shows how high in energy the residuals are perturbative (well past scales where EFT potentials fail).
    - Only a few "large" eigenvalues $\longrightarrow$ treat residual in UPA.