Theoretical developments in the studies of deuteron induced reactions

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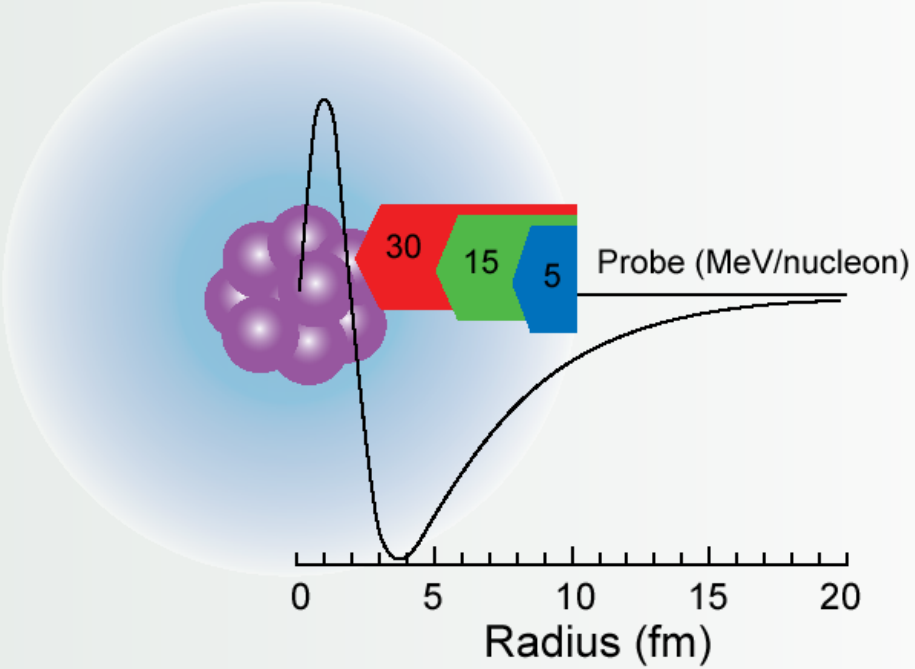
Probing structure through (d,p)

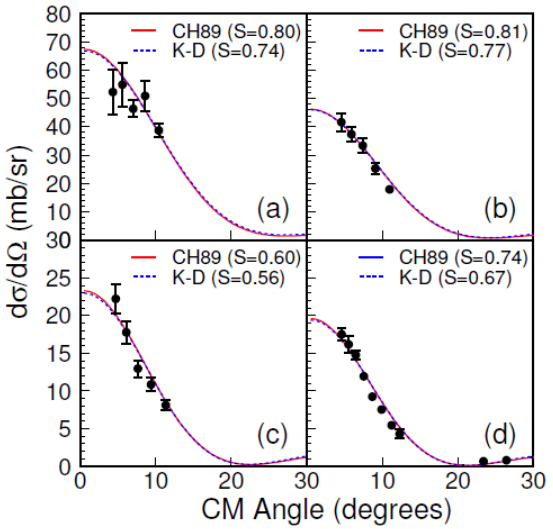
- A(d,p)B reactions probe the overlap function of final nucleus
- angular distributions provide angular momentum of final state
- different beam energies probe different regions of space



The overlap function for $^{19}C \rightarrow n+^{18}C$ in arbitrary units. The radial sensitivity of the $^{18}C(d,p)^{19}C$ cross section is represented by the colored bars for different beam energies.

Example of using (d,p) to probe halos

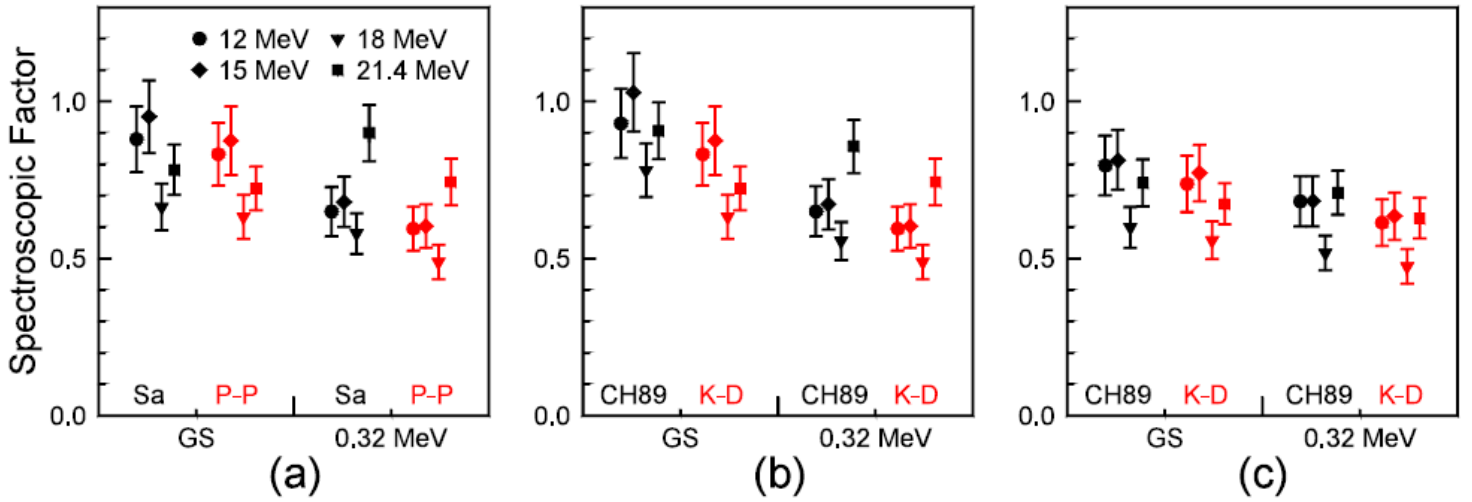
^{10}Be(d,p)^{11}Be @ 12-21 MeV



DWBA
entrance channel

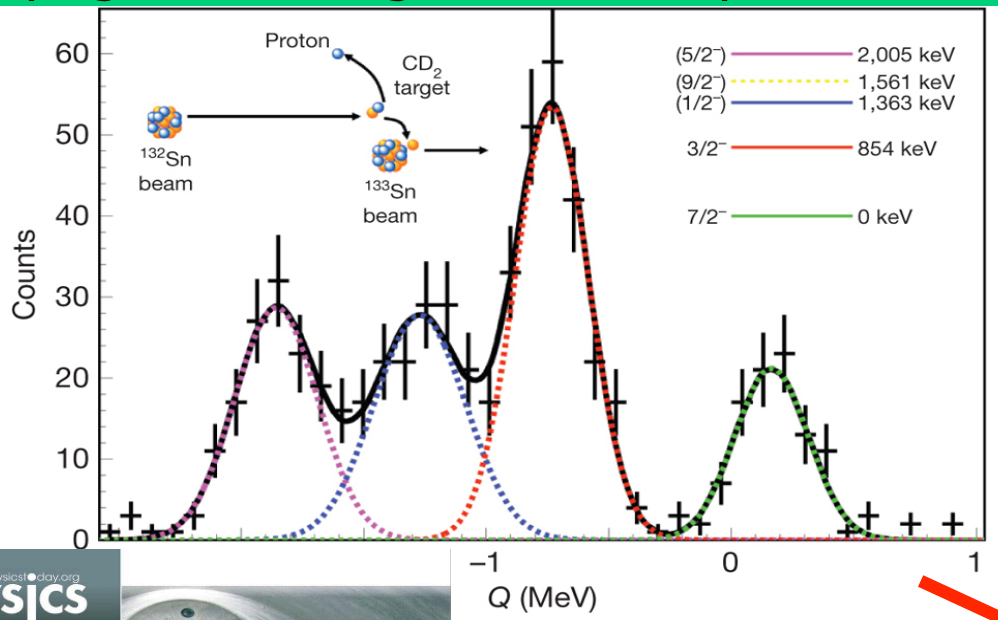
DWBA
exit channel

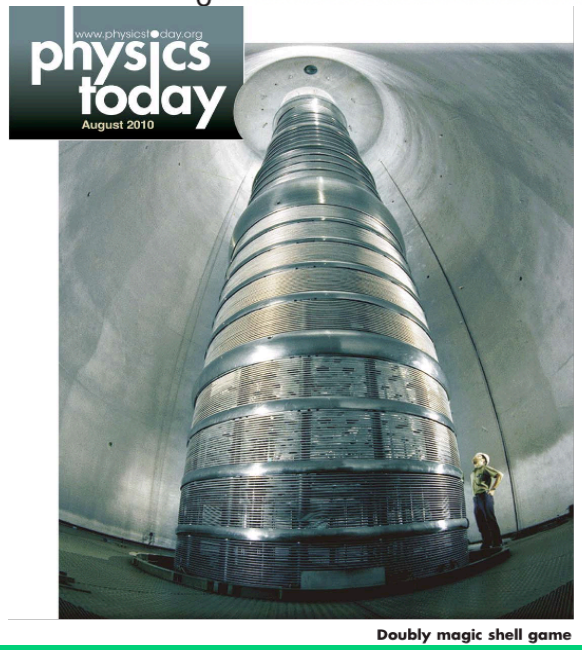
ADWA



Schmitt et al, PRL 108, 192701 (2012), PRC 88, 064612 (2013)

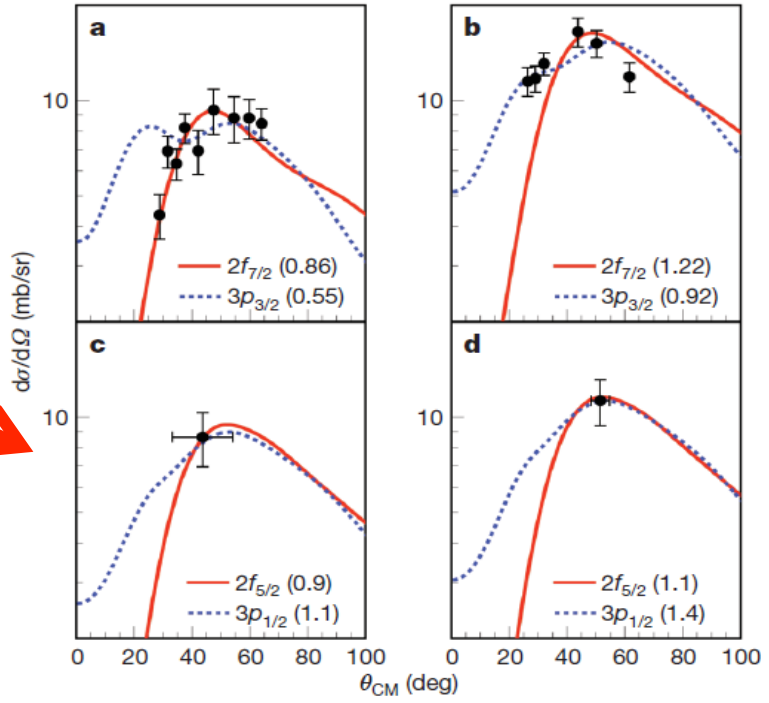
Example of using (d,p) as a probe of magicity:
studying double magic nuclei away from stability





Doubly magic shell game

d(^{132}Sn,^{133}Sn)p@5 MeV/u



What is the error bar from the reaction model?

K. Jones et al, Nature 465 (2010) 454, PRC 84, 034601 (2011)

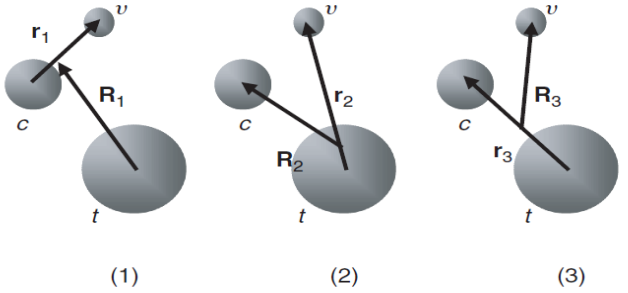
overview

1) benchmarking reaction theories
2) Faddeev AGS including Coulomb without screening
3) non-locality in reactions

differences between three-body methods

ADWA:
- only one Jacobi component
- elastic and breakup fully coupled (no rearrangement)
- adiabatic approximation for breakup
- only applicable to obtain transfer cross sections
- runs on desktop – practical

Johnson and Tandy NP (1974)



3 jacobi coordinate sets

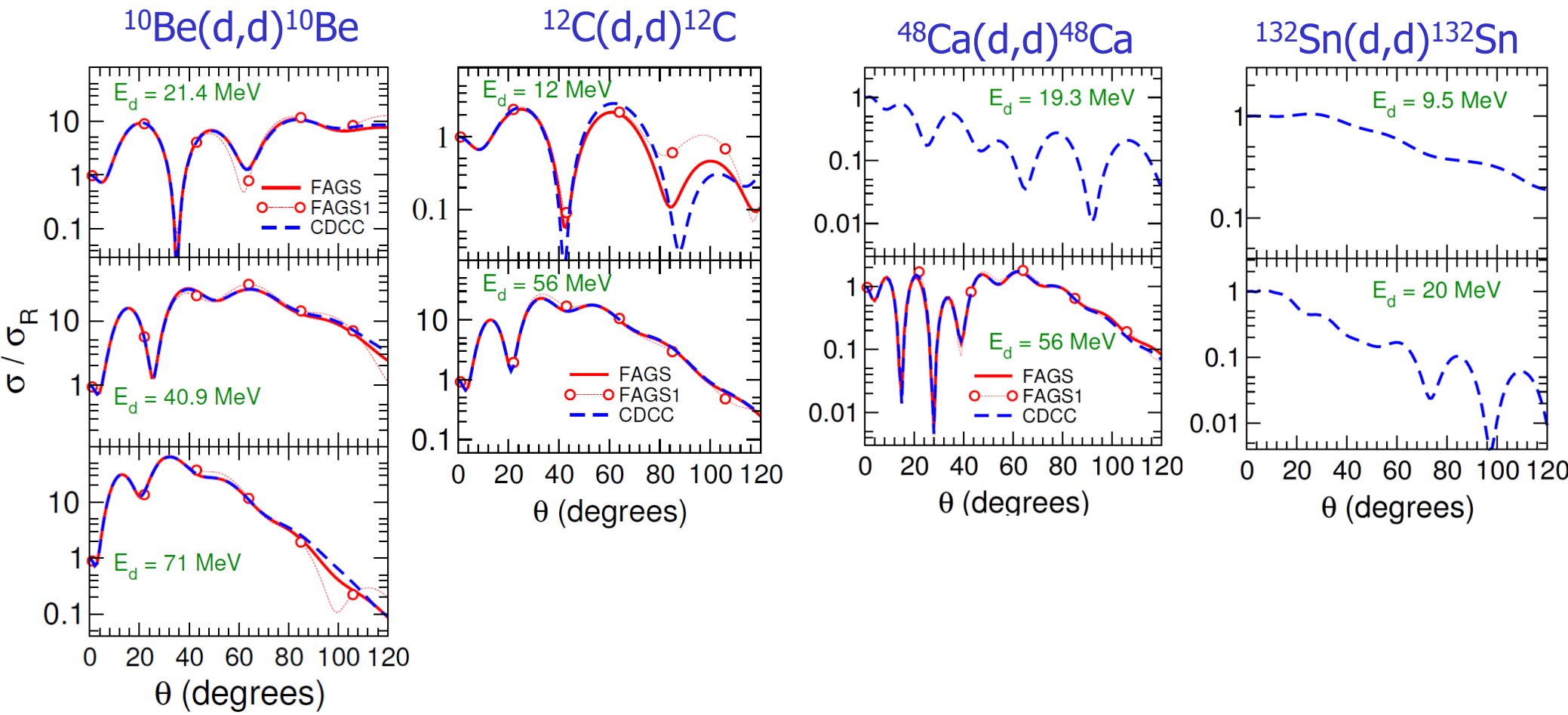
CDCC:
- only one Jacobi component
- elastic and breakup fully coupled (no rearrangement)
- computationally expensive

Austern, Kamimura, Rawistcher, Yahiro etc, Prog. Theo. Phys (1986)

Faddeev AGS: **EXACT**
- all three Jacobi components are included
- elastic, breakup and rearrangement channels are fully coupled
- computationally expensive

Deltuva and Fonseca, Phys. Rev. C79, 014606 (2009).

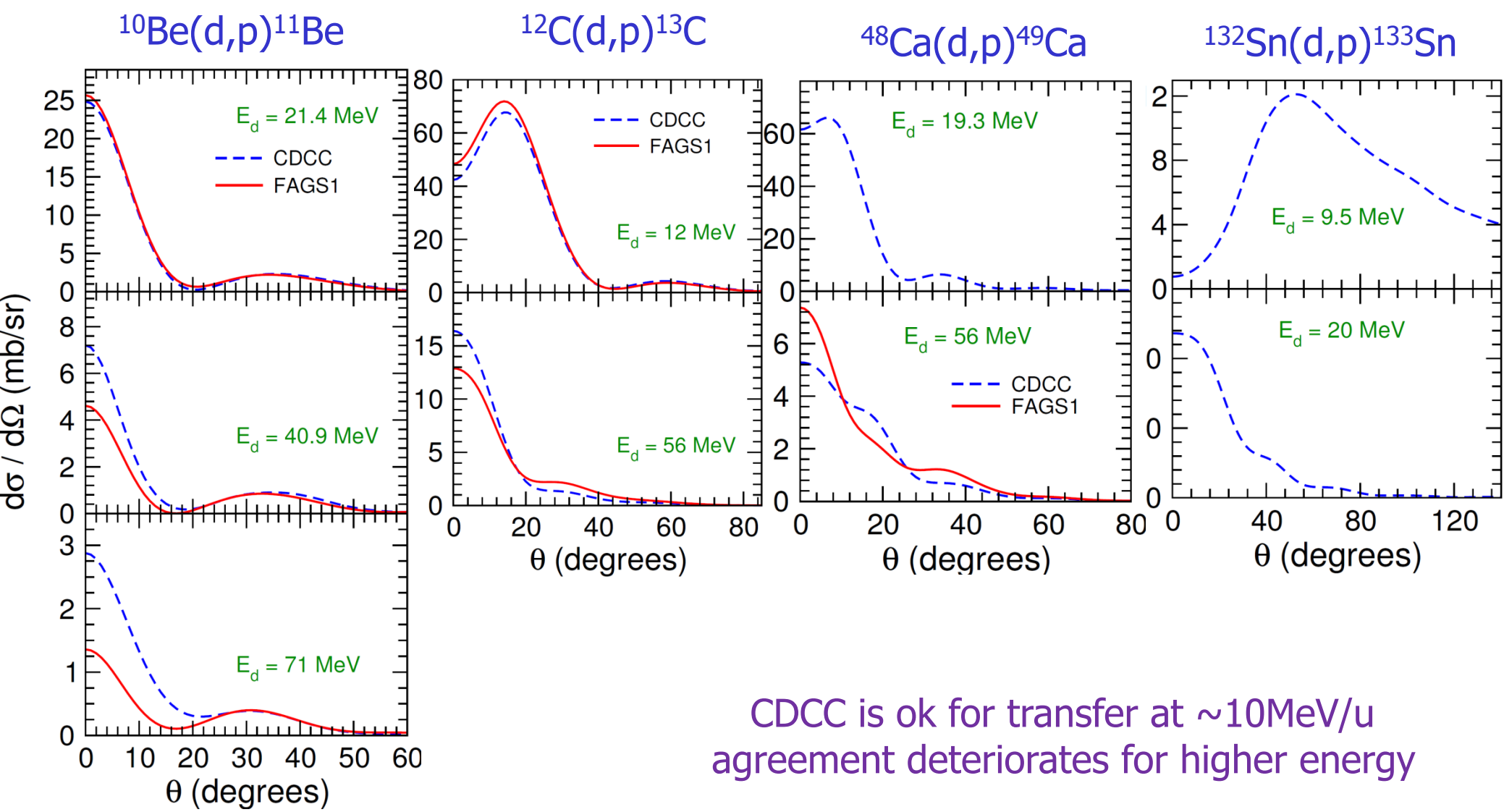
Comparing elastic scattering



CDCC provides a good approximation for elastic scattering

Upadhyay, Deltuva and Nunes, PRC 85, 054621 (2012)

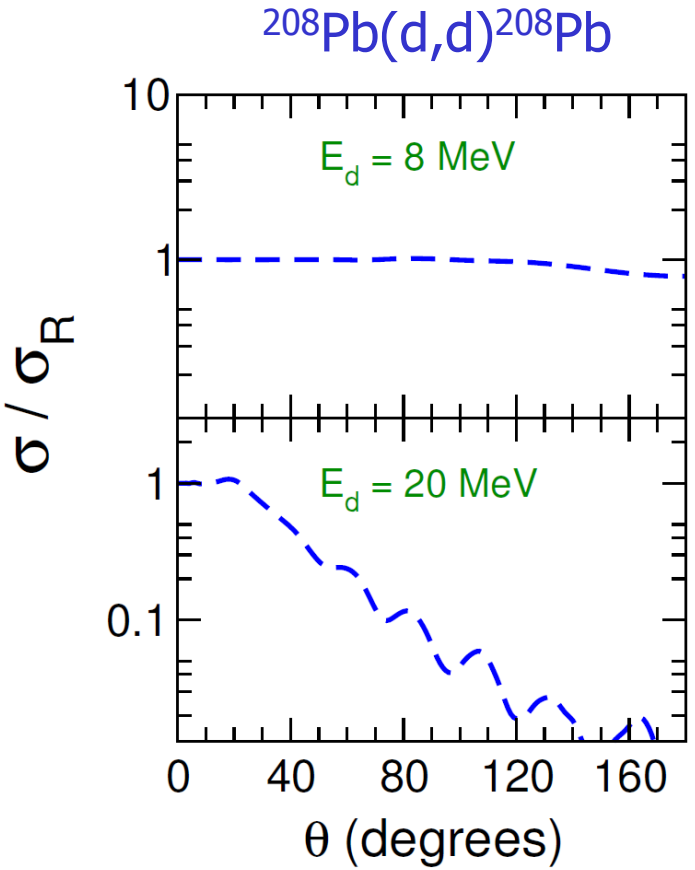
Comparing transfer

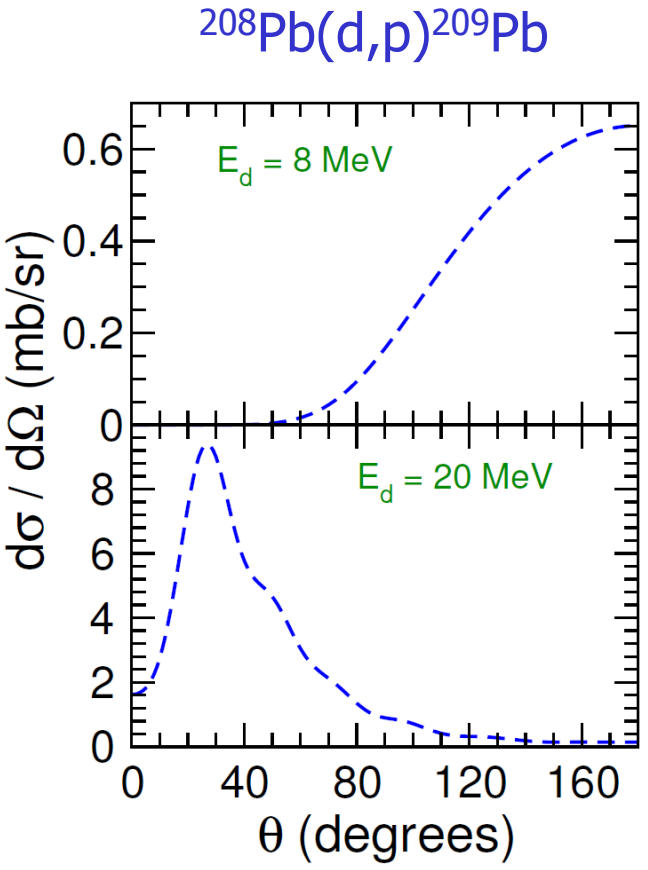


CDCC is ok for transfer at ~10MeV/u
agreement deteriorates for higher energy

Upadhyay, Deltuva and Nunes, PRC 85, 054621 (2012)

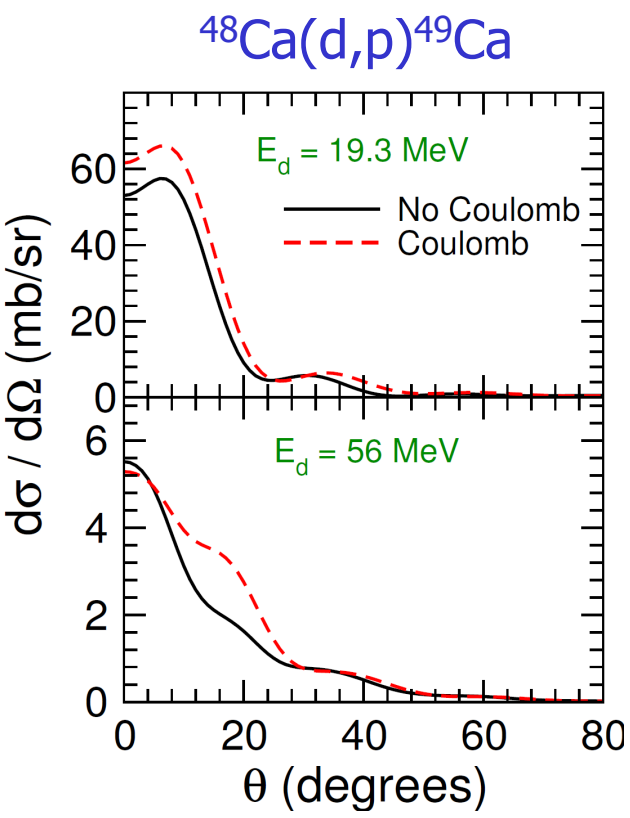
CDCC results for Pb

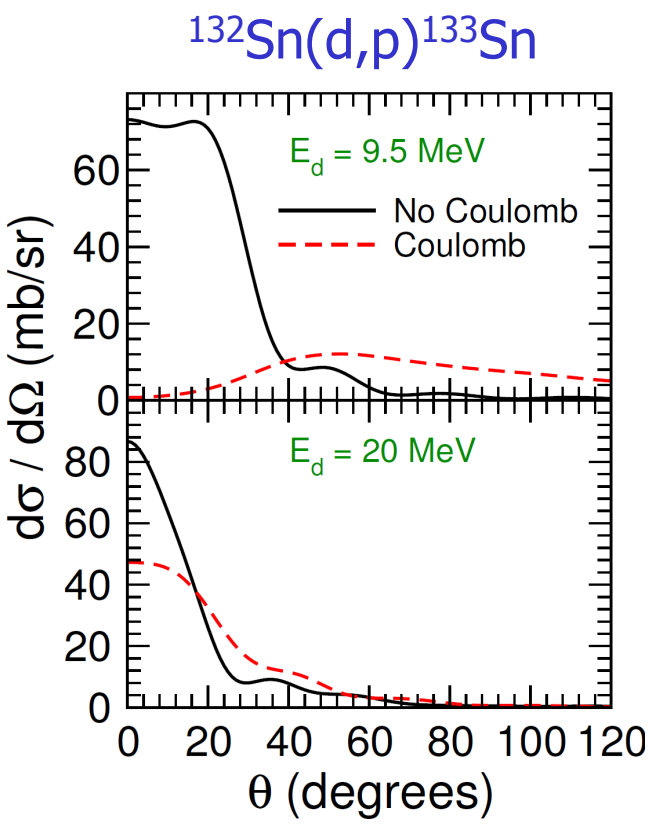


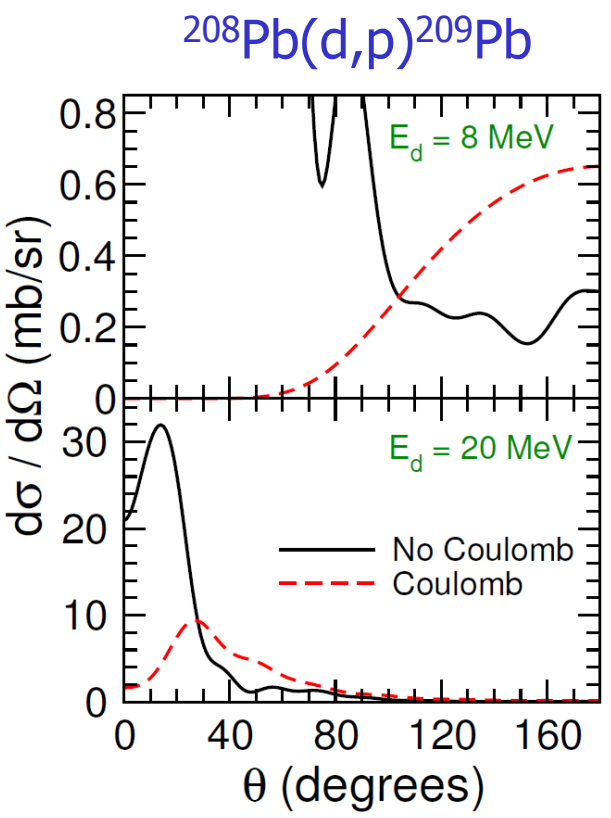


No comparison with Faddeev possible!

The effects of Coulomb on transfer

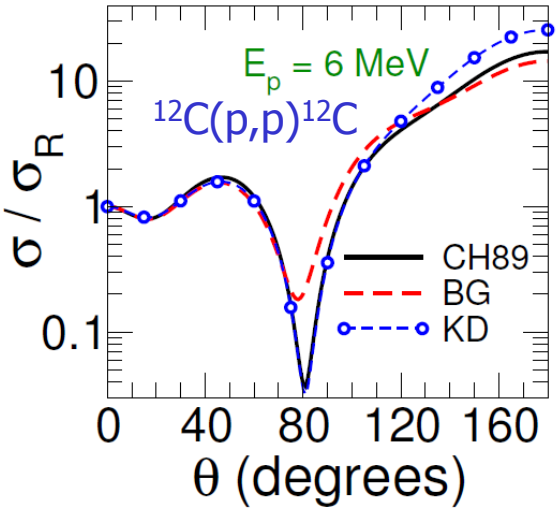


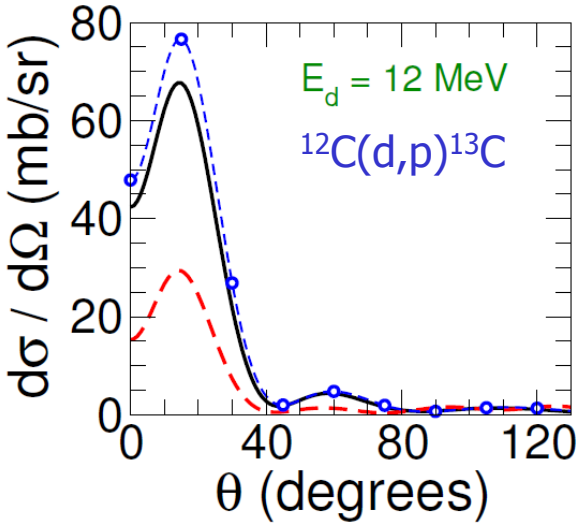


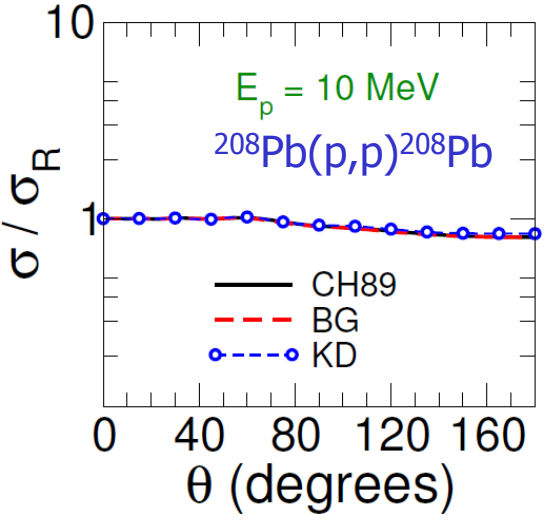


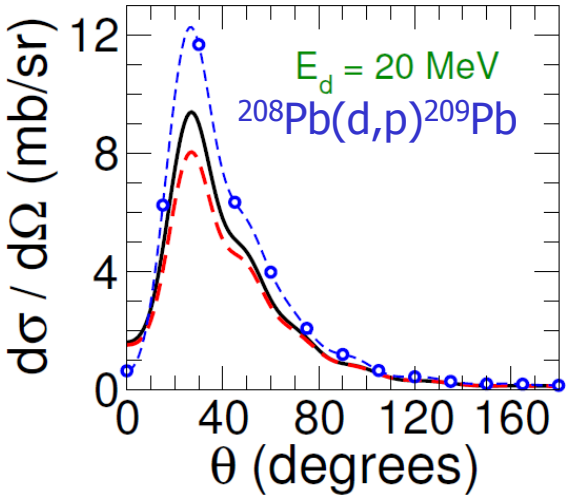
Coulomb effects on transfer can be very large…
New method needs to accurately include Coulomb!

The dependence on the optical potential









- Constraining p-A elastic reduces uncertainties but remaining uncertainty not neglegible
- Important to include good optical potential information

overview

1) benchmarking reaction theories
2) Faddeev AGS including Coulomb without screening
3) non-locality in reactions

The three-body d+A problem with Coulomb

Faddeev AGS with screened Coulomb Deltuva et al., PRC71, 054004

- equations written in the plane wave basis
- screening radius increases with increasing Z target
- larger number of partial waves needed for convergence
- integral equation solvers break down

Faddeev AGS including unscreened Coulomb

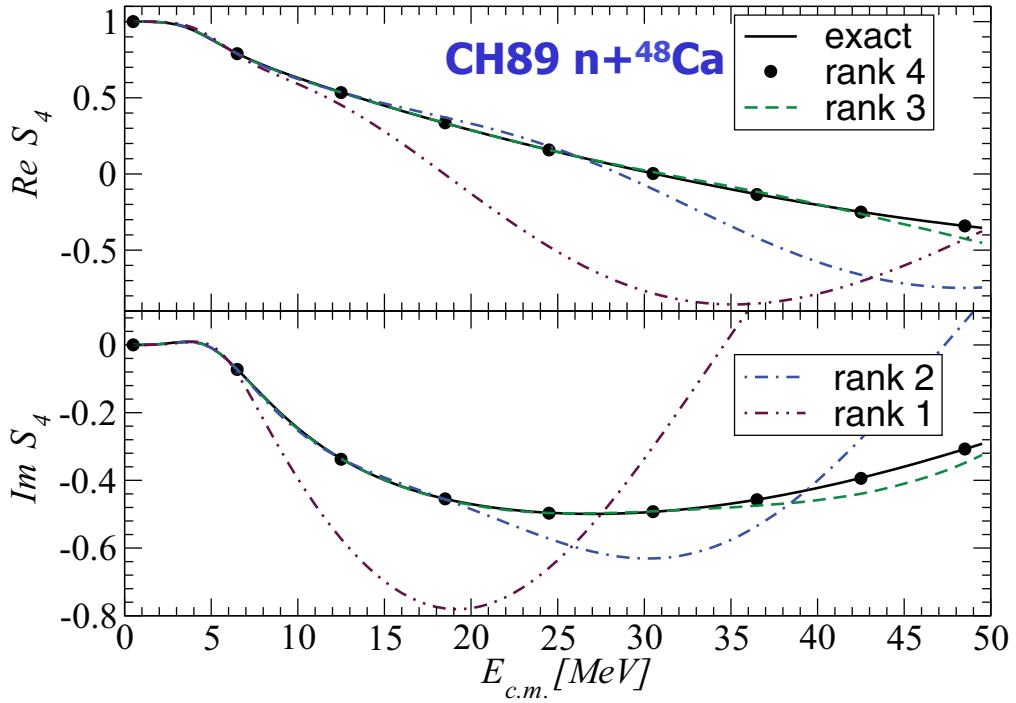
Mukhamedzhanov et al., PRC86, 034001

- equations written in the momentum space Coulomb distorted basis
- no screening of interactions
- assumes interactions are separable
- challenge to calculate the Coulomb distorted nuclear form factors

Three-body including Coulomb without screening

TORUS collaboration

1) Determine separable form for the interactions (optical potentials!)
2) Compute the Coulomb distorted nuclear form factors
3) Solve the corresponding AGS equations



Separable form for the t-matrix

$$t(E) = \sum_{i,j} u\left|f_{l,k_{E_i}}\right\rangle \tau_{ij}(E) \left\langle f^*_{l,k_{E_j}}\right| u$$

Hlophe et al., PRC 88, 064608 (2013)

Coulomb distorted nuclear form factors

The Coulomb distorted basis:

nuclear form factor

$$u_l^C(p) = \int_0^\infty \frac{dq\ q^2}{2\pi^2}\ u_l(q)\ (\psi_{l,p}^C)^\star(q)$$

Momentum space Coulomb wfn

$$\psi_{l,p}^C(q) = -\frac{2\pi\, e^{\eta\pi/2}}{pq} \lim_{\gamma\to+0} \frac{d}{d\gamma}\left\{\left[\frac{q^2-(p+i\gamma)^2}{2pq}\right]^{i\eta} (\zeta^2-1)^{-i\frac{\eta}{2}}\, Q_l^{i\eta}(\zeta)\right\}$$

$$Q_l^{i\eta}(\zeta) = \frac{e^{-\pi\eta}}{2}\left\{\Gamma(i\eta)\left(\frac{\zeta+1}{\zeta-1}\right)^{\frac{i\eta}{2}} {}_2F_1\left(-l, l+1; 1-i\eta; \frac{1-\zeta}{2}\right)\right.$$

$$\left. +\Gamma(-i\eta)\frac{\Gamma(l+1+i\eta)}{\Gamma(l+1-i\eta)}\left(\frac{\zeta-1}{\zeta+1}\right)^{\frac{i\eta}{2}} {}_2F_1\left(-l, l+1; 1+i\eta; \frac{1-\zeta}{2}\right)\right\}$$

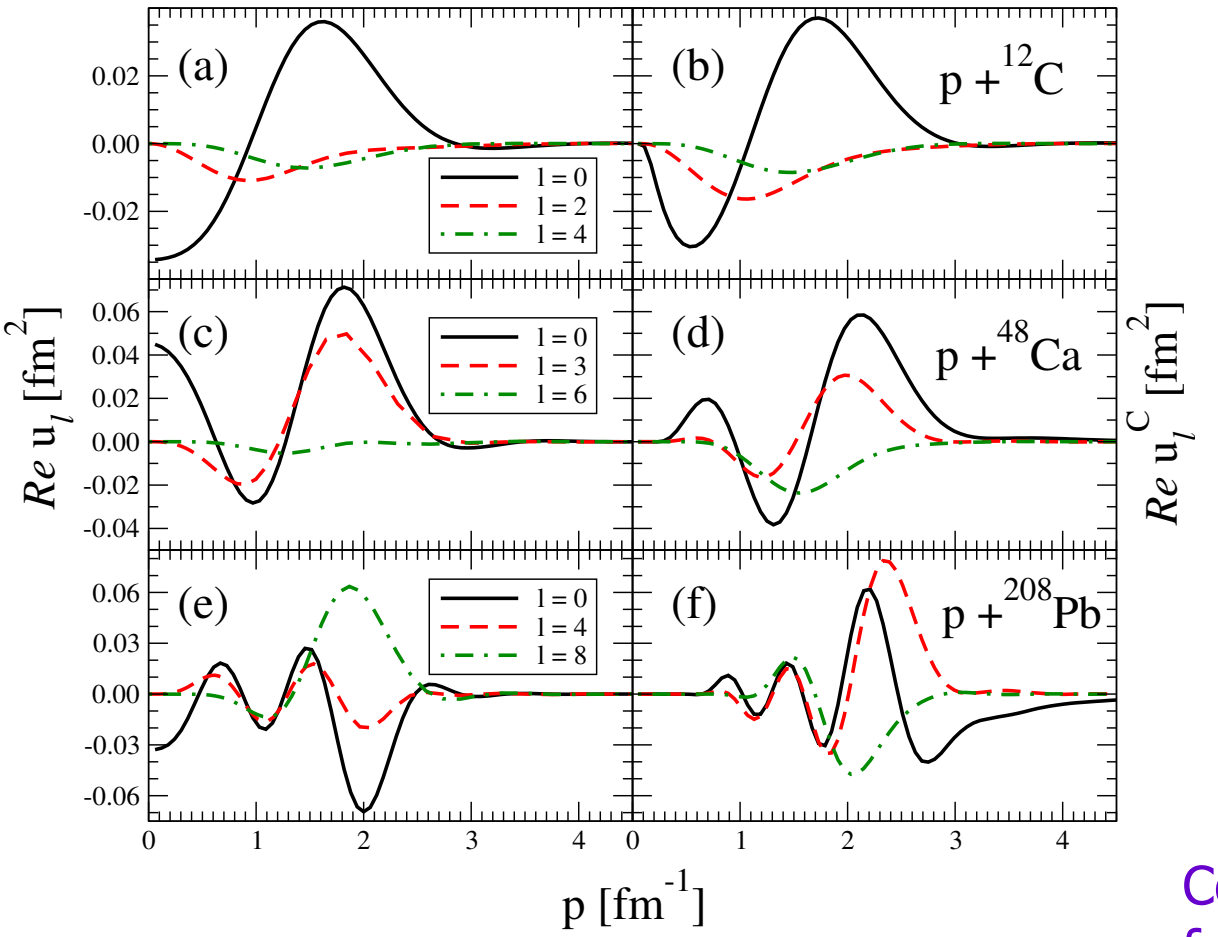
$$\zeta = (p^2+q^2)/2pq$$

Oscillating singularity at p=q

$$S(p-q) = \lim_{\gamma\to+0} \frac{1}{(p-q+i\gamma)^{1+i\eta}}$$

Nuclear form factors with and without Coulomb

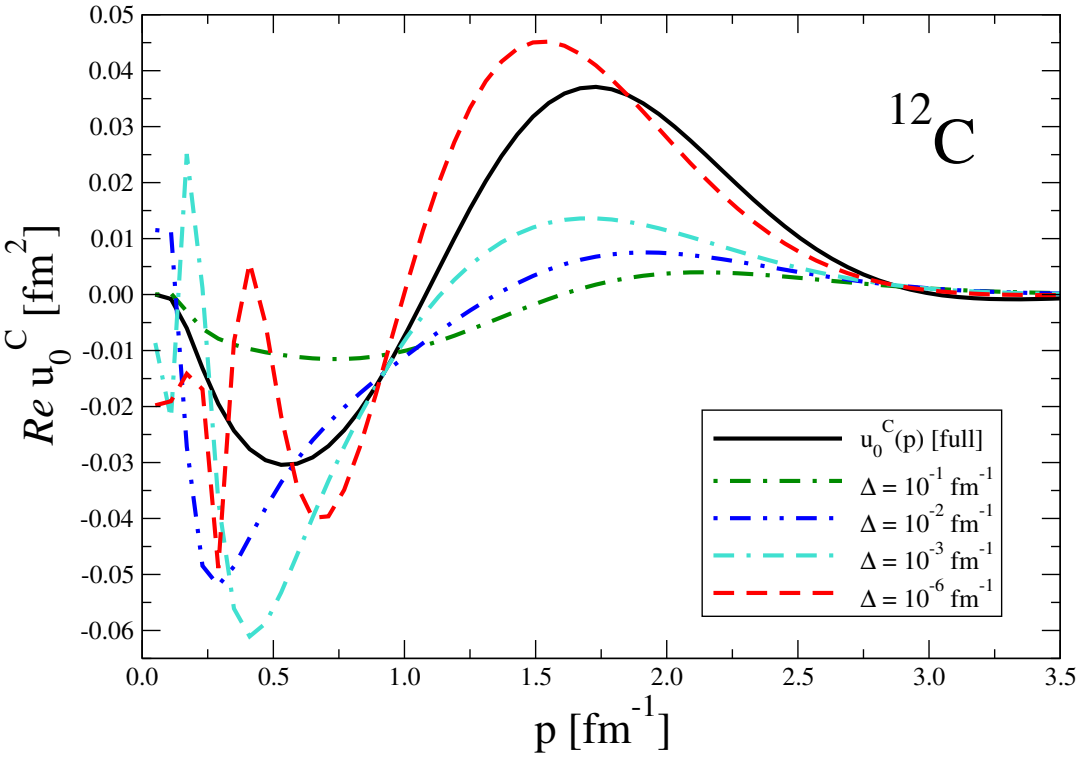
TORUS collaboration



Coulomb form factor shifted away from k=0 but still die off fast; less extended than Yamaguchi

Coulomb distorted nuclear form factors: the pole

TORUS collaboration



The treatment of the pole is critical
Regularization procedure developed by Gel'fand and Shilov

overview

1) benchmarking reaction theories
2) Faddeev AGS including Coulomb without screening
3) non-locality in reactions

Non-local potential?

- Phenomenological optical potentials are usually made local
- microscopically derived optical potentials are non-local
    - Does non-locality make a difference in the reaction?
    - Can we constrain non-locality with reactions?

Non-local potential: what we did

Solve the single channel scattering problem with non-local optical potential
Solve the single channel bound state problem with non-local mean field

$$\frac{\hbar^2}{2\mu}\nabla^2\Psi(\mathbf{r})+E\Psi(\mathbf{r})=U_o(\mathbf{r})\Psi(\mathbf{r})+\int U^{NL}(\mathbf{r},\mathbf{r}')\Psi(\mathbf{r}')d\mathbf{r}'$$

Construct the (d,p) matrix element within DWBA

Perey and Buck type non-locality

$$U^{NL}(\mathbf{r},\mathbf{r}')=U_{WS}^{NL}\left(\left|\frac{\mathbf{r}+\mathbf{r}'}{2}\right|\right)\frac{\exp\left(-\left|\frac{\mathbf{r}-\mathbf{r}'}{\beta}\right|^2\right)}{\pi^{3/2}\beta^3}$$

F. Perey and B. Buck, Nucl. Phys. 32, 353 (1962).

If the local momentum approximation is valid

$$\psi_\ell^{PCF}(r)=F(r)\psi_\ell^{Loc}$$

$$F(r)=\left[1-\frac{\mu\beta^2}{2\hbar^2}\left(U^{LE}(r)-U_o(r)\right)\right]^{-1/2}$$

N. Austern, Phys. Rev. 137, 752 (1965)

Non-local potentials: effect on (p,d)

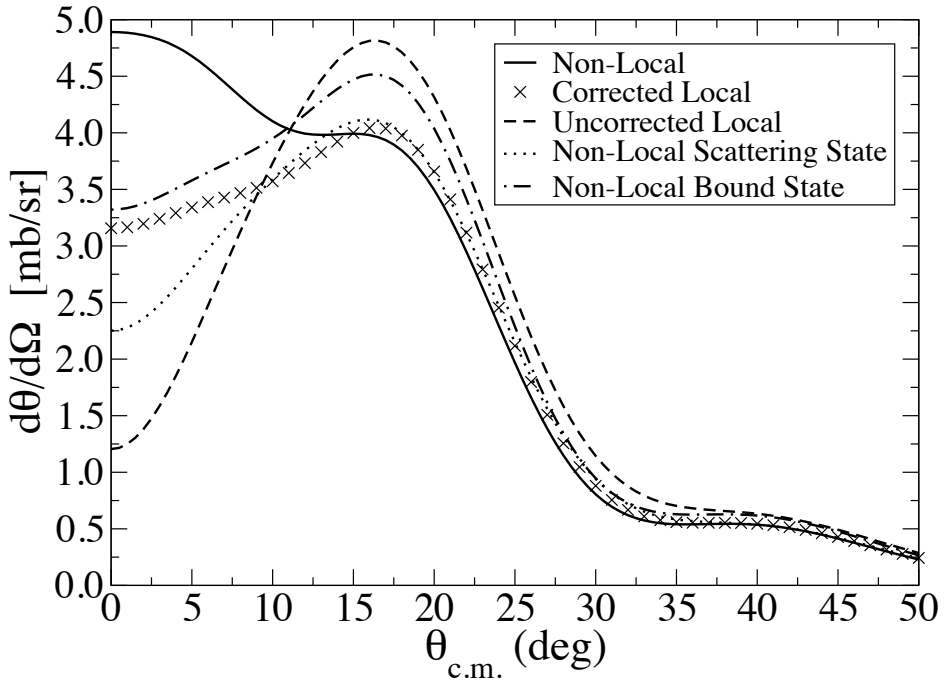


FIG. 5: Angular distributions for ^{49}Ca$(p,d)^{48}$Ca at 50.0 MeV: inclusion of non-locality in both the proton distorted wave and the neutron bound state (solid line), using LEP, then applying the correction factor to both the scattering and bound states (crosses), using the LEP without applying any corrections (dashed line); including non-locality only to the proton distorted wave (dotted line), and including non-locality in the neutron bound state only (dot-dashed line).

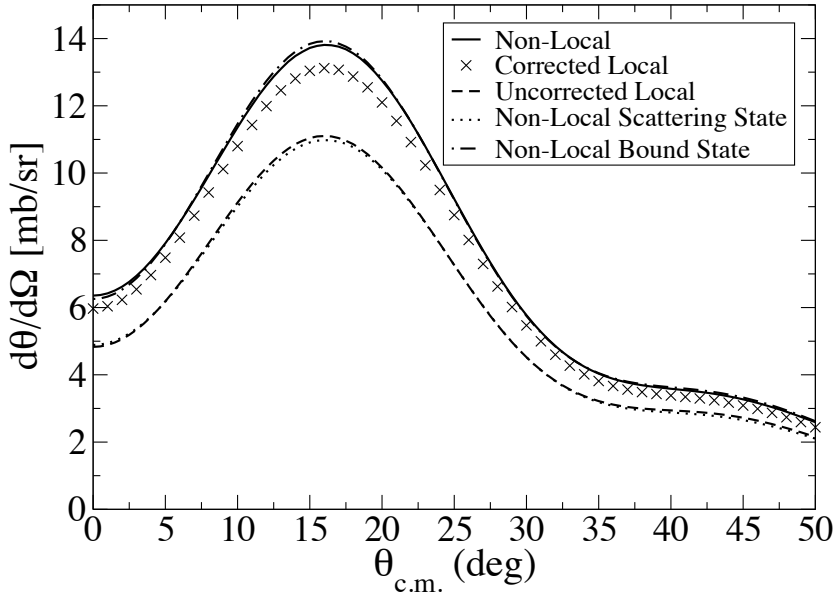


FIG. 7: Angular distributions for ^{133}Sn$(p,d)^{132}$Sn at 20.0 MeV (descriptions of each line is given in the caption of Fig.5).

Titus and Nunes, PRC submitted

Non-local potentials: effect in (p,d) reactions

$E_{lab} = 20$ MeV	Corrected Relative to Local	Non-Local Relative to Local
$^{17}\mathrm{O}(1d_{5/2})(p,d)$	7.1%	18.8%
$^{17}\mathrm{O}(2s_{1/2})(p,d)$	20.1%	26.5%
$^{41}\mathrm{Ca}(p,d)$	11.4%	21.9%
$^{49}\mathrm{Ca}(p,d)$	10.4%	17.3%
$^{127}\mathrm{Sn}(p,d)$	17.5%	17.3%
$^{133}\mathrm{Sn}(p,d)$	18.2%	24.4%
$^{209}\mathrm{Pb}(p,d)$	19.4%	20.8%

$E_{lab} = 50$ MeV	Corrected Relative to Local	Non-Local Relative to Local
$^{17}\mathrm{O}(1d_{5/2})(p,d)$	17.0%	35.4%
$^{17}\mathrm{O}(2s_{1/2})(p,d)$	0.2%	12.7%
$^{41}\mathrm{Ca}(p,d)$	2.9%	5.8%
$^{49}\mathrm{Ca}(p,d)$	−16.0%	−17.1%
$^{127}\mathrm{Sn}(p,d)$	10.1%	4.5%
$^{133}\mathrm{Sn}(p,d)$	−6.7%	−16.9%
$^{209}\mathrm{Pb}(p,d)$	8.6%	8.6%

Titus and Nunes, PRC submitted

Summary and Outlook

- Comparisons CDCC and ADWA versus Faddeev
  - strong disagreement for transfer and breakup
  - current implementation of Faddeev AGS limited to Z ~ 20
    - need better approach

- Faddeev AGS in the Coulomb distorted basis
  - separable forms for optical potentials was developed
  - Coulomb distorted nuclear form factors are now implemented
  - next: implement the corresponding AGS equations

- Impact of non-locality in nuclear reactions
  - DWBA tests using Perey and Buck show strong sensitivity to non-locality (up to 30% change in cross section)
  - need to upgrade best reaction theories to handle non-local interactions
  - use state-of-the-art ab-initio methods with correlations to derive non-local optical potentials

thankyou!

Collaborators:

Collaborators from TORUS:
Neelam Upadhyay (MSU, now at LSU)
Charlotte Elster, Linda Hlophe, Vasily Eremenko (Ohio),
Ian Thompson and Jutta Escher (LLNL)
Goran Arbanas (ORNL)

Arnas Deltuva (Lisbon)

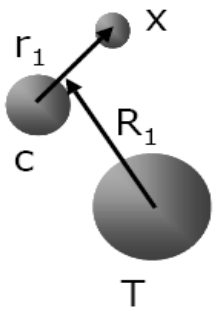
Luke Titus (MSU)

Kate Jones and Kyle Schmitt (Univ. Tennessee)

supported by NSF, DOE-NT, NNSA

backup

reaction methods: CDCC versus Faddeev formalism

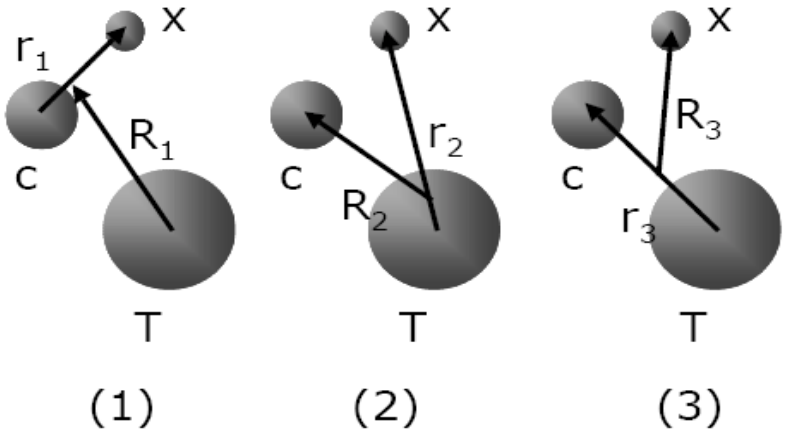


CDCC Formalism

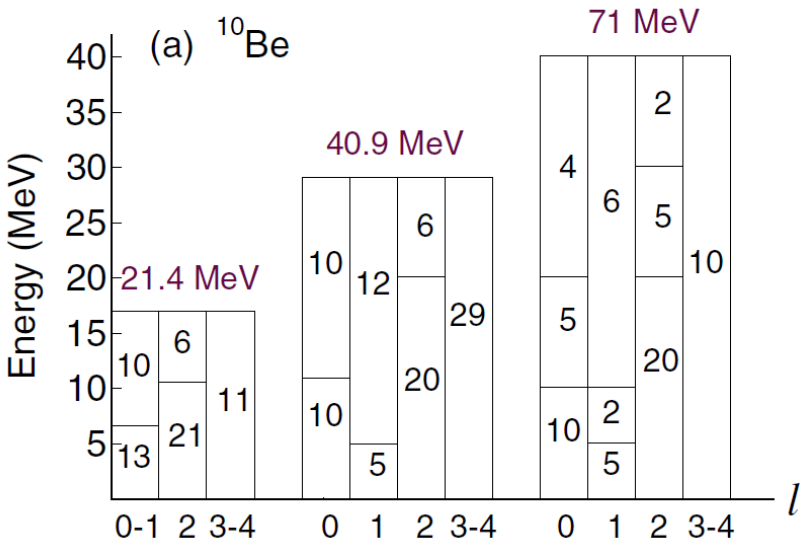
$$[H_{3\mathrm{b}} - E]\Psi^{(1)}(\mathbf{r}_1, \mathbf{R}_1) = 0$$

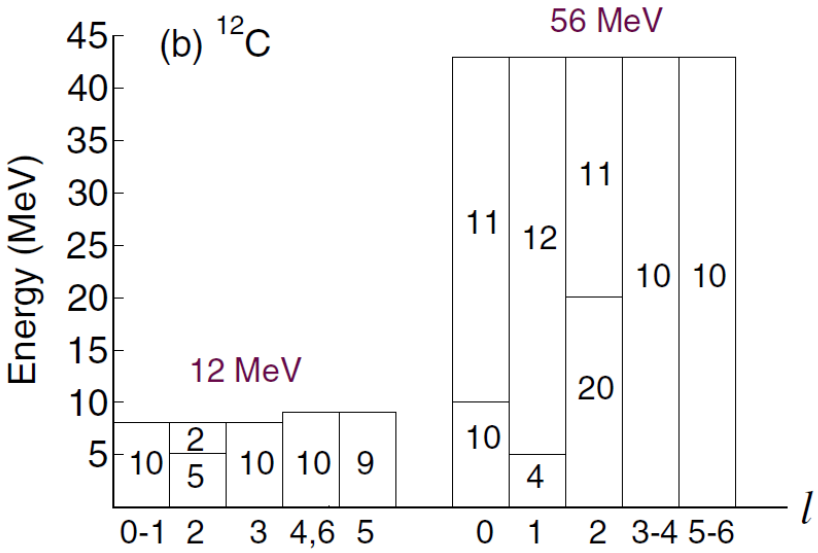
Faddeev Formalism

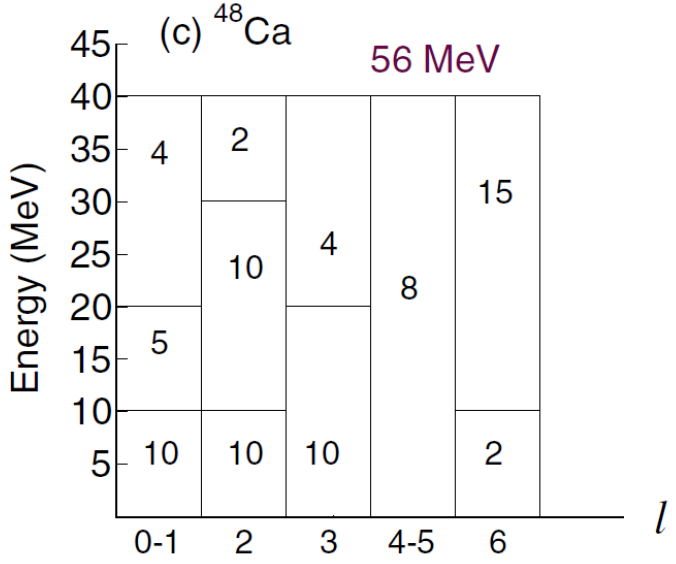
$$\begin{aligned}
(E - T_1 - V_{xc})\Psi^{(1)} &= V_{xc}(\Psi^{(2)} + \Psi^{(3)}) \\
(E - T_2 - V_{ct})\Psi^{(2)} &= V_{ct}(\Psi^{(3)} + \Psi^{(1)}) \\
(E - T_3 - V_{tx})\Psi^{(3)} &= V_{tx}(\Psi^{(1)} + \Psi^{(2)})
\end{aligned}$$



CDCC model space







Upadhyay, Deltuva and Nunes, PRC 85, 054621 (2012)

Faddeev calculations: details

Label	$U_{\rm pA}$	$U_{\rm nA}$	nA-bound
FAGS	$E_d/2$	$E_d/2$	no
FAGS1	$E_d/2$	$E_d/2$	yes
FAGS2	E_p	$E_d/2$	yes

TABLE III: Types of Faddeev-AGS calculations being performed, the labels used, the energies at which the associated interactions were determined and whether a neutron-nucleus potential supports a bound state.

Upadhyay, Deltuva and Nunes, PRC 85, 054621 (2012)

Sensitivity to interactions

At low energies, L dependence of NN interaction important
At high energies, spin-orbit in optical potential important

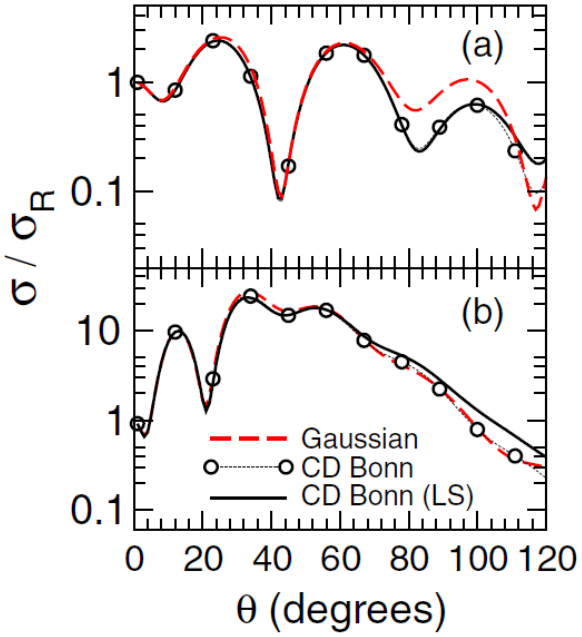


FIG. 11: Elastic distributions for FAGS1 calculations for ^{12}C (d, d) ^{12}C reaction at: (a)E_d = 12 MeV and (b) E_d = 56 MeV.

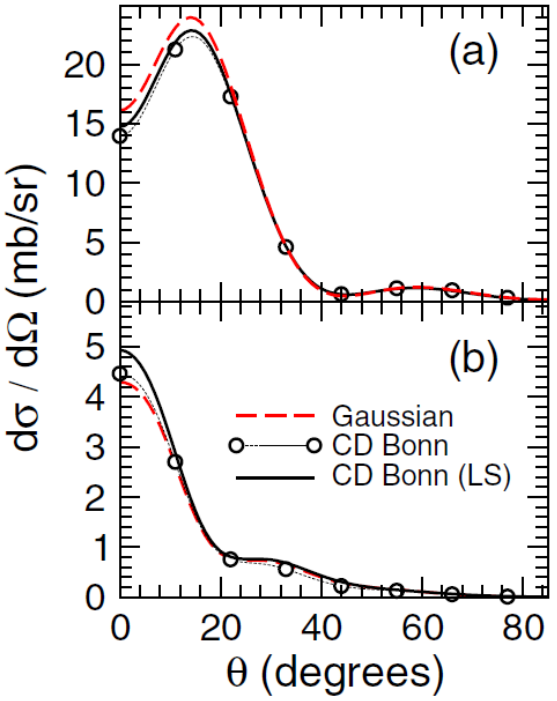
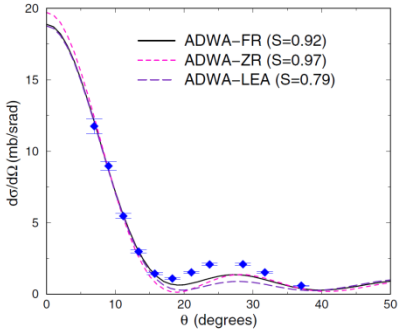
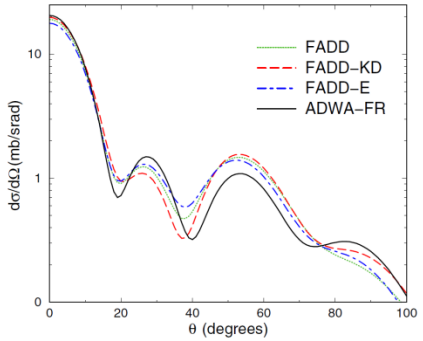


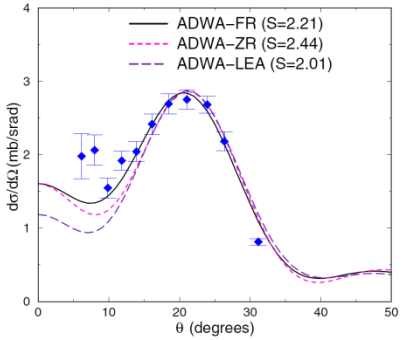
FIG. 12: Transfer angular distributions for FAGS1 calculations for ^{12}C (d, p) ^{13}C reaction at: (a) E_d = 12 MeV and (b) E_d = 56 MeV.

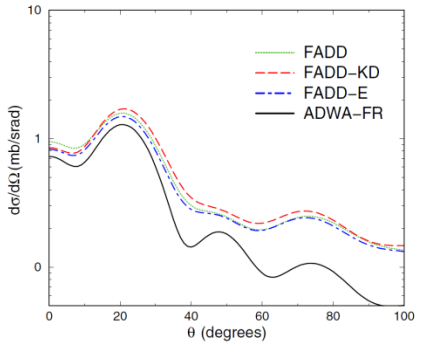
Upadhyay, Deltuva and Nunes, PRC 85, 054621 (2012)

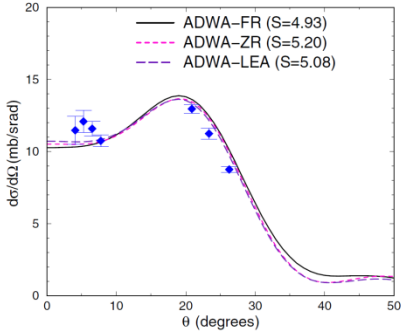
transfer data for Ar isotopes











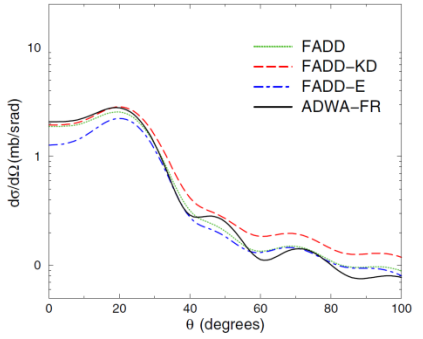


FIG. 1: (Color online) Angular distributions for: (a) ^{34}Ar(p,d)^{33}Ar(g.s.) E_p = 33 MeV, (b) ^{36}Ar(p,d)^{35}Ar(g.s.) E_p = 33 MeV and (c) ^{46}Ar(p,d)^{45}Ar(g.s.) E_p = 33 MeV. Comparison of full finite range (solid) with the zero-range approximation (dashed), and the local energy approximation (long-dashed). All distributions have been multiplied to scale the data by the indicated spectroscopic factor S.

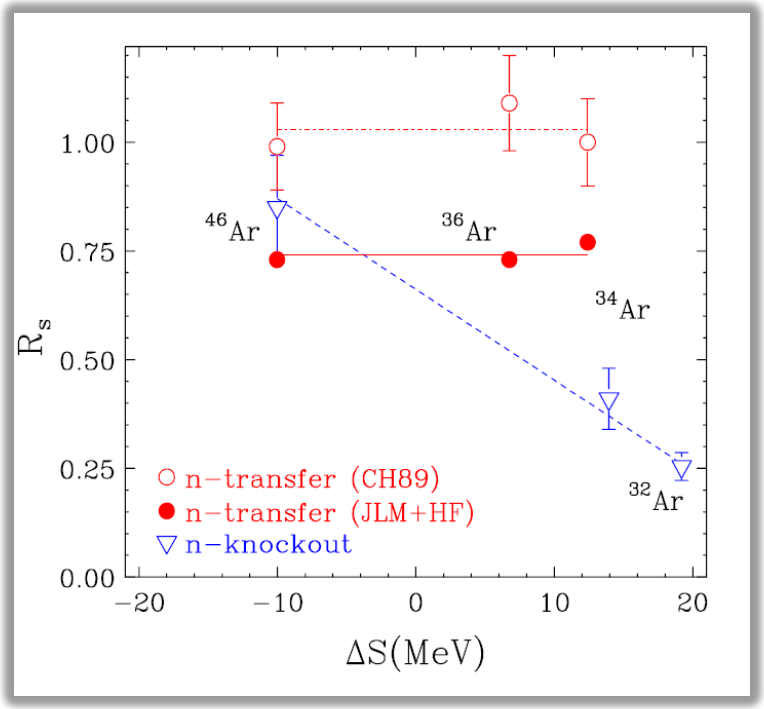
3. 2: (Color online) Angular distributions for: (a) Ar(p,d)^{33}Ar(g.s.) E_p = 33 MeV, (b) ^{36}Ar(p,d)^{35}Ar(g.s.) = 33 MeV and (c) ^{46}Ar(p,d)^{45}Ar(g.s.) E_p = 33 MeV. ll three-body Faddeev calculations (FADD) with the finite ige adiabatic model (ADWA-FR). More detail in the text.

- finite range adiabatic methods are used to obtained spectroscopic factors
- Faddeev calculations are used to determined error in reaction theory

Errors	$\epsilon_{th}(^{34}\mathrm{Ar})$	$\epsilon_{th}(^{36}\mathrm{Ar})$	$\epsilon_{th}(^{46}\mathrm{Ar})$
Optical potential	8 %	7%	4%
Faddeev	6 %	19%	11%
Experiment	8%	8%	8%
Total	13 %	22 %	14 %

[FN, Deltuva, Hong, PRC83, 034610 (2011)]

transfer versus knockout



[Jenny Lee et al, PRL 2009]

[Gade et al, Phys. Rev. Lett. 93, 042501]

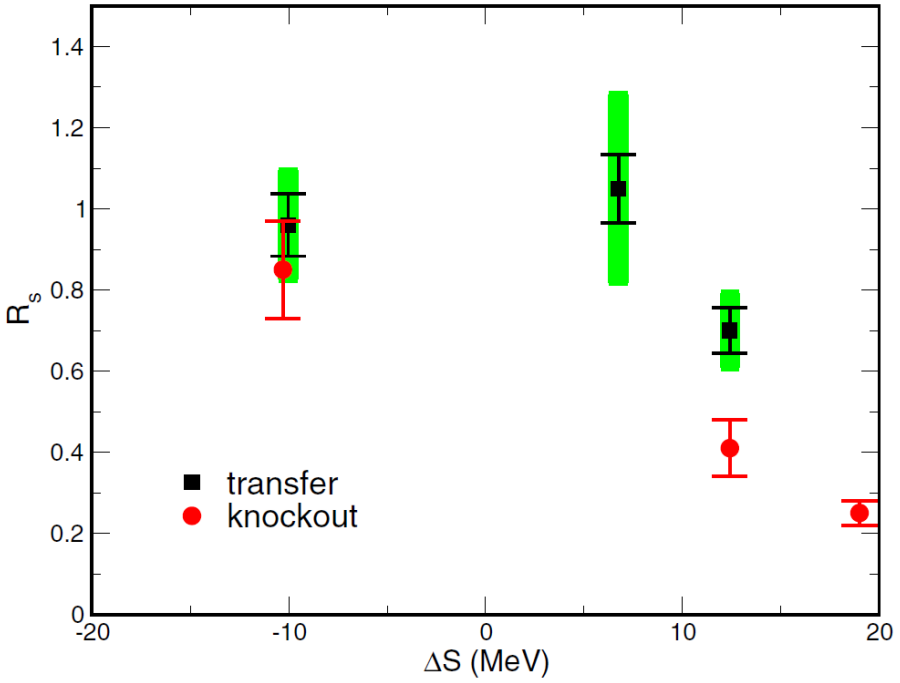
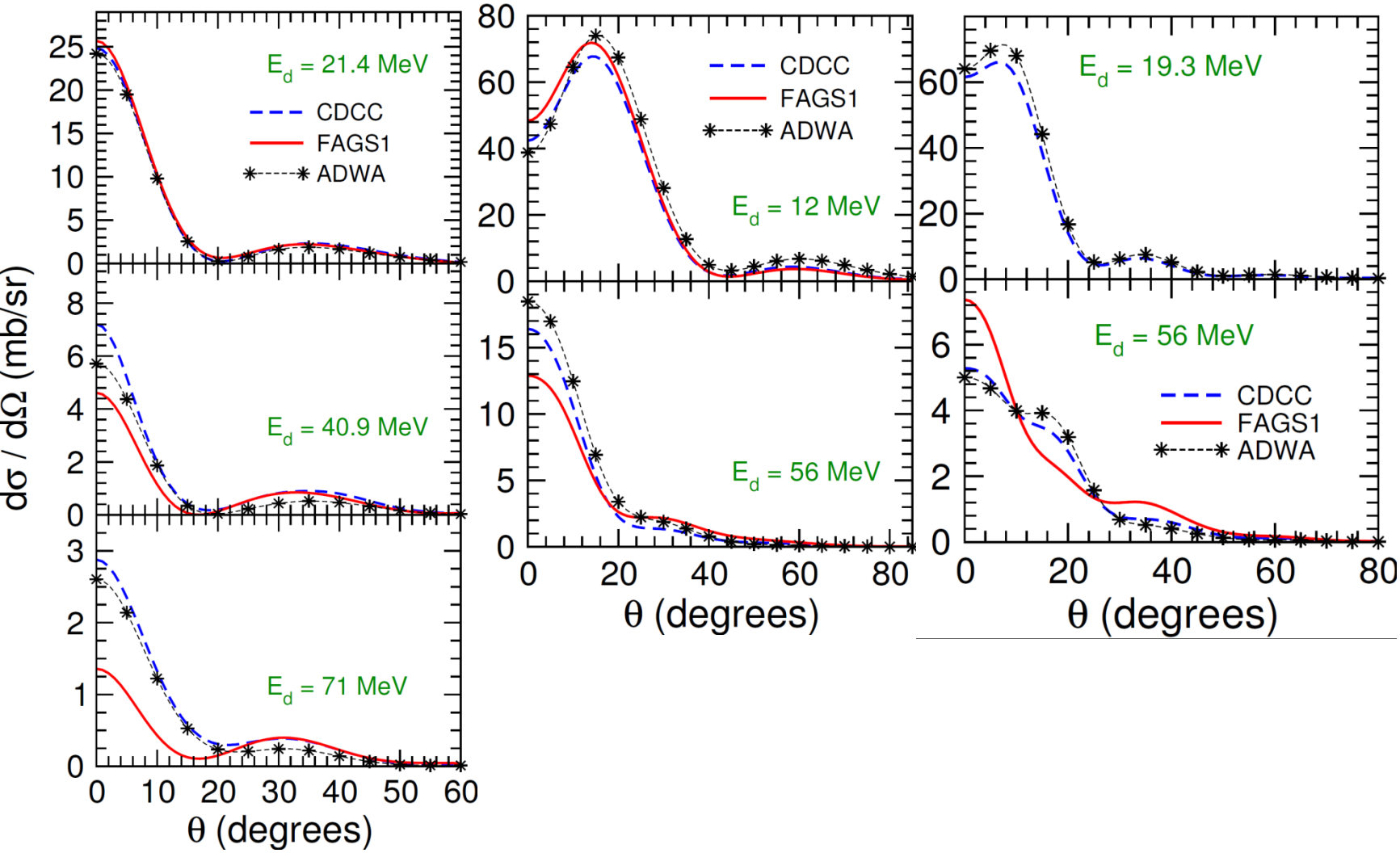


FIG. 3: (Color online) Reduction factors $Rs = SF(ADWA - FR)/SF(LB - SM)$ as a function of the difference between the neutron and proton separation energies ΔS. The squares and circles correspond to values extracted using transfer or knockout respectively. The bars correspond to the total uncertainty including both experimental and theoretical errors evaluated for the transfer reactions.

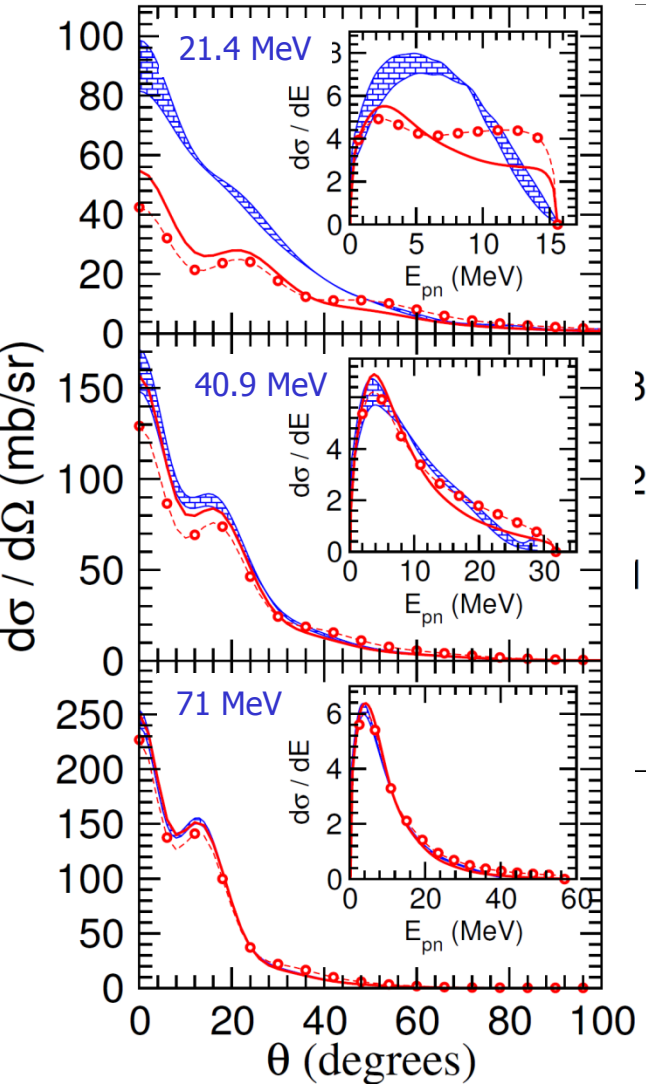
[FN, Deltuva, Hong, PRC83, 034610 2011]

Comparing transfer models CDCC, ADWA and Faddeev

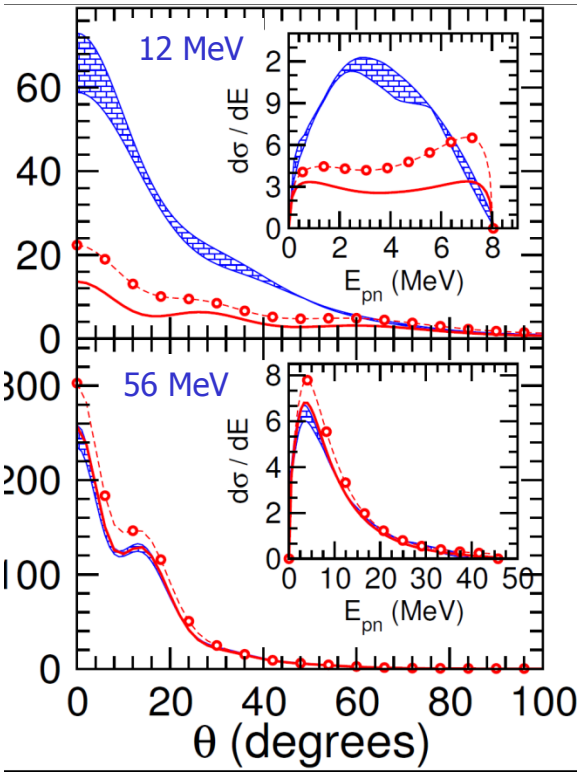


Comparing breakup

^{10}Be(d,pn)^{10}Be

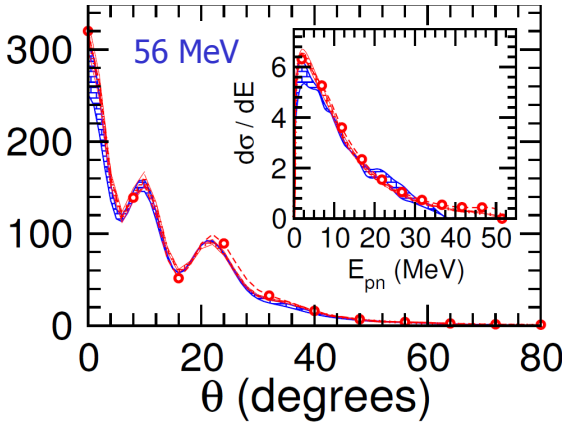


^{12}C(d,pn)^{12}C



blue – CDCC
red – Faddeev
(All calculations without Coulomb)

^{48}Ca(d,pn)^{48}Ca

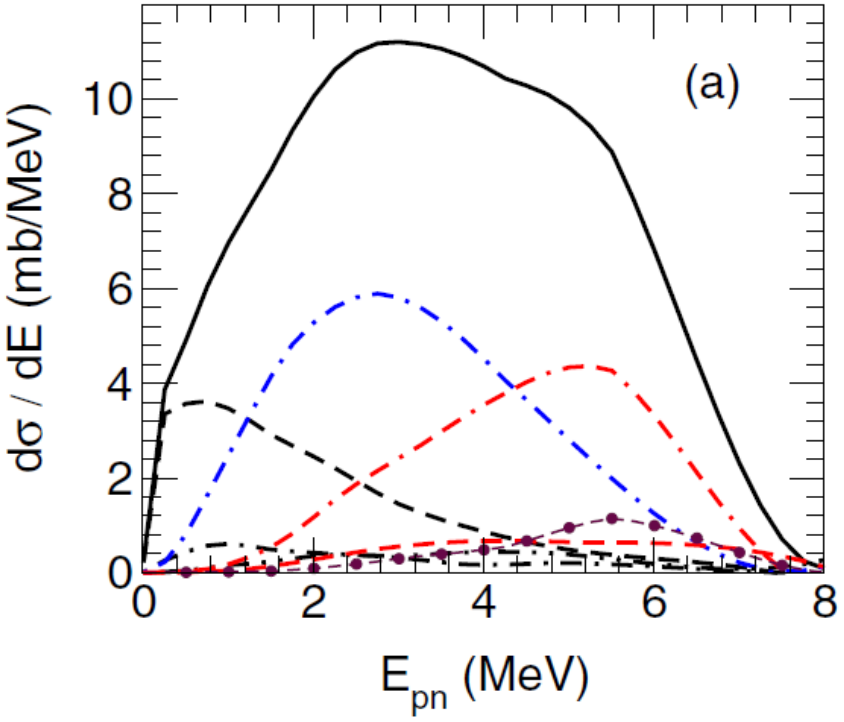


CDCC is reliable at higher energy
Very poor description ~10MeV/u

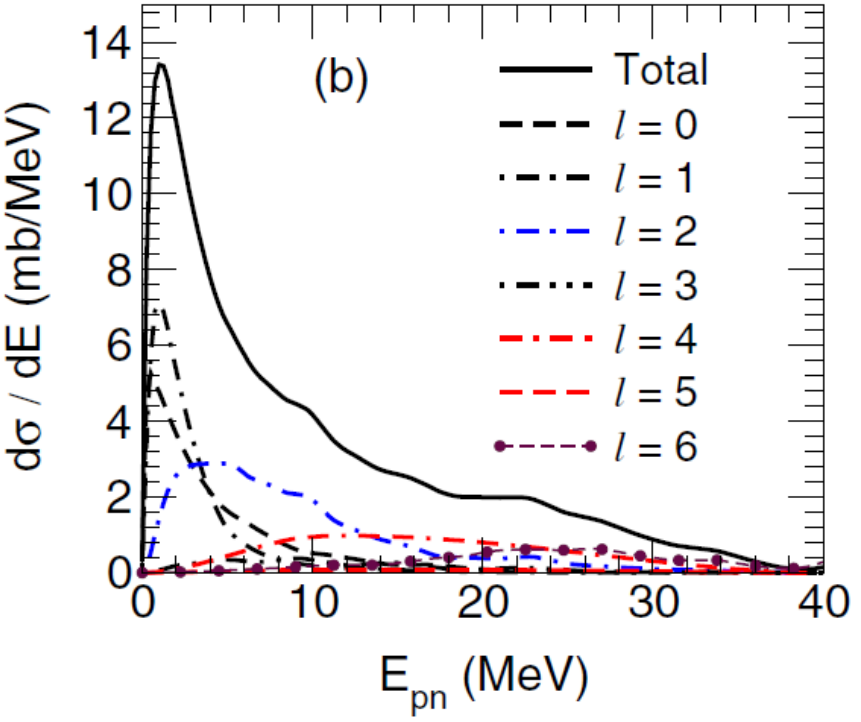
Upadhyay, Deltuva and Nunes, PRC 85, 054621 (2012)

Technical challenges: CDCC model space

$^{12}C(d,pn)^{12}C$ @ 12 MeV



$^{48}Ca(d,pn)^{48}Ca$ @ 56 MeV



- Contribution of np partial waves to breakup
- CDCC convergence is very slow at low energy

(No predictions for low energy breakup on ^{48}Ca, ^{132}Sn, ^{208}Pb)