Chiral nuclear forces with local regulators

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Nuclear Structure & Reactions: Experimental and Ab Initio Theoretical Perspectives
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With V. Bernard, E. Epelbaum, A. Gasparyan, U.-G. Meißner

LENPIC

Low Energy Nuclear Physics International Collaboration

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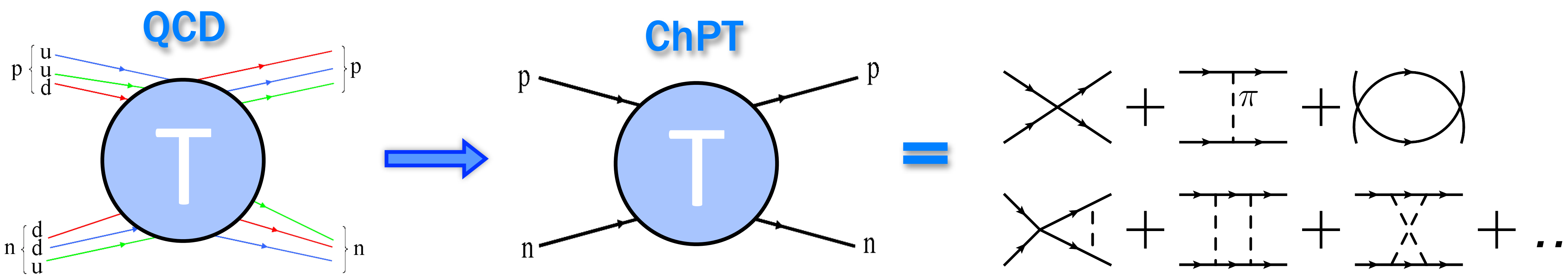
Outline

- Nuclear forces in chiral EFT
- Role of Δ(1232) resonance
- NN with local regulators
- Long-range part of three-nucleon forces up to N^4LO
- PWD of the local three-nucleon forces
- Summary & Outlook

From QCD to nuclear physics

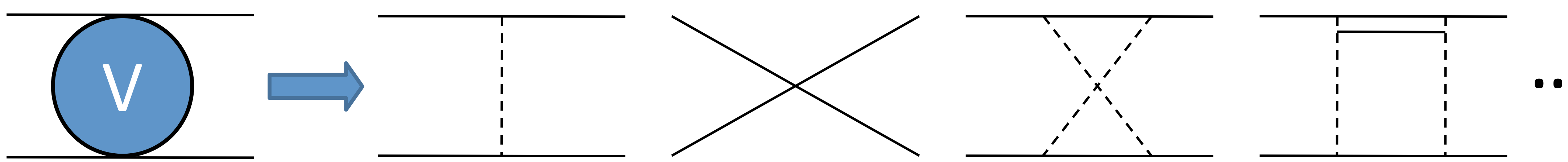


- **NN interaction is strong**: resummations/nonperturbative methods needed
- $1/m_N$ - expansion: nonrelativistic problem ($|\vec{p}_i| \sim M_\pi \ll m_N$) ⟹ the QM A-body problem

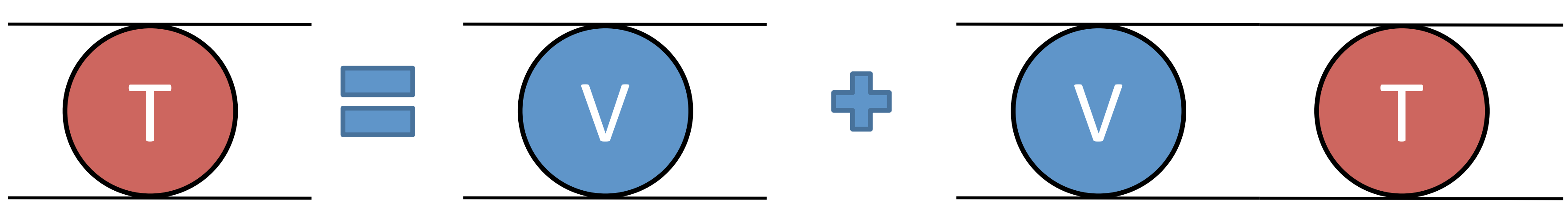
$$\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\textit{derived within ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

Weinberg '91

- Construct effective potential perturbatively

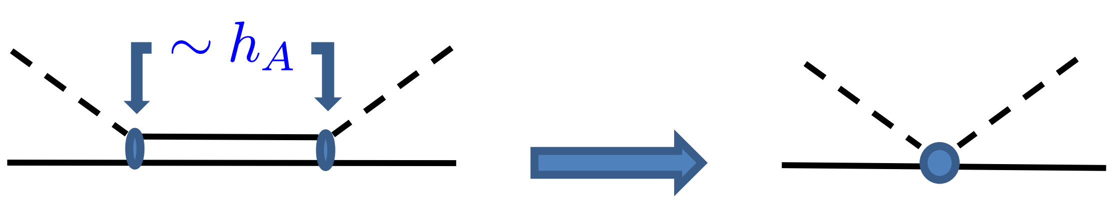


- Solve Lippmann-Schwinger equation nonperturbatively



EFT with explicit Δ(1232)

- Standard chiral expansion: $Q \sim M_\pi \ll \Delta \equiv m_\Delta - m_N = 293\ \mathrm{MeV}$
- Small scale expansion: $Q \sim M_\pi \sim \Delta \ll \Lambda_\chi$ *(Hemmert, Holstein & Kambor '98)*
- Delta contributions encoded in LECs
  (Bernard, Kaiser & Meißner ´97)

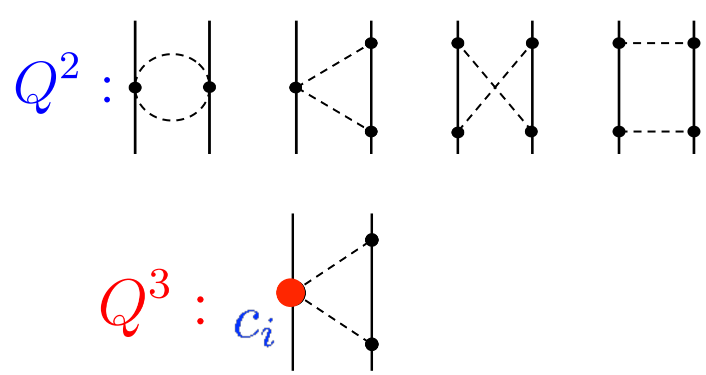


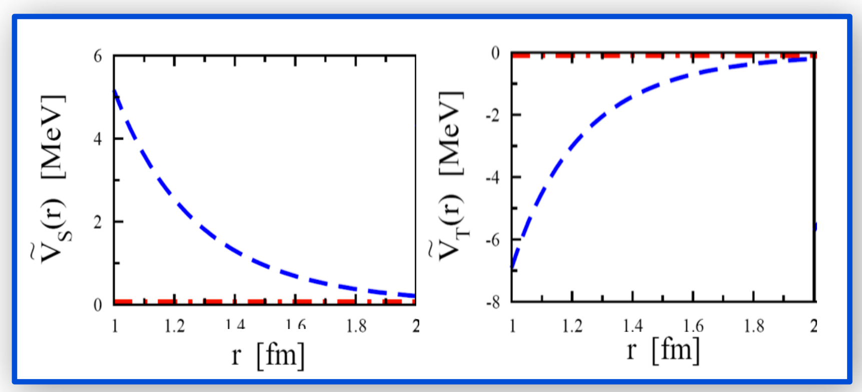
Delta-resonance saturation

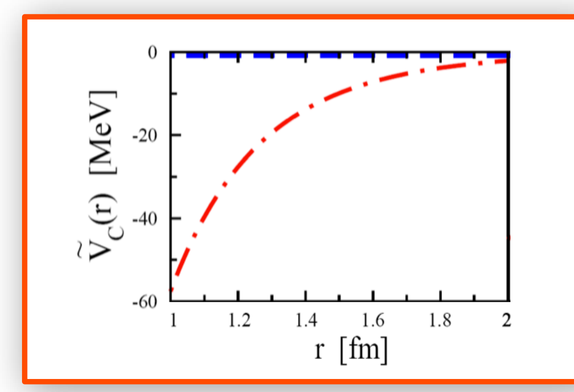
$$c_3 = -2c_4 = c_3(\not\Delta) - \boxed{\frac{4h_A^2}{9\Delta}}$$

Enlargement due to Delta contribution

- Convergence of EFT potential







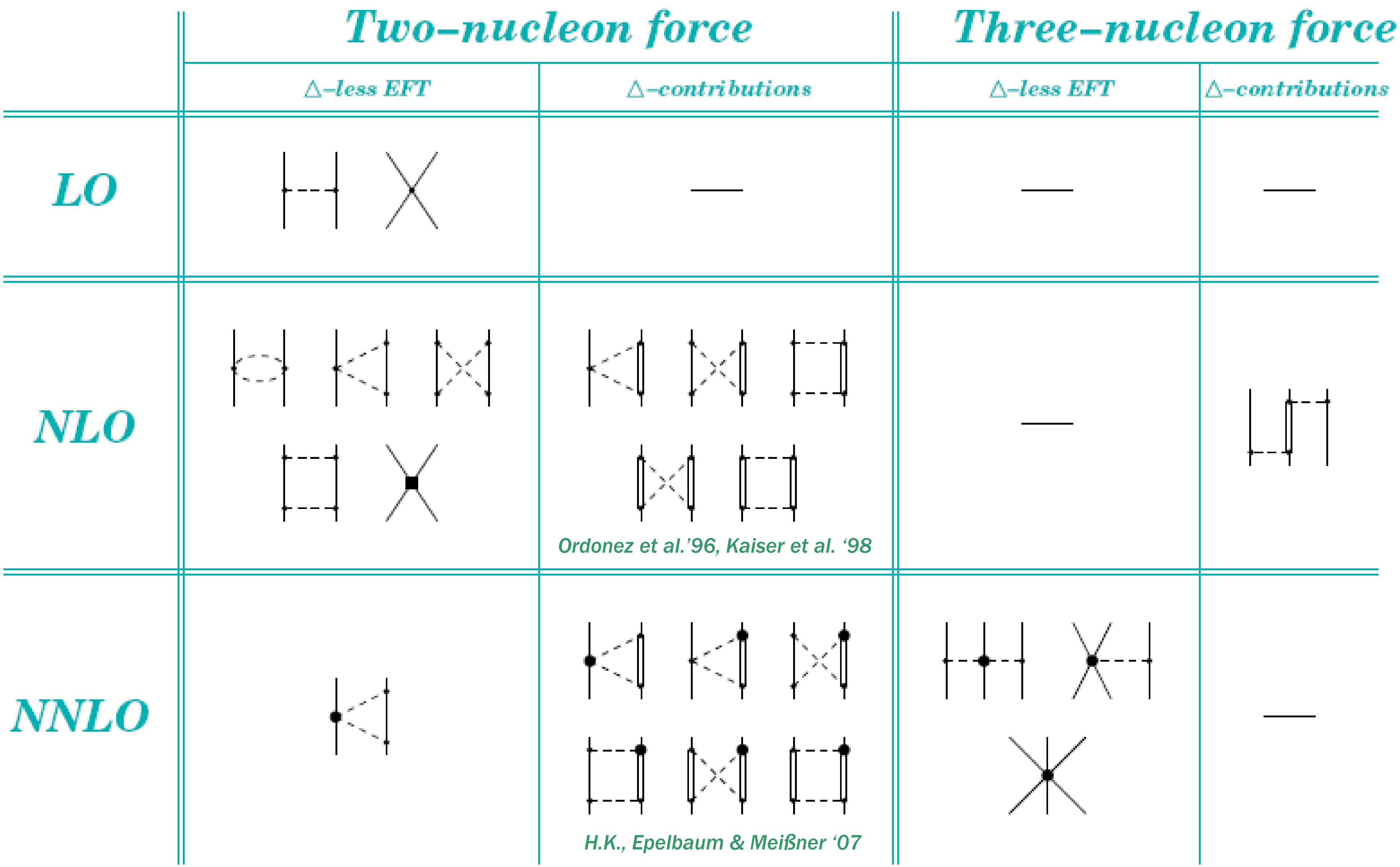
The subleading contributions are larger than the leading one!

Expectation from inclusion of Δ explicitely

- more natural size of LECs
- better convergence
- applicability at higher energies

Few-nucleon forces with the Delta

Isospin-symmetric contributions

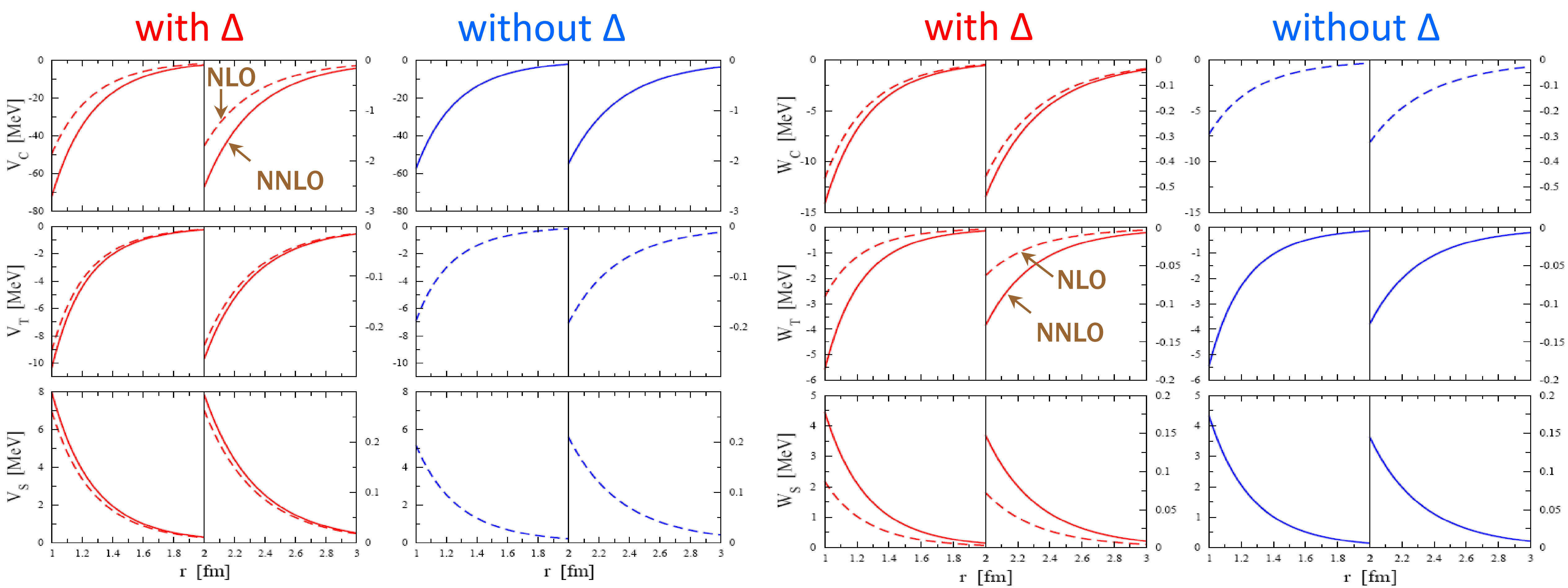


NN potential with explicit Δ

Epelbaum, H.K., Meißner, Eur. Phys. J. A32 (2007) 127

$$V_{\text{eff}} = V_C + W_C \vec{\tau}_1\cdot\vec{\tau}_2 + [V_S + W_S \vec{\tau}_1\cdot\vec{\tau}_2]\vec{\sigma}_1\cdot\vec{\sigma}_2 + [V_T + W_T \vec{\tau}_1\cdot\vec{\tau}_2](3\,\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2)$$

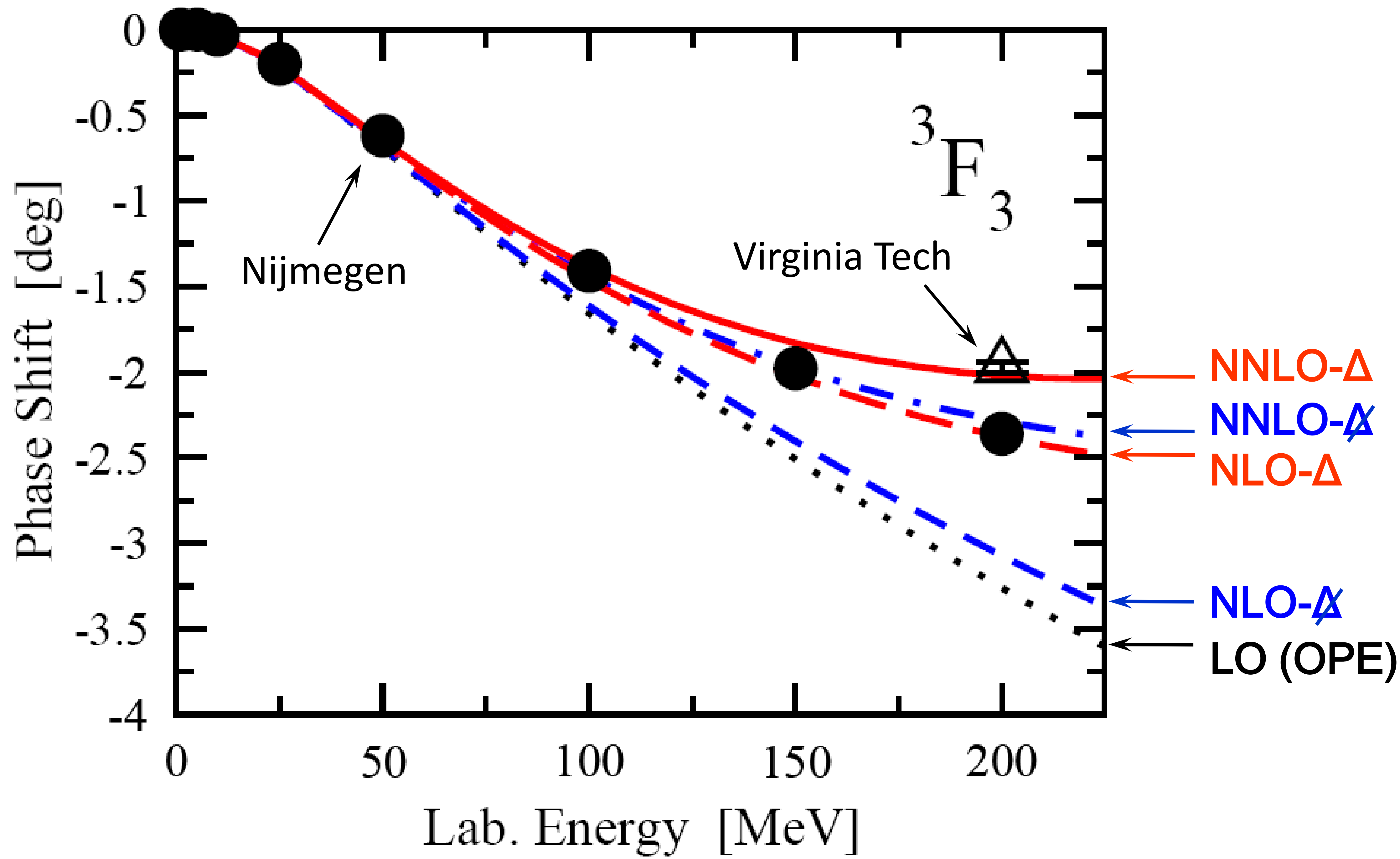
Chiral 2π- exchange potential up to NNLO



Advantages when Δ is included explicitly

- Dominant contributions already at NLO
- Much better convergence in all potentials

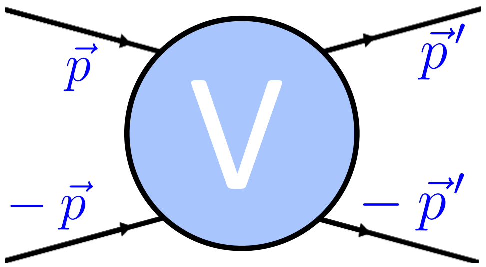
$^3F_3$ partial waves up to NNLO with and without Δ



(calculated in the first Born approximation)

NN forces with local regulator

For numerical studies chiral nuclear forces need to be regularized



- Usually nonlocal regulator used (does not mix partial waves)

$$V_{\rm ChPT}(\vec{p},\vec{p}\,') \to \exp\left(-\frac{p^6}{\Lambda^6}\right) V_{\rm ChPT}(\vec{p},\vec{p}\,') \exp\left(-\frac{p'^6}{\Lambda^6}\right)$$ *EGM ´02*

- For many-body method like QMC local version of the force is needed

$$V_{\rm ChPT}(\vec{p},\vec{p}\,') = \sum_i V_{\rm local}^{(i)}(\vec{p}-\vec{p}\,')\ {\rm Polynomial}^{(i)}(\vec{p},\vec{p}\,')$$

Sources of non-locality: contact interactions; $1/m_N$-corrections

$$V_{\rm local}^{(i)}(\vec{p}-\vec{p}\,') \to \delta(\vec{r}\,'-\vec{r}) \left[\tilde{V}_{\rm local}^{(i)}(\vec{r}) = \tilde{V}_{\rm long}^{(i)}(\vec{r}) + \tilde{V}_{\rm cont}^{(i)}(\vec{r})\right]$$

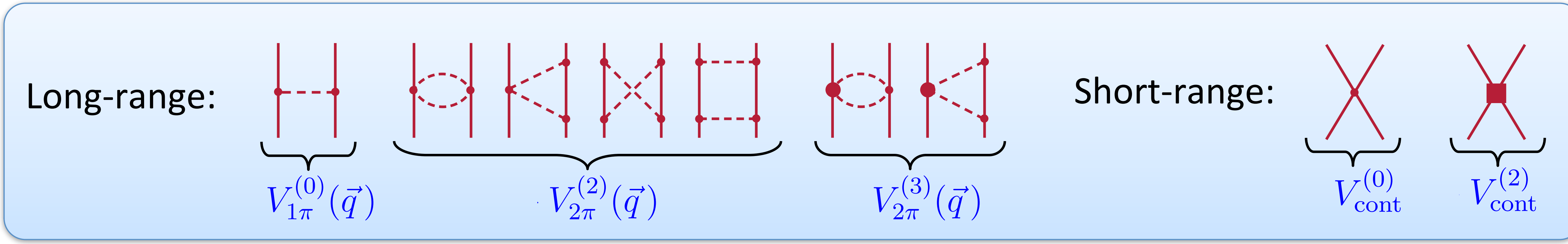
Introduce local regulator in coordinate space via e.g.

$$\tilde{V}_{\rm long}^{(i)}(\vec{r}) \to \tilde{V}_{\rm long}^{(i)}(\vec{r}) \left(1-\exp\left(-(r/R_0)^4\right)\right) \quad \text{and} \quad \delta(\vec{r}) \to \frac{1}{\pi\Gamma(3/4)R_0^3} \exp\left(-(r/R_0)^4\right)$$

$$\Lambda = 450 \ldots 600\,{\rm MeV} \longleftrightarrow R_0 = 1.0 \ldots 1.2\,{\rm fm}$$

For local regularization in momentum space: *Gazit, Quaglioni, Navratil ´09*

Construction of local N^{2}LO potential



- There are 9 isospin-concerving contact terms whose choice is not unique. Standard:

$$V_{\rm cont}^{(0)} = C_S + C_T(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

$$V_{\rm cont}^{(2)} = C_1 q^2 + C_2 k^2 + C_3 q^2(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + C_4 k^2(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + \frac{iC_5}{2}(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{q} \times \vec{k} + C_6(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) + C_7(\vec{\sigma}_1 \cdot \vec{k})(\vec{\sigma}_2 \cdot \vec{k})$$

where $\vec{q} = \vec{p}\,' - \vec{p}, \quad \vec{k} = (\vec{p} + \vec{p}\,')/2$

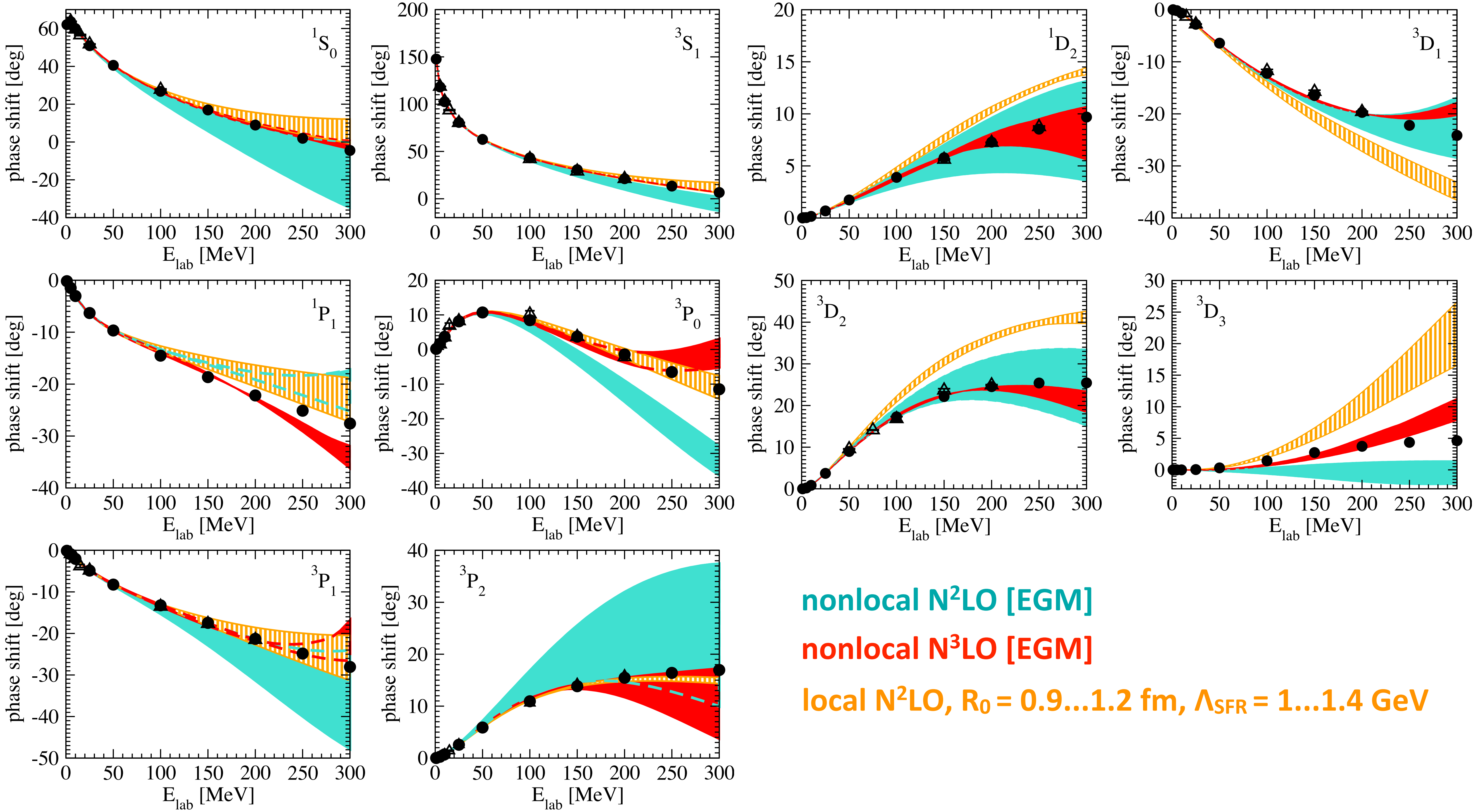
In the standard choice there are no isospin-matrices in the operator basis

- One can choose instead a quasi-local basis:

$$V_{\rm cont}^{(2)} = C_1 q^2 + C_2 q^2(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) + C_3 q^2(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + C_4 q^2(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) + \frac{iC_5}{2}(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{q} \times \vec{k}$$
$$+ \; C_6(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) + C_7(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$$

The LECs are determined from NN S-, P-waves and the mixing angle ε_1

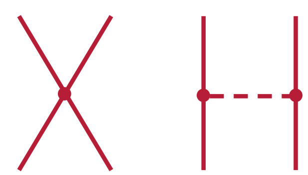
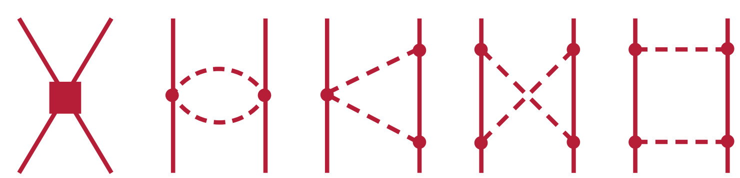
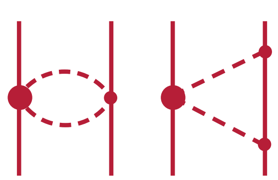
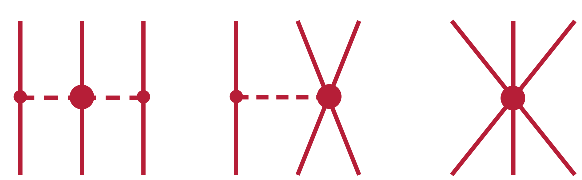
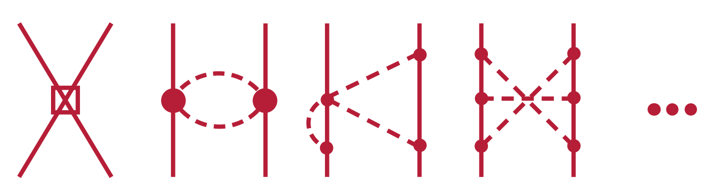
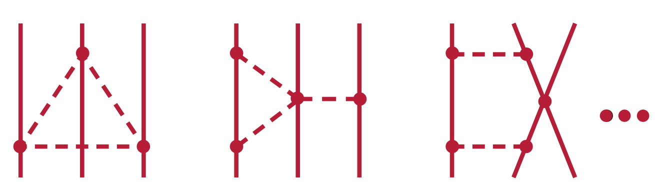
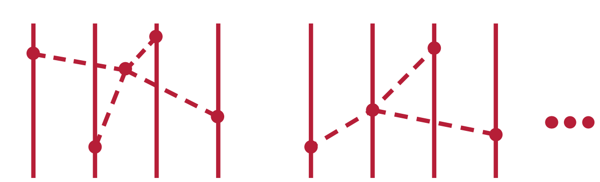
np phase shifts



QMC applications: *Gezerlis et al. PRL 111 (2013) 032501* → *talk by Gezerlis*

Nuclear forces up to N^3LO

dimensional analysis counting

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)		—	—
NLO (Q^2)		—	—
N^2LO (Q^3)			—
N^3LO (Q^4)			

- converged
- accurate description of NN at least up to E_{lab} ~ 200 MeV

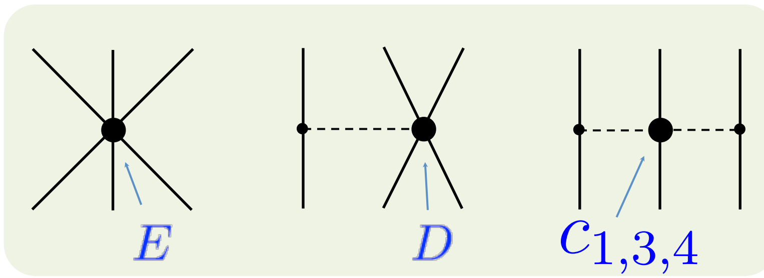
- not yet converged
- higher orders in progress
- impact on few- & many-N systems?

- converged ??

Three-nucleon forces

- Three-nucleon forces in chiral EFT start to contribute at N^2LO

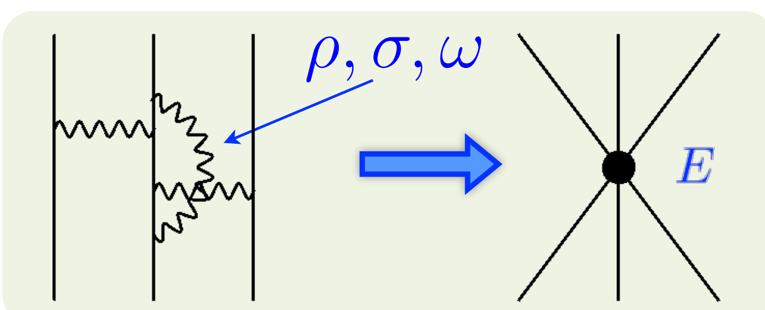
(Friar & Coon ´86; U. van Kolck ´94; Epelbaum et al. ´02; Nogga et al. ´05; Navratil et al. ´07)



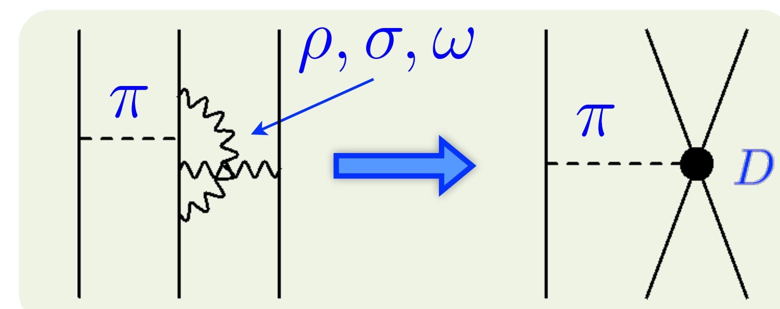
$c_{1,3,4}$ from the fit to πN-scattering data

D, E from $^3\mathrm{H}, ^4\mathrm{He}, ^{10}\mathrm{B}$ binding energy + coherent nd - scattering length

- LECs D and E incorporate short-range contr.



Resonance saturation interpretation of LECs



- Delta contributions encoded in LECs

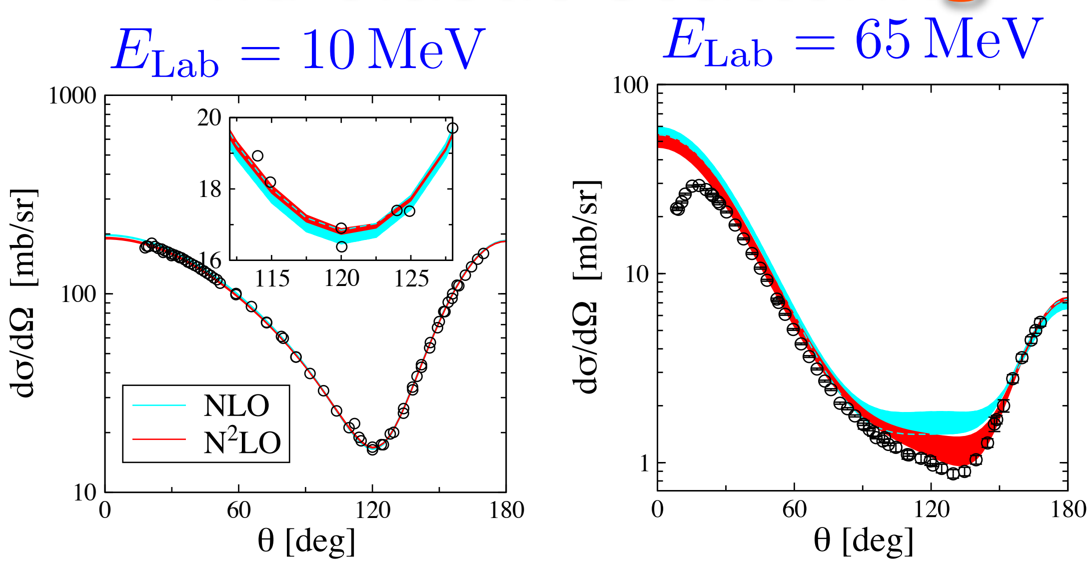
(Bernard, Kaiser & Meißner '97)

Delta-resonance saturation

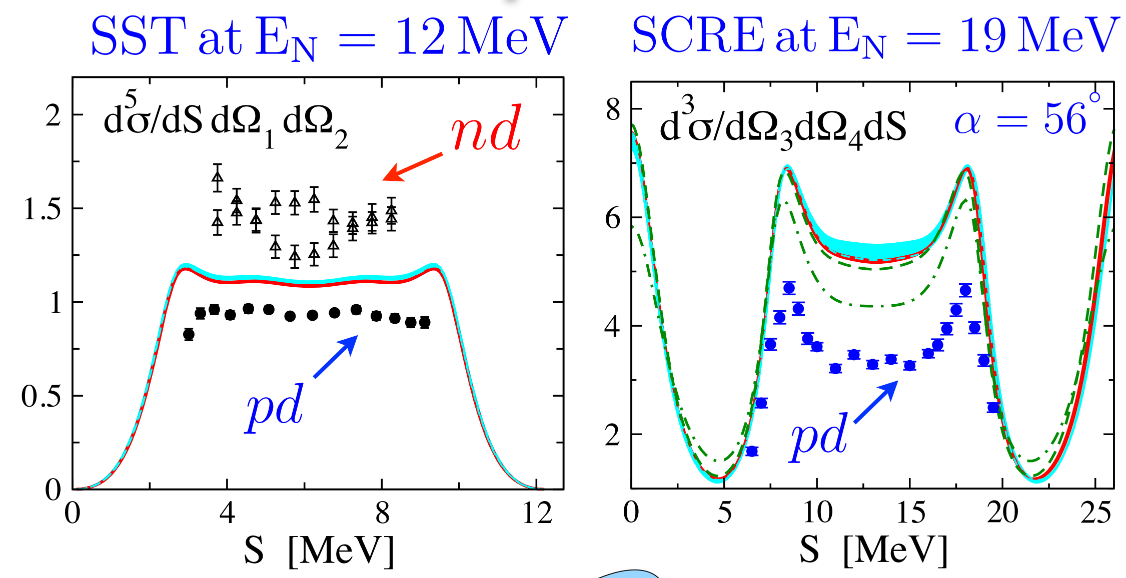
$$c_3 = -2c_4 = c_3(\not{\Delta}) - \boxed{\frac{4h_A^2}{9\Delta}}$$

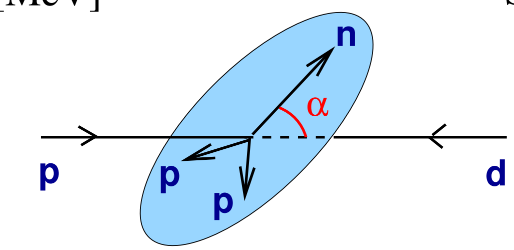
Enlargement due to Delta contribution

nd elastic scattering

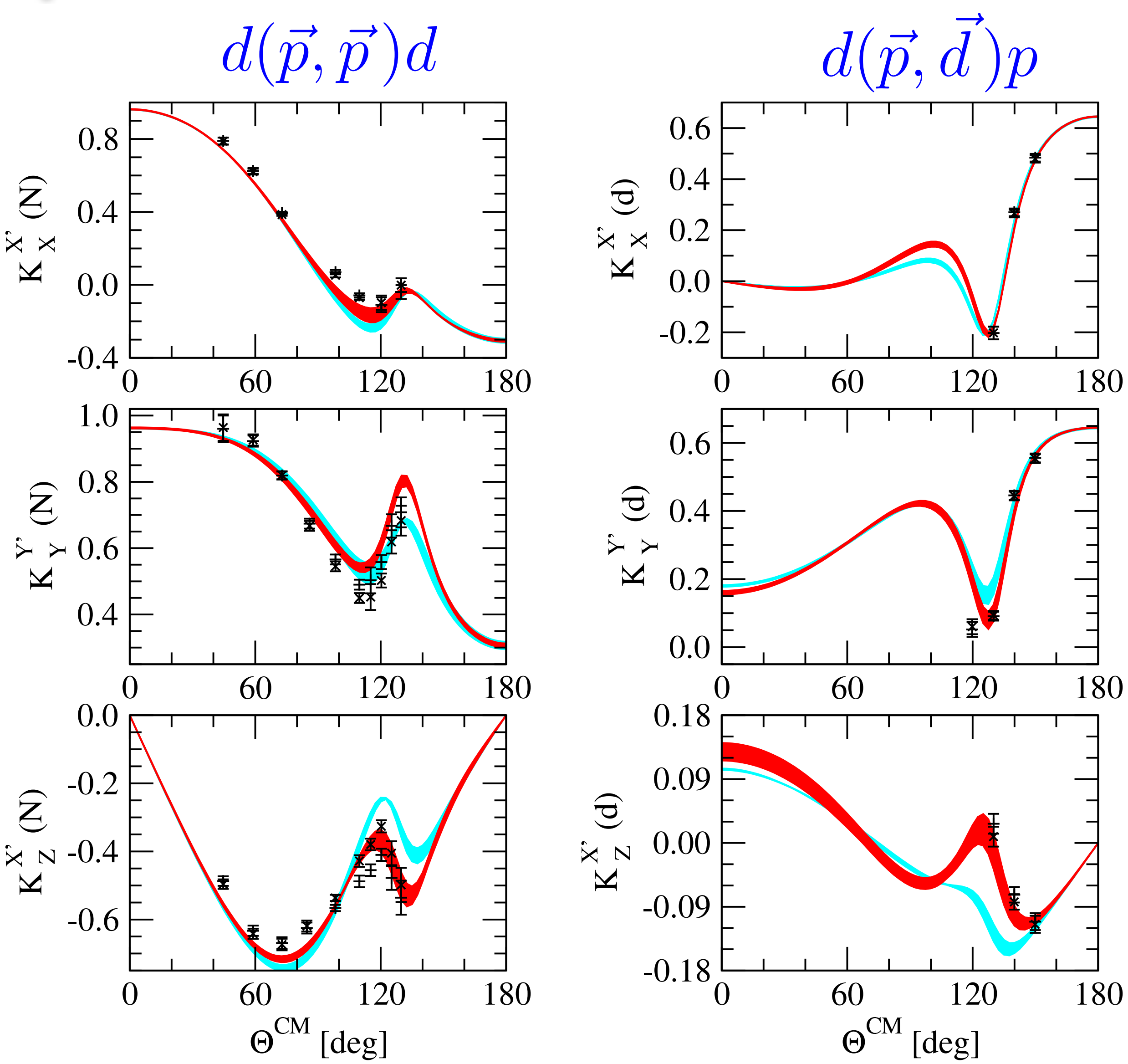


nd break-up $[\mathrm{mb\,MeV^{-1}sr^{-2}}]$





polarization transfer: $E_p^{\mathrm{Lab}} = 22.7\,\mathrm{MeV}$



For references see recent reviews:

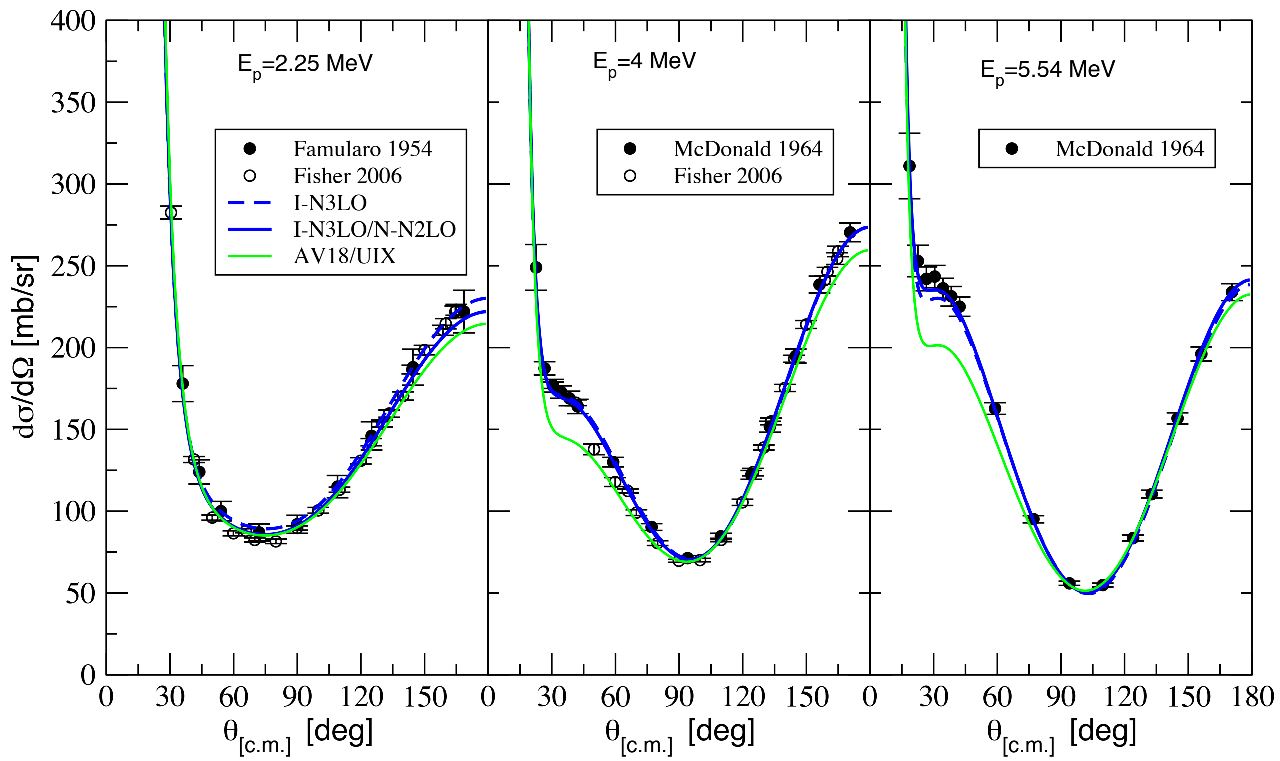
- Epelbaum, Prog. Part Nucl. Phys. 57 (06) 654
- Epelbaum, Hammer, Meißner, Rev. Mod. Phys. 81 (09) 1773
- Entem, Machleidt, Phys. Rept. 503 (11) 1
- Epelbaum, Meißner, Ann. Rev. Nucl. Part. Sci. 62 (12) 159
- Kalantar et al. Rep. Prog. Phys. 75 (12) 016301

- Generally good description of data. But some discrepancies survive. E.g. break-up observables for SCRE/SST configuration at low energy
- Hope for improvement at N^3LO

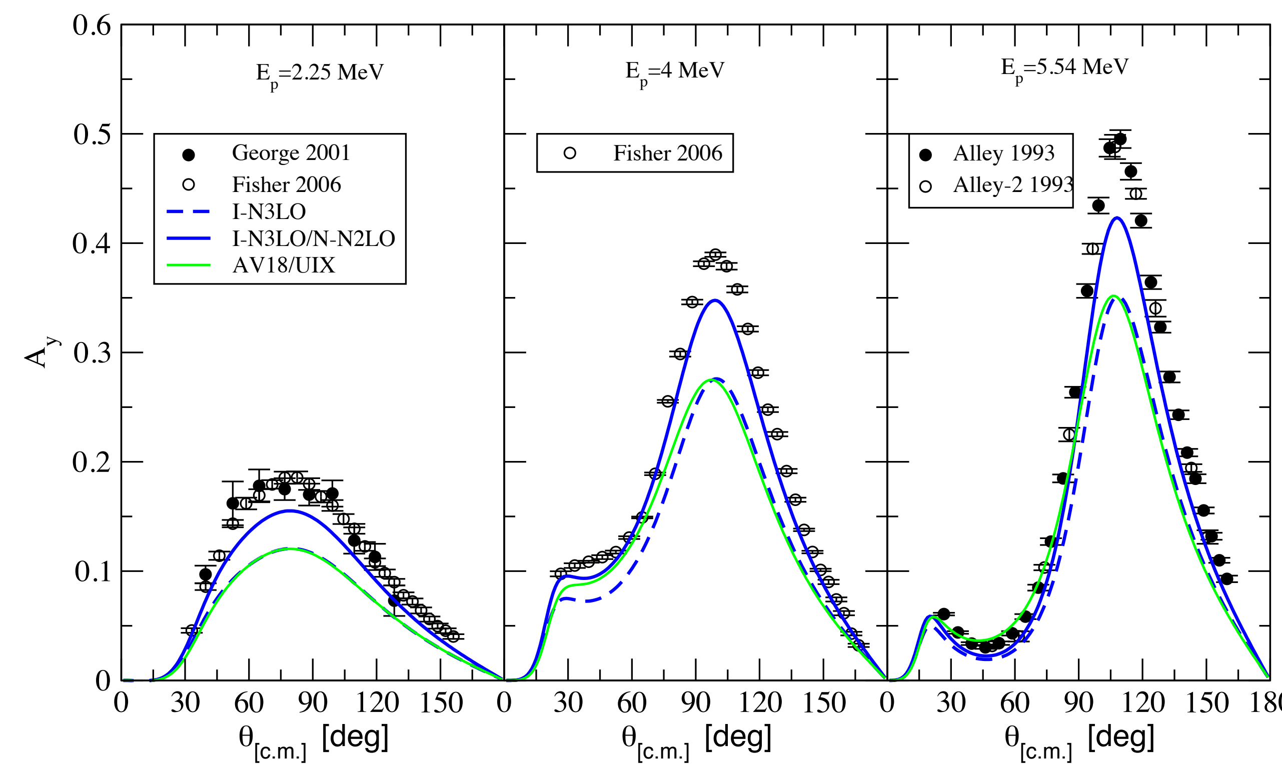
Proton-^{3}He elastic scattering

Viviani, Girlanda, Kievsky, Marcucci, Rosati EPJ Web Conf. 3 (2010) 05011

p-^{3}He differential cross section at low energies



proton vector analyzing power A_y-puzzle



As in n-d scattering case N^{2}LO 3NF's are not enough to resolve underprediction of A_y

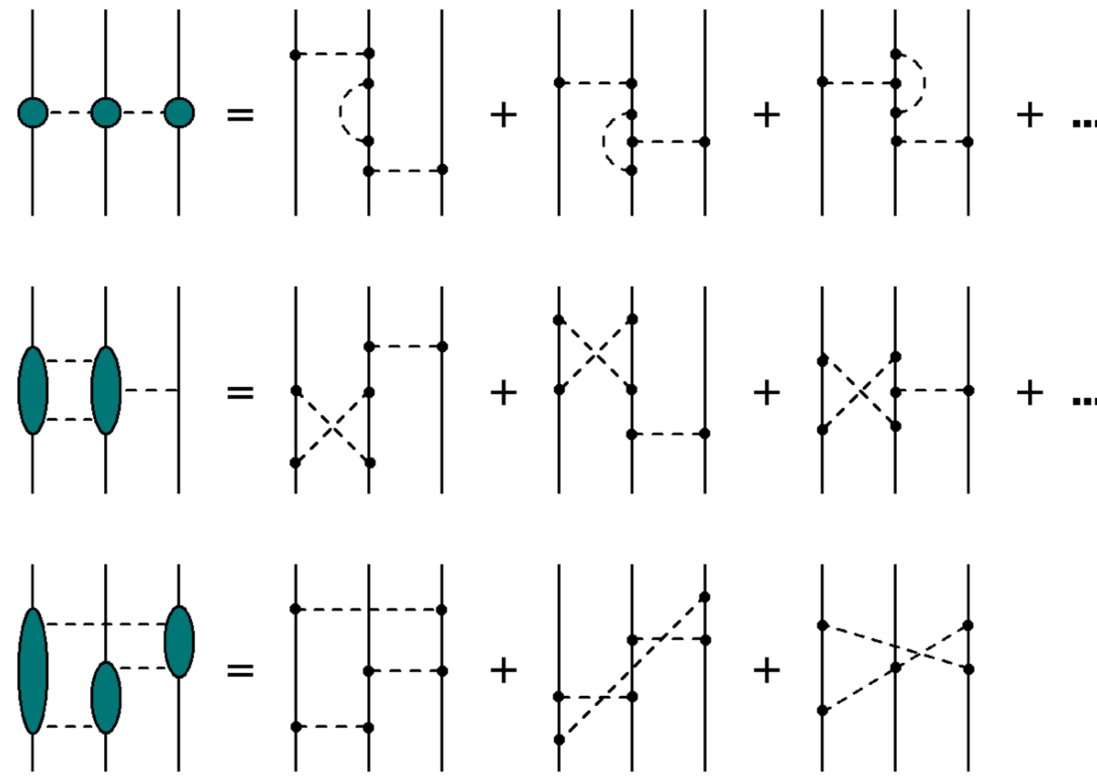
→ Hope for improvement at higher orders

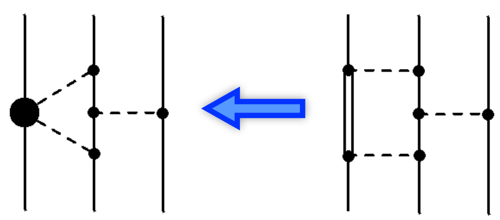
Three-nucleon forces

- Three-nucleon forces at N^3LO

Long range contributions *Bernard, Epelbaum, HK, Meißner ´08; Ishikawa, Robilotta ´07*

- No additional free parameters
- Expressed in terms of g_A, F_π, M_π
- Rich isospin-spin-orbit structure
- Δ(1232)-contr. are important

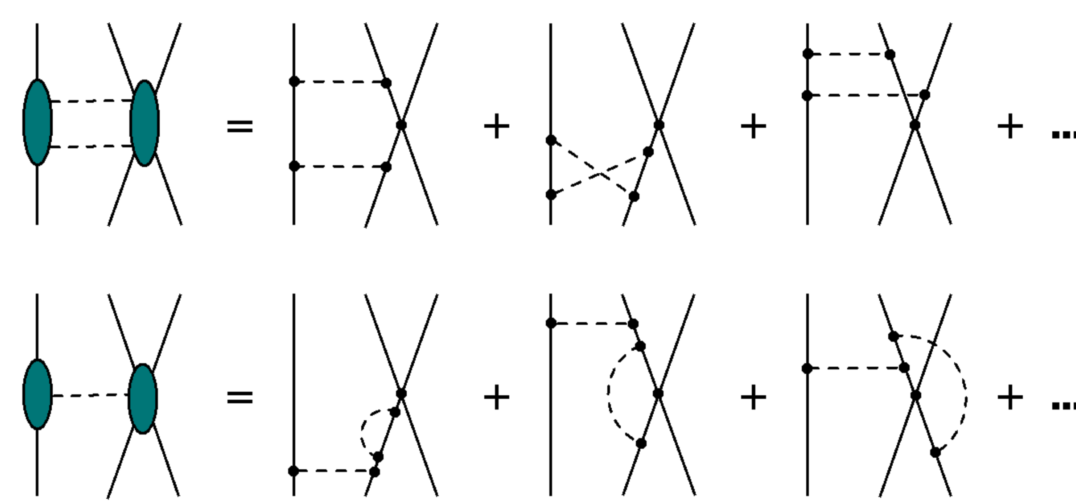


Large $c_i \sim$ 

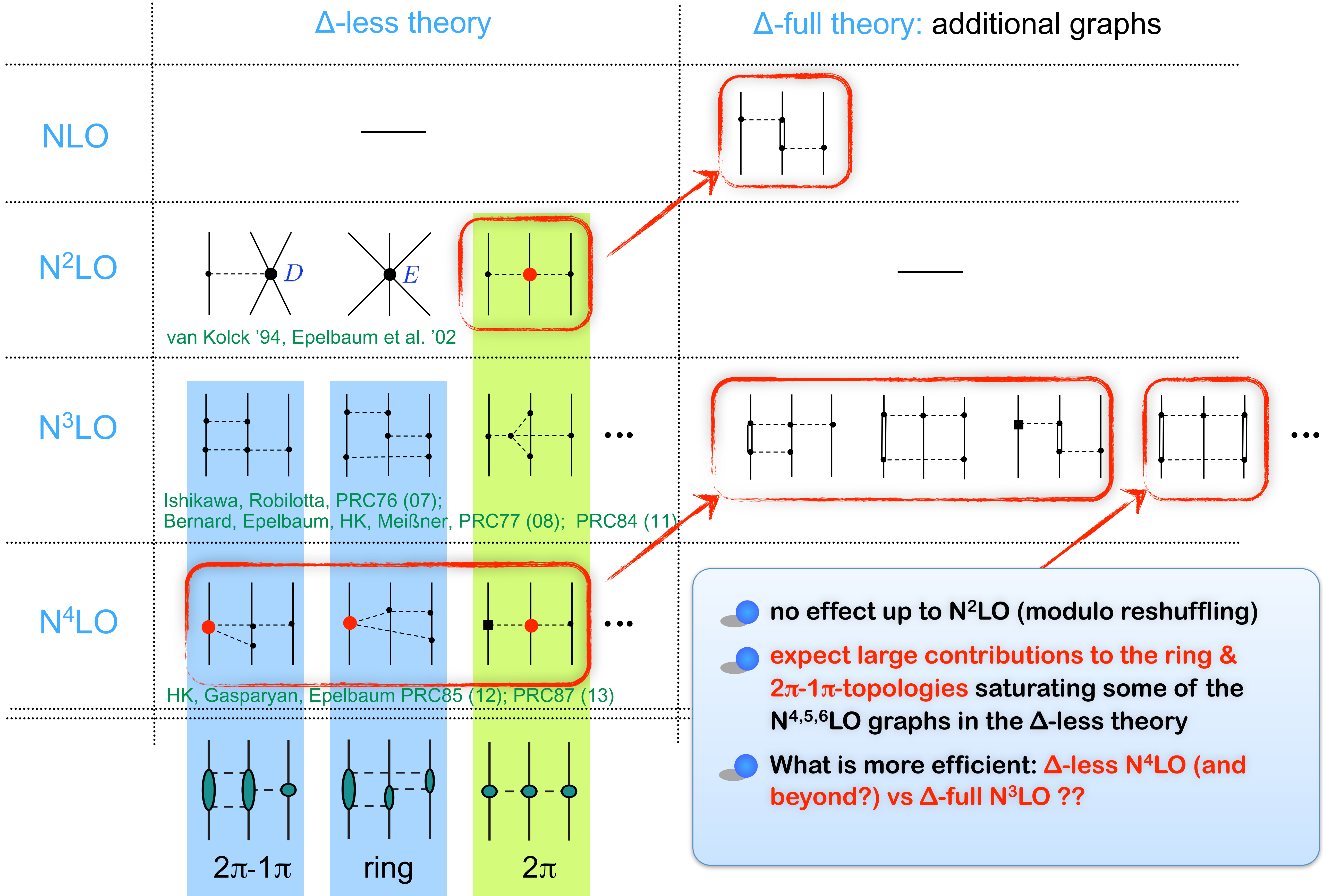
Shorter range contributions *Bernard, Epelbaum, HK, Meißner ´11*

- LECs needed for shorter range contr.

  g_A, F_π, M_π, C_T
- Central NN contact interaction does not contribute
- Unique expressions in the static limit for a renormalizable 3NF

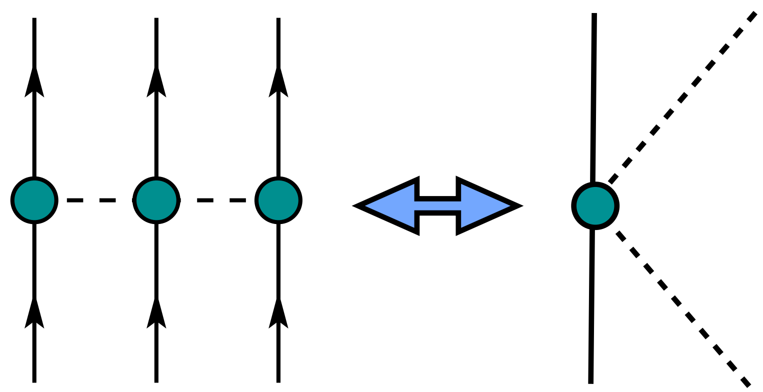


Small scale expansion of 3NF



- no effect up to N^2LO (modulo reshuffling)
- expect large contributions to the ring & 2π-1π-topologies saturating some of the $N^{4,5,6}LO$ graphs in the Δ-less theory
- What is more efficient: Δ-less N^4LO (and beyond?) vs Δ-full N^3LO ??

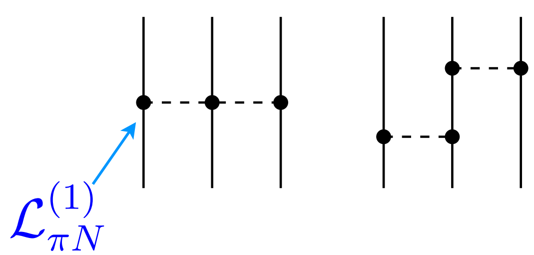
Two-pion-exchange 3NF



- Two-pion-exchange 3NF is connected to pion-nucleon scattering amplitude
  Ishikawa, Robilotta ´07
  - The same linear combinations of LECs
  - The same renormalization

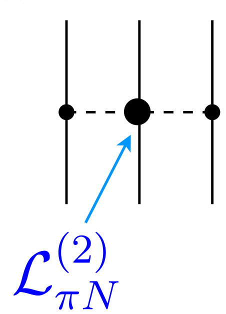
$$V_{2\pi} \;=\; \frac{\vec\sigma_1\cdot\vec q_1\,\vec\sigma_3\cdot\vec q_3}{[q_1^2+M_\pi^2]\,[q_3^2+M_\pi^2]}\Big(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,\mathcal{A}(q_2)+\boldsymbol{\tau}_1\times\boldsymbol{\tau}_3\cdot\boldsymbol{\tau}_2\,\vec q_1\times\vec q_3\cdot\vec\sigma_2\,\mathcal{B}(q_2)\Big)$$

NLO - contr.

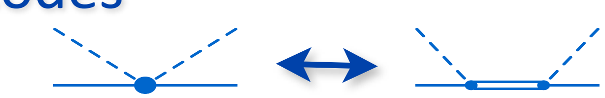


← yield vanishing 3NF contributions

N²LO - contr.



← first nonvanishing 3NF, encodes information about the Δ:

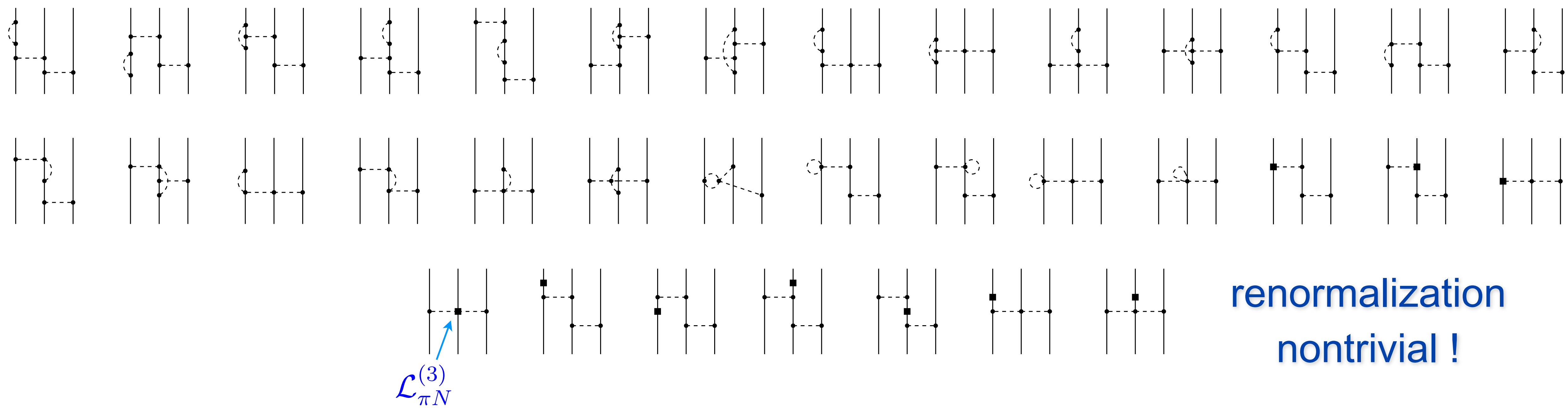


$$\mathcal{A}^{(3)}(q_2)=\frac{g_A^2}{8F_\pi^4}\Big((2c_3-4c_1)M_\pi^2+c_3q_2^2\Big)\,,\qquad \mathcal{B}^{(3)}(q_2)=\frac{g_A^2c_4}{8F_\pi^4}$$

U. van Kolck ´94

Two-pion-exchange 3NF

N³LO - contr. (leading 1 loop)



renormalization nontrivial !

$$\mathcal{A}^{(4)}(q_2) = \frac{g_A^4}{256\pi F_\pi^6}\left[A(q_2)\left(2M_\pi^4+5M_\pi^2 q_2^2+2q_2^4\right)+\left(4g_A^2+1\right)M_\pi^3+2\left(g_A^2+1\right)M_\pi q_2^2\right],$$

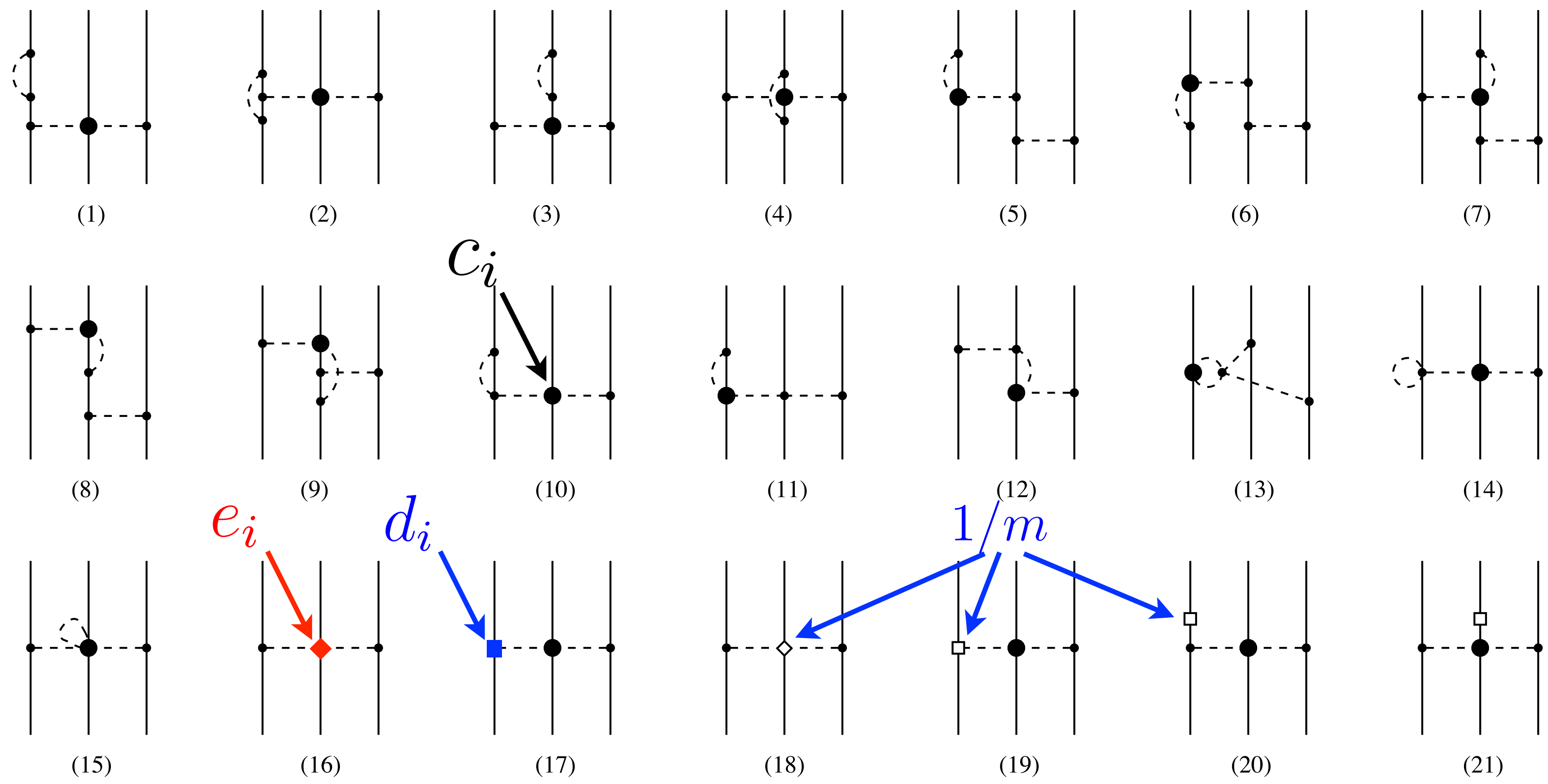
$$\mathcal{B}^{(4)}(q_2) = -\frac{g_A^4}{256\pi F_\pi^6}\left[A(q_2)\left(4M_\pi^2+q_2^2\right)+(2g_A^2+1)M_\pi\right]$$

Ishikawa, Robilotta ´07,
Bernard, Epelbaum, HK, Meißner ´07

- No unknown parameters at this order
- Everything is expressed in terms of loop function $A(q) = \frac{1}{2q}\arctan\frac{q}{2M_\pi}$
- Additional unitarity transformations required for proper renormalization

Two-pion-exchange 3NF

N4LO - contr. (subleading 1 loop) *Epelbaum, Gasparyan, HK,´12*

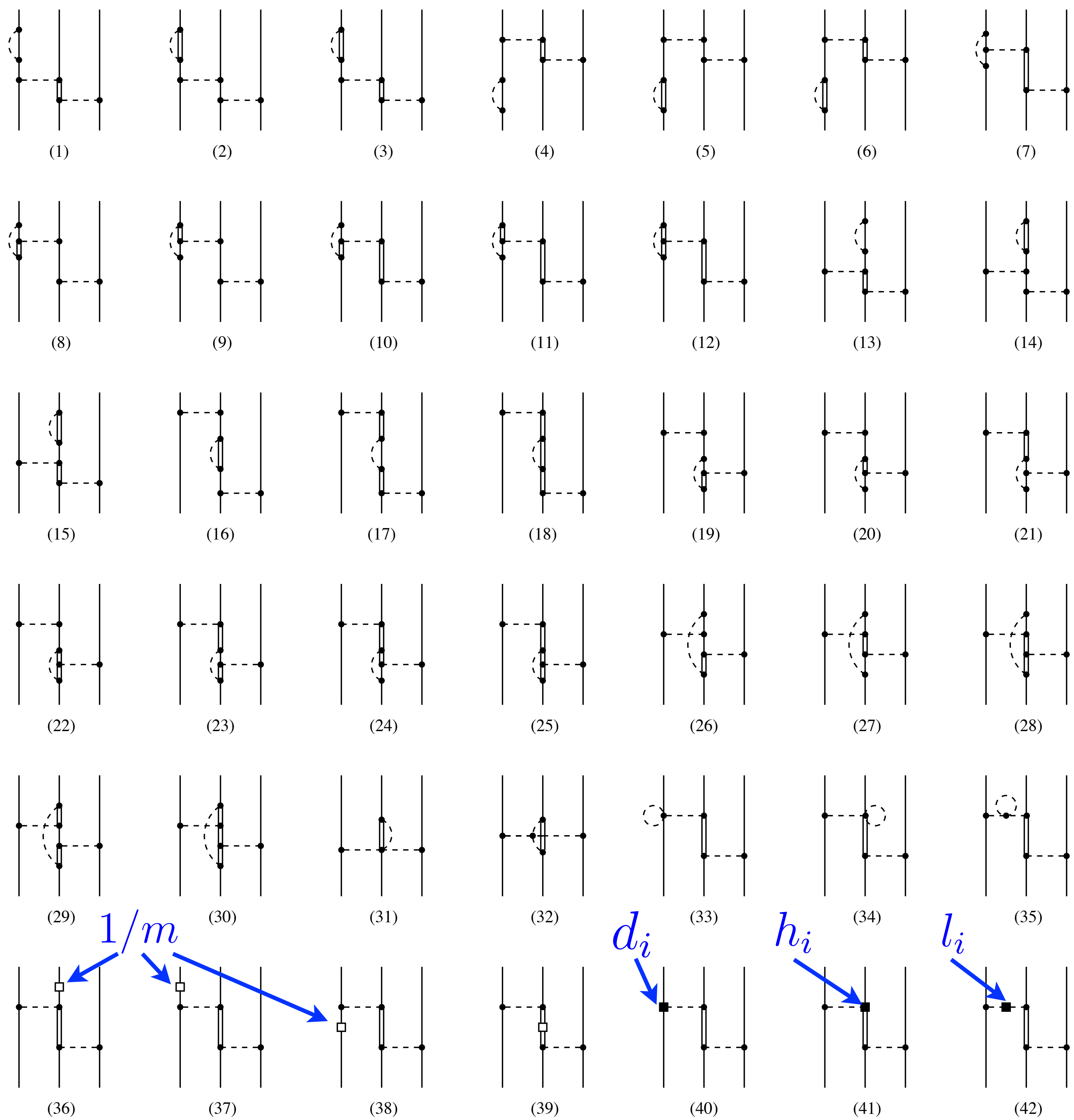


c_i´s LECs from $\mathcal{L}_{\pi N}^{(2)}$, d_i´s LECs from $\mathcal{L}_{\pi N}^{(3)}$, e_i´s LECs from $\mathcal{L}_{\pi N}^{(4)}$: fitted to πN - scattering data

- Leading Δ - contributions are taken into account through c_i´s
- Vanishing $1/m$ - contributions at this order

Two-pion-exchange 3NF

N[3]LO - delta- contr. (subleading 1 loop) *Epelbaum, Gasparyan, HK, forthcoming*



Additional LECs in the diagrams

$d_i \in \mathcal{L}_{\pi N}^{(3)}, \quad h_i \in \mathcal{L}_{\pi N\Delta}^{(3)}, \quad l_i \in \mathcal{L}_{\pi\pi}^{(4)}$

After renormalization the only additional LECs are

- Leading order πNΔ-constant

$$h_A \simeq \frac{3\, g_A}{2\sqrt{2}} \leftarrow \text{Large-N}_c$$

- Leading order πΔΔ-constant

$$g_1 \simeq \frac{9\, g_A}{5} \leftarrow \text{Large-N}_c$$

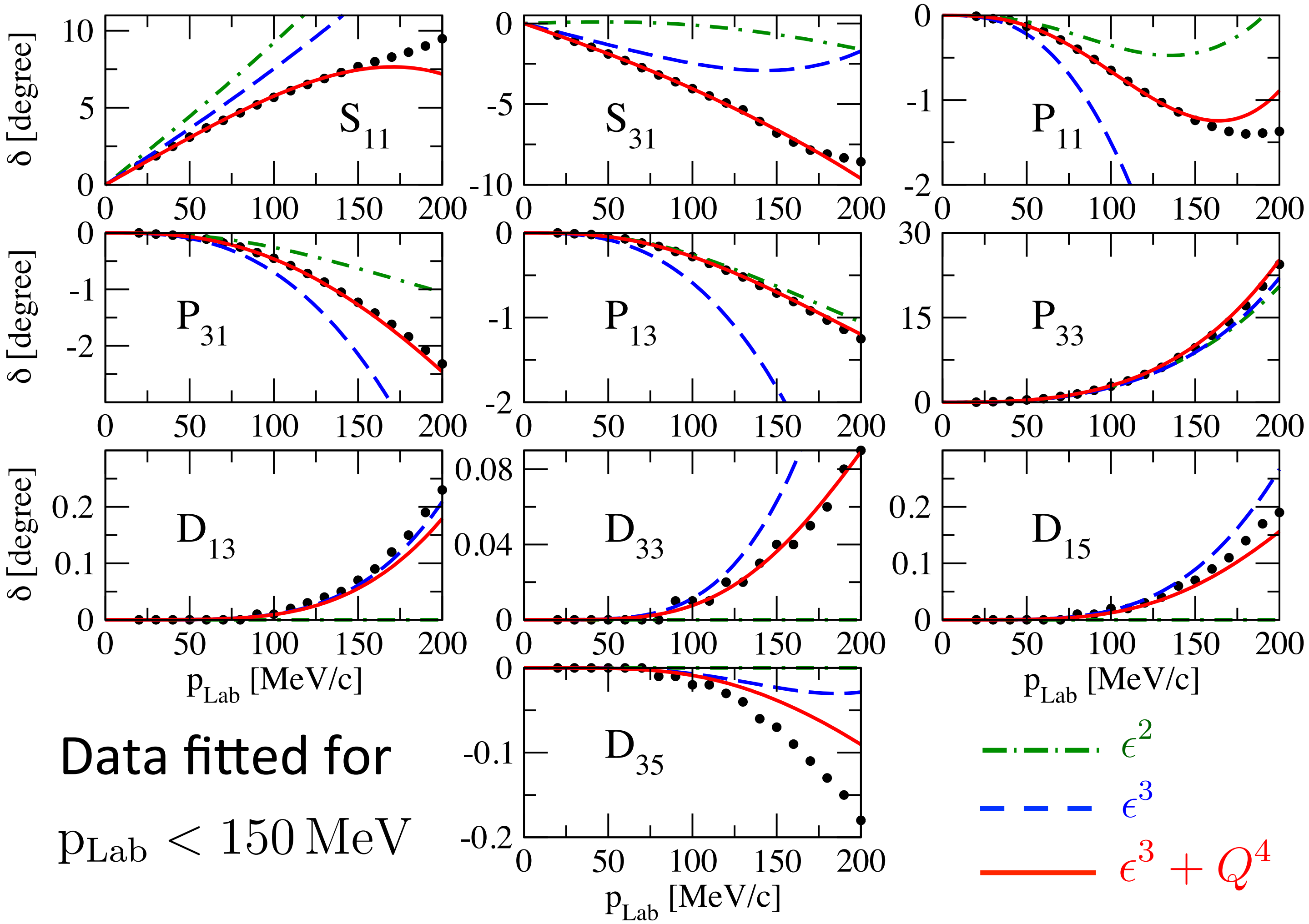
- Δ-resonance saturation of N[4]LO 3NF checked, explicitly

Preliminary

Pion-nucleon scattering

Heavy baryon SSE calculation up to ε^3: *Fettes & Meißner ´01; Epelbaum, Gasparyan, HK, in preparation*

Karlsruhe-Helsinki (KH) PWA: R. Koch Nucl. Phys. A 448 (1986) 707



Data fitted for $p_{\mathrm{Lab}} < 150\,\mathrm{MeV}$

- Size of the LECs are consistent with resonance saturation

$$c_1(\Delta) = 0, \; c_2(\Delta) = -c_3(\Delta) = 2\, c_4(\Delta) = \frac{4h_A^2}{9\Delta}$$

$$(\bar{d}_1 + \bar{d}_2)(\Delta) = -\bar{d}_3(\Delta) = -\frac{1}{2}(\bar{d}_{14} - \bar{d}_{15})(\Delta) = \frac{h_A^2}{9\Delta^2}$$

$$\bar{e}_{14}(\Delta) = \frac{h_A^2}{864\, F_\pi^2\, \pi^2\, \Delta}\left(7 + 10\,\log\left(\frac{2\,\Delta}{M_\pi}\right)\right), \ldots$$

- LECs which appear in 3NF up to N^{4}LO are of natural size

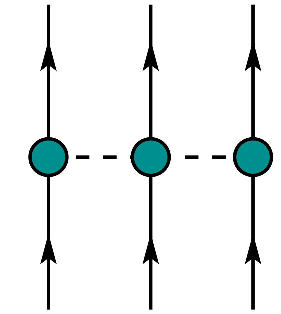
	c_1	c_2	c_3	c_4	$\bar{d}_1 + \bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14} - \bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
$Q^1 + Q^2 + Q^3 + Q^4$: Fit to KH [60]	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-10.41	6.08	-0.37	3.26
$\epsilon^1 + \epsilon^2 + \epsilon^3 + Q^4$: Fit to KH[60]	-0.95	1.90	-1.78	1.50	2.40	-3.87	1.21	-5.25	-0.24	-6.35	2.34	-0.39	2.81
Delta-resonance saturation contribution	0	2.81	-2.81	1.40	2.39	-2.39	0	-4.77	1.87	-4.15	4.15	-0.17	1.32

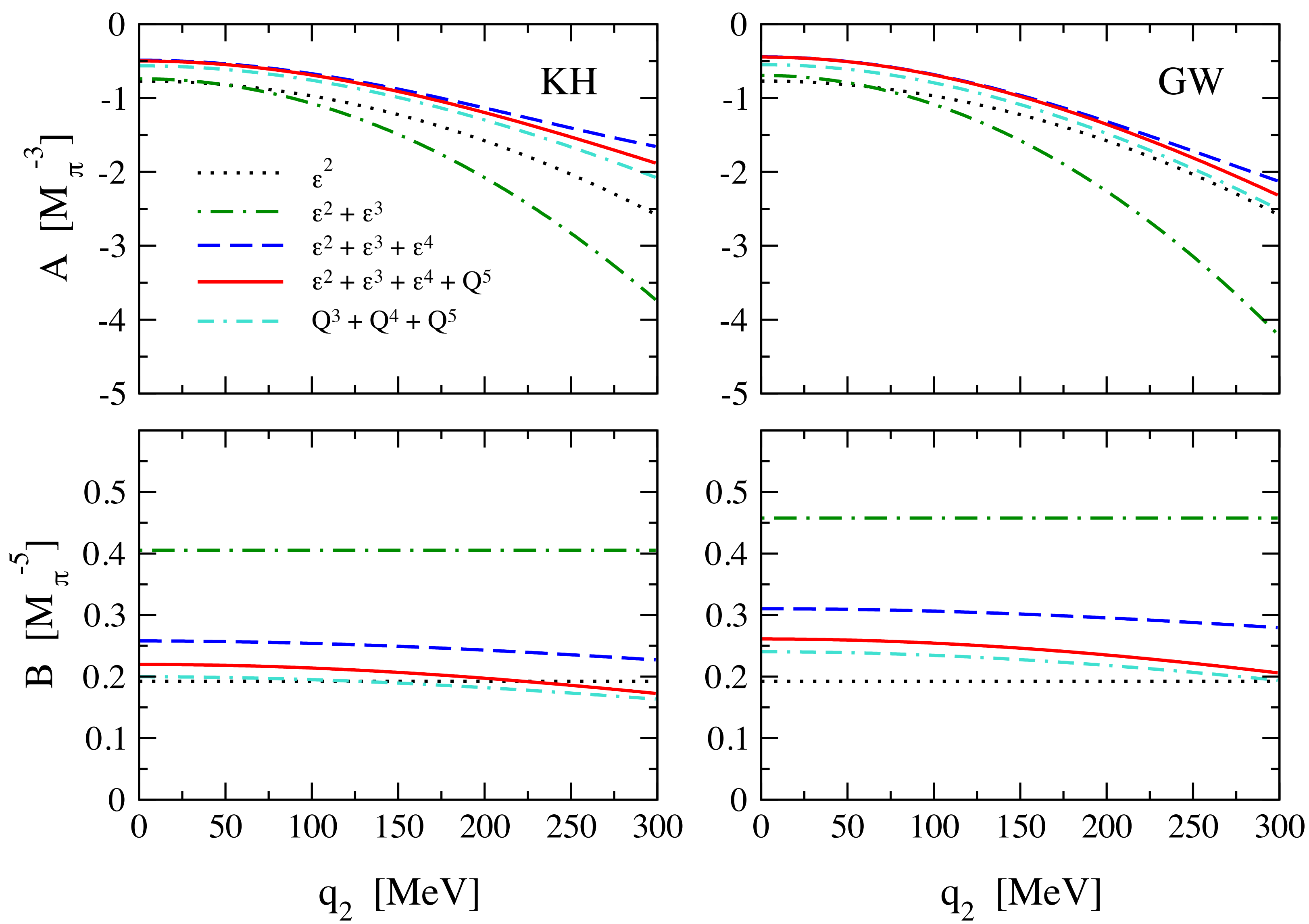
Two-pion-exchange 3NF

Preliminary

Epelbaum, Gasparyan, HK. forthcoming

$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$

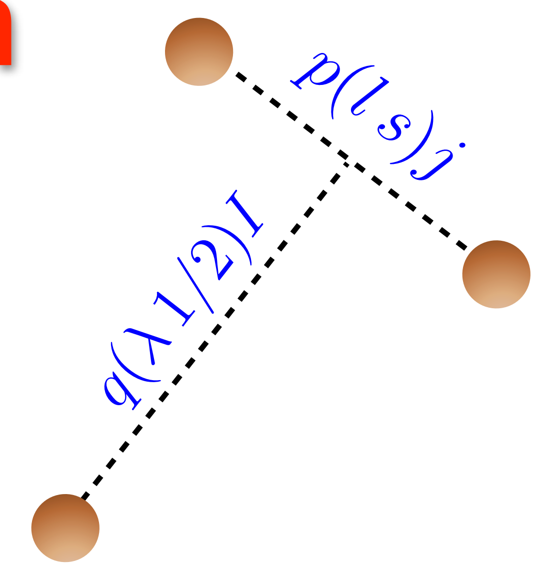




- Similar results for TPE-3NF in N^{3}LO-Δ and N^{4}LO Δ-less approaches

Partial wave decomposition

Golak et al. Eur. Phys. J. A 43 (2010) 241



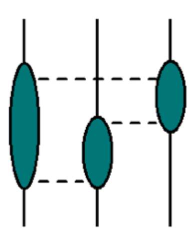
- Faddeev equation is solved in the partial wave basis

$$|p, q, \alpha\rangle \equiv |pq(ls)j(\lambda\frac{1}{2})I(jI)JM_J\rangle\,|(t\frac{1}{2})TM_T\rangle$$

- Too many terms for doing PWD by hand ⟹ Automatization

$$\underbrace{\langle p'q'\alpha'|V|pq\alpha\rangle}_{\text{matrix} \sim 10^5 \times 10^5} = \int \underbrace{d\hat{p}'\,d\hat{q}'\,d\hat{p}\,d\hat{q}}_{\text{can be reduced to 5 dim. integral}} \sum_{m_l,\dots} \Big(\text{CG coeffs.}\Big)\Big(Y_{l,m_l}(\hat{p})\,Y_{l',m_l'}(\hat{p}')\,\dots\Big)\underbrace{\langle m'_{s_1}m'_{s_2}m'_{s_3}|V|m_{s_1}m_{s_2}m_{s_3}\rangle}_{\text{depends on spin \& isospin}}$$

- Ring-diagram-contr. expensive to calculate on the fly



We prestore ring-contr. to 3nf's on a fine momentum grid ⟹ Numerical interpolation of ring terms

- PWD matrix-elements can be used to produce matrix-elements in harmonic oscillator basis

Straightforward implementation of high order 3nf's in many-body calc.
within No-Core Shell Model

PWD for local forces

$$\langle m_s'|\vec{\sigma}\cdot\vec{p}\,|m_s\rangle = \sum_{\mu=-1}^{1} p\,Y_{1\mu}^*(\hat{p})\sqrt{\frac{4\pi}{3}}\langle m_s'|\vec{\sigma}\cdot\vec{e}_\mu\,|m_s\rangle$$

← momentum-independent part

$$\langle m_{s_1}' m_{s_2}' m_{s_3}'|V|m_{s_1} m_{s_2} m_{s_3}\rangle = \sum_{\mu' s} \langle m_{s_1}' m_{s_2}' m_{s_3}'|\text{Spin matrices}\,\&\,\vec{e}_\mu\text{'s}|m_{s_1} m_{s_2} m_{s_3}\rangle (Y_{1\mu}'s)$$

$$\times\ V((\vec{p}\,' - \vec{p}\,)^2, (\vec{q}\,' - \vec{q}\,)^2, (\vec{p}\,' - \vec{p}\,)\cdot(\vec{q}\,' - \vec{q}\,))$$

can be reduced to 3 dim. integral

$$\langle p' q' \alpha'|V|p\,q\,\alpha\rangle = \sum_{m_l\ldots} (\,\text{CG coeffs.}) \int \overbrace{d\hat{p}'\,d\hat{q}'\,d\hat{p}\,d\hat{q}}\, Y_{l_1'm_1'}^*(\hat{p}'\,) Y_{l_2'm_2'}^*(\hat{q}'\,) Y_{l_1 m_1}^*(\hat{p}\,) Y_{l_2 m_2}^*(\hat{q}\,)$$

$$\times\ V((\vec{p}\,' - \vec{p}\,)^2, (\vec{q}\,' - \vec{q}\,)^2, (\vec{p}\,' - \vec{p}\,)\cdot(\vec{q}\,' - \vec{q}\,))$$

Numerical implementation: *talk by Kai Hebeler*

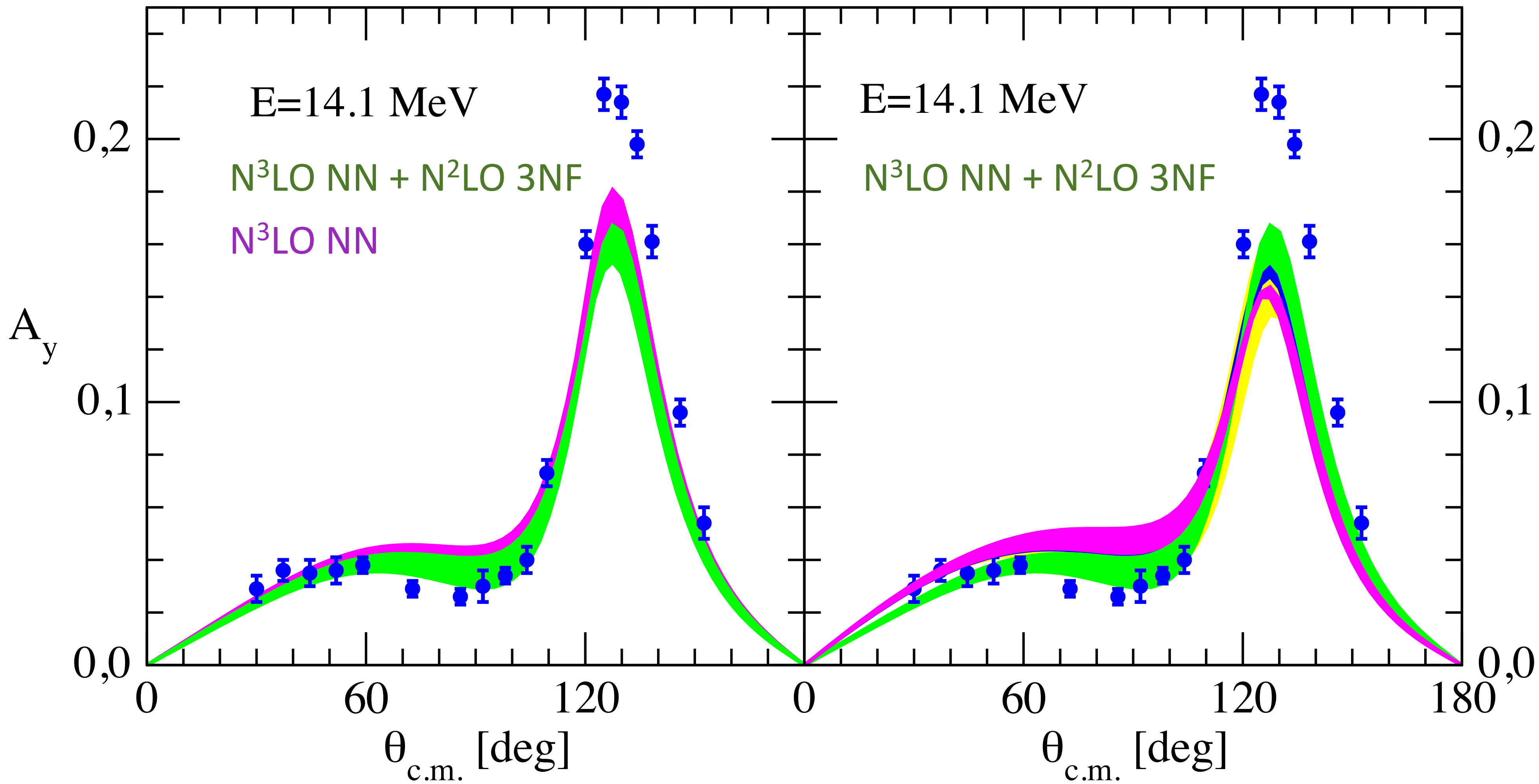
Matrix elements for local part of N^3LO 3nf are calculated upto

$$J^{\max} = 9/2 \quad \& \quad J_2^{\max} = 6$$

Full N^3LO calculations of medium mass nuclei are possible

A_y-puzzle in elastic nd scattering

Witala et al. Proceedings of Few Body 20



Right panel: X = N³LO NN + N²LO 3NF + N³LO 3NF (1π-cont.) + N³LO 3NF (cont.)

= X + N³LO 3NF (2π-exch.)

= X + N³LO 3NF (2π-exch. & 2π-1π-exch.)

= X + N³LO 3NF (2π-exch. & 2π-1π-exch. & ring)

Incomplete results: N³LO 3NF (2π-cont. & 1/m-corr.) are missing

Summary

- Chiral NN forces with local regulators are studied upto N^2LO
  - → QMC calculations with chiral forces
- Long-range part of 3NFs is analyzed up to N^4LO Δ-less/N^3LO-Δ
- Optimized version of PWD for local 3NF's
  - → Matrix elements calculated upto $J^{\max} = 9/2 \quad \& \quad J_2^{\max} = 6$

Outlook

- N^3LO few-body calculations of Nd scattering and breakup reactions
- Chiral NN forces with local regulators upto N^3LO
- N^4LO Δ-less/N^3LO-Δ calc. of shorter range part of 3NF

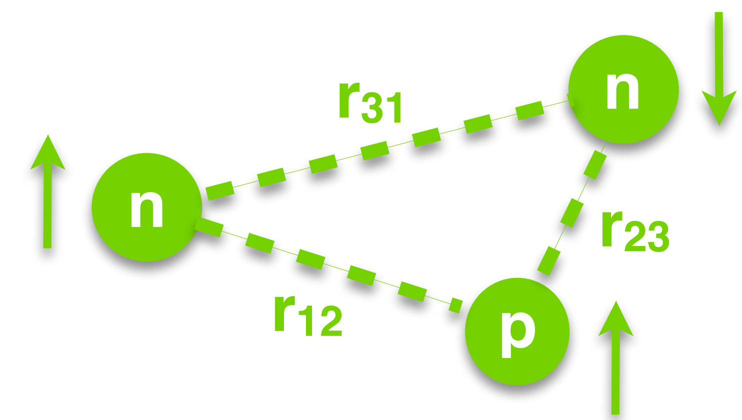
Most general structure of a local 3NF

Epelbaum, Gasparyan, HK, PRC87 (2013) 054007

Up to N^{4}LO, the computed contributions are local $\longrightarrow$ it is natural to switch to r-space.

A meaningful comparison requires a complete set of independent operators

$$
\begin{aligned}
\tilde{\mathcal{G}}_1 &= 1\,, \\
\tilde{\mathcal{G}}_2 &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\,, \\
\tilde{\mathcal{G}}_3 &= \vec{\sigma}_1 \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_4 &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \vec{\sigma}_1 \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_5 &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \vec{\sigma}_1 \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_6 &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)\,, \\
\tilde{\mathcal{G}}_7 &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_8 &= \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_9 &= \hat{r}_{23} \cdot \vec{\sigma}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\,, \\
\tilde{\mathcal{G}}_{10} &= \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{11} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{12} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{13} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{14} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{15} &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \hat{r}_{13} \cdot \vec{\sigma}_1\, \hat{r}_{13} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{16} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_2\, \hat{r}_{12} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{17} &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{18} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \vec{\sigma}_3\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_{19} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_3 \cdot \hat{r}_{23}\, \hat{r}_{23} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\,, \\
\tilde{\mathcal{G}}_{20} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{23}\, \vec{\sigma}_2 \cdot \hat{r}_{23}\, \vec{\sigma}_3 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_{21} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{13}\, \vec{\sigma}_3 \cdot \hat{r}_{13}\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_{22} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{23}\, \vec{\sigma}_3 \cdot \hat{r}_{12}\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,,
\end{aligned}
$$



Building blocks:

$\boldsymbol{\tau}_1,\ \boldsymbol{\tau}_2,\ \boldsymbol{\tau}_3,\ \vec{\sigma}_1,\ \vec{\sigma}_2,\ \vec{\sigma}_3,\ \vec{r}_{12},\ \vec{r}_{23}$

Constraints:

- Locality
- Isospin symmetry
- Parity and time-reversal invariance

$$\longrightarrow \quad V_{3\mathrm{N}} = \sum_{i=1}^{22} \mathcal{G}_i\, F_i(r_{12}, r_{23}, r_{31}) \;+\; 5 \text{ perm.}$$

Redundant operators (for $\tilde{\mathcal{G}}_{20}$, $\tilde{\mathcal{G}}_{21}$)

Schat, Phillips, PRC88 (2013) 034002

Epelbaum, Gasparyan, HK, Schat, forthcoming

Computational strategy

- d-dim one loop tensor integrals by Passarino-Veltman reduction

$$\int \frac{d^d l}{(2\pi)^d} \frac{l_{\mu_1} \dots l_{\mu_n}}{[l^2 - M^2][(l+p)^2 - M^2]} = T^{(1)}_{\mu_1 \dots \mu_n}(p) f_1(p^2) \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2 - M^2][(l+p)^2 - M^2]} + T^{(2)}_{\mu_1 \dots \mu_n}(p) f_2(p^2) \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2 - M^2]}$$

Tensors in p

$f_1(p^2)$ and $f_2(p^2)$ include in general non-physical singularities which cancel in final result

- Dimensional-shift reduction Davydychev '91

Combinatorial factors

$$\int \frac{d^d l}{(2\pi)^d} \frac{l_{\mu_1} \dots l_{\mu_n}}{[l^2 - M^2][(l+p)^2 - M^2]} = \sum_{ij} T^{(i)}_{\mu_1 \dots \mu_n}(p) \int \frac{d^{d+2i} l}{(2\pi)^{d+2i}} \frac{c_{ij}}{[l^2 - M^2]^{n_{ij}}[(l+p)^2 - M^2]^{m_{ij}}}$$

- Partial integration techniques provide recursion relations

$$\int \frac{d^d l}{(2\pi)^d} \frac{\partial}{\partial l_\mu} f(l) = 0$$ ⟸ Connection betw. Dimensional-shift and Passarino-Veltman red.

Implement Heavy-Baryon extension of these techniques in Mathematica/FORM

Explicit decoupling

Don‘t positive powers of Δ possibly spoil the convergence?

Small scale expansion parameter $\Delta/\Lambda_\chi \sim \frac{1}{3}$ is not that small!

Manifest decoupling through the choice of renormalization conditions (no positive powers of Δ)

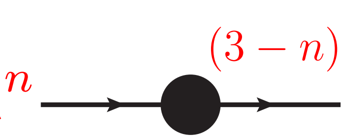
Decoupling theorem due to Appelquist & Carrazone Phys. Rev. 11 (1974) 2856

$$\mathcal{L}_{\pi N}^{\rm SSE} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \Delta \mathcal{L}_{\pi N}^{(2)} + \Delta^2 \mathcal{L}_{\pi N}^{(1)} + \mathcal{O}(\epsilon^4)$$

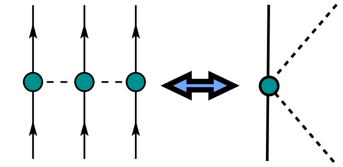
Choose finite part of these LECs such that

$$\lim_{\Delta \to \infty} \text{Green Function} < \infty$$

Bernard, Fearing, Hemmert, Meißner NPA635 (1998) 121

$\lim_{\Delta \to \infty} \Big[$  $+ \sum_{n=1}^{3} \Delta^n$  $\Big] = 0$

Pion-nucleon scattering



Heavy baryon calculation up to order q^4 *Fettes, Meißner Nucl. Phys. A676 (2000) 311*

1/m power counting used in FM work ⇒ $\frac{p}{m} \sim \frac{q}{\Lambda_\chi}$

- Difference in Weinberg's power counting for NN ⇒ $\frac{p}{m} \sim \left(\frac{q}{\Lambda_\chi}\right)^2$

Refit of d_i and e_i LECs is needed

$$\pi^a(q_1) + N(p_1) \to \pi^b(q_2) + N(p_2)$$

$$T^{ba}_{\pi N} = \frac{E+m}{2m}\left(\delta^{ba}\left[g^+(\omega,t) + i\,\vec{\sigma}\cdot\vec{q}_2\times\vec{q}_1\,h^+(\omega,t)\right] + i\,\epsilon^{bac}\tau^c\left[g^-(\omega,t) + i\vec{\sigma}\cdot\vec{q}_2\times\vec{q}_1\,h^-(\omega,t)\right]\right)$$

CMS kinematics: $\omega = q_1^0 = q_2^0, \quad E = E_1 = E_2 = \sqrt{\vec{q}^{\,2} + m^2}, \quad \vec{q}_1^{\,2} = \vec{q}_2^{\,2} = \vec{q}^{\,2}, \quad t = (q_1 - q_2)^2$

Partial wave amplitudes: $f^{\pm}_{l\pm}(s) = \frac{E+m}{16\pi\sqrt{s}}\int_{-1}^{1} dz\left[g^{\pm}P_l(z) + \vec{q}^{\,2}h^{\pm}\left(P_{l\pm1}(z) - zP_l(z)\right)\right]$

In the isospin basis: $f^{1/2}_{l\pm} = f^+_{l\pm} + 2f^-_{l\pm}, \quad f^{3/2}_{l\pm} = f^+_{l\pm} - f^-_{l\pm}$

Absence of inelasticity below the two-pion production threshold

$$\delta^I_{l\pm}(s) = \arctan\left(|\vec{q}|\mathcal{R}e\, f^I_{l\pm}(s)\right)$$