Five- and six-nucleon scattering from QCD-based interaction

Nuclear Structure & Reactions workshop TRIUMF Vancouver, February 17th 2014.

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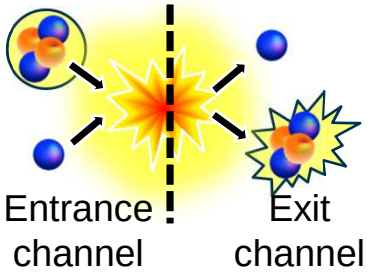
Ab initio NCSM/RGM: formalism for binary clusters

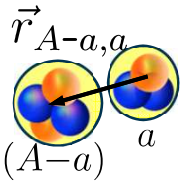
S. Quaglioni and P. Navrátil, PRL101 (2008); PRC79 (2009)

Ex: n-^{4}He scattering



- Starts from:

$$\Psi^{(A)}_{RGM} = \sum_{\nu} \int d\vec{r}\; g_{\nu}(\vec{r})\; \hat{A}_{\nu} \left| \Phi^{(A-a,a)}_{\nu\vec{r}} \right\rangle \Longleftrightarrow \psi^{(A-a)}_{\alpha_1} \psi^{(a)}_{\alpha_2} \delta\left(\vec{r} - \vec{r}_{A-a,a}\right)$$



Relative wave function (unknown) — Antisymmetrizer — Channel basis

Cluster expansion technique

- Schrödinger equation on channel basis:

$$H\Psi^{(A)}_{RGM} = E\Psi^{(A)}_{RGM} \Longrightarrow \sum_{\nu} \int d\vec{r}\; \left[H_{\nu'\nu}(\vec{r}',\vec{r}) - E\; N_{\nu'\nu}(\vec{r}',\vec{r}) \right] g_{\nu}(\vec{r}) = 0$$

- RGM accounts for: 1) interaction (Hamiltonian kernel), 2) Pauli principle (Norm kernel) between clusters.
- NCSM accounts for: internal structure of clusters.
- Together with the same microscopic nuclear interaction.

Demonstrated capability to describe binary-cluster reactions starting from NN interactions

☑ Nucleon-nucleus collisions

- ✓ n-^{3}H, p-^{3}He, N-^{4}He, n-^{10}Be scattering with N^{3}LO NN (mod. Lee-Suzuki eff. Int.)
- ✓ Nucleon scattering on ^{3}H, ^{3,4}He,^{7}Li,^{7}Be,^{12}C,^{16}O with SRG-N^{3}LO
- ✓ ^{7}Be(p,γ)^{8}B radiative capture with SRG-N^{3}LO

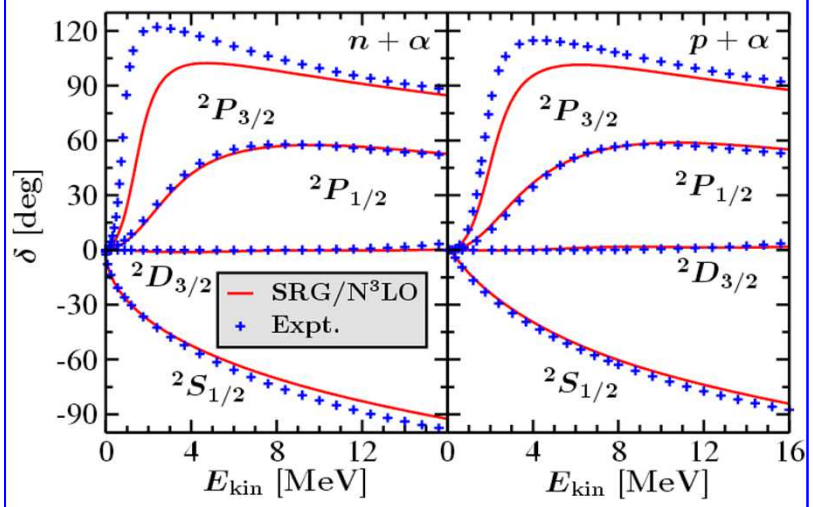
☑Deuterium-nucleus collisions

- ✓ d-^{4}He scattering and ^{6}Li structure with SRG-N^{3}LO

☑(d,N) transfer reactions

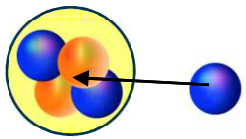
- ✓ ^{3}H(d,n)^{4}He and ^{3}He(d,p)^{4}He reactions with SRG-N^{3}LO

n-He^{4} phase-shift NN only



λ dependent

Good reproduction, but still incomplete



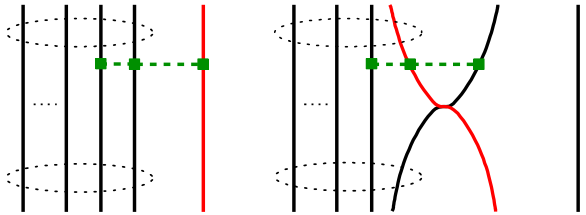
n-^{4}He scattering

Including the NNN force into the NCSM/RGM approach

nucleon-nucleus formalism

$$\left\langle \Phi^{J^\pi T}_{\nu' r'} \left| \hat{A}_{\nu'} V^{NNN} \hat{A}_{\nu} \right| \Phi^{J^\pi T}_{\nu r} \right\rangle = \left\langle \; (A-1) \;\; r' \;\; (a'=1) \; \left| V^{NNN} \left(1 - \sum_{i=1}^{A-1} \hat{P}_{iA} \right) \right| \; (a=1) \;\; r \;\; (A-1) \; \right\rangle$$

$$\mathcal{V}^{NNN}_{\nu'\nu}(r,r') = \sum R_{n'l'}(r') R_{nl}(r) \left[\frac{(A-1)(A-2)}{2} \left\langle \Phi^{J^\pi T}_{\nu' n'} | V_{A-2A-1A}(1 - 2P_{A-1A}) | \Phi^{J^\pi T}_{\nu n} \right\rangle \right.$$

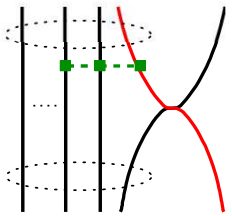


(a) (b)

Direct potential:

$$\propto {}_{SD}\left\langle \psi^{(A-1)}_{\alpha'_1} \left| a^+_i a^+_j a_l a_k \right| \psi^{(A-1)}_{\alpha_1} \right\rangle_{SD}$$

$$\left. - \frac{(A-1)(A-2)(A-3)}{2} \left\langle \Phi^{J^\pi T}_{\nu' n'} | P_{A-1A} V_{A-3A-2A-1} | \Phi^{J^\pi T}_{\nu n} \right\rangle \right] .$$



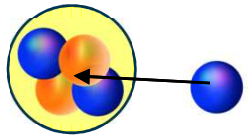
(c)

Exchange potential:

$$\propto {}_{SD}\left\langle \psi^{(A-1)}_{\alpha'_1} \left| a^+_h a^+_i a^+_j a_m a_l a_k \right| \psi^{(A-1)}_{\alpha_1} \right\rangle_{SD}$$

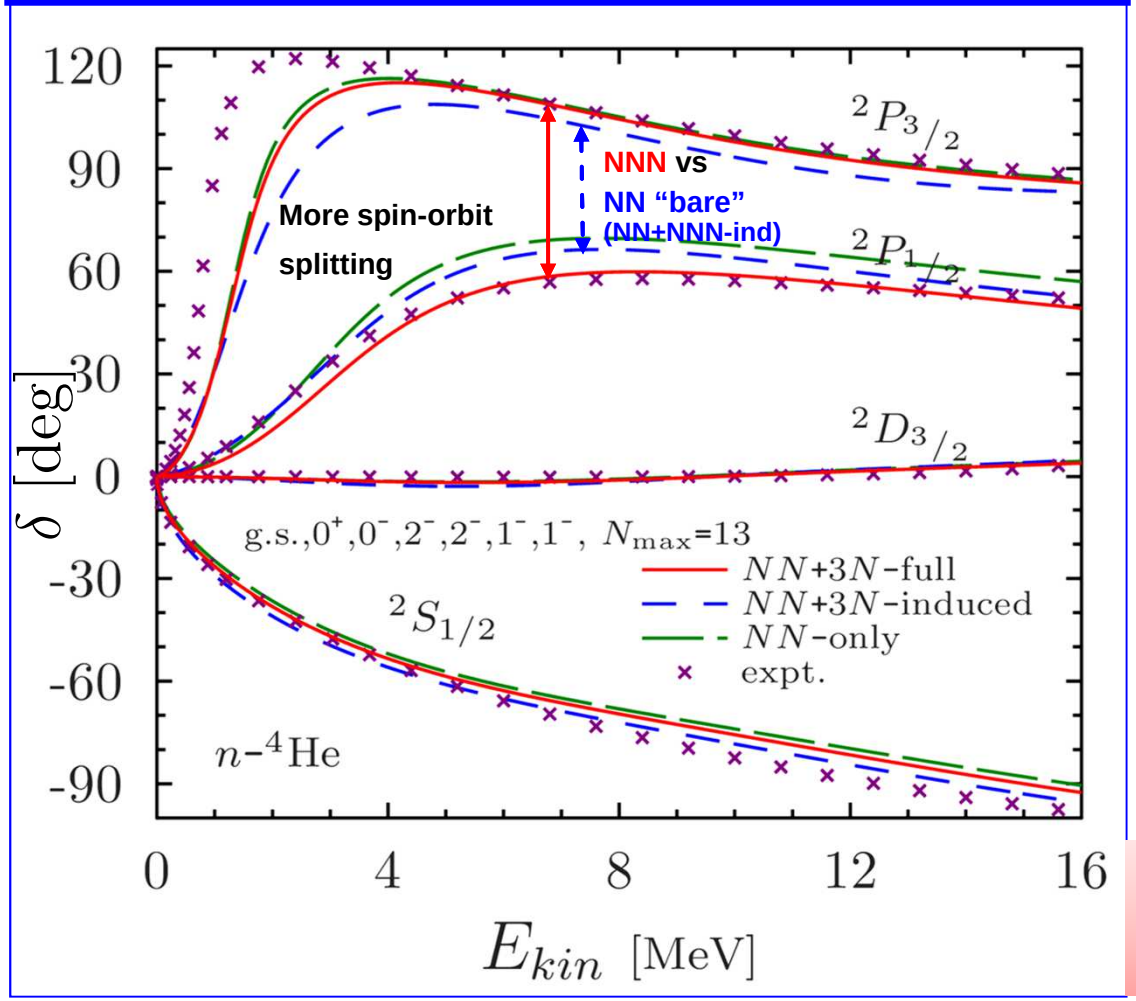
n-^{4}He scattering: NN versus NNN interactions

G. Hupin, J. Langhammer *et al.* PRC88 (2013)



n-^{4}He scattering

Three scenarii of nuclear Hamiltonians



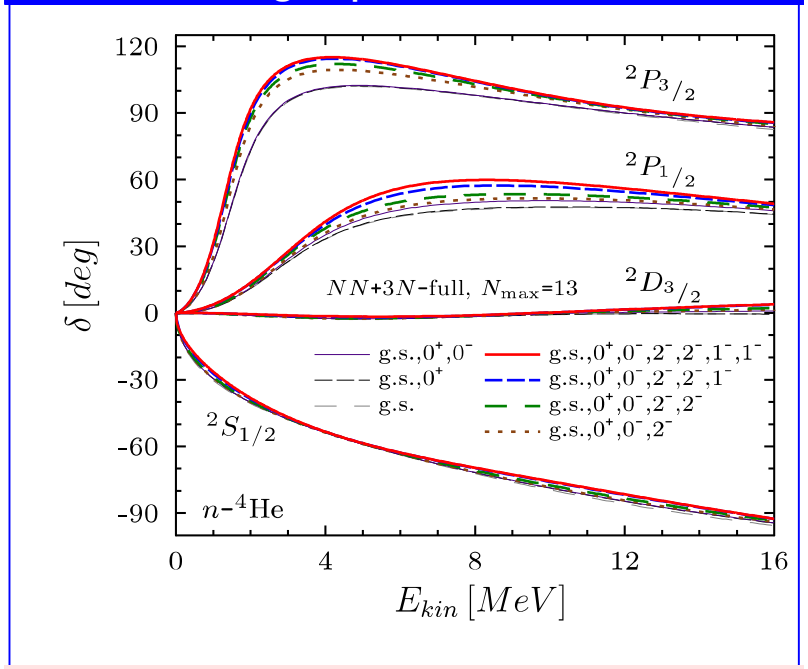
- The NNN interactions influence mostly the P waves.
- The largest splitting between *P* waves is obtained with NN+NNN.
- The agreement of the $P_{3/2}$ phase-shifts between NN-only and NN+NNN forces is accidental.

Comparison between NN+NNN -ind and NN+NNN at Nmax=13 with six ^{4}He states.

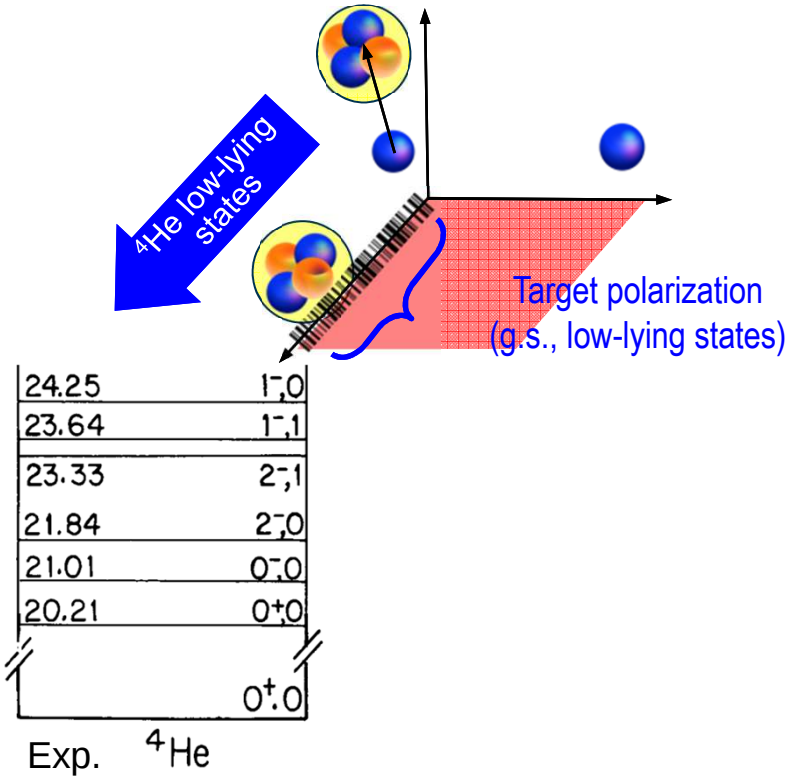
n-^{4}He scattering: study of the RGM convergence in the NNN case

G. Hupin, J. Langhammer *et al.* PRC88 (2013)

Convergence with respect to the target polarization



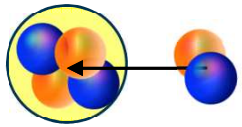
Convergence of the phase-shifts as a function of the ^{4}He excited states.



- We have included the first 6 low-lying states of ^{4}He.
- Convergence is difficult to assess.

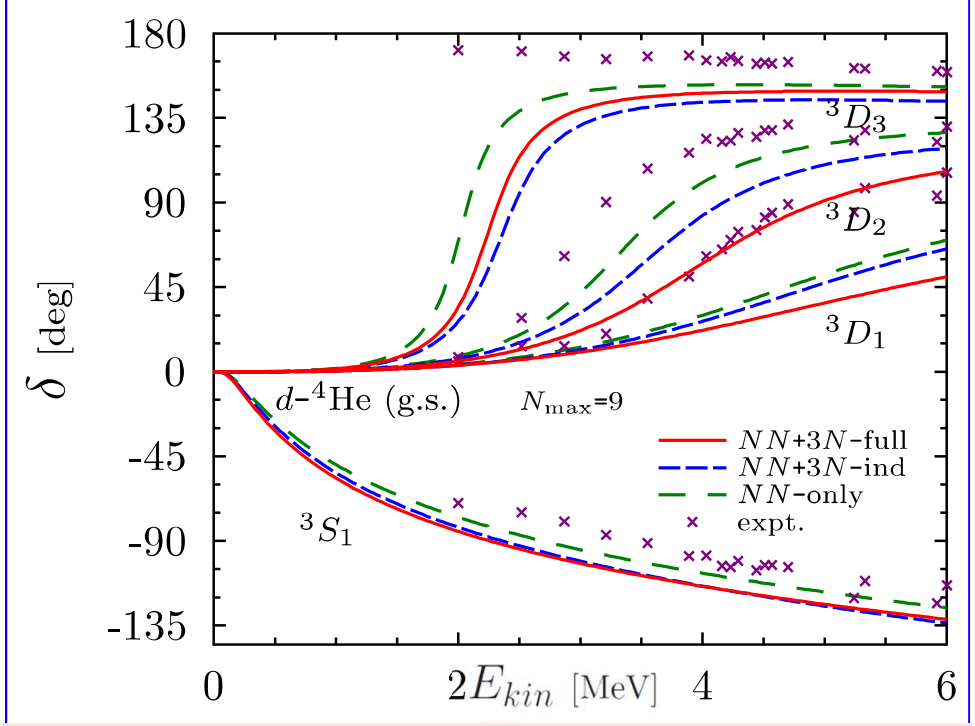
^{4}He(d,d)^{4}He with NN+NNN interaction

G. Hupin, S. Quaglioni and P. Navrátil, work in progress

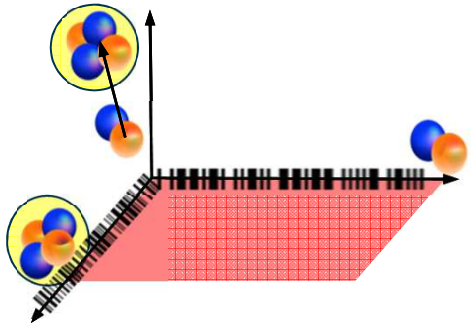


d-^{4}He scattering

Comparison of the d-α phase-shifts with different interactions



d-^{4}He(g.s.) scattering phase-shifts for NN, NN+NNN-induced and NN+NNN potential with λ=2.0 fm^{-1}.

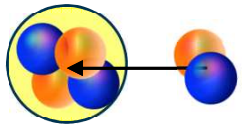


E_B [MeV]	NN-only	NN+NNN-ind	NN+NNN-full
^6Li	−0.76	−0.53	−0.38

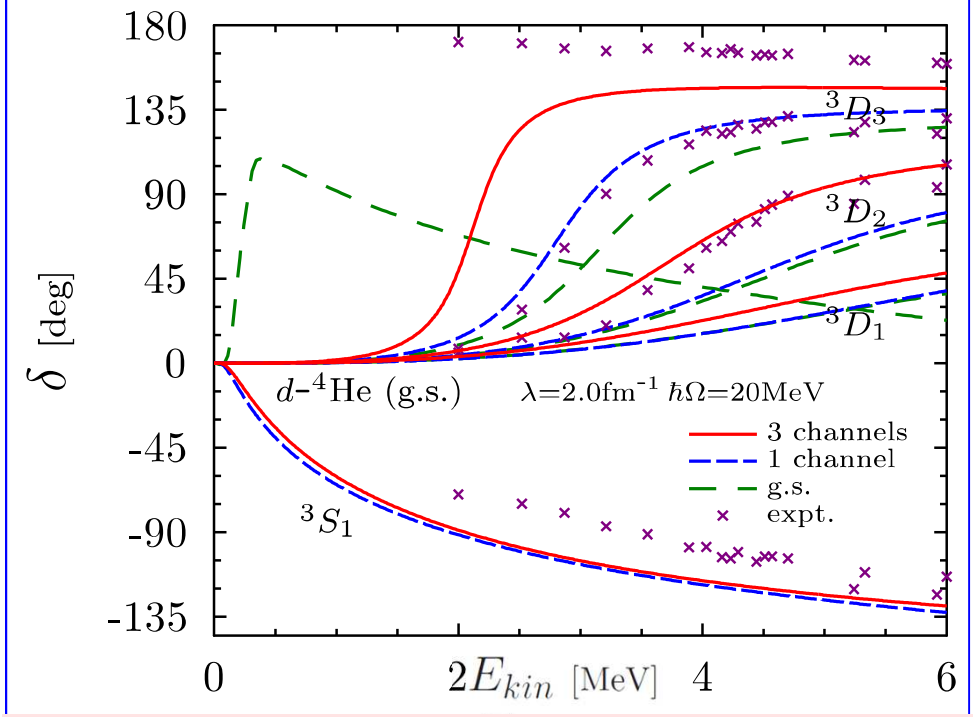
- ^{6}Li loses binding energy with NNN interactions.
- The largest splitting between $^3D_3$ and $^3D_2$ partial waves is obtained with NN+NNN.

Study of the influence of *d** continuum

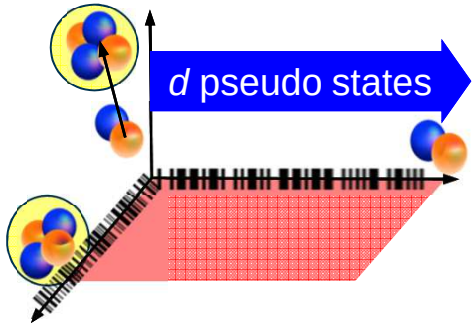
G. Hupin, S. Quaglioni and P. Navrátil, work in progress



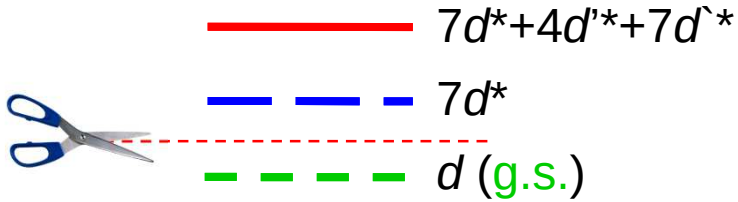
Comparison of the *d*-α phase-shifts with different interactions



d-^{4}He(g.s.) scattering phase-shifts with NN+NNN interaction and λ=2.0 fm^{-1} at Nmax=7.



$^3S_1$-$^3D_1$ channel, $^3D_2$ channel…



Pseudo-states in each deuteron channels

d (g.s.,$^3S_1$-$^3D_1$, $^3D_2$, $^3D_3$-$^3G_3$)

To overcome the difficulty: couple NCSM and NCSM/RGM (NCSMC)

S. Baroni, P. Navrátil and S. Quaglioni PRL110 (2013)

- Methods develop in this presentation to solve the many body problem

$$\Psi^{(A)}_{NCSM} = \sum_{\lambda} c_{\lambda} \left| A\lambda J^{\pi} T \right\rangle$$

Mixing coefficients (unknown)

A-body harmonic oscillator states

$$\left| A\lambda J^{\pi} T \right\rangle_{SD} \varphi_{00}\left(\vec{R}^{A}_{c.m.}\right)$$

Second quantization

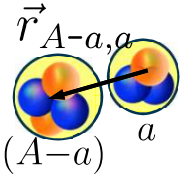
Can address bound and low-lying resonances (short range correlations)

$$\Psi^{(A)}_{RGM} = \sum_{\nu} \int d\vec{r}\, g_{\nu}(\vec{r})\, \hat{A}_{\nu} \left| \Phi^{(A-a,a)}_{\nu\vec{r}} \right\rangle$$

Relative wave function (unknown)

Antisymmetrizer

Channel basis



$$\psi^{(A-a)}_{\alpha_1} \psi^{(a)}_{\alpha_2} \delta\left(\vec{r} - \vec{r}_{A-a,a}\right)$$

Cluster expansion technique

Design to account for the continuum of scattering state (long range correlations)

- The many body <u>quantum</u> problem best describe by superposition of both

$$\Psi^{(A)}_{NCSMC} = \sum_{\lambda} c_{\lambda} \left| A\lambda J^{\pi} T \right\rangle + \sum_{\nu} \int d\vec{r}\, g_{\nu}(\vec{r})\, \hat{A}_{\nu} \left| \Phi^{(A-a,a)}_{\nu\vec{r}} \right\rangle$$

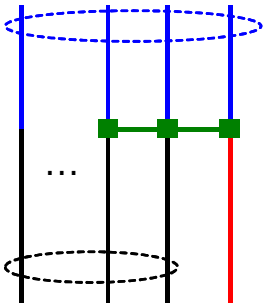
NCSMC

Including the NNN force into the NCSMC approach

nucleon-nucleus formalism

$$\left\langle A\lambda J^\pi T \middle| \mathcal{V}^{NNN}_{A-2,A-1,A} \middle| \Phi^{J^\pi T}_{\nu n} \right\rangle = \left\langle \; (A-1) \;\; r' \;\; (a'=1) \; \middle| \; V^{NNN} \; \middle| \; \right\rangle$$

A-body compound system

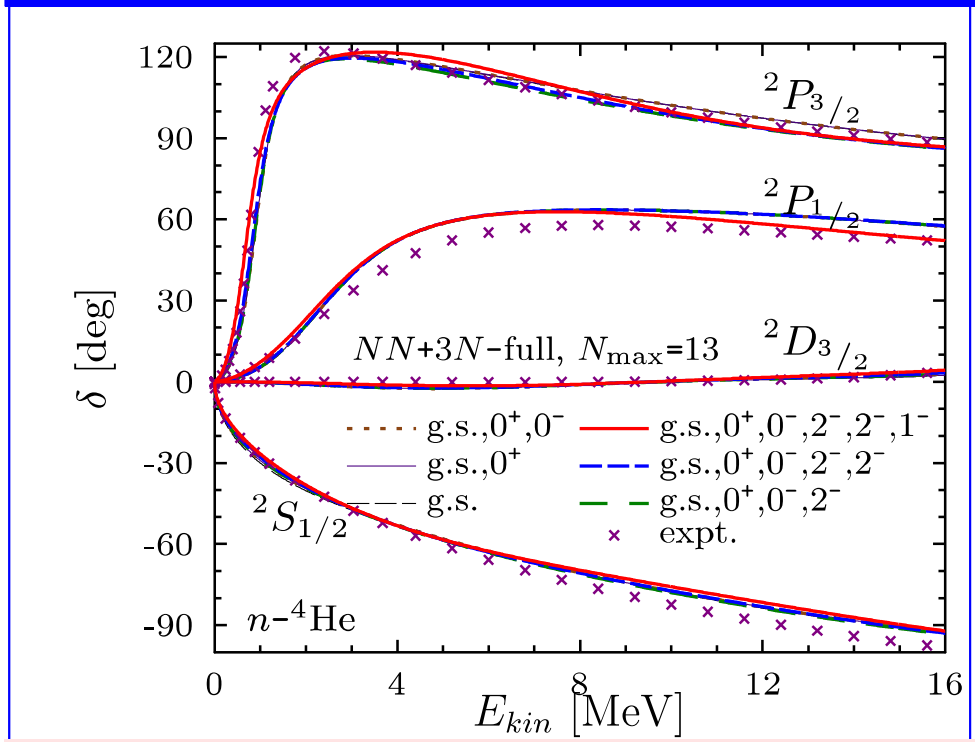


Target and projectile in relative motion

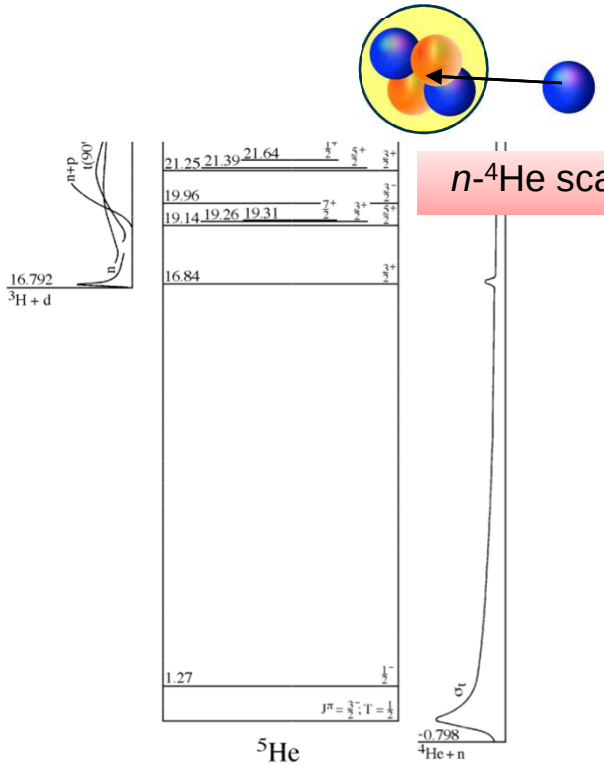
n-^{4}He scattering with NCSMC

G. Hupin, S. Quaglioni and P. Navrátil, work in progress

Study of the convergence with respect to the # of ^{4}He low-lying states



n-^{4}He scattering phase-shifts for NN+NNN potential with λ=2.0 fm^{-1} and 8 low-lying state of ^{5}He.



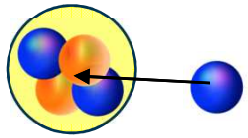
n-^{4}He scattering

Experimental low-lying states of the A=5 nucleon systems.

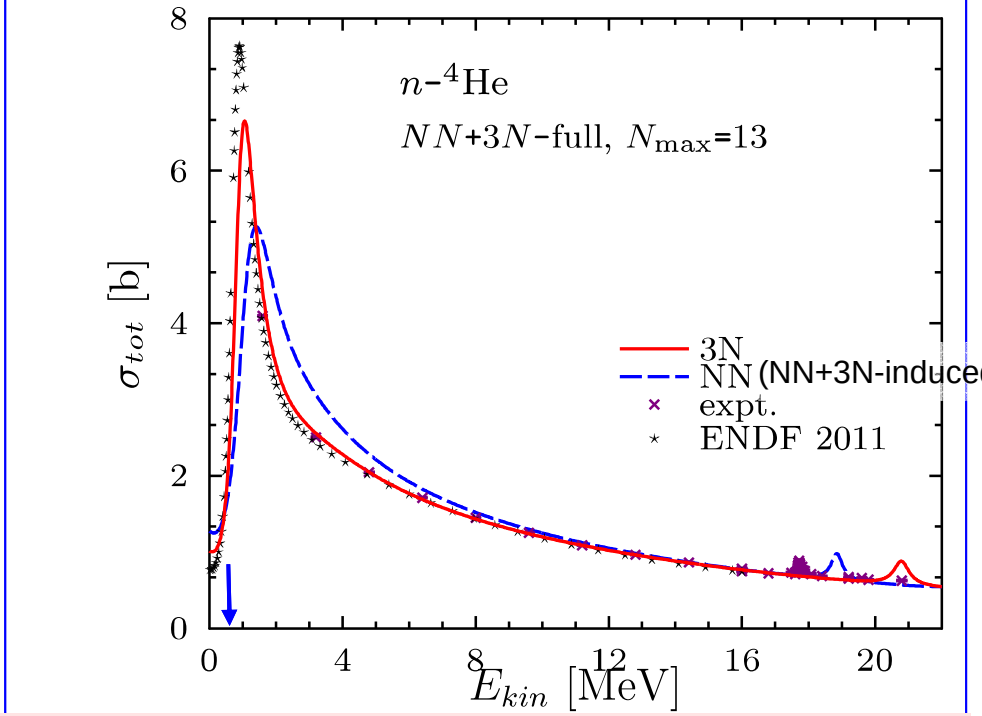
- The convergence pattern looks good.
- The experimental phase-shifts are well reproduced.

How *n*-^{4}He elastic cross-sections compare ?

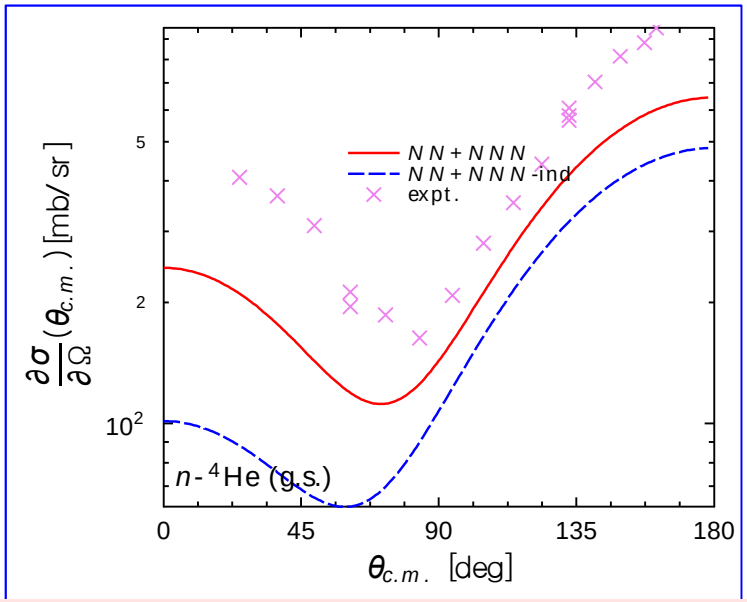
G. Hupin, S. Quaglioni and P. Navrátil, work in progress



Comparison of the elastic cross-section between NN and NN+3N with ^{4}He (g.s.)



n-^{4}He elastic cross-section for NN+3N-induced, NN+3N potentials compared to expt. and ENDF evaluation .

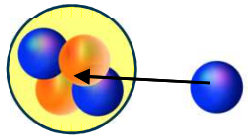


Differential cross-section at $E_{neutron}$ =0.84 MeV between NN+3N-ind and NN+3N.

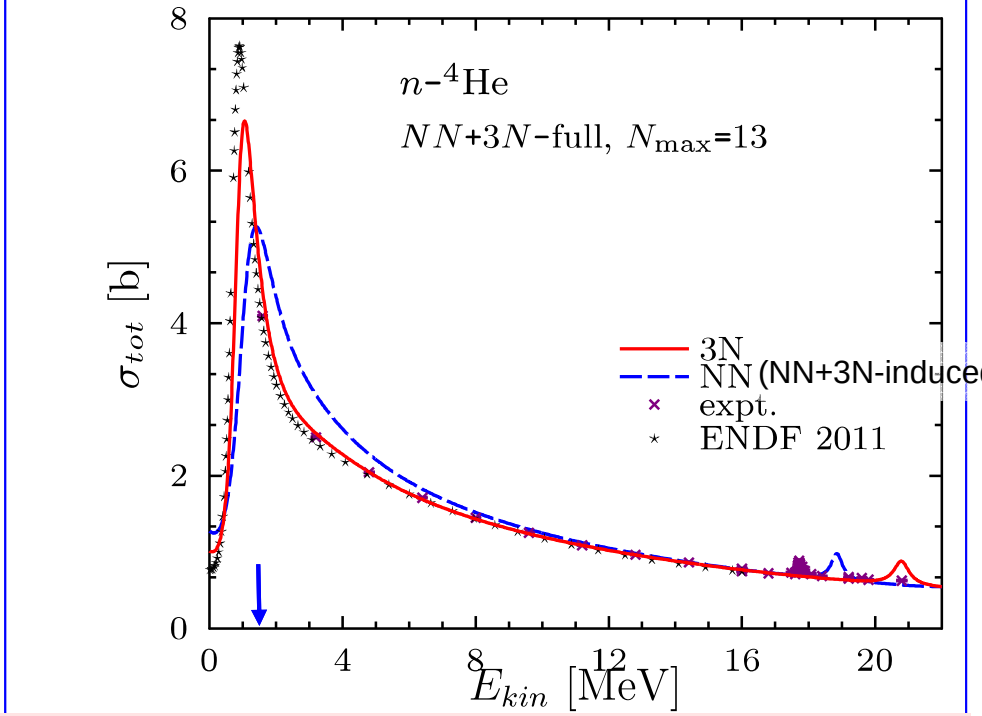
- A better agreement with experiment is obtained with NN+NNN.
- The NNN force is essential to get the resonance right.

How *n*-^{4}He elastic cross-sections compare ?

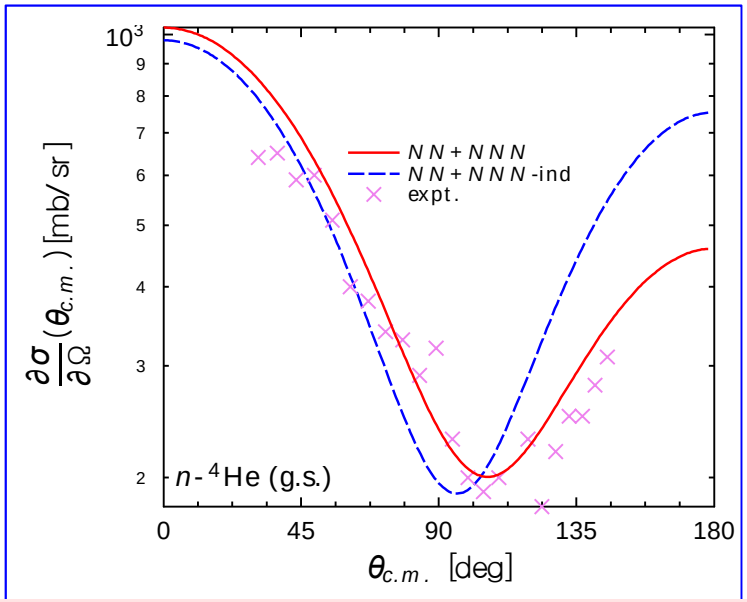
G. Hupin, S. Quaglioni and P. Navrátil, work in progress



Comparison of the elastic cross-section between NN and NN+3N with ^{4}He (g.s.)



n-^{4}He elastic cross-section for NN+3N-induced, NN+3N potentials compared to expt. and ENDF evaluation .

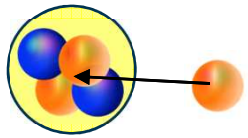


Differential cross-section at $E_{neutron}$ =1.79 MeV between NN+3N-ind and NN+3N.

- A better agreement with experiment is obtained with NN+NNN.
- The NNN force is essential to get the resonance right.

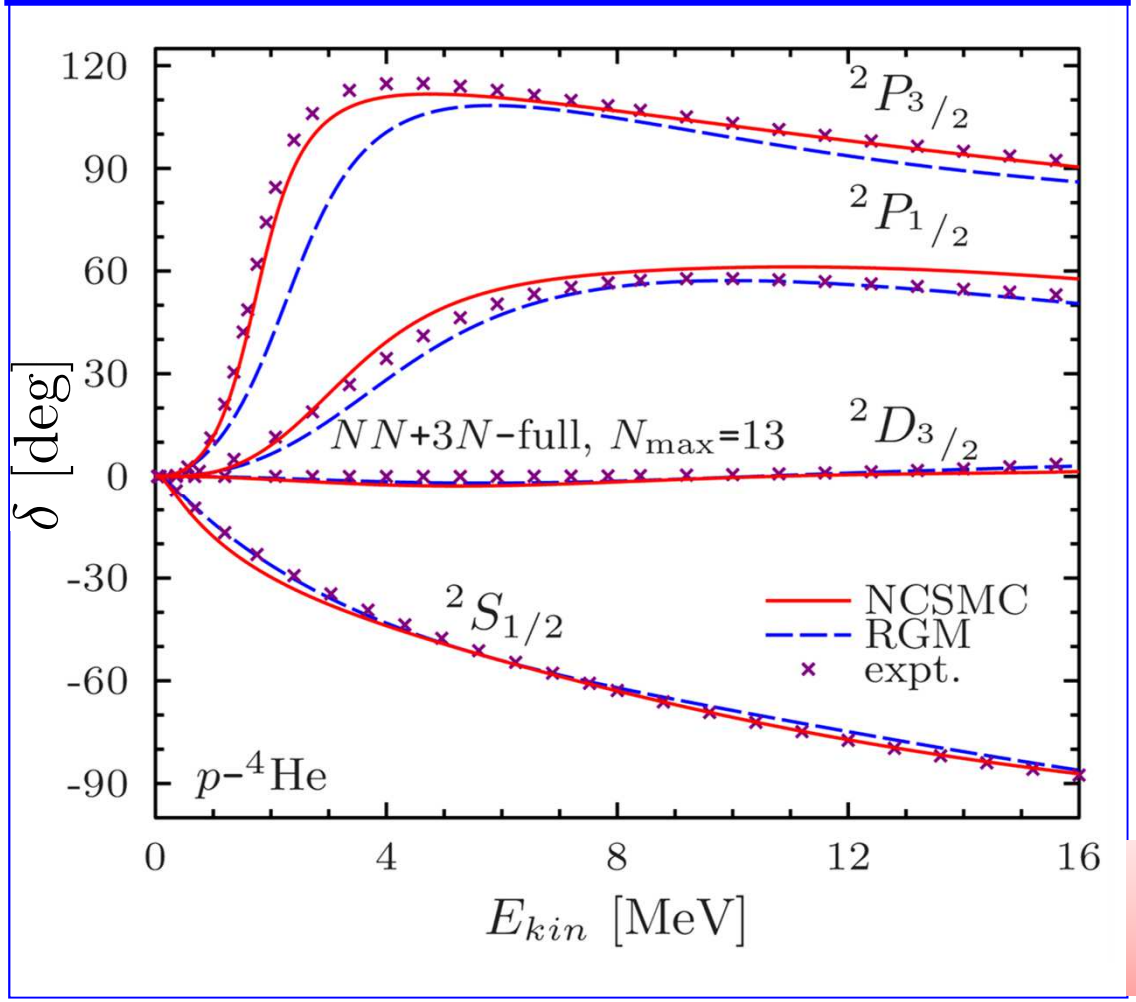
p-^{4}He scattering: NCSM/RGM and NCSMC

G. Hupin, S. Quaglioni and P. Navrátil, work in progress



p-^{4}He scattering

NN+NNN NCSM/RGM and NCSMC



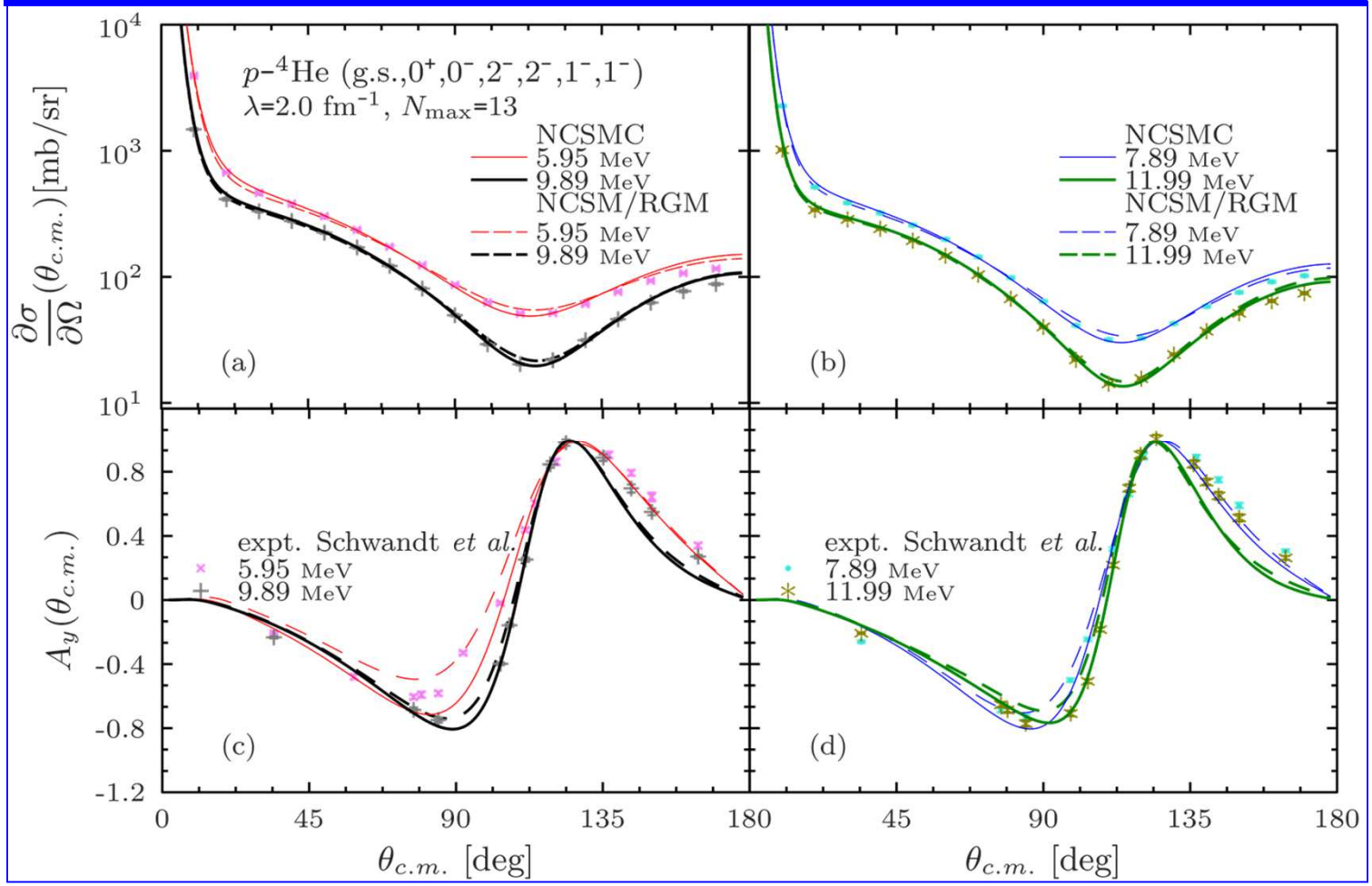
- Preliminary calculation, the NCSMC results are obtained with only the ^{4}He ground state.
- We can see an improvement in the reproduction of the experimental phase-shifts.

Comparison between NCSM/RGM and NCSMC at Nmax=13 and λ=2.0 with NN+NNN.

Analyzing power and differential cross section

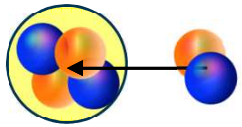
G. Hupin, S. Quaglioni and P. Navrátil, work in progress

p-^{4}He reaction observables compared to experiment



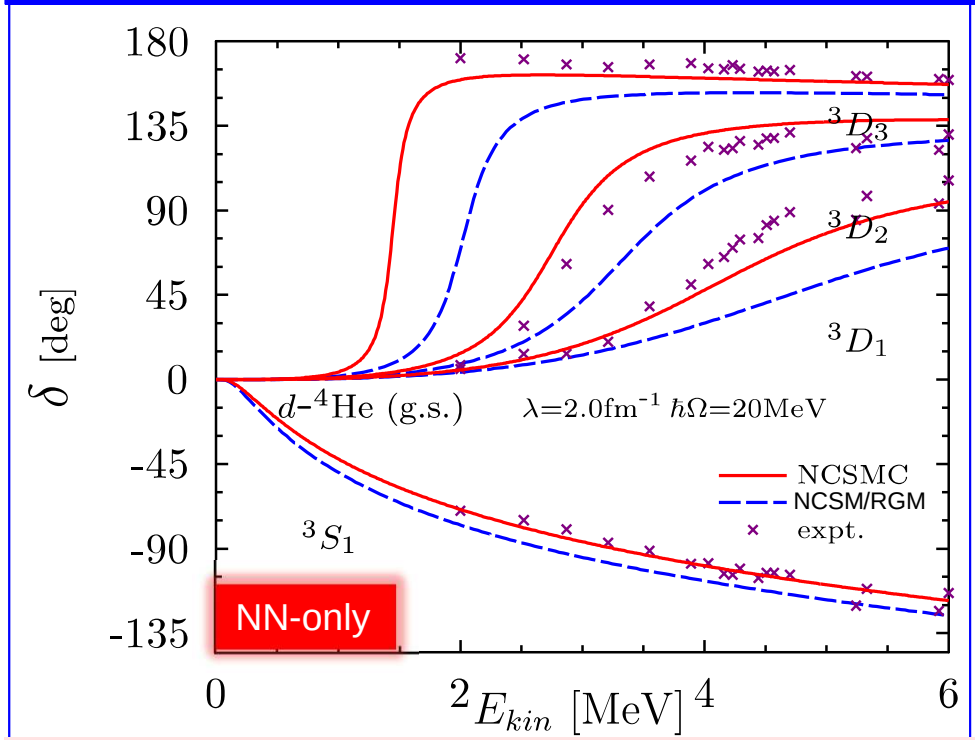
^{4}He(d,d)^{4}He with NCSMC

G. Hupin, S. Quaglioni and P. Navrátil, work in progress

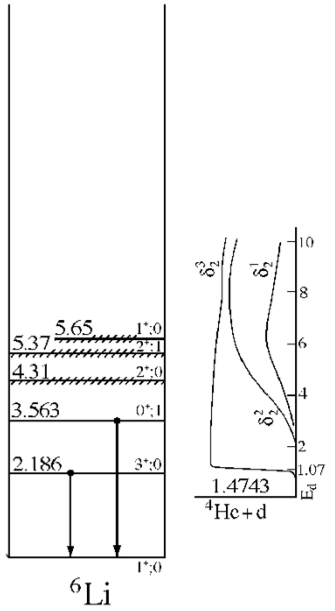


d-^{4}He scattering

Effects of the short-range correlation of the compound nuclei on the d-α phase-shifts



d-^{4}He(g.s.) scattering phase-shifts for NN+NNN potential using NCSM/RGM and NCSMC.

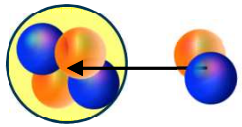


Experimental bound and low-lying states of the A=6 nucleon systems.

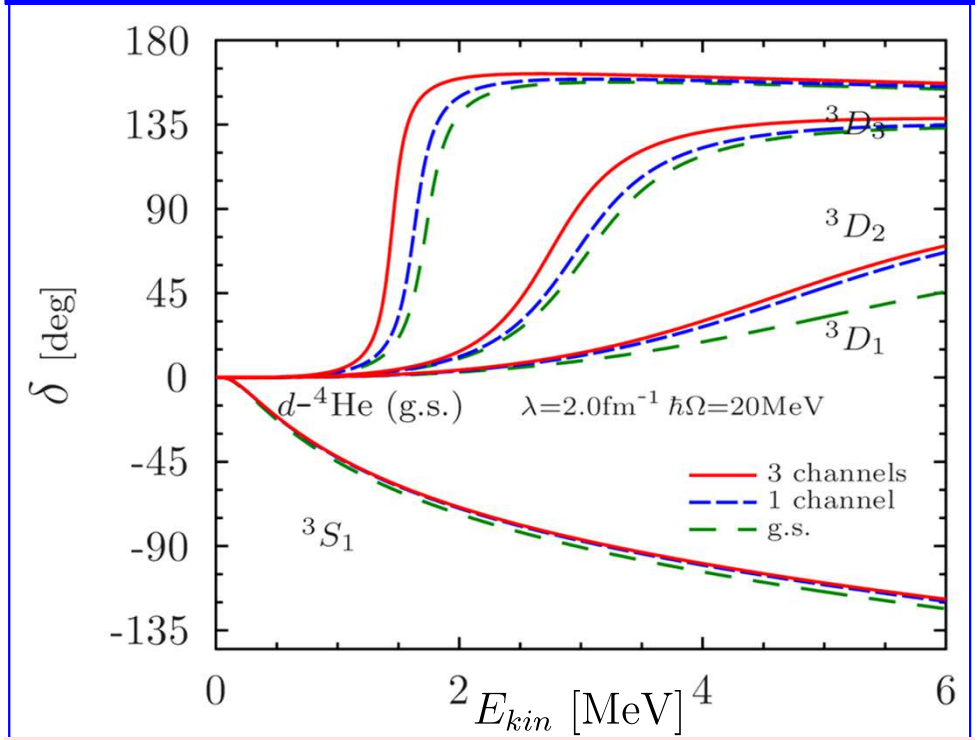
- Preliminary results in a small model space (Nmax=9).
- The coupling to the compound nuclei addresses some missing correlation.
- Some splitting between the $^3D_3$ and $^3D_2$ phase-shifts is missing.

^{4}He(d,d)^{4}He with NN+NNN interaction

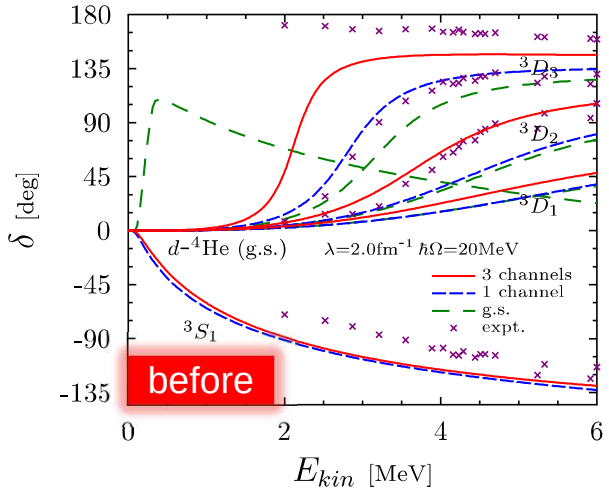
G. Hupin, S. Quaglioni and P. Navrátil, work in progress



Comparison of the d-α phase-shifts wrt the number of d^* pseudo-states



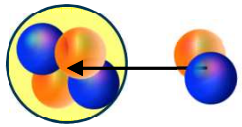
d-^{4}He(g.s.) scattering phase-shifts for NN-only with different numbers of deuteron pseudo-states.



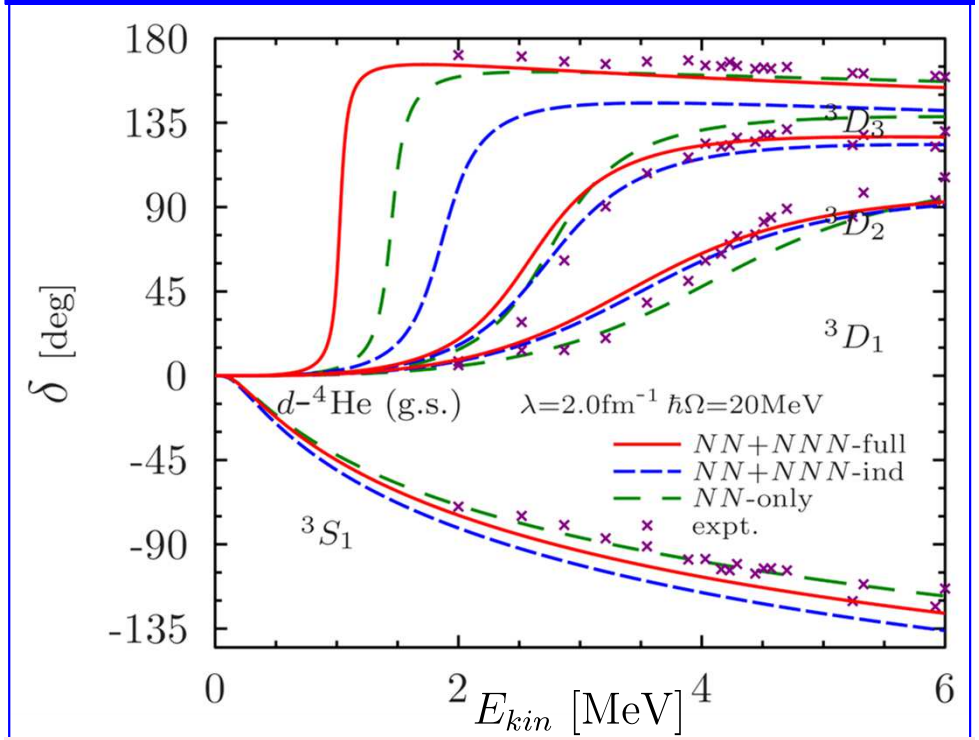
- The NCMSC weakens the dependence on the d^* pseudo-states.
- Residual dependence could be attributed to the missing breakup channel.

^{4}He(d,d)^{4}He with NN+NNN interaction

G. Hupin, S. Quaglioni and P. Navrátil, work in progress



Comparison of the d-α phase-shifts with different interactions

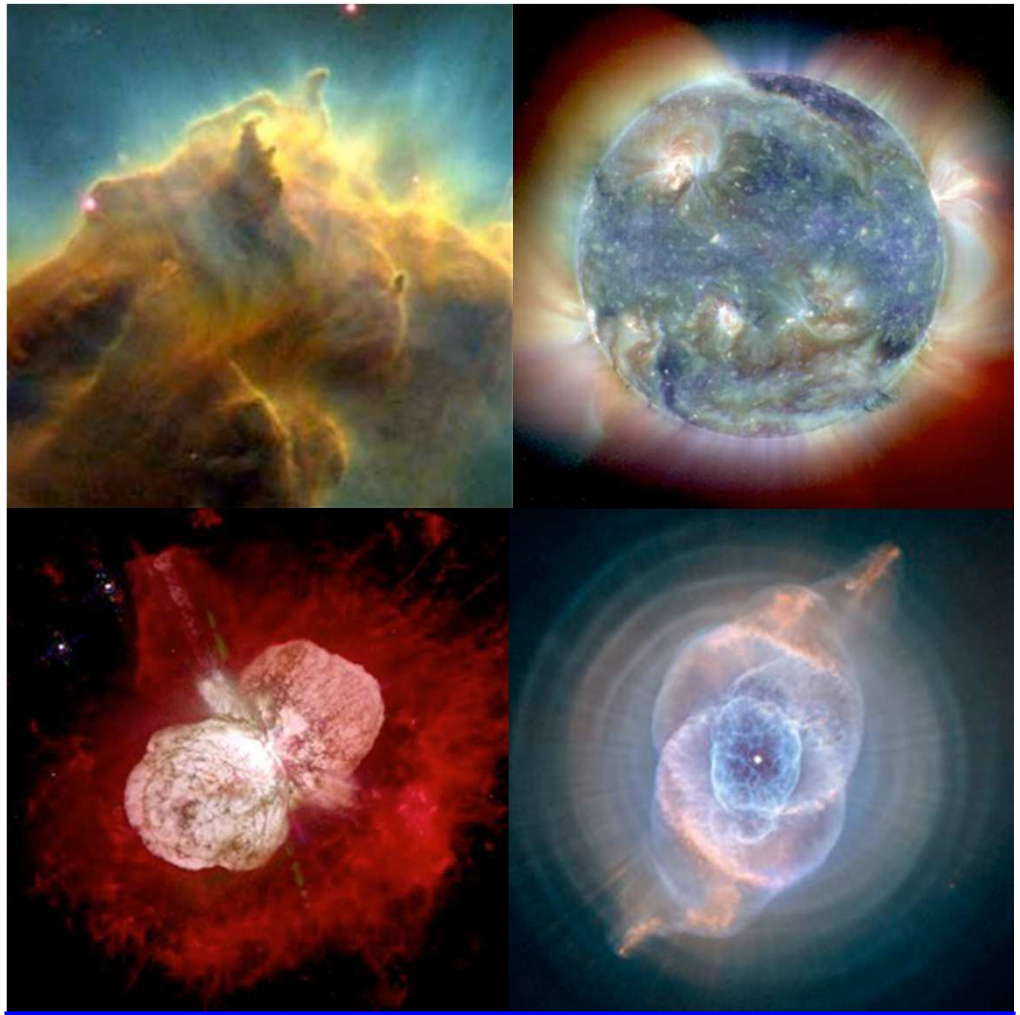


d+^{4}He(g.s.) scattering phase-shifts for NN-only, NN+NNN-induced, NN+NNN potential with λ=2 fm^{-1}.

E_B [MeV]	NN-only	NN+NNN-ind	NN+NNN-full
^6Li	-1.43	-1.15	-1.73

- Preliminary results in a small model space (Nmax=9).
- The $^3D_3$ resonance is not quite reproduced but the 3N force is helping to get the correct position.

Conclusions and Outlook



Evolution of stars, birth, main sequence, death

- We are extending the *ab initio* NCSM/RGM approach to describe low-energy reactions with two- and three-nucleon interactions.
- We are able to describe:
  - Nucleon-nucleus collisions with NN+NNN interaction
  - Deuterium-nucleus collisions with NN+NNN interaction
  - NCSMC for single- and two-nucleon projectile
- Work in progress
  - Fusion reactions with our best complete *ab initio* approach
  - The present NNN force is "incomplete", need to go to N^3LO
  - Scattering of heavier target