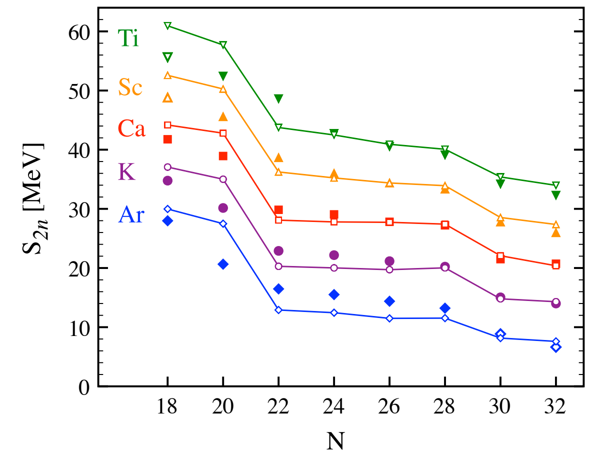
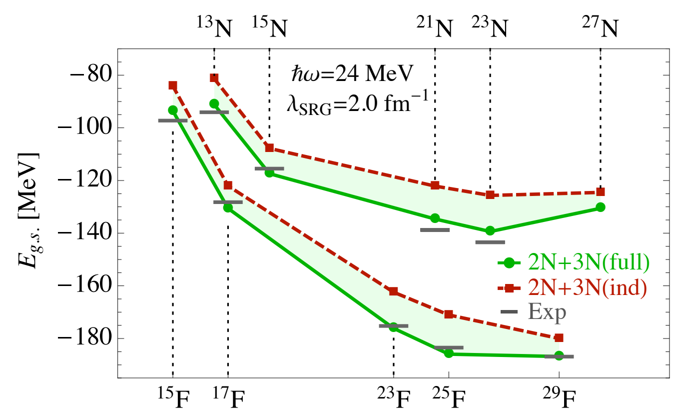
Nuclear Struclear Structure and Reactions: Experimental and Ab-Initio Perspetives
TRIUMF, 18-21 February 2014

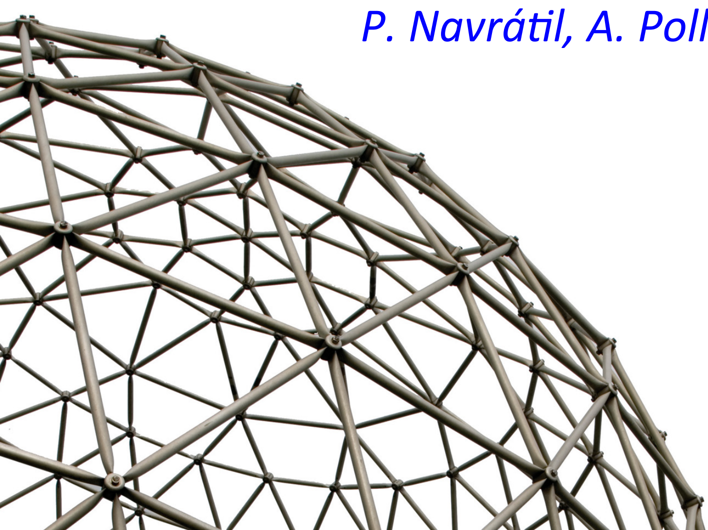
Ab-Initio Studies of Three-Body Interactions around O and Ca isotopes



Carlo Barbieri
University of Surrey

A. Carbone, A. Cipollone, T. Duguet, V. Somà,
P. Navrátil, A. Polls, A. Rios-Huguet





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Outilne:

- Dyson (*closed-shell*) and Gorkov (*open shells*) *formalisms of GF*
- Inclusion of 3NF and fluorine/nitrogen driplines
- Spectral functions and masses around ^{A}O and ^{A}Ca
- Applications of SCGF to reactions—optical models

V. Somà, A. Cipollone, CB, P. Navrátil, T. Duguet, arXiv:1312.2068 [nucl-th]

*A. Carbone, A. Cipollone, CB, A. Rios, A. Polls, Phys. Rev. C**88**, 054326 (2013)*
*A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)*

V. Somà, CB, and T. Duguet, , arXiv:1311.1989 [nucl-th] – Phys. Rev C, in print
*V. Somà, CB, and T. Duguet, Phys. Rev. C **87**, 011303R (2013)*
*V. Somà, T. Duguet, and CB, Phys. Rev. C **84**, 064317 (2011)*

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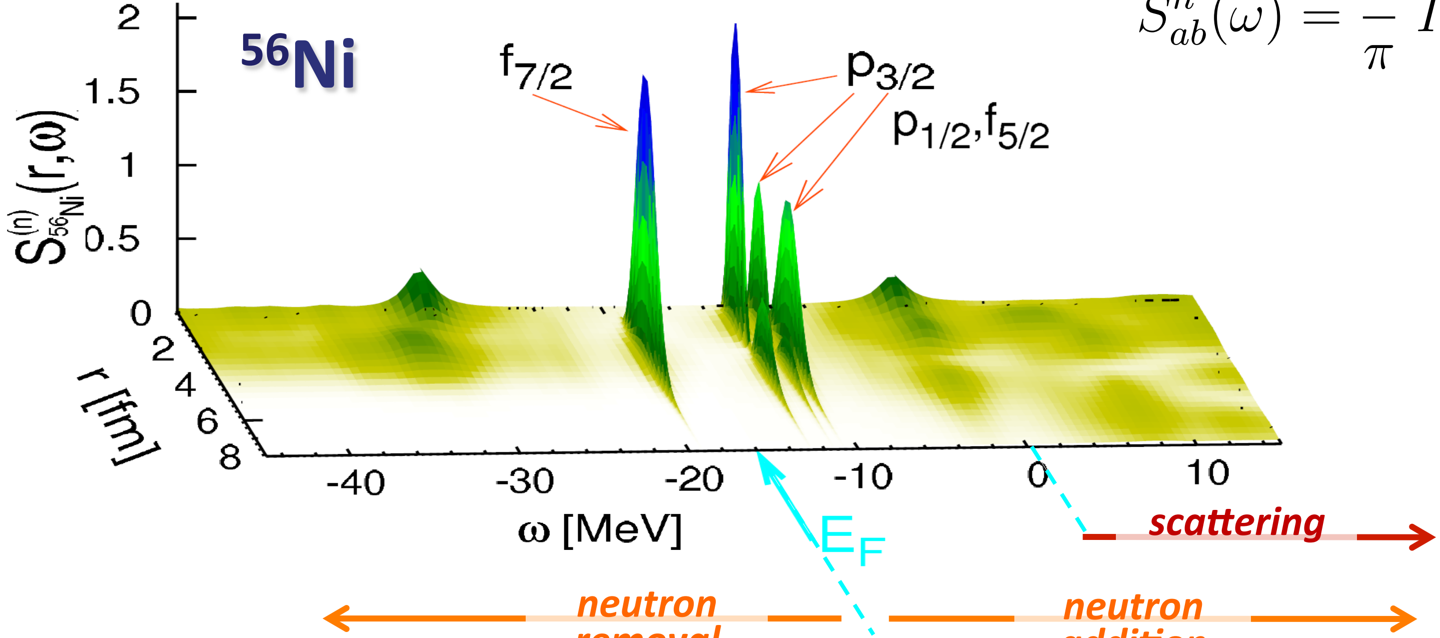
Green's functions in many-body theory

One-body Green's function (or propagator) describes the motion of quasi-particles and holes:

$$g_{\alpha\beta}(E) = \sum_n \frac{\langle \Psi_0^A | c_\alpha | \Psi_n^{A+1} \rangle \langle \Psi_n^{A+1} | c_\beta^\dagger | \Psi_0^A \rangle}{E - (E_n^{A+1} - E_0^A) + i\eta} + \sum_k \frac{\langle \Psi_0^A | c_\beta^\dagger | \Psi_k^{A-1} \rangle \langle \Psi_k^{A-1} | c_\alpha | \Psi_0^A \rangle}{E - (E_0^A - E_k^{A-1}) - i\eta}$$

…this contains <u>all the structure information</u> probed by nucleon transfer (spectral function):

$$S_{ab}^h(\omega) = \frac{1}{\pi} \, Im \, g_{ab}(\omega)$$



[CB, M.Hjorth-Jensen, Pys. Rev. C**79**, 064313 (2009); CB, Phys. Rev. Lett. **103**, 202502 (2009)]

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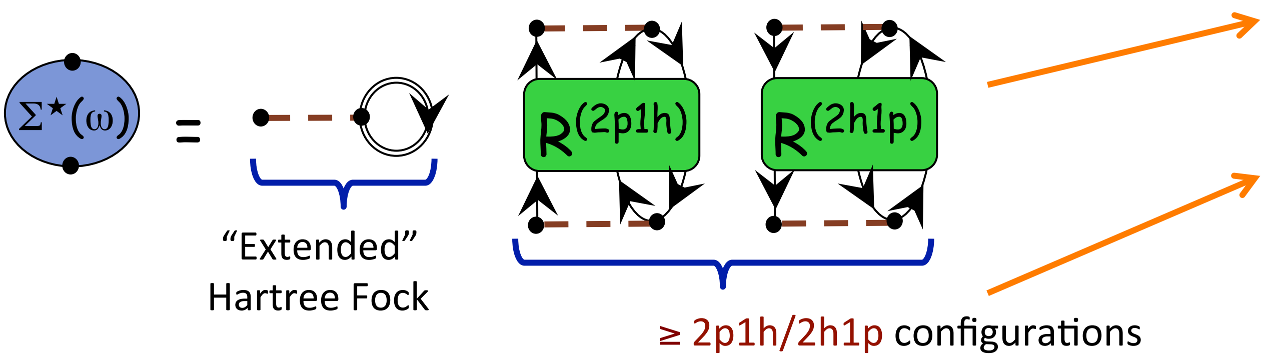
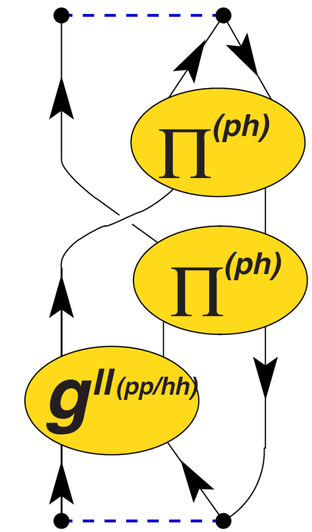
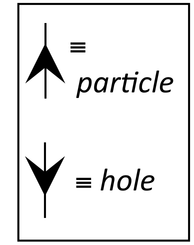
Faddeev-RPA in two words…

Self-energy (optical potential):



Faddeev-RPA:



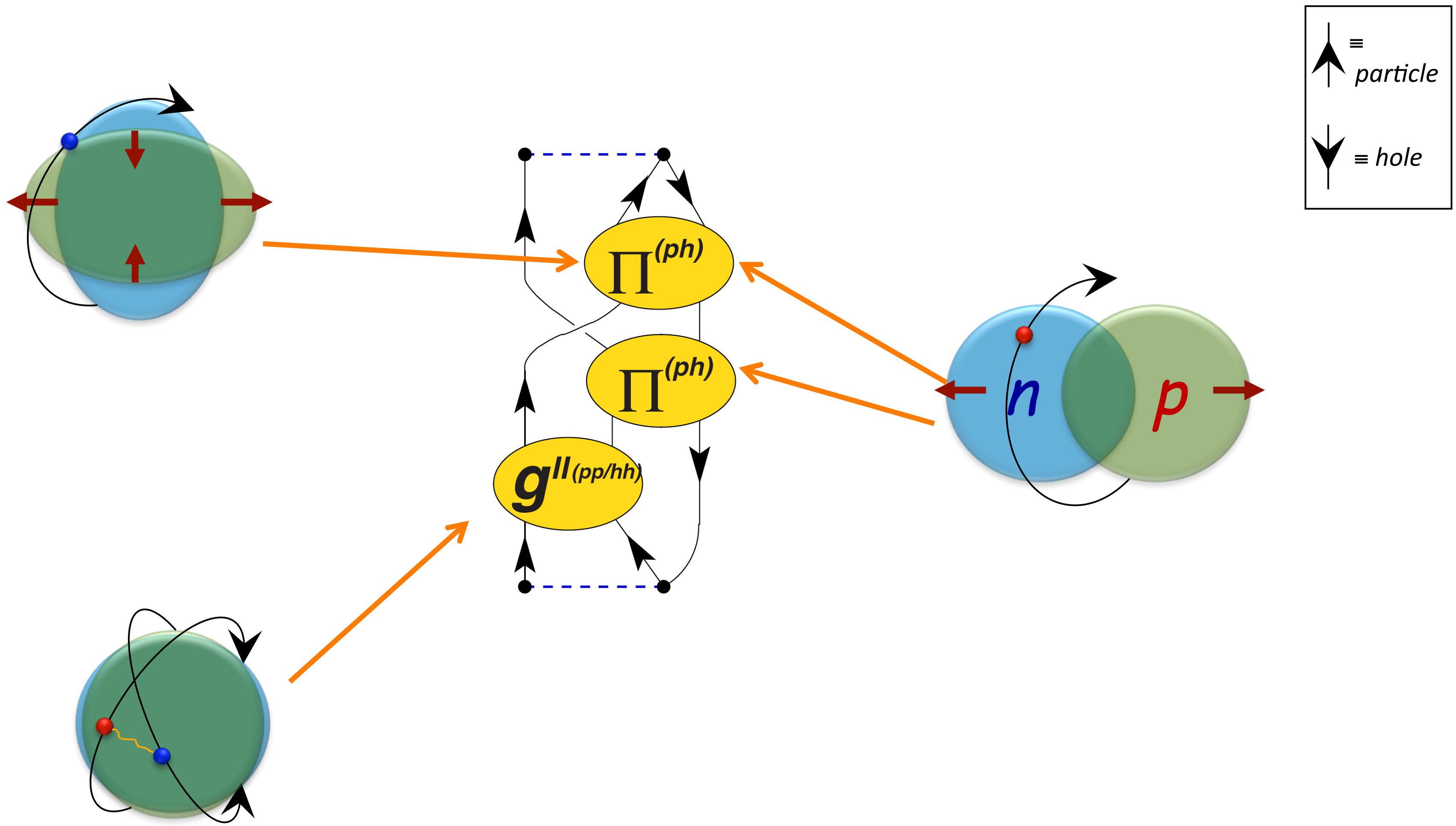


Phys.Rev.C63, 034313 (2001)
Phys.Rev.C65, 064313 (2002)
Phys.Rev.A76, 052503 (2007)

- A complete expansion requires *all types* of particle-vibration coupling:
  - ✓ $g^{II}(\omega)$ → pairing effects, two-nucleon transfer
  - ✓ $\Pi^{(ph)}(\omega)$ → collective motion, using RPA or beyond
  - ✓ Pauli exchange effects
- The Self-energy $\Sigma^\star(\omega)$ yields *both* single-particle states and scattering
- Finite nuclei: → require high-performance computing

Faddeev-RPA in two words…

Particle vibration coupling is the main cause driving the distribution of particle strength—a least close to the Fermi surface…

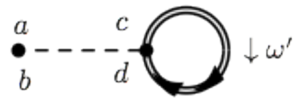


Open-shells: 1st & 2nd order Gorkov diagrams

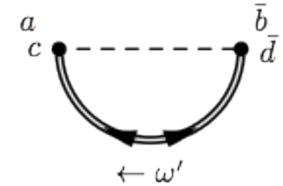
V. Somà, CB, T. Duguet, , arXiv:1311.1989 [nucl-th] – *PRC*, in print
V. Somà, CB, T. Duguet, Phys. Rev. C **87**, 011303R (2013)
V. Somà, T. Duguet, CB, Phys. Rev. C **84**, 064317 (2011)

- 1st order ➟ energy-independent self-energy

$\Sigma^{11\,(1)}_{ab} =$

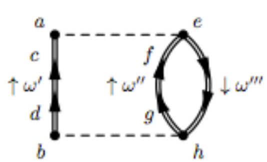


$\Sigma^{12\,(1)}_{ab} =$



- 2nd order ➟ energy-dependent self-energy

$\Sigma^{11\,(2)}_{ab}(\omega) =$



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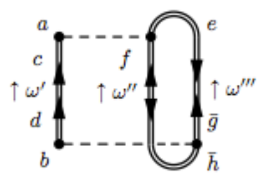
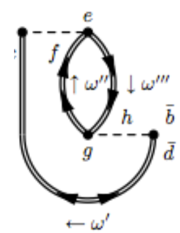


$\Sigma^{12\,(2)}_{ab}(\omega) =$



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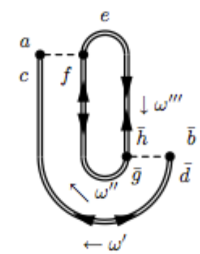

- Gorkov equations ⟶ eigenvalue problem

$$\sum_b \begin{pmatrix} t_{ab} - \mu_{ab} + \Sigma^{11}_{ab}(\omega) & \Sigma^{12}_{ab}(\omega) \\ \Sigma^{21}_{ab}(\omega) & -t_{ab} + \mu_{ab} + \Sigma^{22}_{ab}(\omega) \end{pmatrix}\Bigg|_{\omega_k} \begin{pmatrix} \mathcal{U}^k_b \\ \mathcal{V}^k_b \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}^k_a \\ \mathcal{V}^k_a \end{pmatrix}$$

$$\mathcal{U}^{k*}_a \equiv \langle \Psi_k | \bar{a}^\dagger_a | \Psi_0 \rangle$$
$$\mathcal{V}^{k*}_a \equiv \langle \Psi_k | a_a | \Psi_0 \rangle$$

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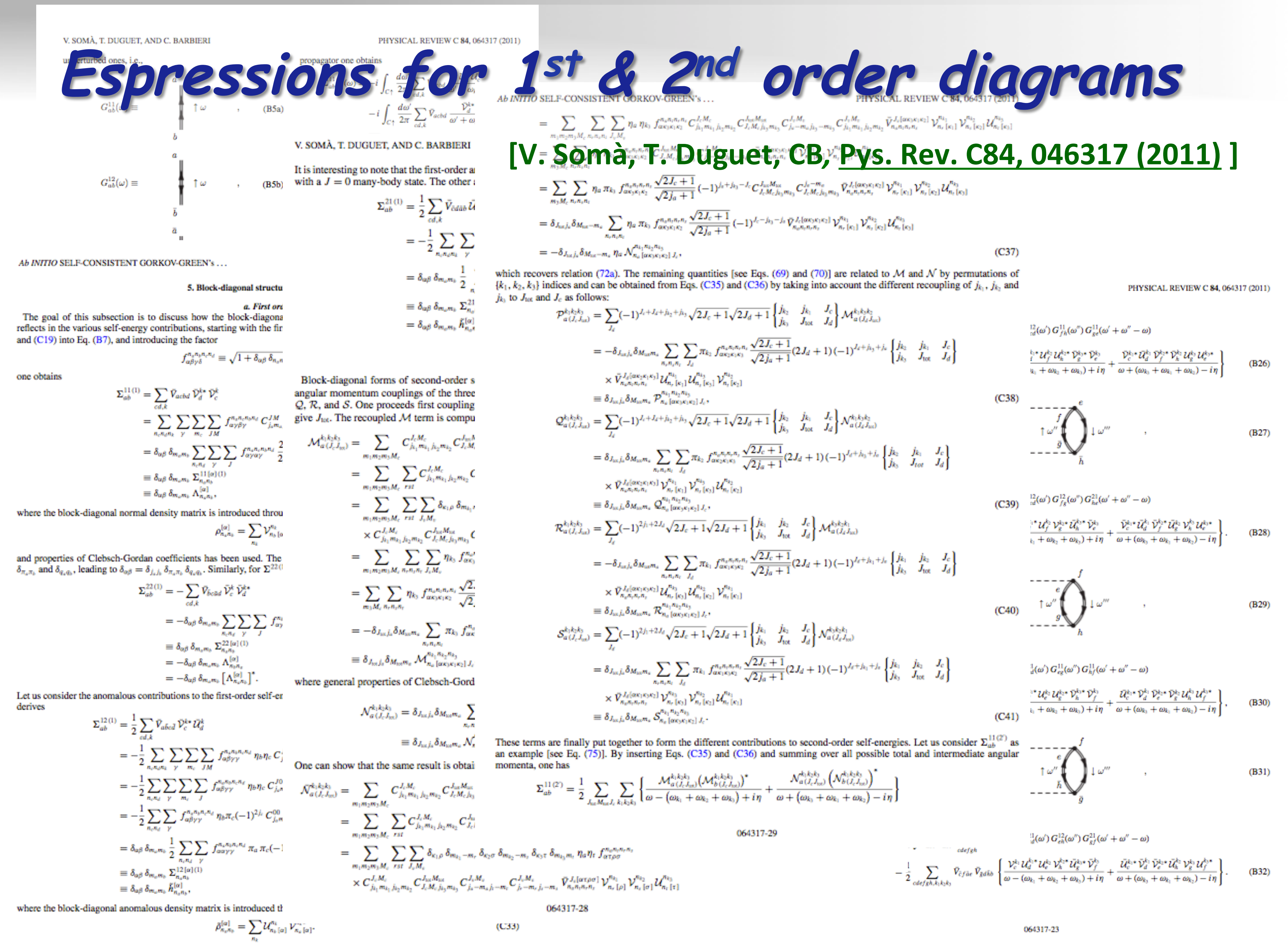
Espressions for 1st & 2nd order diagrams

[V. Somà, T. Duguet, CB, Pys. Rev. C84, 046317 (2011)]

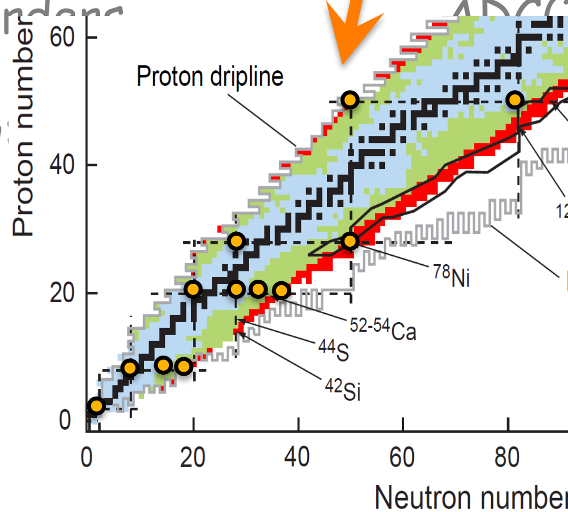


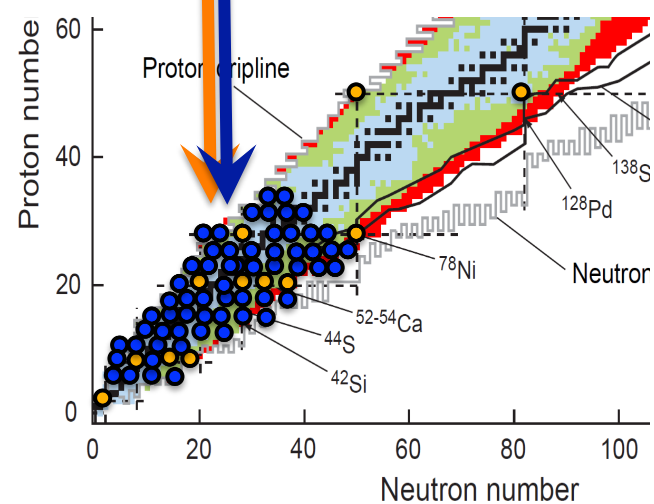
Approaches in GF theory

Truncation scheme:	Dyson formulation (closed shells)	Gorkov formulation (semi-magic)
1st order:	Hartree-Fock	HF-Bogolioubov
2nd order:	2nd order	2nd order (w/ pairing)
. . .	. . .	
3rd and all-orders sums, P-V coupling:	ADC(3) FRPA etc…	G-ADC(3) …work in progress

Approaches in GF theory

Truncation scheme:	Dyson formulation (closed shells)	Gorkov formulation (semi-magic)
1^{st} order:	Hartree-Fock	HF-Bogolioubov
2^{nd} order:	2^{nd} order	2^{nd} order (w/ pairing)
. . .	. . .	
3^{rd} and all-orders sums, P-V coupling:	ADC(3)	





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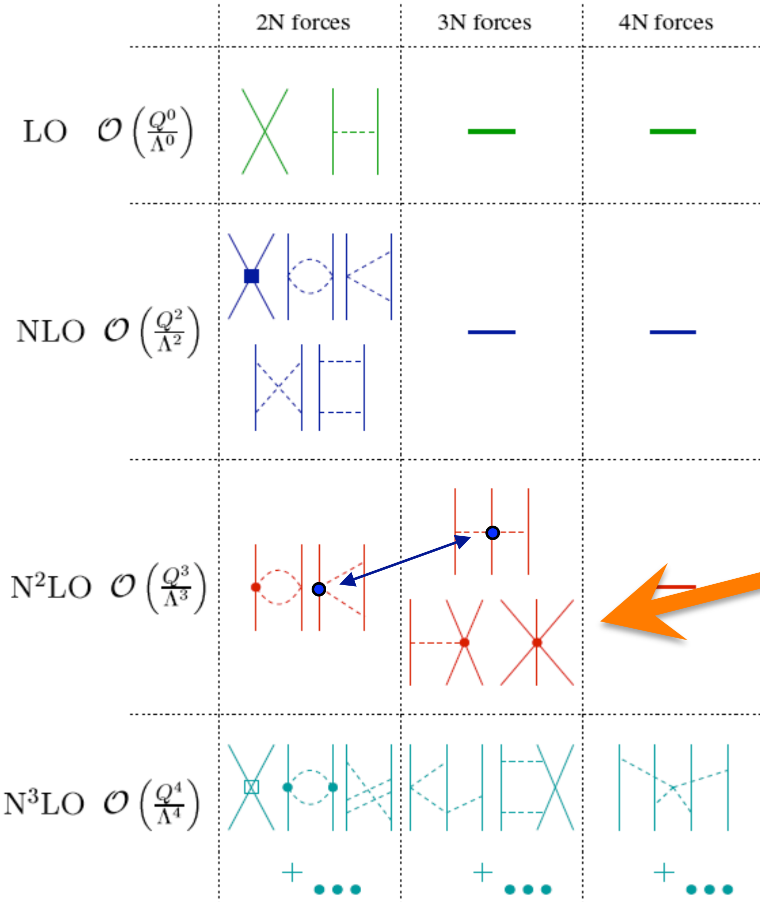
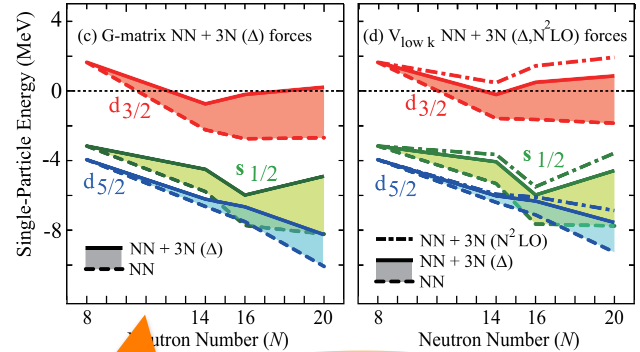
Modern realistic nuclear forces

Chiral EFT for nuclear forces:



(3NFs arise naturally at N2LO)

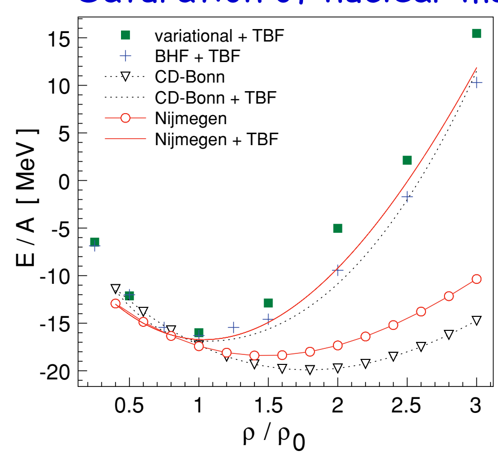
Single particle spectrum at E_{fermi}:



[T. Otsuka et al., Phys Rev. Lett **105**, 32501 (2010)]

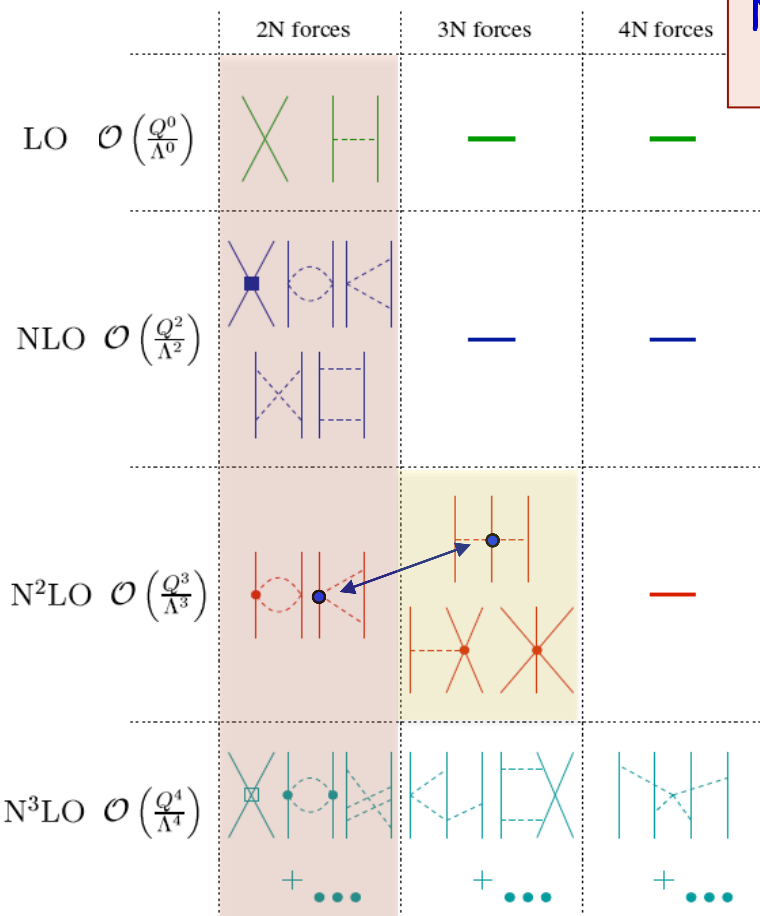
Need at LEAST 3NF!!!
("cannot" do RNB physics without…)

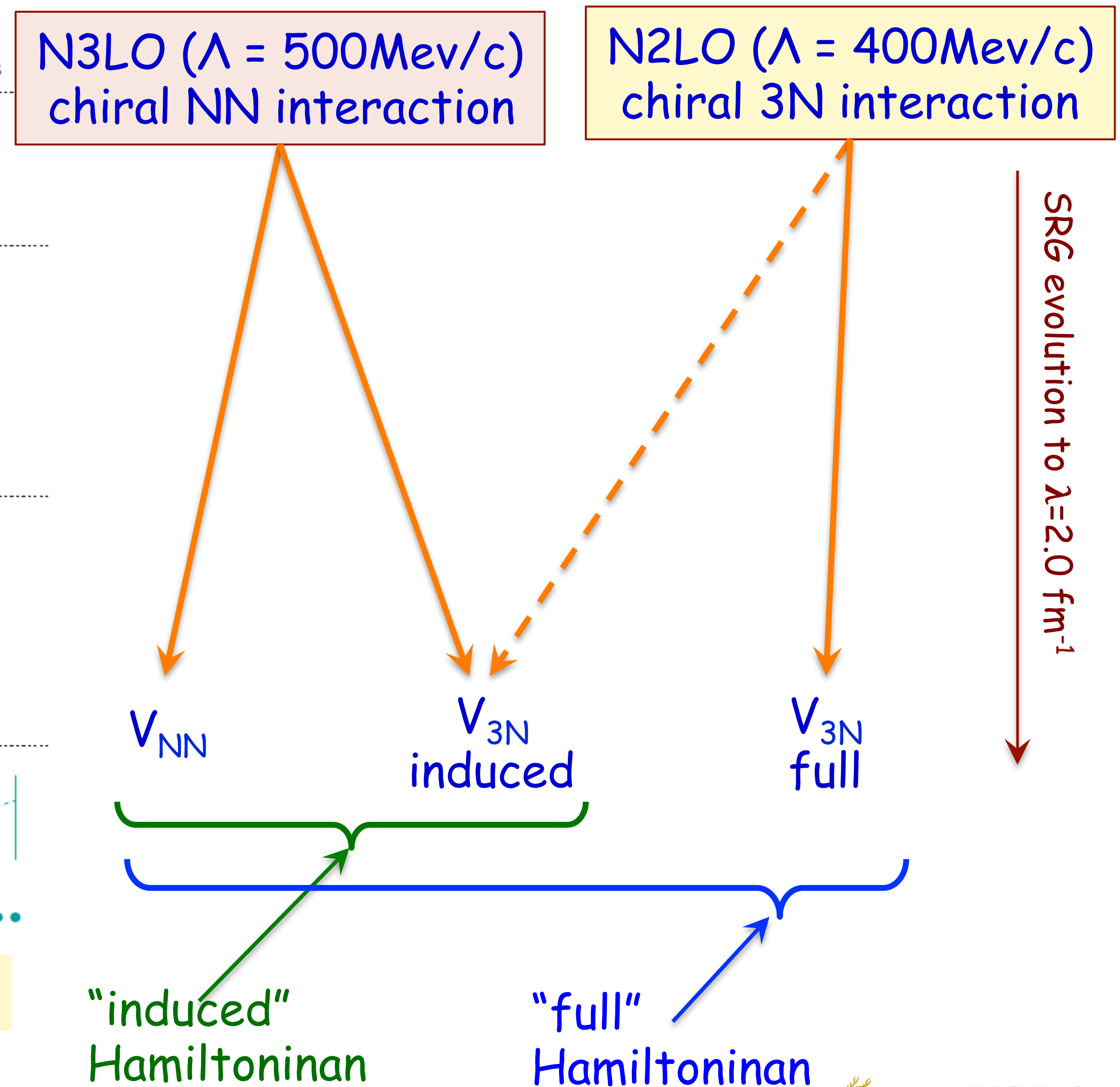
Saturation of nuclear matter:



[V. Somà, Phys Rev. C **78**, 054003 (2008))]

Chiral Nucler forces SRG evolved





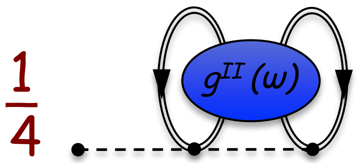
(3NFs arise naturally at N2LO)

[Jurgenson, Navrátil, Furnstahl,
Phys. Rev. Lett. **103**, 082501 (2009);
Hebeler, Phys. Rev. C 85, 021002 (2012)]

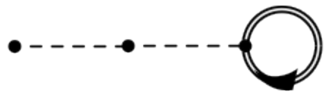
Inclusion of NNN forces

A. Carbone, CB, et al., Phys. Rev. C88, 054326 (2013)

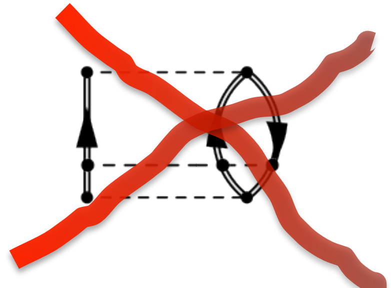
- NNN forces can enter diagrams in three different ways:



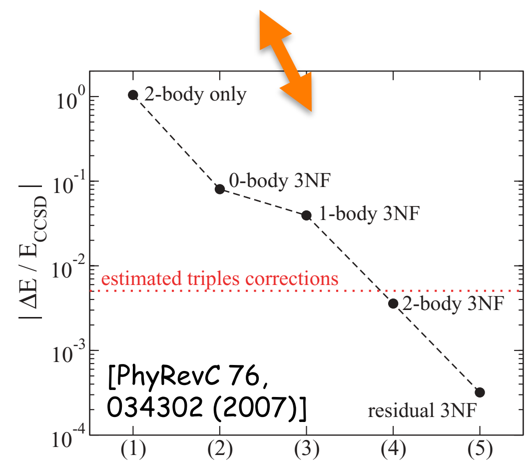
Correction to external 1-Body interaction



Correction to *non-contracted* 2-Body interaction



pure 3-body contribution (small)



- Contractions are with *fully correlated density matrices* (BEYOND a normal ordering…)

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Inclusion of NNN forces

A. Carbone, CB, et al., Phys. Rev. C**88**, 054326 (2013)

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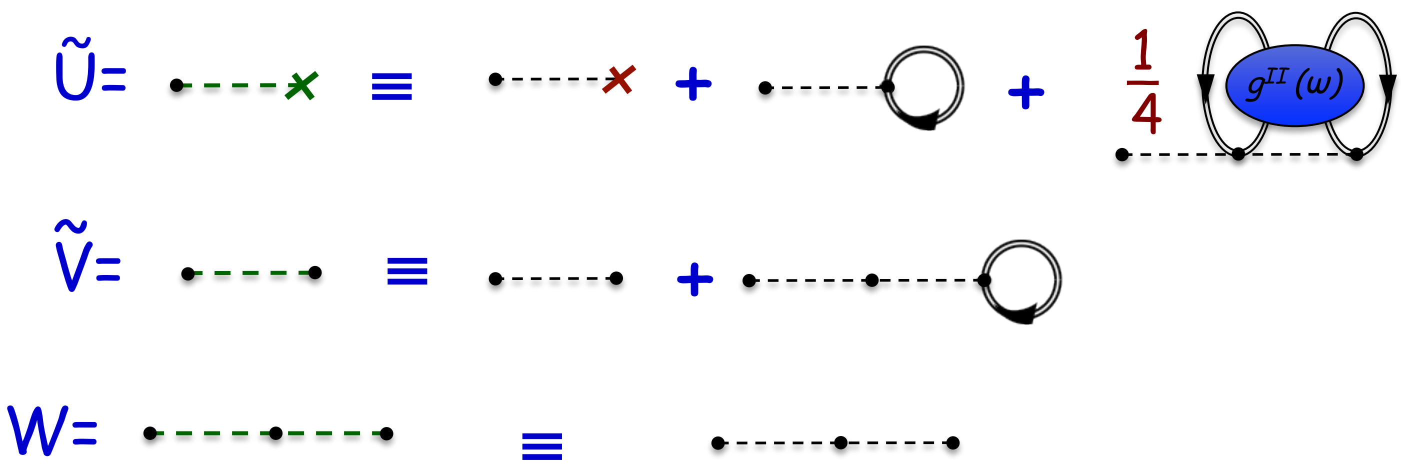
→ Define new 1- and 2-body interactions and
use <u>only</u> interaction-irreducible diagrams



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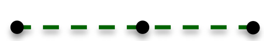
$$\tilde{U} = \sum_{\alpha\beta} \left[-U_{\alpha,\beta} - i\hbar \sum_{\gamma\delta} V_{\alpha\gamma,\beta\delta})\, G_{\delta\gamma}(t-t^+) + \frac{i\hbar}{4} \sum_{\substack{\gamma\epsilon \\ \delta\eta}} W_{\alpha\gamma\epsilon,\beta\delta\eta}\, G^{II}_{\delta\eta,\gamma\epsilon}(t-t^+) \right] a^\dagger_\alpha a_\beta$$

$$\tilde{V} = \frac{1}{4} \sum_{\substack{\alpha\gamma \\ \beta\delta}} \left[V_{\alpha\gamma,\beta\delta} - i\hbar \sum_{\epsilon\eta} W_{\alpha\gamma\epsilon,\beta\delta\eta}\, G_{\eta\epsilon}(t-t^+) \right] a^\dagger_\alpha a^\dagger_\gamma a_\delta a_\beta$$

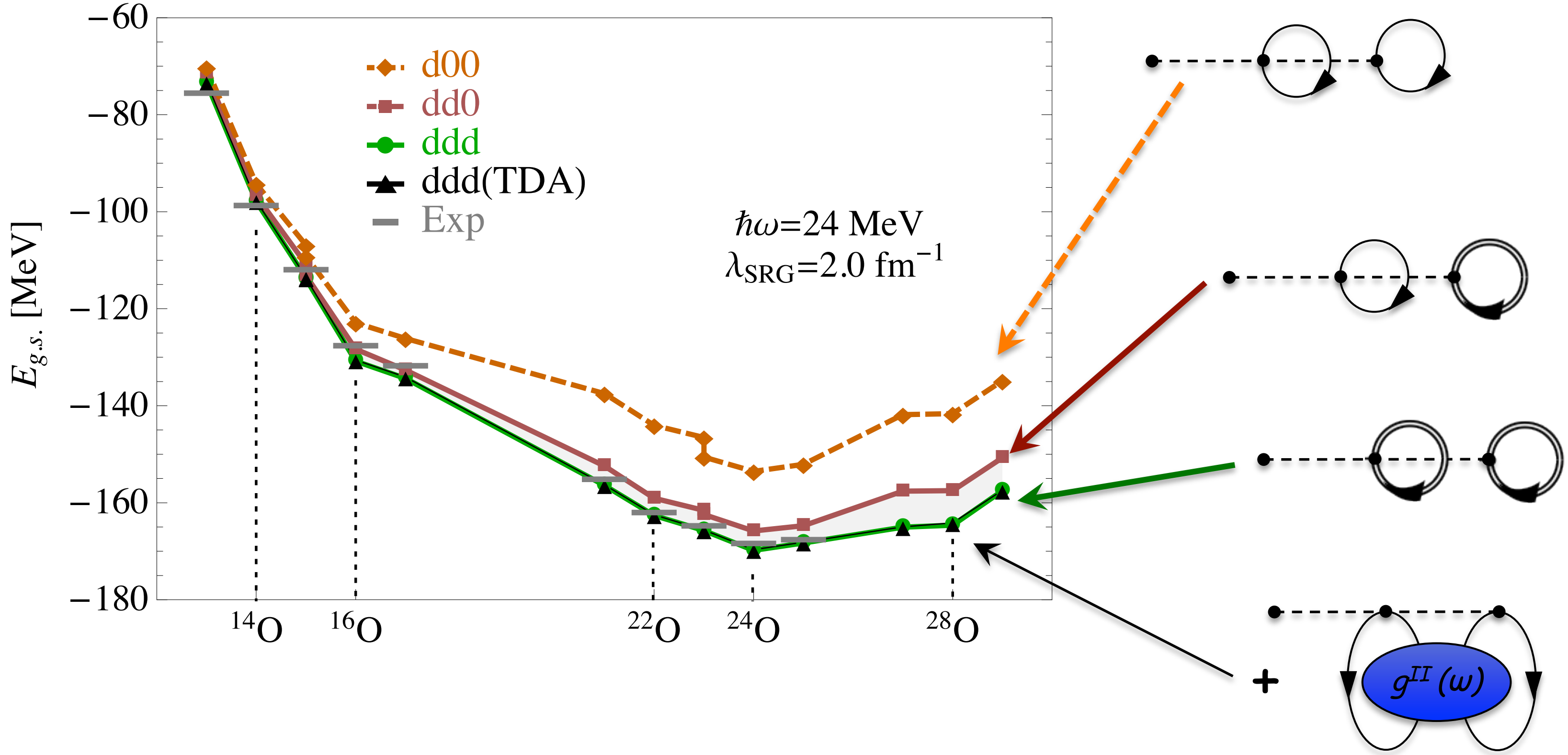
W=  ≡ 

- Contractions are with <u>*fully correlated density matrices*</u> (BEYOND a normal ordering…)

NNN forces in FRPA/FTDA formalism

A. Cipollone, CB

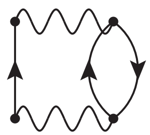
→ Ladder contributions to static self-energy are negligible (in oxygen)



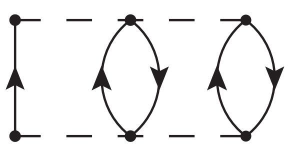
Inclusion of NNN forces

A. Carbone, CB, et al., Phys. Rev. C ***88****, 054326 (2013)*

- *Second order* PT diagrams with 3BFs:



(a)



(b)

effectively:

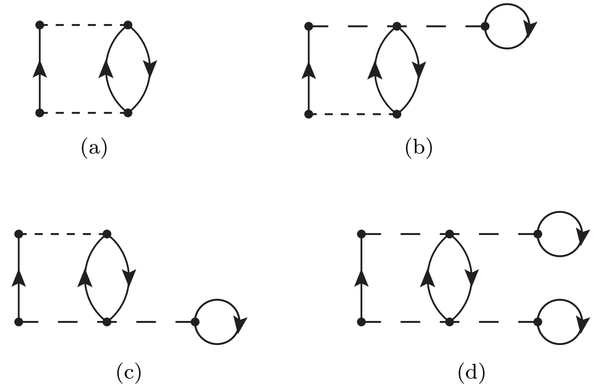
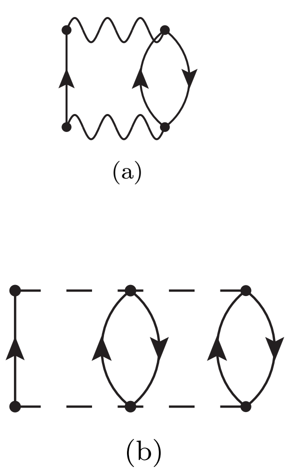


FIG. 4. The one *interaction irreducible* diagrams (a) and the three *interaction reducible* ones (b, c and d) that are contained in Fig. 3a.

Inclusion of NNN forces

A. Carbone, CB, et al., Phys. Rev. C88, 054326 (2013)

- *Second order* PT diagrams with 3BFs:



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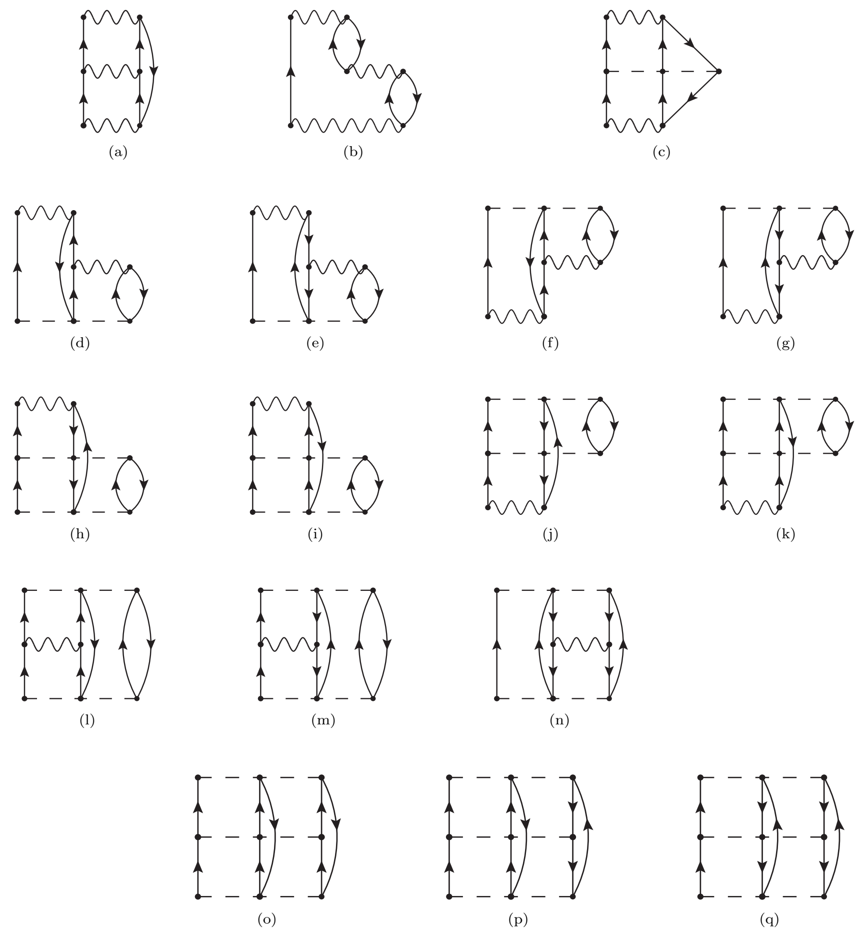

FIG. 5. 1PI, *skeleton* and *interaction irreducible* self-energy diagrams appearing at 3^{rd}-order in perturbative expansion (7), making use of the effective hamiltonian of Eq. (9).

Inclusion of NNN forces

A. Carbone, CB, et al., Phys. Rev. C88, 054326 (2013)

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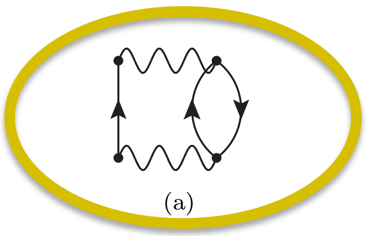
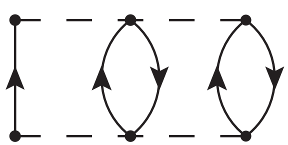
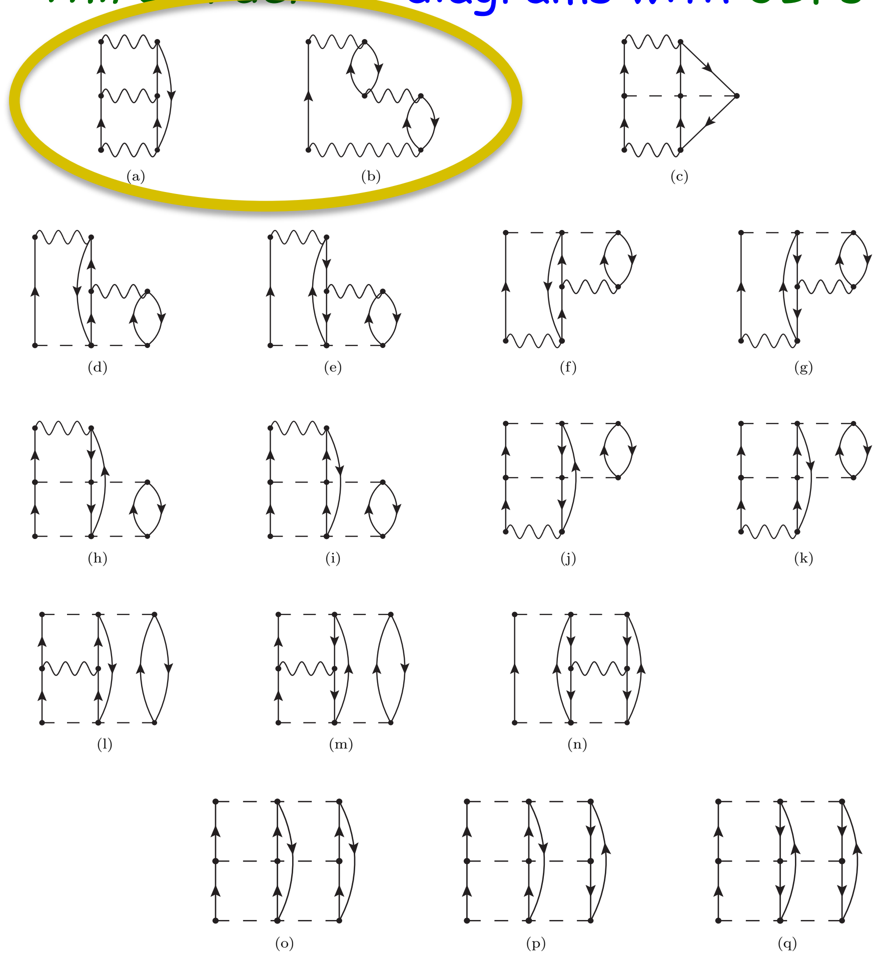


(a)



(b)

- *Third order* PT diagrams with 3BFs:



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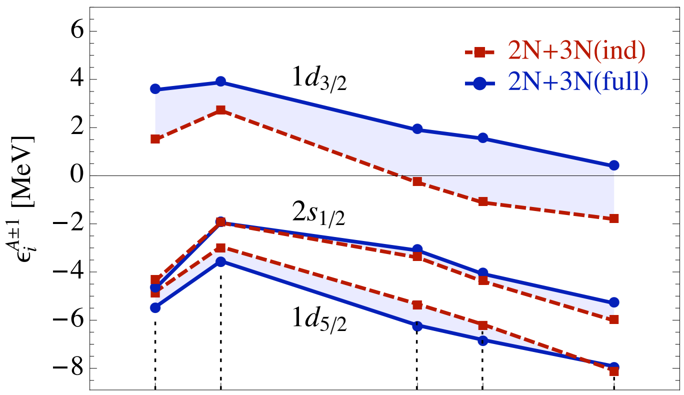
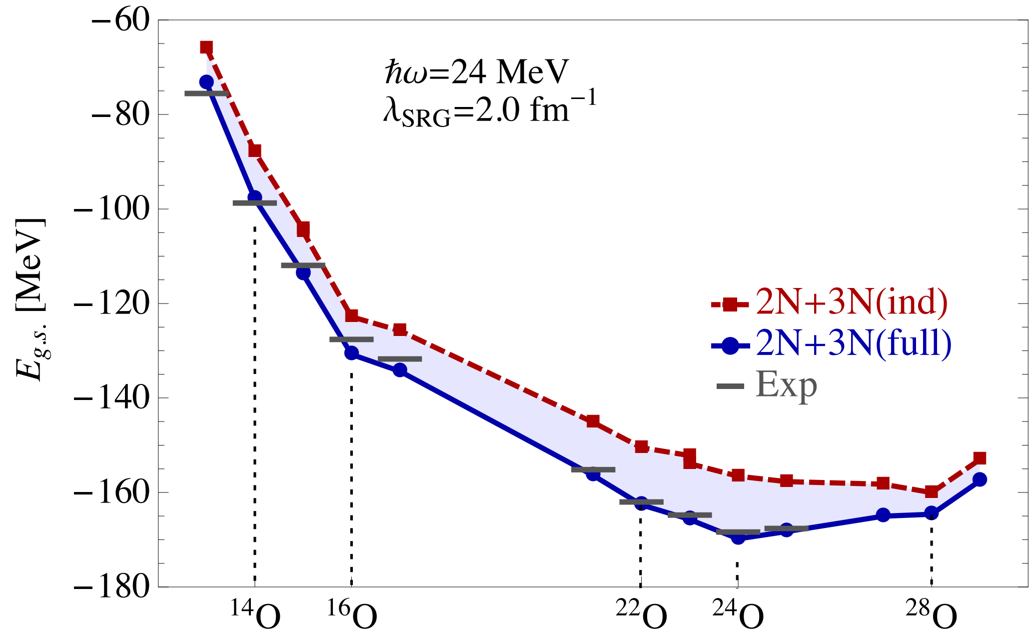
Results for the N-O-F chains

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)





→ 3NF crucial for reproducing binding energies and driplines around oxygen

→ *d3/2 raised* by genuine 3NF

→ cf. microscopic shell model [Otsuka et al, PRL**105**, 032501 (2010).]

N3LO (Λ = 500Mev/c) chiral NN interaction evolved to 2N + 3N forces (2.0fm^{-1})
N2LO (Λ = 400Mev/c) chiral 3N interaction evolved (2.0fm^{-1})

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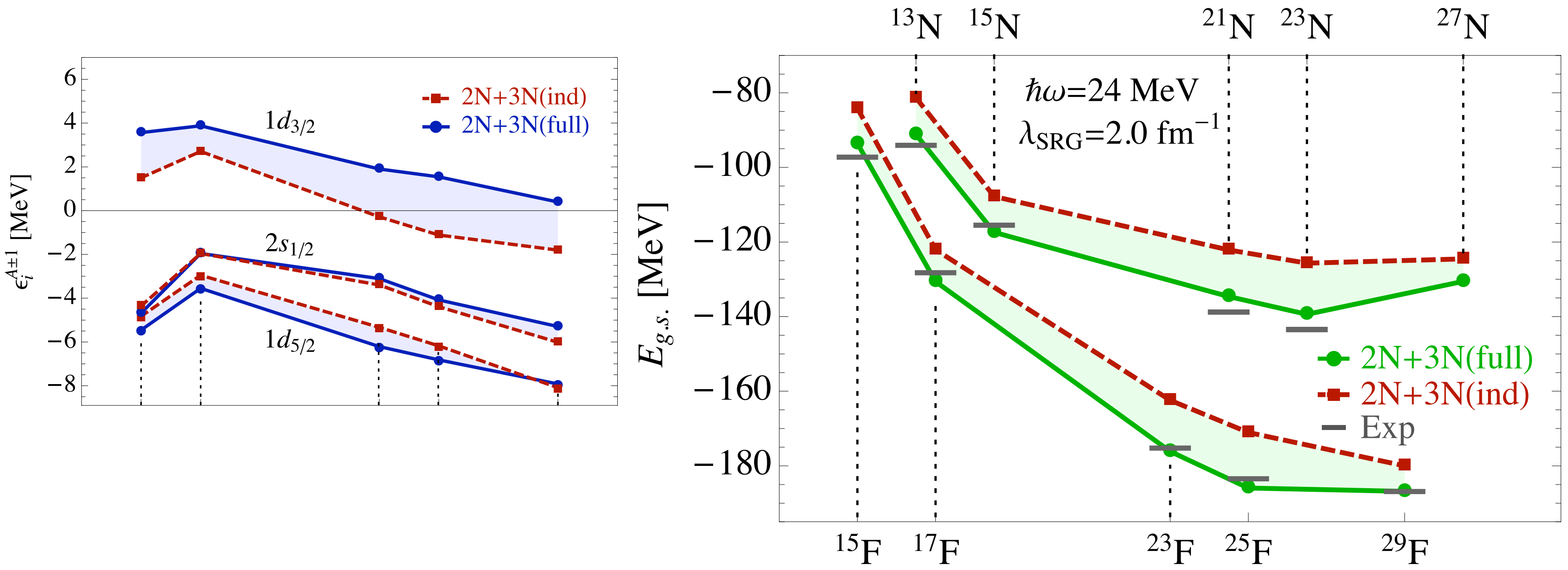

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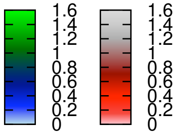
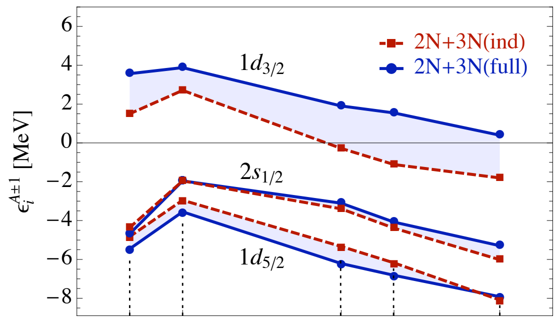
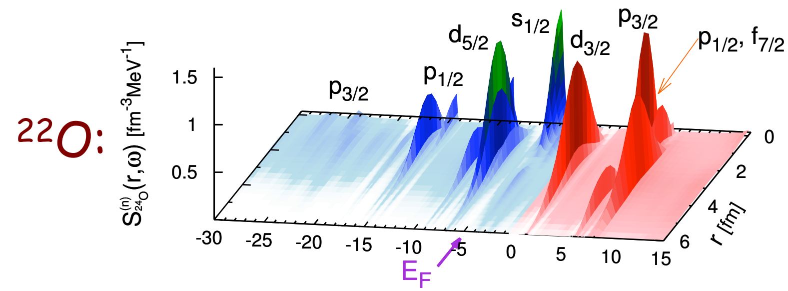
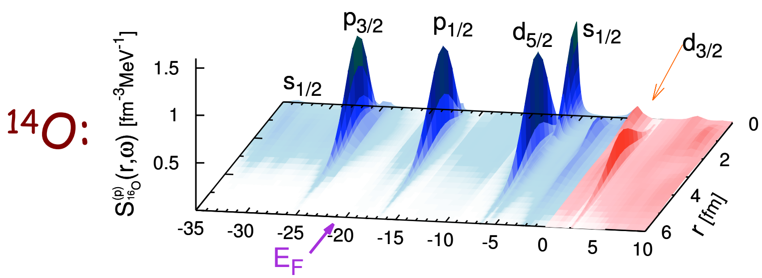
Neutron spectral function of Oxygens

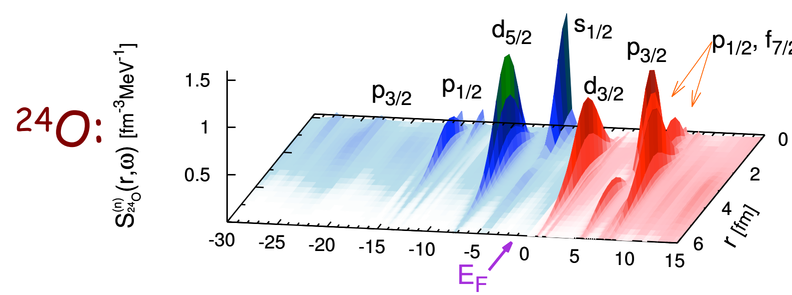
A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)

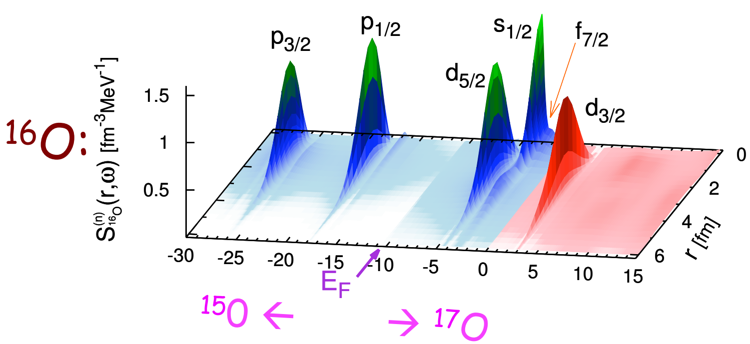


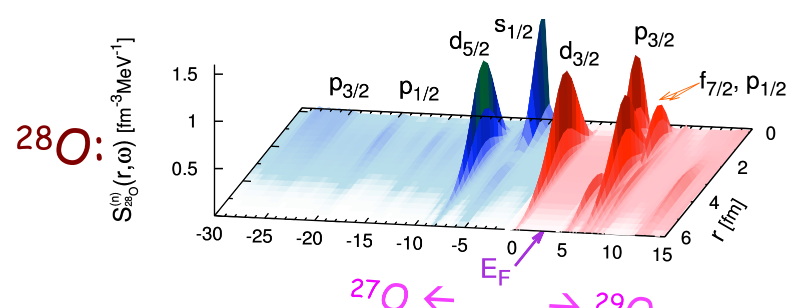








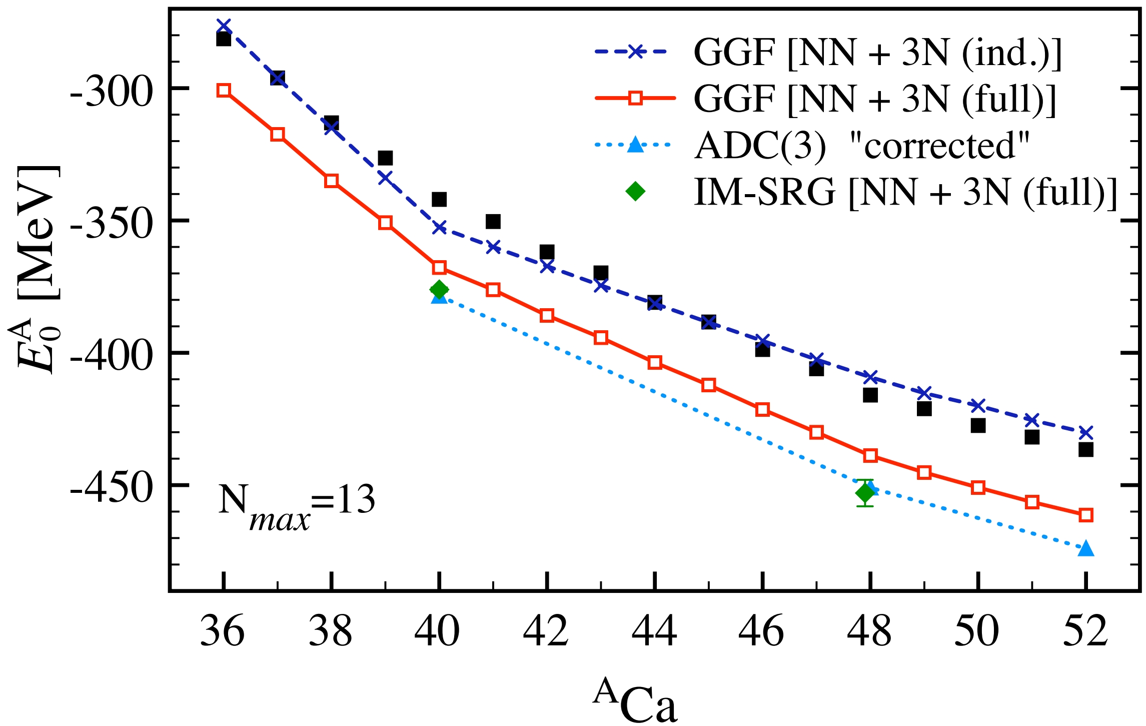




Calcium isotopic chain

V. Somà, CB *et al.*,
arXiv:1312.2068 [nucl-th]

Ab-initio calculation of the whole Ca chain with NN+3N forces



→ *induced* and full3NF investigated

→ genuine (N2LO) 3NF needed to correct the energy curvature

→ Full 3NF give a correct trend but overbind!

→ convergence worsens after A≈52

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Calcium isotopic chain

V. Somà, CB *et al.*,
arXiv:1312.2068 [nucl-th]

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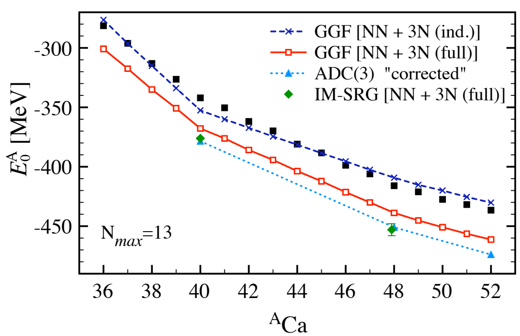

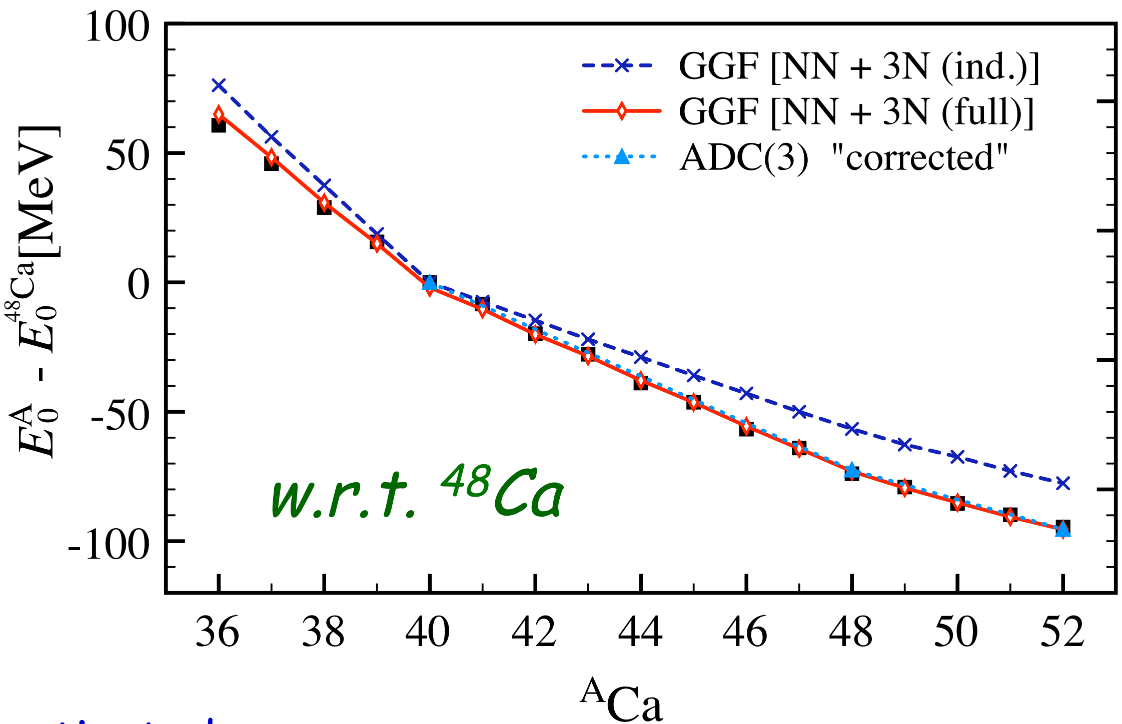

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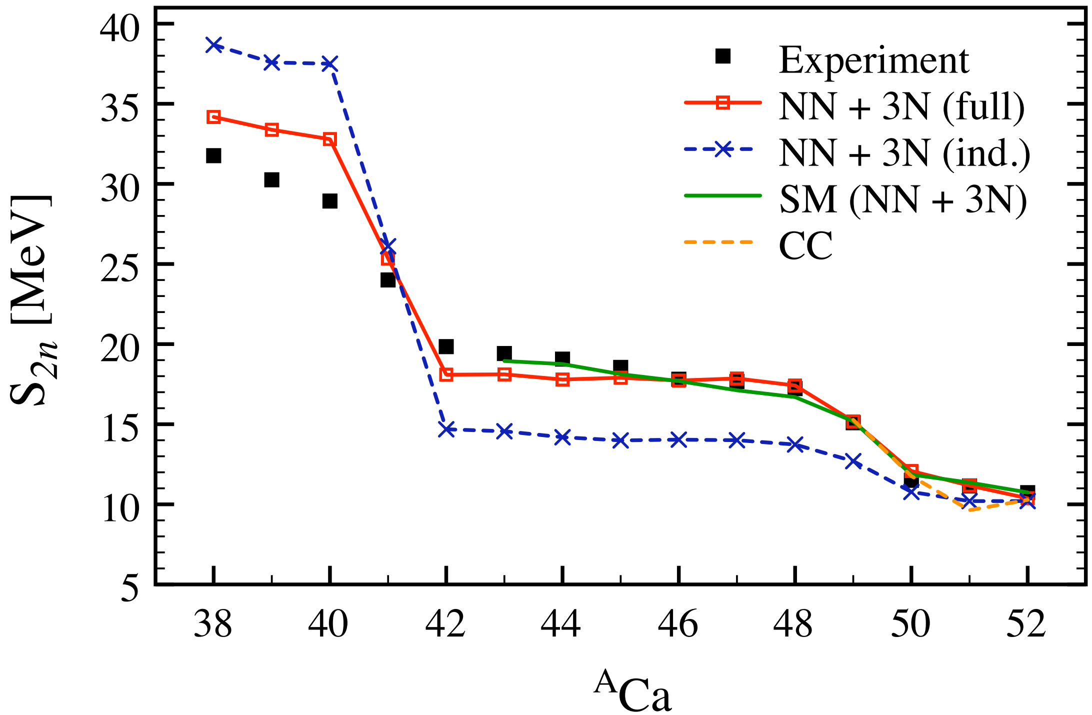
Two-neutron separation energies



→ *induced* and full3NF investigated

→ genuine (N2LO) 3NF needed to reproduce S_{2n}

→ N=20 and Z=20 gaps overestimated!

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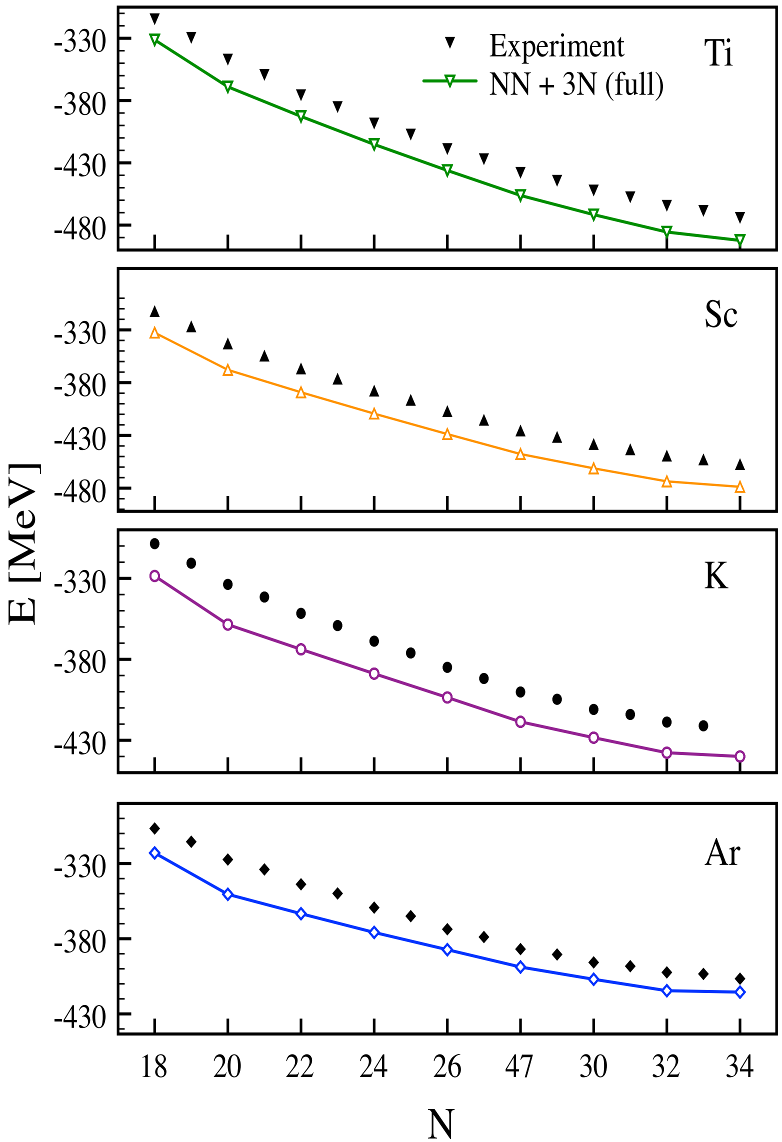
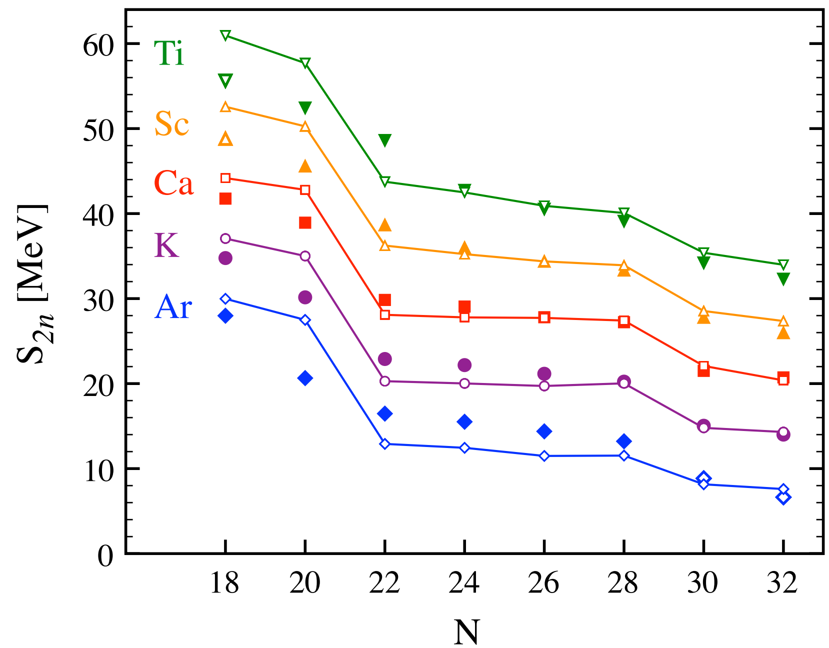
Neighbouring chains: Ar, K, Sc, Ti

V. Somà, CB *et al.*, arXiv:1312.2068 [nucl-th]





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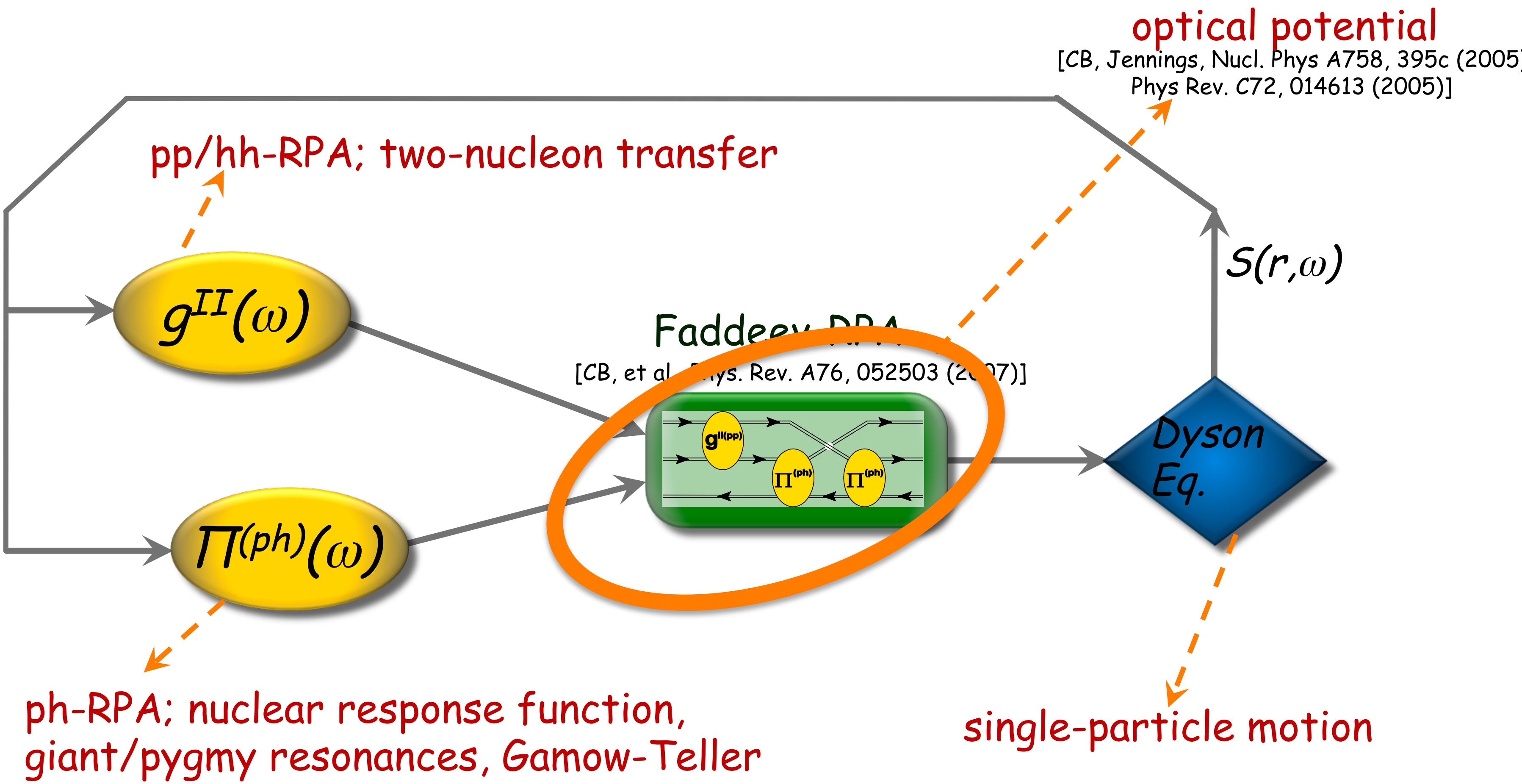
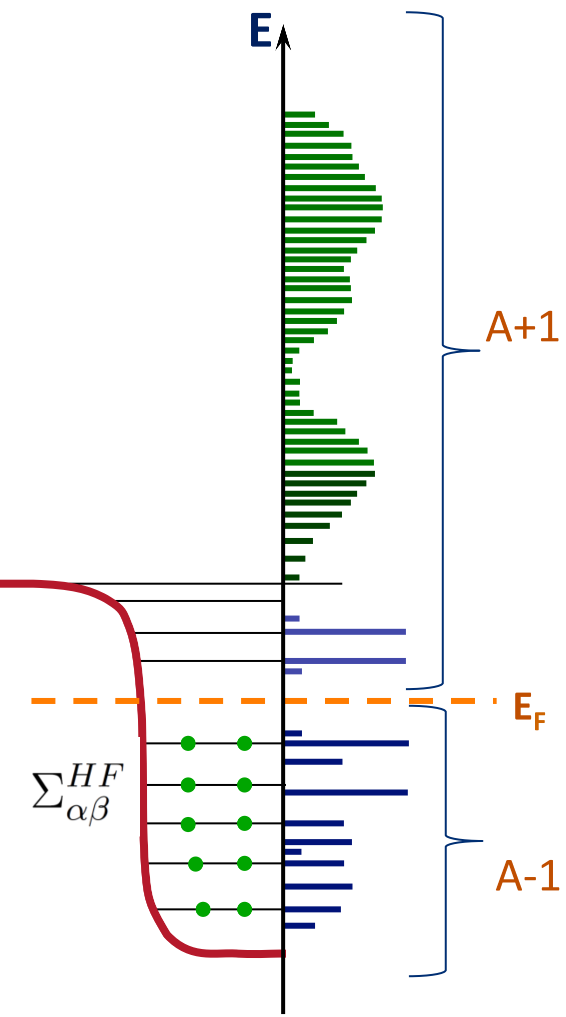
Optical Potentials Based on the Nuclear Self-energy

Self-Consistent Green's Function Approach



Nucleon elastic scattering



The irreducible self-energy is a nucleon-nucleus optical potential [see e.g. Mahaux and Sartor, Adv. Nucl. Phys. 20, (1991)]

$$\Sigma^{\star}(\mathbf{r},\mathbf{r}';\varepsilon) = \Sigma^{HF}_{\alpha\beta} - \frac{1}{\pi}\int_{\varepsilon_T^{>}}^{\infty} dE' \, \frac{Im\ \Sigma^{\star}(\mathbf{r},\mathbf{r}';E')}{\varepsilon - E' + i\eta} + \frac{1}{\pi}\int_{-\infty}^{\varepsilon_T^{<}} dE' \, \frac{Im\ \Sigma^{\star}(\mathbf{r},\mathbf{r}';E')}{\varepsilon - E' - i\eta}$$

mean-field

resonances
beyond mean-field

➔ This provides *consistent* overlaps and scattering wave functions

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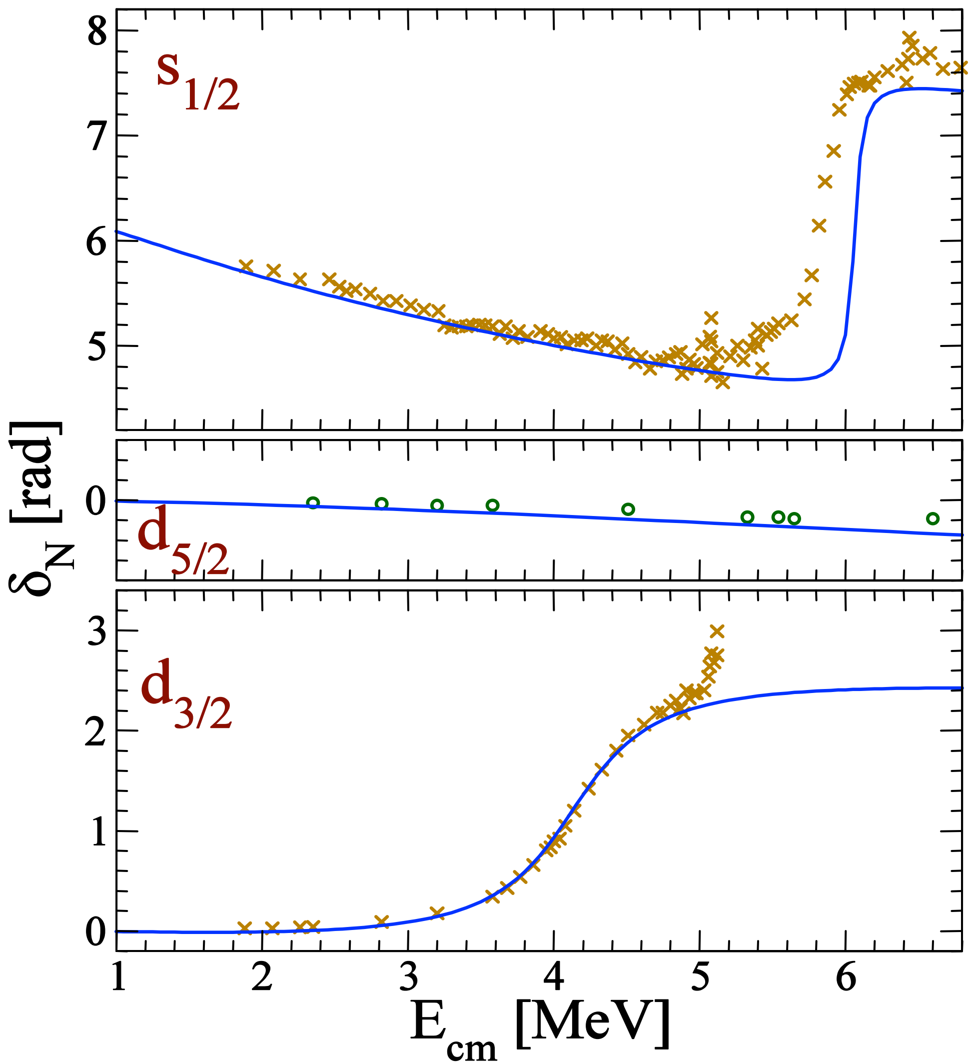
p-^{16}O phase shifts – positive parity waves

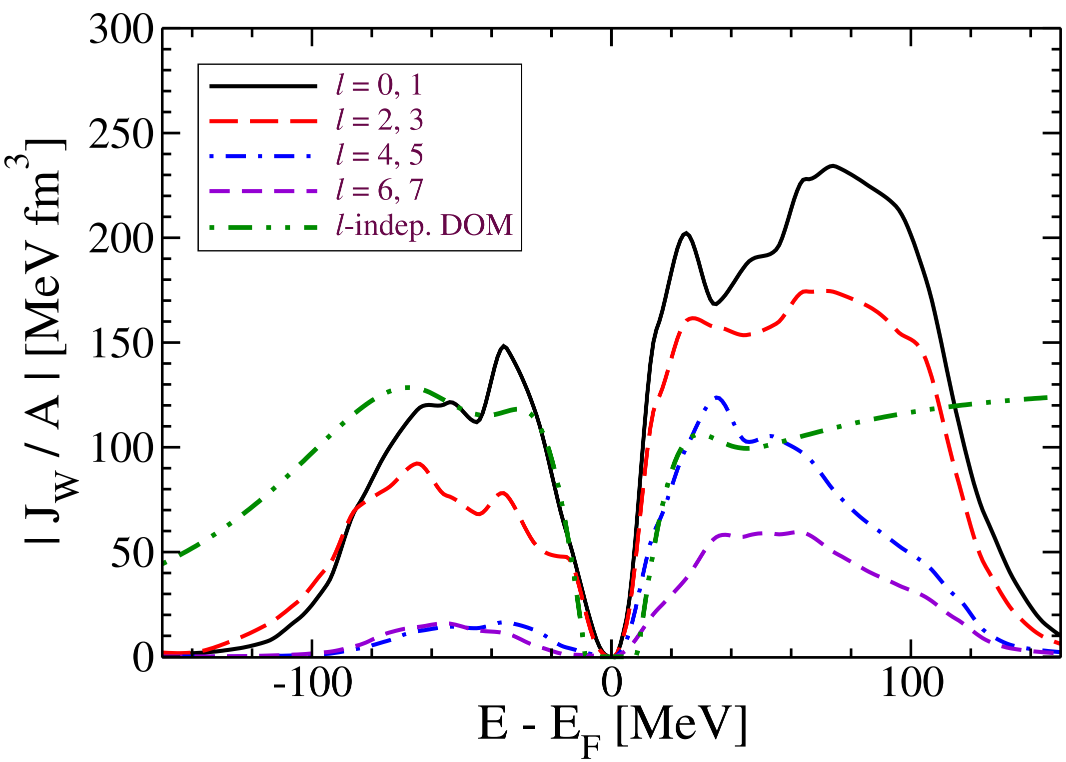
[C.B., B.Jennings,
Phys. Rev. C**72**, 014613 (2005)]



- AV18 interaction
- The phase shift are in agreement with the experiment!
- BUT does not reproduce phase shifts and bound state energies at the same time
  → need for improved H / 3NF
- Non-MF resonances "OK"

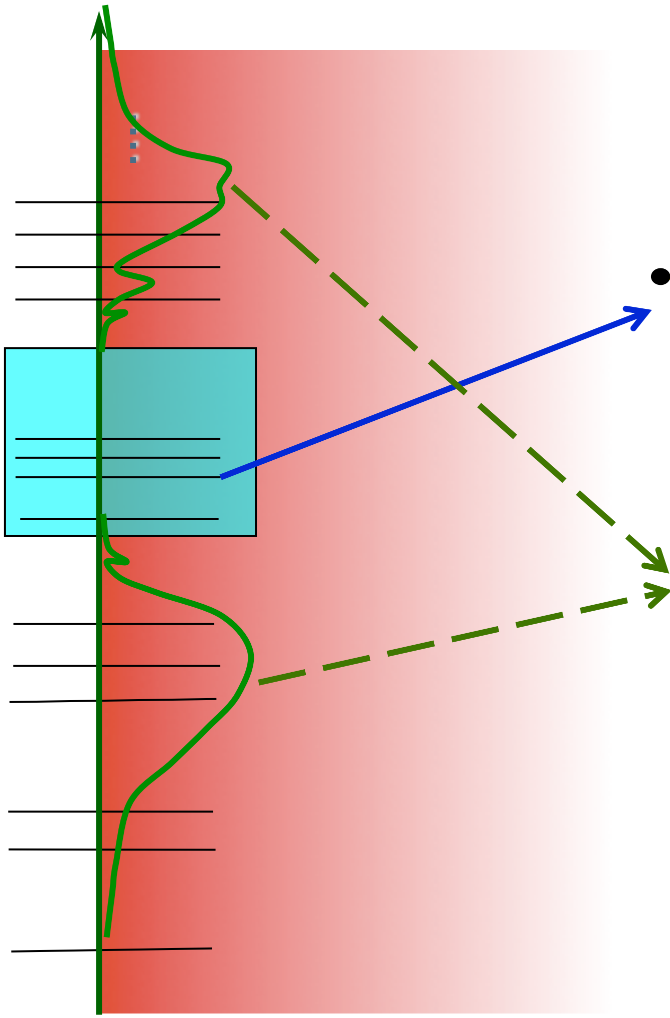
Convergence of Ab-Initio Calculated Optical Potentials

- J_w: integral over the imaginary optical potential (overall absorption)
- angular momentum dependence (non locality) not negligible!
  → in particular below E_F

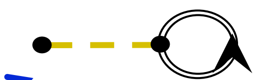


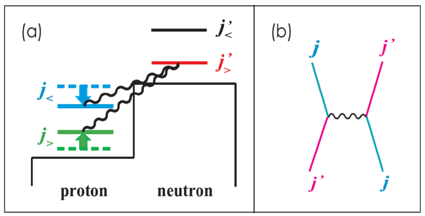
S. Waldecker, CB, W.Dickhoff – Phys. Rev. C84, 034616 (2011)

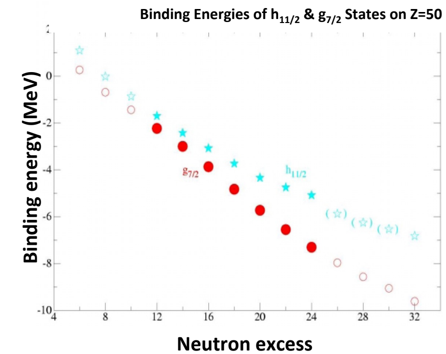
Correlations in sp energies and strengths



Single particle energies – driven by tensor + 3N force...
(see works by T. Otsuka PRL2005, 2010)



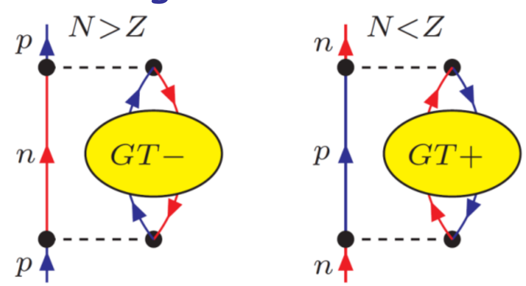




Quenching of spectral strength (spect. factor) – driven by coupling to collective modes...

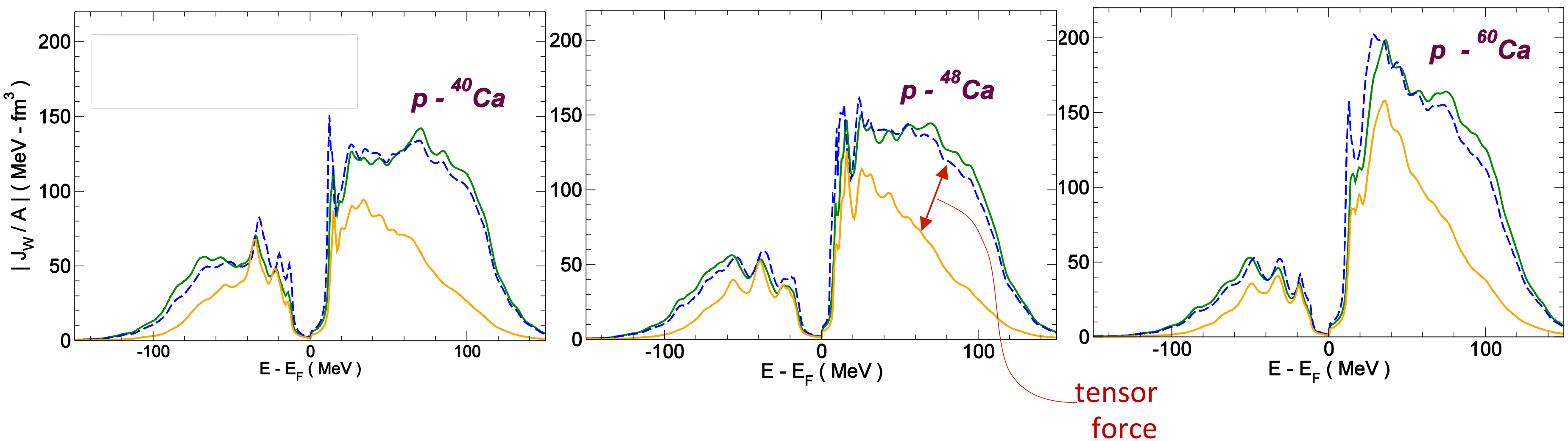
•Role of tensor force??

•Collective, charge exchange effects???



Microscopic Optical Potential from FRPA

- absorption away from E_F is enhanced by the tensor force
- little effects from charge exchange (e.g. p-^{48}Ca <-> n-^{48}Sc)



J_W: integral over the imaginary opt. pot (overall absorption)

Full FRPA result (w/ av18)
Charge-exchange d.o.f. suppressed
Tensor force suppressed

S. Waldecker, CB, W.Dickhoff – Phys. Rev. C84, 034616 (2011)

Collaborators

A. Cipollone, A. Rios

V. Somà, T. Duguet

A. Carbone

P. Navratil

A. Polls

W.H. Dickhoff, S. Waldecker

D. Van Neck, M. Degroote

M. Hjorth-Jensen

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energie atomique • energies alternatives

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Washington University in St. Louis

Center for Molecular Modeling

Conclusions

• The GORKOV formulation permits for the first time *ab-initio* calculations of binding energies, spectral quantities, and so on… for open-shell semi-magic nuclei: this means MANY of THEM, in the MID-MASS region, and previously out of reach for ab-initio.

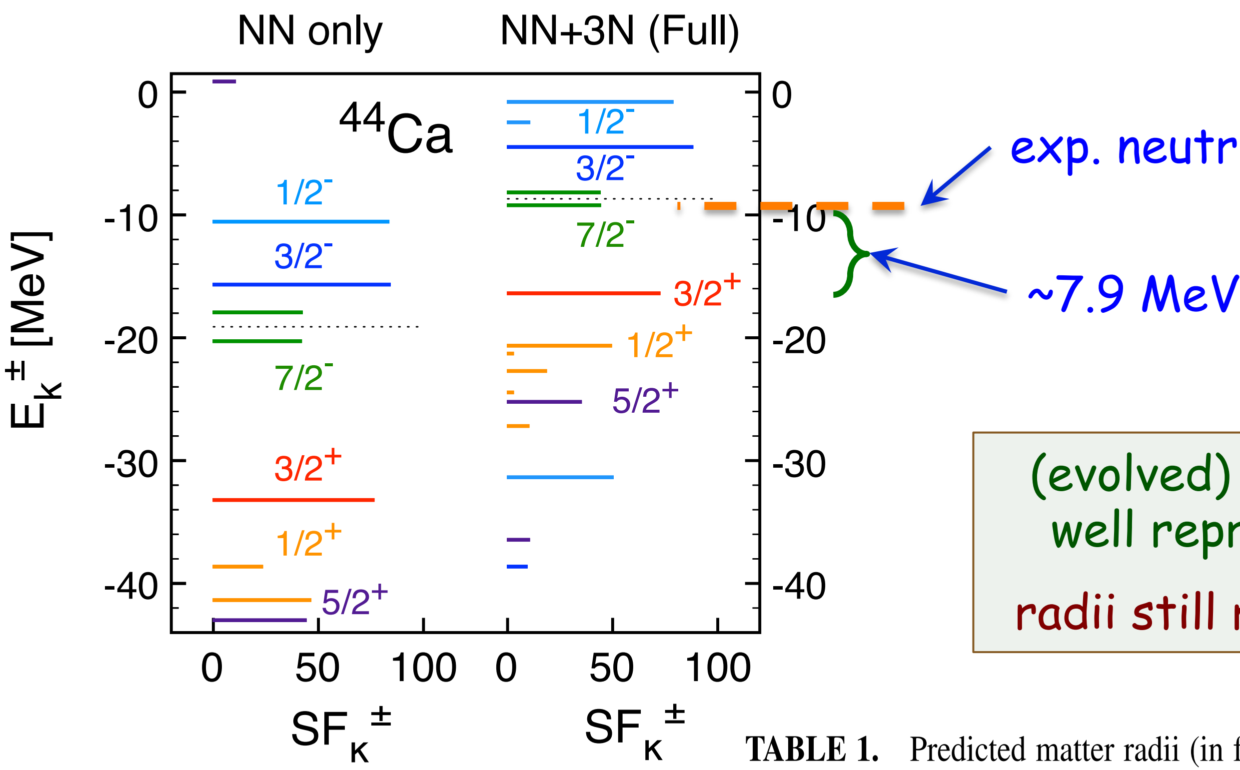
•*Consistent prediction of s.p, spectral distribution and scattering*

•*Performance of chiral nuclear forces for finite nuclei:*

- *Leading* three nucleon forces (NNLO) are ALWAYS needed to explain the proper trends. They equally set the driplines of O, N, and F.
- N, O, F region: binding energies are predicted OK (<1%), radii small.
- Ar, K, Ca, Sc, Ti region: overbound by ~1MeV/A.
- *N*=20, *Z*=20 gaps and separations among major shells are exaggerated.
- Absorption in optical potentials above the girant resonances dominated by tensor force.

Evolved chiral 3NF and the Ca isotopes

A. Cipollone, CB, V.Somà, P. Navratil



(evolved) 3NF interactions well reproduces energies

radii still remain a challenge

TABLE 1. Predicted matter radii (in fm) for ^{16}O and ^{44}Ca form SRG evolved 2N-only interactions and by including induced and full 3NF. Experiment are charge radii.

	2NF only	2+3NF(ind.)	2+3NF(full)	Experiment
^{16}O:	2.10	2. 41	2.38	2.718±0.210 [19]
^{44}Ca:	2.48	2.93	2.94	3.520±0.005 [20]

CB *et al.*, arXiv:1211.3315 [nucl-th]

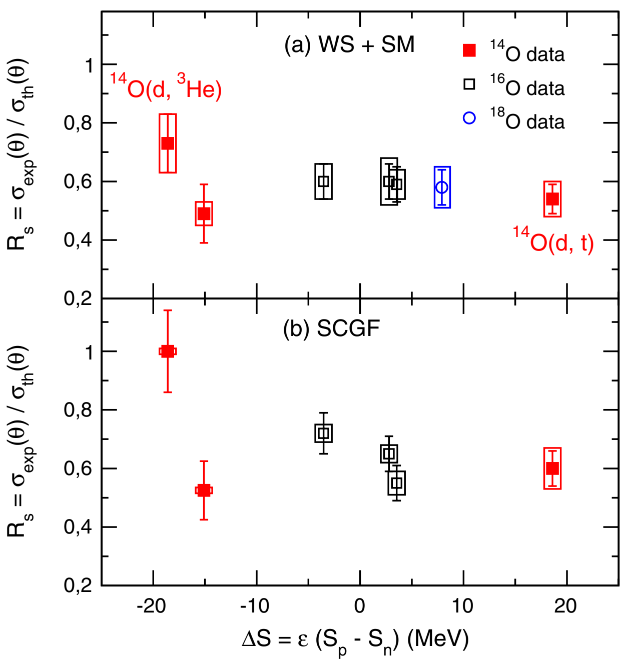
N3LO (Λ = 500Mev/c) chiral NN interaction evolved to 2N + 3N forces (2.0fm^{-1})
N2LO (Λ = 400Mev/c) chiral 3N interaction evolved (2.0fm^{-1})

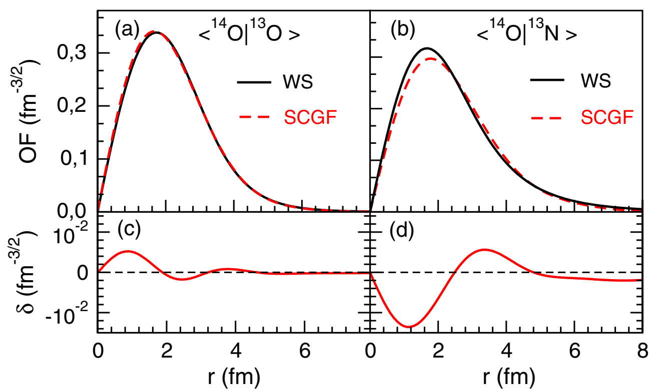
Single nucleon transfer in the oxygen chain

[F. Flavigny et al, PRL**110**, 122503 (2013)]

→ Analysis of $^{14}O(d,t)^{13}O$ and $^{14}O(d,^{3}He)^{13}N$ transfer reactions @ SPIRAL

Reaction	E^* (MeV)	J^π	R_{rms}^{HFB} (fm)	r_0 (fm)	C^2S_{exp} (WS)	C^2S_{th} $0p+2\hbar\omega$	R_s (WS)	C^2S_{exp} (SCGF)	C^2S_{th} (SCGF)	R_s (SCGF)
^{14}O (d, t) ^{13}O	0.00	$3/2^-$	2.69	1.40	1.69 (17)(20)	3.15	0.54(5)(6)	1.89(19)(22)	3.17	0.60(6)(7)
^{14}O (d, ^3He) ^{13}N	0.00	$1/2^-$	3.03	1.23	1.14(16)(15)	1.55	0.73(10)(10)	1.58(22)(2)	1.58	1.00(14)(1)
	3.50	$3/2^-$	2.77	1.12	0.94(19)(7)	1.90	0.49(10)(4)	1.00(20)(1)	1.90	0.53(10)(1)
^{16}O (d, t) ^{15}O	0.00	$1/2^-$	2.91	1.46	0.91(9)(8)	1.54	0.59(6)(5)	0.96(10)(7)	1.73	0.55(6)(4)
^{16}O (d, ^3He) ^{15}N [19,20]	0.00	$1/2^-$	2.95	1.46	0.93(9)(9)	1.54	0.60(6)(6)	1.25(12)(5)	1.74	0.72(7)(3)
	6.32	$3/2^-$	2.80	1.31	1.83(18)(24)	3.07	0.60(6)(8)	2.24(22)(10)	3.45	0.65(6)(3)
^{18}O (d, ^3He) ^{17}N [21]	0.00	$1/2^-$	2.91	1.46	0.92(9)(12)	1.58	0.58(6)(10)			





- Overlap functions and strengths from GF
- Rs independent of asymetry