The Hoyle state and its relatives:
The C12 continuum

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**“Nuclear Structure & Reactions:
Experimental and Ab Initio Theoretical Perspectives”**

**TRIUMF, Vancouver, Canada
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HELMHOLTZ ASSOCIATION

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Overview

Unitary Correlation Operator Method

Fermionic Molecular Dynamics

Cluster States in ^{12}C

- **FMD and microscopic cluster model**
- **electron scattering data – form factors**

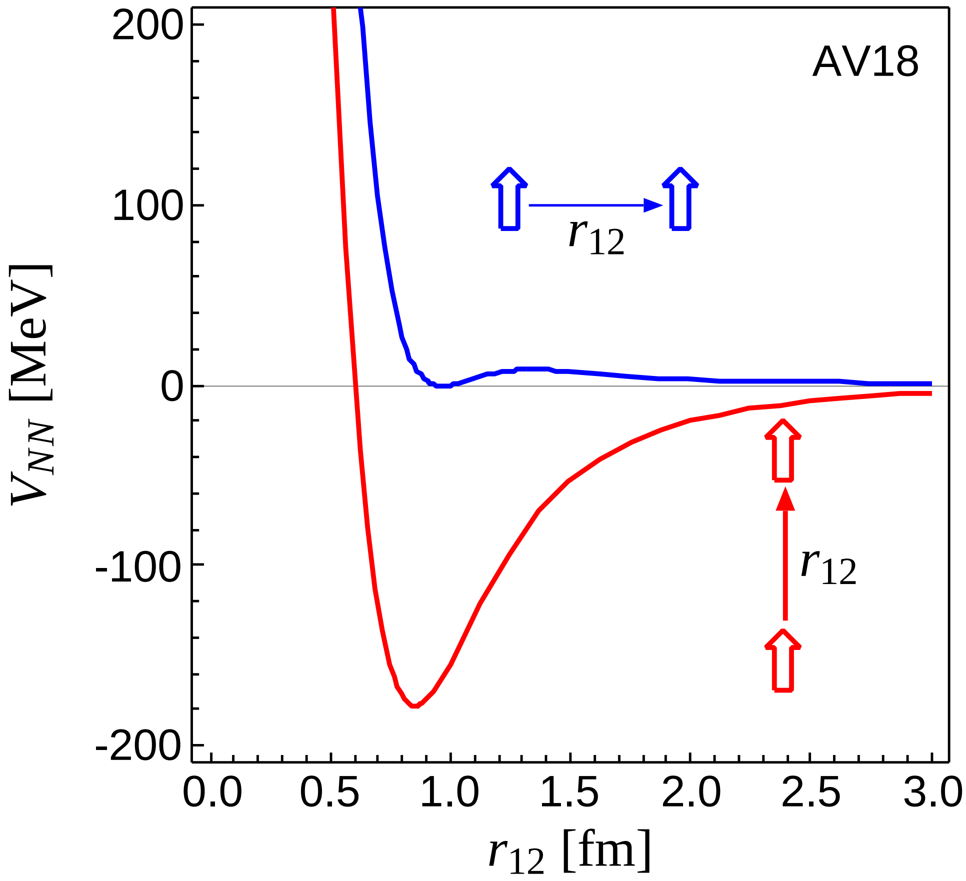
Resonances and Scattering States in ^{12}C

- **include ^{8}Be+α configurations**
- **R-matrix method**

Nuclear Force

Argonne V18 (T=0)

spins aligned parallel or perpendicular to the relative distance vector



- strong repulsive core: nucleons can not get closer than ≈ 0.5 fm

➽ **central correlations**

- strong dependence on the orientation of the spins due to the tensor force

➽ **tensor correlations**

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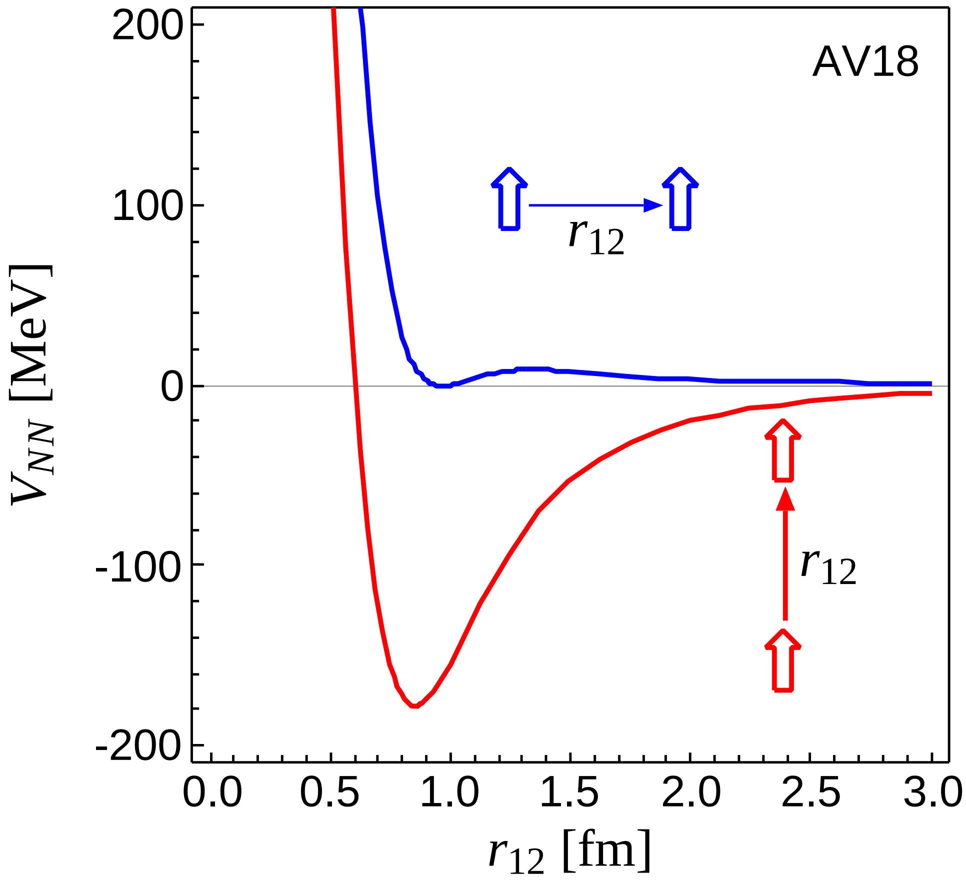

- strong repulsive core: nucleons can not get closer than ≈ 0.5 fm

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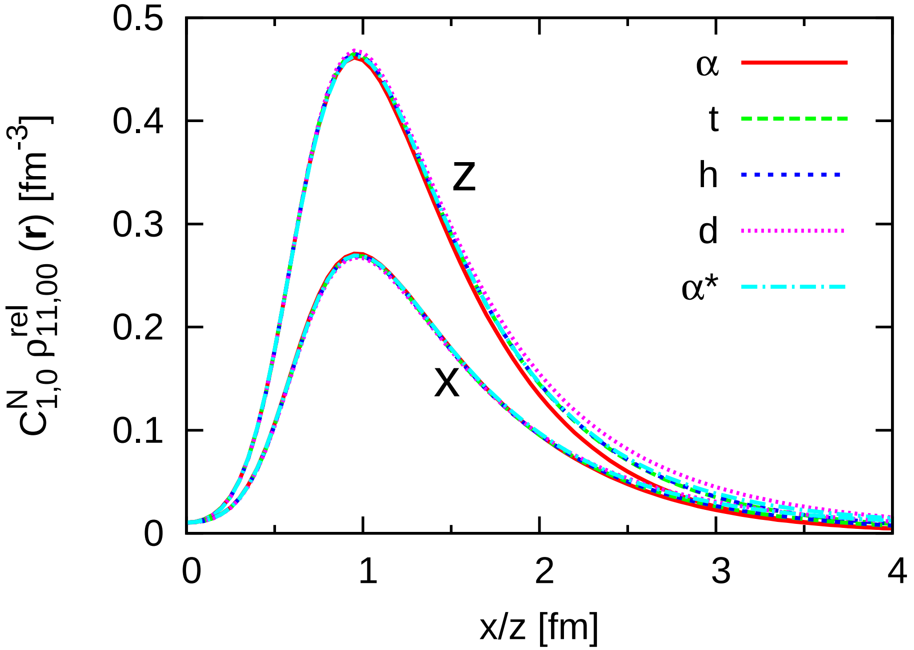
➽ **tensor correlations**

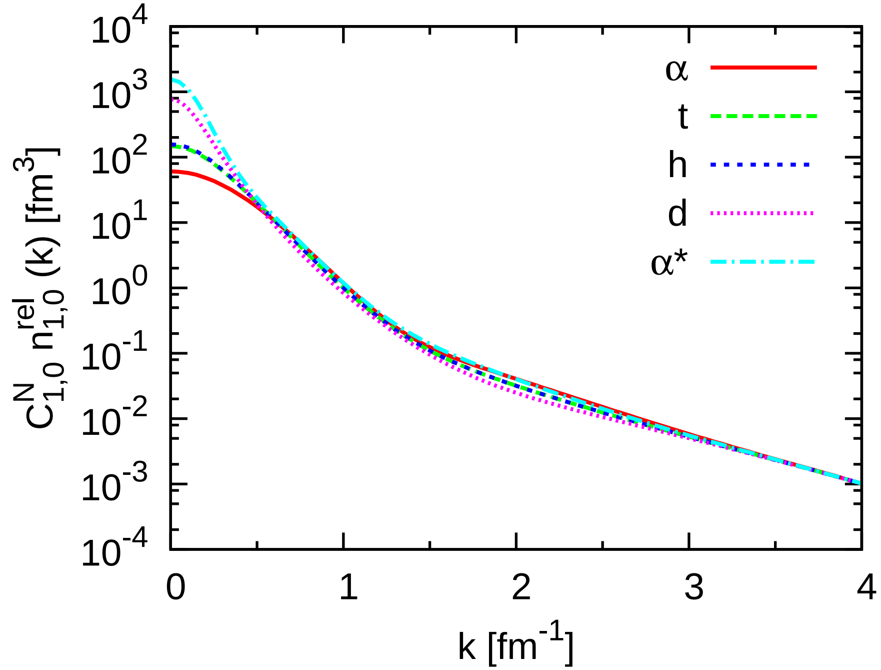
the nuclear force will induce **strong short-range correlations** in the nuclear wave function

Two-body densities in $A = 2, 3, 4$ Nuclei — AV8'

coordinate space
$S = 1, M_S = 1$, $T = 0$



momentum space
$S = 1$, $T = 0$



- normalize two-body density in coordinate space at r=1.0 fm
- normalized two-body densities in coordinate space are identical at short distances for all nuclei
- use the **same** normalization factor in momentum space – high momentum tails agree for all nuclei

Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

Unitary Correlation Operator Method

Correlation Operator

- induce short-range (two-body) central and tensor correlations into the many-body state

$$\underset{\sim}{C} = \underset{\sim}{C}_\Omega \underset{\sim}{C}_r = \exp\Big[-i\sum_{i<j}\underset{\sim}{g}_{\Omega,ij}\Big]\exp\Big[-i\sum_{i<j}\underset{\sim}{g}_{r,ij}\Big] \quad , \quad \underset{\sim}{C}^\dagger \underset{\sim}{C} = \underset{\sim}{1}$$

- correlation operator should conserve the symmetries of the Hamiltonian and should be of finite-range, correlated interaction **phase shift equivalent** to bare interaction by construction

Correlated Operators

- correlated operators will have contributions in higher cluster orders

$$\underset{\sim}{C}^\dagger \underset{\sim}{O} \underset{\sim}{C} = \underset{\sim}{\hat{O}}^{[1]} + \underset{\sim}{\hat{O}}^{[2]} + \underset{\sim}{\hat{O}}^{[3]} + \ldots$$

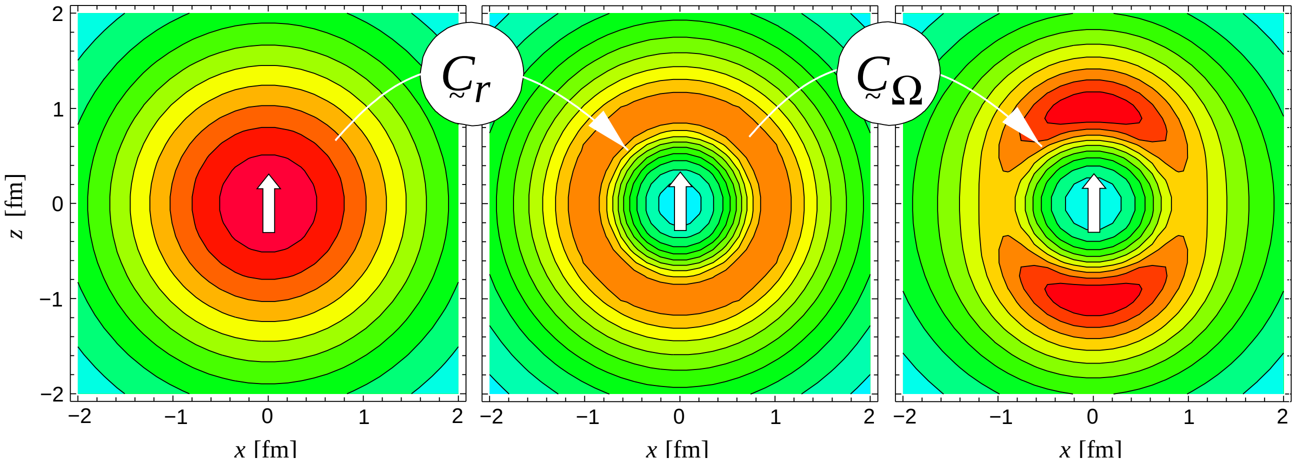
- two-body approximation: correlation range should be small compared to mean particle distance

Correlated Interaction

$$\underset{\sim}{C}^\dagger \, (\underset{\sim}{T} + \underset{\sim}{V}) \, \underset{\sim}{C} = \underset{\sim}{T} + \underset{\sim}{V}_{\mathrm{UCOM}} + \underset{\sim}{V}^{[3]}_{\mathrm{UCOM}} + \ldots$$

Correlations and Energies

two-body densities



$\rho^{(2)}_{S,T}(\mathbf{r}_1 - \mathbf{r}_2) \quad S = 1, M_S = 1, T = 0$

central correlator $\underset{\sim}{C}_{\mathbf{r}}$
shifts density out of the repulsive core

tensor correlator $\underset{\sim}{C}_{\mathbf{\Omega}}$
aligns density with spin orientation

Neff and Feldmeier, Nucl. Phys. **A713** (2003) 311

Correlations and Energies

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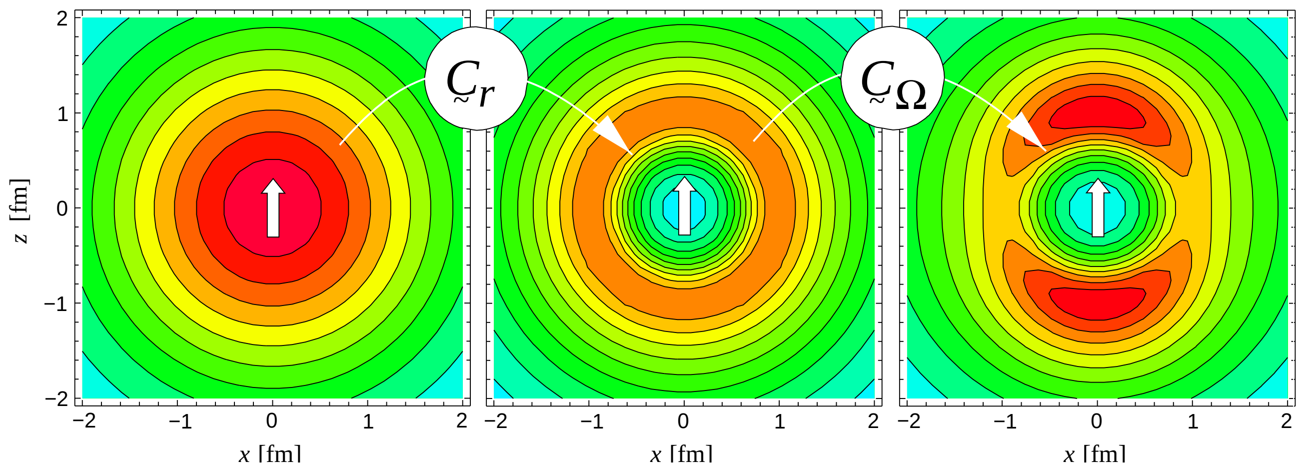

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tensor correlator $\underset{\sim}{C}_{\boldsymbol{\Omega}}$
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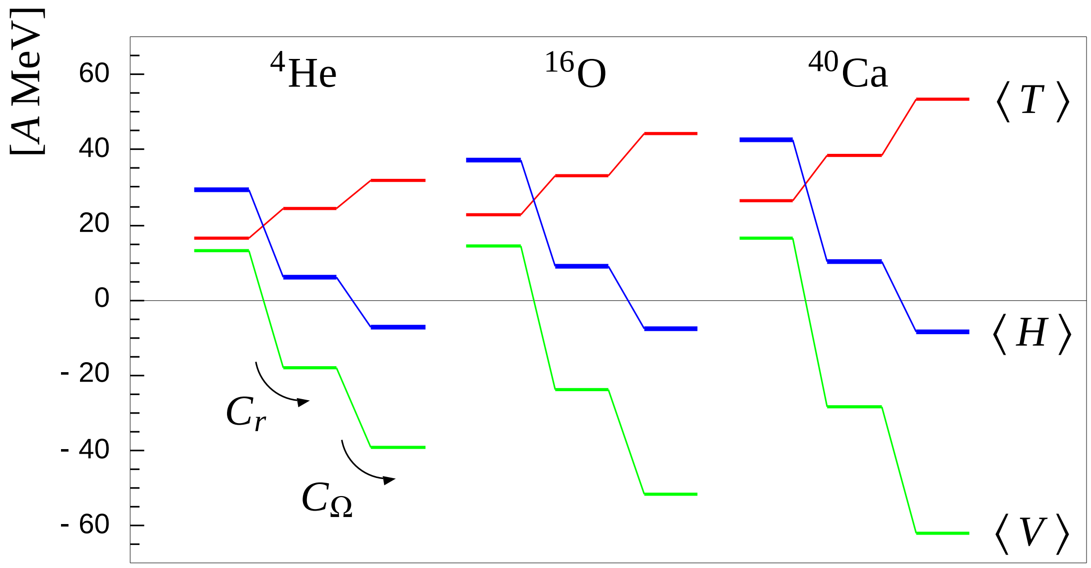
both central and tensor correlations are essential for binding

energies

$0\hbar\omega$ Harmonic Oscillator



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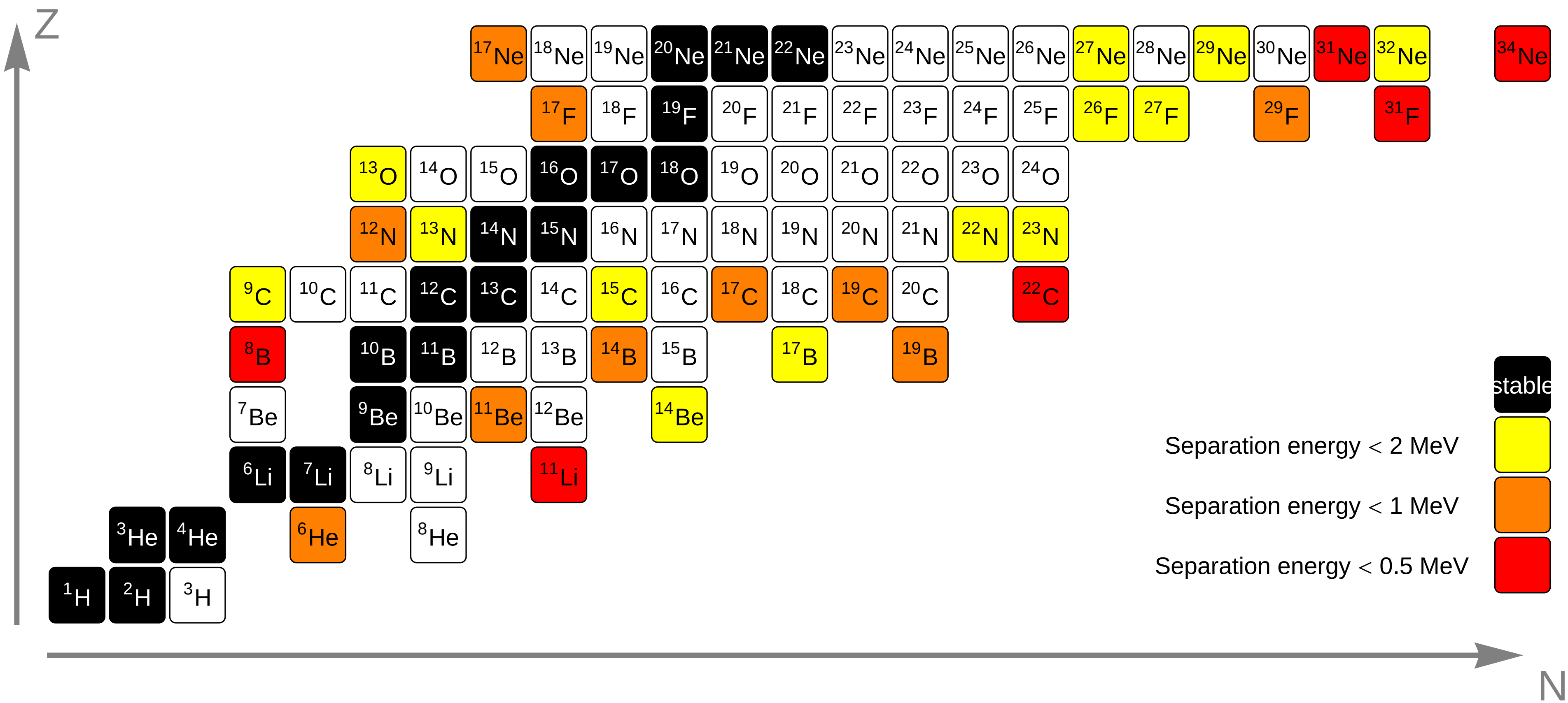
Exotica: Special Challenges



- states close to one-nucleon, two-nucleon or cluster thresholds can have well developed **halo** or **cluster** structure
- these are hard to tackle in the harmonic oscillator basis

Fermionic Molecular Dynamics

Fermionic

Slater determinant

$$\left| Q \right\rangle = \underset{\sim}{\mathcal{A}} \Big(\left| q_1 \right\rangle \otimes \cdots \otimes \left| q_A \right\rangle \Big)$$

- antisymmetrized A-body state

Feldmeier, Schnack, Rev. Mod. Phys. **72** (2000) 655

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Molecular

single-particle states

$$\left\langle \mathbf{x} \middle| q \right\rangle = \sum_i c_i \exp\left\{ -\frac{(\mathbf{x} - \mathbf{b}_i)^2}{2 a_i} \right\} \otimes \left| \chi^{\uparrow}_i, \chi^{\downarrow}_i \right\rangle \otimes \left| \xi \right\rangle$$

- Gaussian wave-packets in phase-space (complex parameter $\mathbf{b}_i$ encodes mean position and mean momentum), spin is free, isospin is fixed
- width a_i is an independent variational parameter for each wave packet
- use one or two wave packets for each single particle state

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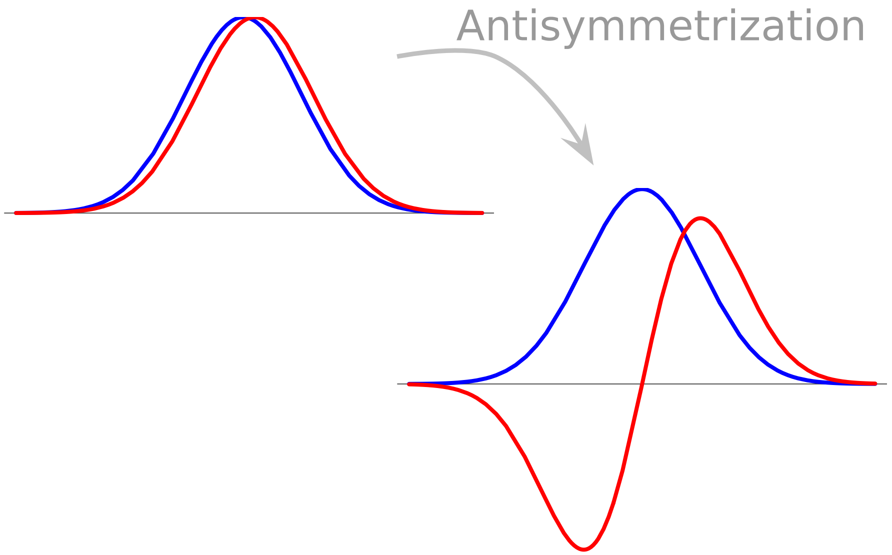

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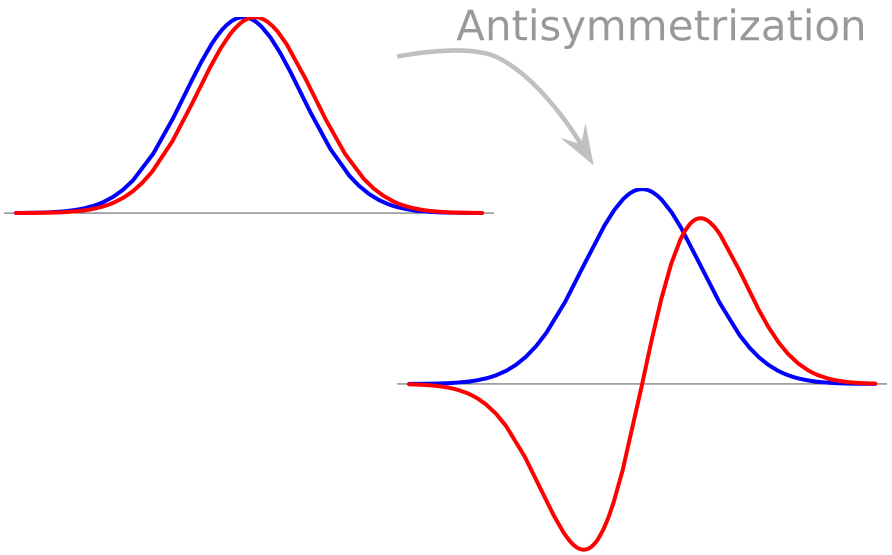

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see also
Antisymmetrized Molecular Dynamics
Horiuchi, Kanada-En'yo, Kimura, . . .

Feldmeier, Schnack, Rev. Mod. Phys. **72** (2000) 655

Neff, Feldmeier, Nucl. Phys. **A738** (2004) 357

Operator Representation of V_{UCOM}

$$\underset{\sim}{C}^{\dagger}(\underset{\sim}{T}+\underset{\sim}{V})\underset{\sim}{C} \quad = \quad \underset{\sim}{T}$$

one-body kinetic energy

$$+\sum_{ST}\hat{V}_{c}^{ST}(r)+\frac{1}{2}\left(\underset{\sim}{p_r}^{2}\,\hat{V}_{p^2}^{ST}(r)+\hat{V}_{p^2}^{ST}(r)\,\underset{\sim}{p_r}^{2}\right)+\hat{V}_{l^2}^{ST}(r)\,\underset{\sim}{\mathbf{l}}^{2}$$

central potentials

$$+\sum_{T}\hat{V}_{ls}^{T}(r)\,\underset{\sim}{\mathbf{l}}\cdot\underset{\sim}{\mathbf{s}}+\hat{V}_{l^2ls}^{T}(r)\,\underset{\sim}{\mathbf{l}}^{2}\,\underset{\sim}{\mathbf{l}}\cdot\underset{\sim}{\mathbf{s}}$$

spin-orbit potentials

$$+\sum_{T}\hat{V}_{t}^{T}(r)\,\underset{\sim}{S}_{12}(\mathbf{r},\mathbf{r})+\hat{V}_{trp_\Omega}^{T}(r)\,\underset{\sim}{p_r}\,\underset{\sim}{S}_{12}(\mathbf{r},\mathbf{p_\Omega})+\hat{V}_{tll}^{T}(r)\,\underset{\sim}{S}_{12}(\mathbf{l},\mathbf{l})+$$
$$\hat{V}_{tp_\Omega p_\Omega}^{T}(r)\,\underset{\sim}{S}_{12}(\mathbf{p_\Omega},\mathbf{p_\Omega})+\hat{V}_{l^2tp_\Omega p_\Omega}^{T}(r)\,\underset{\sim}{\mathbf{l}}^{2}\underset{\sim}{S}_{12}(\mathbf{p_\Omega},\mathbf{p_\Omega})$$

tensor potentials

bulk of tensor force mapped onto central part of correlated interaction

tensor correlations also change the spin-orbit part of the interaction

Weber, Feldmeier, Hergert, Neff, arXiv:1311.7610

PAV, VAP and Multiconfiguration

Projection After Variation (PAV)

- mean-field may break symmetries of Hamiltonian
- restore inversion, translational and rotational symmetry by projection on parity, linear and angular momentum

$$\underset{\sim}{P}^{\pi} = \frac{1}{2}(1 + \pi \underset{\sim}{\Pi})$$

$$\underset{\sim}{P}^{J}_{MK} = \frac{2J+1}{8\pi^2} \int d^3\Omega \, {D^{J}_{MK}}^{\star}(\Omega) \, \underset{\sim}{R}(\Omega)$$

$$\underset{\sim}{P}^{\mathbf{P}} = \frac{1}{(2\pi)^3} \int d^3X \; \exp\{-i(\underset{\sim}{\mathbf{P}} - \mathbf{P})\cdot\mathbf{X}\}$$

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Variation After Projection (VAP)

- effect of projection can be large
- **Variation after Angular Momentum and Parity Projection** (VAP) for light nuclei
- combine VAP with **constraints** on **radius**, **dipole** moment, **quadrupole** moment, . . . to generate additional configurations

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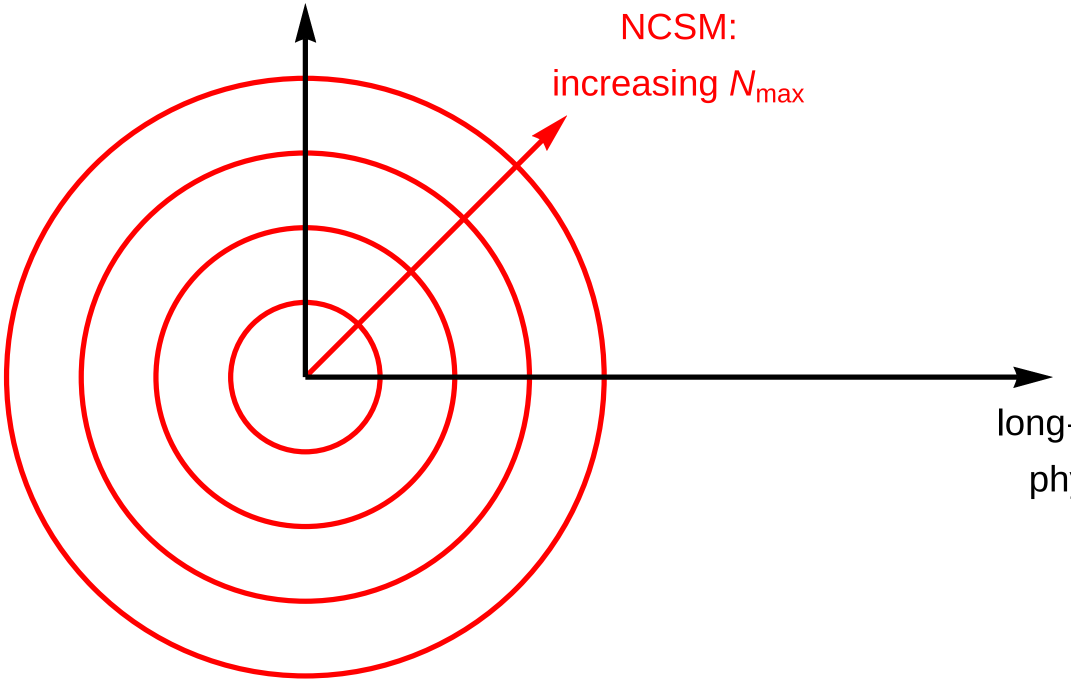
Multiconfiguration Calculations

- **diagonalize** Hamiltonian in a set of projected intrinsic states

$$\left\{ \left| Q^{(a)} \right\rangle , \quad a = 1, \ldots, N \right\}$$

$$\sum_{K'b} \left\langle Q^{(a)} \middle| \underset{\sim}{H} \underset{\sim}{P}^{J^\pi}_{KK'} \underset{\sim}{P}^{\mathbf{P}=0} \middle| Q^{(b)} \right\rangle \cdot c^{\alpha}_{K'b} = E^{J^\pi \alpha} \sum_{K'b} \left\langle Q^{(a)} \middle| \underset{\sim}{P}^{J^\pi}_{KK'} \underset{\sim}{P}^{\mathbf{P}=0} \middle| Q^{(b)} \right\rangle \cdot c^{\alpha}_{K'b}$$

FMD vs NCSM model spaces



- NCSM allows good description of short-range physics, but long-range behavior suffers from harmonic oscillator asymptotics

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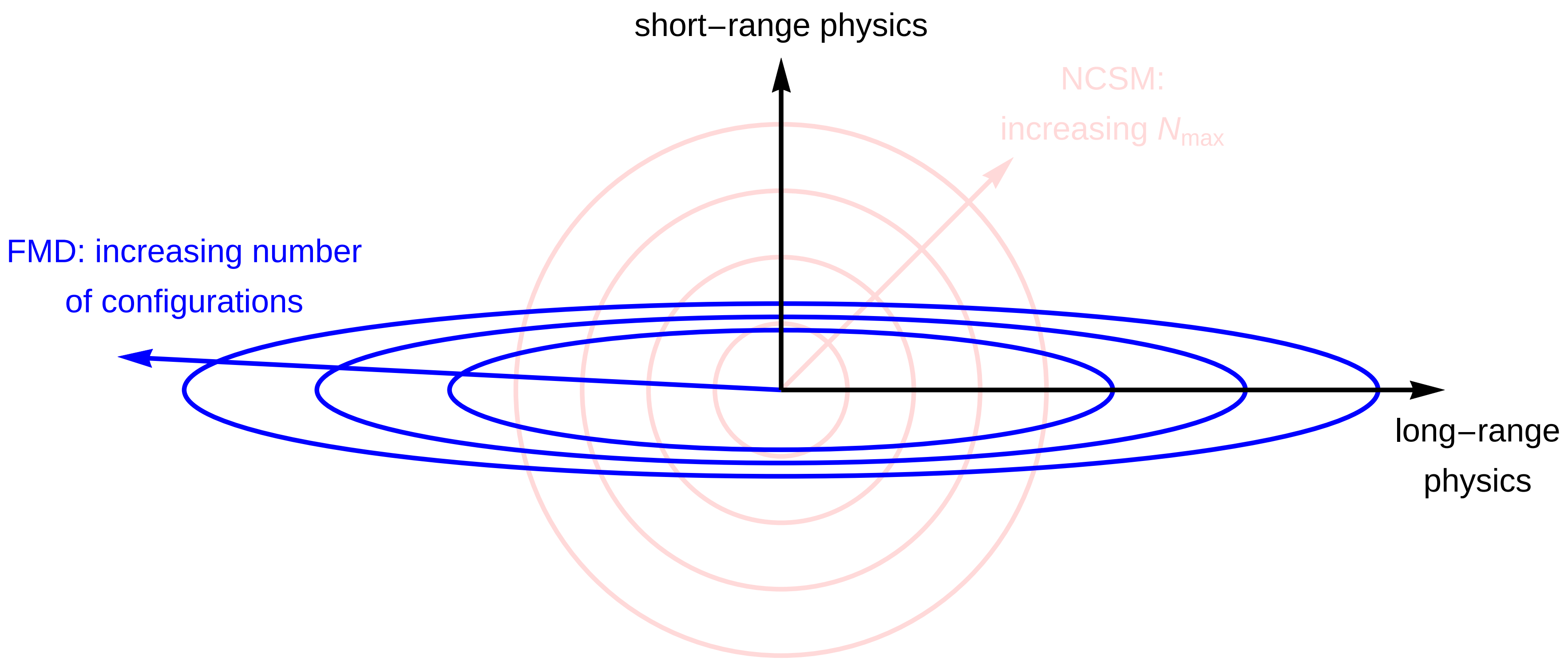
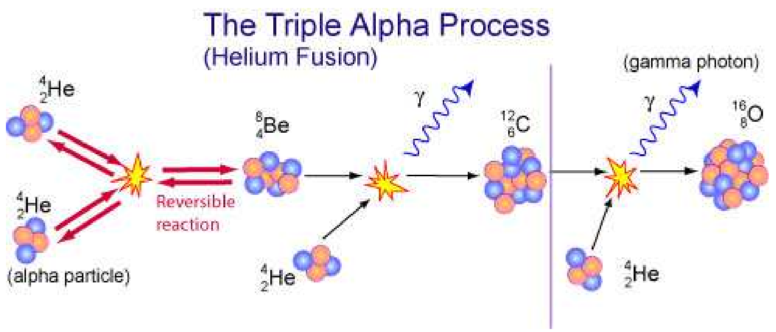


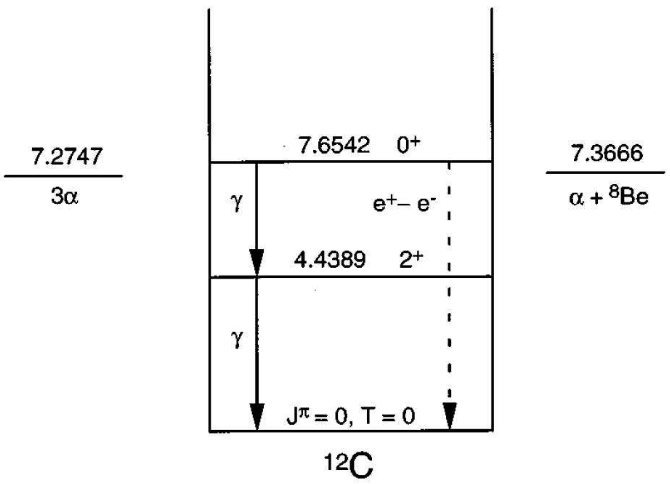
- NCSM allows good description of short-range physics, but long-range behavior suffers from harmonic oscillator asymptotics
- FMD allows to describe long-range physics by superposition of localized cluster configurations, but limited in description of short-range physics

Cluster States in ^{12}C



Astrophysical Motivation

- **Helium burning: triple alpha-reaction**

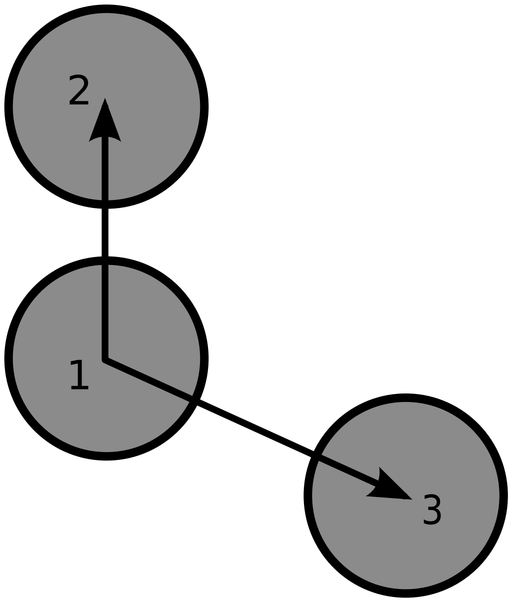


Structure

- **Is the Hoyle state a pure α-cluster state ?**
- **Other excited 0^+ and 2^+ states**

➽ **Compare FMD results to microscopic α-cluster model**
➽ **Intrinsic structure from two-body densities**
➽ **Analyze wave functions in harmonic oscillator basis**

Microscopic α-Cluster Model



$$R_{12} = (2, 4, \ldots, 10)\,\mathrm{fm}$$
$$R_{13} = (2, 4, \ldots, 10)\,\mathrm{fm}$$
$$\cos(\vartheta) = (1.0, 0.8, \ldots, -1.0)$$

alltogether 165 configurations

Kamimura, Nuc. Phys. **A351** (1981) 456

Funaki et al., Phys. Rev. C **67** (2003) 051306(R)

Basis States

- describe Hoyle State as a system of 3 ^{4}He nuclei

$$\left| \Psi_{3\alpha}(\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3); JMK\pi \right\rangle = P^J_{MK} P^\pi \mathcal{A} \left\{ \left| \psi_\alpha(\mathbf{R}_1) \right\rangle \otimes \left| \psi_\alpha(\mathbf{R}_2) \right\rangle \otimes \left| \psi_\alpha(\mathbf{R}_3) \right\rangle \right\}$$

Volkov Interaction

- simple central interaction
- parameters adjusted to give reasonable α binding energy and radius, $\alpha - \alpha$ scattering data, adjusted to reproduce ^{12}C ground state energy

✘ only reasonable for ^{4}He, ^{8}Be and ^{12}C nuclei

‘BEC’ wave functions

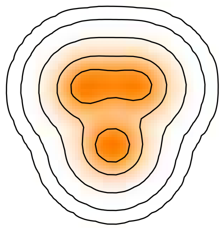
- interpretation of the Hoyle state as a Bose-Einstein Condensate of α-particles by Funaki, Tohsaki, Horiuchi, Schuck, Röpke
- same interaction and α-cluster parameters used

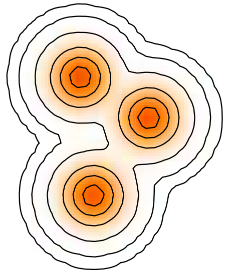
FMD

Basis States

- 20 FMD states obtained in Variation after Projection on 0^+ and 2^+ with constraints on the radius
- 42 FMD states obtained in Variation after Projection on parity with constraints on radius and quadrupole deformation
- 165 α-cluster configurations

➽ projected on angular momentum and linear momentum

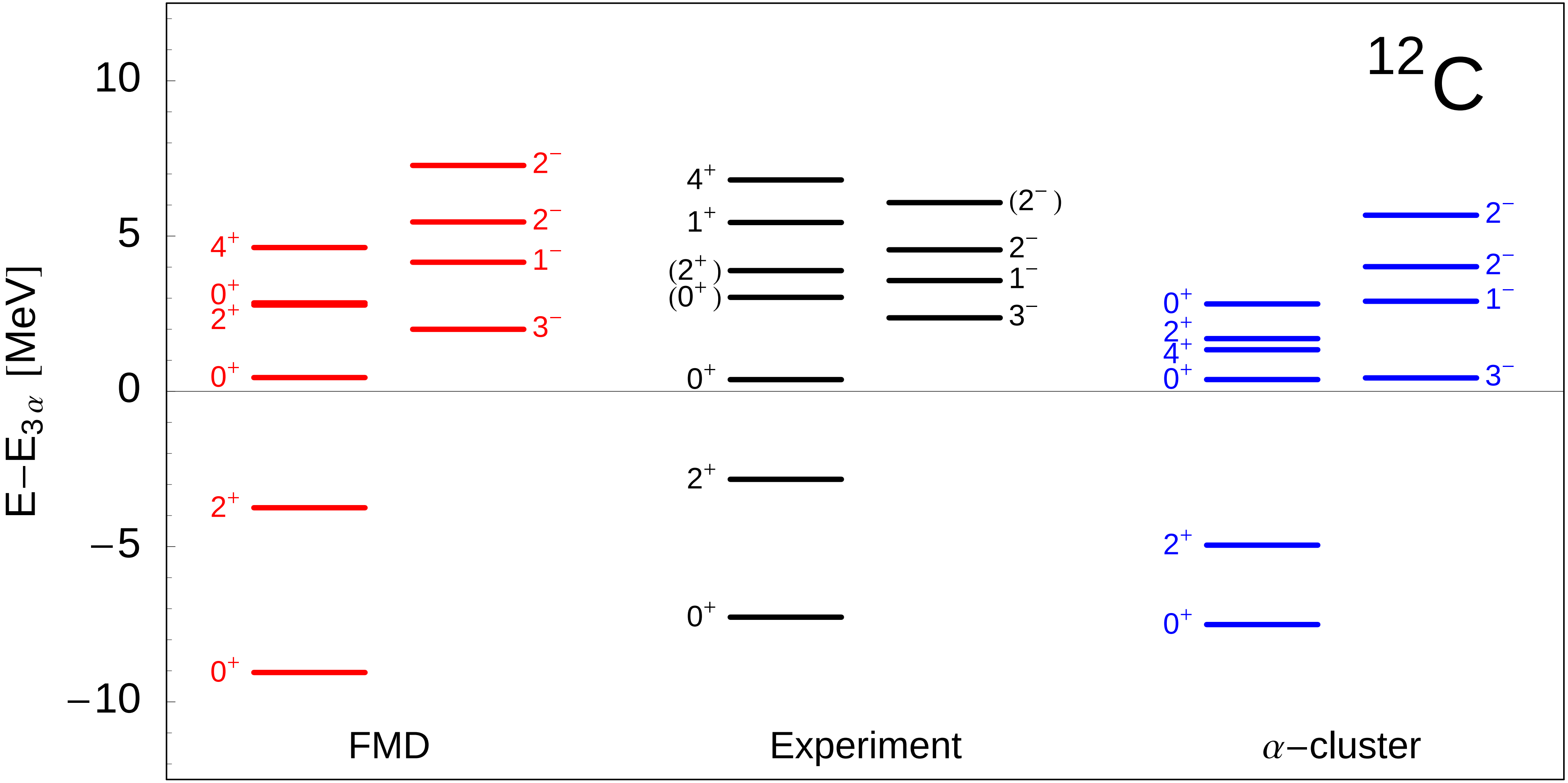




Interaction

- UCOM interaction (I_ϑ=0.30 fm^3 with phenomenological two-body correction term (momentum-dependent central and spin-orbit) fitted to doubly-magic nuclei
- not tuned for α-α scattering or ^{12}C properties

Comparison


^{12}C
10
5
0
−5
−10
$E - E_{3\alpha}$ [MeV]
2^-
4^+
2^-
1^-
0^+
2^+
3^-
0^+
2^+
0^+
4^+
1^+
(2^+)
(0^+)
0^+
2^+
0^+
(2^-)
2^-
1^-
3^-
0^+
2^+
4^+
0^+
2^+
0^+
2^-
2^-
1^-
3^-
FMD
Experiment
α−cluster

Comparison

	Exp[1]	Exp[2]	FMD	α-cluster	'BEC'[3]
$E(0_1^+)$	-92.16		-92.64	-89.56	-89.52
$E^*(2_1^+)$	4.44		5.31	2.56	2.81
$E(3\alpha)$	-84.89		-83.59	-82.05	-82.05
$E(0_2^+) - E(3\alpha)$	0.38		0.43	0.38	0.26
$E(0_3^+) - E(3\alpha)$	(3.0)	2.7(3)	2.84	2.81	
$E(2_2^+) - E(3\alpha)$	(3.89)	2.76(11)	2.77	1.70	
$r_{charge}(0_1^+)$	2.47(2)		2.53	2.54	
$r(0_1^+)$			2.39	2.40	2.40
$r(0_2^+)$			3.38	3.71	3.83
$r(0_3^+)$			4.62	4.75	
$r(2_1^+)$			2.50	2.37	2.38
$r(2_2^+)$			4.43	4.02	
$M(E0, 0_1^+ \rightarrow 0_2^+)$	5.4(2)		6.53	6.52	6.45
$B(E2, 2_1^+ \rightarrow 0_1^+)$	7.6(4)		8.69	9.16	
$B(E2, 2_1^+ \rightarrow 0_2^+)$	2.6(4)		3.83	0.84	
$B(E2, 2_2^+ \rightarrow 0_1^+)$		0.73(13)	0.46	1.99	

experimental situation for 0^+ and 2^+ states above threshold still not completely settled

[1] Ajzenberg-Selove, Nuc. Phys. **A506**, 1 (1990)
[2] Itoh et al., Nuc. Phys. **A738**, 268 (2004), Zimmermann et al., Phys. Rev. Lett. **110**, 152502 (2013)
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calculated in bound state approximation

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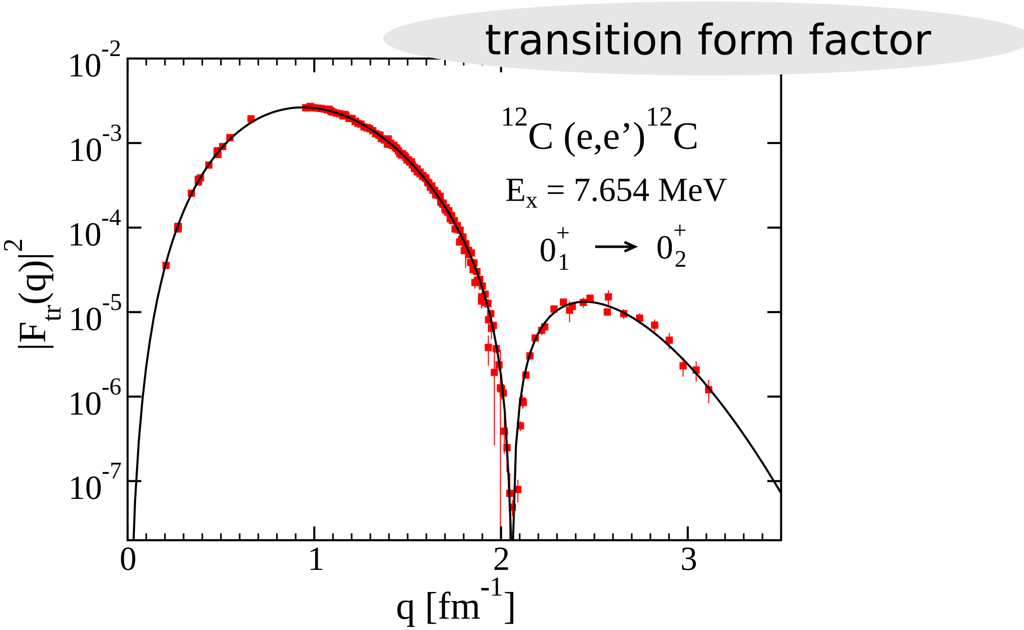
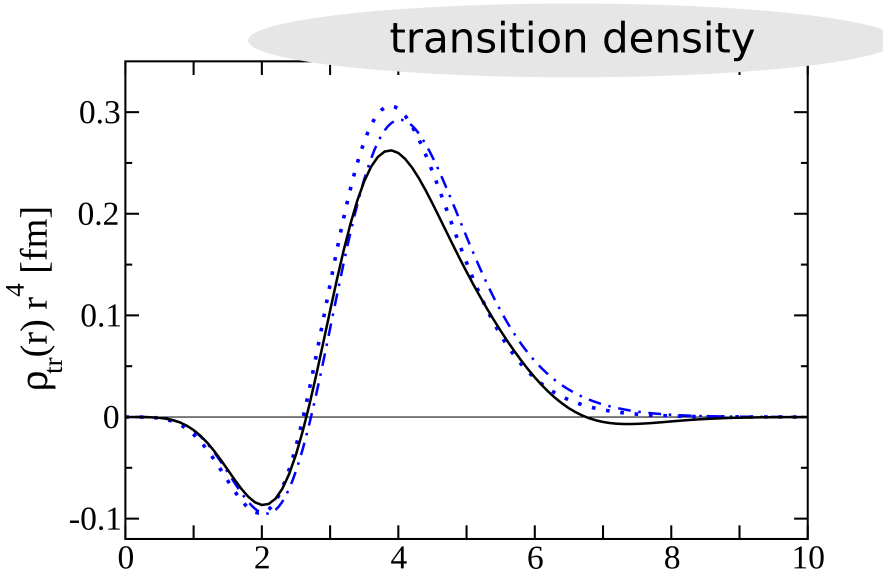
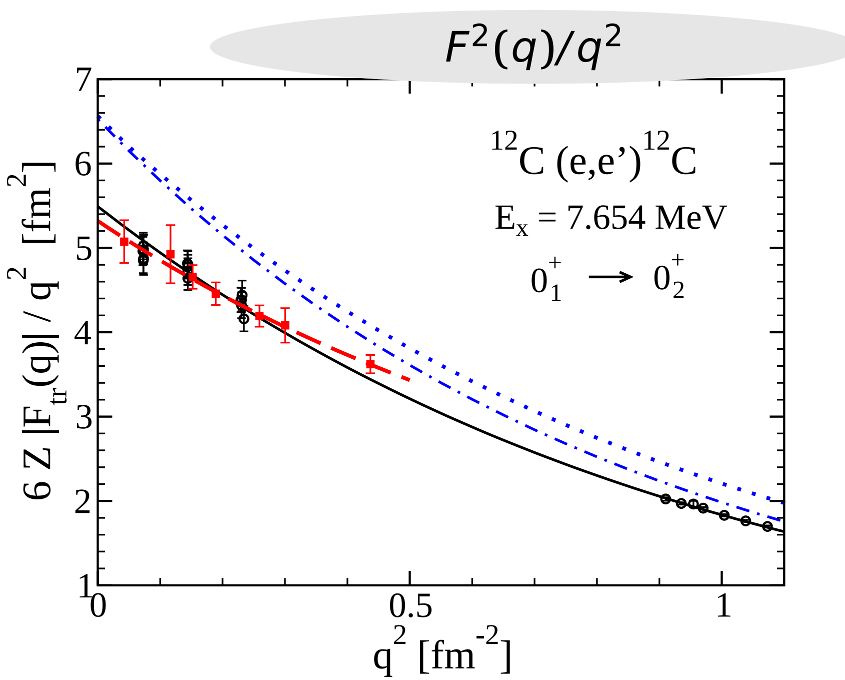
Monopole Matrix Element revisited







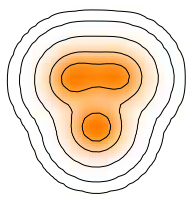
- $M(E0)$ determines the pair decay width
- model-independent self-consistent determination of transition form-factor/density in DWBA
- data at high momentum transfer necessary to constrain matrix element $M(E0) = 5.47 \pm 0.09\, e^2\text{fm}^2$

M. Chernykh, H. Feldmeier, T. Neff, P. von Neumann-Cosel, A. Richter, Phys. Rev. Lett. **105** (2010) 022501

Cluster States in ^{12}C

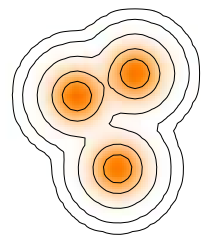
Important Configurations

- Calculate the overlap with FMD basis states to find the most important contributions to the Hoyle state

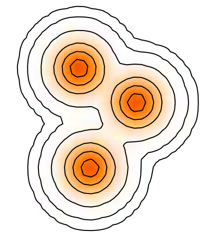


$\left|\left\langle \cdot \mid 0_1^+ \right\rangle\right| = 0.94$

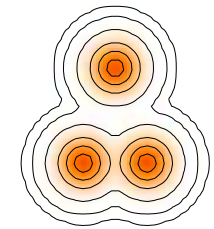
$\left|\left\langle \cdot \mid 2_1^+ \right\rangle\right| = 0.93$



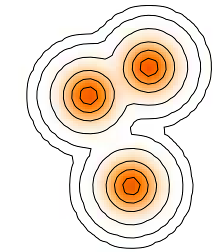
$\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.72$



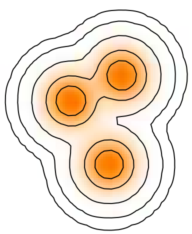
$\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.71$



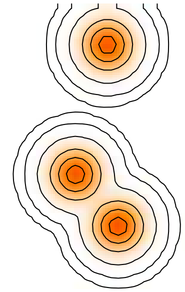
$\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.61$



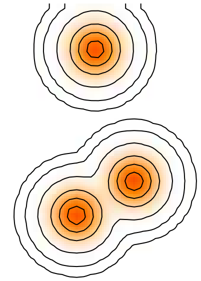
$\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.61$



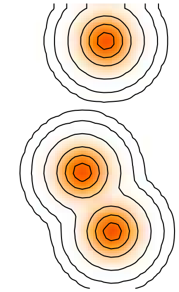
$\left|\left\langle \cdot \mid 3_1^- \right\rangle\right| = 0.83$



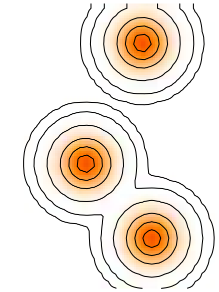
$\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.50$



$\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.49$



$\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.44$



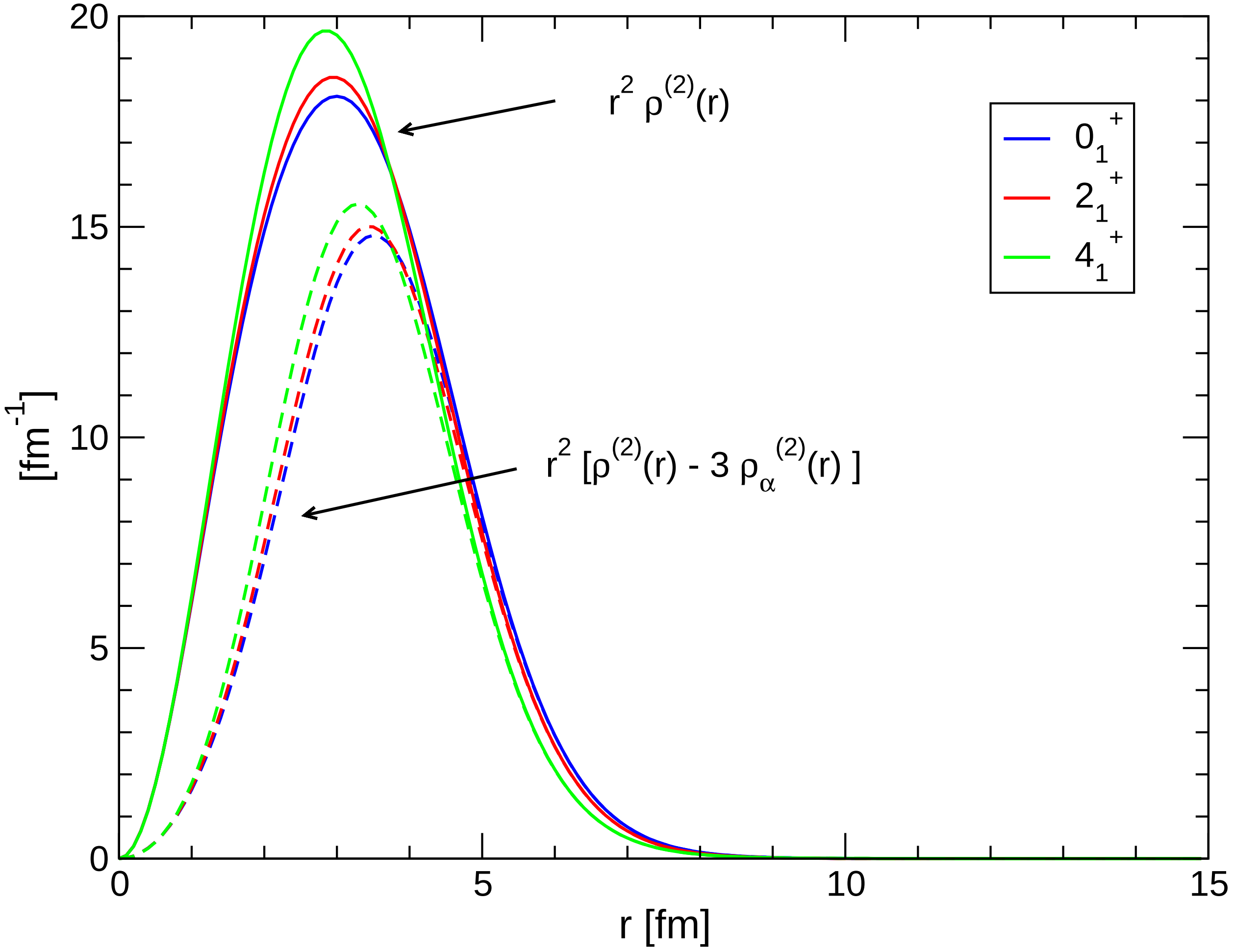
$\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.41$

FMD basis states are not orthogonal!

0_2^+ and 0_3^+ states have no rigid intrinsic structure

Two-body Densities and Intrinsic Structure

ground state band



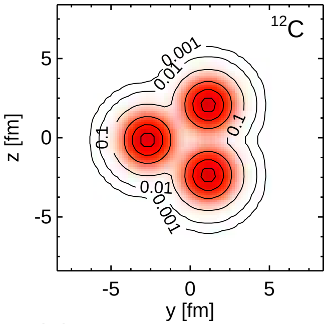
Cluster Model

$$\rho^{(2)}(r) = \langle \Psi \,|\, \sum_{i<j} \delta(\mathbf{r} - \underset{\sim}{\mathbf{r}}_{ij}) \,|\, \Psi \rangle$$

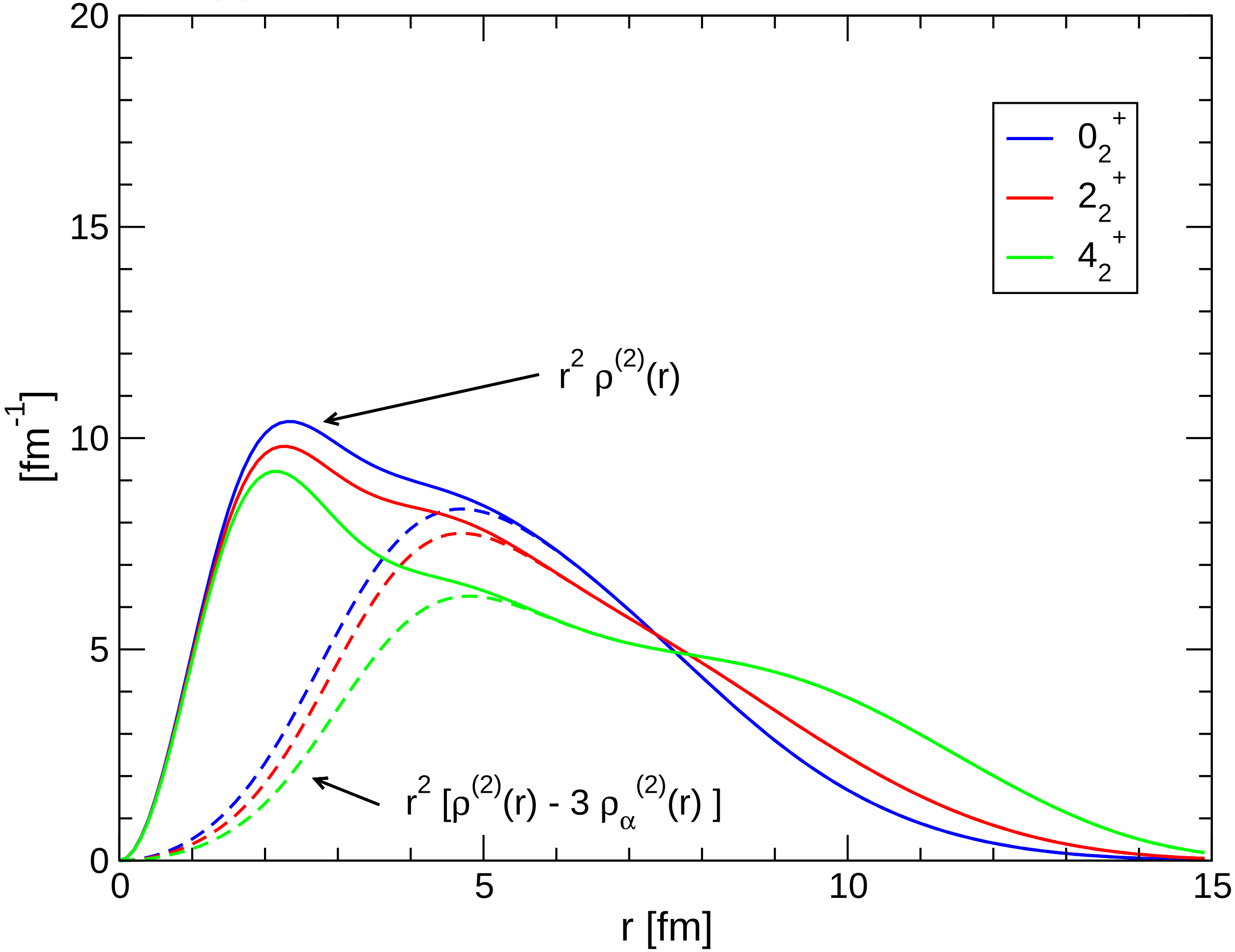
- substract contributions from α's to extract "α-α" correlations
- (substracted) two-body density peaks at 3.5 fm

➸ consistent with **compact triangular structure**

Two-body Densities and Intrinsic Structure



Hoyle state "band"

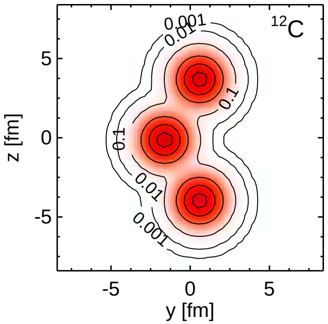


Cluster Model

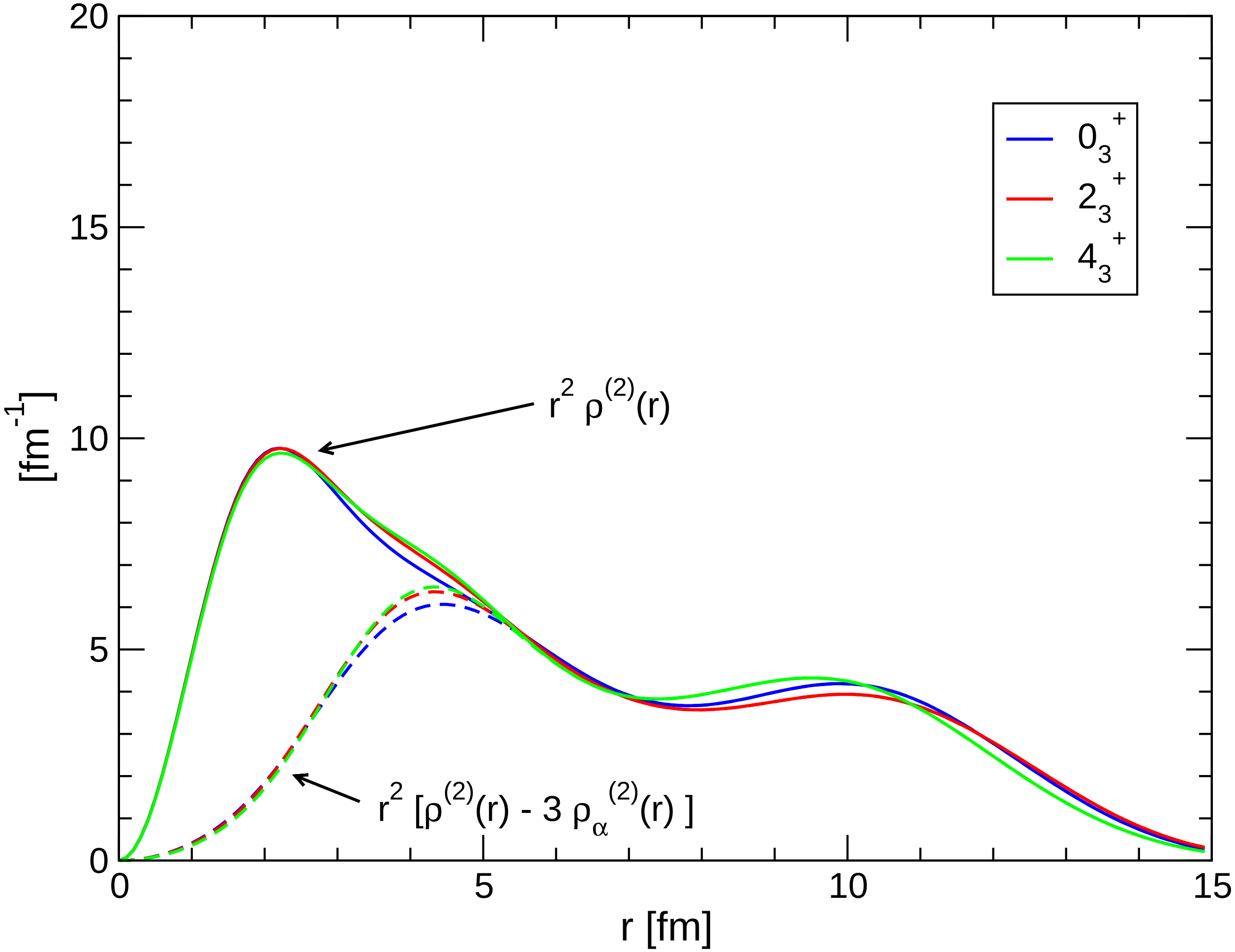
$$\rho^{(2)}(r) = \langle \Psi \,|\, \sum_{i<j} \delta(\mathbf{r} - \mathbf{r}_{ij}) \,|\, \Psi \rangle$$

- substract contributions from α's to extract "α-α correlations"
- Hoyle state two-body density peaks at 5 fm, extended tail
- ➼ consistent with **triangular structure**
- tail in 2_2^+ and 4_2^+ states more pronounced
- ➼ admixture of open triangle configurations

Two-body Densities and Intrinsic Structure



third 0^+ state band



Cluster Model

$$\rho^{(2)}(r) = \langle\, \Psi \,|\, \sum_{i<j} \delta(\mathbf{r} - \underset{\sim}{\mathbf{r}}_{ij}) \,|\, \Psi \,\rangle$$

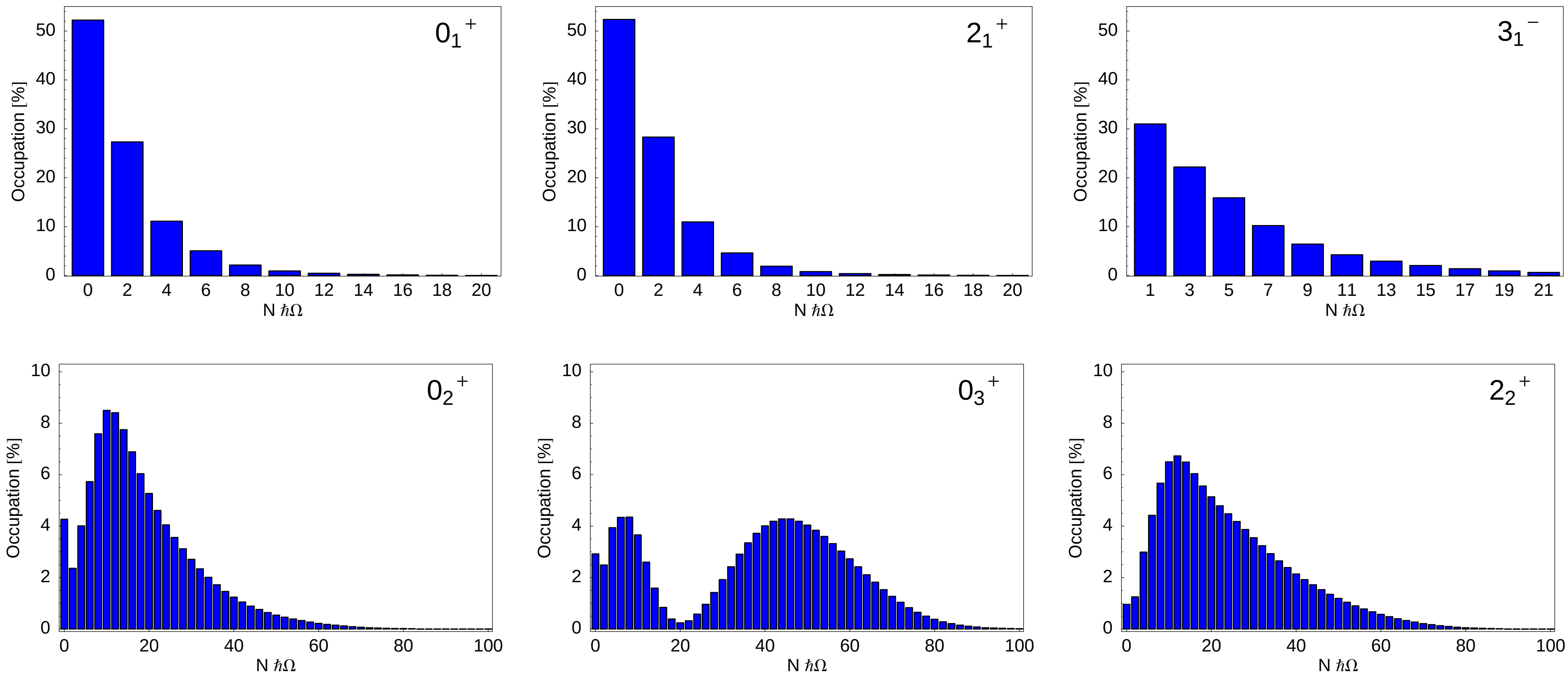
- substract contributions from α's to extract "α-α" correlations
- two-body density peaks at 4.5 fm and 10 fm
- ➽ consistent with **open triangle/chain configuration**

Cluster States in ^{12}C
Harmonic Oscillator $N\hbar\Omega$ Excitations

Y. Suzuki *et al*, Phys. Rev. C **54**, 2073 (1996).

$$\mathrm{Occ}(N) = \left\langle \Psi \,\middle|\, \delta\left(\textstyle\sum_i (\underset{\sim}{H}_i^{HO}/\hbar\Omega - 3/2) - N\right) \,\middle|\, \Psi \right\rangle$$

Cluster Model



Include ^{8}Be-α continuum

How to treat the ^{12}C continuum above the 3-α threshold ?

- **In principle it should be described as a three-body continuum**
- **However ^{8}Be+α configurations are lower in energy than 3-α configurations up to pretty large hyperradii**
- **Approximation: consider ^{8}Be(0^+) and ^{8}Be(2^+) (and additional ^{8}Be pseudo states) as bound states**
- **Could be considered as a microscopic CDCC approach**

^{8}Be-α wave functions

alpha-cluster model calculations with continuum:

Descouvemont, Baye, Phys. Rev. **C36**, 54 (1987)
Arai, Phys. Rev. **C74**, 064311 (2006)
Vasilevsky *et al.*, Phys. Rev. **C85**, 034318 (2012)

^{8}Be wave functions

- α-α configurations up to 9 fm distance, project on 0^+ and 2^+, $M = 0, 1, 2$

$$\left| {}^{8}\text{Be}_{I,K} \right\rangle = P^{I}_{K0} \sum_i \left\{ \left| {}^{4}\text{He}(-R_i/2\mathbf{e}_z) \right\rangle \otimes \left| {}^{4}\text{He}(R_i/2\mathbf{e}_z) \right\rangle \right\} c^{J}_{i}$$

- reproduces ground state energy within 50 keV compared to full calculation

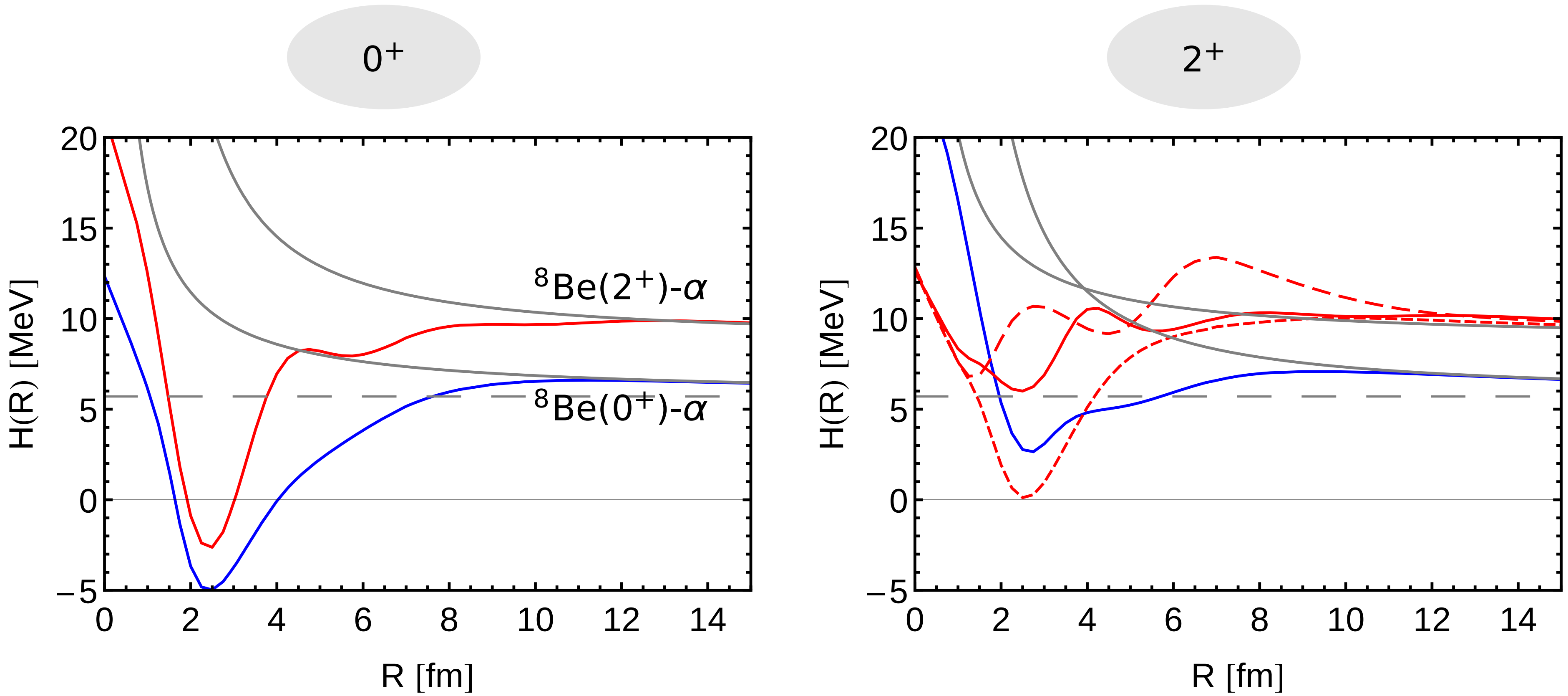
^{12}C configurations

- ^{8}Be(0^+,2^+) and α at distance R
- ^{8}Be(2^+) can have different orientations with respect to distance vector
- ^{8}Be(0^+,2^+)+α configurations have to be projected on total angular momentum

$$\left| {}^{8}\text{Be}_{I,K}, {}^{4}\text{He}; R; JM \right\rangle = P^{J}_{MK} \left\{ \left| {}^{8}\text{Be}_{I,K}(-1/3R\mathbf{e}_z) \right\rangle \otimes \left| {}^{4}\text{He}(2/3R\mathbf{e}_z) \right\rangle \right\}$$

Cluster Model: ^{8}Be-α Continuum

GCM Energy Surfaces

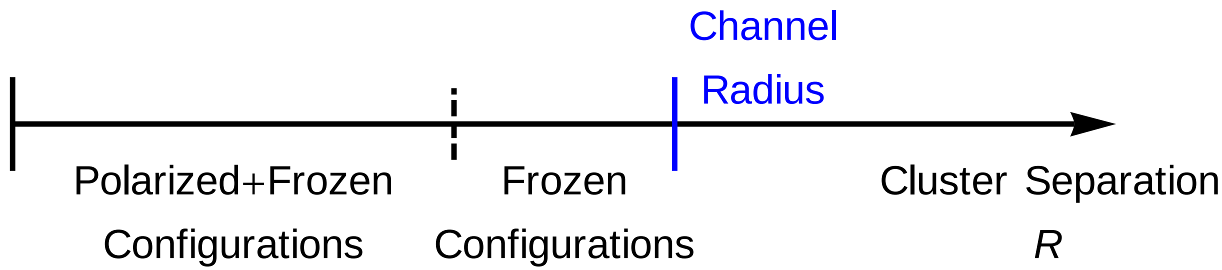


- energy surfaces contain localization energy for relative motion of ^{8}Be and α
- 2^+ energy surface depends strongly on orientation of ^{8}Be 2^+ state – $K = 2$ most attractive

Full calculation: Microscopic R-matrix Method

Model Space

- Internal region in the cluster model: 3-α configurations on a grid
- External region: ^{8}Be($0^+, 2^+$)-α configurations
- Channel radius has to be large: only Coulomb interaction between ^{8}Be and α and Coulomb coupling between different ^{8}Be channels should be small
- Check that results are independent from channel radius: used $a = 16.5$ fm here



Scattering Solutions

- Obtain scattering matrix using multichannel microscopic R-matrix approach
  Descouvemont, Baye, Phys. Rept. 73, 036301 (2010)
- Diagonal phase shifts and inelasticity parameters: $S_{ii} = \eta_i \exp\{2i\delta_i\}$
- Eigenphases: $S = V^{-1}DV, D_{\alpha\alpha} = \exp\{2i\delta_\alpha\}$

Slater determinants and RGM wave functions

- Divide model space into internal and external region at channel radius a
- In internal region wave function is described microscopically with FMD Slater determinants
- In external region wave function is considered as a system of two point-like clusters
- (Microscopic) cluster wave function – Slater determinant

$$\left| Q^{ab}(\mathbf{R}) \right\rangle = \frac{1}{\sqrt{c_{ab}}} \underset{\sim}{\mathcal{A}} \left\{ \left| Q^{a}(-\frac{m_b}{m_a+m_b}\mathbf{R}) \right\rangle \otimes \left| Q^{b}(\frac{m_a}{m_a+m_b}\mathbf{R}) \right\rangle \right\}$$

- Projection on total linear momentum decouples intrinsic motion, relative motion of clusters and total center-of-masss

$$\left| Q^{ab}(\mathbf{R}); \mathbf{P}=0 \right\rangle = \int d^3r \, \tilde{\Gamma}(\mathbf{r}-\mathbf{R}) \left| \Phi^{ab}(\mathbf{r}) \right\rangle \otimes \left| \mathbf{P}_{cm}=0 \right\rangle$$

using RGM basis states

$$\left\langle \boldsymbol{\rho}, \xi_a, \xi_b \,\middle|\, \Phi^{ab}(\mathbf{r}) \right\rangle = \frac{1}{\sqrt{c_{ab}}} \mathcal{A} \left\{ \delta(\boldsymbol{\rho}-\mathbf{r}) \Phi^{a}(\xi_a) \Phi^{b}(\xi_b) \right\}$$

RGM norm kernel

$$n^{ab}(\mathbf{r},\mathbf{r}') = \left\langle \Phi^{ab}(\mathbf{r}) \,\middle|\, \Phi^{ab}(\mathbf{r}') \right\rangle$$

Slater determinants and RGM wave functions

- Relative motion in Slater determinant described by Gaussian

$$\tilde{\Gamma}(\mathbf{r}-\mathbf{R}) = \left(\frac{\beta_{\mathrm{rel}}}{\pi^2 a_{\mathrm{rel}}}\right)^{3/4} \exp\left(-\frac{(\mathbf{r}-\mathbf{R})^2}{2a_{\mathrm{rel}}}\right)$$

with

$$a_{\mathrm{rel}} = \frac{a_a A_b + a_b A_a}{A_a A_b}, \quad \beta_{\mathrm{rel}} = \frac{a_a a_b}{a_a A_b + a_b A_a}$$

- Overlap of full wave function with RGM cluster basis

$$\psi(\mathbf{r}) = \int \mathrm{d}^3 r' \, n^{1/2}(\mathbf{r},\mathbf{r}') \left\langle \Phi(\mathbf{r}') \middle| \Psi \right\rangle$$

- Match asymptotics to Whittaker, outgoing Coulomb or Coulomb functions

$$\psi_{\mathrm{b}}(r) = A\,\frac{1}{r} W_{-\eta,L+1/2}(2\kappa r), \qquad \psi_{\mathrm{Gamow}}(r) = A\frac{1}{r} O_L(\eta, kr)$$

$$\psi_{\mathrm{scatt}}(r) = \frac{1}{r}\left\{I_L(\eta,kr) - e^{2i\delta} O_L(\eta,kr)\right\}$$

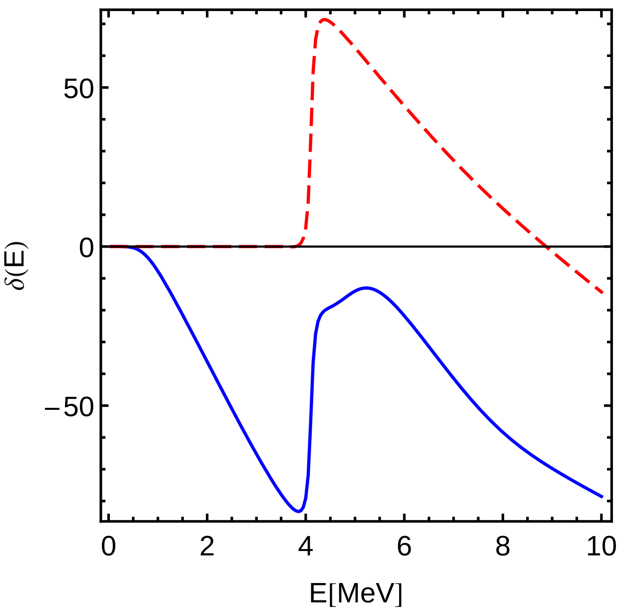
with

$$\kappa = \sqrt{-2\mu E_b}, \quad k = \sqrt{2\mu E}, \quad \eta = \mu \frac{Z_a Z_b e^2}{k}$$

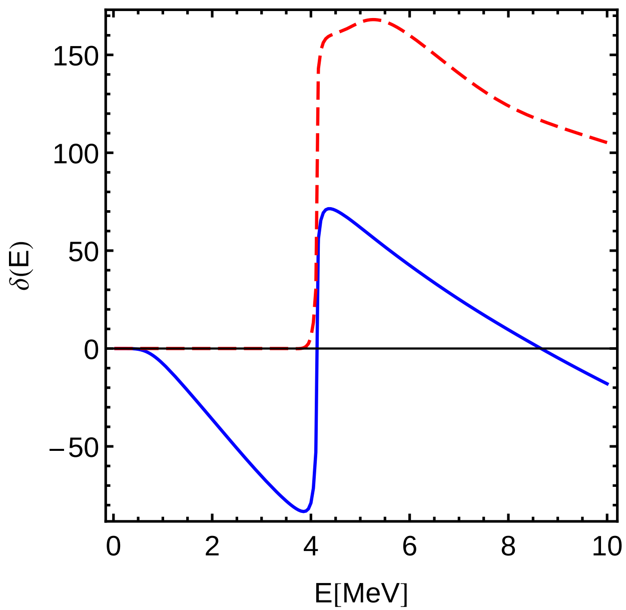
Cluster Model: ^{8}Be-α Continuum

0^+ Phase shifts

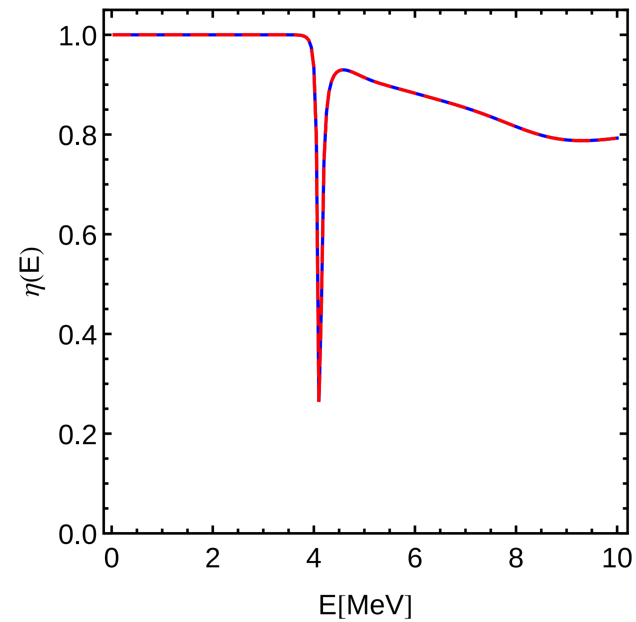
Eigenphaseshifts



Phaseshifts



Inelasticities



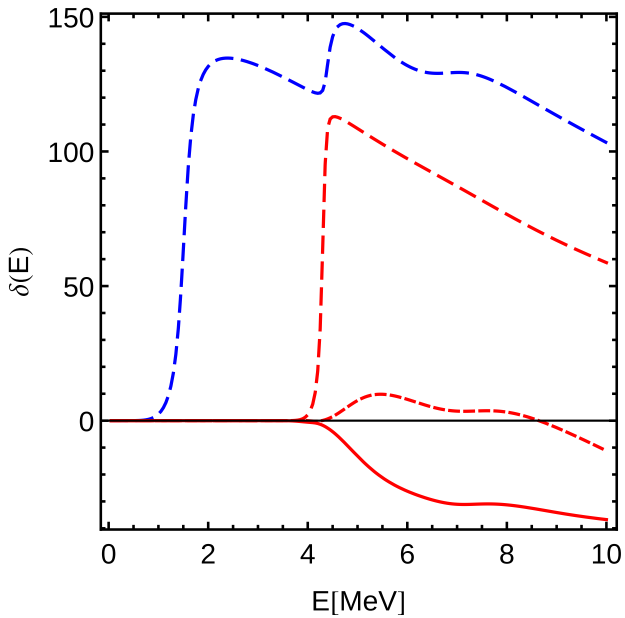
Gamow states

	E [MeV]	Γ [MeV]	
0_2^+	0.29	$1.78 \cdot 10^{-5}$	
0_3^+	4.11	0.12	
0_4^+	4.76	1.57	(?)

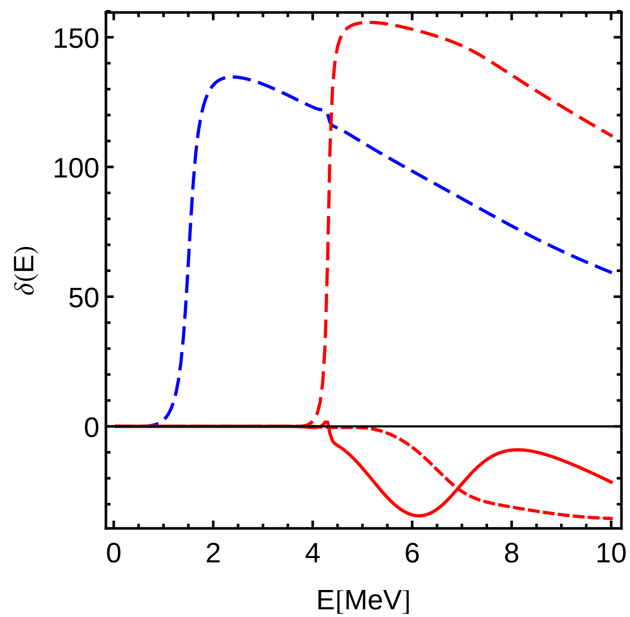
- non-resonant background
- strong coupling between ^{8}Be(0^+) and ^{8}Be(2^+) channel at 4.1 MeV
- Hoyle state not resolved in phase shifts
- stability of broad resonance with respect to channel radius ?

2^+ Phase shifts

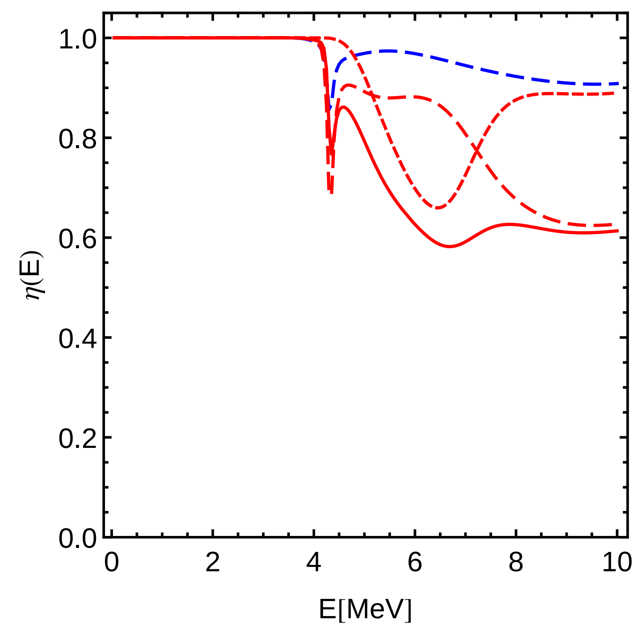
Eigenphaseshifts



Phaseshifts



Inelasticities



Gamow states

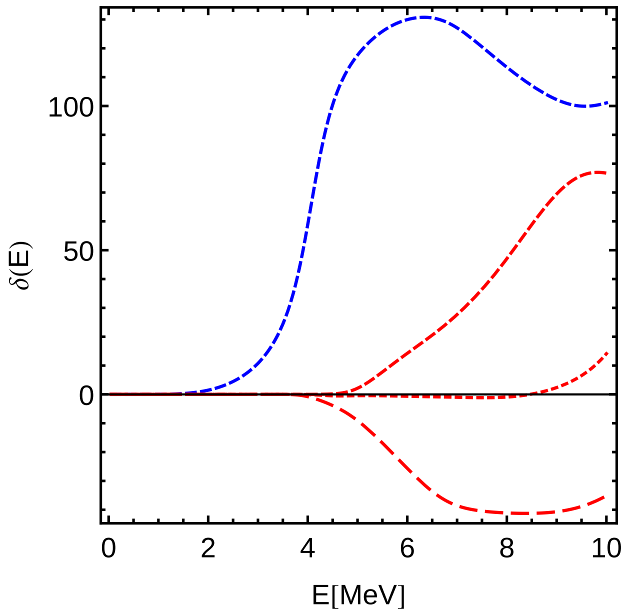
	E [MeV]	Γ [MeV]
2^+_2	1.51	0.32
2^+_3	4.31	0.14
...		

- non-resonant background
- strong $L = 2$ ^{8}Be(0^+) and ^{8}Be(2^+) resonances

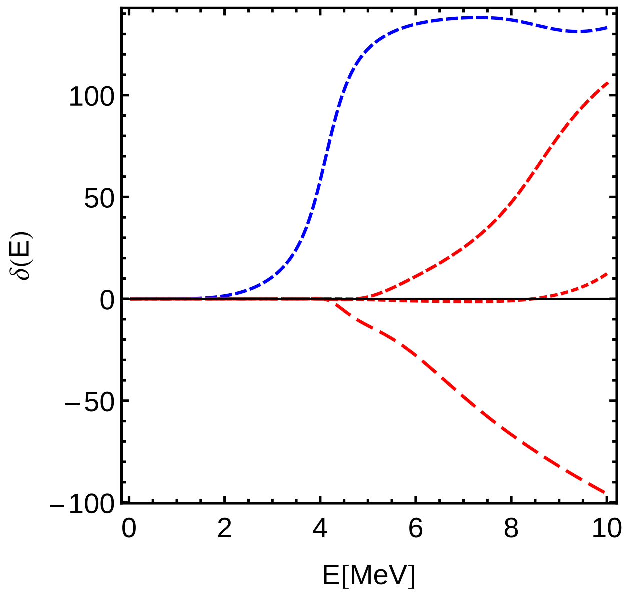
Cluster Model: ^{8}Be-α Continuum

4^+ Phase shifts

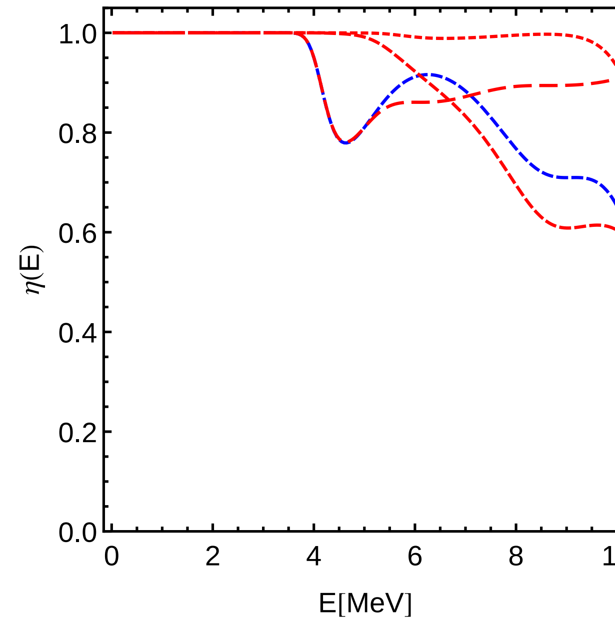
Eigenphaseshifts



Phaseshifts



Inelasticities



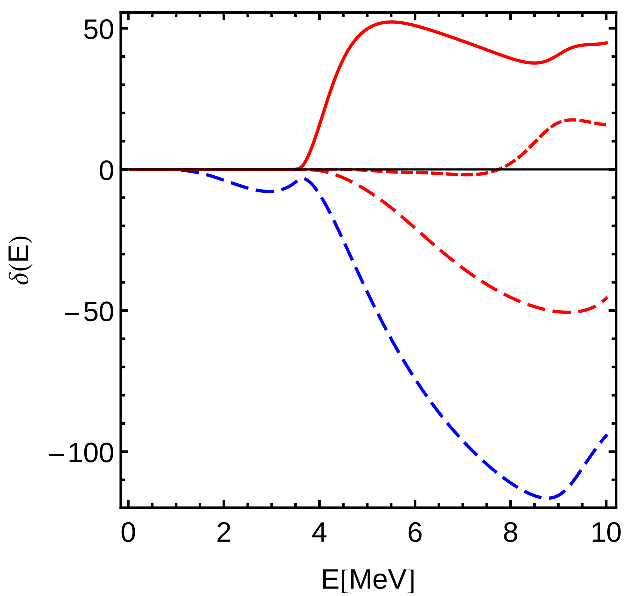
Gamow states

	E [MeV]	Γ [MeV]
4^+_1	1.17	$8.07 \cdot 10^{-6}$
4^+_2	4.06	0.98
. . .		

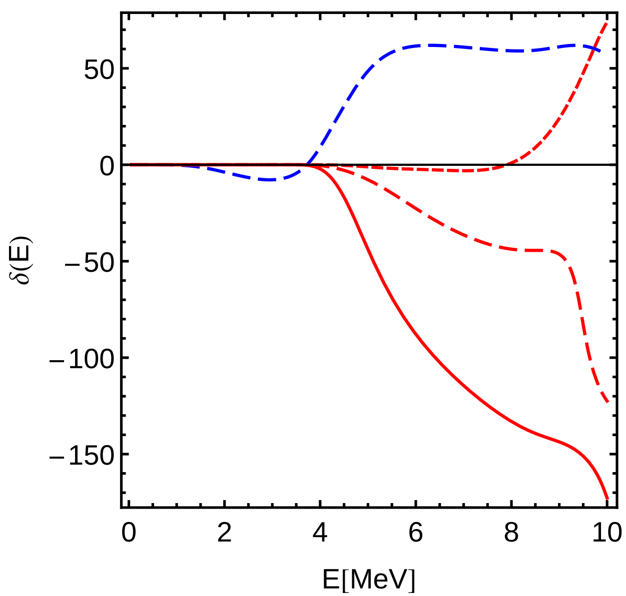
- 4_1 state very narrow, not resolved in phase shifts
- 4^+_2 state mostly ^{8}Be(0+)

Cluster Model: ^{8}Be-α Continuum
3^{-} Phase shifts

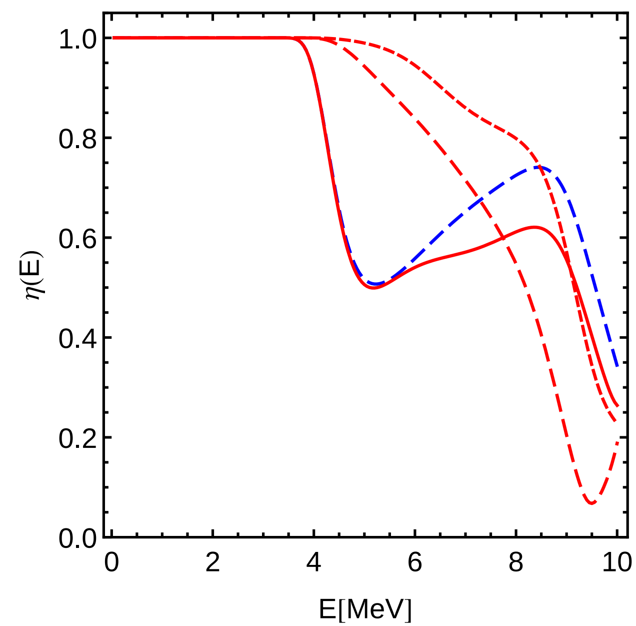
Eigenphaseshifts



Phaseshifts



Inelasticities



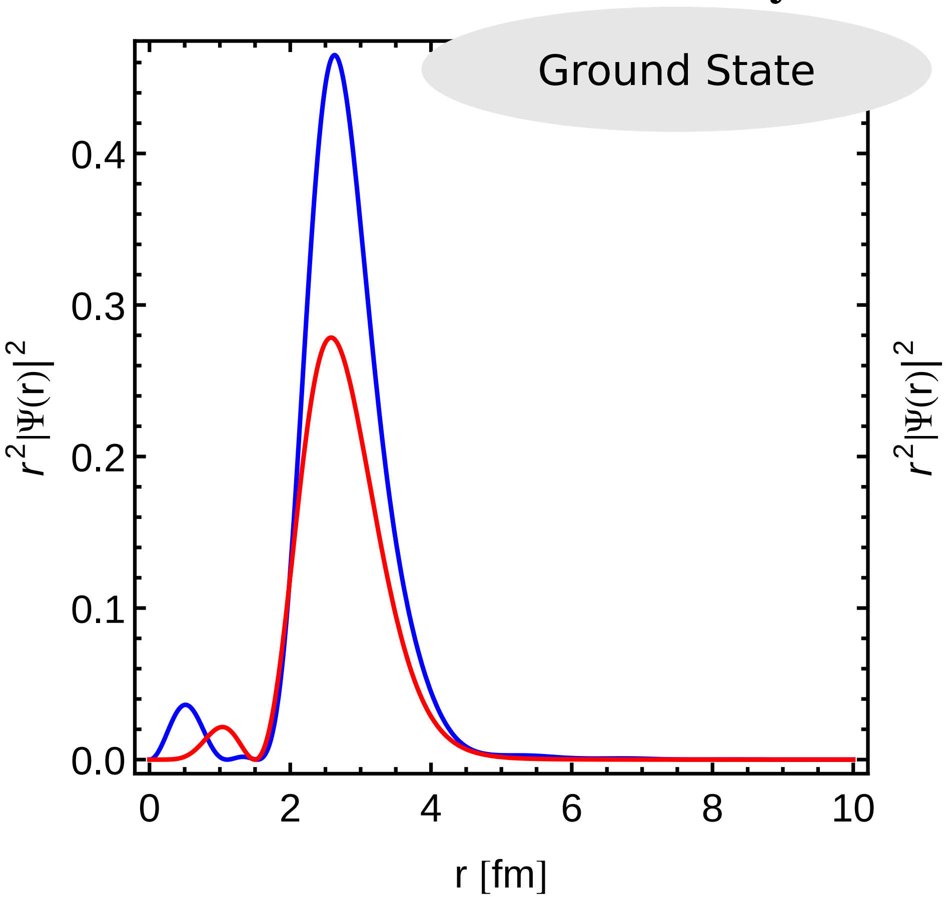
Gamow states

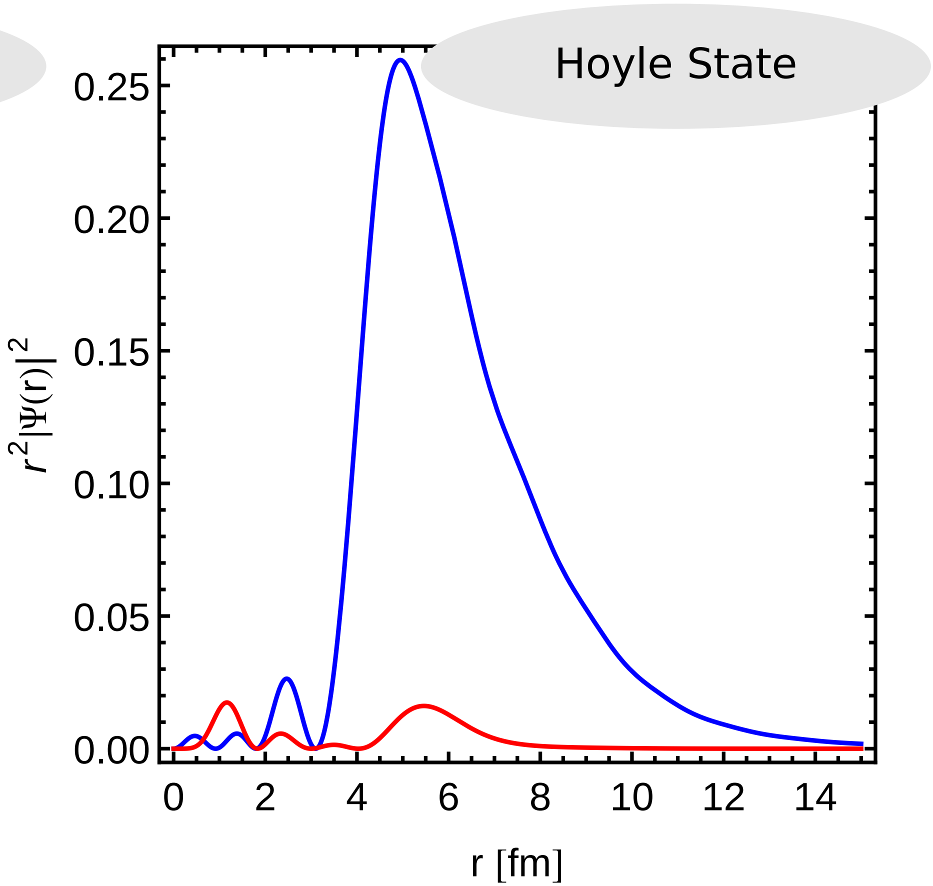
	E [MeV]	Γ [MeV]
3^-_1	0.54	$4.46{\cdot}10^{-6}$
...		

- 3^-_1 state very narrow, not resolved in phase shifts

Overlap functions

$$\psi(\mathbf{r}) = \int d^3r'\, n^{1/2}(\mathbf{r},\mathbf{r}') \langle\, \Phi(\mathbf{r}') \,|\, \Psi \,\rangle$$





- Ground state overlap with ^{8}Be(0^+)+α and ^{8}Be(2^+)+α configurations of similar magnitude
- Hoyle state overlap dominated by ^{8}Be(0^+)+α configurations, large spatial extension

Work in Progress:
FMD calculation with ^{8}Be-α Continuum

UCOM interaction

- **Correlation functions from SRG $\lambda = 1.5$fm^{-1}**
- **Increase strength of spin-orbit force to partially account for omitted three-body forces**

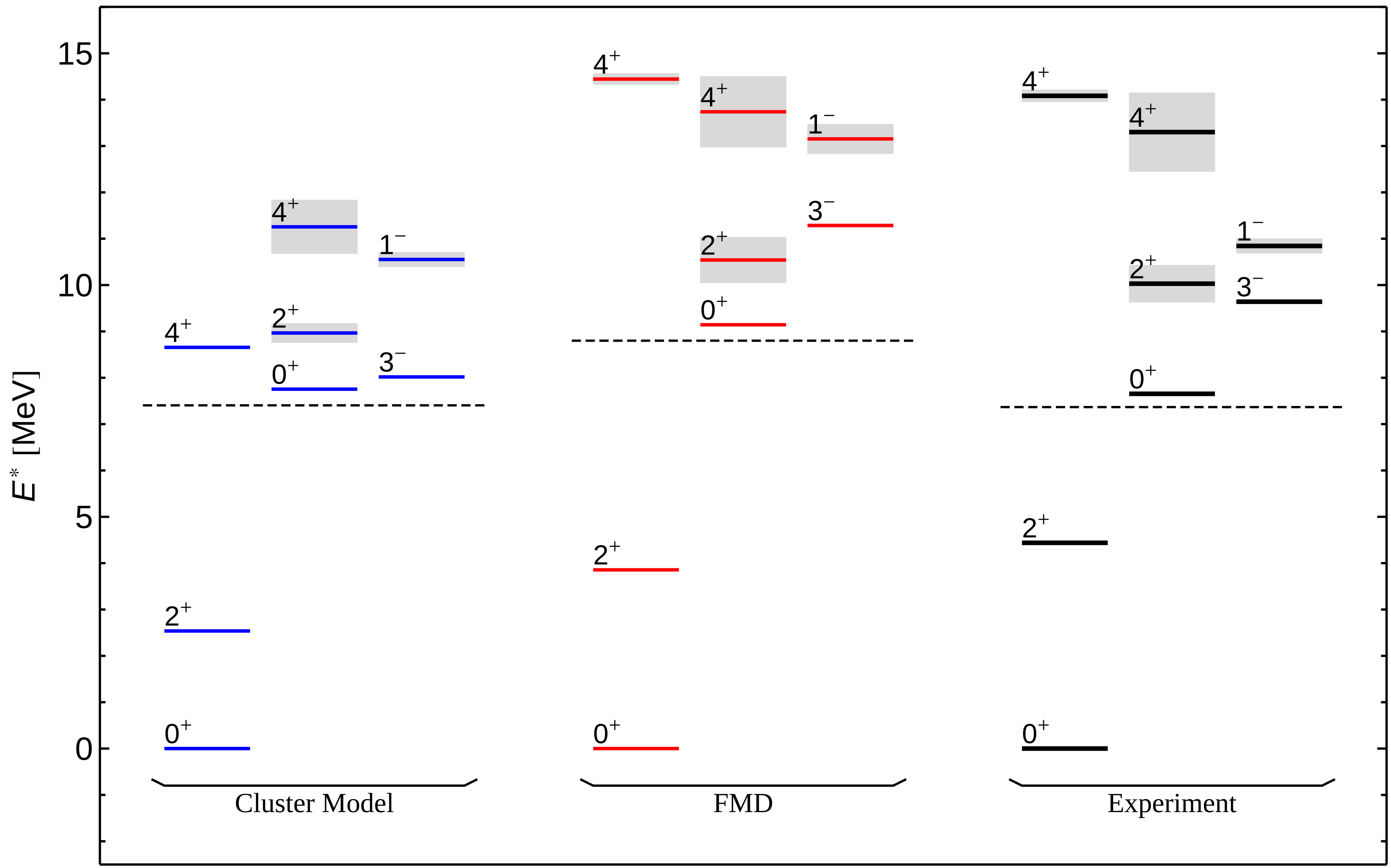
^{8}Be-α Continuum

- **To get a reasonable description of ^{8}Be it is essential to include polarized configurations**

➽ **Calculate strength distributions**

➽ **Investigate non-cluster states: non-natural parity states, $T = 1$ states, M1 transitions, ^{12}B and ^{12}N β-decay into ^{12}C, . . .**

Spectra (preliminary)



- FMD: ^{8}Be wave functions still relatively poor

Summary

Unitary Correlation Operator Method

- Explicit description of short-range central and tensor correlations

Fermionic Molecular Dynamics

- Gaussian wave-packet basis contains HO shell model and Brink-type cluster states

Cluster States in ^{12}C

- Consistent description of ground state band and clustered states including the Hoyle state
- Test Hoyle state structure with electron scattering

The ^{12}C Continuum

- Include ^{8}Be(0^+,2^+)+α continuum in cluster model
- Hoyle state can be understood as ^{8}Be(0^+)+α
- First results for FMD with continuum

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Robert Roth (TUD), Yasuyuki Suzuki (Niigata), Dennis Weber (GSI)