

Core Excitations and Transfer Reactions in Exotic Nuclei

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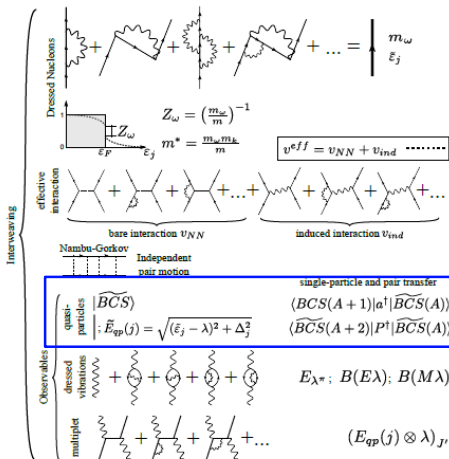
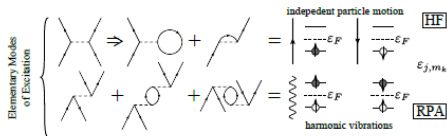
TRIUMF, February 18th, 2014

Study of the interplay between nuclear elementary modes of excitation (single-particle motion, surface vibrations and pairing modes), probed with transfer reactions.

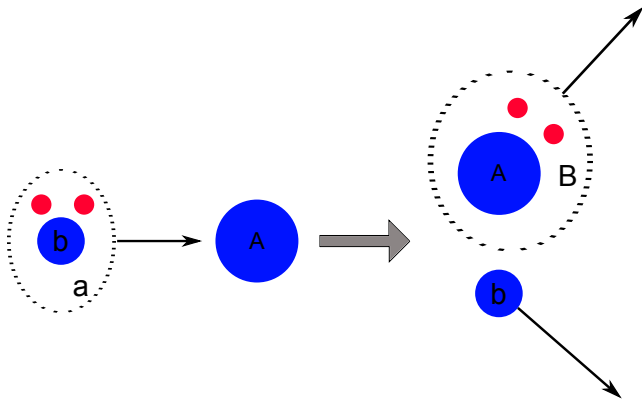
Outline:

- Structure: nuclear elementary modes of excitation.
- Reaction formalism: two-particle transfer in 2-step DWBA.
- Two-particle transfer in stable nuclei: a probe of pairing correlations.
- Coupling of pairing modes with surface modes in exotic nuclei: some results and perspectives.

interweaving of elementary modes of excitation: NFT

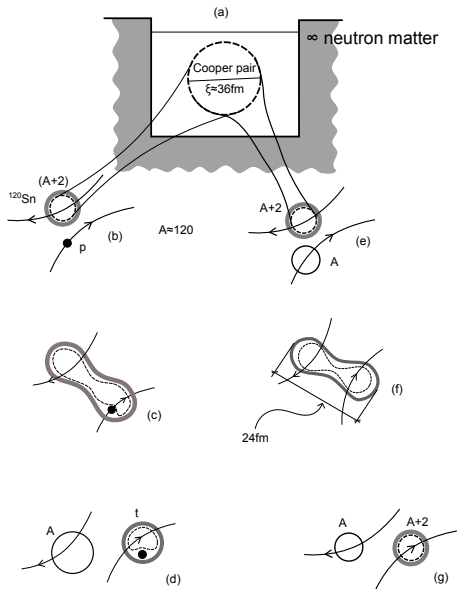


Two-Nucleon Transfer

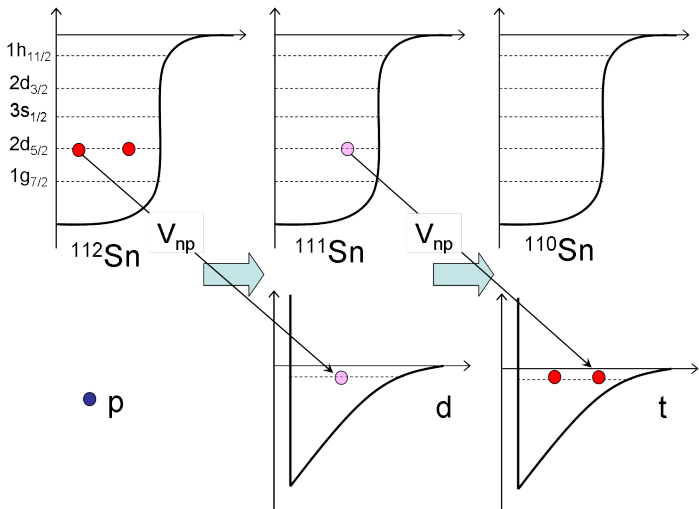


- Reaction $A + a(\equiv b + 2) \longrightarrow a + B(\equiv A + 2)$.
- Measure of the **pairing correlations** between the transferred nucleons.
- Need to correctly **account for the correlated wavefunction**.

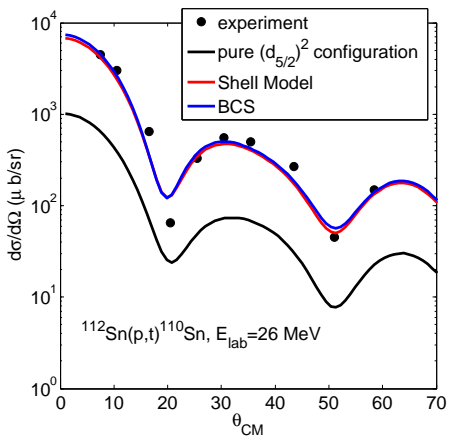
Pair transfer reaction mechanism: coherence length



Example: $^{112}\text{Sn}(p,t)^{110}\text{Sn}$ in 2-step DWBA



Probing pairing with 2-transfer: $^{112}\text{Sn}(p,t)^{110}\text{Sn}$



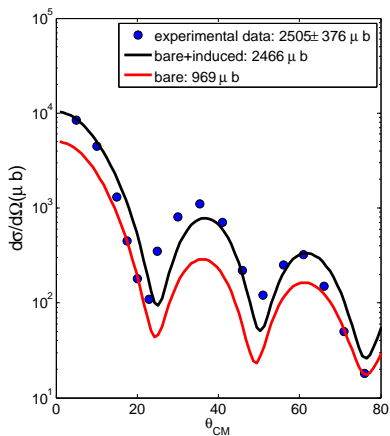
enhancement factor with respect to the transfer of uncorrelated neutrons:

$$\epsilon = 20.6$$

Experimental data and shell model wavefunction from Guazzoni *et al.*
PRC **74** 054605 (2006)

experiment very well reproduced with mean field (BCS) wavefunctions

$^{122}\text{Sn}(p, t)^{120}\text{Sn}$ (gs): role of induced interaction



Differential cross section worked out making use of **two different structure calculations**:

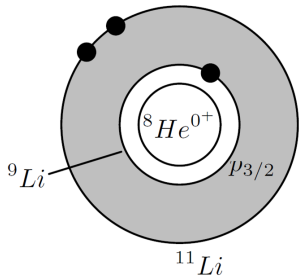
- Skyrme in $p - h$ channel (**mean field**) + **collective vibrations** + bare v_{14} Argonne interaction and particle-vibration coupling (**induced interaction**) in $p - p$ channel (black line),
- Skyrme in $p - h$ channel (**mean field**) + **bare** v_{14} Argonne in $p - p$ channel (red line),

compared with experimental data.

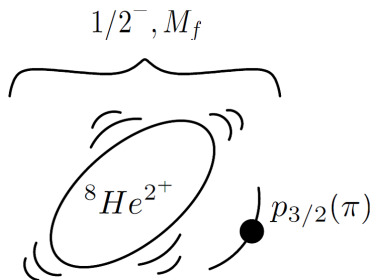
$^{122}\text{Sn}(p, t)^{120}\text{Sn}$ at 26 MeV. Data from Guazzoni *et.al.* (1999).

Transfer in drip-line nuclei ${}^1\text{H}({}^{11}\text{Li}, {}^9\text{Li}){}^3\text{H}$

We will try to draw information about the halo structure of ${}^{11}\text{Li}$ from the reactions ${}^1\text{H}({}^{11}\text{Li}, {}^9\text{Li}){}^3\text{H}$ and ${}^1\text{H}({}^{11}\text{Li}, {}^9\text{Li}^*(2.69\text{ MeV})){}^3\text{H}$ (I. Tanihata et al., Phys. Rev. Lett. **100**, 192502 (2008))

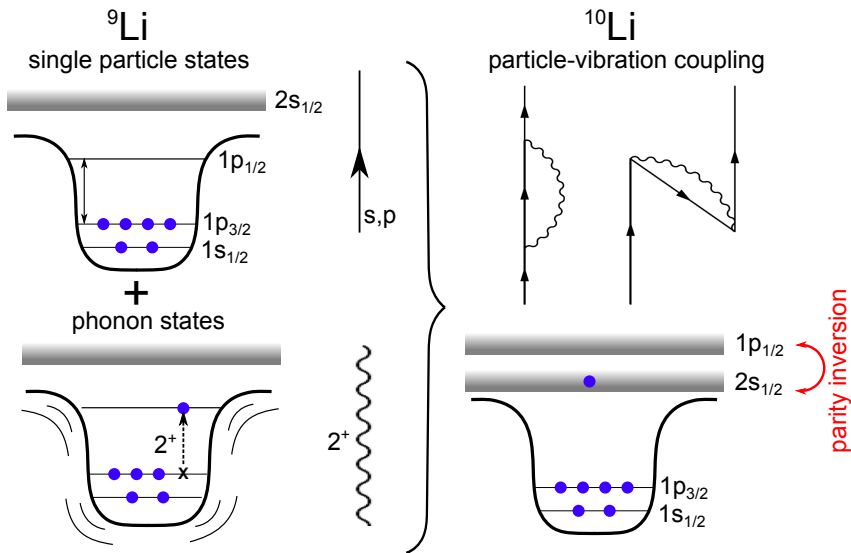


Schematic depiction of ${}^{11}\text{Li}$



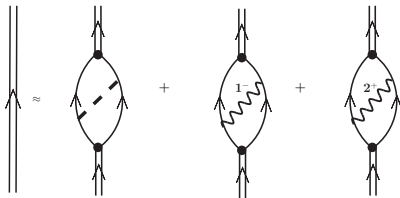
First excited state of ${}^9\text{Li}$

Beyond mean field: particle–vibration coupling



Structure of the ^{11}Li ($3/2^-$) ground state

$^{11}\text{Li} = {}^9\text{Li}$ core + 2-neutron halo (single Cooper pair). According to Barranco *et al.* (2001), the two neutrons correlate by means of the **bare interaction** (accounting for $\approx 20\%$ of the ^{11}Li binding energy) and by exchanging 1^- and 2^+ **phonons** ($\approx 80\%$ of the binding energy)

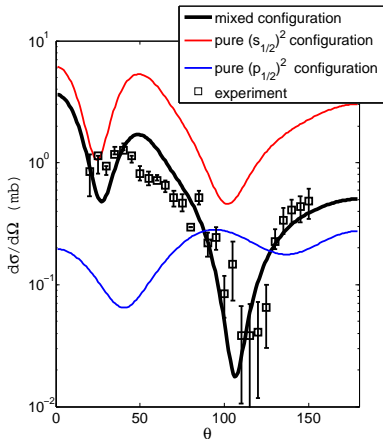


Within this model, the ^{11}Li **wavefunction** can be written as

$$|\tilde{0}\rangle = 0.45|s_{1/2}^2(0)\rangle + 0.55|p_{1/2}^2(0)\rangle + 0.04|d_{5/2}^2(0)\rangle \\ + 0.70|(ps)_{1-} \otimes 1^-; 0\rangle + 0.10|(sd)_{2+} \otimes 2^+; 0\rangle.$$

highly renormalized single particle states coupled to **excited states of the core**

Transition to the ground state of ${}^9\text{Li}$



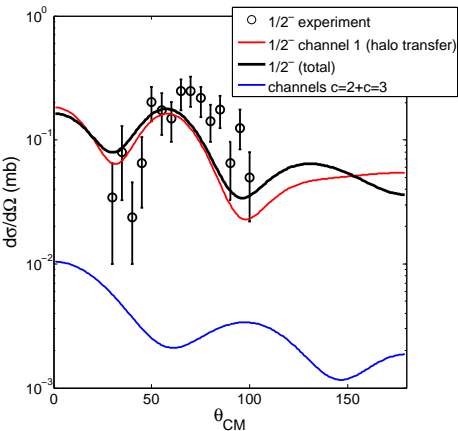
differential cross section calculated with three ${}^{11}\text{Li}$ ground state model wavefunctions:

- pure $(s_{1/2})^2$ configuration
- pure $(p_{1/2})^2$ configuration
- $20\%(s_{1/2})^2 + 30\%(p_{1/2})^2$ configuration (Barranco *et al.* (2001)).

compared with experimental data.

${}^1\text{H}({}^{11}\text{Li}, {}^9\text{Li}){}^3\text{H}$ at 33 MeV. Data from Tanihata *et al.* (2008).

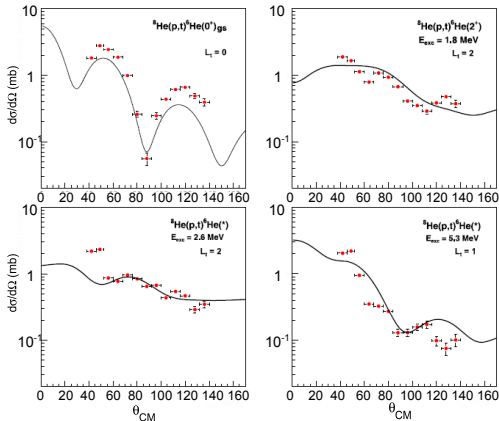
Transition to the first $1/2^-$ (2.69 MeV) excited state of ^9Li



differential cross section calculated with the [Barranco *et. al.* \(2001\)](#) ^{11}Li ground state wavefunction, compared with experimental data. According to this model, the ^9Li excited state is found after the transfer reaction because it is already present in the ^{11}Li ground state.

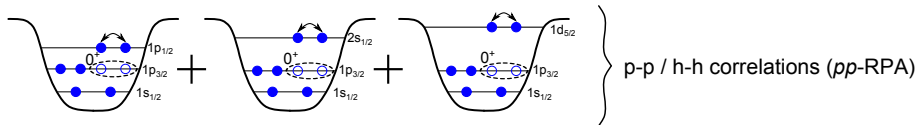
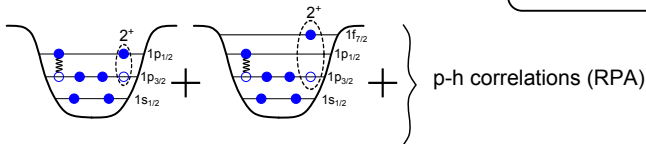
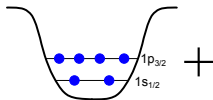
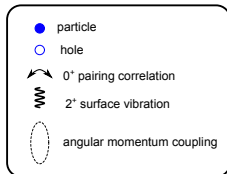
$^1\text{H}(^{11}\text{Li}, ^9\text{Li}^*(2.69 \text{ MeV}))^3\text{H}$ at 33 MeV. Data from [Tanihata *et.al.* \(2008\)](#).

Two-neutron transfer with ^8He

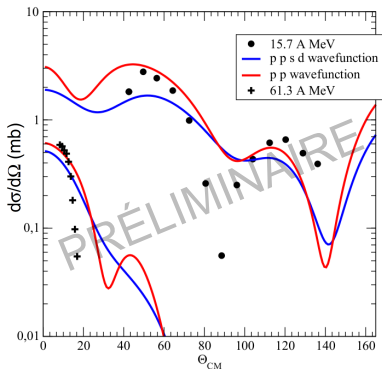


- X. Mougeot *et al.* PLB **718**, 441 (2012) $^8\text{He}(p,t)^6\text{He}(\text{gs}), ^8\text{He}(2^+)$ with SPIRAL and MUST2;
- Coupled Reaction Channels (CRC) analysis by N .Keeley.

Neutronic Structure of ^8He

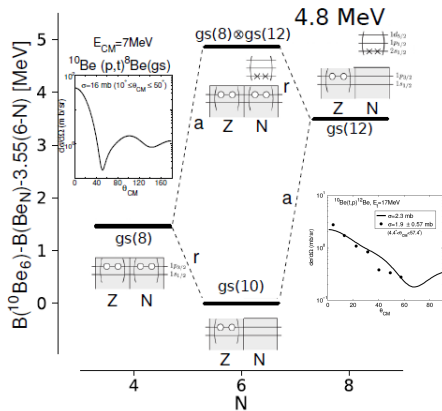


${}^8\text{He}(p, t)$ reaction in 2-step DWBA



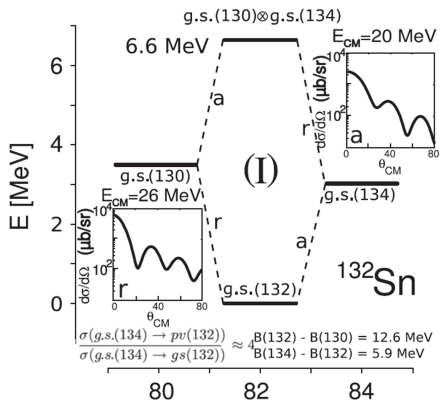
- Sensitive to ${}^8\text{He}$ structure.
- Nuclear Field Theory calculations for ${}^8\text{He}(\text{g.s.}), {}^6\text{He}(\text{g.s.}, 2^+)$ (${}^6\text{He}$ as a pair removal mode of ${}^8\text{He}$?).
- Consistent description of elastic and one-neutron transfer channels and the overlap ${}^8\text{He}(\text{g.s.})/{}^6\text{He}(2^+)$ is essential.

Pairing vibrations in exotic nuclei: ^{10}Be



$^8\text{Be}(p,t)^{10}\text{Be}$ and $^{10}\text{Be}(t,p)^{12}\text{Be}$ reactions can probe the pairing vibrations around ^{10}Be ($N = 6$ shell closure).
 $^{10}\text{Be}(t,p)^{12}\text{Be}$ data by Fortune *et al*, PRC **50** (1994) 1355.

Pairing vibrations in exotic nuclei: ^{132}Sn

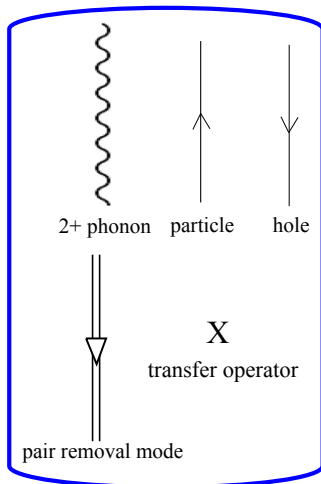
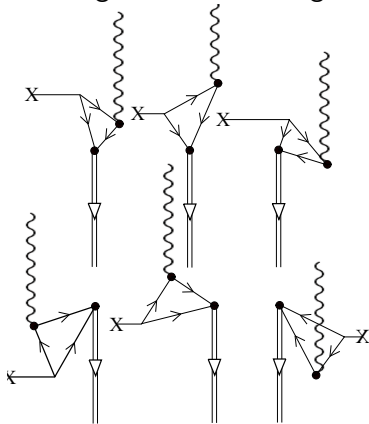


$^{132}\text{Sn}(p,t)^{130}\text{Sn}$ and $^{134}\text{Sn}(p,t)^{132}\text{Sn}$ reactions can probe the predicted pairing vibrations of the exotic double magic nucleus ^{132}Sn .
Foreseen experiments at GANIL with SPIRAL2

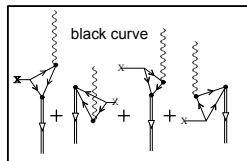
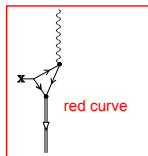
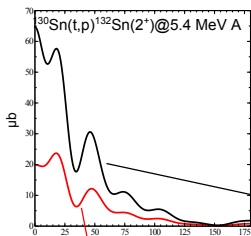
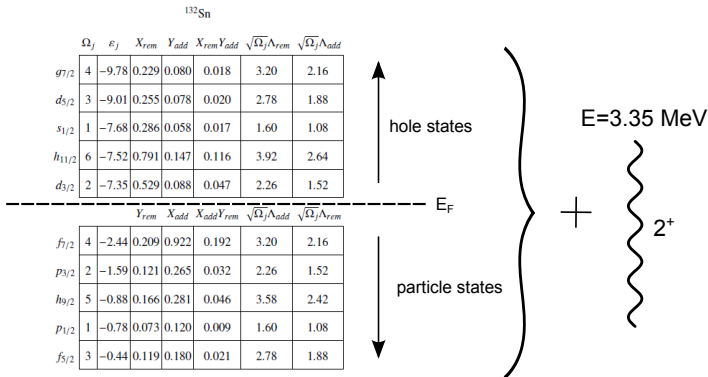
Coupling of pairing vibrations with phonons

Population of excited 2^+ state with (t, p) reaction

Diagrams contributing



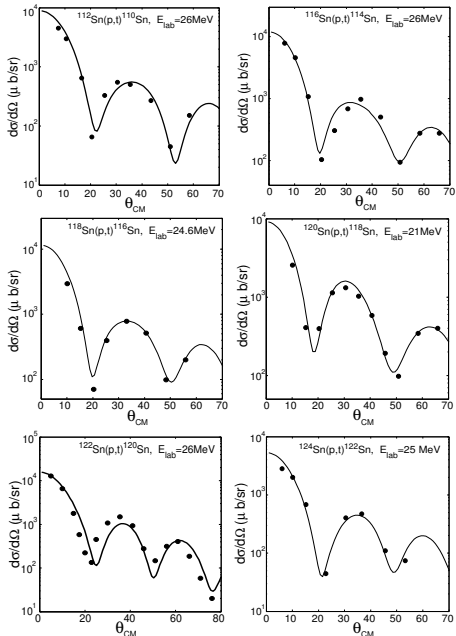
$^{130}\text{Sn}(t,p)^{132}\text{Sn}(2^+)$



- Atomic nuclei can be consistently described as **interacting elementary modes of excitation (NFT)**.
- The **interweaving** of these modes can be studied at profit with **transfer reactions**.
- We have presented examples of the study of **coupling between pairing and collective modes** with the help of **two-nucleon transfer reaction** within a 2-step DWBA formalism.
- **Good agreement** with experiment is obtained essentially **without adjustable parameters**.

Thank You!

2-transfer in well bound nuclei $^A\text{Sn}(p,t)^{A-2}\text{Sn}$



Comparison with the experimental data available so far for **superfluid tin isotopes**

Potel *et al.*, PRL **107**, 092501 (2011)

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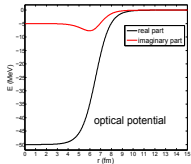
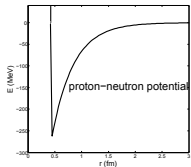
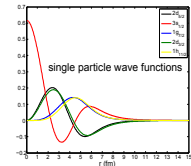
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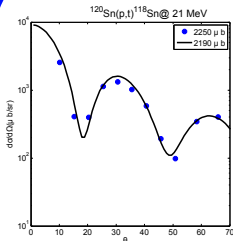
Reaction formalism, between structure and experiment

structure

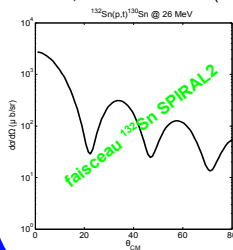


formalisme de réaction

calcul réaction + expérience

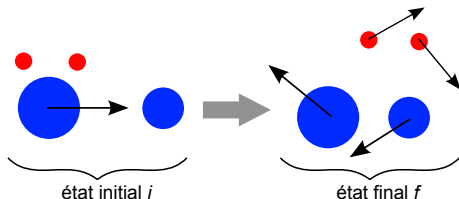


Potel et al., PRL **107** 092501 (2011)



Reaction mechanism:

2-step DWBA



- lowest order in the interaction potential,
- explicitly incorporates microscopic structure inputs,
- adapted to a variety of reaction channels

Transition amplitude

Matrix element of interaction potential between initial (i) and final (f) states

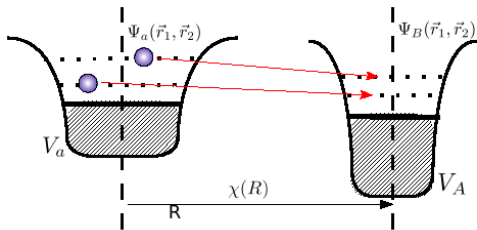
$$\langle \chi_f(R)\phi_f(\xi) | V(\xi) | \chi_i(R)\phi_i(\xi) \rangle$$

can be applied to **1- and 2-nucleon transfer** and **knock-out**

Elements of the calculation

$\Psi_a(\vec{r}_1, \vec{r}_2)$, $\Psi_B(\vec{r}_1, \vec{r}_2)$: **internal wave functions** of the transferred nucleons in each nucleus

$\chi(R)$: **distorted wave** describing the relative motion in the optical potential $U(R) = V(R) + iW(R) \left(\frac{P_R^2}{2\mu} + U(R) \right) \chi(R) = E_{CM}\chi(R)$

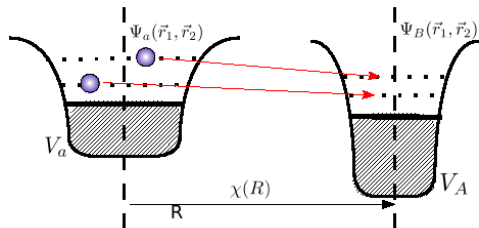


V_A, V_a : **mean field potentials** of the two nuclei

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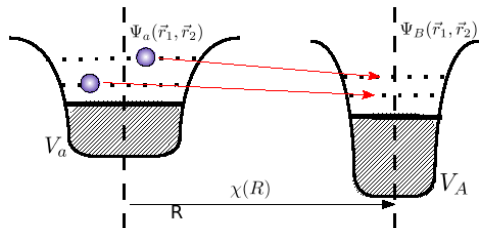
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V_A (V_a) is the **interaction potential** that transfers the nucleons from one nucleus to the other in the **prior (post)** representation

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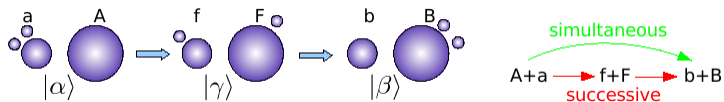


V_A, V_a : **mean field potentials** of the two nuclei

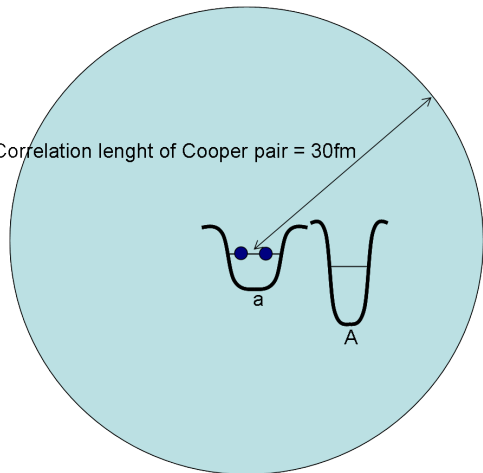
V_A (V_a) is the **interaction potential** that transfers the nucleons from one nucleus to the other in the **prior (post)** representation

it is a **single particle potential!!**

simultaneous and successive contributions



Correlation length of Cooper pair = 30fm



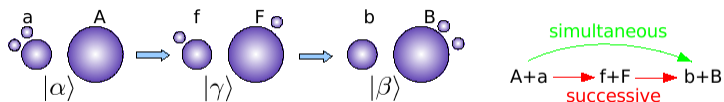
$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2) \times$$

$$\phi_A(\xi_A) \chi_{aA}(\mathbf{r}_{aA})$$

$$|\beta\rangle = \phi_b(\xi_b) \phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2) \times$$

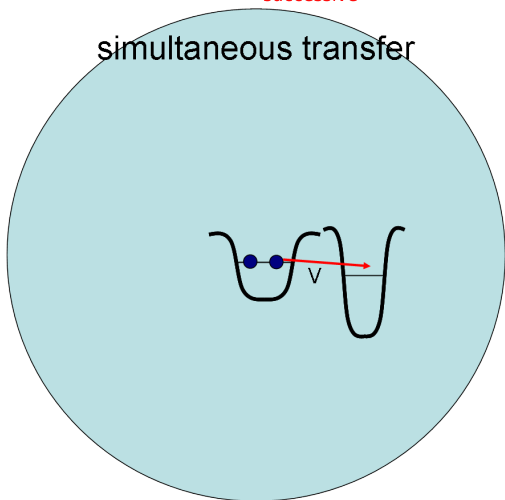
$$\chi_{bB}(\mathbf{r}_{bB})$$

simultaneous and successive contributions

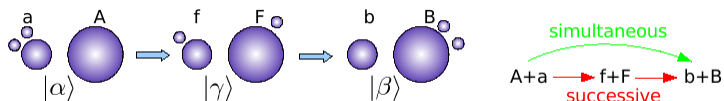


simultaneous transfer

$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2) \times \phi_A(\xi_A) \chi_{aA}(\mathbf{r}_{aA})$$
$$|\beta\rangle = \phi_b(\xi_b) \phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2) \times \chi_{bB}(\mathbf{r}_{bB})$$

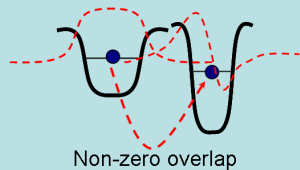


simultaneous and successive contributions

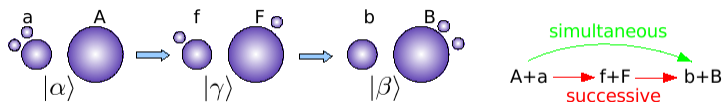


simultaneous transfer

$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2) \times \phi_A(\xi_A) \chi_{aA}(\mathbf{r}_{aA})$$
$$|\beta\rangle = \phi_b(\xi_b) \phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2) \times \chi_{bB}(\mathbf{r}_{bB})$$

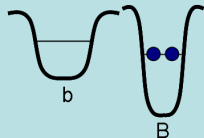


simultaneous and successive contributions

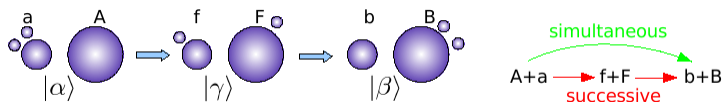


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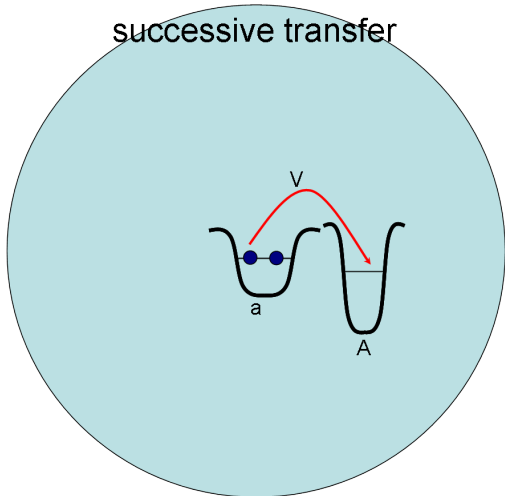


simultaneous and successive contributions

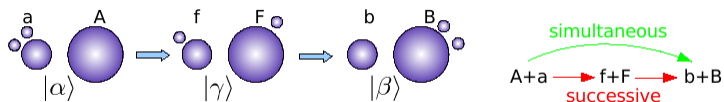


successive transfer

$$| \alpha \rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2) \times \phi_A(\xi_A) \chi_{aA}(\mathbf{r}_{aA})$$
$$| \beta \rangle = \phi_b(\xi_b) \phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2) \times \chi_{bB}(\mathbf{r}_{bB})$$

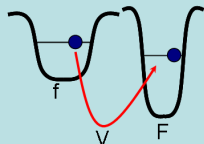


simultaneous and successive contributions

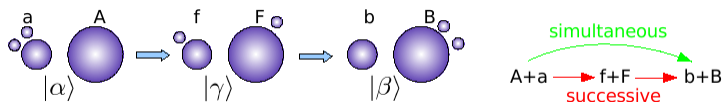


successive transfer

$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2) \times \phi_A(\xi_A) \chi_{aA}(\mathbf{r}_{aA})$$
$$|\beta\rangle = \phi_b(\xi_b) \phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2) \times \chi_{bB}(\mathbf{r}_{bB})$$

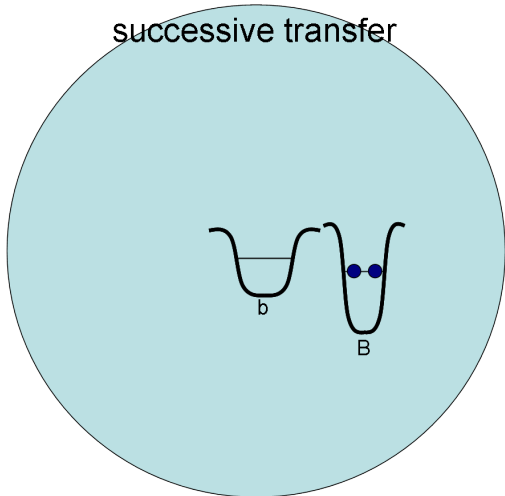


simultaneous and successive contributions

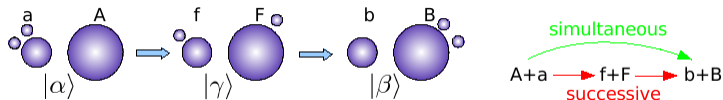


successive transfer

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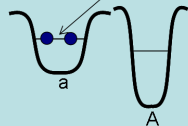
simultaneous and successive contributions



$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2) \times \phi_A(\xi_A) \chi_{aA}(\mathbf{r}_{aA})$$

$$|\beta\rangle = \phi_b(\xi_b) \phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2) \times \chi_{bB}(\mathbf{r}_{bB})$$

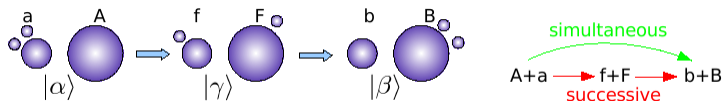
Correlation length of Cooper pair = 30fm



Because of the large correlation length of the Cooper pair, pairing correlations are maintained during the whole process

Two particle transfer in second order DWBA

Some details of the calculation of the differential cross section for two-nucleon transfer reactions



$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T_{succ}^{(2)}(j_i, j_f) - T_{NO}^{(2)}(j_i, j_f) \right)$$

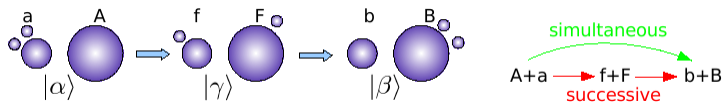
$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(4\pi \hbar^2)^2} \frac{k_f}{k_i} |T_{2NT}|^2$$

Simultaneous transfer

$$T^{(1)}(j_i, j_f) = 2 \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*} \chi_{bB}^{(-)*}(\mathbf{r}_{bB}) \\ \times v(\mathbf{r}_{b1}) [\Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1) \Psi^{j_i}(\mathbf{r}_{b2}, \sigma_2)]_{\mu}^{\Lambda} \chi_{aA}^{(+)}(\mathbf{r}_{aA})$$

Two particle transfer in second order DWBA

Some details of the calculation of the differential cross section for two-nucleon transfer reactions



$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T_{succ}^{(2)}(j_i, j_f) - T_{NO}^{(2)}(j_i, j_f) \right)$$

Successive transfer

$$T_{succ}^{(2)}(j_i, j_f) = 2 \sum_{K, M} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'_1 \sigma'_2}} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*}$$

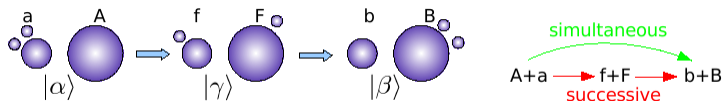
$$\times \chi_{bB}^{(-)*}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]_M^K$$

$$\times \int d\mathbf{r}'_{fF} d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} G(\mathbf{r}_{fF}, \mathbf{r}'_{fF}) [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_M^K$$

$$\times \frac{2\mu_{fF}}{\hbar^2} v(\mathbf{r}'_{f2}) [\Psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_{\mu}^{\Lambda} \chi_{aA}^{(+)}(\mathbf{r}'_{aA})$$

Two particle transfer in second order DWBA

Some details of the calculation of the differential cross section for two-nucleon transfer reactions



$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T_{succ}^{(2)}(j_i, j_f) - T_{NO}^{(2)}(j_i, j_f) \right)$$

Non-orthogonality term

$$\begin{aligned} T_{NO}^{(2)}(j_i, j_f) = & 2 \sum_{K, M} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'_1 \sigma'_2}} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*} \\ & \times \chi_{bB}^{(-)*}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]_M^K \\ & \times \int d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_M^K \\ & \times [\Psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_\mu^\Lambda \chi_{aA}^{(+)}(\mathbf{r}'_{aA}) \end{aligned}$$

Cancellation of simultaneous and non-orthogonal contributions

very schematically, the *first order* (*simultaneous*) contribution is

$$T^{(1)} = \langle \beta | V | \alpha \rangle,$$

while the second order contribution can be separated in a *successive* and a *non-orthogonality* term

$$\begin{aligned} T^{(2)} &= T_{\text{succ}}^{(2)} + T_{\text{NO}}^{(2)} \\ &= \sum_{\gamma} \langle \beta | V | \gamma \rangle G \langle \gamma | V | \alpha \rangle - \sum_{\gamma} \langle \beta | \gamma \rangle \langle \gamma | V | \alpha \rangle. \end{aligned}$$

If we sum over a *complete basis* of intermediate states γ , we can apply the closure condition and $T_{\text{NO}}^{(2)}$ *exactly cancels* $T^{(1)}$

the transition potential being *single particle*, two-nucleon transfer is a *second order process*.

Reaction and structure models

Structure:

$$\Phi_i(\mathbf{r}_1, \sigma_1, \mathbf{r}_2, \sigma_2) = \sum_{j_i} B_{j_i} [\psi^{j_i}(\mathbf{r}_1, \sigma_1) \psi^{j_i}(\mathbf{r}_2, \sigma_2)]_{\mu}^{\Lambda}$$
$$\Phi_f(\mathbf{r}_1, \sigma_1, \mathbf{r}_2, \sigma_2) = \sum_{j_f} B_{j_f} [\psi^{j_f}(\mathbf{r}_1, \sigma_1) \psi^{j_f}(\mathbf{r}_2, \sigma_2)]_0^0$$

Reaction:

$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T_{succ}^{(2)}(j_i, j_f) - T_{NO}^{(2)}(j_i, j_f) \right)$$
$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(4\pi \hbar^2)^2} \frac{k_f}{k_i} |T_{2NT}|^2$$

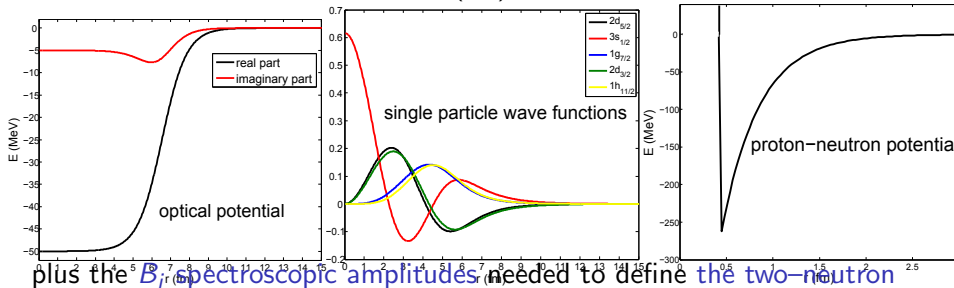
with:

$$T^{(1)}(j_i, j_f) = 2 \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*} \chi_{bB}^{(-)*}(\mathbf{r}_{bB})$$
$$\times v(\mathbf{r}_{b1}) [\psi^{j_i}(\mathbf{r}_{b1}, \sigma_1) \psi^{j_i}(\mathbf{r}_{b2}, \sigma_2)]_{\mu}^{\Lambda} \chi_{aA}^{(+)}(\mathbf{r}_{aA})$$

etc...

Ingredients of the calculation

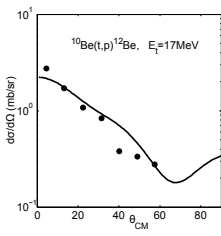
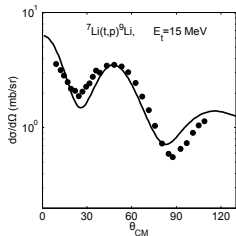
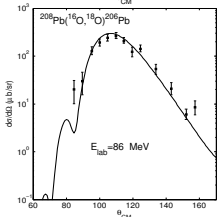
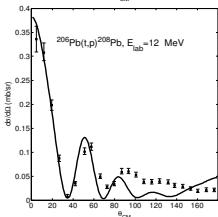
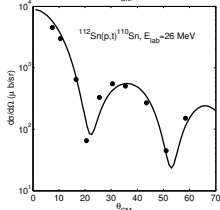
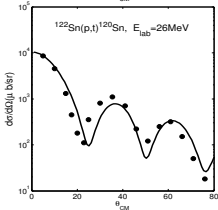
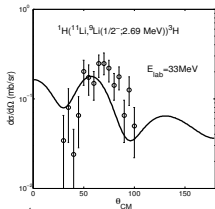
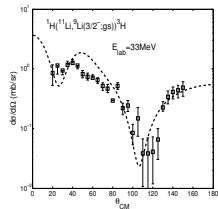
Structure input for, e.g., the $^{112}\text{Sn}(p,t)^{110}\text{Sn}$ reaction:



plus the B_j spectroscopic amplitudes needed to define the two-neutron wavefunction:

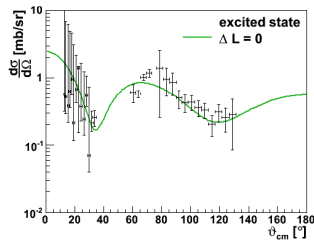
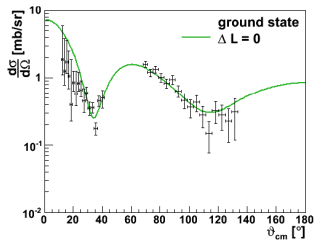
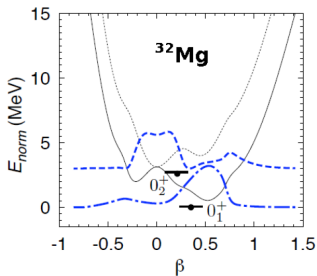
$$\Phi(\mathbf{r}_1, \sigma_1, \mathbf{r}_2, \sigma_2) = \sum_j B_j [\psi^j(\mathbf{r}_1, \sigma_1) \psi^j(\mathbf{r}_2, \sigma_2)]_0^0$$

Examples of calculations



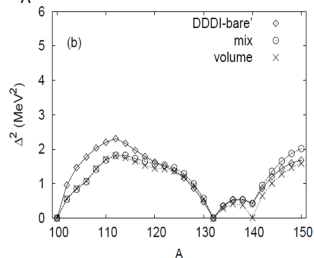
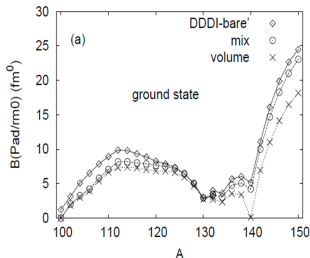
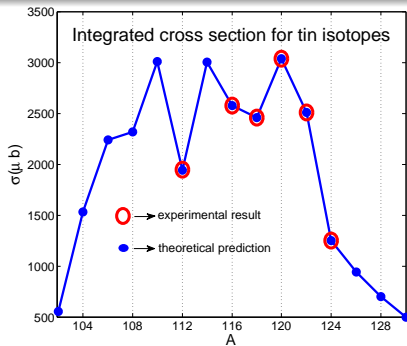
good results obtained for halo nuclei,
 population of excited states,
 superfluid nuclei,
 normal nuclei (pairing vibrations),
 heavy ion reactions...
 Potel *et al.*, arXiv:0906.4298.

Shape coexistence and 2-neutron transfer



- Recent $t(^{32}\text{Mg},p)^{30}\text{Mg}$ @ 1.8 MeV.A at ISOLDE (Wimmer *et.al.*) reaction.
- Shape coexistence (low-lying 0^+ excited state).
- Ground state and first excited 0^+ populated with 2-neutron transfer

$^A\text{Sn}(p,t)^{A-2}\text{Sn}$, superfluid isotopic chain

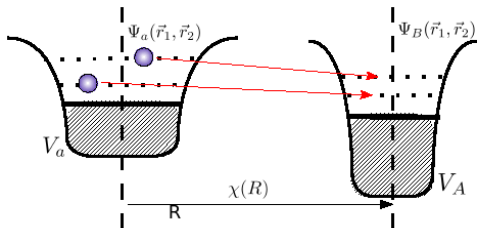


Shimoyama and Matsu, nucl-th/1106.1715

Elements of the calculation

$\Psi_a(\vec{r}_1, \vec{r}_2)$, $\Psi_B(\vec{r}_1, \vec{r}_2)$: **internal wave functions** of the transferred nucleons in each nucleus

$\chi(R)$: **distorted wave** describing the relative motion in the optical potential $U(R) = V(R) + iW(R) \left(\frac{P_R^2}{2\mu} + U(R) \right) \chi(R) = E_{CM}\chi(R)$

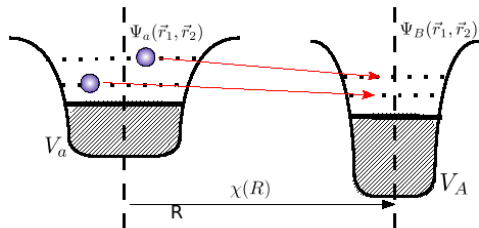


V_A, V_a : **mean field potentials** of the two nuclei

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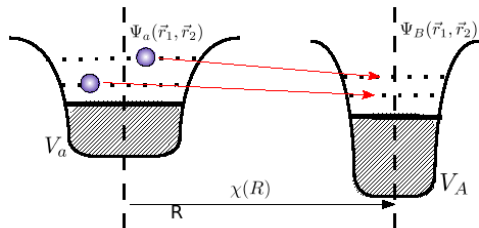
V_A, V_a : **mean field potentials** of the two nuclei

V_A (V_a) is the **interaction potential** that transfers the nucleons from one nucleus to the other in the **prior (post)** representation

Elements of the calculation

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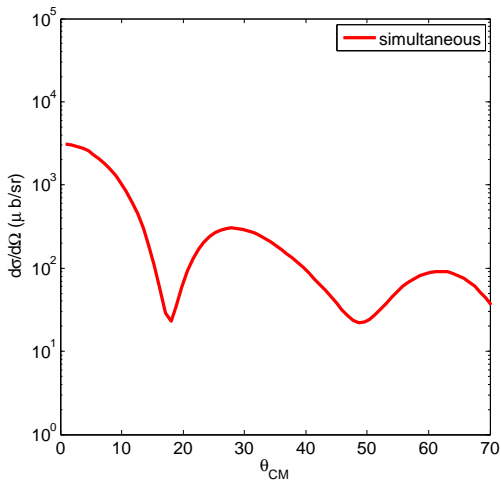


V_A, V_a : **mean field potentials** of the two nuclei

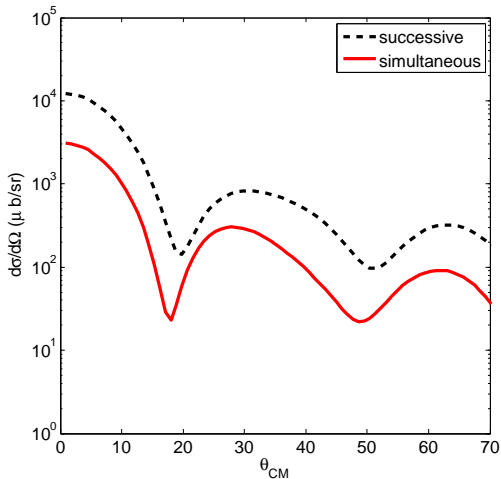
V_A (V_a) is the **interaction potential** that transfers the nucleons from one nucleus to the other in the **prior (post)** representation

it is a **single particle potential!!**

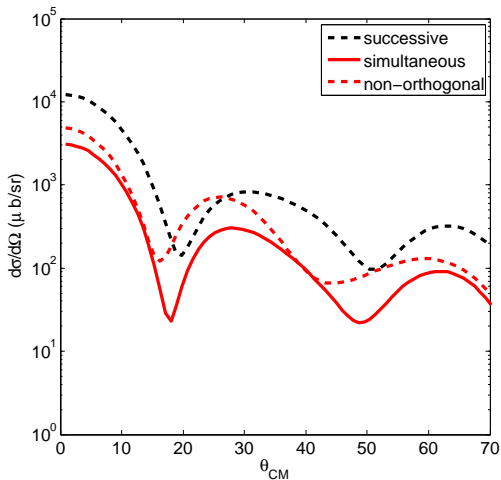
Contributions to the $^{112}\text{Sn}(p,t)^{110}$ total cross section



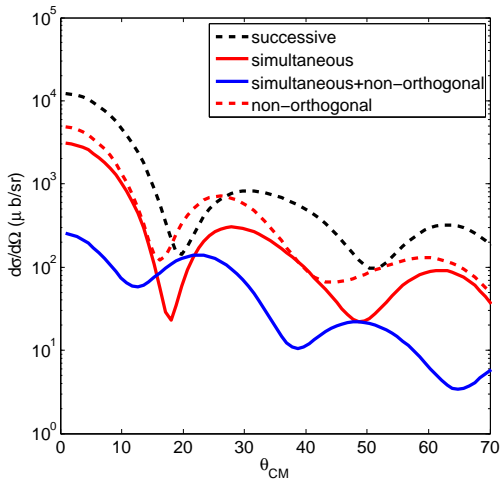
Contributions to the $^{112}\text{Sn}(p,t)^{110}$ total cross section



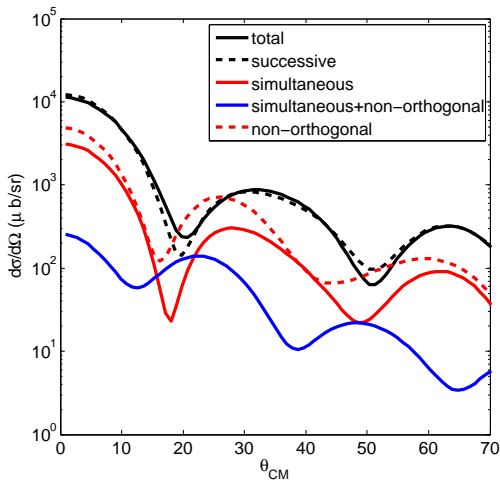
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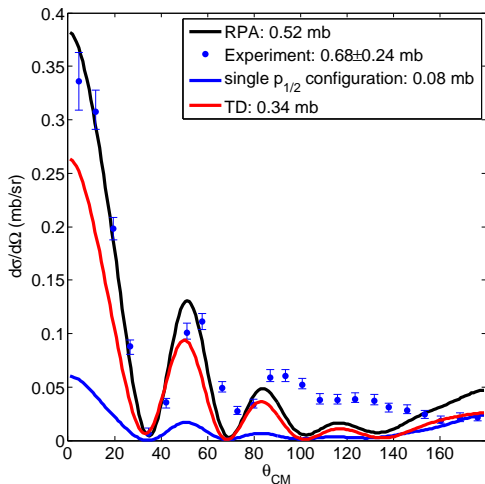
Contributions to the $^{112}\text{Sn}(p,t)^{110}$ total cross section



Essentially a **successive** process!

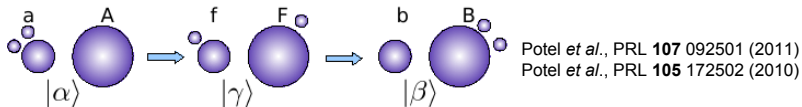
$^{206}\text{Pb}(t, p)^{208}\text{Pb}$ (gs): pairing in normal nuclei

$^{206}\text{Pb}(t, p)^{208}\text{Pb}$ at 12 MeV. Data from Bjerregaard *et.al.* (1966)



state nlj	B_{nlj}	
	pp RPA	(TDA)
$1h_{9/2}$	0.15	(0.14)
$2f_{7/2}$	0.21	(0.26)
$1i_{13/2}$	0.29	(0.28)
$3p_{3/2}$	0.23	(0.22)
$2f_{5/2}$	0.32	(0.31)
$3p_{1/2}$	0.89	(0.85)
$2g_{9/2}$	0.18	(—)
$1i_{11/2}$	0.15	
$1j_{15/2}$	0.13	
$3d_{5/2}$	0.06	
$4s_{1/2}$	0.06	
$2g_{7/2}$	0.10	
$3d_{3/2}$	0.05	

Two particle transfer in 2-step DWBA

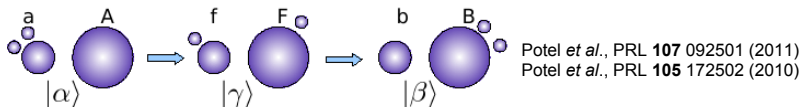


$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T_{succ}^{(2)}(j_i, j_f) - T_{NO}^{(2)}(j_i, j_f) \right)$$

Simultaneous transfer

$$T^{(1)}(j_i, j_f) = 2 \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*} \chi_{bB}^{(-)*}(\mathbf{r}_{bB}) \\ \times v(\mathbf{r}_{b1}) [\Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1) \Psi^{j_i}(\mathbf{r}_{b2}, \sigma_2)]_{\mu}^{\Lambda} \chi_{aA}^{(+)}(\mathbf{r}_{aA})$$

Two particle transfer in 2-step DWBA

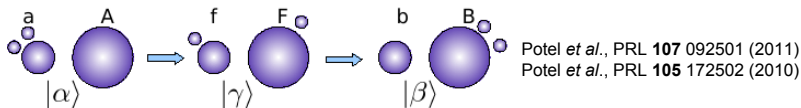


$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T_{succ}^{(2)}(j_i, j_f) - T_{NO}^{(2)}(j_i, j_f) \right)$$

Successive transfer

$$\begin{aligned}
 T_{succ}^{(2)}(j_i, j_f) = & 2 \sum_{K, M} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'_1 \sigma'_2}} \int d\mathbf{r}_{FF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*} \\
 & \times \chi_{bB}^{(-)*}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]_M^K \\
 & \times \int d\mathbf{r}'_{FF} d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} G(\mathbf{r}_{FF}, \mathbf{r}'_{FF}) [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_M^K \\
 & \times \frac{2\mu_{FF}}{\hbar^2} v(\mathbf{r}'_{f2}) [\Psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_{\mu}^{\Lambda} \chi_{aA}^{(+)}(\mathbf{r}'_{aA})
 \end{aligned}$$

Two particle transfer in 2-step DWBA



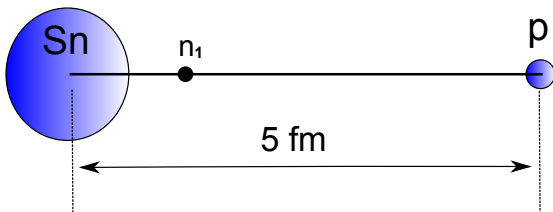
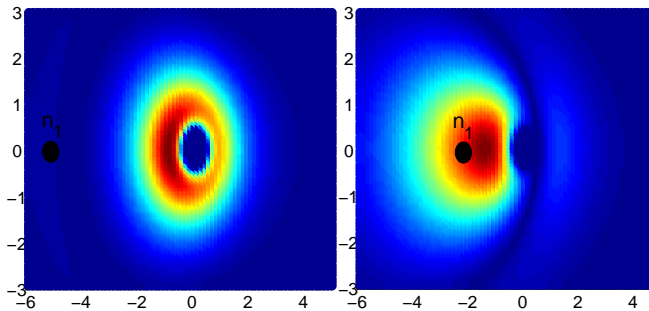
$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T_{succ}^{(2)}(j_i, j_f) - T_{NO}^{(2)}(j_i, j_f) \right)$$

Non-orthogonality term

$$\begin{aligned} T_{NO}^{(2)}(j_i, j_f) = & 2 \sum_{K, M} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'_1 \sigma'_2}} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*} \\ & \times \chi_{bB}^{(-)*}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]_M^K \\ & \times \int d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_M^K \\ & \times [\Psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_\mu^\Lambda \chi_{aA}^{(+)}(\mathbf{r}'_{aA}) \end{aligned}$$

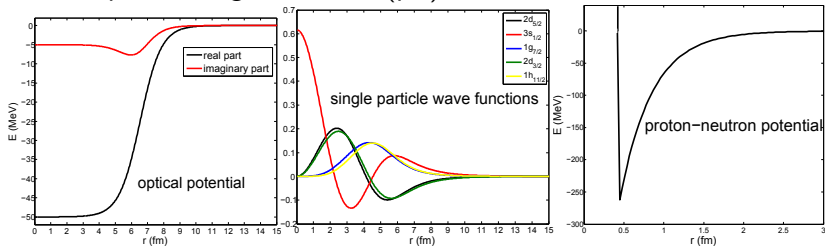
Non-local, correlated form factor

$$F(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{Ap}) = \phi_f(\mathbf{r}_{p1}, \mathbf{r}_{p2}) V_{pn}(\mathbf{r}_{p1}) V_{pn}(\mathbf{r}_{p2}) \phi_i(\mathbf{r}_{A1}, \mathbf{r}_{A2})$$



Ingredients of the calculation

Structure input for, e.g., the $^{112}\text{Sn}(p,t)^{110}\text{Sn}$ reaction:



plus the B_j spectroscopic amplitudes needed to define the two-neutron wavefunction:

$$\Phi(\mathbf{r}_1, \sigma_1, \mathbf{r}_2, \sigma_2) = \sum_j B_j [\psi^j(\mathbf{r}_1, \sigma_1) \psi^j(\mathbf{r}_2, \sigma_2)]_0^0$$