Insights into the Fermi-matrix element and superallowed beta-decay

Nuclear structure and reactions: Experimental and ab-initio theoretical perspectives

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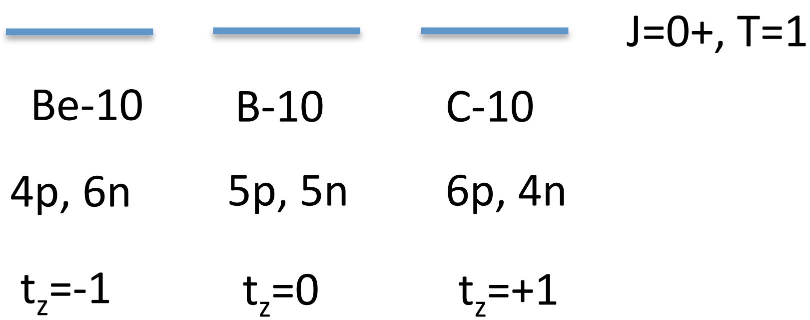
Background nuclear physics…

In order to make the talk more accessible let's bring everyone to the same page.

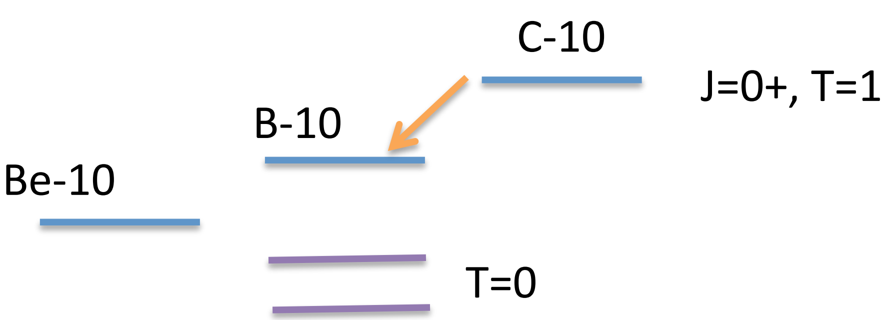
No Coulomb + nuclear force components equal



<u>Isobaric analogue</u> states are nuclear states that appear in mirror-nuclei when p and n's are interchanged.
They can be labeled by the isospin "T".

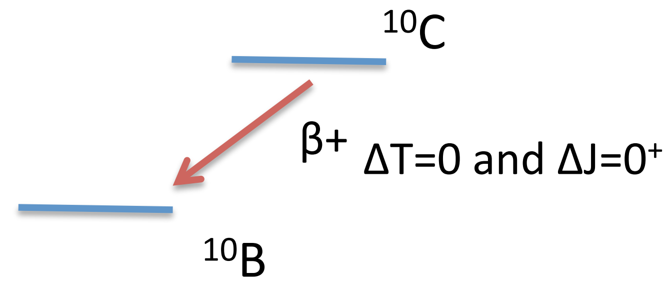
<u>Isospin</u> is much like spin (SU(2)).
The third component t_z=1/2(Z-N)

Coulomb + nuclear interaction



<u>Superallowed Fermi</u> transitions are beta-decays between J=0+ T=1 isobaric analog states.

Motivation



Superallowed Fermi β-decay transitions provide <u>excellent tests of electroweak theory</u>.

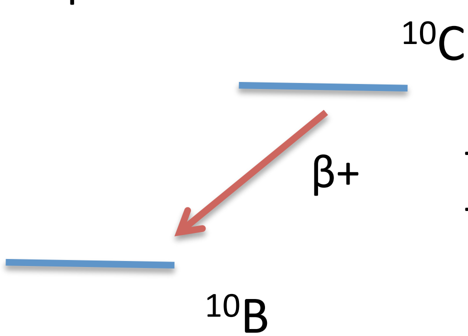
These transitions involve only the vector part of the weak interaction.

If the conserved-vector-current hypothesis is true then for pure Fermi transitions “ft” should be independent of nucleus (i.e. Gv is not renormalized in the nuclear medium).

$$ft = \frac{K}{G_V^2 |M_F|^2}$$

K fundamental constants and Gv is the vector coupling const.

Explanation of terms



t = partial lifetime of state (for branching)

f = statistical phase-space factor (Fermi function integral)

$$M_F = \langle \psi_f | T_{\pm} | \psi_i \rangle$$

T is the Isospin raising or lowering operator (changes neutron to proton or Vice versa.)

Corrections to measured "ft" values

But experimentally measured "ft's" are not nucleus-independent.

For std. model tests one needs to apply two nucleus-dependent corrections to ft.

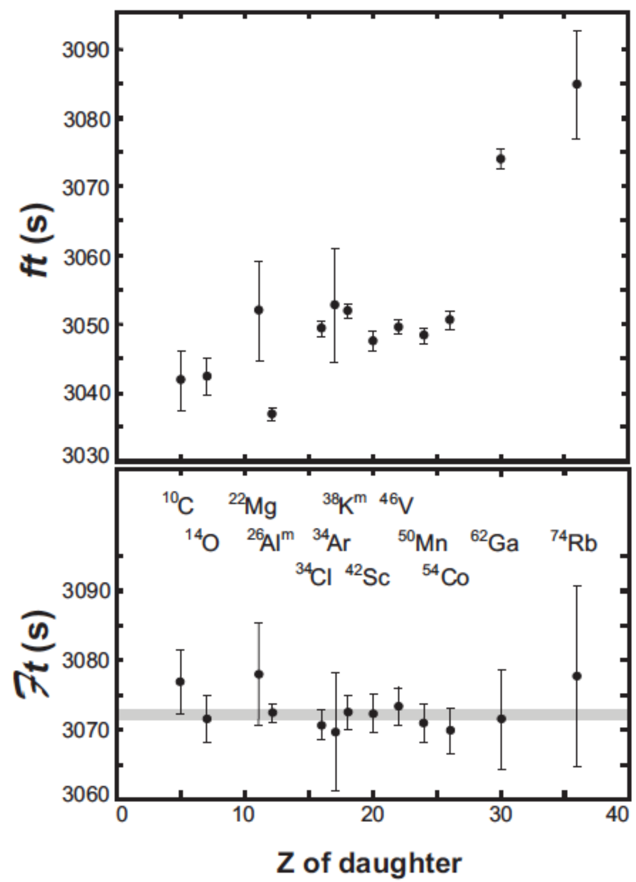
$$\mathcal{F}t = ft(1+\delta_R+\Delta_R)(1-\delta_C)$$

QED radiative corrections

Fermi-matrix element correction arising from nuclear structure due to isospin-breaking effects.

The nucleus-independent Ft values are then used to determine the CKM mixing matrix element between up and down quarks.

$$|v_{ud}|^2 = \frac{\pi^3 \ln 2}{\mathcal{F}t} \frac{\hbar^7}{G_F^2 m_e^5 c^4}$$



See Fig 1. of Hardy & Towner PRC 70, 055502 (2009) and references in caption.

The CKM matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Gives the relative probability that a down-type quark will weak decay into an up-type quark.
d' is a superposition of down, strange and bottom.

The CKM matrix assumes three generations of quarks and is a fundamental part of the electroweak theory. It is believed to be unitary.

$$\sum_k |V_{ik}|^2 = \sum_i |V_{ik}|^2 = 1$$

Numerically, the values of the CKM matrix are below. V_{ud} is the largest component and thus you should spend most of your effort there.

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}$$

The sum of the squares of the top row give 0.9999 ±0.0006

These numbers come from the PDG 2010 revised by A. Ceccucci, Z.Ligeti and Y.Sakai, in Section 11.

Determining V_{ud} in the CKM matrix

$$V_{ud} = G_V/G_F$$ Fundamentally, ratio of vector-to-Fermi coupling constants.

Corrected "ft" values from nuclear structure:

$$\mathcal{F}t = ft(1+\delta_R+\Delta_R)(1-\delta_C)$$

$$ft = \frac{K}{G_V^2|M_F|^2}$$

$$|v_{ud}|^2 = \frac{\pi^3 \ln 2}{\mathcal{F}t}\frac{\hbar^7}{G_F^2 m_e^5 c^4}$$

Provided you know G_F
(from pure leptonic decays such as muon decay)

$$|V_{ud}| = 0.97425 \pm 0.00022$$

Nuclear structure value taking into account uncertainties in the radiative corrections as well as structure part.
Avg. of 13 superallowed Fermi transitions (20 measured)[1].

One can also determine Vud from neutron beta decay or pion decay[2].

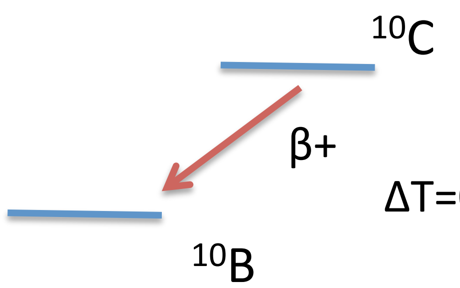
$$V_{ud} = 0.9746(18)$$ Neutrons: have to consider axial-vector part too.

$$V_{ud} = 0.9749(26)$$ Pions: branching ratio of 10^{-8} that must be considered.

1) Hardy and Towner, PRC 70, 055502 (2009)
2) A. Ceccucci, Z.Ligeti and Y.Sakai, PDG Feb 2010 Section 11

The isospin-mixing correction δ_c

Quantify isospin-symmetry breaking in a "heavier" nucleus by M_F.



$\Delta T=0$ and $\Delta J=0^+$

Why is isospin broken?

Coulomb force (protons≠neutrons)
NN scattering lengths pp≠pn≠nn

Fundamentally due to quark interactions
These effects are encoded in the potential at various orders.

If isospin were exact for T=1: $M_F^2 = 2$

We need to consider a small deviation

$$\delta_c = \frac{|M_F^2 - 2|}{2}$$

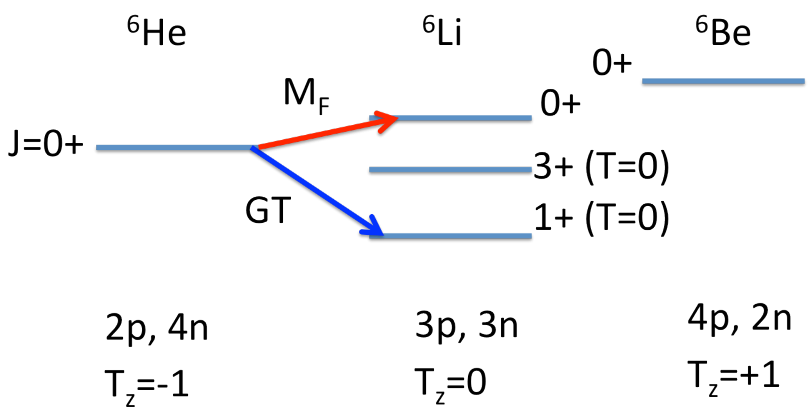
Typically δ_c ~ 0.1% for light nuclei.

Corrections come from Coulomb as well as mixing of various isospin states and 1p-1h excitations.

Isobaric mass multiplet equation

- The IMME predicts parabolic energy dependence of similar isospin states in a mass multiplet.

$$E(A,T,T_z) = a + bT_z + cT_z^2$$



- The IMME coefficients have a physical interpretation in terms of isovector (b) and isotensor (c) components.

$$b = \frac{E(^6\mathrm{Be}) - E(^6\mathrm{He})}{2}$$

$$c = \frac{E(^6\mathrm{Be}) + E(^6\mathrm{He})}{2} + E(^6\mathrm{Li})$$

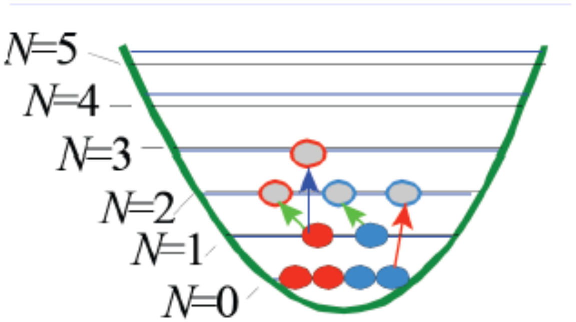
- "b" and "c" coefficients are experimentally measured. We set out to calculate these quantities too to judge convergence of various observables.

The No-Core Shell Model (NCSM)

Starting Hamiltonian is <u>translationally invariant.</u>

$$H_A = \frac{1}{A}\sum_{i<j}^{A}\frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i<j}^{A} V_{\mathrm{NN},ij}$$

NCSM has two parameters: Nmax and Ω



Provided interaction is “soft” we don't need to do any renormalization of interaction,

It's that “simple”.

If we now use a single-particle basis, we have to <u>remove the spurious CM states.</u>

Advantage in m-scheme: Antisymmetry is easy to implement.
Disadvantage in m-scheme: Number of basis states is much larger than JT basis

A=6 : Bare N3LO NN interaction

Begin by calculating M_F for A=6:

Computationally much *easier than A=10.*

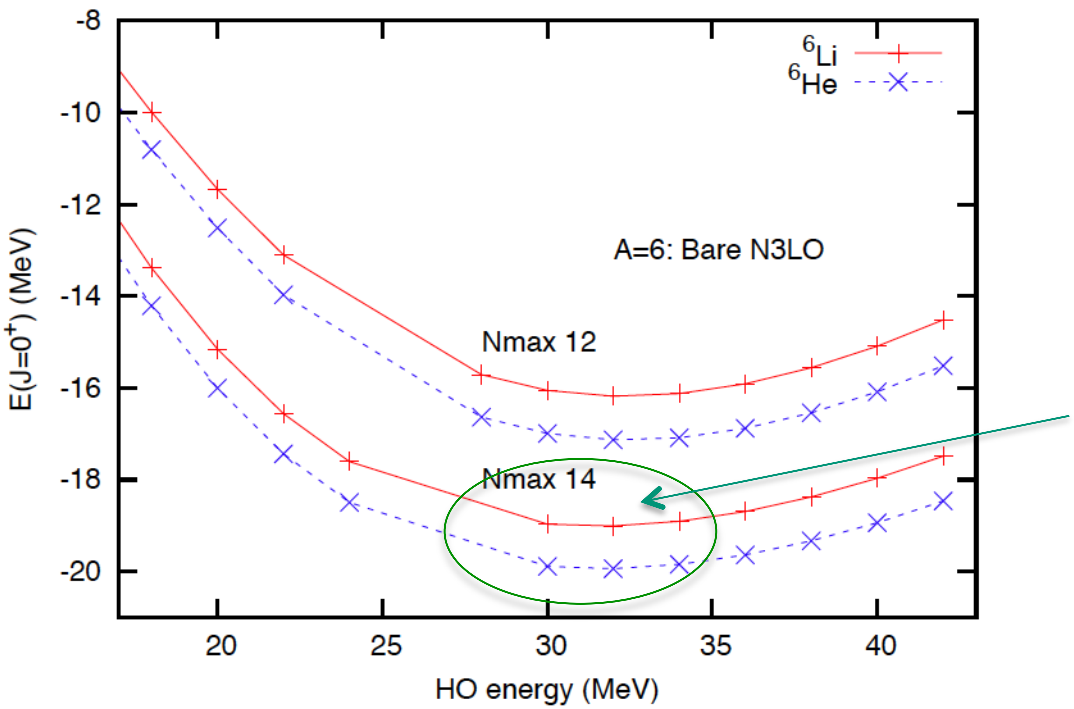
Extrapolation methods can be tested.

Guides us for A=10 calculation.

Bare interaction (are you crazy)?

No. A=6 can be calculated in the NCSM with the bare interaction by extrapolating.

What does SRG do to the isospin parts of the interaction?



J=0+ energies of He6 (blue)/Li6(red).

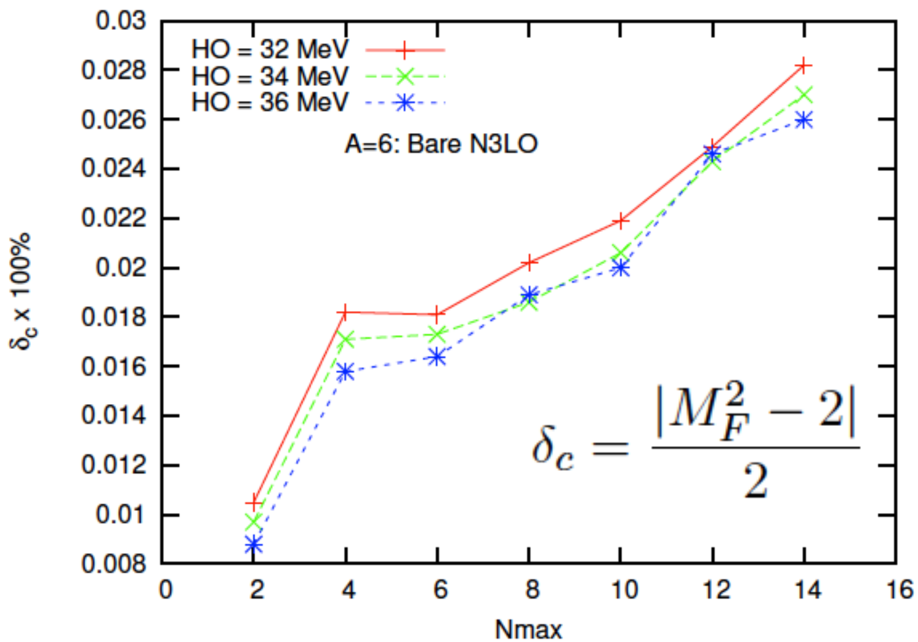
Conventional wisdom indicates that one should calculate observables In this region here (variational min.)

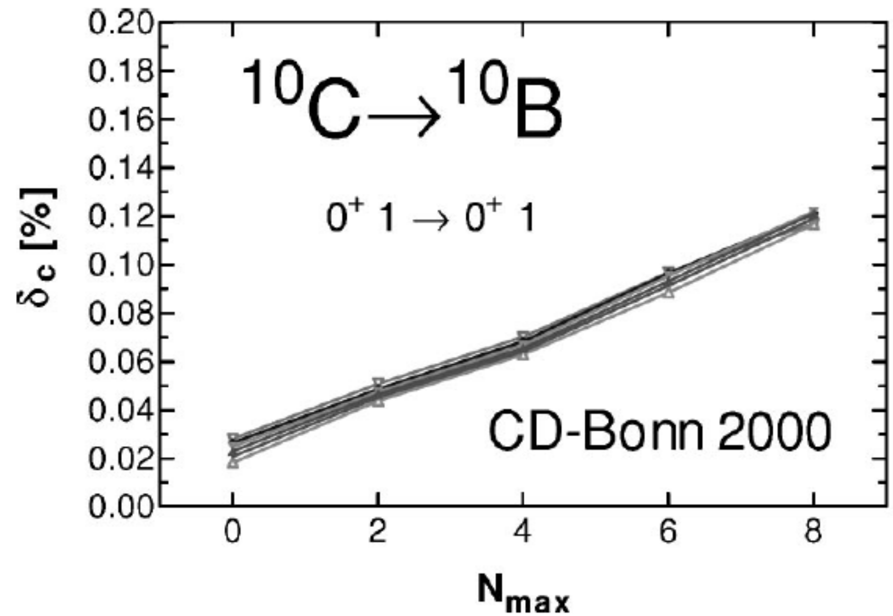
But…is that really true?

At Nmax=14 the bare calculation is about 5 MeV away from extrapolated gs[1].

1) Navratil and Caurier, PRC 69, 014311 (2004)

Isospin-mixing correction (bare int)





The Fermi-matrix element at the variational minima does not seem to converge!
Can't extrapolate reliably.
What to do?

Linear scaling with Nmax is known from older calculations
E. Caurier, P. Navratil, W.E. Ormand, J.P. Vary
PRC **66**, 024314 (2002)

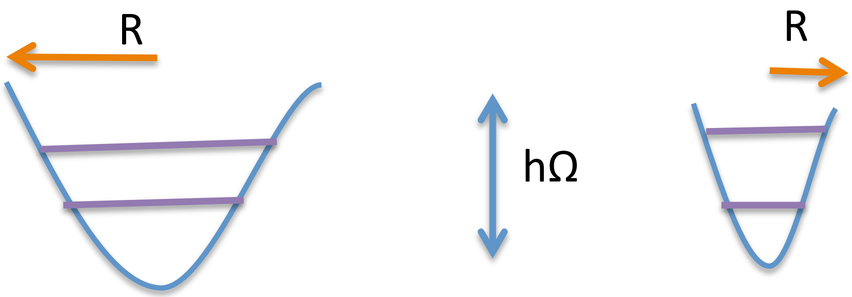
Approach convergence by new means.

What can δ_c depend on?

Coulomb force obviously!

Dependence quantified through Coulomb-energy (i.e. charge radii) or “b” coeff. (1^{st} order).

But if δ_c depends on radii there should be dependence on hΩ.



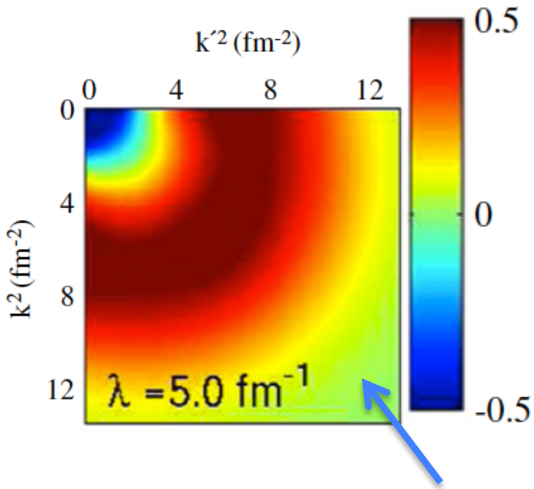
HO well

“Squeezed” nucleus

δ_c also has contributions from states mixing – subtle hΩ dependence.

And then there are the various contributions from isovector (b) and isotensor (c) components of the NN force.

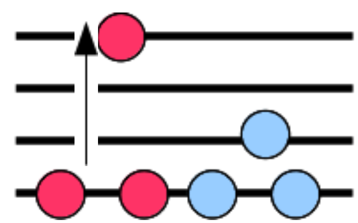
Unified Extrapolations (UV+ir)



Nuclear interaction is expressed in terms of matrix elements: In order to correctly capture the physics in the interaction you need both UV and ir convergence.

Harmonic oscillator basis regulators

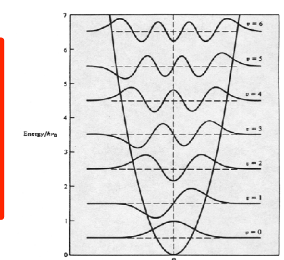
$$\Lambda = \sqrt{m_N (N + 3/2)\hbar\Omega}$$



NN interaction at high momentum is (super)-exponentially regulated.
For Chiral N3LO: UV ~ 800 MeV/c

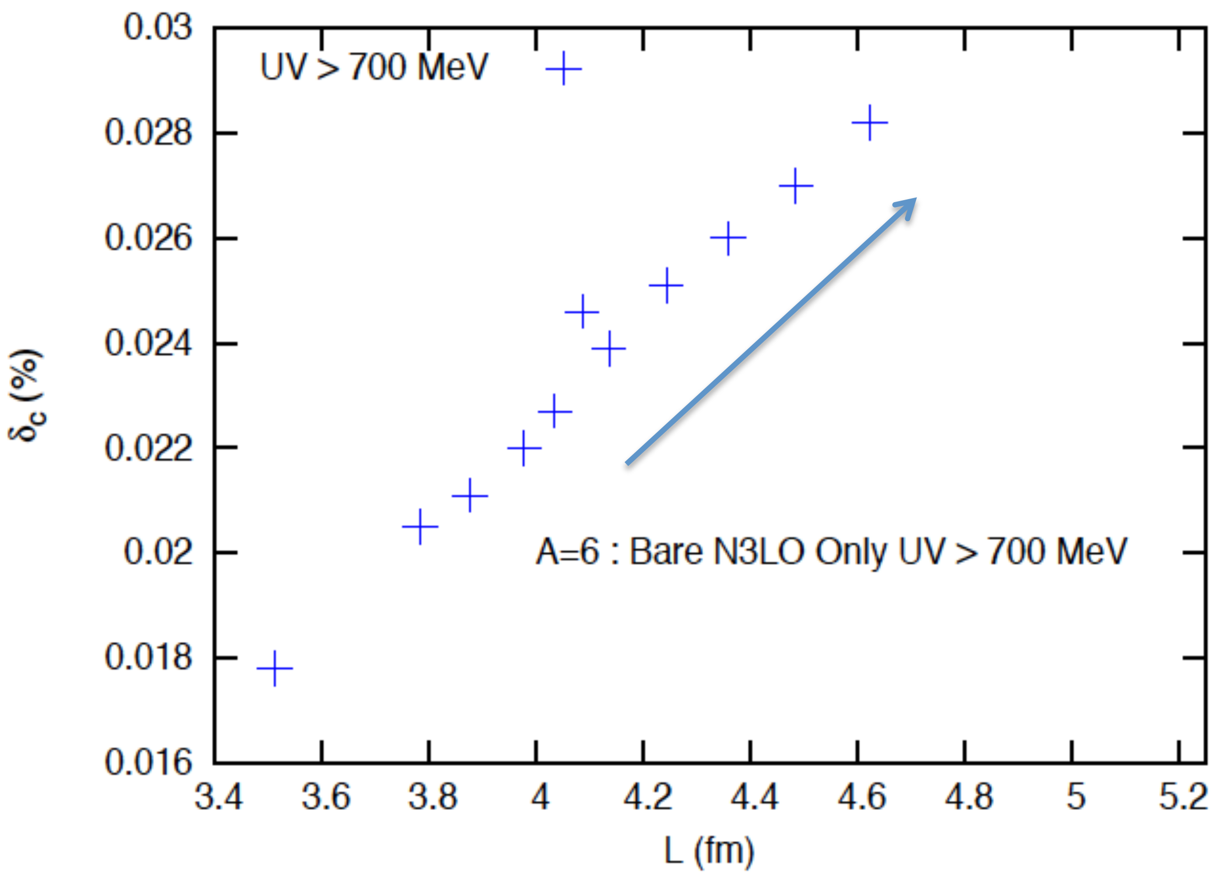
NN int. is also fit to low-energy scattering data, e.g. scattering lengths or the Deuteron.
All NN int's should have ir ~ 20-45 MeV/c

$$\lambda_{ir} = \lambda_{sc} = \sqrt{\frac{m_N \hbar\Omega}{N + 3/2}}$$



Coon, Avetian, Kruse, van Kolck, Maris, Vary, PRC 86 054002 (2012)
Furnstahl, Hagen, Papenbrock, PRC 86, 031301 (2012)
More, Ekström, Furnstahl, Hagen, Papenbrock, PRC 87, 044326 (2013)

UV converged δ_c extrapolation



Arizona (i.e. Sid, MKGK prefer)

$$\lambda_{ir} = \lambda_{sc} = \sqrt{\frac{m_N \hbar\Omega}{N + 3/2}}$$

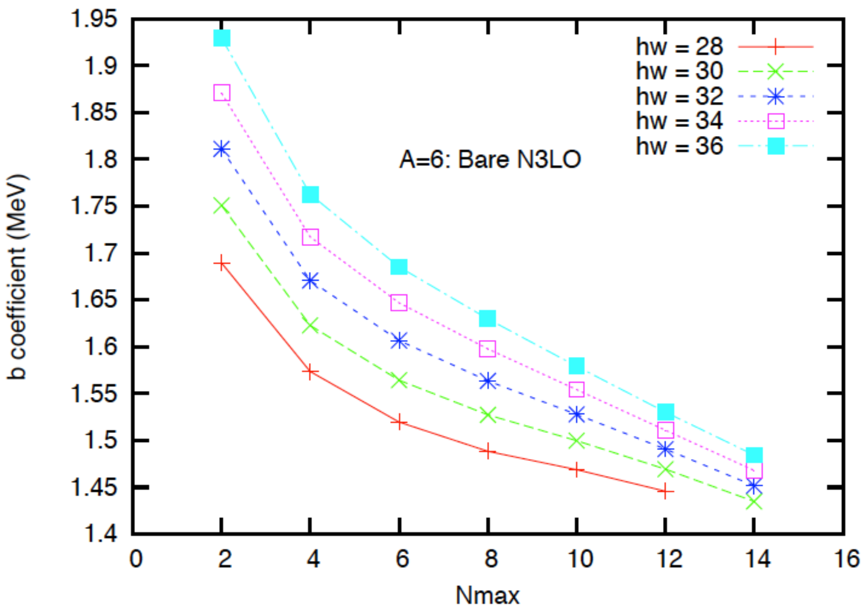
But of course you could use

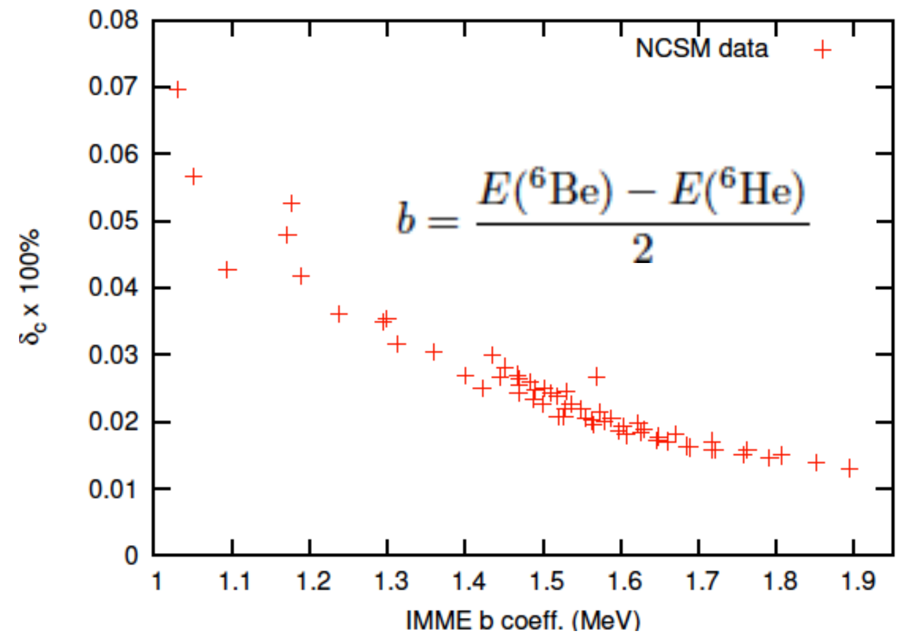
$$L_i = \sqrt{N_{\max} + 3/2 + ib}$$

As a first-attempt, we take approximately UV converged points for δ_c and then extrapolate into the ir region (large "L"). Unfortunately this procedure does not seem to converge either (for later: but might be ok.

δ_c as a function of the b coefficient

Strategy: Can we extrapolate other observables to indirectly determine δ_c?
Perhaps we can correlate multiple observables to make a consistent prediction of δ_c.





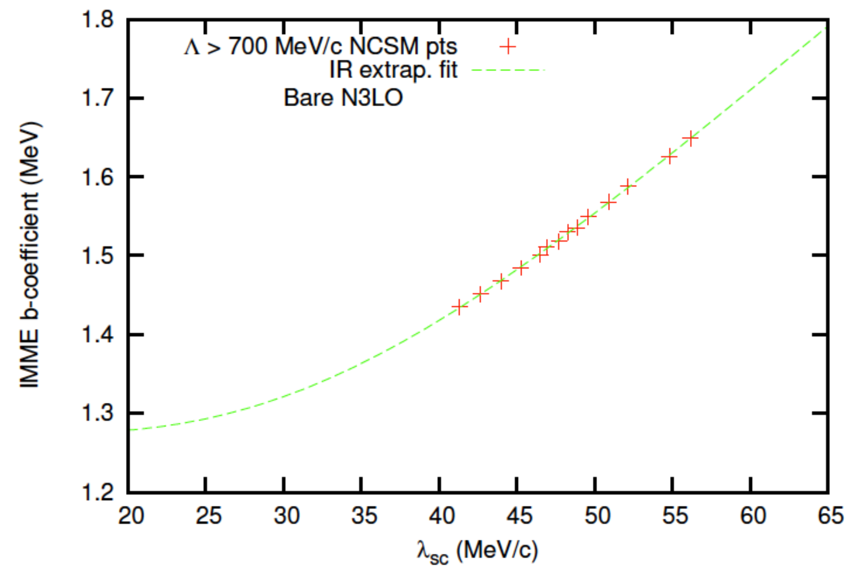
b as a function of Nmax:
hΩ dependence and no obvious form of extrapolating function.
Now what?

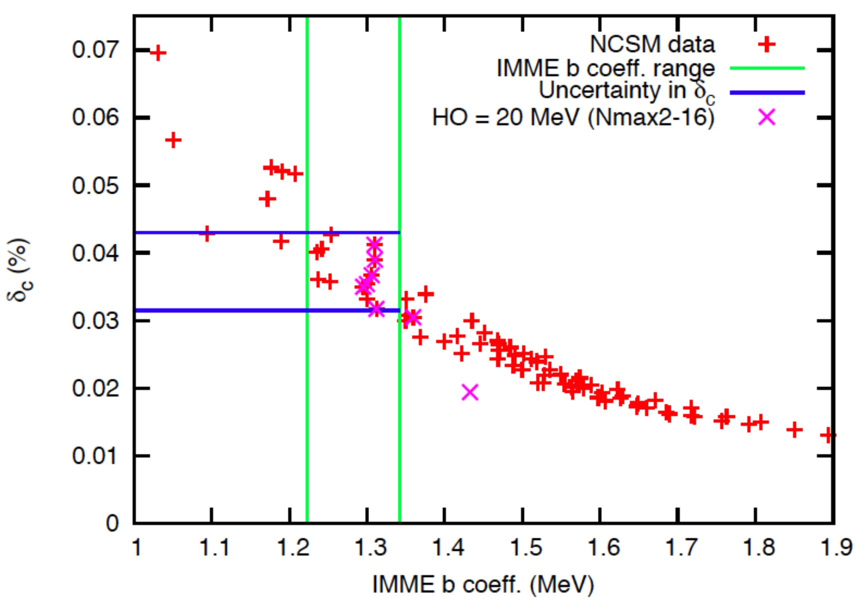
NCSM parameters:
Nmax = 4 – 14
HO = 12-42 MeV (Nmax ≤ 10)
HO = 28-42 MeV (Nmax 12-14)

Extrapolating the b coefficient

The b coef. is extrapolated for various UV values.
UV = 650,675,…,725 MeV/c.
UV convergence for b ~ 700 MeV/c[1]
Note: "Blind" to Nmax and hΩ.

$$b(\lambda_{sc}) \doteq A\exp(-B/\lambda_{sc}) + b(\lambda_{sc}=0)$$





δ_c values consistent with extrapolated b's
δ_c = 0.035% with 10% error.
Bracketed points are all hΩ~20 MeV (?).

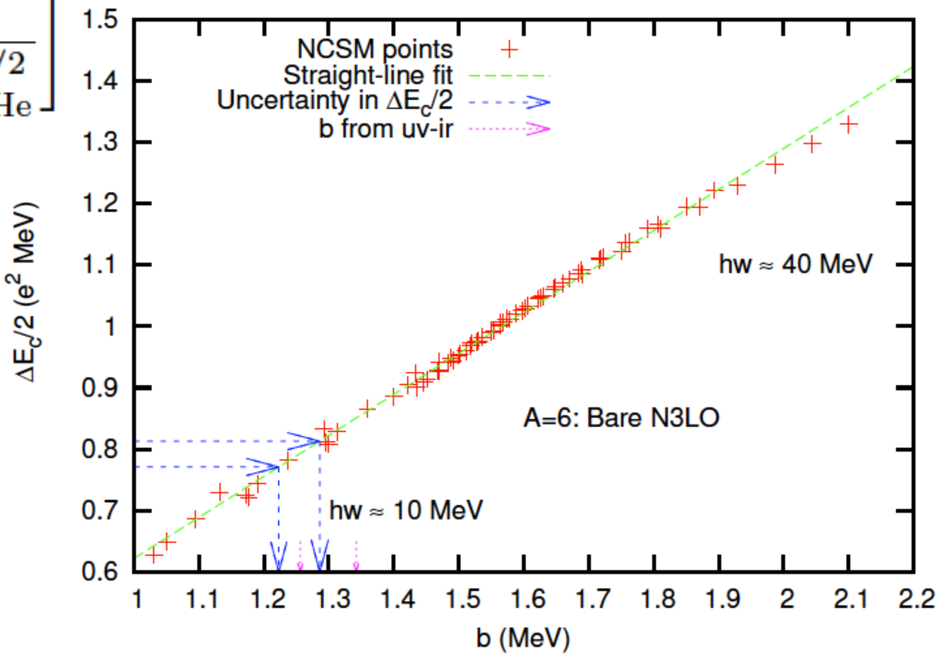
1) Also known from He3-H3 studies by MKGK

Coulomb-energy and b: Consistent?

Argument: If you determine b then you have a range for δ_c that is acceptable.
Question: Did you determine b correctly?

Extrapolate rms charge-radii of He-6 and Be-6 to determine Coulomb-energy diff.

$$\Delta E_c(^6\mathrm{Be} - {}^6\mathrm{He}) = \left(\frac{3}{5}\right)^2 e^2 \left[\frac{16}{\langle r_p^2\rangle^{1/2}_{^6\mathrm{Be}}} - \frac{4}{\langle r_p^2\rangle^{1/2}_{^6\mathrm{He}}}\right]$$



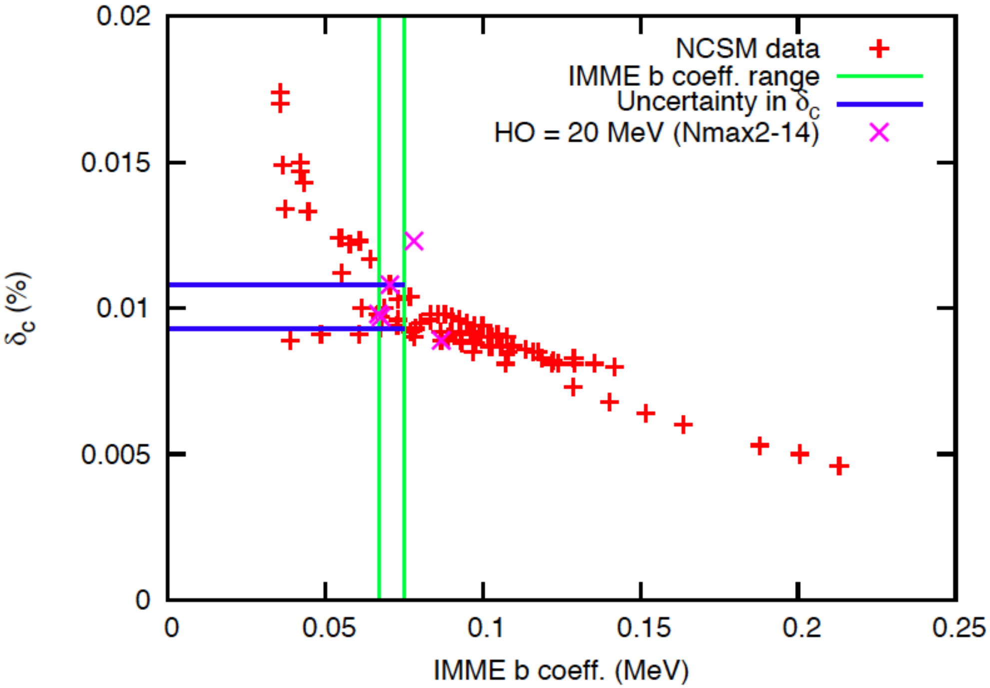
The radii are extrapolated with UV/ir:

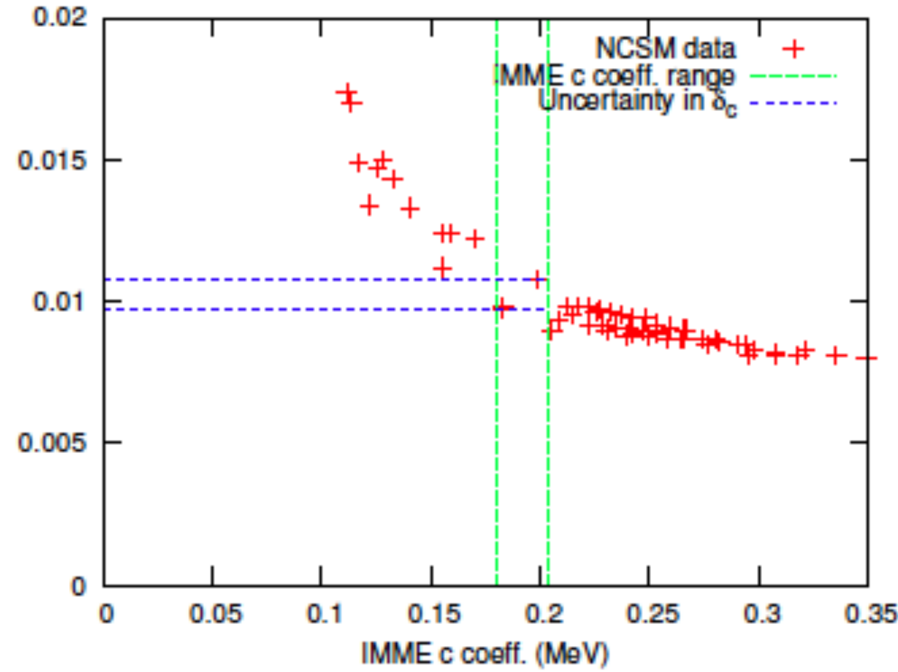
$$\langle r_p^2\rangle_L = \langle r_p^2\rangle_\infty \left[1 - c_0\beta^3 e^{-\beta}\right]$$

Short answer: Yes, *consistent*.

No Coulomb (only NN parts)

How large are the contributions to δ_c from Coulomb/strong interaction?
As a first step, we neglect the Coulomb interaction and redo A=6 calculations.





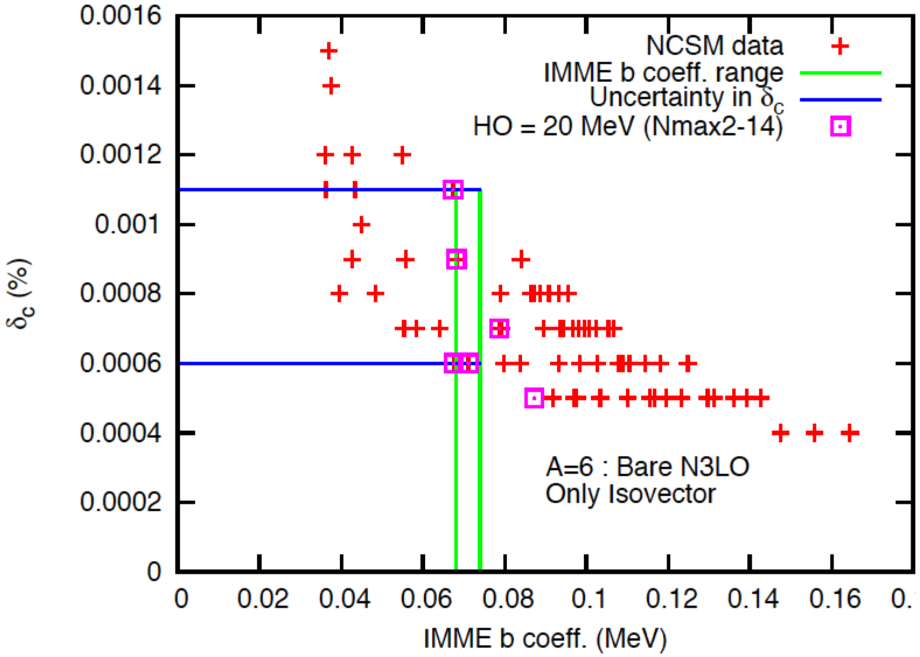
Extrapolate b with UV/ir as before.

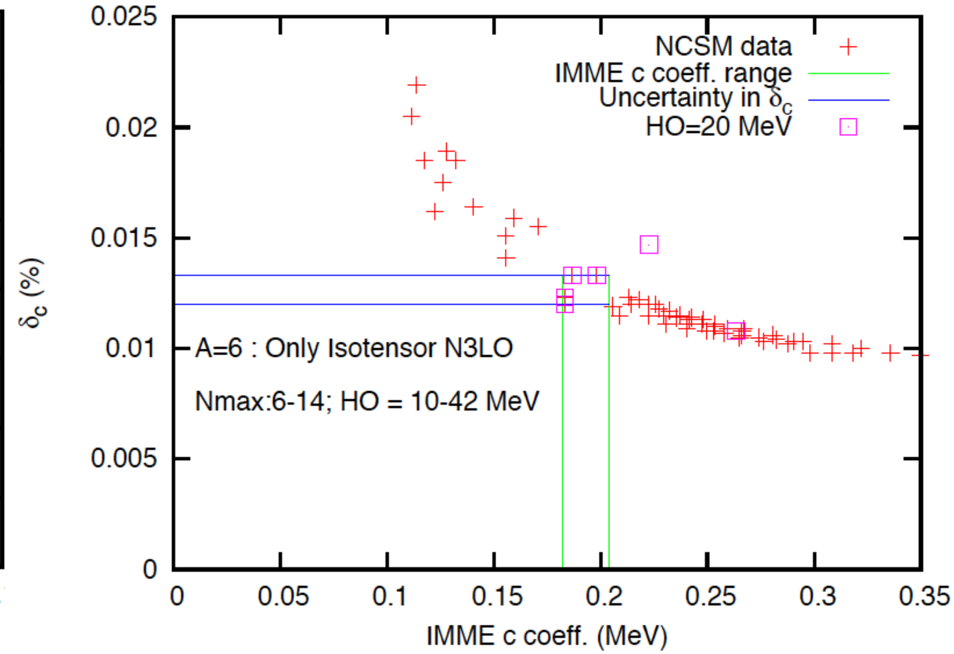
We can also extrapolate c.

b and c extrapolations give the same range for δ_c (consistent).
δ_c is about 1/3 the value of the full bare interaction – 10 % error on δ_c.

Isovector and isotensor components

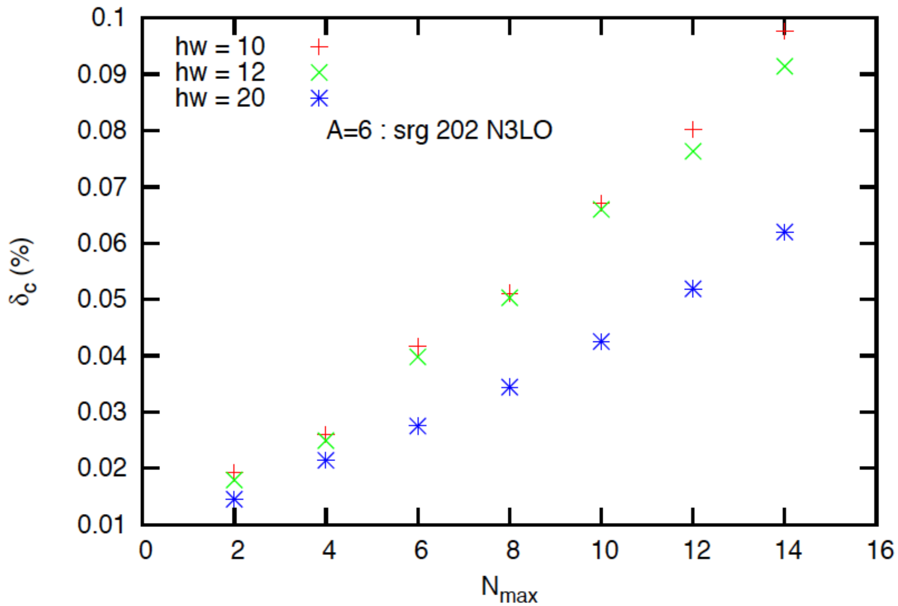
Now turn off the isovector and isotensor parts separately to see which part of the interaction gives rise to isospin-breaking. There is no Coulomb interaction present.





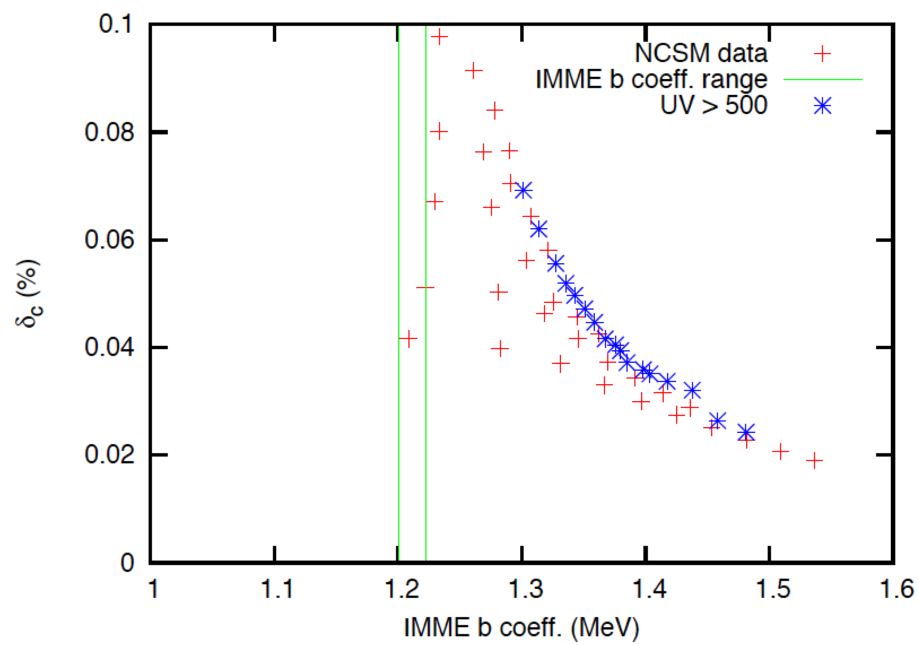
Notice that δ_c is receiving only contributions from the isotensor part of the interaction. The value of δ_c is about 15-20% larger than when both components are present (prev. slide) but is still about a factor 1/3 smaller than the bare+coulomb interaction.

And what about SRG interactions?



δ_c increases with Nmax once more, but note that it is about 1/3 in magnitude than the bare interaction. Coulomb has also been srg'ed.

Unfortunately the correlation between extrapolated 'b' coefficient and δ_c is now completely inconsistent. 'b' has been extrapolated to 40 keV of experimental value. δ_c does follow some sort of 'universal' curve for UV converged points. Convergence with ir has been tested but leads to inconsistent results.



Conclusion

Really too early to say anything just yet – need to complete this work.

Unfortunately subtleties are present:

δ_c depends on a number of parameters/observables. Do them ***consistently***!

SRG results are confusing but perhaps are due to the RG itself.

$\Delta T=1$ matrix elements in Argonne calculation (we don't consider them).

BUT we are figuring out how to present a truly ab-initio approach to isospin-mixing δ_c

Extrapolation techniques (UV/ir)

Correlation of observables to make consistent predictions.

Uncertainty quantification (theory errors).

Vital for determining δ_c in A=10 system which we have started.

Thanks to the following people:

Erich Ormand (LLNL)

Calvin Johnson (SDSU)

Sid Coon (UofA)

Coulomb-energy, b-coefficients and rms charge-radii…

- "b" coefficient is connected to the Coulomb-displacement energy.
- The Coulomb-energy in a uniformly charged sphere $E_c = \frac{3}{5}\frac{(Ze)^2}{R}$
- Writing the "b" coefficient in terms of Coulomb-displacement energies

$$\Delta E_c(^6\mathrm{Be} - {}^6\mathrm{He}) = \left(\frac{3}{5}\right)^2 e^2 \left[\frac{16}{\langle r_p^2\rangle^{1/2}_{^6\mathrm{Be}}} - \frac{4}{\langle r_p^2\rangle^{1/2}_{^6\mathrm{He}}}\right]$$

- Note that I replaced "R" by the rms charge radius since that is what we measure and calculate.
- Thus we need the rms radii as well!

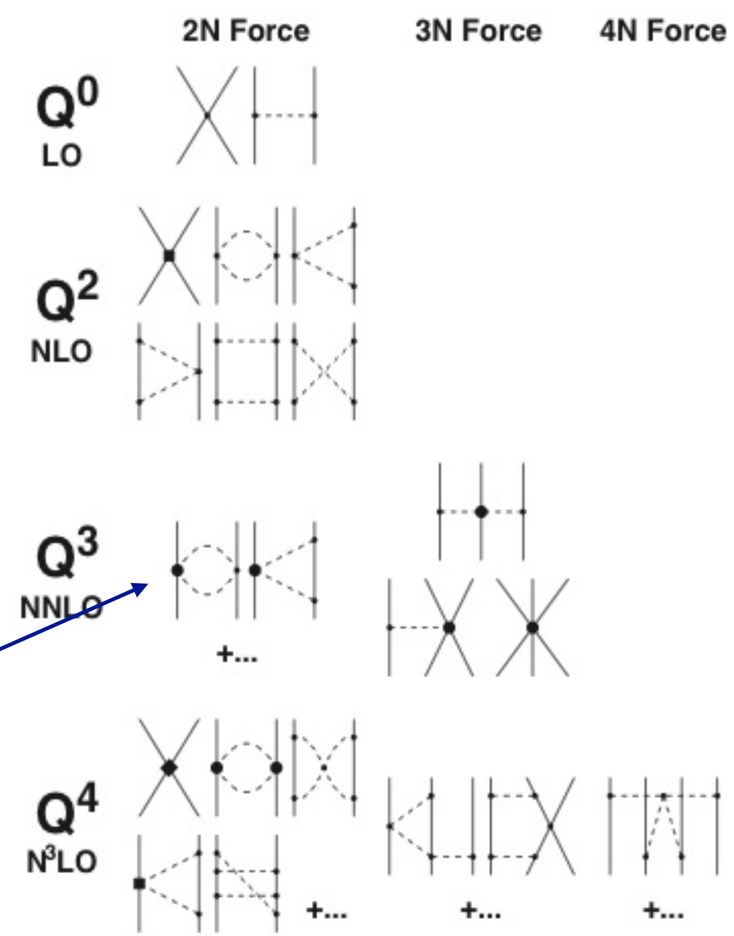
Chiral Effective Field theory

Low-energy theory of QCD in which the degrees of freedom are now nucleons and pions.

Therefore based on QCD symmetries.

Systematic power-expansion* (Weinberg) in powers of momentum over "QCD" scale.

Short-range physics is integrated out, leading to Low-energy constants (LEC's) that need to be determined exp.



*** But Weinberg counting is not renormalization-group invariant!**