Microscopic approach of the nucleus-nucleus bremsstrahlung

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Experimental and Ab Initio Theoretical Perspectives

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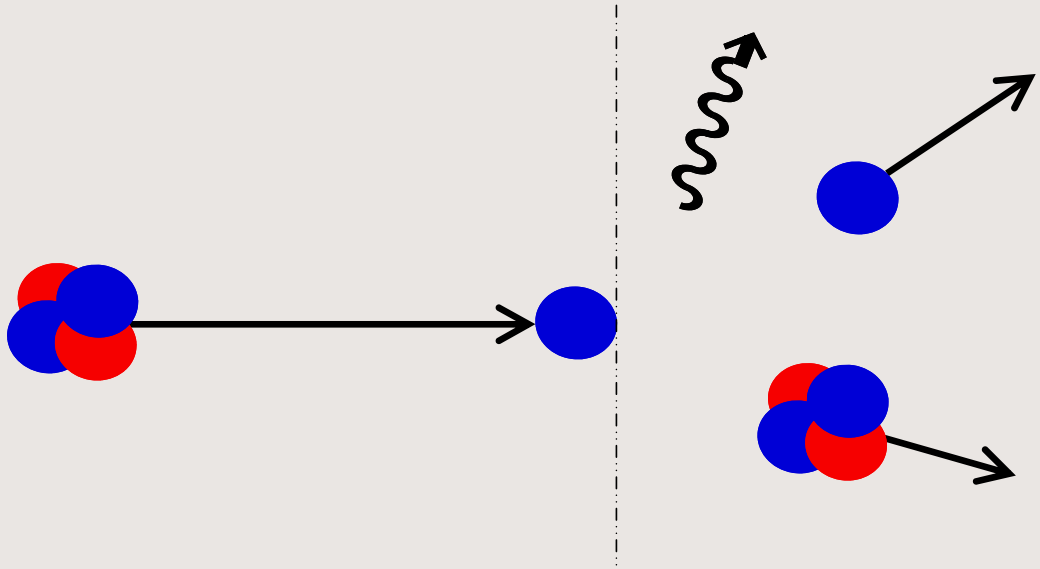
Introduction

- The nucleus-nucleus bremsstrahlung is a specific bremsstrahlung where the photon emission is induced by a collision between two nuclei or a nucleus and a neutron.



A part of the kinetic energy between the nuclei is converted to a photon.

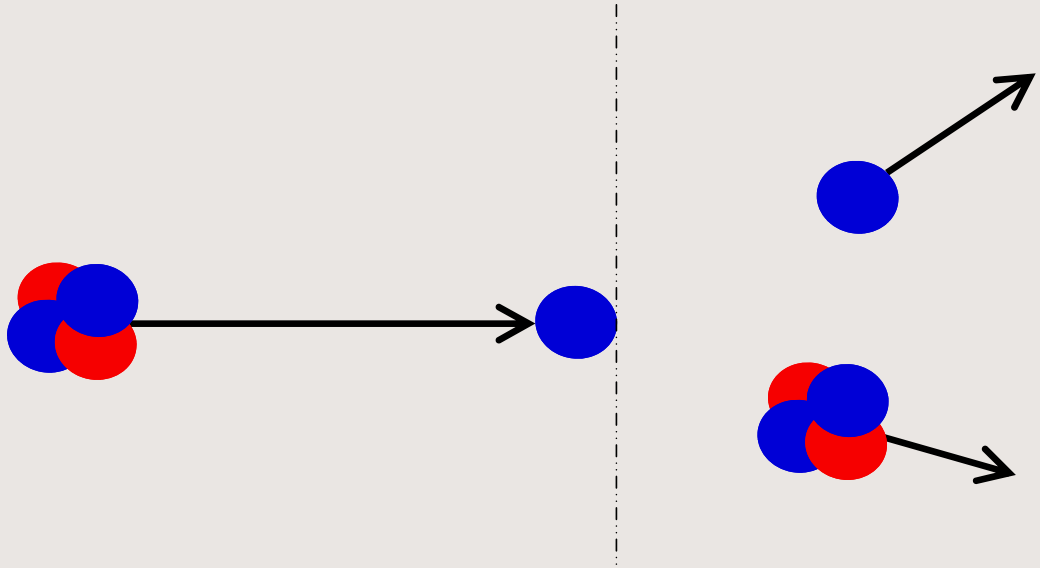
Introduction

The study of the nucleus-nucleus bremsstrahlung should enable us

- to describe the radiative transitions between unstable states (cf. recent measurements of "4^+-to-2^+" gamma transitions in ^{8}Be from the $\alpha(\alpha, \alpha\alpha\gamma)$ bremsstrahlung)
  [V. M. Datar *et al.*, Phys. Rev. Lett. 94, (2005) 122502]
  [V. M. Datar *et al.*, Phys. Rev. Lett. 111 (2013) 062502]
- to describe the $t(d, n\gamma)\alpha$ radiative transfer reaction (perspective to diagnose plasmas in fusion experiments from this reaction)
  [T. J. Murphy *et al.*, Rev. Sci. Instrum. 72 (2001) 773]
- to provide the cluster wave functions (phase-equivalent potentials can lead to different bremsstrahlung cross sections)
  [D. Baye, P. Descouvemont, and M. Kruglanski, Nucl. Phys. A 550 (1992) 250]

Introduction

- Since the electromagnetic forces are much weaker than the nuclear ones, the electromagnetic emission process can be seen as a small perturbation of the elastic scattering.



$\Rightarrow$ necessity to have a fair description of the elastic scattering
$\Rightarrow$ necessity to follow *ab initio* and effective approaches

Outline

- Describing the elastic scattering by a microscopic approach
  - effective and realistic internucleon interactions
  - resonating-group method (RGM) and the no-core shell model/RGM
  - Application to the $\alpha + N$ scattering and comparison with the experiment
- Describing the bremsstrahlung by a microscopic approach
  - Electric transition multipole operators from the nuclear current
  - Siegert approach: electric transition multipole operators from the charge density
  - Comments about the calculation of the matrix elements
  - Application to the $\alpha + N$ bremsstrahlung (in the effective approach)
  - (Very) preliminary results for the $\alpha + p$ bremsstrahlung with the NCSM/RGM

Microscopic approach

For an A-nucleon system, in non-relativistic microscopic model, all physical quantities are derived from the internal many-body Schrödinger equation

$$H\Psi = \left(\sum_{i=1}^{A} \frac{p_i^2}{2m_N} + \sum_{i>j=1}^{A} v_{ij} + \sum_{i>j>k=1}^{A} v_{ijk} - T_{\mathrm{c.m.}} \right) \Psi = E_T \Psi,$$

where

- $p_i^2/2m_N$ is the kinetic energy of nucleon i
- v_{ij} is a two-body interaction between nucleons i and j
- v_{ijk} is a three-body interaction between nucleons i, j, and k
- $T_{\mathrm{c.m.}}$ is the kinetic energy of the center of mass

Internucleon interaction

Two types of internucleon interaction are considered:

- an effective NN interaction (Minnesota potential)
  - central and spin-orbit part
  - soft potential adapted to simple cluster wave function
  - one or two parameters fitted to reproduce the elastic phase shifts of the considered collision
- a realistic NN(+NNN) interactions (chiral NN+NNN interaction renormalized by SRG)
  - reproduces very well the NN phase shifts
  - "no free parameter"
  - requires a quite large model space to converge

Resonating-group method

In the RGM, the wave function is expanded as a sum of cluster functions,

$$\mathcal{A}\Phi_1(\boldsymbol{\xi}^{(1)})\Phi_2(\boldsymbol{\xi}^{(2)})g(\boldsymbol{\rho})$$

Resonating-group method

In the RGM, the wave function is expanded as a sum of cluster functions,

$$\Psi^{JM\pi;TM_T} = \sum_\nu \mathcal{A}\Phi_\nu^{JM\pi;TM_T}\left(\boldsymbol{\xi}^{(1)},\boldsymbol{\xi}^{(2)},\Omega_\rho\right)\frac{g_\nu^{J\pi;TM_T}(\rho)}{\rho}$$

where

$$\Phi_\nu^{JM\pi;TM_T}\left(\boldsymbol{\xi}^{(1)},\boldsymbol{\xi}^{(2)},\Omega_\rho\right) = \left[\left[\Phi_{\nu_1}^{I_{\nu_1}\pi_{\nu_1};T_{\nu_1}}\left(\boldsymbol{\xi}^{(1)}\right)\Phi_{\nu_2}^{I_{\nu_2}\pi_{\nu_2};T_{\nu_2}}\left(\boldsymbol{\xi}^{(2)}\right)\right]^{I;TM_T}Y_\ell(\Omega_\rho)\right]^{JM\pi}$$

and $\nu = \nu_1\nu_2 I\ell$. Two types of internal wave functions of the clusters are considered

- $\Phi_{\nu_1}^{I_{\nu_1}\pi_{\nu_1};T_{\nu_1}}$ and $\Phi_{\nu_2}^{I_{\nu_2}\pi_{\nu_2};T_{\nu_2}}$ are the goundstate waves functions in the harmonic-oscillator shell model (Ex: α described by a $(0s)^4$ state)
- $\Phi_{\nu_1}^{I_{\nu_1}\pi_{\nu_1};T_{\nu_1}}$ and $\Phi_{\nu_2}^{I_{\nu_2}\pi_{\nu_2};T_{\nu_2}}$ are the NCSM eigenstates of the clusters for the considered nuclear interaction ($\rightarrow$ NCSM/RGM).

Resonating-group method

Inserting the RGM expansion in the variational form of the Schrödinger equation

$$\langle \delta\Psi^{J M \pi; T M_T} | H - E_T | \Psi^{J M \pi; T M_T} \rangle = 0,$$

leads to the RGM equations

$$\sum_\nu \int r^2 [\mathcal{H}^{J M \pi; T M_T}_{\nu' \nu}(r', r) - E_T \mathcal{N}^{J M \pi; T M_T}_{\nu' \nu}(r', r)] \frac{g_\nu^{J \pi; T M_T}(r)}{r} dr = 0,$$

where

$$\begin{aligned}
\mathcal{H}^{J M \pi; T M_T}_{\nu' \nu}(r', r) &= \langle \Phi^{J M \pi; T M_T}_{\nu'} \frac{\delta(\rho - r')}{\rho r'} | \mathcal{A}^\dagger H \mathcal{A} | \Phi^{J M \pi; T M_T}_{\nu} \frac{\delta(\rho - r)}{\rho r} \rangle \\
\mathcal{N}^{J M \pi; T M_T}_{\nu' \nu}(r', r) &= \langle \Phi^{J M \pi; T M_T}_{\nu'} \frac{\delta(\rho - r')}{\rho r'} | \mathcal{A}^\dagger \mathcal{A} | \Phi^{J M \pi; T M_T}_{\nu} \frac{\delta(\rho - r)}{\rho r} \rangle
\end{aligned}$$

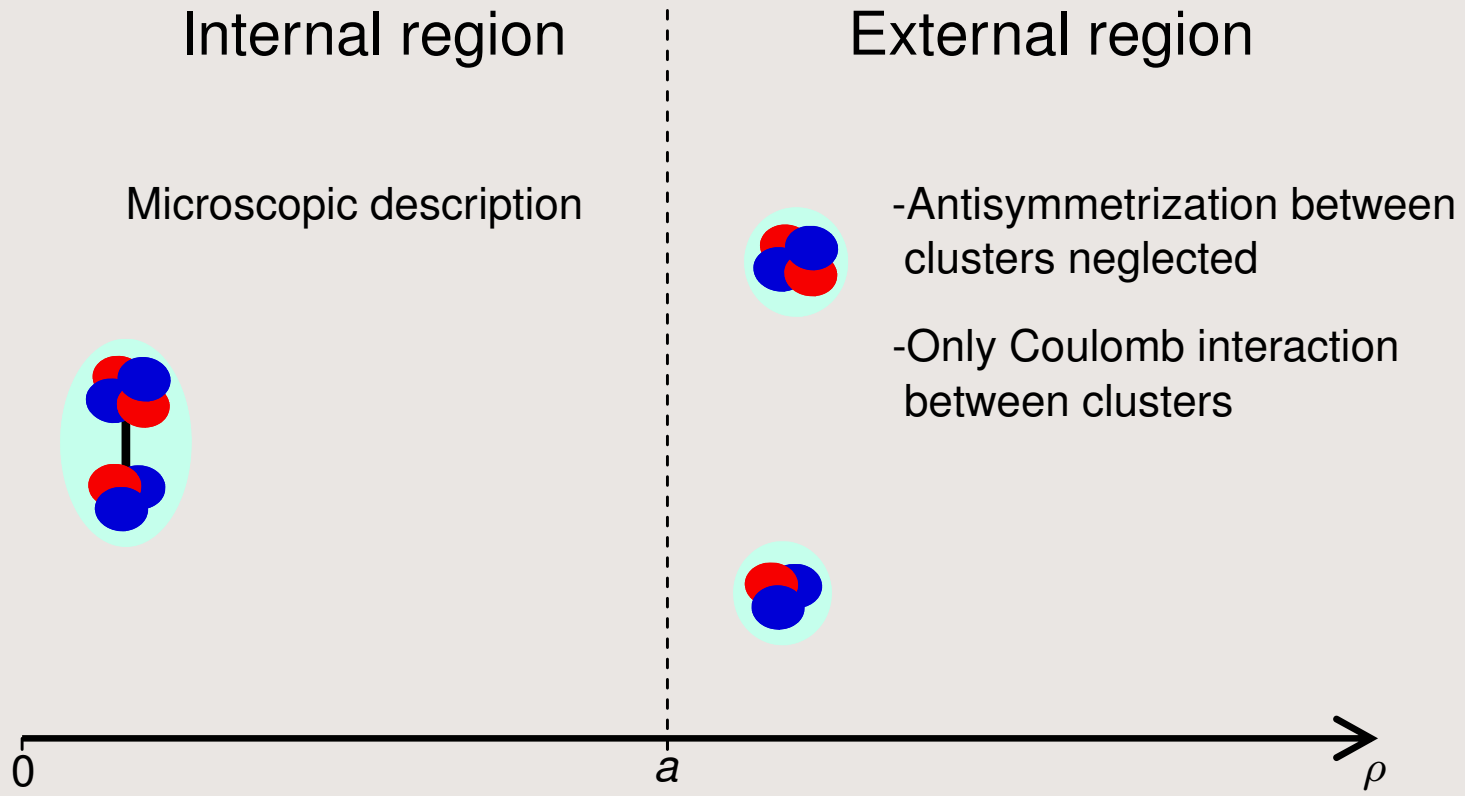
Comment

By expanding $g_\nu^{J \pi; T M_T}$ as a sum of projected Gaussians, RGM⇒Generator-coordinate method (GCM)⇒ Kernels are matrix elements between Slater determinants (cf. Daniel Baye's talk)

References

Y. C. Tang, *Topics in Nuclear Physics II*, Lecture Notes in Physics, Vol. 145, Springer, Berlin, 1981, p. 571
H. Horiuchi, Prog. Theor. Phys. Suppl. 62, 90 (1977)
S. Quaglioni and P. Navrátil, Phys. Rev. C 79, 044606 (2009)

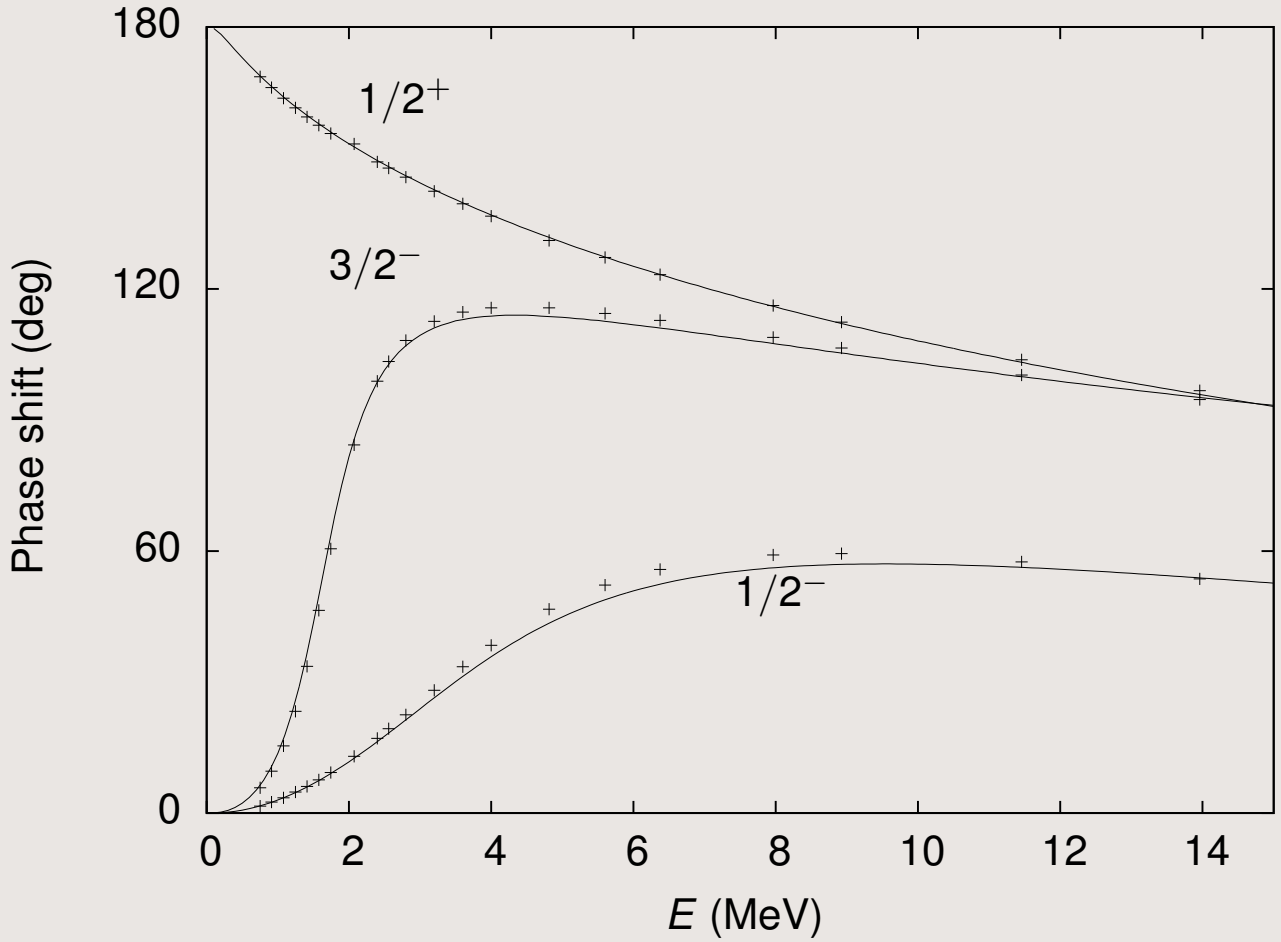
Microscopic *R*-matrix method (MRM)



[D. Baye, P.-H. Heenen, and M. Libert-Heinemann, Nucl. Phys. A 291 (1977) 230]

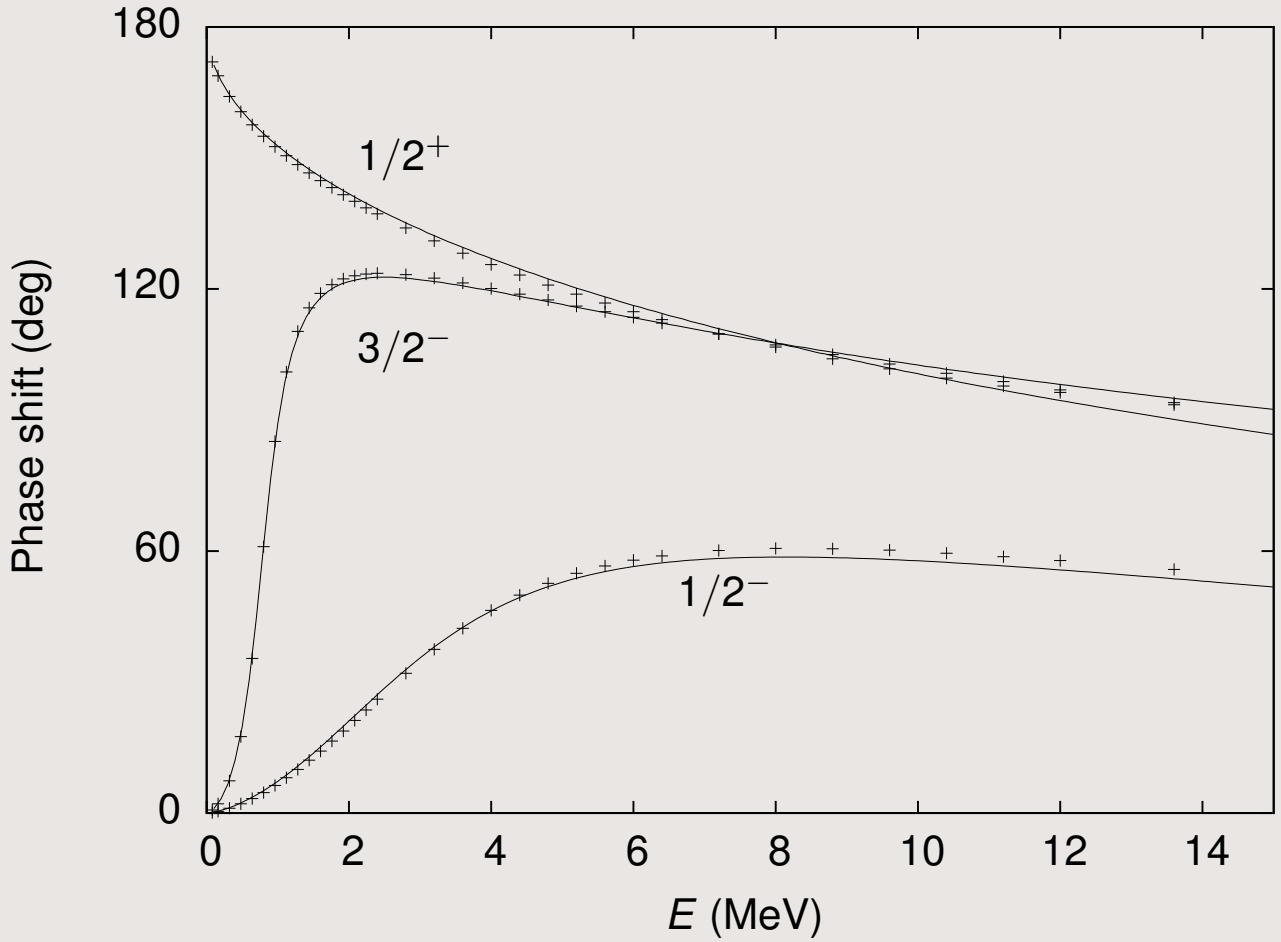
[P. Descouvemont and D. Baye, Rep. Prog. Phys. 73 (2010) 036301]

$\alpha + p$ phase shifts (effective potential)



Experimental data from [Satchler, Owen, Elwyn, Morgan and Walter, Nucl. Phys. A 112 (1968) 1]

$\alpha + n$ phase shifts (effective potential)



Experimental data from [Morgan and Walter, Phys. Rev. 168 (1968) 1114]

Bremsstrahlung cross sections

The calculation of bremsstrahlung cross sections is based on the matrix elements

$$\langle \Psi_f^-(\Omega_f) | \mathcal{M}^\sigma_{\lambda\mu} | \Psi_i^+ \rangle,$$

where

- Ψ_i^+ is the initial incoming state in the z direction
- $\Psi_f^-(\Omega_f)$ is the final outgoing state
- $\mathcal{M}^\sigma_{\lambda\mu}$ are the electromagnetic transition multipole operators, which are defined by

$$\mathcal{M}^\sigma_{\lambda\mu} = (-i)^\sigma \sqrt{\frac{\lambda}{\lambda+1}} \frac{(2\lambda+1)!!}{k_\gamma^\lambda c} \int \boldsymbol{J}(\boldsymbol{r}) \cdot \boldsymbol{A}^\sigma_{\lambda\mu}(\boldsymbol{r}) \mathrm{d}\boldsymbol{r},$$

  where $\boldsymbol{J}$ is the intrinsic nuclear current density and $\boldsymbol{A}^\sigma_{\lambda\mu}$ are the electromagnetic multipole operators.
- Since the electric transitions dominate at low photon energies, the magnetic transitions are not considered here.

Electric transitions

$$\mathcal{M}^{\mathrm{E}}_{\lambda\mu} = (-i)^{\sigma}\sqrt{\frac{\lambda}{\lambda+1}}\frac{(2\lambda+1)!!}{k_{\gamma}^{\lambda}c}\int \boldsymbol{J}(\boldsymbol{r})\cdot\boldsymbol{A}^{\mathrm{E}}_{\lambda\mu}(\boldsymbol{r})\mathrm{d}\boldsymbol{r},$$

- The nuclear current is caused by the motion of the nucleons and also by the motion of the mesons which are responsible for the nucleon-nucleon (NN) interaction

Difficulties

- $\boldsymbol{J}$ depends on the considered NN potential (More complex is the NN potential, more complex is the current)
- $\boldsymbol{J}$ is not defined unequivocally.

These difficulties can be bypassed at low-photon energies by using an extended Siegert theorem, which enables us to reduce the nuclear current dependence.

References

K.-M. Schmitt, P. Wilhelm, H. Arenhovel, A. Cambi, B. Mosconi, and P. Ricci, Phys. Rev. C41, 841 (1990).

JDE, D. Baye, Phys. Rev. C88 (2013) 024602.

Extended Siegert theorem

$$\mathcal{M}^{\mathrm{E}}_{\lambda\mu} = \sqrt{\frac{\lambda}{\lambda+1}}\frac{(2\lambda+1)!!}{k_\gamma^\lambda c}\int \boldsymbol{J}(\boldsymbol{r})\cdot\boldsymbol{A}^{\mathrm{E}}_{\lambda\mu}(\boldsymbol{r})\mathrm{d}\boldsymbol{r},$$

where

$$\boldsymbol{A}^{\mathrm{E}}_{\lambda\mu}(\boldsymbol{r}) = \frac{i}{k_\gamma\sqrt{\lambda(\lambda+1)}}\left(k_\gamma^2\boldsymbol{r}+\boldsymbol{\nabla}\frac{\partial}{\partial r}r\right)j_\lambda(k_\gamma r)Y_{\lambda\mu}(\Omega)$$

- The deletion of the current dependence of the electric transition at low photon energies relies on the fact that $\boldsymbol{A}^{\mathrm{E}}_{\lambda\mu}$ is reduced to a gradient term at the long-wavelength approximation,

$$\boldsymbol{A}^{\mathrm{E}}_{\lambda\mu}(\boldsymbol{r}) \underset{k_\gamma\to 0}{\longrightarrow} \frac{i\sqrt{\lambda+1}k_\gamma^{\lambda-1}}{\sqrt{\lambda}(2\lambda+1)!!}\boldsymbol{\nabla}r^\lambda Y^\mu_\lambda(\Omega).$$

- BUT the long-wavelength approximation cannot be done. Since the initial and final states are in the continuum, the wave functions are not square-integrable and the long-wavelength approximation leads to divergent integrals.

Extended Siegert theorem

- To reduce the current dependence without applying the long-wavelength approximation, the idea is to introduce an approximate electric transition multipole operator, denoted by $\widetilde{\mathcal{M}}^{\mathrm{E}}_{\lambda\mu}$, in which $\boldsymbol{A}^{\mathrm{E}}_{\lambda\mu}$ is approximate only by a gradient term

$$\widetilde{\mathcal{M}}^{\mathrm{E}}_{\lambda\mu} = \sqrt{\frac{\lambda}{\lambda+1}}\frac{(2\lambda+1)!!}{k_\gamma^\lambda c}\int \boldsymbol{J}(\boldsymbol{r})\cdot\boldsymbol{\nabla}\Phi_{\lambda\mu}(\boldsymbol{r})\mathrm{d}\boldsymbol{r},$$

  where

$$\frac{||\boldsymbol{\nabla}\Phi_{\lambda\mu}||}{||\boldsymbol{A}^{\mathrm{E}}_{\lambda\mu}||} \underset{k_\gamma\to 0}{\longrightarrow} 1.$$

Possible choice:

$$\Phi_{\lambda\mu}(\boldsymbol{r}) = \frac{i\sqrt{\lambda+1}}{k_\gamma\sqrt{\lambda}} j_\lambda(k_\gamma r) Y_{\lambda\mu}(\Omega).$$

After integrating by parts and by using the continuity equation

$$\boldsymbol{\nabla}\cdot\boldsymbol{J}(\boldsymbol{r}) + \frac{i}{\hbar}[H,\rho(\boldsymbol{r})] = 0,$$

where ρ is the charge density, the operator $\widetilde{\mathcal{M}}^{\mathrm{E}}_{\lambda\mu}$ can be written as

$$\widetilde{\mathcal{M}}^{\mathrm{E}}_{\lambda\mu} = i\sqrt{\frac{\lambda}{\lambda+1}}\frac{(2\lambda+1)!!}{k_\gamma^\lambda \hbar c}\int [H,\rho(\boldsymbol{r})]\Phi_{\lambda\mu}(\boldsymbol{r})\mathrm{d}\boldsymbol{r}.$$

Extended Siegert theorem

$$\widetilde{\mathcal{M}}^{\mathrm{E}}_{\lambda\mu} = i\sqrt{\frac{\lambda}{\lambda+1}}\frac{(2\lambda+1)!!}{k_\gamma^\lambda \hbar c}\int [H,\rho(\boldsymbol{r})]\Phi_{\lambda\mu}(\boldsymbol{r})\mathrm{d}\boldsymbol{r}.$$

In the calculation of the matrix elements between Ψ_f^- and Ψ_i^+, $\widetilde{\mathcal{M}}^{\mathrm{E}}_{\lambda\mu}$ is equivalent to

$$\widetilde{\mathcal{M}}^{\mathrm{E(S)}}_{\lambda\mu} = -i\sqrt{\frac{\lambda}{\lambda+1}}\frac{(2\lambda+1)!!}{k_\gamma^{\lambda-1}}\int \rho(\boldsymbol{r})\Phi_{\lambda\mu}(\boldsymbol{r})\mathrm{d}\boldsymbol{r}.$$

- $\widetilde{\mathcal{M}}^{\mathrm{E}}_{\lambda\mu}$ and $\widetilde{\mathcal{M}}^{\mathrm{E(S)}}_{\lambda\mu}$ lead to the same results if consistent current and charge densities and exact eigenstates of the Hamiltonian are used.
- $\widetilde{\mathcal{M}}^{\mathrm{E(S)}}_{\lambda\mu}$ depends on the charge density but not on the current density.

Extended Siegert theorem

Back to the "exact" electric transition multipole operator. It can be written as

$$\mathcal{M}^{\mathrm{E}}_{\lambda\mu} = \widetilde{\mathcal{M}}^{\mathrm{E}}_{\lambda\mu} + (\mathcal{M}^{\mathrm{E}}_{\lambda\mu} - \widetilde{\mathcal{M}}^{\mathrm{E}}_{\lambda\mu}).$$

By analogy, the Siegert form can be defined as

$$\mathcal{M}^{\mathrm{E(S)}}_{\lambda\mu} = \widetilde{\mathcal{M}}^{\mathrm{E(S)}}_{\lambda\mu} + (\mathcal{M}^{\mathrm{E}}_{\lambda\mu} - \widetilde{\mathcal{M}}^{\mathrm{E}}_{\lambda\mu}).$$

- $\mathcal{M}^{\mathrm{E}}_{\lambda\mu}$ and $\mathcal{M}^{\mathrm{E(S)}}_{\lambda\mu}$ lead exactly to the same results if consistent current and charge densities are considered and the exact eigenstates of the Hamiltonian are used.
- At low photon energies, the contribution of $\widetilde{\mathcal{M}}^{\mathrm{E(S)}}_{\lambda\mu}$ dominates
- The operator $\mathcal{M}^{\mathrm{E(S)}}_{\lambda\mu}$ should be preferred in microscopic calculations because
  1. it leads to easier calculations than $\mathcal{M}^{\mathrm{E}}_{\lambda\mu}$
  2. the charge density is better known than the current density
  3. to avoid derivatives of the wave functions which are known less accurately than the wave function itself

Charge and current densities

- Charge and current densities for free nucleons are considered

$$
\begin{aligned}
\rho(\boldsymbol{r}) &= e\sum_{j=1}^{A}(\frac{1}{2}-t_{j3})\delta(\boldsymbol{r}_j-\boldsymbol{R}_{\text{c.m.}}-\boldsymbol{r}),\\
\boldsymbol{J}(\boldsymbol{r}) &= \frac{e}{2m_N}\sum_{j=1}^{A}(\frac{1}{2}-t_{j3})\left[\boldsymbol{p}_j-A^{-1}\boldsymbol{P}_{\text{c.m.}},\delta(\boldsymbol{r}_j-\boldsymbol{R}_{\text{c.m.}}-\boldsymbol{r})\right]_+\\
&+ \frac{e}{2m_N}\sum_{j=1}^{A}g_{sj}\boldsymbol{\nabla}\times\delta(\boldsymbol{r}_j-\boldsymbol{R}_{\text{c.m.}}-\boldsymbol{r})\boldsymbol{s}_j,
\end{aligned}
$$

where $[\boldsymbol{a},\boldsymbol{b}]_+$ is a shorthand notation for $\boldsymbol{a}\cdot\boldsymbol{b}+\boldsymbol{b}\cdot\boldsymbol{a}$ and g_{sj} is the gyromagnetic factor.

Electric transition multipole operators

- The non-Siegert electric transition multipole operators are given explicitly by

$$\mathcal{M}^{\mathrm{E}}_{\lambda\mu} = \frac{ie(2\lambda+1)!!}{m_N c(\lambda+1)k_\gamma^{\lambda+1}} \sum_{j=1}^{A} \Bigg[\left(\frac{1}{2} - t_{j3}\right) \boldsymbol{\chi}_{\lambda\mu}(k_\gamma, \boldsymbol{r}) \cdot \left(\boldsymbol{p}_j - A^{-1}\boldsymbol{P}_{\mathrm{c.m.}}\right) - \frac{1}{2} k_\gamma^2 g_{sj} (\boldsymbol{r} \times \boldsymbol{\nabla}) \phi_{\lambda\mu}(k_\gamma \boldsymbol{r}) \cdot \boldsymbol{s}_j \Bigg]_{\boldsymbol{r}=\boldsymbol{r}_j - \boldsymbol{R}_{\mathrm{c.m.}}},$$

where

$$\boldsymbol{\chi}_{\lambda\mu}(k_\gamma, \boldsymbol{r}) = \left(k_\gamma^2 \boldsymbol{r} + \boldsymbol{\nabla}\frac{\partial}{\partial r} r\right) \phi_{\lambda\mu}(k_\gamma \boldsymbol{r}),$$

$$\phi_{\lambda\mu}(k_\gamma \boldsymbol{r}) = j_\lambda(k_\gamma r) Y_{\lambda\mu}(\Omega).$$

Electric transition multipole operators

- The Siegert electric transition multipole operators are given explicitly by

$$\mathcal{M}^{\mathrm{E(S)}}_{\lambda\mu} = \frac{e(2\lambda+1)!!}{k_\gamma^\lambda} \sum_{j=1}^{A} \left(\frac{1}{2} - t_{j3}\right) \phi_{\lambda\mu}\left[k_\gamma\left(\boldsymbol{r}_j - \boldsymbol{R}_{\mathrm{c.m.}}\right)\right]$$

$$+ \frac{ie(2\lambda+1)!!}{2m_N c(\lambda+1)k_\gamma^{\lambda+1}} \sum_{j=1}^{A} \Bigg\{ \left(\frac{1}{2} - t_{j3}\right)$$

$$\left[\boldsymbol{\chi}_{\lambda\mu}(k_\gamma, \boldsymbol{r}) - (\lambda+1)\boldsymbol{\nabla}\phi_{\lambda\mu}(k_\gamma \boldsymbol{r}), \boldsymbol{p}_j - A^{-1}\boldsymbol{P}_{\mathrm{c.m.}}\right]_+$$

$$- k_\gamma^2 g_{sj}(\boldsymbol{r} \times \boldsymbol{\nabla})\phi_{\lambda\mu}(k_\gamma \boldsymbol{r}) \cdot \boldsymbol{s}_j \Bigg\}_{\boldsymbol{r}=\boldsymbol{r}_j - \boldsymbol{R}_{\mathrm{c.m.}}}.$$

Calculation of the matrix elements

The reduced matrix element of $\mathcal{M}_\lambda$ ($\mathcal{M}^{\mathrm{E}}_\lambda$ or $\mathcal{M}^{\mathrm{E(S)}}_\lambda$) is approximated with a good accuracy by

$$\begin{aligned}\langle \Psi^{J_f\pi_f;T_f}||\mathcal{M}_\lambda||\Psi^{J_i\pi_i;T_i}\rangle &= \langle \Psi^{J_f\pi_f;T_f}_{\mathrm{int}}||\mathcal{M}_\lambda||\Psi^{J_i\pi_i;T_i}_{\mathrm{int}}\rangle_{\mathrm{int}} + \langle \Psi^{J_f\pi_f;T_f}_{\mathrm{ext}}||\mathcal{M}_\lambda||\Psi^{J_i\pi_i;T_i}_{\mathrm{ext}}\rangle_{\mathrm{ext}} \\ &\approx \langle \Psi^{J_f\pi_f;T_f}_{\mathrm{int}}||\widehat{\mathcal{M}}_\lambda||\Psi^{J_i\pi_i;T_i}_{\mathrm{int}}\rangle_{\mathrm{int}} + \langle \Psi^{J_f\pi_f;T_f}_{\mathrm{ext}}||\mathcal{M}^{as}_\lambda||\Psi^{J_i\pi_i;T_i}_{\mathrm{ext}}\rangle_{\mathrm{ext}}\end{aligned}$$

where $\widehat{\mathcal{M}}_\lambda$ is the long-wavelength approximation of $\mathcal{M}_\lambda$ and the superscript as designates the asymptotic form of the multipole operator.

- Over the external region, the antisymmetrization between clusters can be neglected.
- The long-wavelength approximation can be done for the matrix elements between the internal parts of the wave functions because they are square-integrable

$$\widehat{\mathcal{M}}^{\mathrm{E}}_{\lambda\mu} = \frac{ie}{m_N c k_\gamma}\sum_{j=1}^{A}\left(\frac{1}{2}-t_{j3}\right)\left[\boldsymbol{\nabla} r^\lambda Y^\mu_\lambda(\Omega)\right]_{\boldsymbol{r}_j-\boldsymbol{R}_{\mathrm{c.m.}}}\cdot\left(\boldsymbol{p}_j - A^{-1}\boldsymbol{P}_{\mathrm{c.m.}}\right)$$

$$\widehat{\mathcal{M}}^{\mathrm{E(S)}}_{\lambda\mu} = e\sum_{j=1}^{A}\left(\frac{1}{2}-t_{j3}\right)\left[r^\lambda Y^\mu_\lambda(\Omega)\right]_{\boldsymbol{r}_j-\boldsymbol{R}_{\mathrm{c.m.}}}$$

Eλ operator in the RGM/GCM

- In the GCM, the calculation of the matrix elements of the electric multipole can be done by using the single-nucleon coordinates
- The evaluation of the Eλ matrix elements requires the calculation of matrix elements of the one-body operator

$$\widehat{\mathcal{M}}^{\mathrm{E}}_{\lambda\mu} = \frac{ie}{m_N c k_\gamma} \sum_{j=1}^{A} \left(\frac{1}{2} - t_{j3}\right) \left[\boldsymbol{\nabla} r_j^\lambda Y_\lambda^\mu(\Omega_j)\right] \cdot \boldsymbol{p}_j$$

in the non-Siegert approach and

$$\widehat{\mathcal{M}}^{\mathrm{E(S)}}_{\lambda\mu} = e \sum_{j=1}^{A} \left(\frac{1}{2} - t_{j3}\right) \left[r_j^\lambda Y_\lambda^\mu(\Omega_j)\right]$$

in the Siegert approach evaluated between Slater determinants.

- $\Rightarrow$ systematic calculation

References

D. Baye and P. Descouvemont, Nucl. Phys. A 407 (1983) 77
D. Baye and P. Descouvemont, Nucl. Phys. A 443 (1985) 302
JDE, D. Baye, Phys. Rev. C88 (2013) 024602.

E1 operator in the NCSM/RGM

E1 operator in the Siegert approach can be written in vector notation as

$$
\begin{aligned}
\boldsymbol{E1} &= e\sum_{j=1}^{A}\left(\frac{1}{2}-t_{j3}\right)\left(\boldsymbol{r}_j-\boldsymbol{R}_{\text{c.m.}}\right) \\
&= e\sum_{j=1}^{A_1}\left(\frac{1}{2}-t_{j3}\right)\left(\boldsymbol{r}_j-\boldsymbol{R}^{(1)}_{\text{c.m.}}\right)+e\sum_{j=A_1+1}^{A}\left(\frac{1}{2}-t_{j3}\right)\left(\boldsymbol{r}_j-\boldsymbol{R}^{(2)}_{\text{c.m.}}\right)+eZ^{(1)}_{\text{eff}}\boldsymbol{\rho} \\
&= \boldsymbol{E1}(1)+\boldsymbol{E1}(2)+eZ^{(1)}_{\text{eff}}\boldsymbol{\rho}
\end{aligned}
$$

where

$$
Z^{(\lambda)}_{\text{eff}} = Z_1\left(\frac{A_2}{A}\right)^{\lambda}+Z_2\left(\frac{-A_1}{A}\right)^{\lambda}
$$

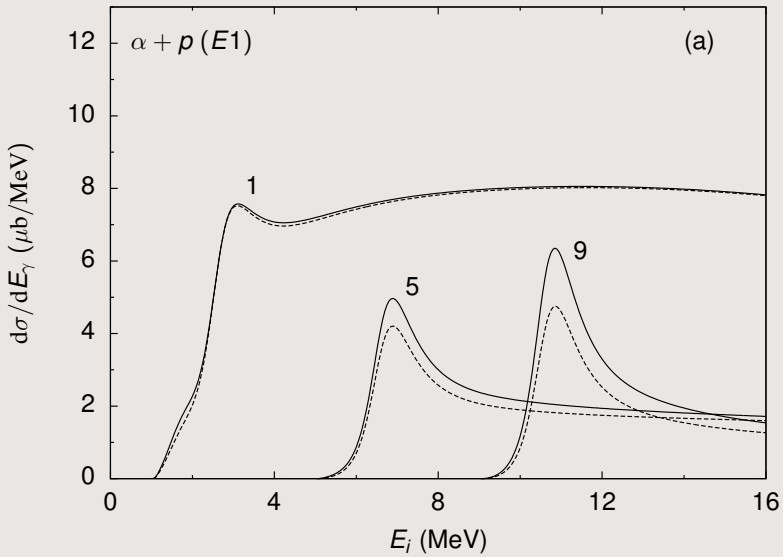
E1 operator in the NCSM/RGM

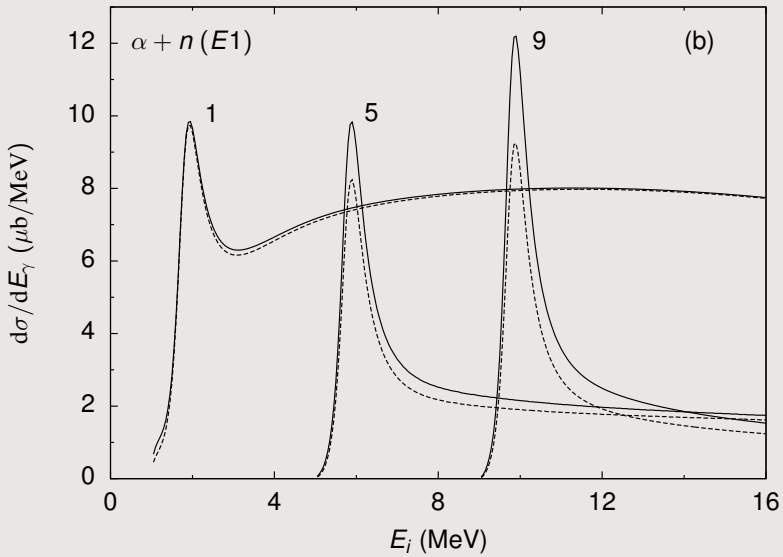
For $\alpha + p$,

$$\begin{aligned} \boldsymbol{E1} &= \boldsymbol{E1}(1) + \boldsymbol{E1}(2) + eZ^{(1)}_{\text{eff}}\boldsymbol{\rho} \\ &= -e\sum_{i=1}^{A-1} \boldsymbol{r}_i t_{i3} + eZ^{(1)}_{\text{eff}}\boldsymbol{\rho} \end{aligned}$$

- matrix element of $eZ^{(1)}_{\text{eff}}\boldsymbol{\rho}$ calculated from the norm kernels (already required for the scattering calculation)
- matrix element of $-e\sum_{i=1}^{A-1} \boldsymbol{r}_i t_{i3}$ calculated from the one-body and two-body density matrix elements of the α (already required for the scattering calculation)
- Required some method for eliminating the center-of-mass motion for an operator which modifies parity, angular momentum, and isospin.
- Required to apply the SRG transform to obtain very accurate results (cf. Sofia Quaglioni's talk).

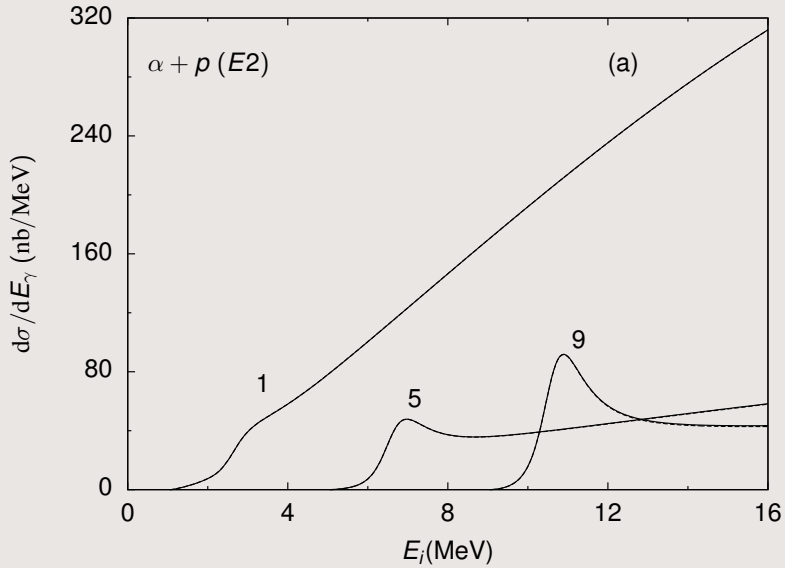
$\alpha + N$ bremsstrahlung

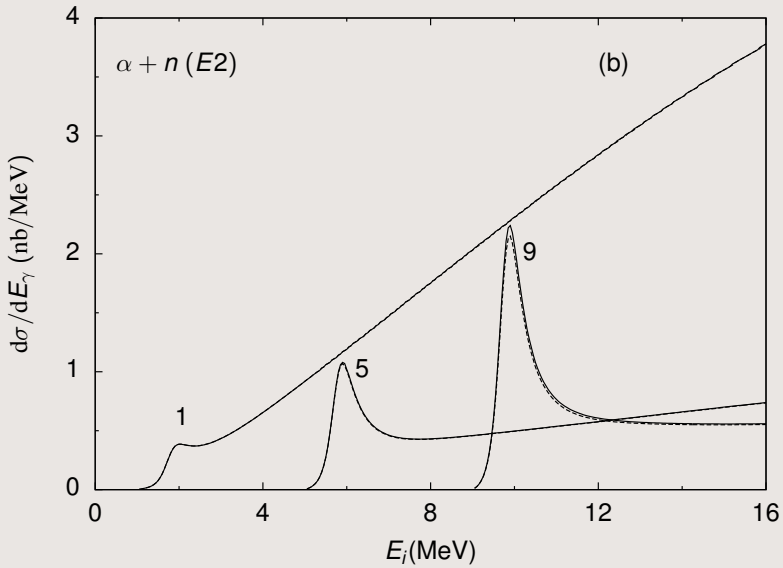




JDE, Phys. Rev. C, *in press*

$\alpha + N$ bremsstrahlung





JDE, Phys. Rev. C, *in press*

Effective charge

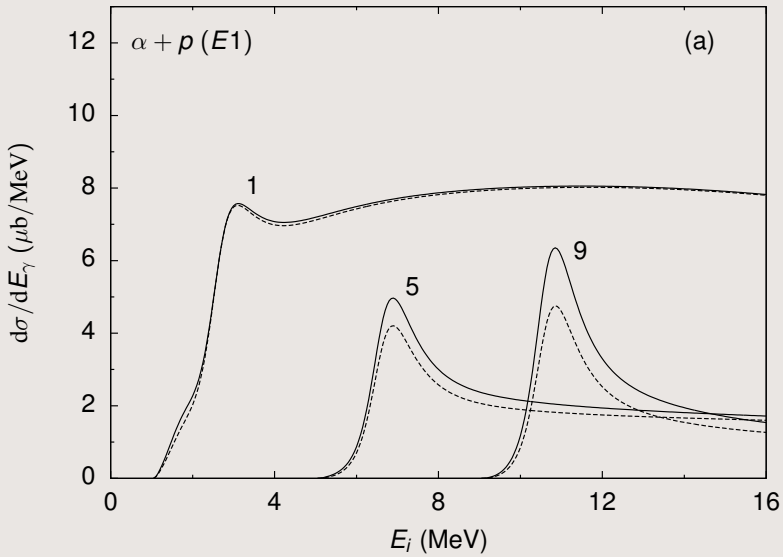
- The ratio of the orders of magnitude of the electric transition contributions can be explained by comparing the effective charges defined by

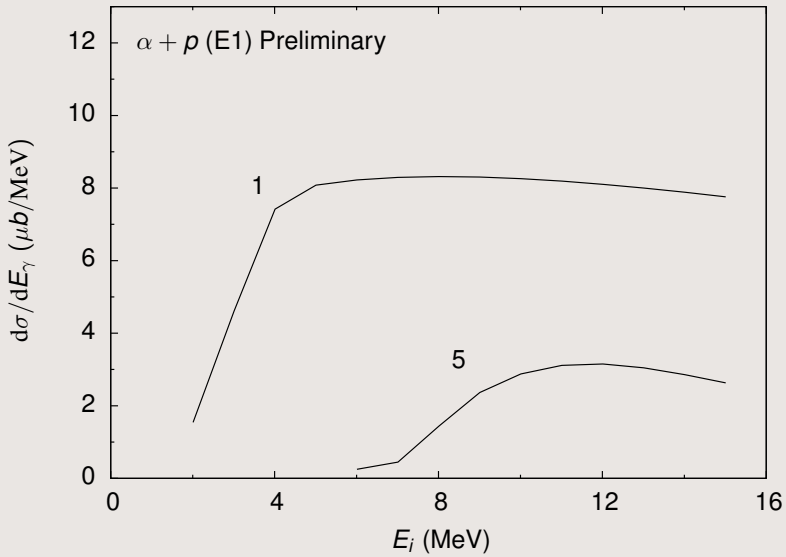
$$Z_{\mathrm{eff}}^{(\lambda)} = Z_1 \left(\frac{A_2}{A}\right)^{\lambda} + Z_2 \left(\frac{-A_1}{A}\right)^{\lambda}$$

- In first approximation, the ratio between the contributions of a given electric transition for the $\alpha + p$ and $\alpha + n$ bremsstrahlung cross sections is given by the square of the ratio between the effective charges

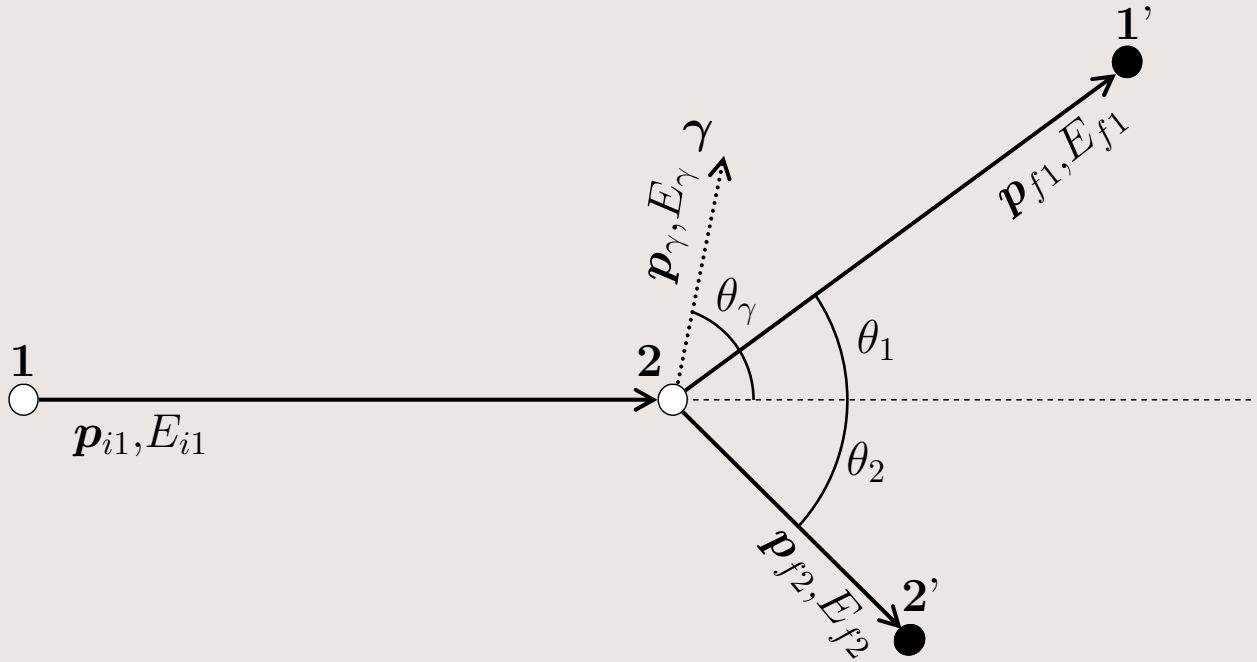
$$\frac{\mathrm{d}\sigma(\alpha p, \mathrm{E}\lambda)}{\mathrm{d}\sigma(\alpha n, \mathrm{E}\lambda)} \approx \left(\frac{Z_{\mathrm{eff},\alpha p}^{(\lambda)}}{Z_{\mathrm{eff},\alpha n}^{(\lambda)}}\right)^2$$

NCSM/RGM


$\alpha + p$ ($E1$)
(a)
1
5
9
$d\sigma/dE_\gamma$ (μb/MeV)
E_i (MeV)
0
2
4
6
8
10
12
0
4
8
12
16

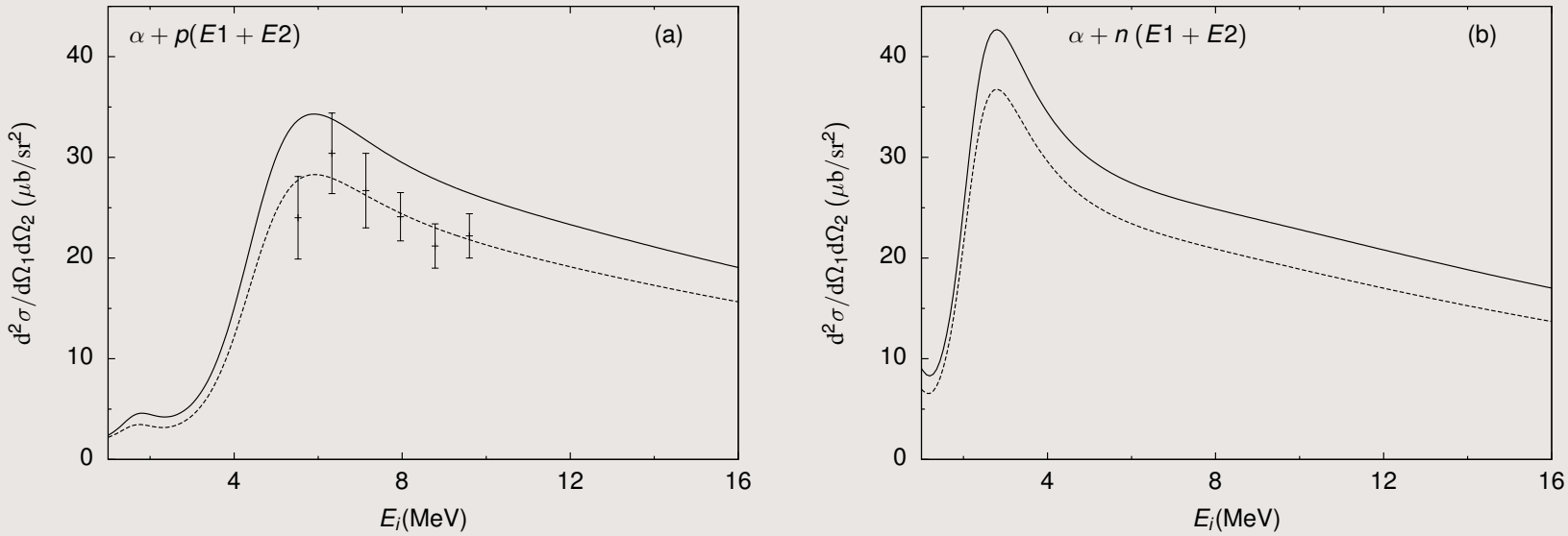

$\alpha + p$ (E1) Preliminary
1
5
$d\sigma/dE_\gamma$ (μb/MeV)
E_i (MeV)
0
2
4
6
8
10
12
0
4
8
12
16

Coplanar configuration



In the so-called Harvard geometry, the photon is undetected.

$\alpha(N, \alpha N)\gamma$ bremsstrahlung



Experimental data from [W. Wölfli, J. Hall, and R. Müller, Phys. Rev. Lett. 27 (1971) 271]

Summary

- A microscopic approach of the nucleus-nucleus bremsstrahlung is presented, which is based on elastic wave functions deduced from an internucleon interaction.
- The microscopic model based on an effective NN interaction (the Minnesota potential) reproduces rather well the experimental bremsstrahlung cross sections for $\alpha + \mathrm{N}$.
- A Siegert approach of bremsstrahlung is developed, which takes partially the meson-exchange currents into account.
- The Siegert operator leads to less complicated calculations than the non-Siegert one, which makes easier the development of *ab initio* bremsstrahlung models (*in progress*).

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et en physique des particules

Thank you!
Merci

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