

New Directions in Valence-Space Methods for Exotic sd-Shell Nuclei

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References

- Otsuka, Suzuki, JDH, Schwenk, Akaishi PRL **105**, 032501 (2010)
- JDH, Menendez, Schwenk, EPJA **49**, 39 (2013)
- Caesar et al. (R3B), Simonis, JDH, Menendez, Schwenk PRC **88**, 034313 (2013)
- Bogner, Hergert, JDH, Schwenk, Binder, Calci, Langhammer, Roth, arXiv:1402.1407



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für Bildung
und Forschung



Drip Lines and Magic Numbers: The Nuclear Landscape Toward the Extremes

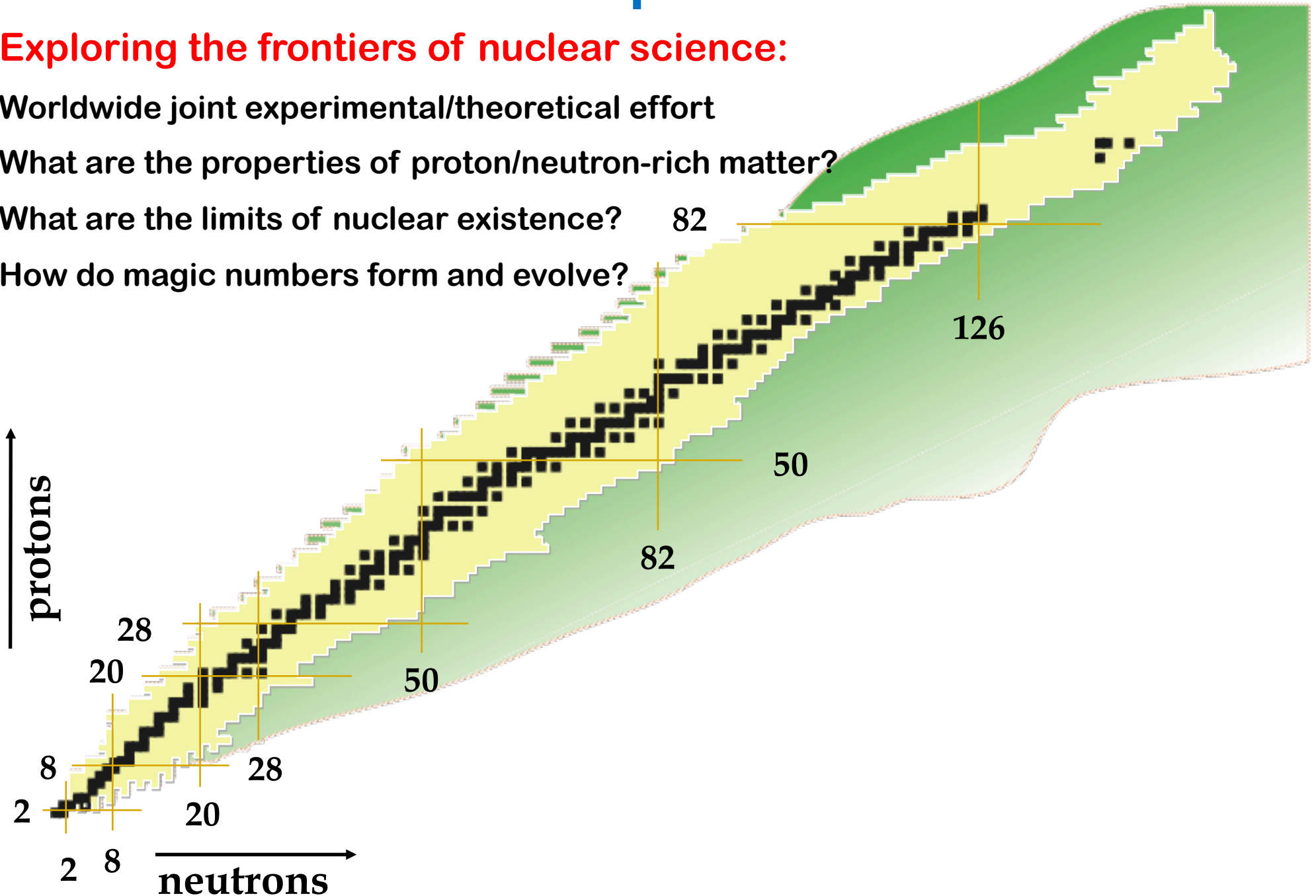
Exploring the frontiers of nuclear science:

Worldwide joint experimental/theoretical effort

What are the properties of proton/neutron-rich matter?

What are the limits of nuclear existence?

How do magic numbers form and evolve?



Drip Lines and Magic Numbers: The Nuclear Landscape Toward the Extremes

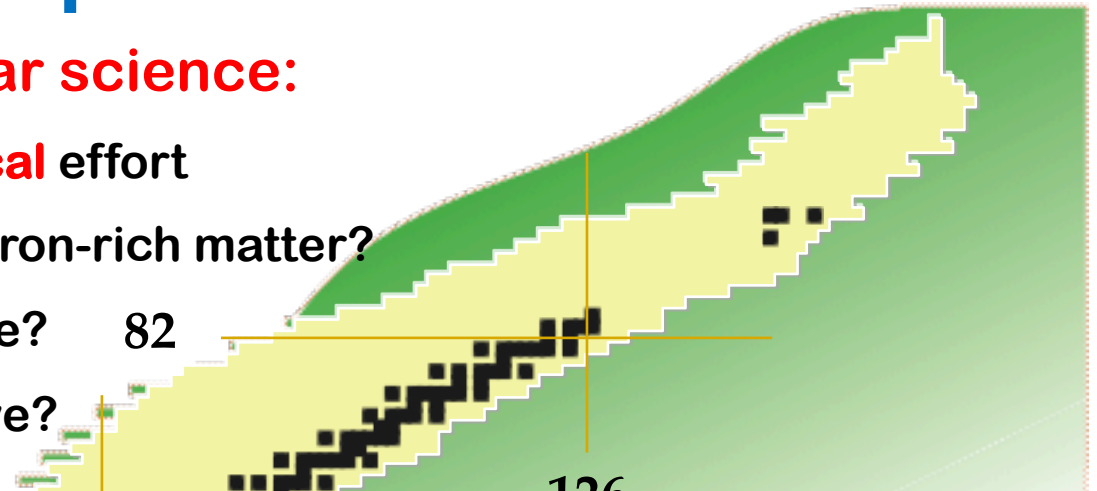
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What are the properties of proton/neutron-rich matter?

What are the limits of nuclear existence? 82

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Advances in many-body methods

Green's Function Monte Carlo

Hyperspherical Harmonics

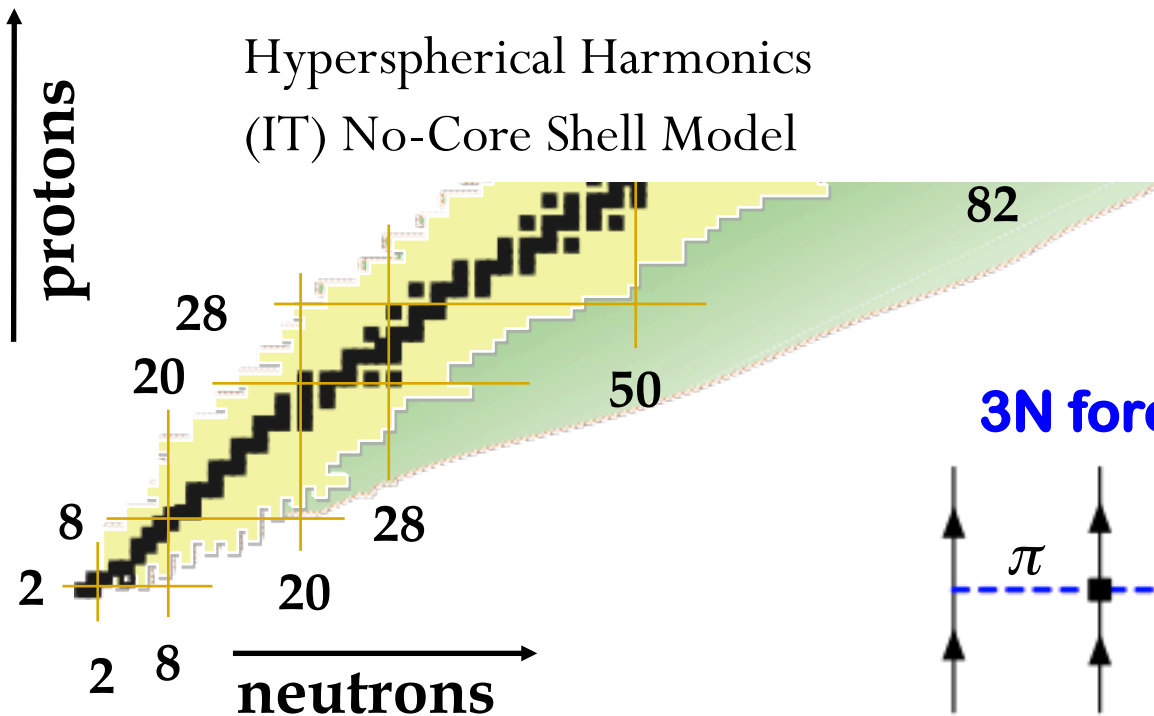
(IT) No-Core Shell Model

Coupled Cluster

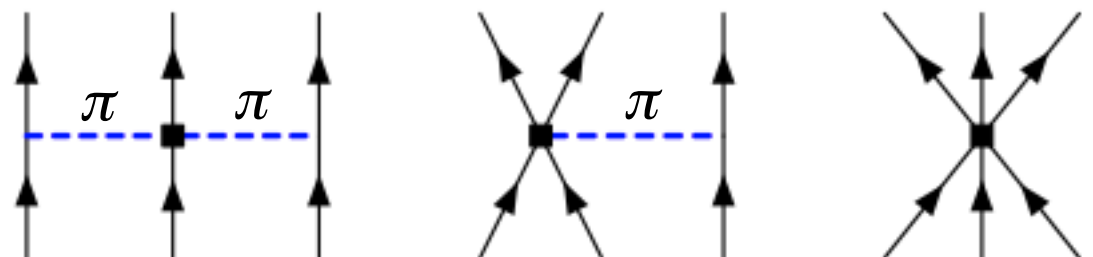
In-Medium SRG

Many-Body Perturbation Theory

Self-Consistent Green's Function



3N forces essential for exotic nuclei



Drip Lines and Magic Numbers: 3N Forces in Medium-Mass Nuclei

Exploring the frontiers of nuclear science:

Worldwide joint experimental/theoretical effort

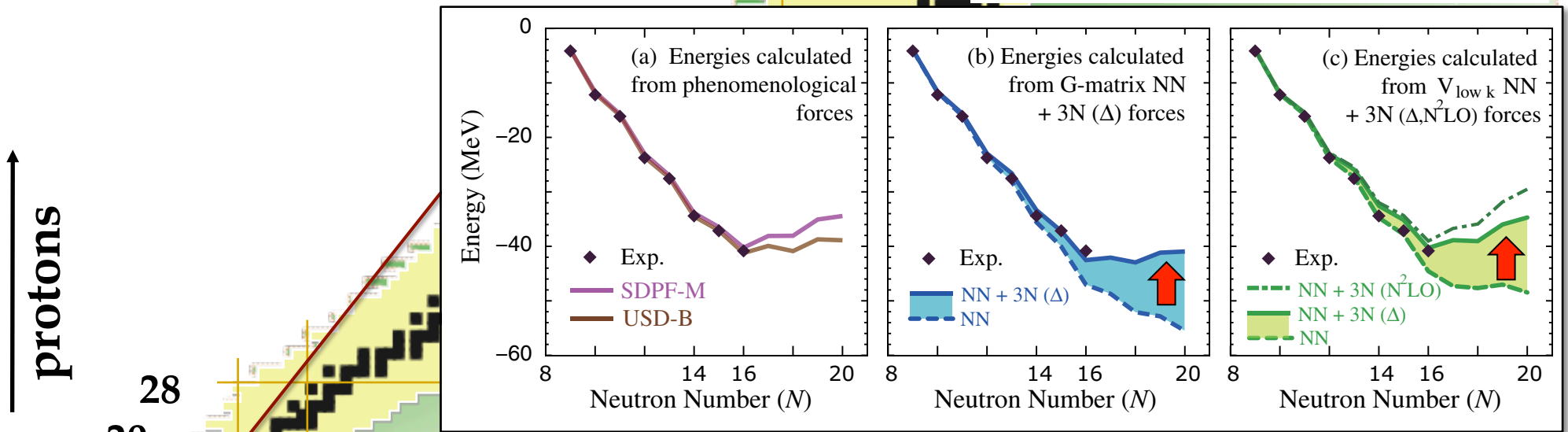
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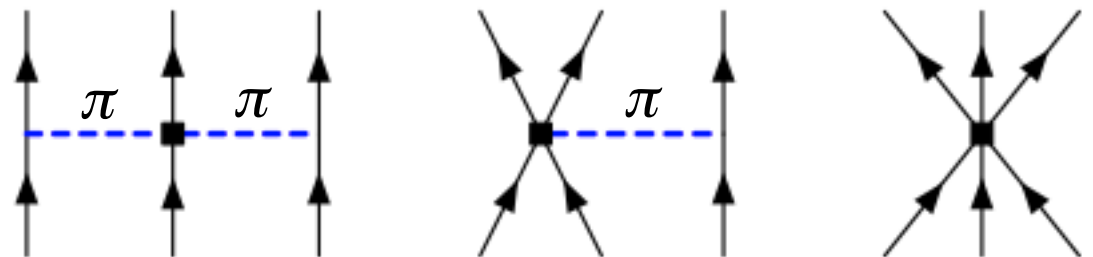
How do magic numbers form and evolve?

82

Heaviest oxygen isotope



Otsuka, Suzuki, JDH, Schwenk, Akaishi, PRL (2010)



Drip Lines and Magic Numbers: 3N Forces in Medium-Mass Nuclei

Exploring the frontiers of nuclear science:

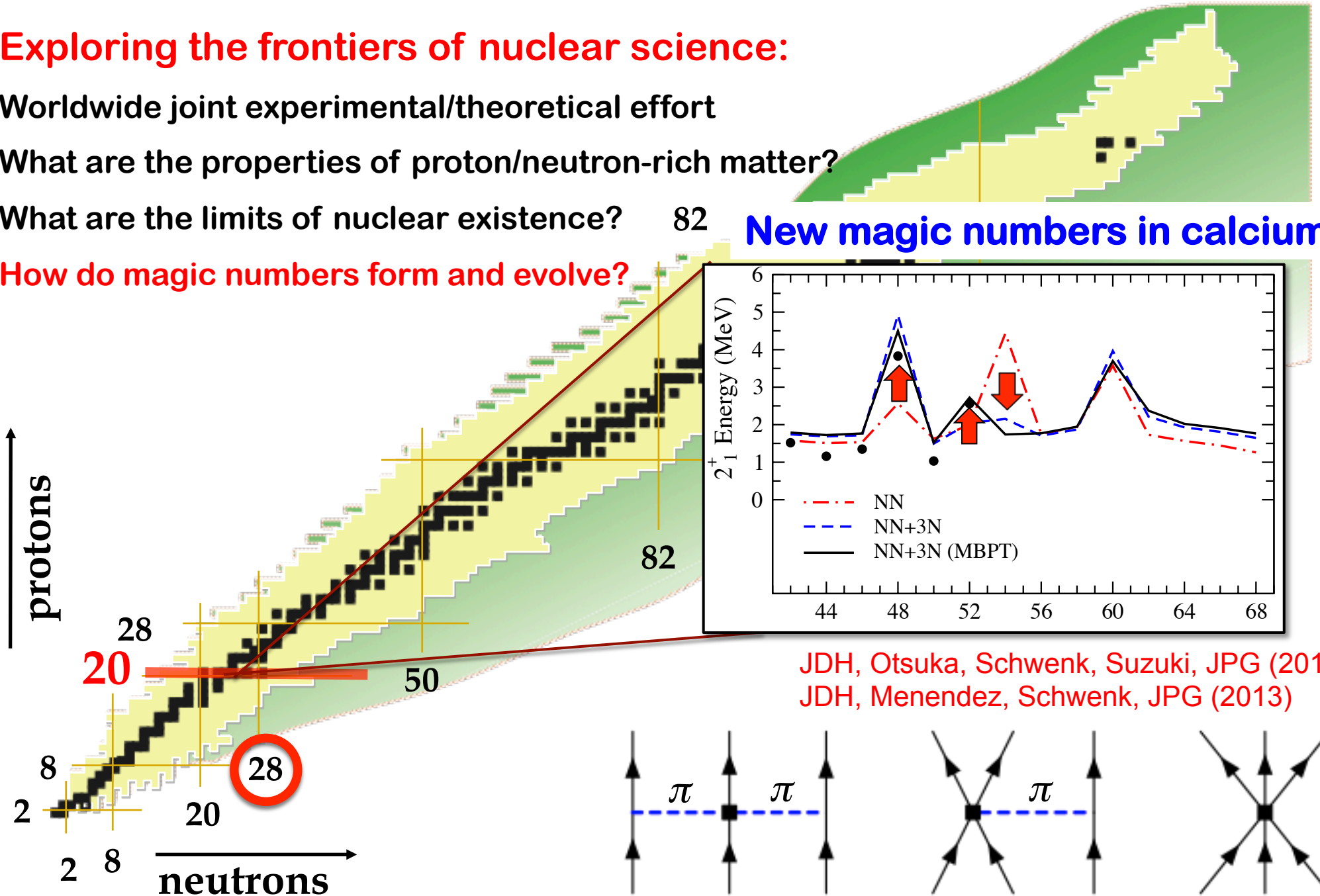
Worldwide joint experimental/theoretical effort

What are the properties of proton/neutron-rich matter?

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How do magic numbers form and evolve?

New magic numbers in calcium



JDH, Otsuka, Schwenk, Suzuki, JPG (2012)

JDH, Menendez, Schwenk, JPG (2013)

Oxygen Isotopes

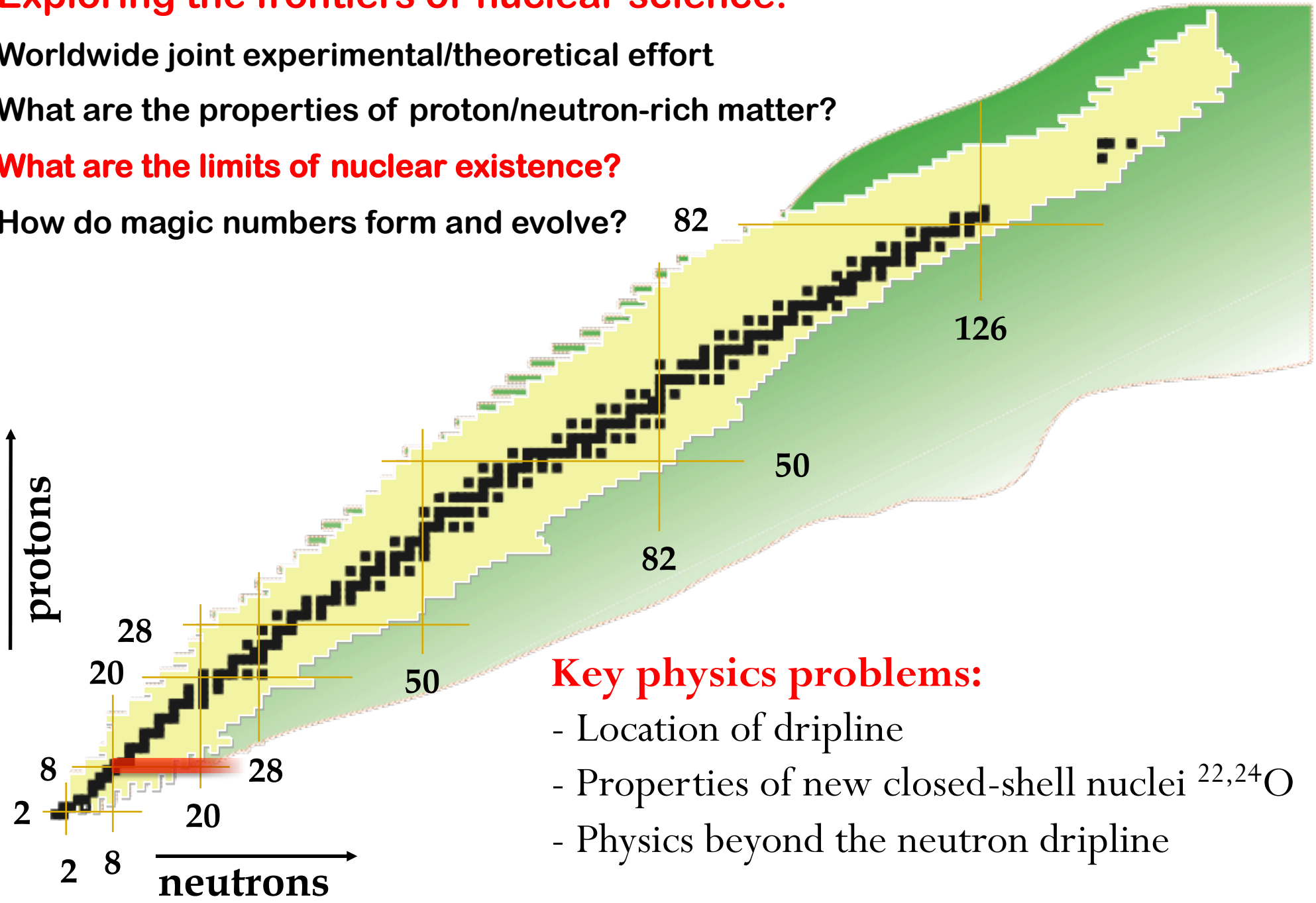
Exploring the frontiers of nuclear science:

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What are the properties of proton/neutron-rich matter?

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Key physics problems:

- Location of dripline
- Properties of new closed-shell nuclei $^{22,24}\text{O}$
- Physics beyond the neutron dripline

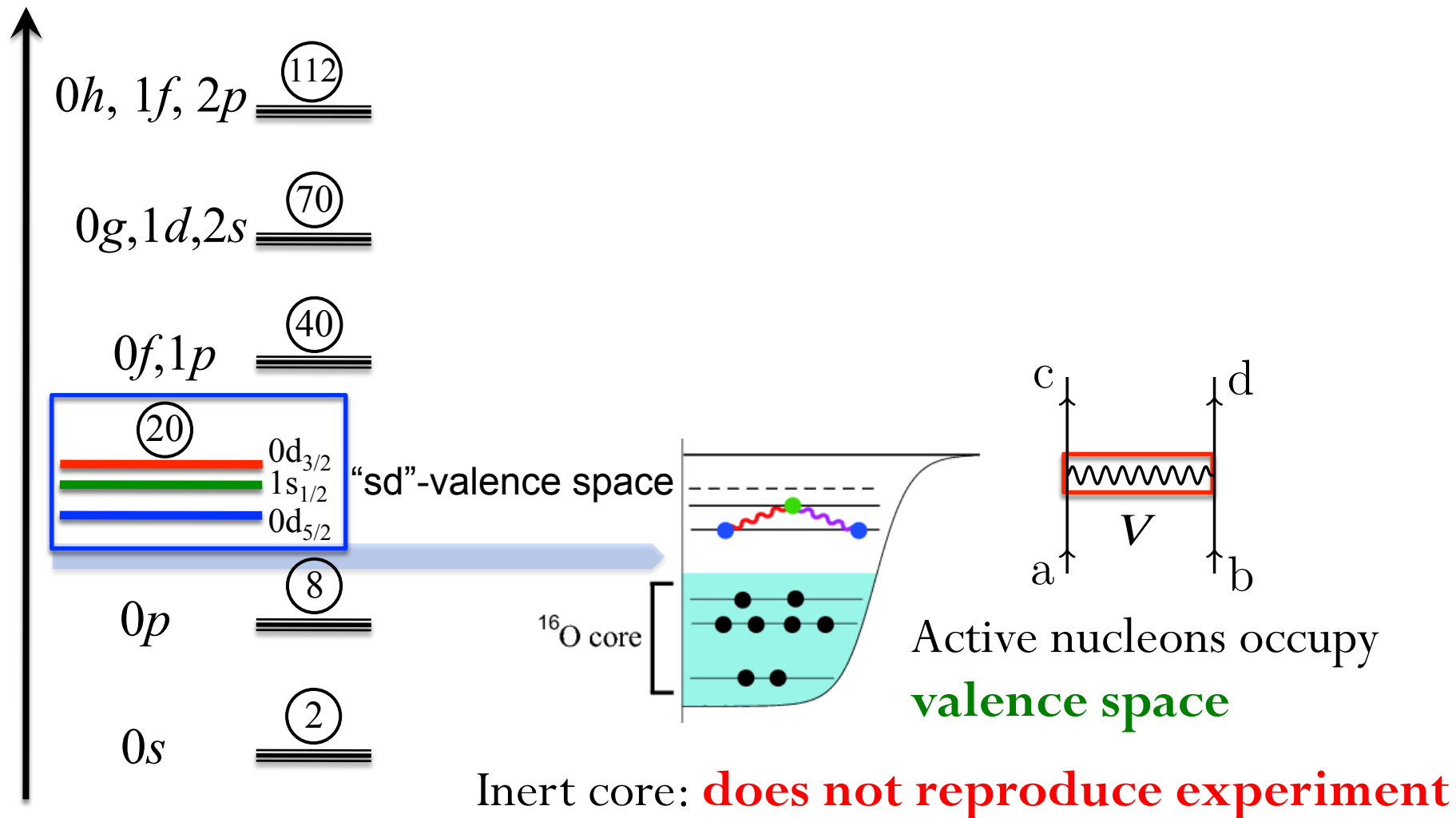
The Nuclear Many-Body Problem

Nuclei understood as many-body system starting from closed shell, add nucleons

Calculate **valence-space** Hamiltonian inputs from nuclear forces

Interaction matrix elements

Single-particle energies (SPEs)



The Nuclear Many-Body Problem

Nuclei understood as many-body system starting from closed shell, add nucleons

Calculate **valence-space** Hamiltonian inputs from nuclear forces

Interaction matrix elements

Single-particle energies (SPEs)

$0h, 1f, 2p$ (112)

Solution: allow breaking of core

$0g, 1d, 2s$ (70)

Effective Hamiltonian for valence nucleons

$0f, 1p$ (40)

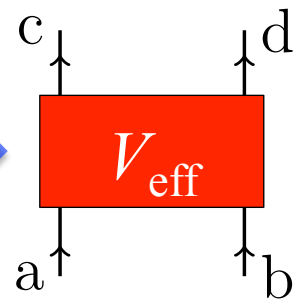
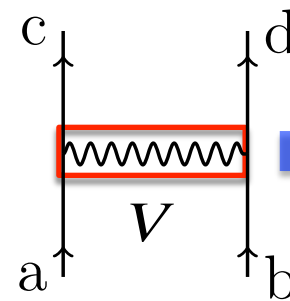
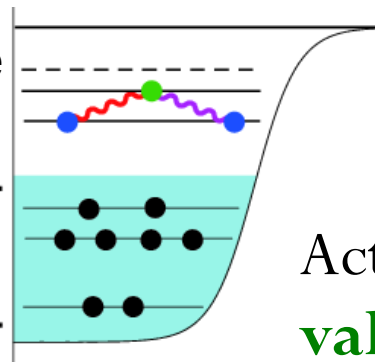
(20)
 $0d_{3/2}$
 $1s_{1/2}$
 $0d_{5/2}$

"sd"-valence space

$0p$ (8)

^{16}O core

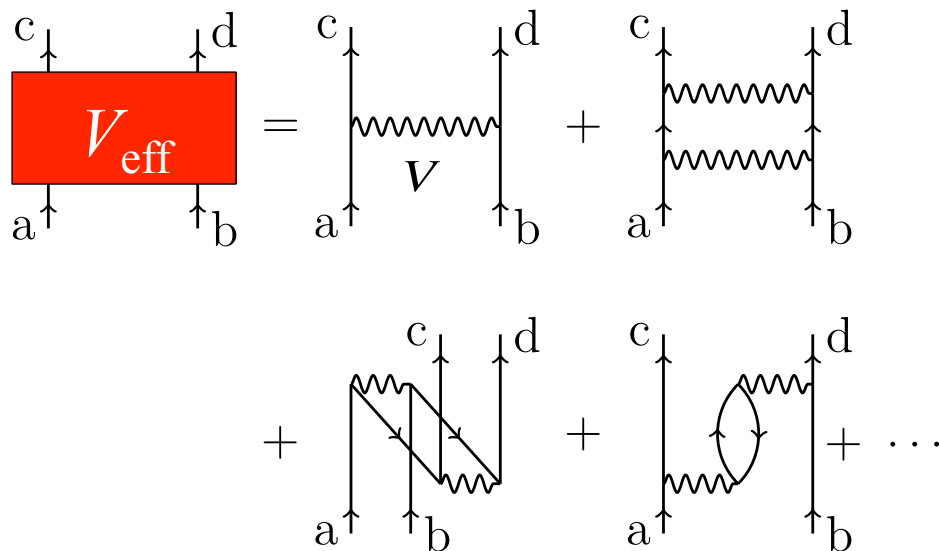
$0s$ (2)



Active nucleons occupy **valence space**

Valence-Space Strategy

- ★ 1) Effective interaction: sum excitations outside valence space to **3rd order**
- ★ 2) Single-particle energies calculated self consistently
- ★ 3) Harmonic-oscillator basis of 13 major shells: **converged**
- 4) NN and 3N forces from chiral EFT – to 3rd-order MBPT
- 5) Explore extended valence spaces



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NN matrix elements

- Chiral N³LO (Machleidt, $\Lambda_{\text{NN}} = 500\text{MeV}$); smooth-regulator $V_{\text{low } k}(\Lambda)$

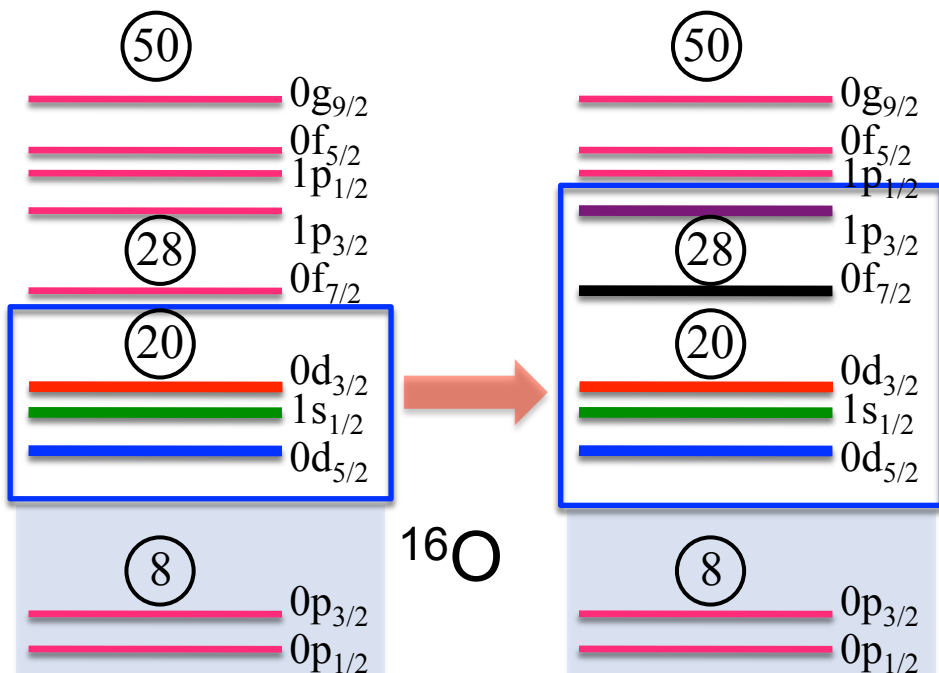
3N force contributions

- Chiral N²LO
 - $c_{\text{D}}, c_{\text{E}}$ fit to properties of light nuclei with $V_{\text{low } k}(\Lambda = \Lambda_{3\text{N}} = 2.0\text{fm}^{-1})$
- Included to 5 major HO shells

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- ★ 5) Explore **extended valence spaces**

Philosophy: diagonalize in largest possible valence space (where orbits relevant)

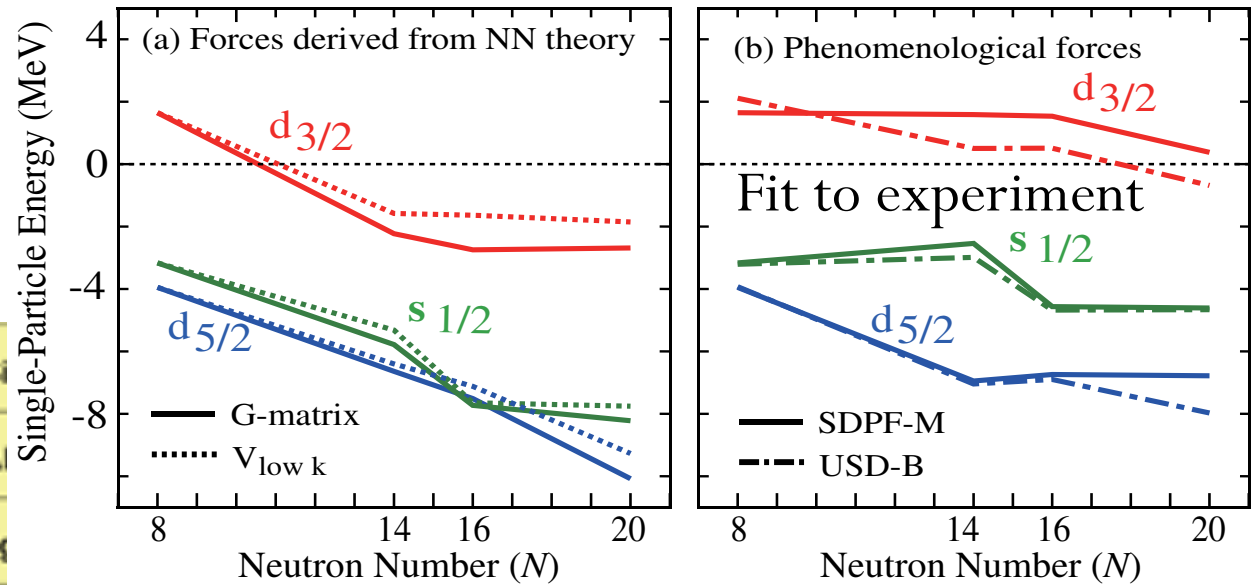
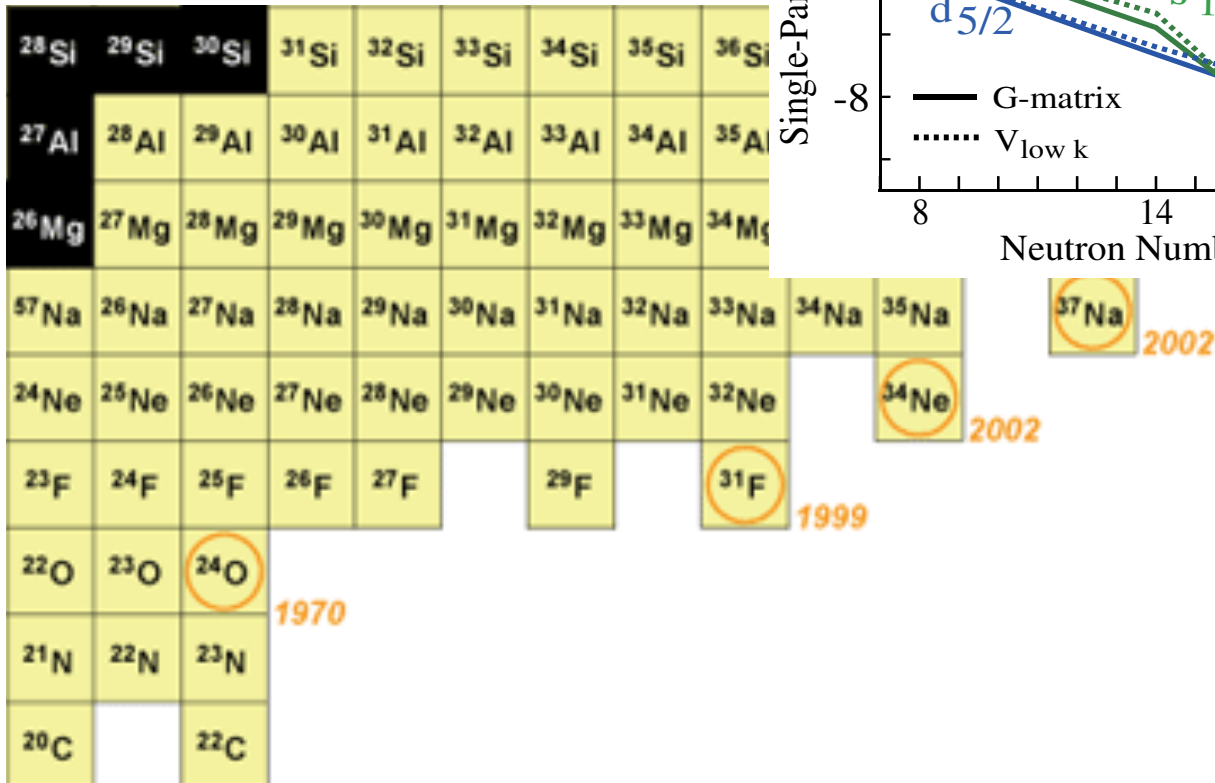


Treats higher orbits nonperturbatively
When important for exotic nuclei?

Limits of Nuclear Existence: Oxygen Anomaly

Microscopic picture:

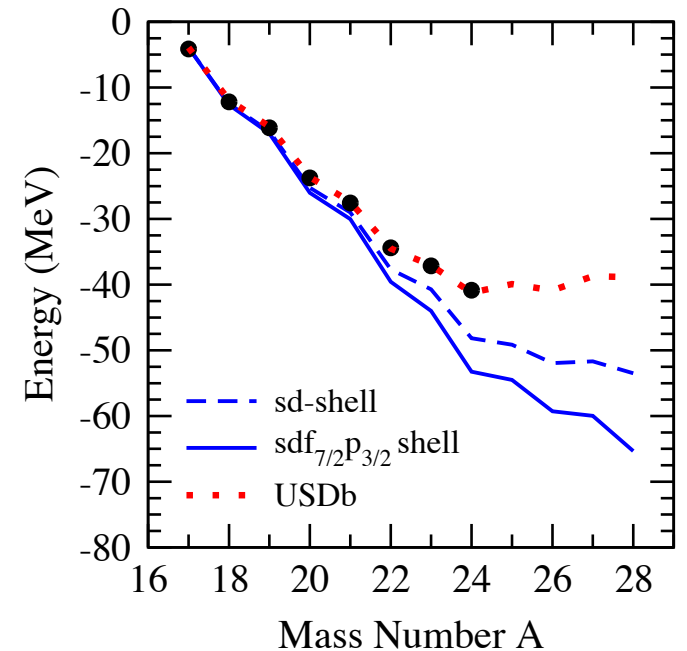
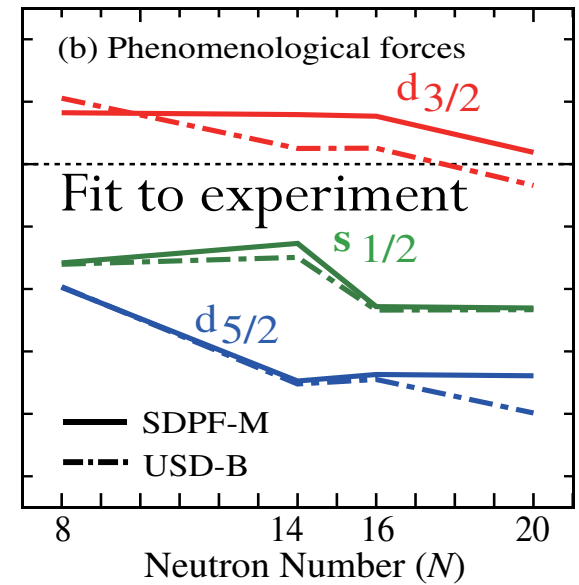
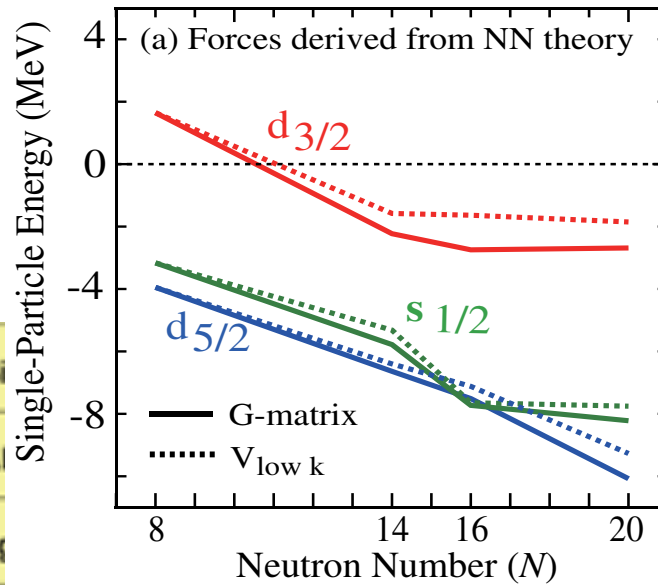
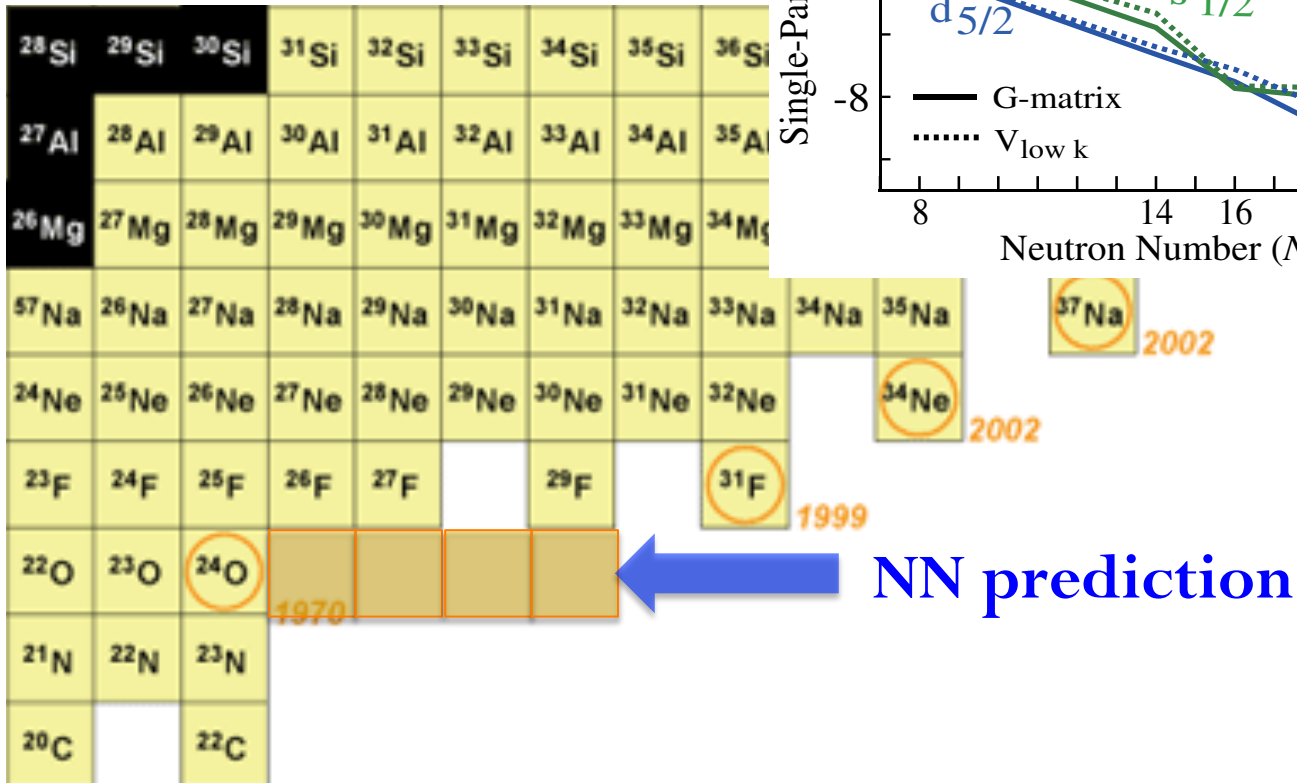
NN-forces too attractive



Limits of Nuclear Existence: Oxygen Anomaly

Microscopic picture:

NN-forces too attractive

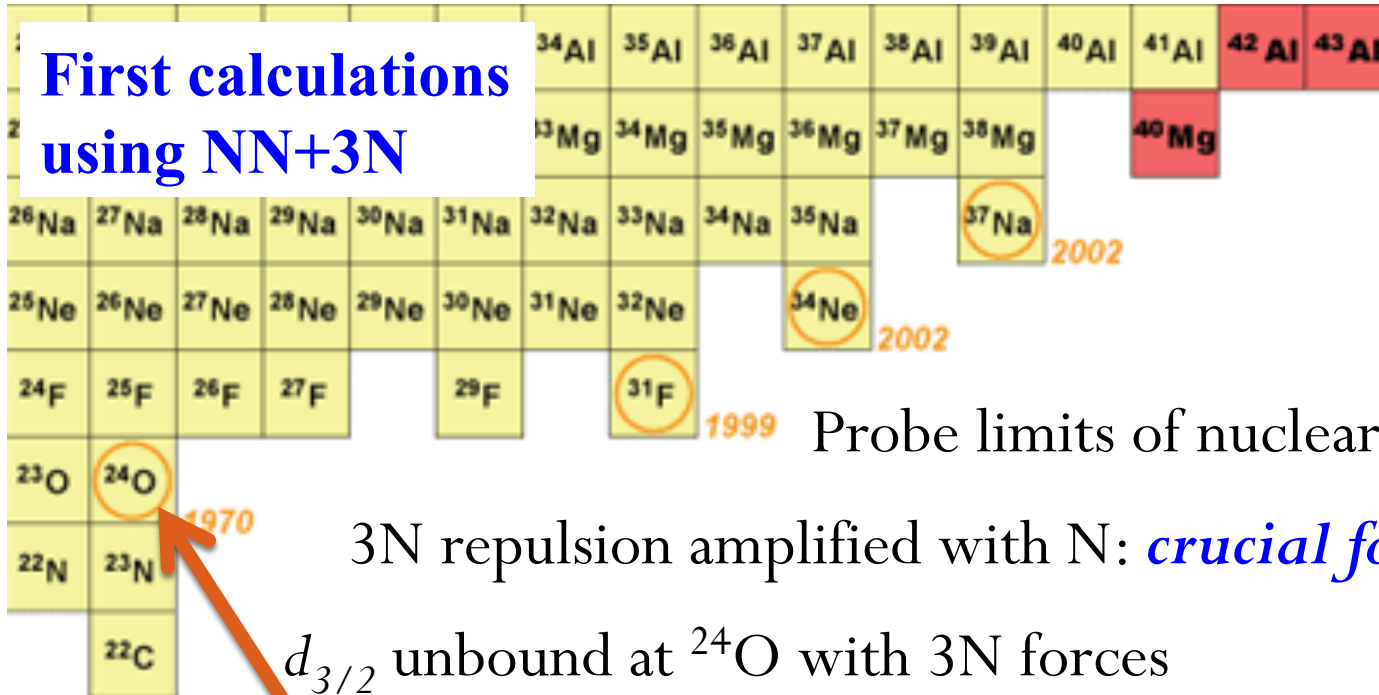


Incorrect prediction of oxygen dripline

Extended-space – more binding

Oxygen Anomaly

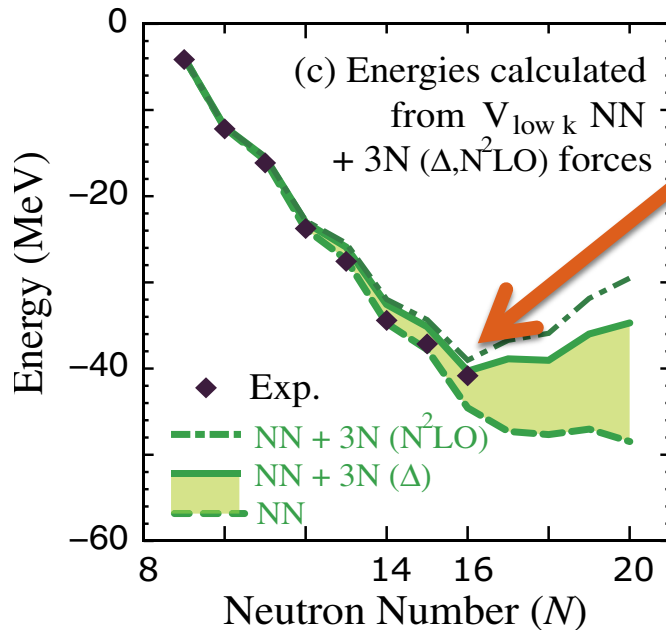
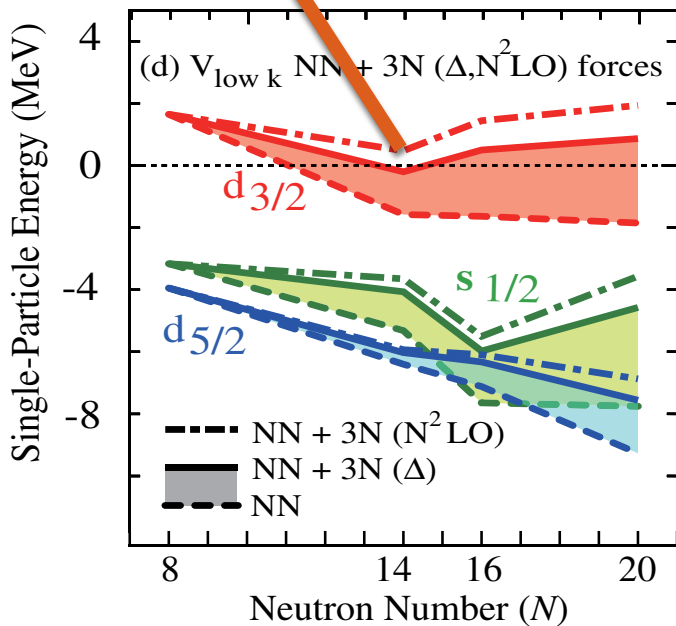
First calculations using NN+3N



Probe limits of nuclear existence with 3N forces

3N repulsion amplified with N: *crucial for neutron-rich nuclei*

$d_{3/2}$ unbound at ^{24}O with 3N forces

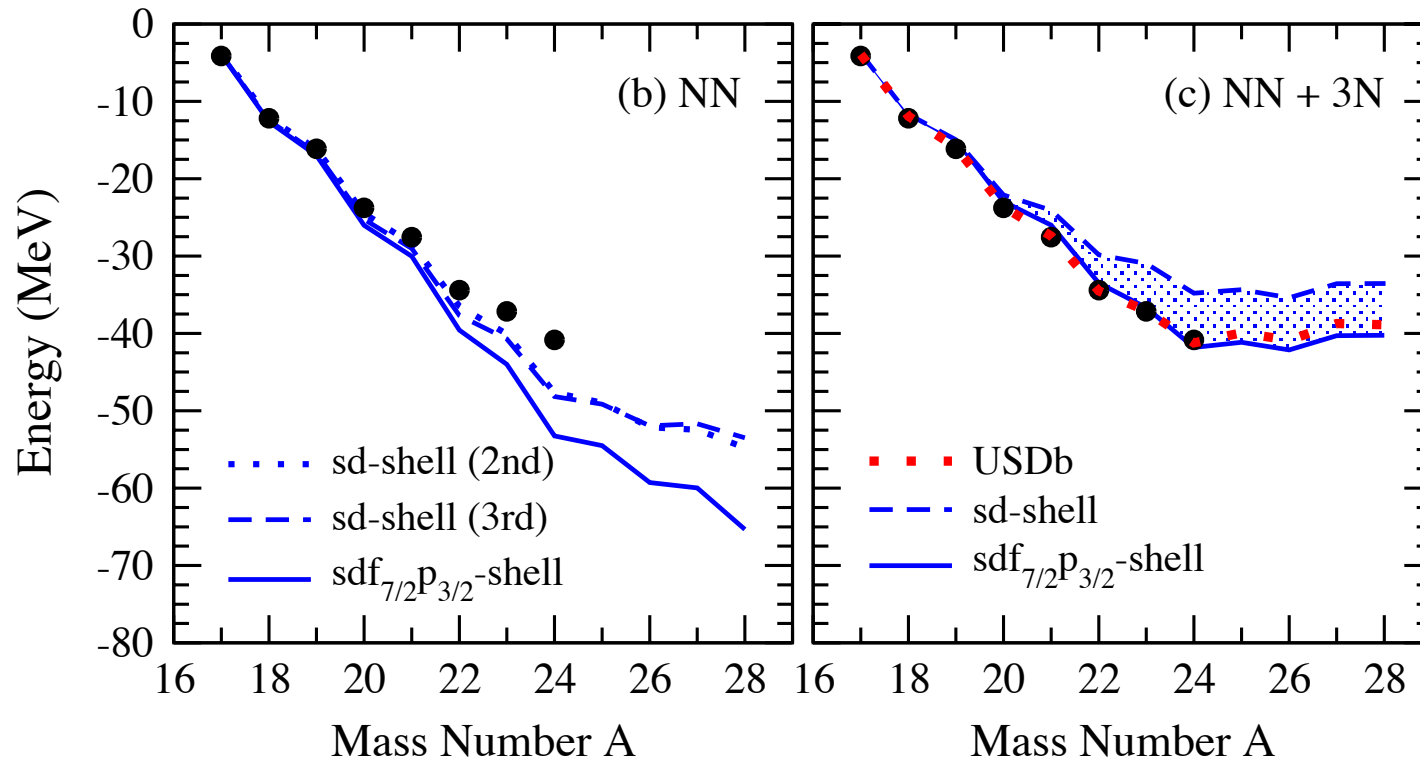


Isotopes unbound beyond ^{24}O

First microscopic explanation of oxygen anomaly

Ground-State Energies of Oxygen Isotopes

Valence-space interaction and SPEs from NN+3N



JDH, Menendez, Schwenk, EPJA (2013)

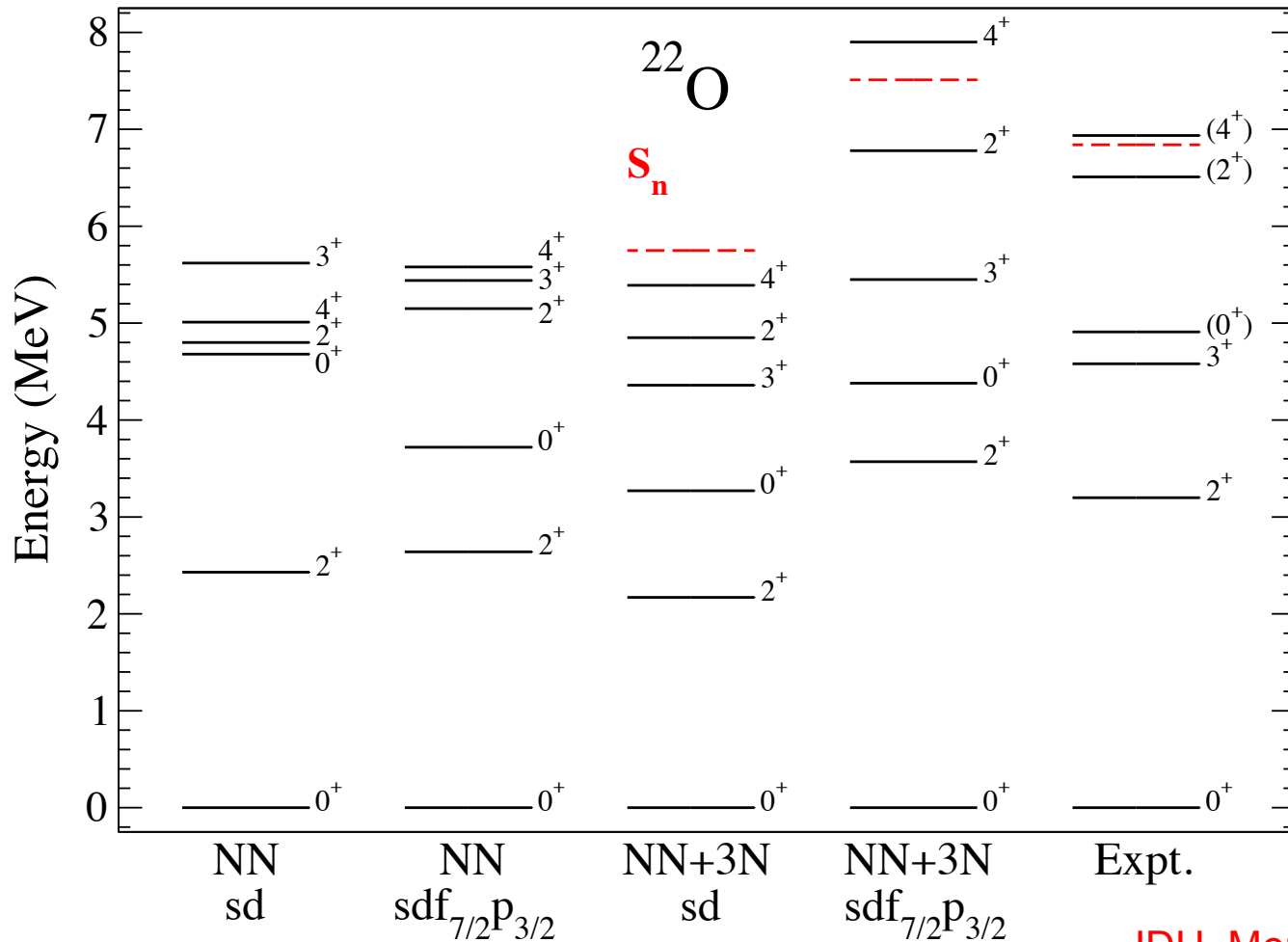
Repulsive character improves agreement with experiment

sd-shell results underbound; improved in **extended space**

Impact on Spectra: ^{22}O

Neutron-rich oxygen spectra with NN+3N

^{22}O : $N=14$ new magic number – not reproduced with NN



NN-only

2⁺ too low

Spectrum too compressed

NN+3N

Extended space essential

Reproduces $N=14$ magic number

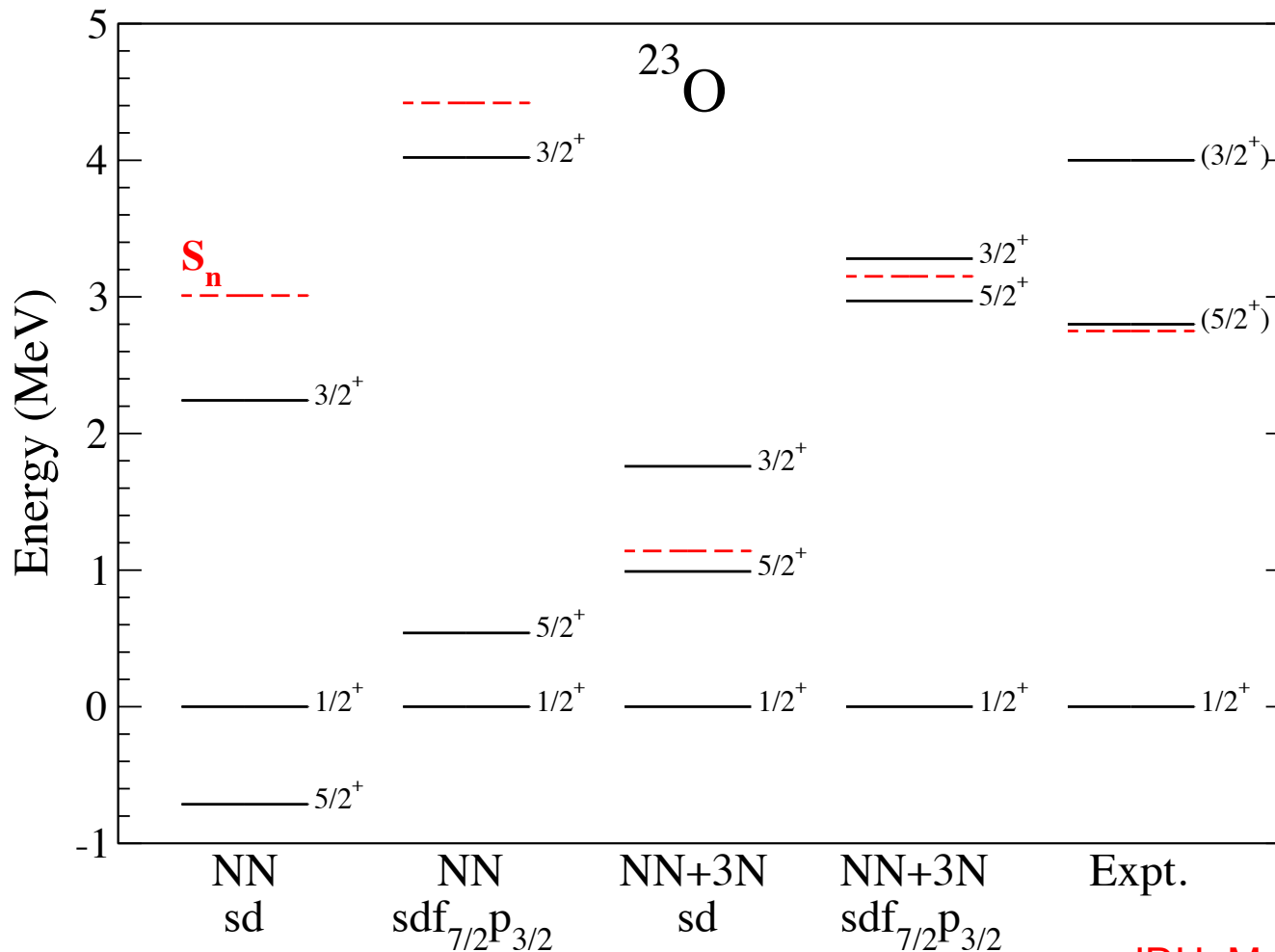
JDH, Menendez, Schwenk, EPJA (2013)

Contributions from 3N and extended valence orbitals important

Impact on Spectra: ^{23}O

Neutron-rich oxygen spectra with NN+3N

$5/2^+$, $3/2^+$ energies reflect $^{22,24}\text{O}$ shell closures



sd-shell NN only

Wrong ground state

$5/2^+$ too low

$3/2^+$ bound

NN+3N

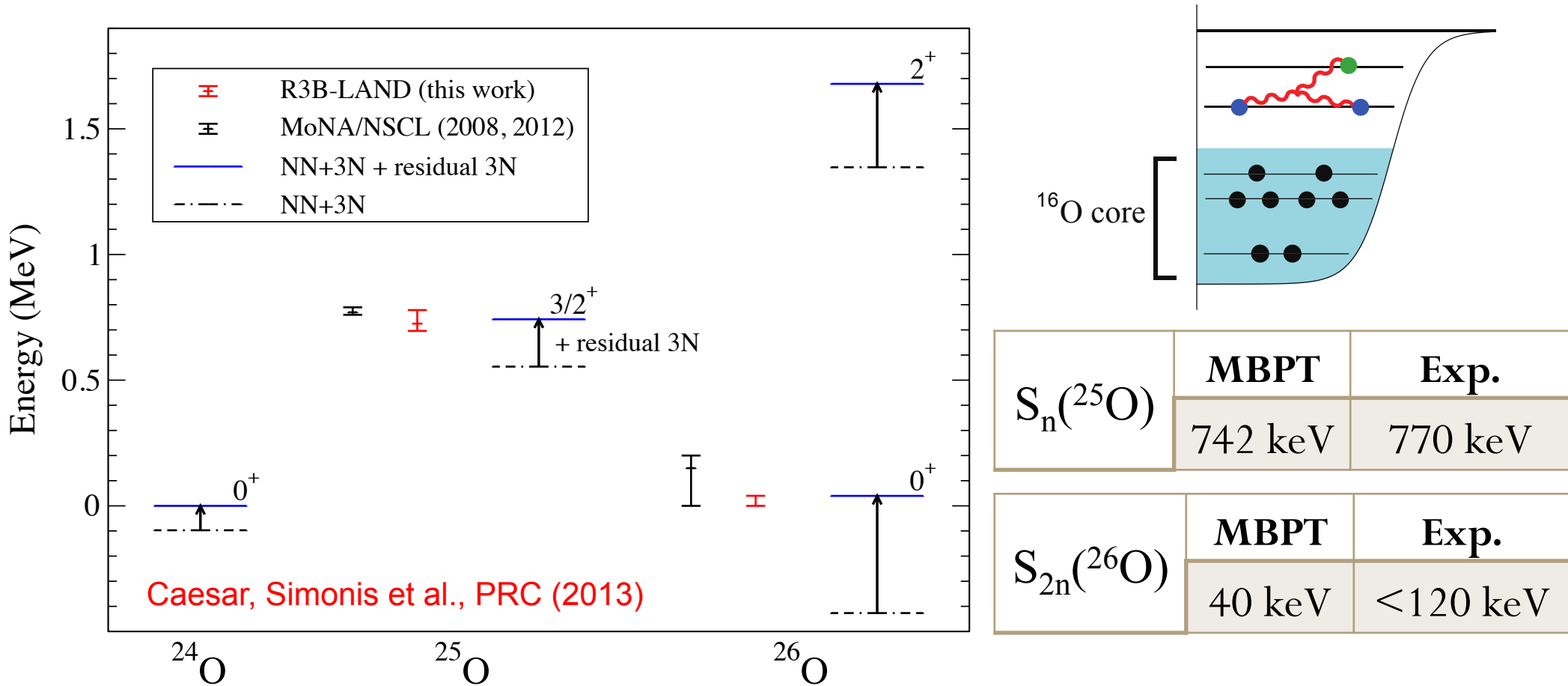
Clear improvement in extended valence space

JDH, Menendez, Schwenk, EPJA (2013)

Experimental Connection: Beyond the Dripline

Hoffman, Kanungo, Lunderberg... PRLs (2008+)

Valence-space Hamiltonian from NN + 3N + **residual 3N**



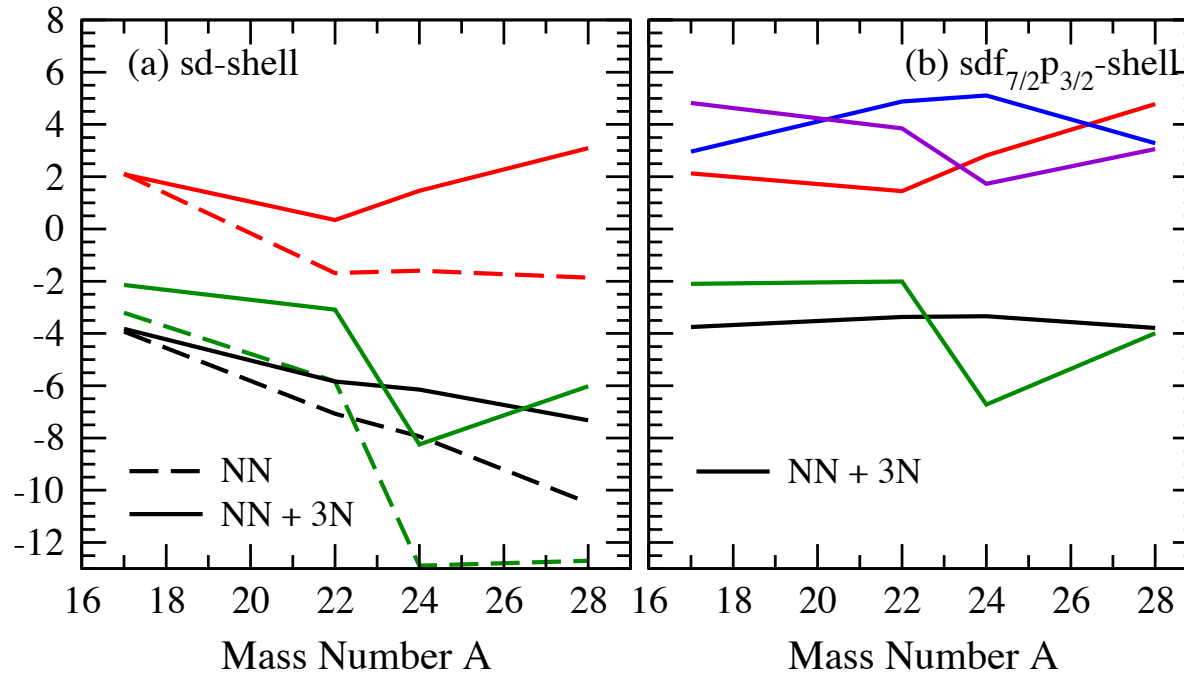
Repulsion more pronounced for neutron-rich systems: 400 keV at ^{26}O

Improved agreement with new data beyond ^{24}O dripline

Future: include coupling to continuum

Evolution of SPEs

SPE evolution with 3N forces in sd and $sd f_{7/2} p_{3/2}$ spaces:



NN+3N extended space:

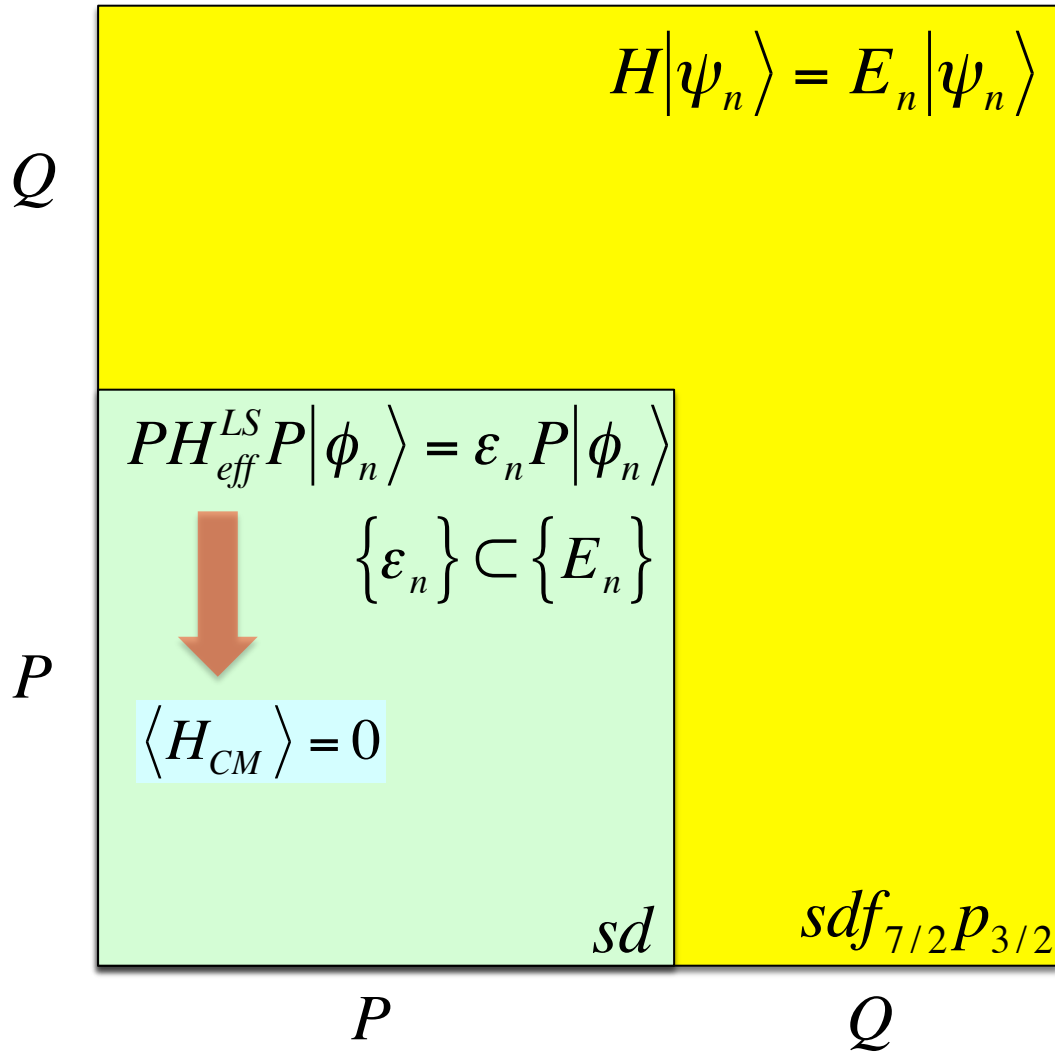
Much different trend than sd -shell

No gap at ^{22}O yet enhanced closed-shell features

Inversion of levels past ^{24}O – essential to include for neutron-rich oxygen

Evaluating Center-of-Mass Contamination

Nonperturbative Lee-Suzuki (LS) transformation from extended space



Transform for **two-body** systems
(*e.g.*, ¹⁸O, ⁴²Ca)

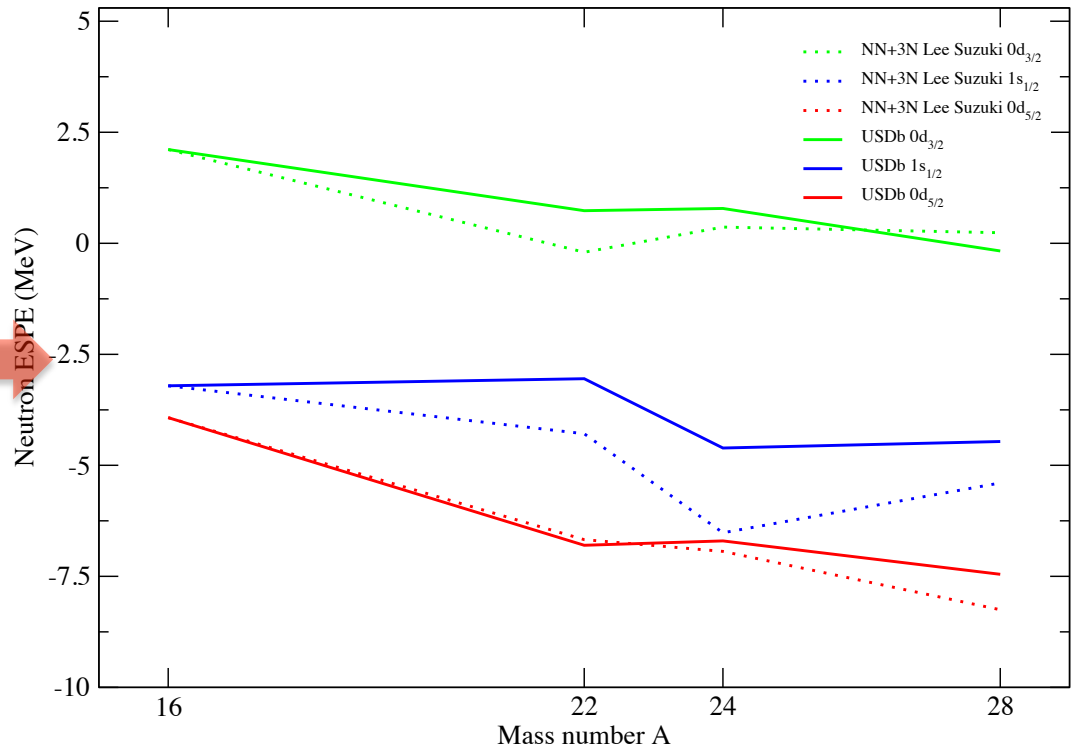
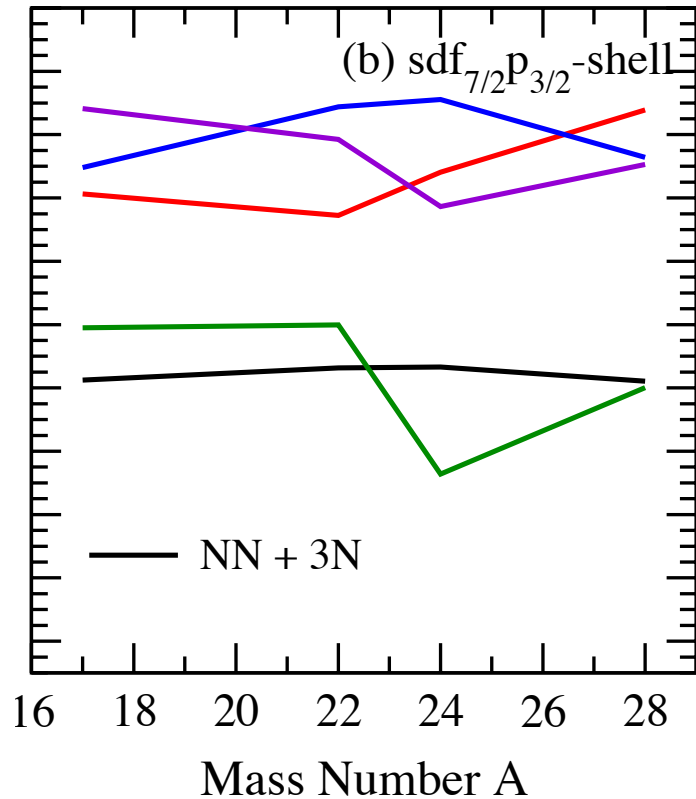
Extended-space spectrum free of CM contamination

Project into standard space
onto eigenenergies from
extended space calculation

Use H_{eff}^{LS} as new two-body Hamiltonian in *sd*-shell valence-space calculations

Evolution of SPEs

SPE evolution of LS-projected sd-shell interaction

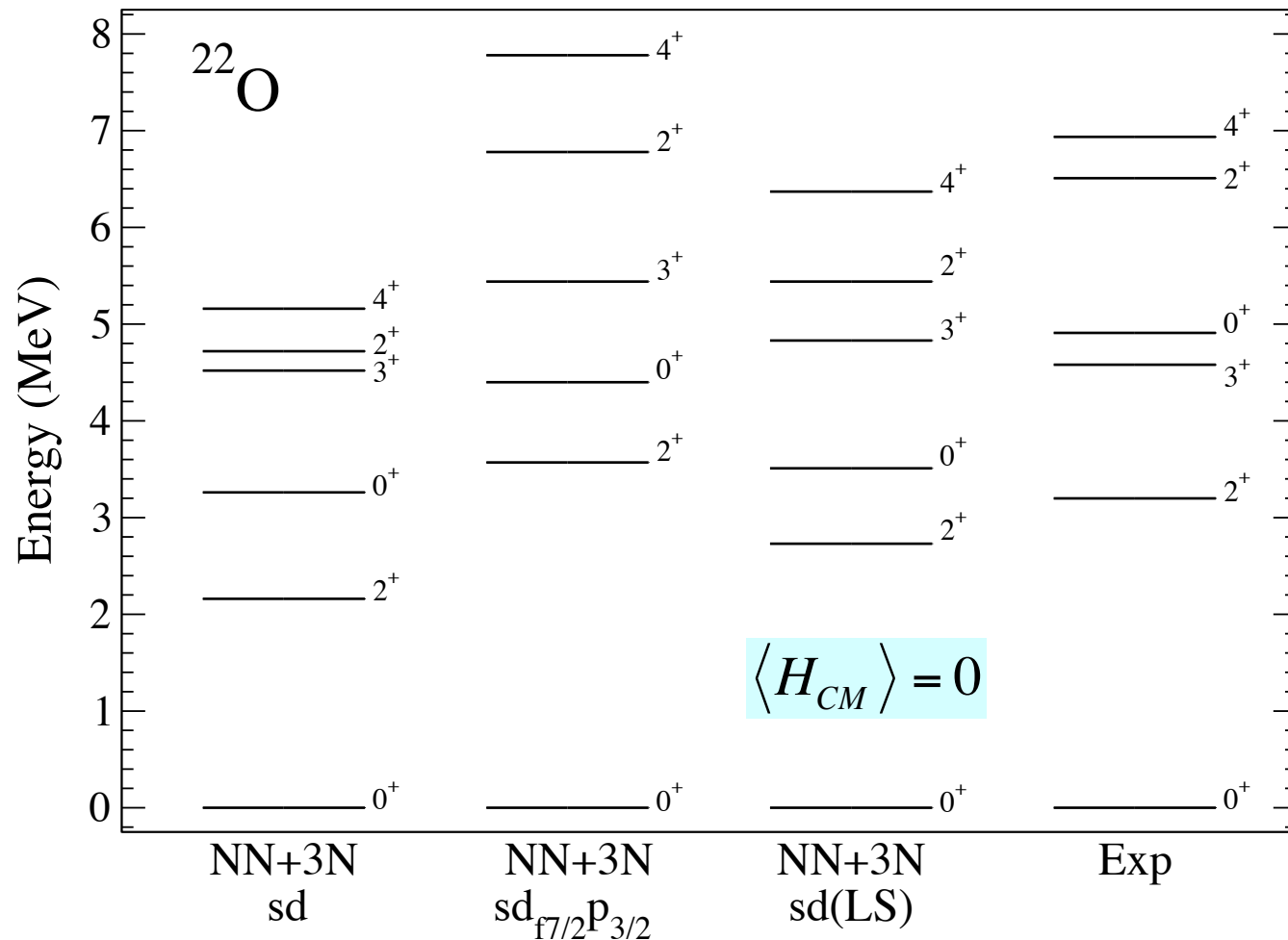


Overall evolution similar to USD – $s_{1/2}$ too attractive

Work in progress: involving $N > 2$ neutrons in extended space

Evaluating Center-of-Mass Contamination

Apply new H_{eff}^{LS} to calculate spectra in neutron-rich oxygen



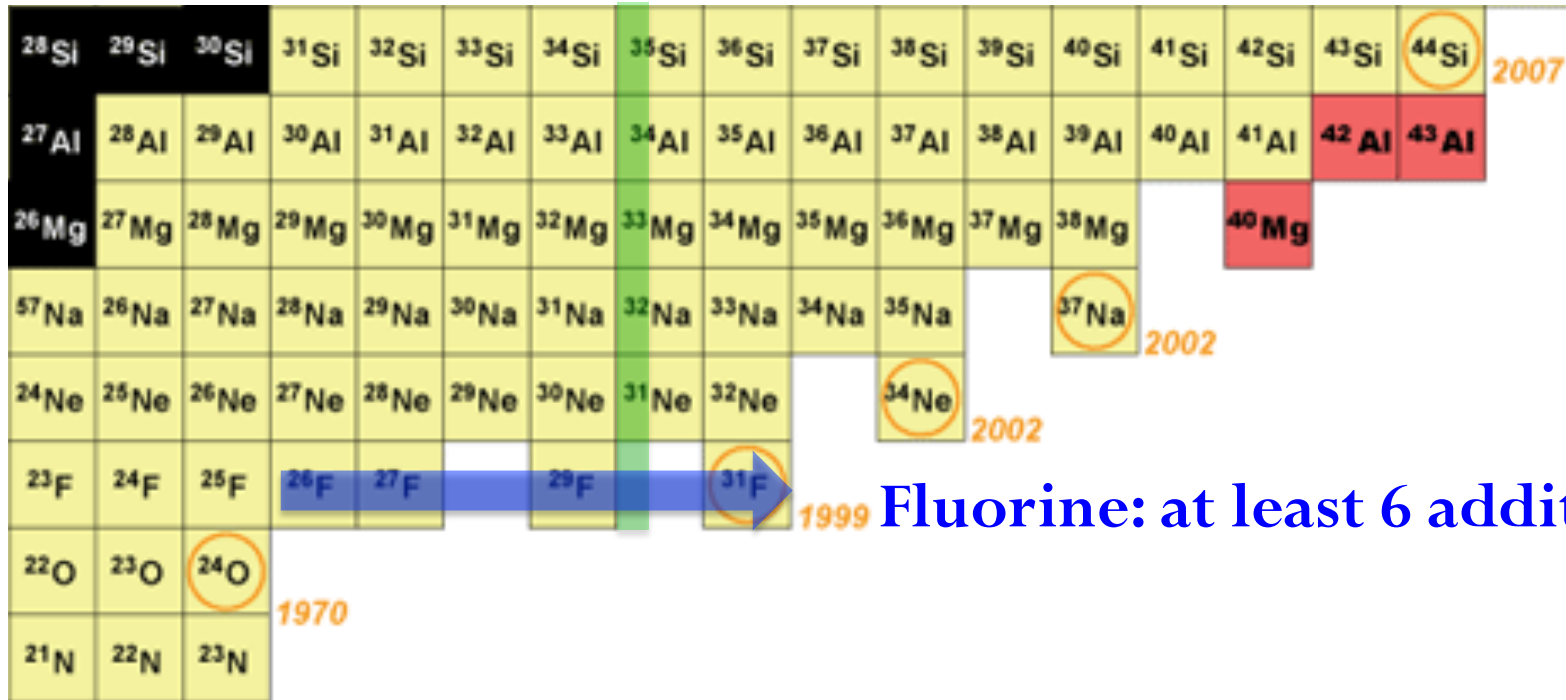
Improvements from standard sd -shell – not due to center of mass

Work in progress: involving $N > 2$ neutrons in extended space

Towards Full sd-Shell with MBPT: Fluorine

Next challenge: **valence protons + neutrons**

Neutron-rich fluorine and neon



Fluorine: at least 6 additional neutrons

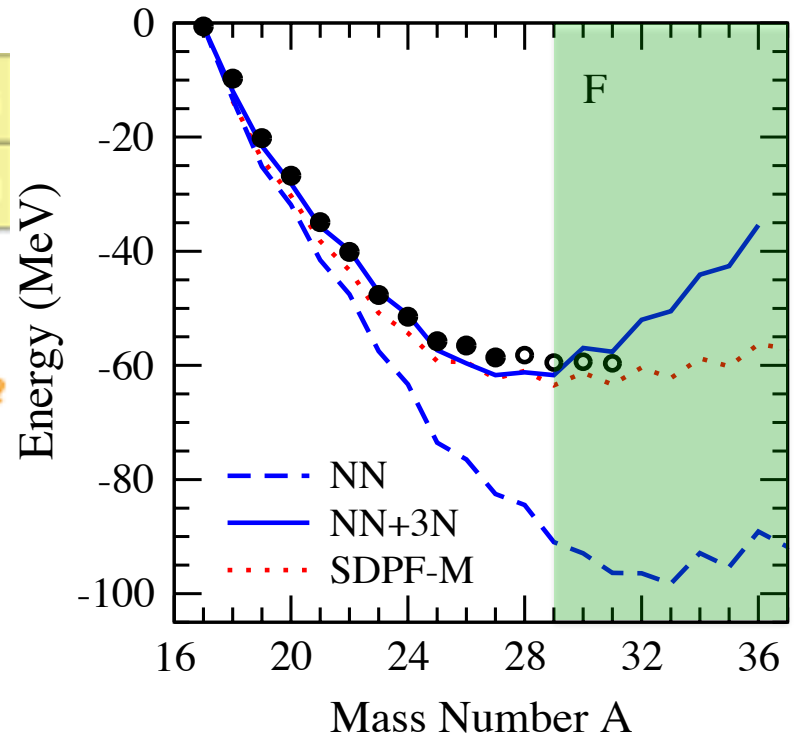
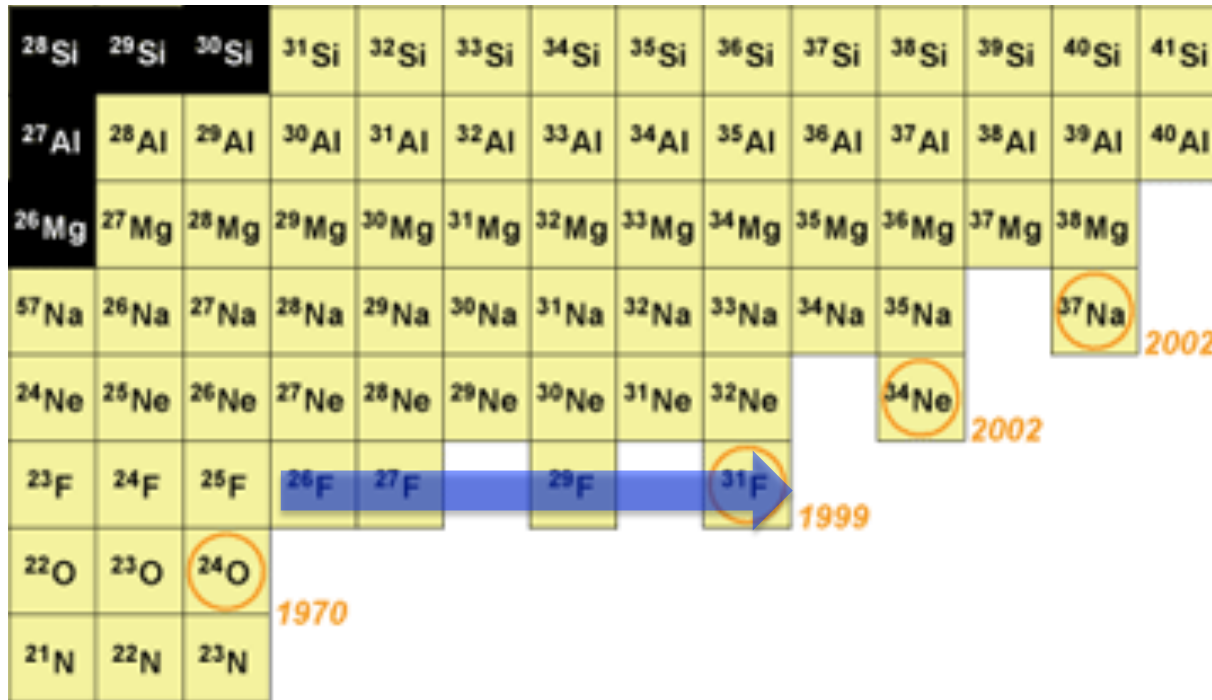
sd shell filled at ²⁹F/³⁰Ne

Need extended-space orbits

Towards Full sd-Shell with MBPT: Fluorine

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Neutron-rich fluorine and neon



JDH, Menendez, Schwenk, in prep.

NN only: severe overbinding

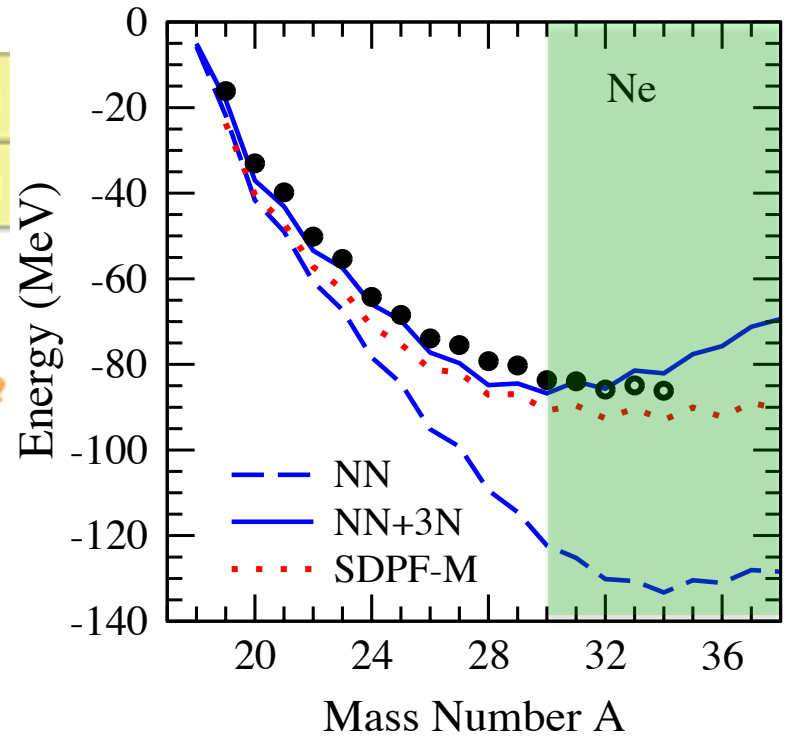
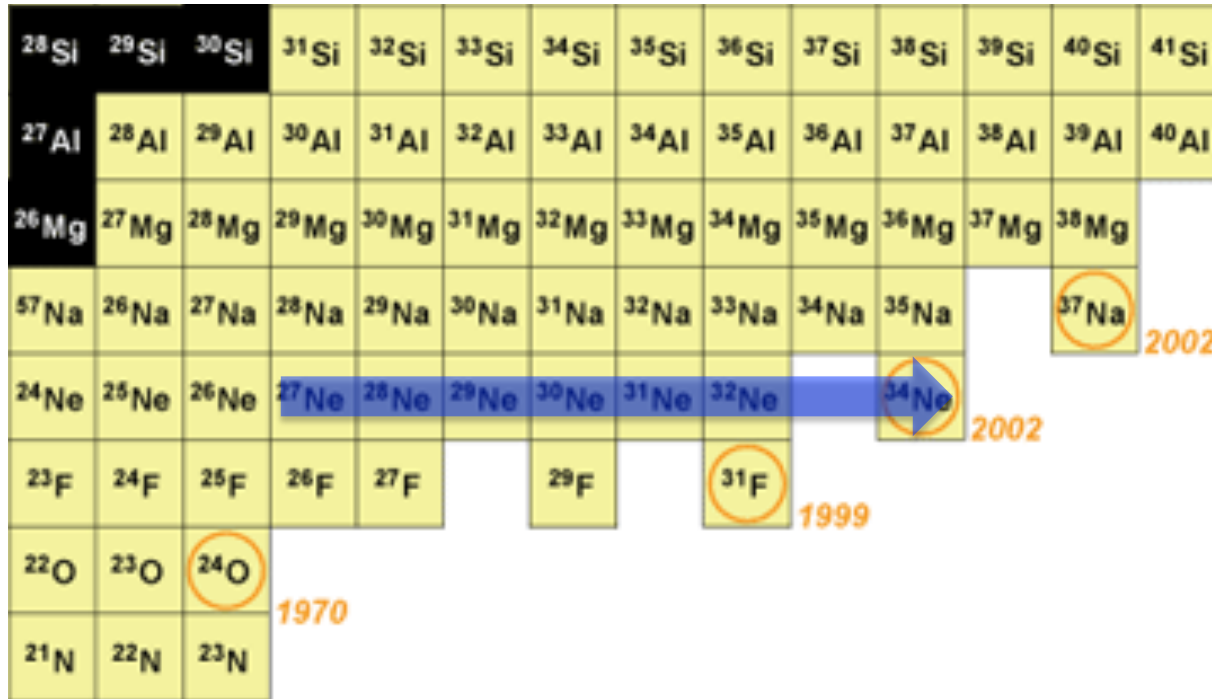
NN+3N: good experimental agreement through ^{29}F

Sharp increase in ground-state energies beyond ^{29}F : incorrect dripline

Towards Full sd-Shell with MBPT: Neon

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Neutron-rich fluorine and neon



JDH, Menendez, Schwenk, in prep.

Similar behavior in Neon isotopes

Revisit cross-shell valence space theory – **non-degenerate valence spaces**

Tsunoda, Hjorth-Jensen, Otsuka

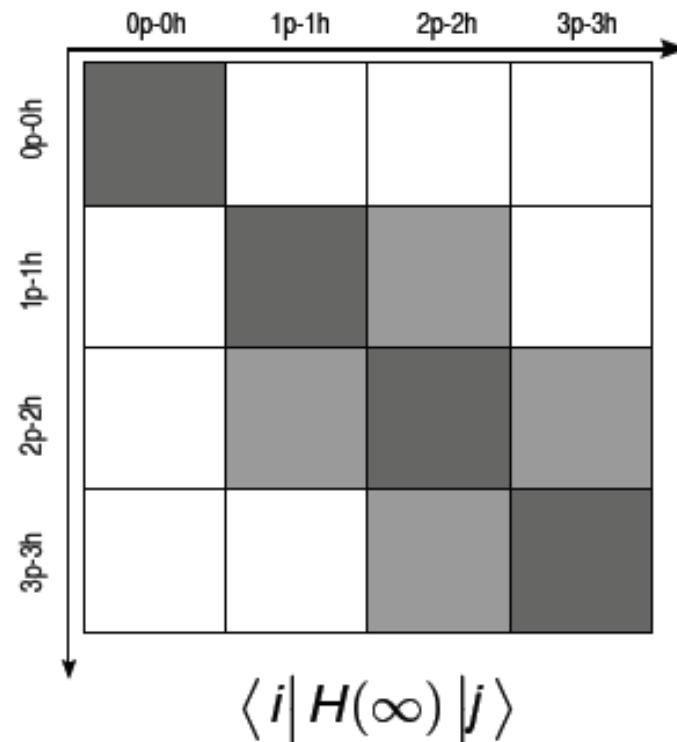
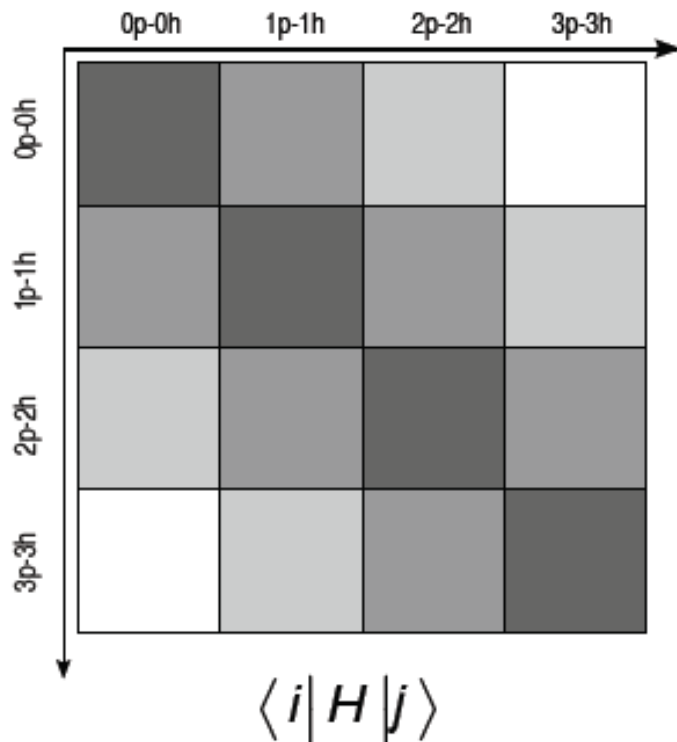
IM-SRG for Valence-Space Hamiltonians

In-Medium SRG applies continuous unitary transformation to drive off-diagonal physics to zero

Tsukiyama, Bogner, Schwenk, PRL (2011)

$$H(s) = U(s)HU^\dagger(s) \equiv H^d(s) + H^{\text{od}}(s) \rightarrow H^d(\infty)$$

Decouples reference state from excitations $\langle npnh | H(\infty) | \Phi_c \rangle = 0$



IM-SRG for Valence-Space Hamiltonians

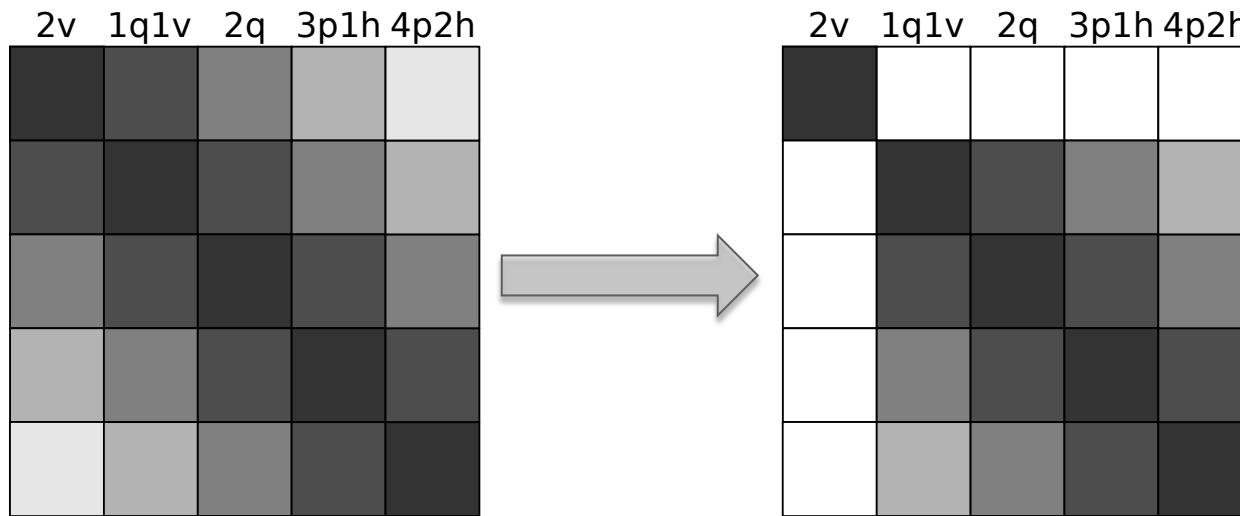
In-Medium SRG applies continuous unitary transformation to drive off-diagonal physics to zero

Tsukiyama, Bogner, Schwenk, PRC (2012)

Open shell systems:

split particle states into valence states, v , and those above valence space, q

Redefine “off-diagonal” to exclude valence particles



$$H(s=0) \rightarrow H(\infty)$$

IM-SRG for Valence-Space Hamiltonians

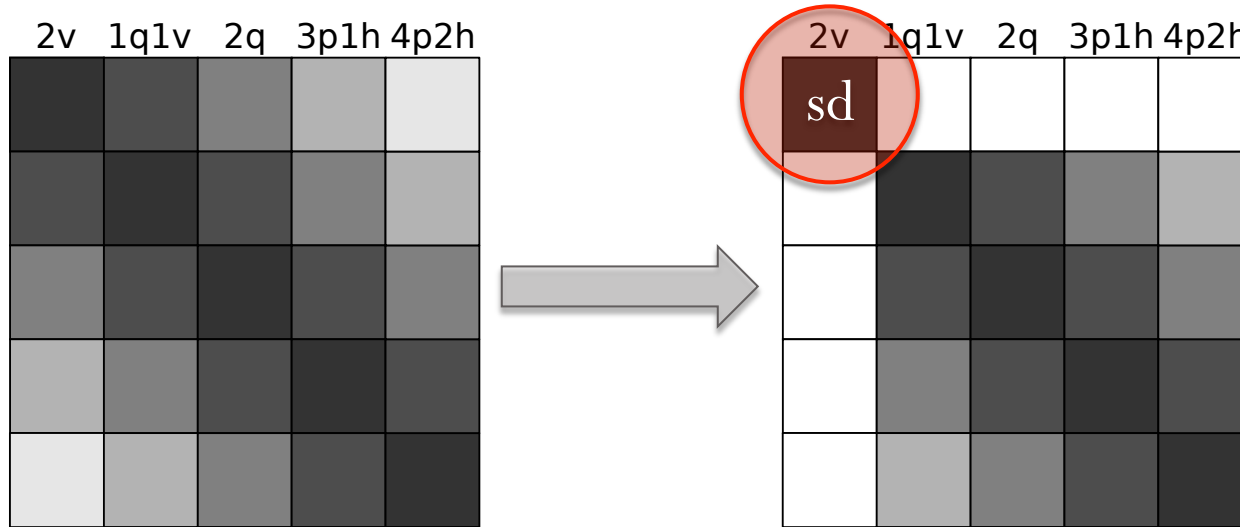
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Open shell systems:

split particle states into valence states, v , and those above valence space, q

Redefine “off-diagonal” to exclude valence particles



$$H(s=0) \rightarrow H(\infty)$$

Defines new effective valence-space Hamiltonian H_{eff}

States outside valence space are decoupled

Nonperturbative Valence-Space Strategy

- 1) Effective interaction: nonperturbative from IM-SRG
- 2) Single-particle energies: nonperturbative from IM-SRG
- 3) Hartree-Fock basis of $e_{\max} = 2n + l = 14$ **converged**
- ★ 4) NN and 3N forces from chiral EFT
- 5) Explore extended valence spaces – in progress

NN matrix elements

- Chiral N³LO (Machleidt, $\Lambda_{\text{NN}} = 500\text{MeV}$); free-space SRG evolution
- Cutoff variation $\lambda_{\text{SRG}} = 1.88 - 2.24\text{fm}^{-1}$
- Vary $\hbar\omega = 20 - 24\text{MeV}$
- Consistently include 3N forces induced by SRG evolution

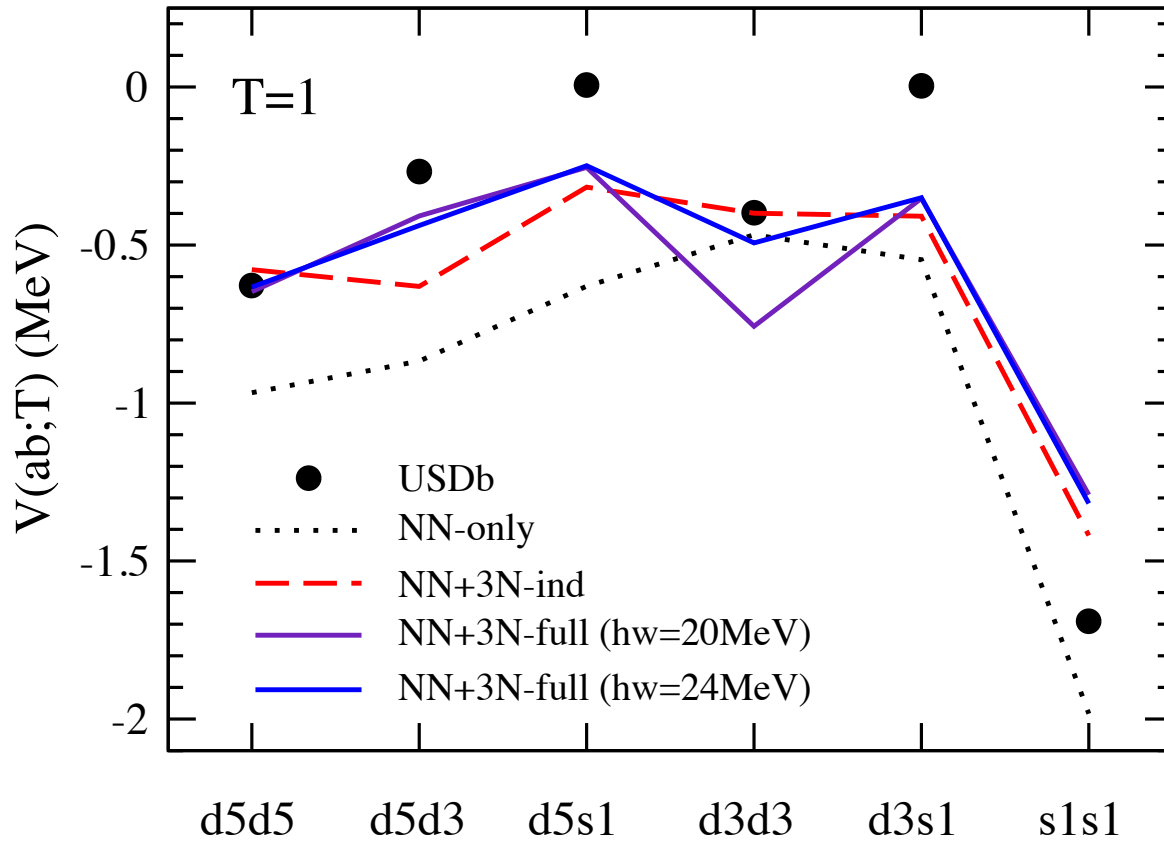
Initial 3N force contributions

- Chiral N²LO $\Lambda_{3\text{N}} = 400\text{MeV}$
- Included with cut: $e_1 + e_2 + e_3 \leq E_{3\max} = 14$

IM-SRG Monopoles

Monopoles: angular average of interaction

Determines interaction of orbit a with b ; evolution of orbital energies



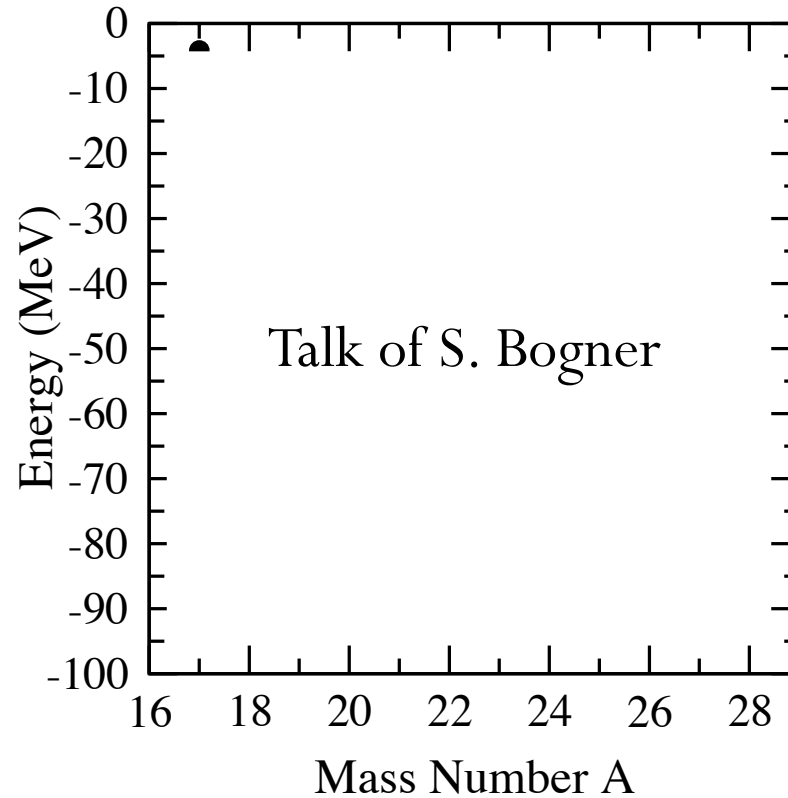
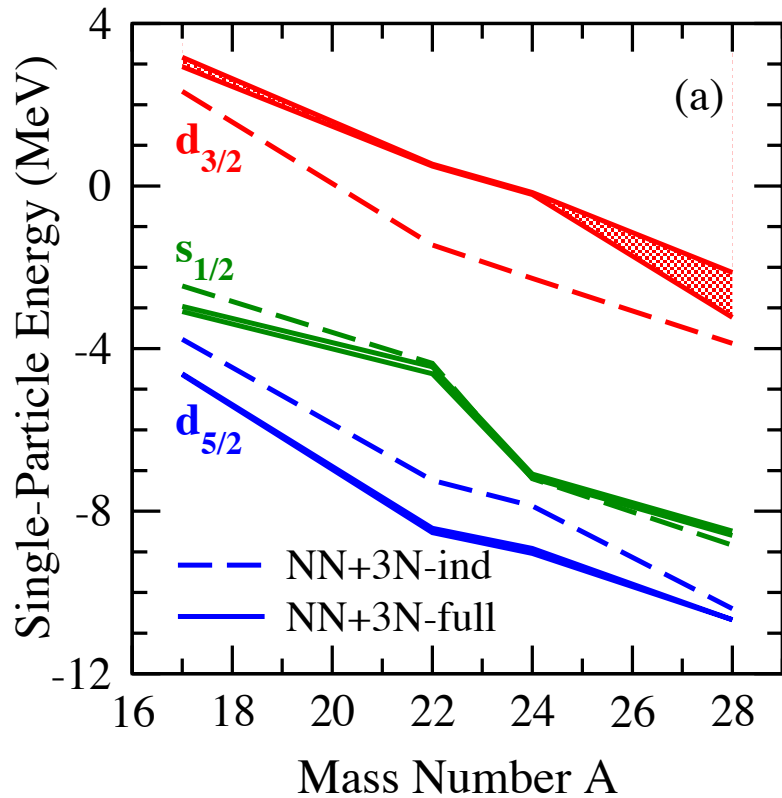
$$V_{ab}^T = \frac{\sum_J (2J+1) V_{abab}^{JT}}{\sum_J (2J+1)}$$

NN-only too attractive

NN+3N-full similar trend to USDb

IM-SRG Oxygen Ground-State Energies

Valence-space interaction and SPEs from NN+3N in *sd*-shell



Bogner et al., arXiv: 1402.1407

Repulsive 3N gives unbound $d_{3/2}$

Good dripline properties?

Conclusion/Outlook

- Nuclear structure theory of medium-mass nuclei with 3N forces, extended spaces
- **Non-empirical valence-space methods**
 - First calculations based on NN+3N forces
 - Extended valence spaces needed
 - Cures NN-only failings: dripline, shell evolution, spectra
 - Residual 3N forces improve predictions beyond dripline
- **New directions**
 - Promising first results for F/Ne ground states to
 - Non-perturbative IM-SRG – excellent binding energies, spectra in sd shell only!
- **Large-space ab-initio methods**
 - Similar improvements with NN+3N as in valence-space methods
 - Agreement between methods encouraging for future – benchmarking valuable!

Acknowledgments

Collaborators



TECHNISCHE
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J. Menendez J. Simonis A. Schwenk

S. Binder, A. Calci, J. Langhammer, R. Roth



S. Bogner



H. Hergert



T. Otsuka



T. Suzuki (Nihon U.)