Chiral Effective Field Theory for Nuclear Forces: The Path to Nuclear Structure from First Principles

R. Machleidt, University of Idaho

Outline

- Historical perspective
- Nuclear forces from chiral EFT: Basic ideas and overview
- Two-nucleon forces (2NF)
- Three-nucleon forces (3NF)
- Outlook

Table 1. Eight Decades of Struggle: The Theory of Nuclear Forces

1935	**Yukawa: Meson Theory**
1950's	*The "Pion Theories"* One-Pion Exchange: o.k. Multi-Pion Exchange: disaster
1960's	Many pions ≡ multi-pion resonances: σ, ρ, ω, ... The One-Boson-Exchange Model
1970's	Refine meson theory: More sophisticated meson-exchange models (Stony Brook, Paris, Bonn)
1980's	Nuclear physicists discover **QCD** Quark Cluster Models
1990's and beyond	Nuclear physicists discover **EFT** Weinberg, van Kolck **Back to Yukawa's Meson Theory!** *But, with Chiral Symmetry*

Table 1. Eight Decades of Struggle: The Theory of Nuclear Forces

1935	Yukawa: Meson Theo
1950's	*The "Pion" Theories"* On[e]-Pion Exchange: o.k. Multi-Pion Exchange: disaster
1960's	Many pions ≡ multi-pion resonances: The [One]-Boson-Exchange Model
1970's	Refine meson theory [more] sophisticated [one]-boson-exchange models (Stony Brook, Paris, Bonn)
1980's	Nuclear physicists discover QCD Quark Cluster Models
1990's	Nuclear physicists discover EFT Weinberg, van Kol[adze]
and beyond	Back to Yukawa's Meson Theory! *But, with Chiral Symmetry*

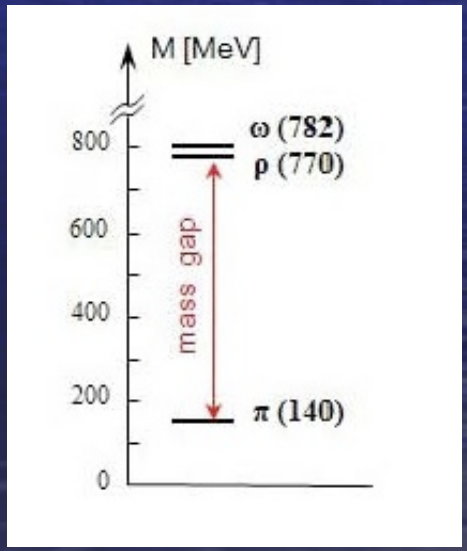
The circle of history is closing.

The ultimate goal of nuclear physics: Understanding nuclei from first principles

- Forces from first principals (QCD)
- *Ab initio* many-body methods

Forces from first principles, i.e., QCD

- QCD at low energy is strong.
- Quarks and gluons are confined into colorless hadrons.
- Nuclear forces are residual forces (similar to van der Waals forces)
- Separation of scales



- Calls for an EFT:

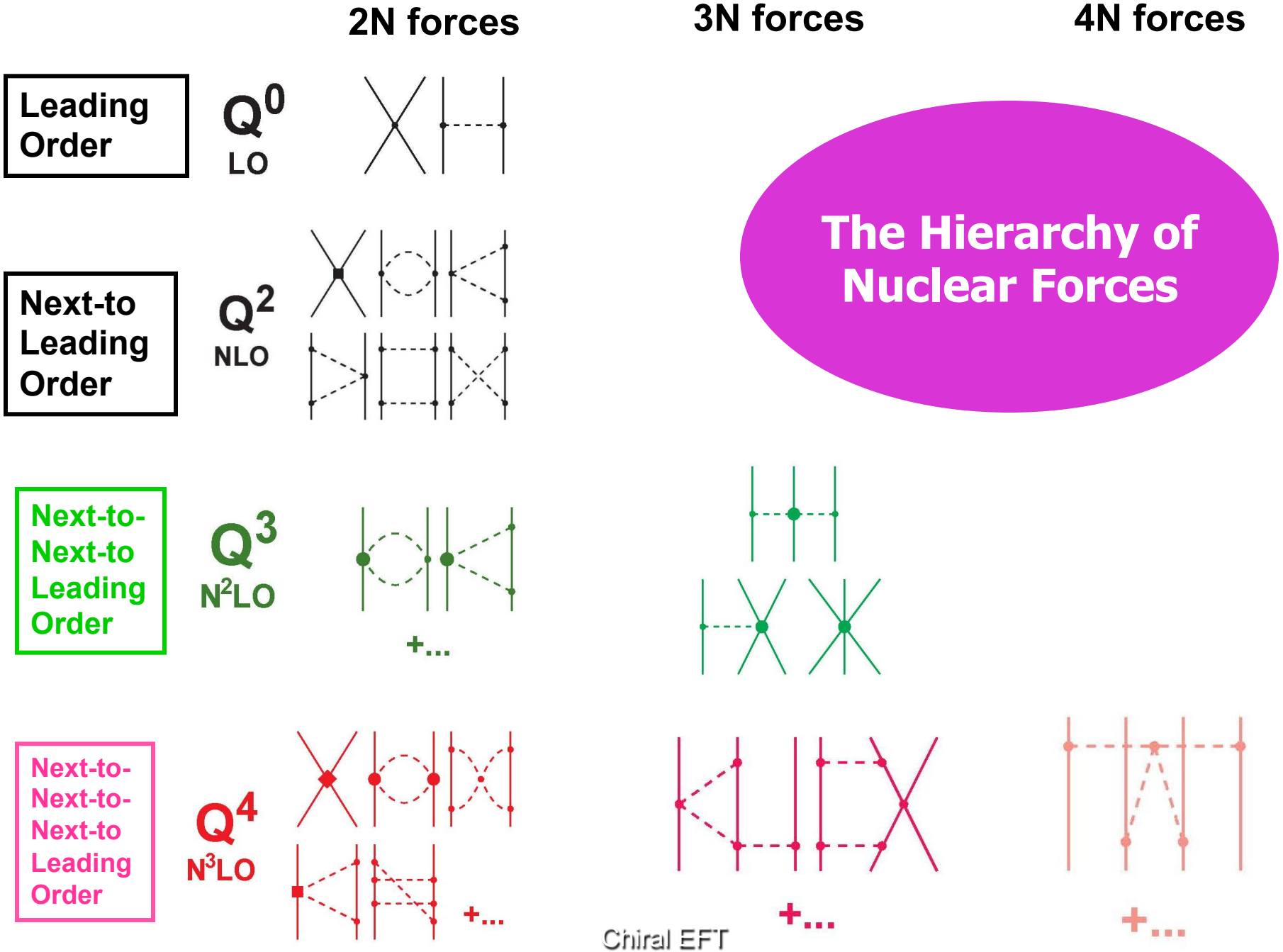
  soft scale: $Q \approx m_\pi$, hard scale: $\Lambda_\chi \approx m_\rho$;
  pions and nucleons are relevant d.o.f.
- Low-momentum expansion: $(Q/\Lambda_\chi)^\nu$

  with ν bounded from below.
- Most general Lagrangian consistent with all symmetries of low-energy QCD, particularly, chiral symmetry which is spontaneously broken.
- Weakly interacting Goldstone bosons = pions.
- π-π and π-N perturbatively
- NN has bound states:

  (i) NN potential perturbatively

  (ii) apply nonpert. in LS equation.

  (Weinberg)


2N forces
3N forces
4N forces
Leading Order
Q^0
LO
Next-to Leading Order
Q^2
NLO
The Hierarchy of Nuclear Forces
Next-to-Next-to Leading Order
Q^3
N^2LO
+...
Next-to-Next-to-Next-to Leading Order
Q^4
N^3LO
+...
+...
+...

2N forces

3N forces

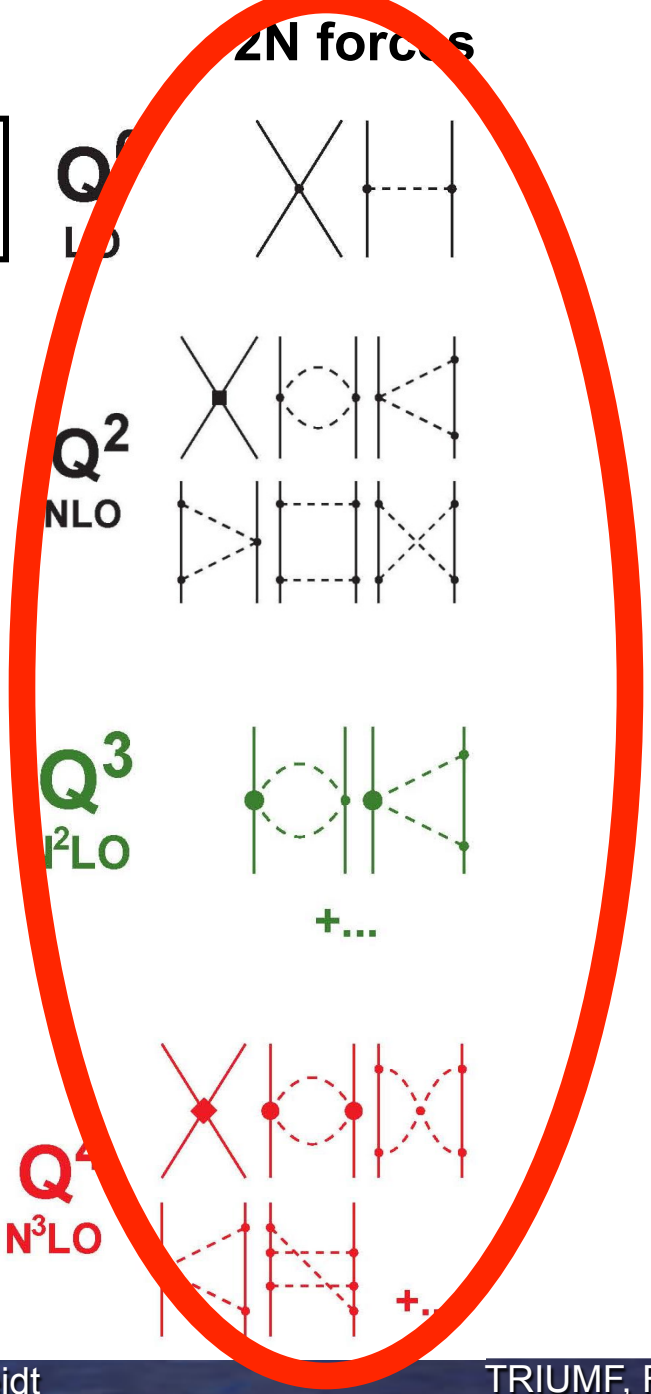
4N forces

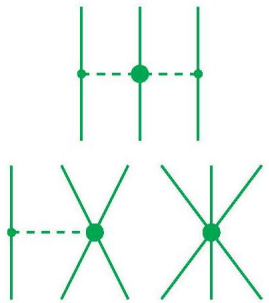
Leading Order

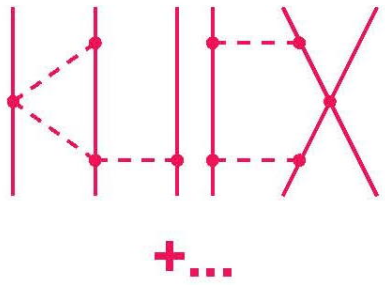
Next-to Leading Order

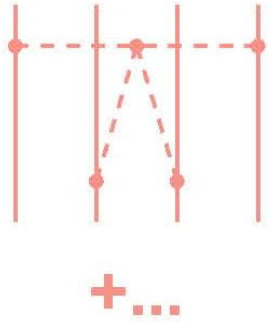
Next-to-Next-to Leading Order

Next-to-Next-to-Next-to Leading Order





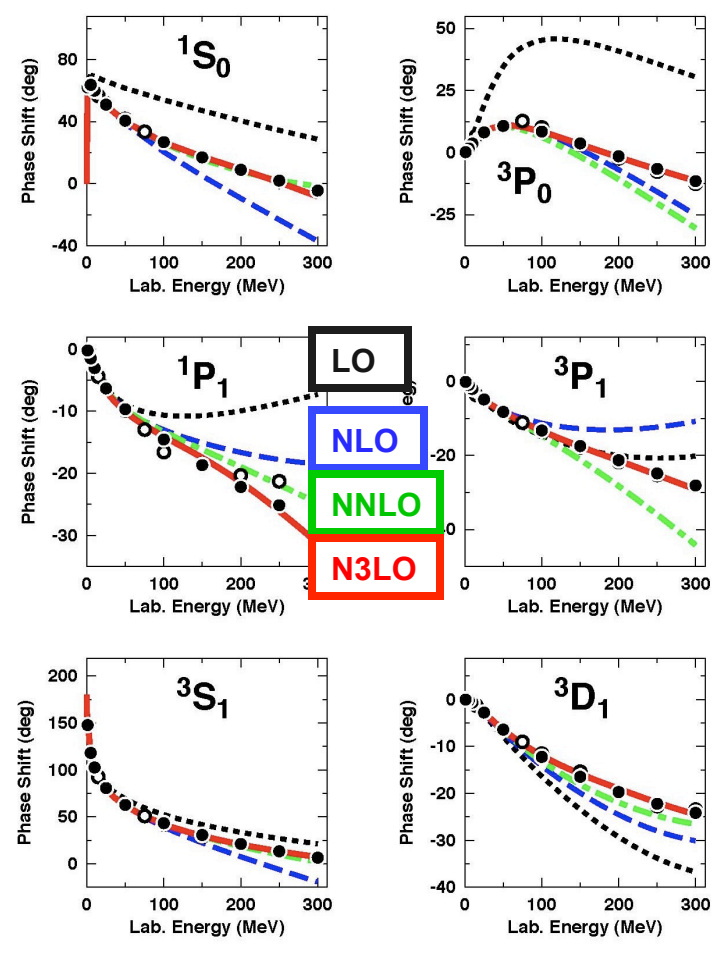


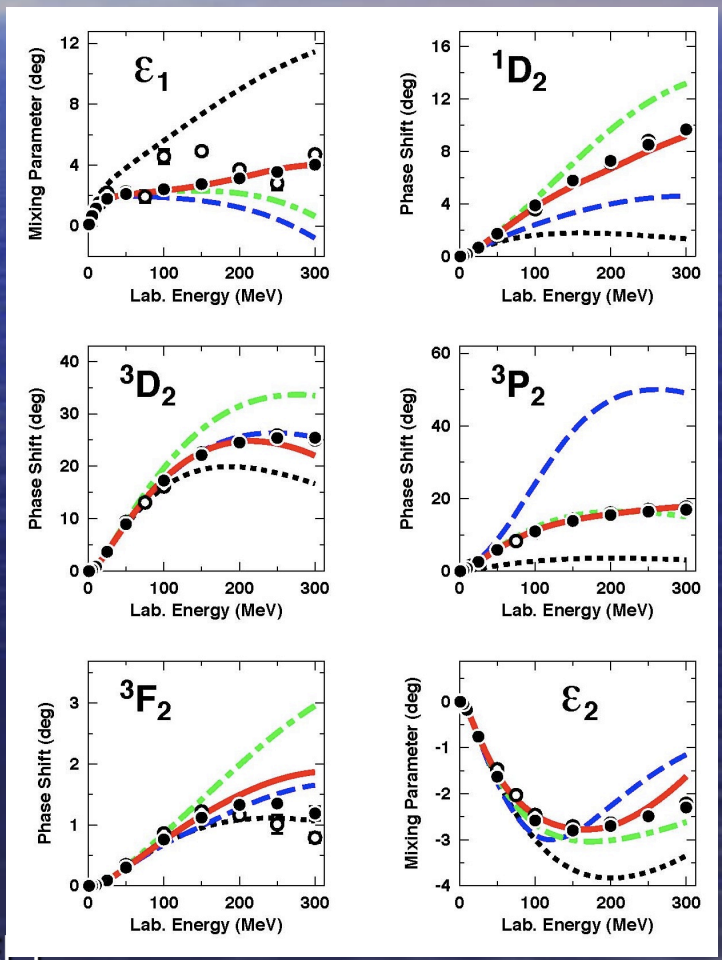


The Hierarchy of Nuclear Forces

NN phase shifts up to 300 MeV

Red Line: N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).
Green dash-dotted line: NNLO Potential, and
blue dashed line: NLO Potential
by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).





χ^2/datum for the reproduction of the 1999 np database

Bin (MeV)	# of data	N^{3}LO	NNLO	NLO	AV18
0–100	1058	1.05	1.7	4.5	0.95
100–190	501	1.08	22	100	1.10
190–290	843	1.15	47	180	1.11
0–290	2402	1.10	20	86	1.04

449 data from TRIUMF (van Oers et al.)

N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).
NNLO and NLO Potentials by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).

Summary: χ^2/datum

- NLO: ≈ 100
- NNLO: ≈ 10
- N3LO: ≈ 1

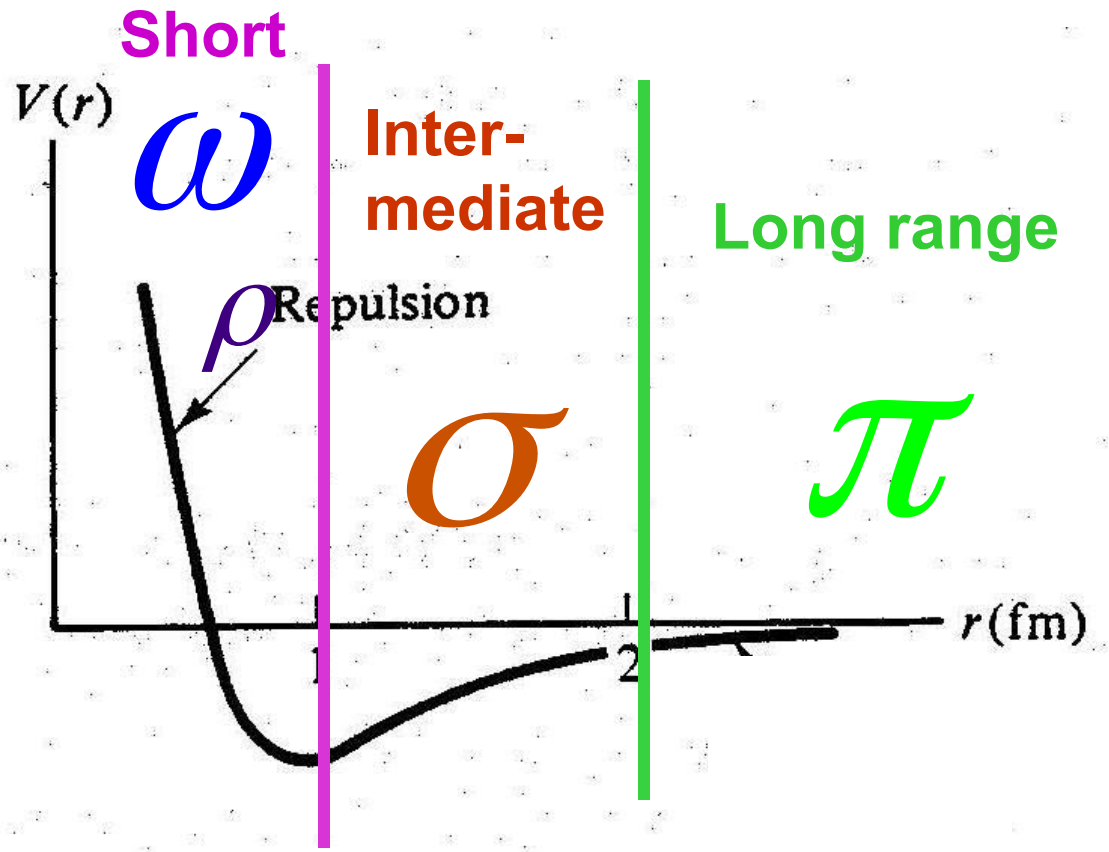
Great rate of convergence!

How does this compare to conventional meson theory?

Main differences

- **Chiral perturbation theory** (ChPT) is an expansion in terms of small momenta.
- **Meson theory** is an expansion in terms of ranges (masses).

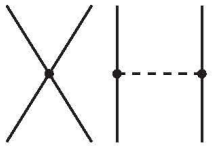
The nuclear force in the meson picture



2N forces

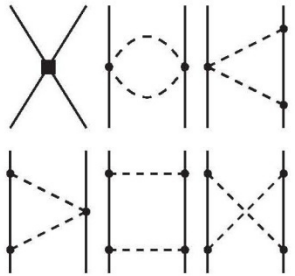
Leading Order

Q^0 LO



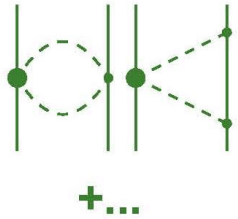
Next-to Leading Order

Q^2 NLO



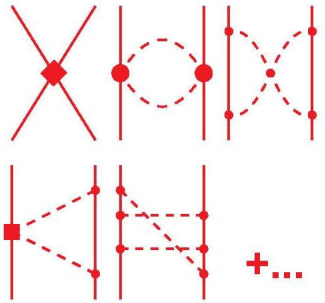
Next-to-Next-to Leading Order

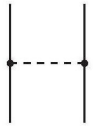
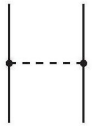
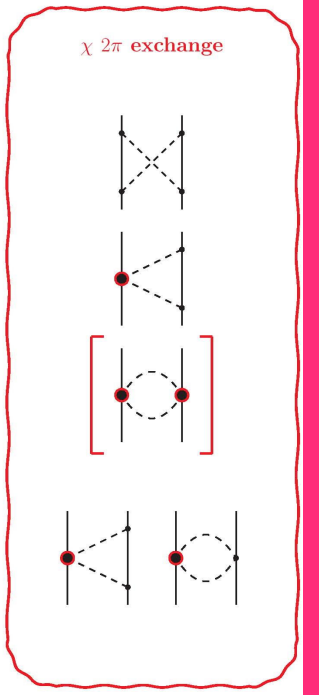
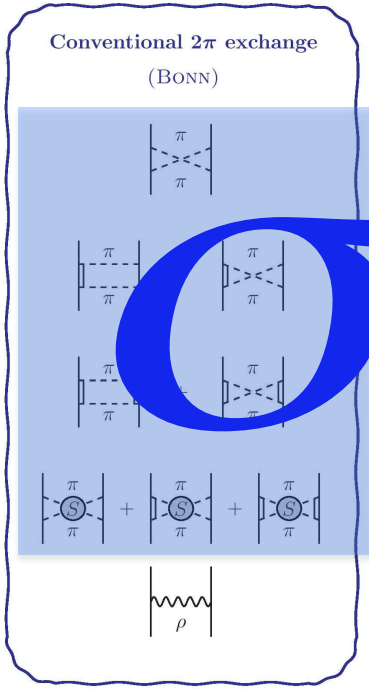
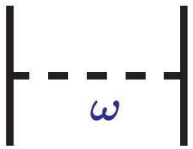
Q^3 N^2LO



Next-to-Next-to-Next-to Leading Order

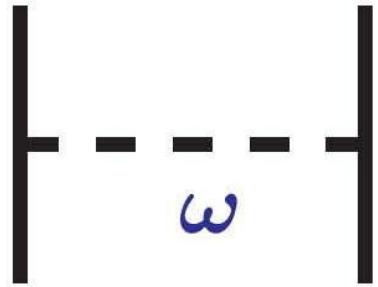
Q^4 N^3LO



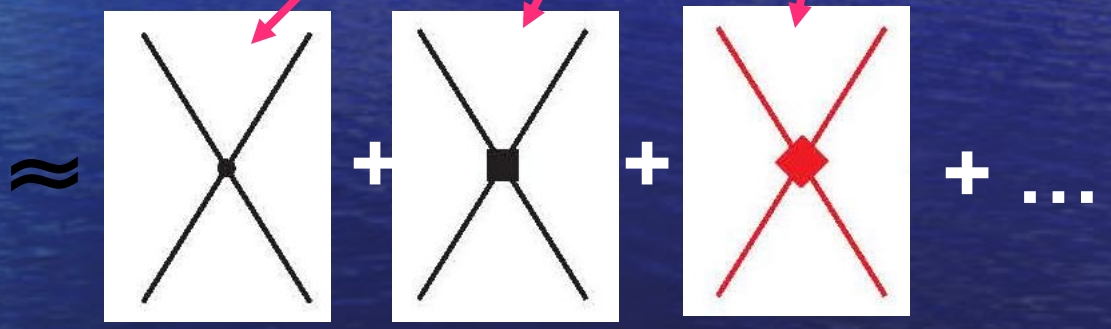
	ChPT	Conventional meson theory
OPE		
TPE		
Short range		

What is the physics of contact terms?

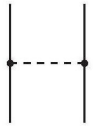
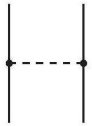
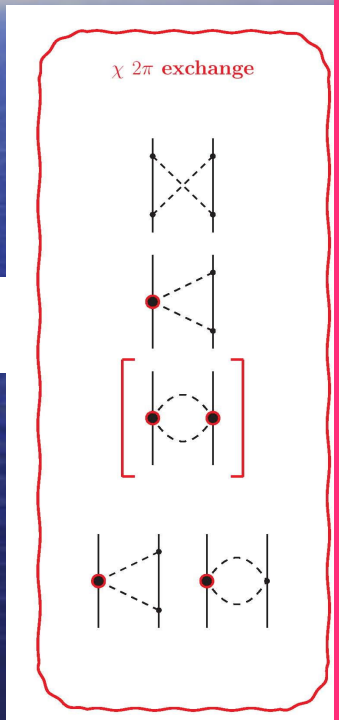
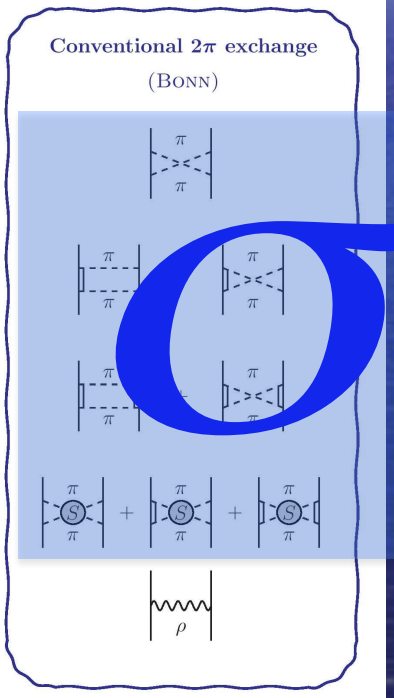
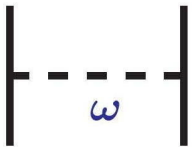
Consider the contribution from the exchange of a heavy meson



$$\frac{1}{m_\omega^2+Q^2} \approx \frac{1}{m_\omega^2}\left[1-\frac{Q^2}{m_\omega^2}+\frac{Q^4}{m_\omega^2}-\ldots\right]$$



Contact terms take care of the short range structures without resolving them.

	ChPT	Conventional meson theory
OPE		
TPE		
Short range		

Question: When everything is so equivalent to conventional meson theory, why not continue to use conventional meson theory?

Answer:

- Chiral EFT has a clear connection to QCD; it IS low-energy QCD.
- In ChPT, there is an organizational scheme ("power counting") that allows to estimate the size of the various contributions and the uncertainty at a given order (i.e., the size of the contributions we left out).
- Moreover, two- and many-body force contributions are generated on an equal footing.
- In conventional meson theory, we don't have this or we have it to a lesser extent.

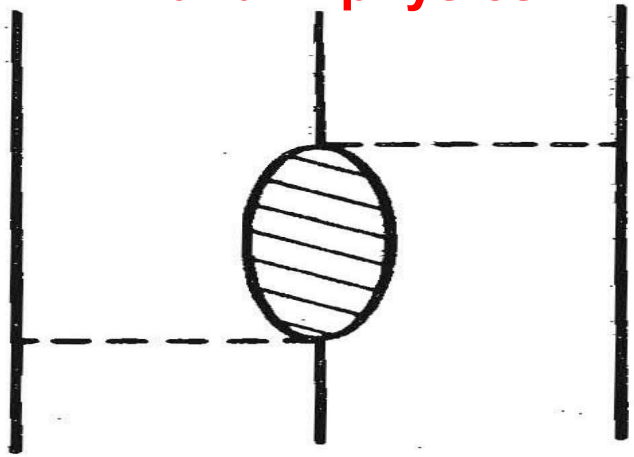
So much about two-nucleon forces (2NFs).
Now let's get ready for three-nucleon forces (3NFs).

Two-body interactions are easy – in physics

and in human life:

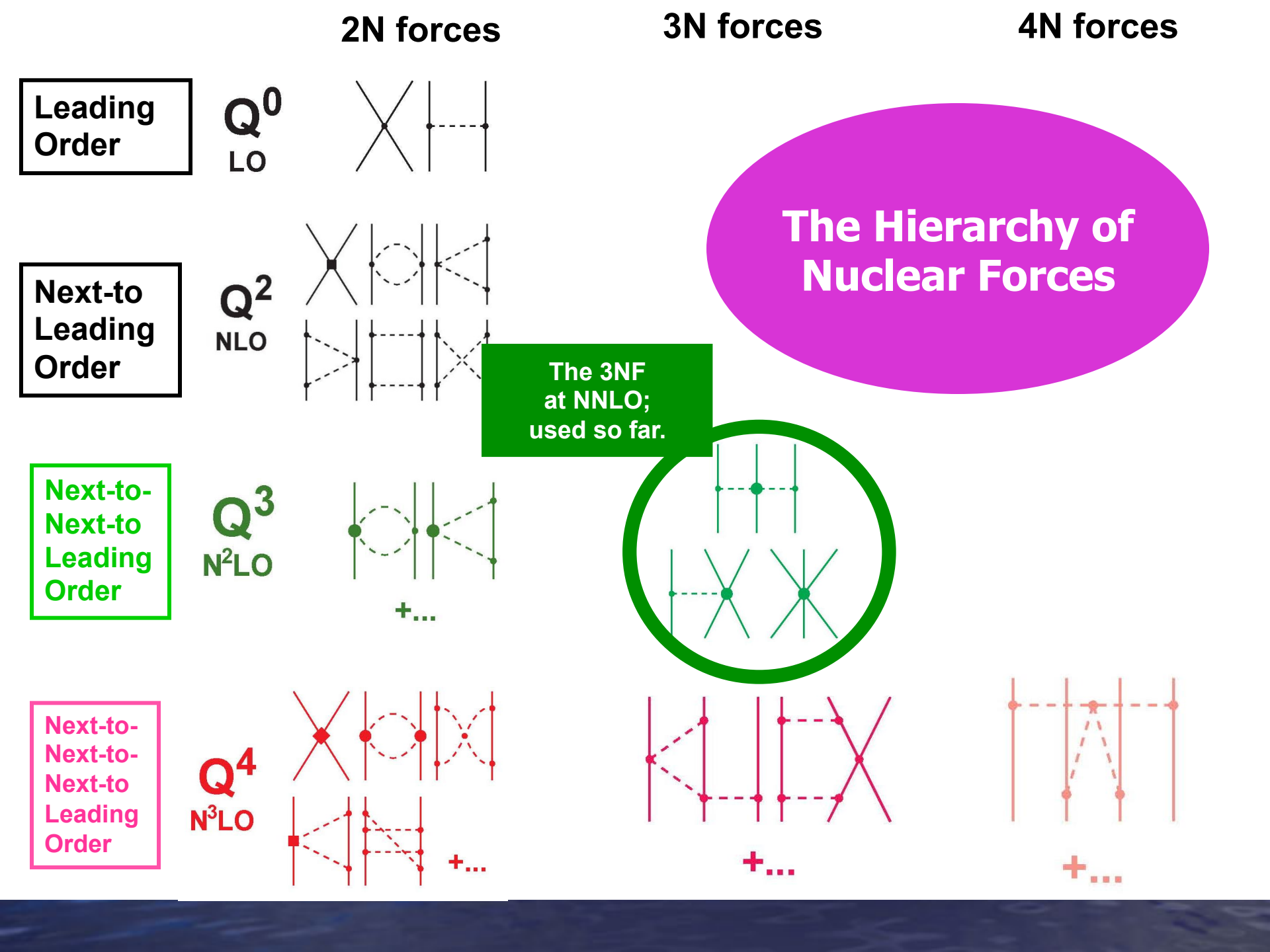
Three-body interactions are difficult –
in human life …

… and in physics



Fujita-Miyazawa, 1957

Status of
Three-body forces
50+ years ago


2N forces
3N forces
4N forces
Leading Order
Q^0
LO
Next-to Leading Order
Q^2
NLO
The Hierarchy of Nuclear Forces
The 3NF at NNLO; used so far.
Next-to-Next-to Leading Order
Q^3
N^2LO
+...
Next-to-Next-to-Next-to Leading Order
Q^4
N^3LO
+...
+...
+...

Now, showing only 3NF diagrams.

Chiral 3N Force

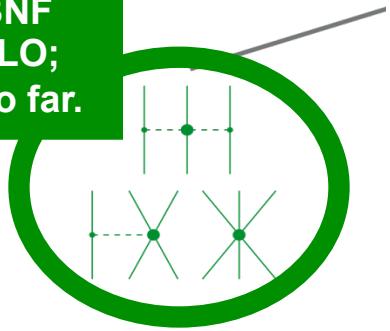
Δ-less

LO
$(Q/\Lambda_\chi)^0$

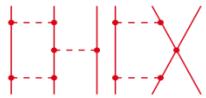
NLO
$(Q/\Lambda_\chi)^2$

The 3NF at NNLO; used so far.

NNLO
$(Q/\Lambda_\chi)^3$

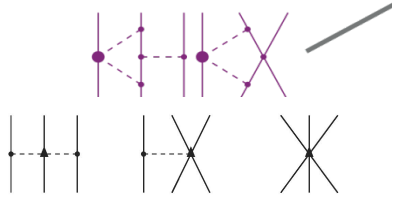


N^{3}LO
$(Q/\Lambda_\chi)^4$



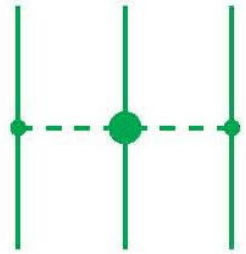
+...

N^{4}LO
$(Q/\Lambda_\chi)^5$



Three-nucleon forces at N2LO

2PE-3NF



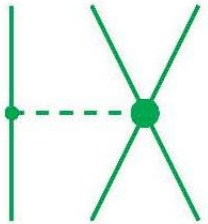
$$V_{\rm TPE}^{\rm 3NF} = \left(\frac{g_A}{2f_\pi}\right)^2 \frac{1}{2} \sum_{i\neq j\neq k} \frac{(\vec{\sigma}_i\cdot\vec{q}_i)(\vec{\sigma}_j\cdot\vec{q}_j)}{(q_i^2+m_\pi^2)(q_j^2+m_\pi^2)} F_{ijk}^{\alpha\beta}\,\tau_i^\alpha\tau_j^\beta$$

with $\vec{q}_i \equiv \vec{p}_i{}' - \vec{p}_i$, where $\vec{p}_i$ and $\vec{p}_i{}'$ are the initial and final momenta of nucleon i, respectively, and

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta}\left[-\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2}\,\vec{q}_i\cdot\vec{q}_j\right] + \frac{c_4}{f_\pi^2}\sum_\gamma \epsilon^{\alpha\beta\gamma}\,\tau_k^\gamma\,\vec{\sigma}_k\cdot[\vec{q}_i\times\vec{q}_j]$$

No new parameters!

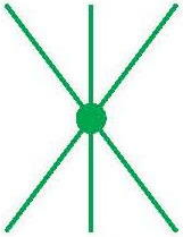
1PE-3NF



$$V_{\rm OPE}^{\rm 3NF} = D\,\frac{g_A}{8f_\pi^2}\sum_{i\neq j\neq k}\frac{\vec{\sigma}_j\cdot\vec{q}_j}{q_j^2+m_\pi^2}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)(\vec{\sigma}_i\cdot\vec{q}_j)$$

One new parameter, D.

Contact-3NF



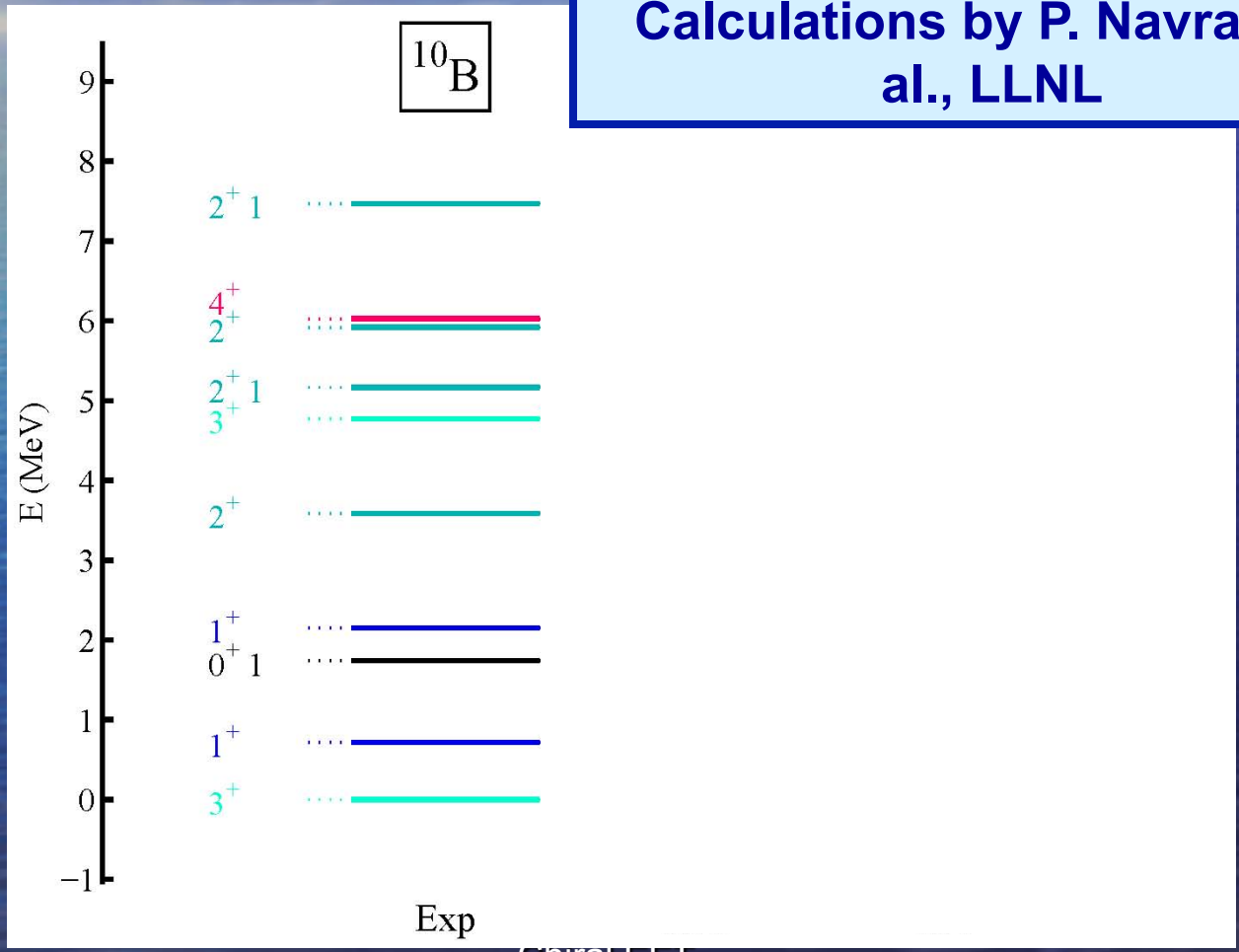
$$V_{\rm ct}^{\rm 3NF} = E\,\frac{1}{2}\sum_{j\neq k}\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k\,.$$

One new parameter, E.

Strategy: Adjust D and E to two few-nucleon observables, e.g., the triton and alpha-particle binding energies. Then predict properties of other light nuclei.

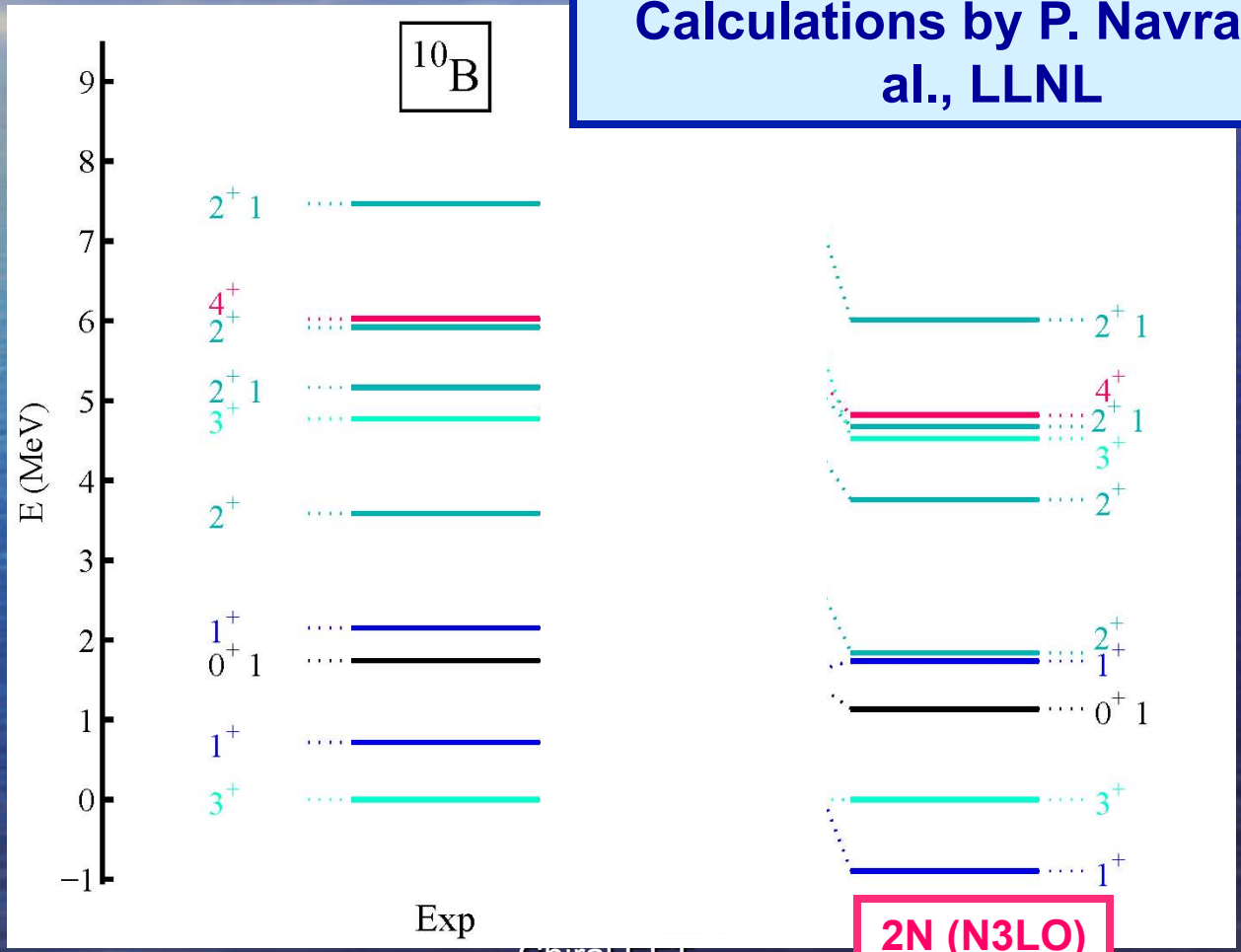
Calculating the properties of light nuclei using chiral 2N and 3N forces

"No-Core Shell Model "
Calculations by P. Navratil et al., LLNL



Calculating the properties of light nuclei using chiral 2N and 3N forces

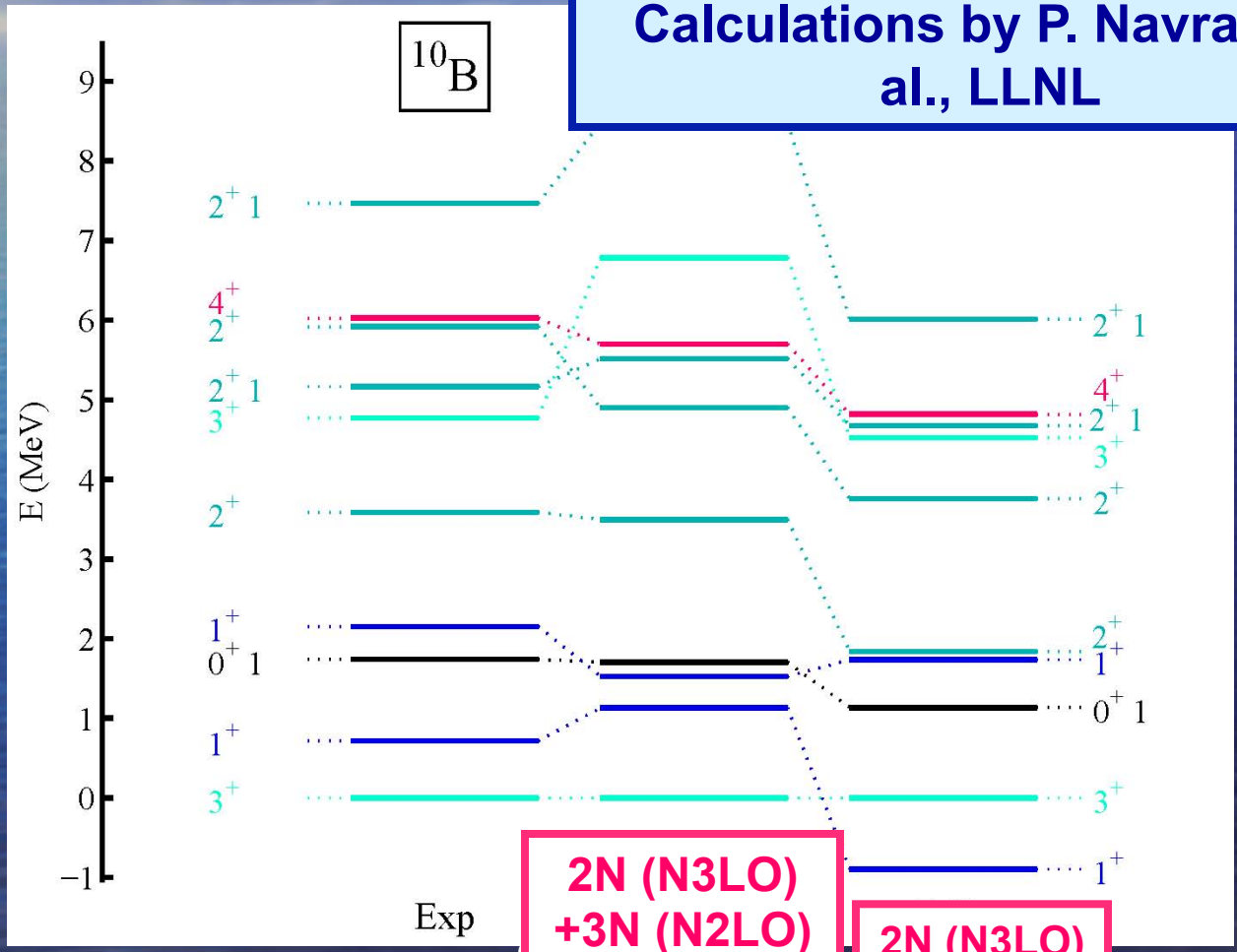
"No-Core Shell Model "
Calculations by P. Navratil et al., LLNL



2N (N3LO) force only

Calculating the properties of light nuclei using chiral 2N and 3N forces

"No-Core Shell Model " Calculations by P. Navratil et al., LLNL



PRL **108,** 242501 (2012) PHYSICAL REVIEW LETTERS week ending 15 JUNE 2012

Continuum Effects and Three-Nucleon Forces in Neutron-Rich Oxygen Isotopes

G. Hagen,[1,2] M. Hjorth-Jensen,[3,4,5] G. R. Jansen,[3] R. Machleidt,[6] and T. Papenbrock[2,1]

PRL **109,** 032502 (2012) PHYSICAL REVIEW LETTERS week ending 20 JULY 2012

Evolution of Shell Structure in Neutron-Rich Calcium Isotopes

G. Hagen,[1,2] M. Hjorth-Jensen,[3,4] G. R. Jansen,[3] R. Machleidt,[5] and T. Papenbrock[1,2]

Oxygen

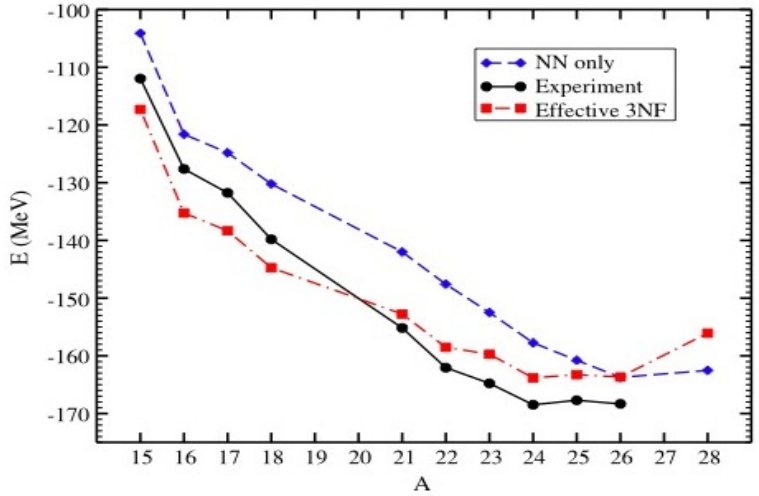


FIG. 1 (color online). Ground-state energy of the oxygen isotope ^{A}O as a function of the mass number A. Black circles: experimental data; blue diamonds: results from nucleon-nucleon interactions; red squares: results including the effects of three-nucleon forces.

Calcium

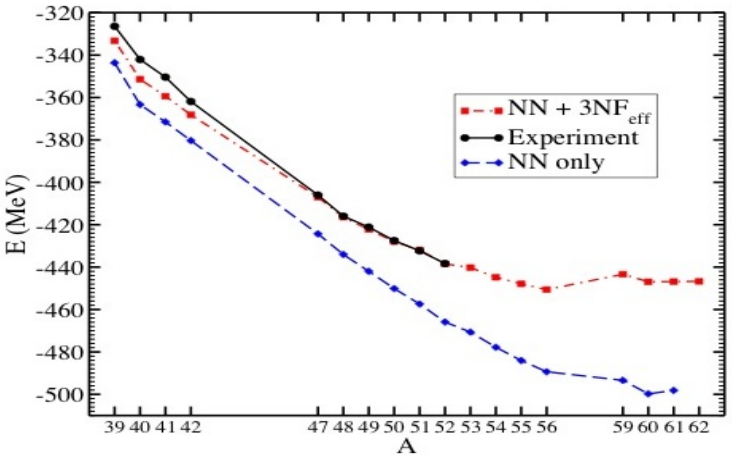


FIG. 1: (Color online) Ground-state energy of the calcium isotopes as a function of the mass number A. Black circles: experimental data; red squares: theoretical results including the effects of three-nucleon forces; blue diamonds: predictions from chiral NN forces alone. The experimental results for ^{51,52}Ca are from Ref. [34].

In-Medium Similarity Renormalization Group with Chiral Two- Plus Three-Nucleon Interactions

H. Hergert,[1] S. K. Bogner,[2] S. Binder,[3] A. Calci,[3] J. Langhammer,[3] R. Roth,[3] and A. Schwenk[4,3]

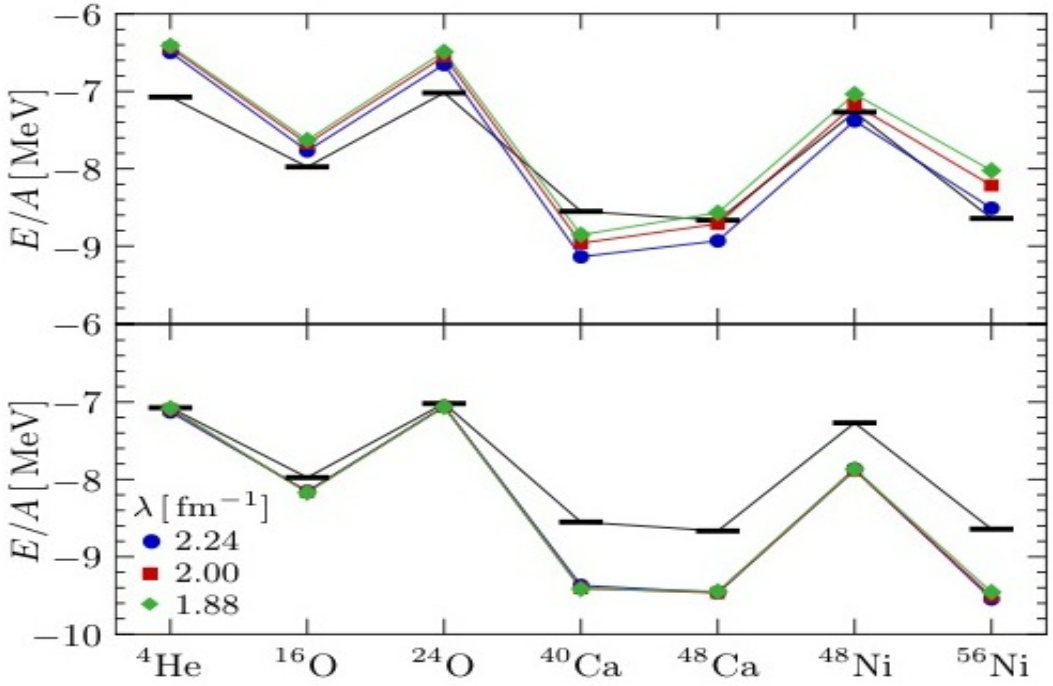


FIG. 7. IM-SRG(2) ground-state energy per nucleon of closed-shell nuclei for $NN + 3N$-induced (top) and $NN + 3N$-full Hamiltonians (bottom) at different resolution scales λ. Energies are determined at optimal $\hbar\Omega$ for $e_{\mathrm{Max}} = 14$. Experimental energies (black bars) are taken from [37].

Analyzing Power Ay

p-d

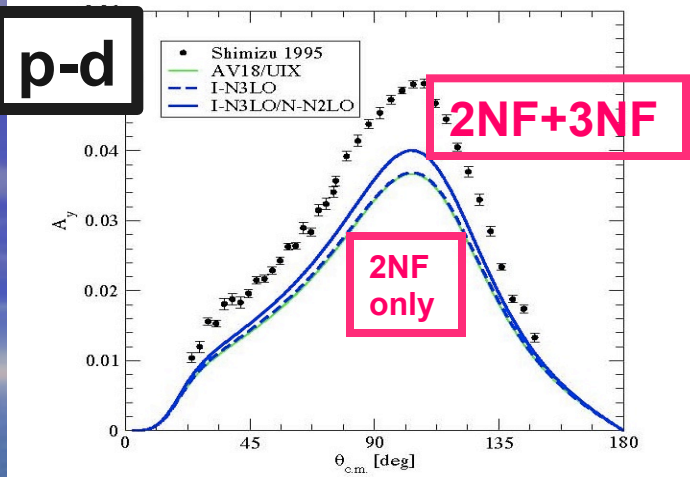


Fig. 9. $p-d$ observable A_y at $E_p = 3$ MeV calculated with the I-N3LO (blue dashed line), I-N3LO/N-N2LO (blue solid line) AV18/UIX (thin green solid line) interaction mod... MeV. The experimental data are from Ref...

Calculations by the Pisa Group

The Ay puzzle is NOT solved by the 3NF at NNLO.

p-^{3}He

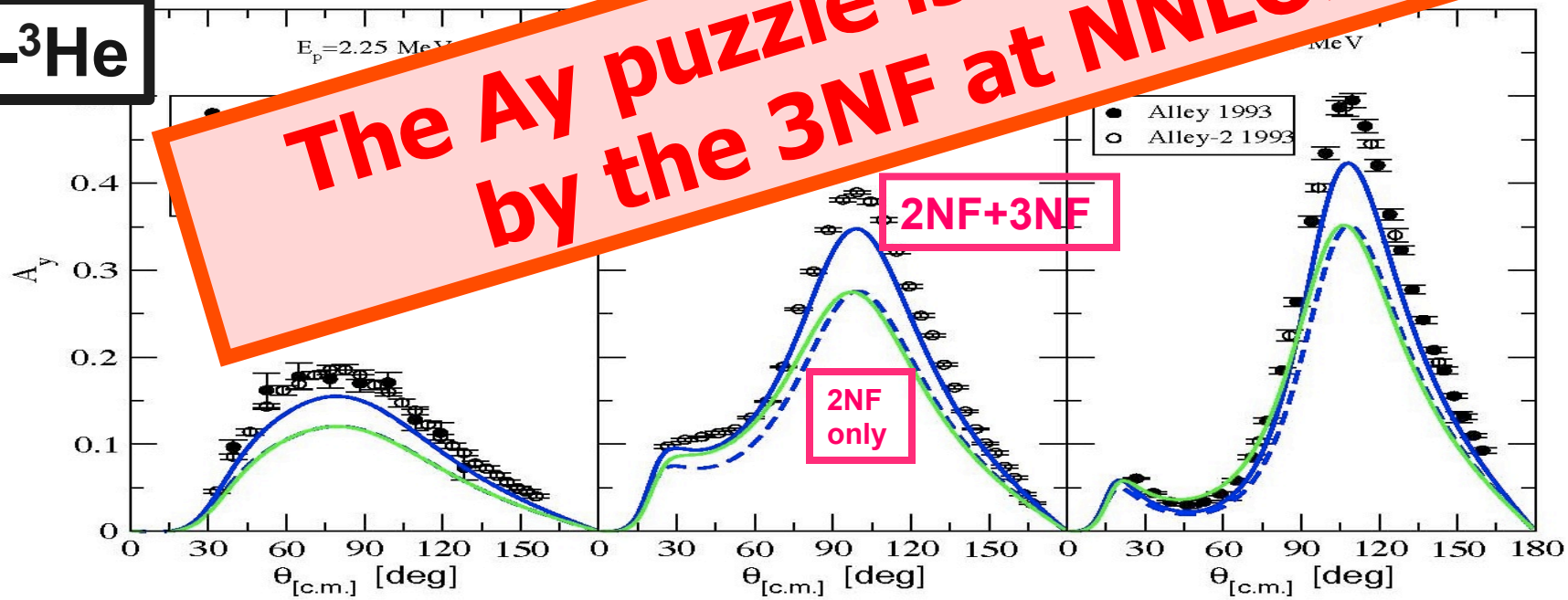


Fig. 6. $p-{}^3$He A_y observable calculated with the I-N3LO (blue dashed line), the I-N3LO/N-N2LO (blue solid line), and the AV18/UIX (thin green solid line) interaction models for three different incident proton energies. The experimental data are from Refs. [37,22,36].

And so,
we need 3NFs beyond NNLO,
because ...

- The 2NF is N3LO;

  consistency requires that all contributions are included up to the same order.

- There are unresolved problems in 3N and 4N scattering, and nuclear structure.

Back to the drawing board.

Chiral 3N Force

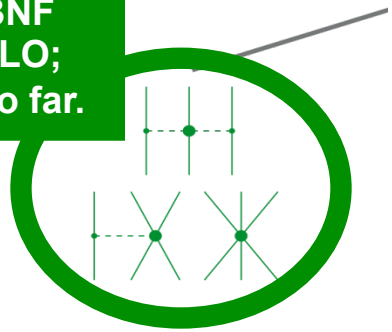
Δ-less

LO
$(Q/\Lambda_\chi)^0$

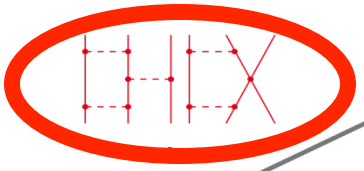
NLO
$(Q/\Lambda_\chi)^2$

The 3NF at NNLO; used so far.

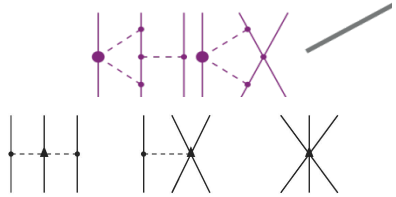
NNLO
$(Q/\Lambda_\chi)^3$



N^{3}LO
$(Q/\Lambda_\chi)^4$



N^{4}LO
$(Q/\Lambda_\chi)^5$



Chiral 3N Force

Δ-less

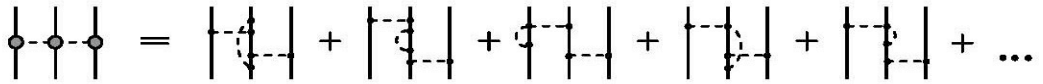
LO

$(Q/\Lambda_\chi)^0$

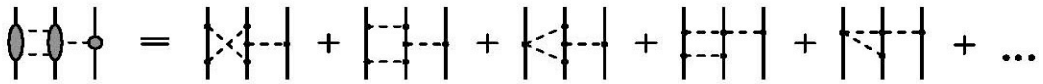
NLO

$(Q/\Lambda_\chi)^2$

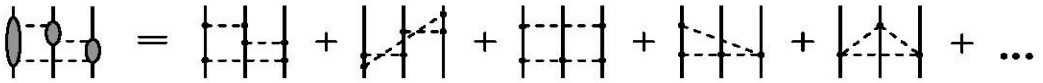
2π–exchange



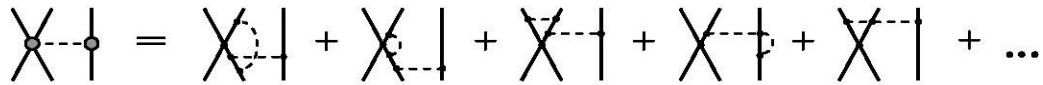
2π-1π–exchange



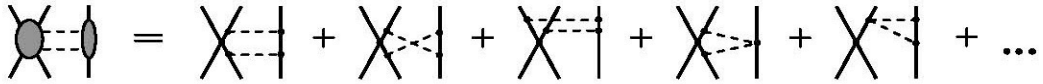
ring diagrams



contact-1π–exchange



contact-2π–exchange



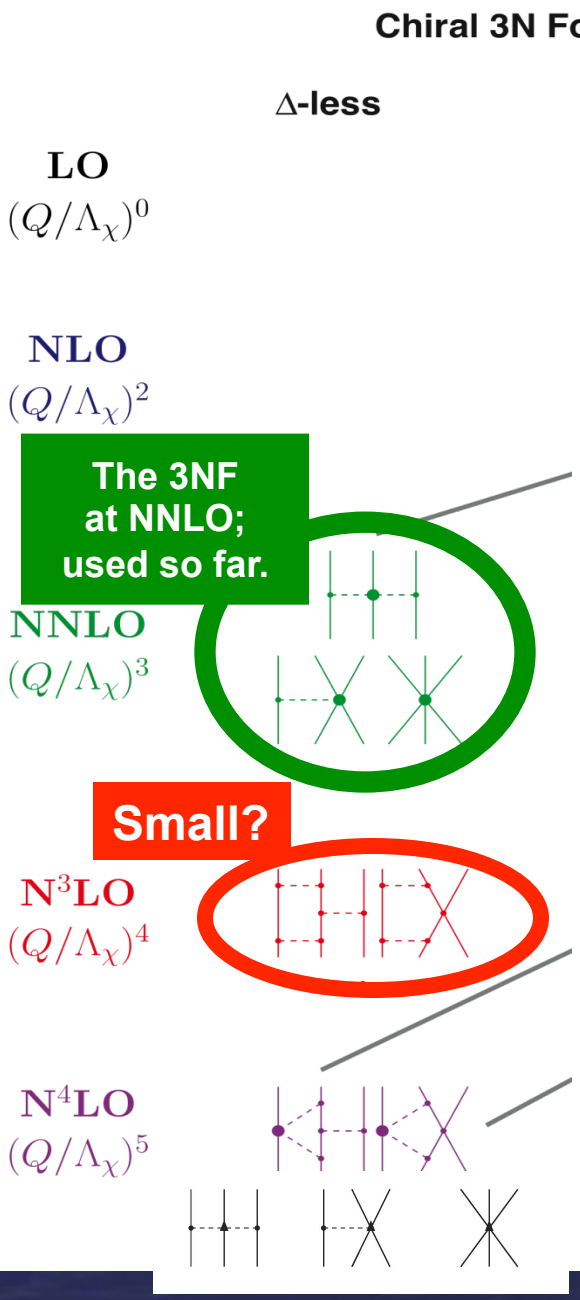
R. Machleidt

Apps of N3LO 3NF:

Triton: Skibinski et al., PRC 84, 054005 (2011).
Not conclusive.

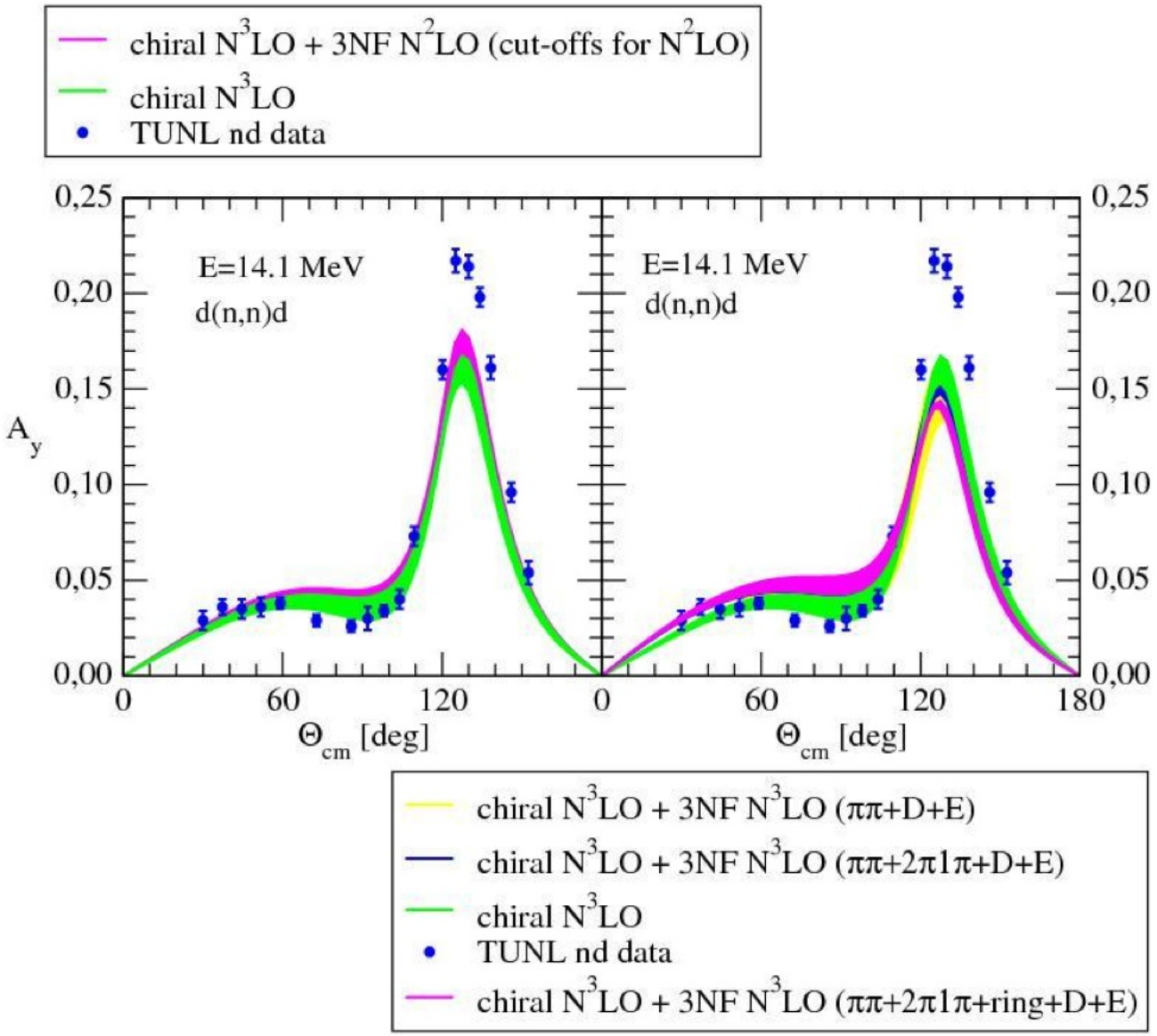
Neutron matter:
Hebeler, Schwenk and co-workers, PRL 110, 032504 (2013).
Not small!(?)

N-d scattering (Ay):
Witala et al.
Small!



A_y puzzle

Calculations by Witala et al.

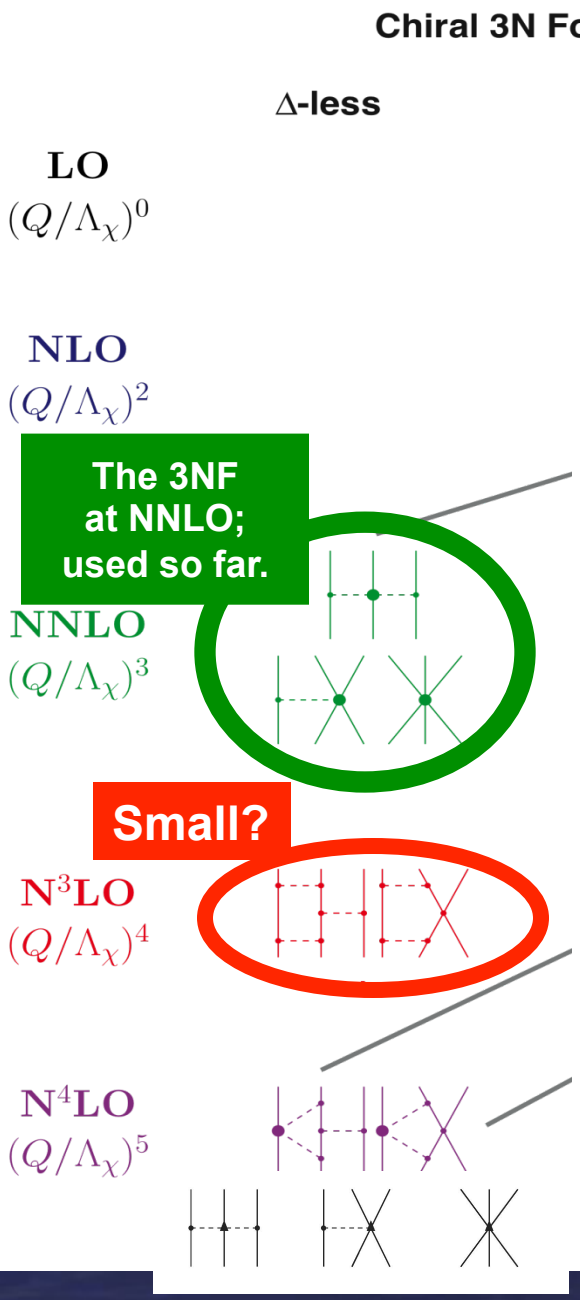


Apps of N3LO 3NF:

Triton: Skibinski et al., PRC 84, 054005 (2011).
Not conclusive.

Neutron matter:
Hebeler, Schwenk and co-workers, PRL 110, 032504 (2013).
Not small!(?)

N-d scattering (Ay):
Witala et al.
Small!



Chiral 3N Force

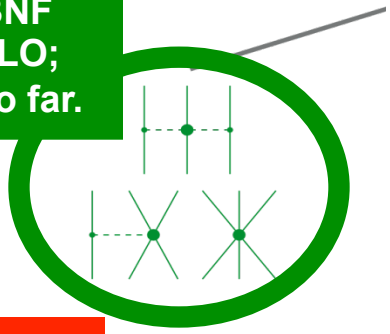
Δ-less

LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

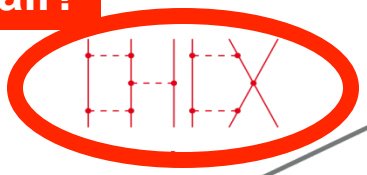
The 3NF at NNLO; used so far.

NNLO
$(Q/\Lambda_\chi)^3$

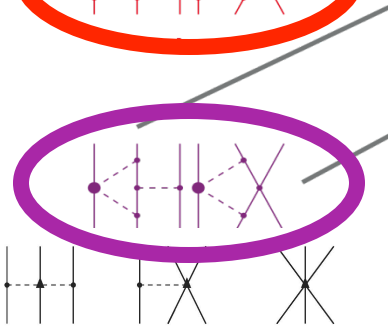


Small?

N^{3}LO
$(Q/\Lambda_\chi)^4$



N^{4}LO
$(Q/\Lambda_\chi)^5$



Chiral 3N Force

Δ-less

LO

$(Q/\Lambda_\chi)^0$

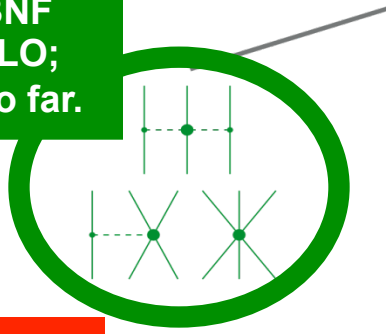
NLO

$(Q/\Lambda_\chi)^2$

The 3NF at NNLO; used so far.

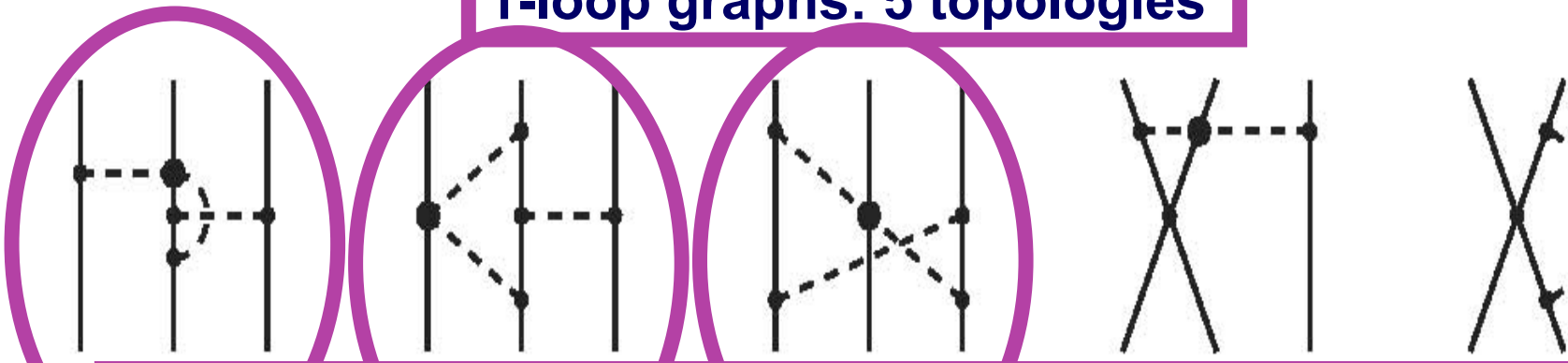
NNLO

$(Q/\Lambda_\chi)^3$



Small?

1-loop graphs: 5 topologies



Chiral 3N Force

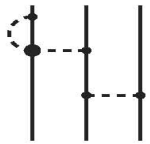
Δ-less

LO
$(Q/\Lambda_\chi)^0$

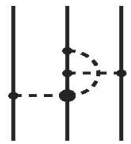
NLO
$(Q/\Lambda_\chi)^2$

The 3NF at NNLO; used so far.

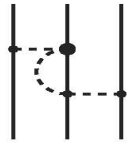
NNLO
$(Q/\Lambda_\chi)^3$

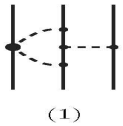


(2)

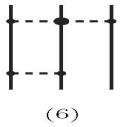


(7)

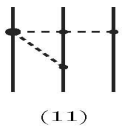




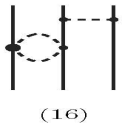
(1)



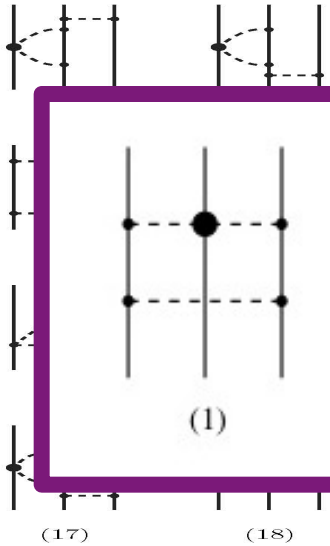
(6)



(11)



(16)



(17) (18)

(1) (2) (3) (4)

ntact-1PE Contac

Chiral 3N Force

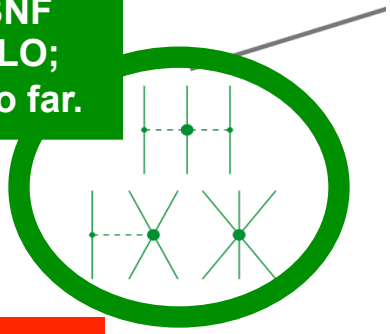
Δ-less

LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

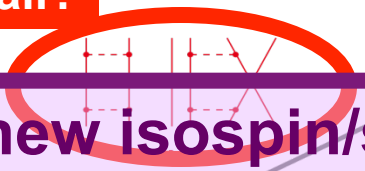
The 3NF at NNLO; used so far.

NNLO
$(Q/\Lambda_\chi)^3$



Small?

N^{3}LO
$(Q/\Lambda_\chi)^4$



Many new isospin/spin/momentum structures.

N^{4}LO
$(Q/\Lambda_\chi)^5$

Large?!

Chiral 3N Force

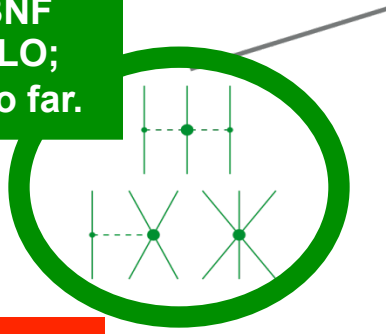
Δ-less

LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

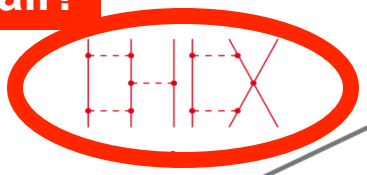
The 3NF at NNLO; used so far.

NNLO
$(Q/\Lambda_\chi)^3$

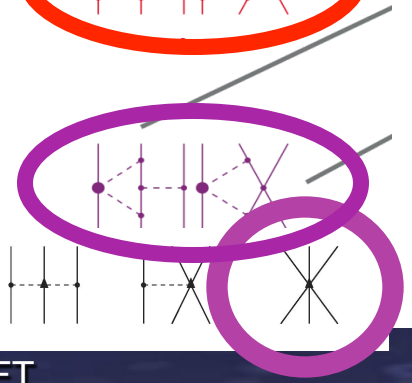


Small?

N^{3}LO
$(Q/\Lambda_\chi)^4$



N^{4}LO
$(Q/\Lambda_\chi)^5$



Chiral 3N Force

Δ-less

LO

$(Q/\Lambda_\chi)^0$

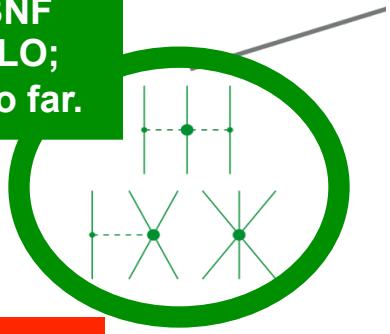
NLO

$(Q/\Lambda_\chi)^2$

The 3NF at NNLO; used so far.

NNLO

$(Q/\Lambda_\chi)^3$



Small?

3NF contacts at N4LO

Girlanda, Kievsky, Viviani, PRC 84, 014001 (2011)

$\mathbf{k}_i = \mathbf{p}_i - \mathbf{p}'_i$ and $\mathbf{Q}_i = \mathbf{p}_i + \mathbf{p}'_i$, $\mathbf{p}_i$ and $\mathbf{p}'_i$ being the initial and final momenta of nucleon i, the potential in momen
space is found to be

$$V = \sum_{i\neq j\neq k} \Big[-E_1\mathbf{k}_i^2 - E_2\mathbf{k}_i^2\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j - E_3\mathbf{k}_i^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j - E_4\mathbf{k}_i^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$$

$$-E_5\left(3\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_i\cdot\boldsymbol{\sigma}_j - \mathbf{k}_i^2\right) - E_6\left(3\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_i\cdot\boldsymbol{\sigma}_j - \mathbf{k}_i^2\right)\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$$

$$+\frac{i}{2}E_7\mathbf{k}_i\times(\mathbf{Q}_i-\mathbf{Q}_j)\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j) + \frac{i}{2}E_8\mathbf{k}_i\times(\mathbf{Q}_i-\mathbf{Q}_j)\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k$$

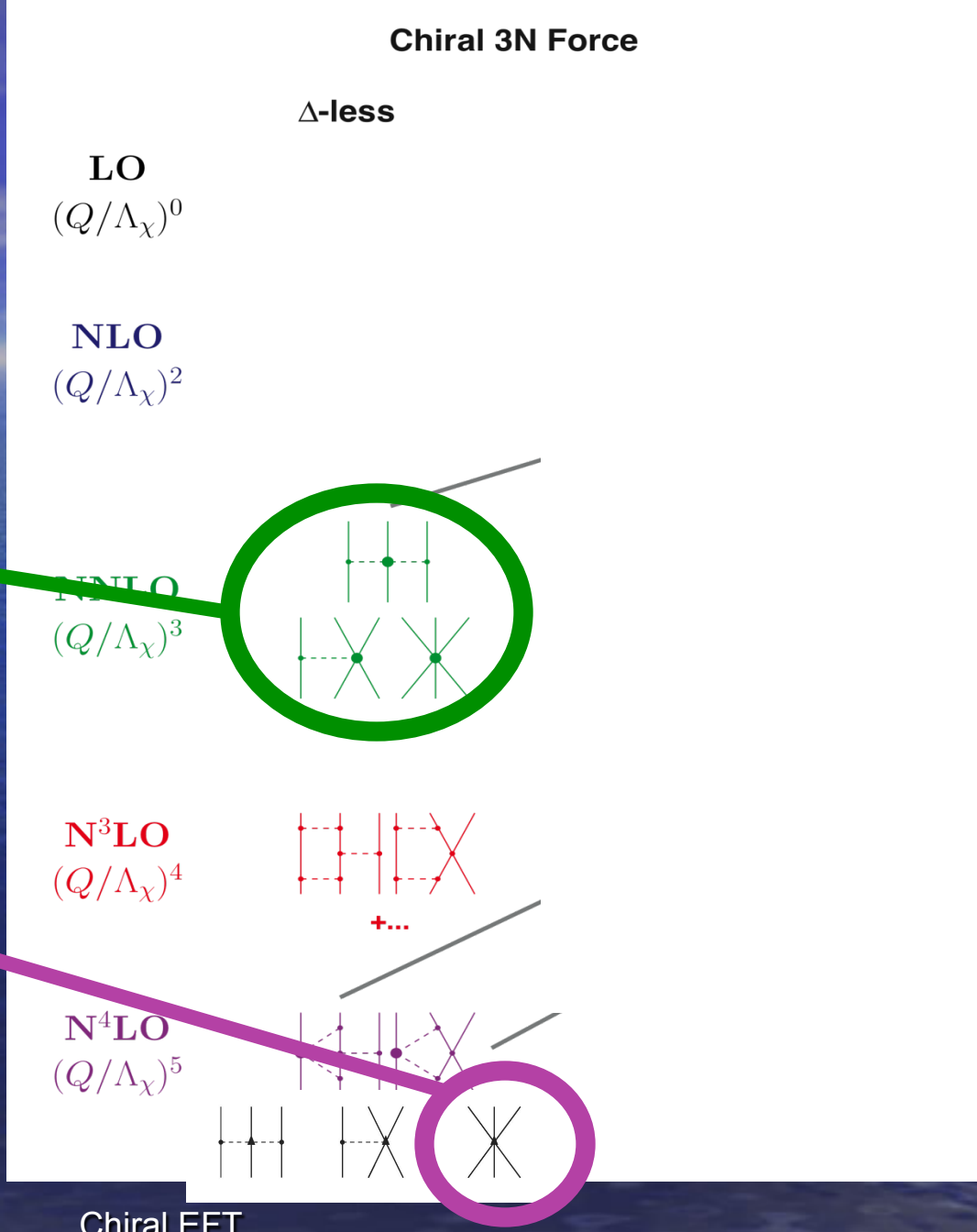
Spin-Orbit Force!

R. Machleidt

A realistic, investigational approach:

- use Δ-less
- include NNLO 3NF
- skip N3LO 3NF

- at N4LO start with contact 3NF, use one term at a time, e.g. spin-orbit
- that may already solve some of your problems.



… and then there
Is also the
Δ-full theory …

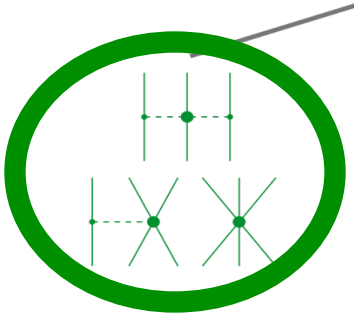
Chiral 3N Force

Δ-less

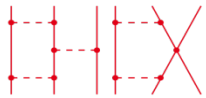
LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

NNLO
$(Q/\Lambda_\chi)^3$

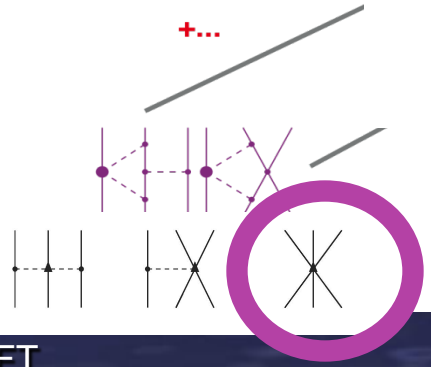


N^{3}LO
$(Q/\Lambda_\chi)^4$



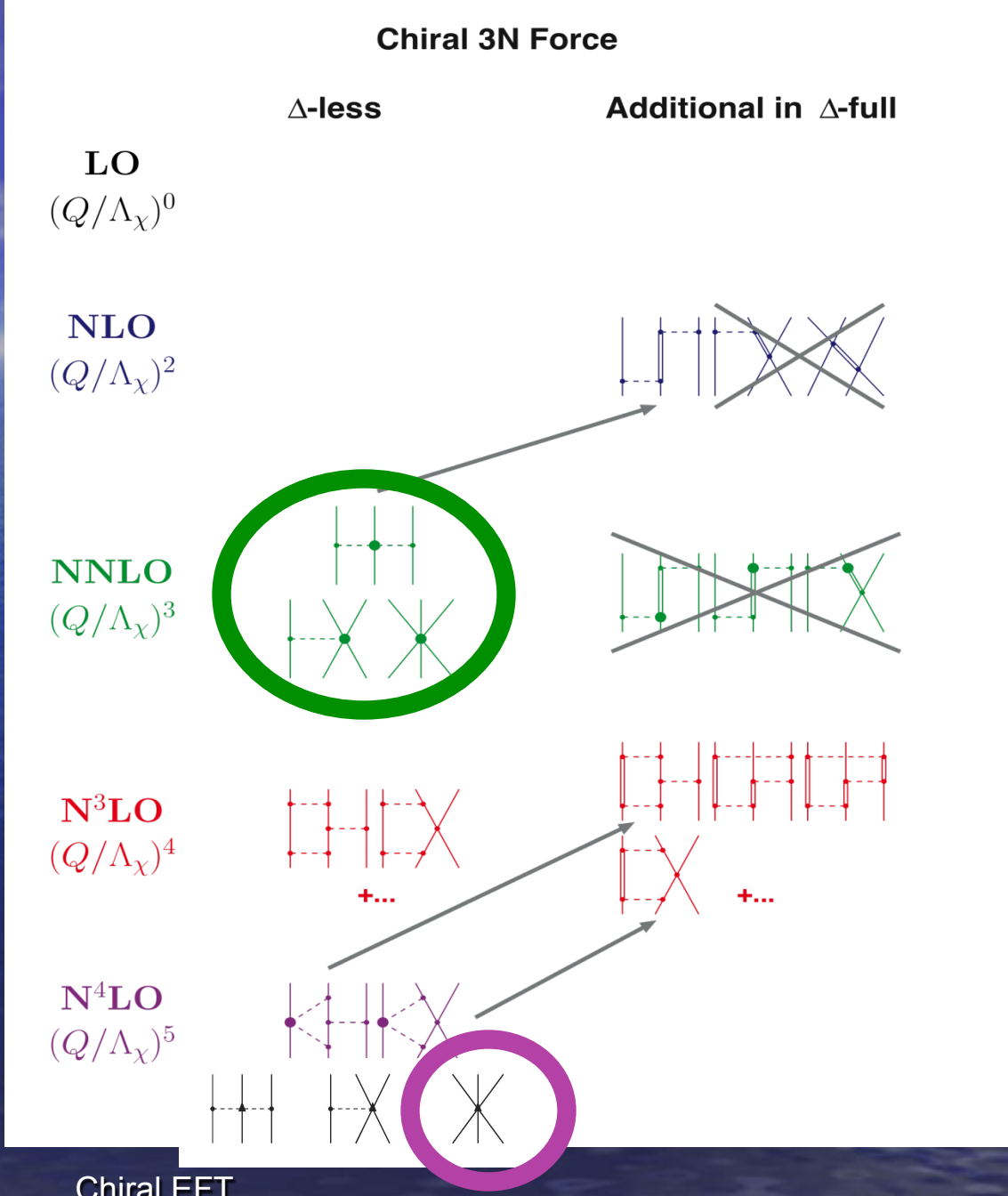
+...

N^{4}LO
$(Q/\Lambda_\chi)^5$



… and then there
Is also the
Δ-full theory …

… but we have no
time left for that.



Conclusions

- **Nuclear physics from first principals is real!**
- **Nuclear forces have been/are being derived from QCD via chiral EFT: 2NFs, 3NFs, 4NFs; order by order.**
- **In parallel, exact few-body and *ab initio* many-body methods have been developed, are tested, and refined, in which those chiral interactions are applied.**
- **New and very promising results are reported almost daily.**
- **But there are still issues.**

  The most intriguing one:

  A full knowledge of the 3NF

Conclusions

- Nuclear physics from first principals is real!
- Nuclear forces have be
  via chiral EFT: 2NFs, 3
- In parallel, exact few-
  methods have been de
  refined, in which thos
- New and very promisi
  daily.
- But there are still issu

  The most intriguing or

  A full knowledge of the 3NF