Short-range correlations studied with unitarily transformed interactions and operators in the NCSM

Thomas Neff Hans Feldmeier Dennis Weber

GSI Helmholtzzentrum für Schwerionenforschung, Darmstadt

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Motivation

Short-range correlations and high-momentum components in wave functions

Subedi *et al.*, Science **320**, 1476 (2008)

Wiringa, Schiavilla, Pieper, Carlson, PRC **85**, 021001(R) (2008)

Interaction dependence – AV18 versus N^{3}LO

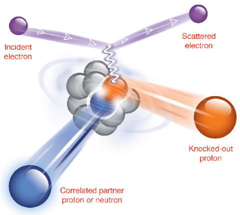
Wiringa, Stoks, Schiavilla, PRC **51**, 38 (1995)

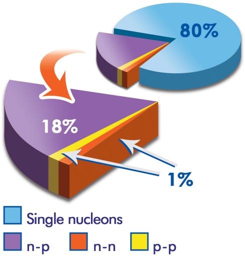
Entem, Machleidt, PRC **68**, 041001 (2003)

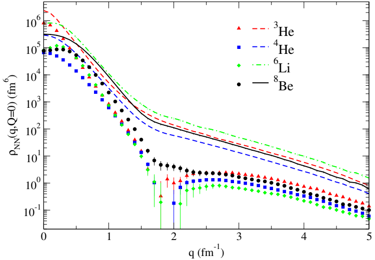
High-momentum components from low-momentum interactions

Anderson, Bogner, Furnstahl, Perry, PRC **82**, 054001 (2010)

Bogner, Roscher, arXiv:1208.1734v1

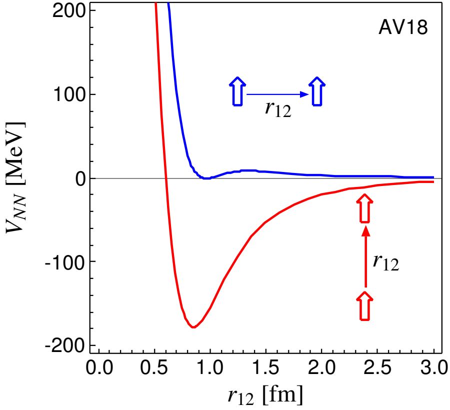






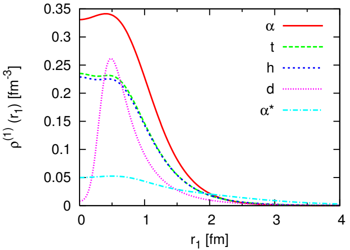
Short-range and tensor correlations

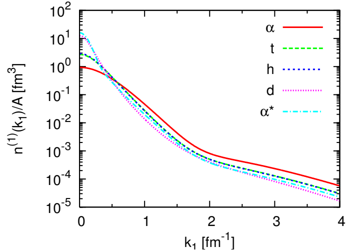
$T = 0$



- strong repulsive core: nucleons can not get closer than ≈ 0.5 fm→ **central correlations**
- strong dependence on the orientation of the spins due to the tensor force → **tensor correlations**
- the nuclear force will induce **strong short-range correlations** in the nuclear wave function

One-body densities for *A*=2,3,4 nuclei





Feldmeier, Horiuchi, Neff, Suzuki, PRC **84**, 054003 (2011)

$$\rho^{(1)}(\boldsymbol{r}_1) = \langle\Psi|\sum_{i=1}^{A}\delta^3(\hat{\boldsymbol{r}}_i - \boldsymbol{r}_1)|\Psi\rangle$$

$$n^{(1)}(\boldsymbol{k}_1) = \langle\Psi|\sum_{i=1}^{A}\delta^3(\hat{\boldsymbol{k}}_i - \boldsymbol{k}_1)|\Psi\rangle$$

- one-body densities calculated from **exact wave functions** (Correlated Gaussian Method) for AV8' interaction
- coordinate space densities reflect different sizes and densities of ^{2}H, ^{3}H, ^{3}He, ^{4}He and the 0_2^+ state in ^{4}He
- similar high-momentum tails in the one-body momentum distributions

Two-body densities

of pairs in given spin-, isospin channels

$$\rho^{(2)}_{SM_S,TM_T}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \langle\Psi|\sum_{i<j}^{A}\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,\delta^3(\hat{\boldsymbol{r}}_i-\boldsymbol{r}_1)\delta^3(\hat{\boldsymbol{r}}_j-\boldsymbol{r}_2)\,|\Psi\rangle$$

$$n^{(2)}_{SM_S,TM_T}(\boldsymbol{k}_1,\boldsymbol{k}_2) = \langle\Psi|\sum_{i<j}^{A}\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,\delta^3(\hat{\boldsymbol{k}}_i-\boldsymbol{k}_1)\delta^3(\hat{\boldsymbol{k}}_j-\boldsymbol{k}_2)\,|\Psi\rangle$$

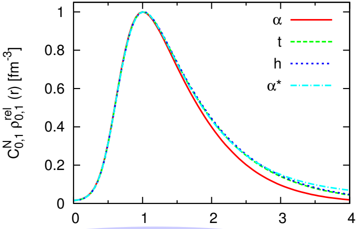
integrated over center-of-mass position $\boldsymbol{R} = \frac{1}{2}(\boldsymbol{r}_1+\boldsymbol{r}_2)$ or the total momentum of the nucleon pair $\boldsymbol{K} = \boldsymbol{k}_1+\boldsymbol{k}_2$:

$$\rho^{\text{rel}}_{SM_S,TM_T}(\boldsymbol{r}) = \langle\Psi|\sum_{i<j}^{A}\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,\delta^3(\hat{\boldsymbol{r}}_i-\hat{\boldsymbol{r}}_j-\boldsymbol{r})\,|\Psi\rangle$$

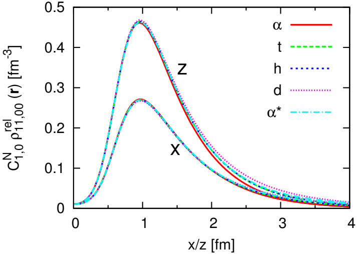
$$n^{\text{rel}}_{SM_S,TM_T}(\boldsymbol{k}) = \langle\Psi|\sum_{i<j}^{A}\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,\delta^3\left(\frac{1}{2}(\hat{\boldsymbol{k}}_i-\hat{\boldsymbol{k}}_j)-\boldsymbol{k}\right)|\Psi\rangle$$

Two-body densities in coordinate space for *A*=2,3,4

$S = 0, T = 1$

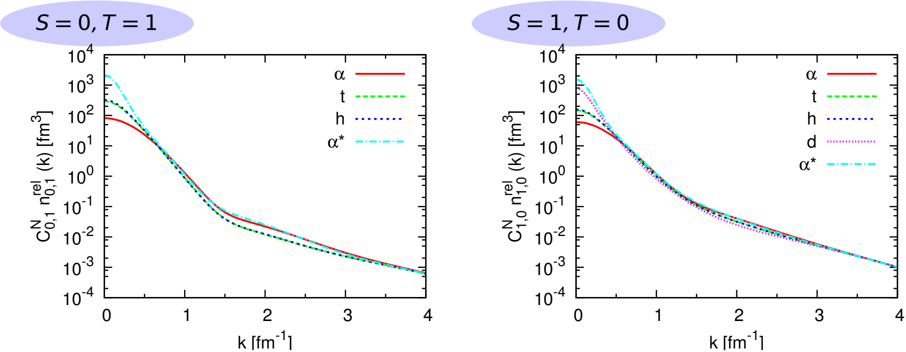


$S = 1, M_S = 1, T = 0$



- two-body densities calculated from **exact wave functions** (Correlated Gaussian Method) for AV8’ interaction
- coordinate space two-body densities reflect correlation hole and tensor correlations
- → normalize two-body density in coordinate space at *r*=1.0 fm
- → normalized two-body densities in coordinate space are **identical at short distances** for all nuclei
- also true for angular dependence in the deuteron channel

Two-body densities in momentum space for A=2,3,4



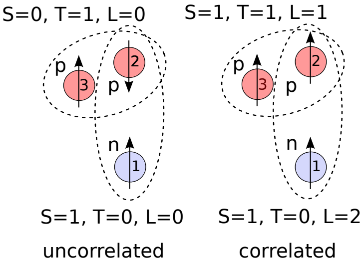
- use **normalization factors fixed in coordinate space**
- two-body densities in momentum space identical for momenta $k > 3\text{fm}^{-1}$
- moderate nucleus dependence in momentum region $1.5\text{fm}^{-1} < k < 3\text{fm}^{-1}$

Feldmeier, Horiuchi, Neff, Suzuki, PRC **84**, 054003 (2011)

Two-body densities reflect many-body correlations

of pairs in the (ST) channels

nucleus	(00)	(01)	(10)	(11)
d	–	–	1	–
t	0.010	1.361	1.490	0.139
h	0.011	1.361	1.489	0.139
α	0.008	2.572	2.992	0.428
α^*	0.034	2.714	2.966	0.286



- occupation in (ST)=(10) almost exactly as in IPM
- (ST)=(01) significantly depopulated in favor of (ST)=(11)
- three-body correlations induced by the two-body tensor force: depopulation of (ST)=(01) channel is the price one has to pay for getting the full binding from the tensor force in the (ST)=(10) channel

Similarity Renormalization Group

What happens if we use SRG evolved interactions and operators ?

- Evolve Hamiltonian and unitary transformation matrix

$$\frac{d\hat{H}_\alpha}{d\alpha} = \left[\hat{\eta}_\alpha, \hat{H}_\alpha\right], \qquad \frac{d\hat{U}_\alpha}{d\alpha} = -\hat{U}_\alpha \hat{\eta}_\alpha$$

- Evolution is done on the N-body level – α-dependence can be used to investigate missing higher-order contributions
- **All operators have to be transformed** consistently

$$\hat{H}_\alpha = \hat{U}^\dagger_\alpha \hat{H} \hat{U}_\alpha, \qquad \hat{B}_\alpha = \hat{U}^\dagger_\alpha \hat{B} \hat{U}_\alpha$$

- Metagenerator

$$\hat{\eta}_\alpha = (2\mu)^2 \left[\hat{T}_{\text{int}}, \hat{H}_\alpha\right]$$

Similarity Renormalization Group

Outline of Calculation

- simultaneous SRG evolution for transformed Hamiltonian and transformation matrix **on the two-body level**

$$\frac{d\hat{H}_\alpha}{d\alpha} = \left[\hat{\eta}_\alpha, \hat{H}_\alpha\right], \qquad \frac{d\hat{U}_\alpha}{d\alpha} = -\hat{U}_\alpha \hat{\eta}_\alpha$$

- Solve many-body problem with SRG transformed Hamiltonian in the NCSM

$$\hat{H}_\alpha \left|\Psi_\alpha\right\rangle = E_\alpha \left|\Psi_\alpha\right\rangle$$

- Calculate two-body densities with "bare" and "effective" density operators

$$\rho_{\text{bare}} = \left\langle \Psi_\alpha \right| \hat{\rho} \left|\Psi_\alpha\right\rangle, \qquad \rho_{\text{effective}} = \left\langle \Psi_\alpha \right| \hat{U}_\alpha^\dagger \hat{\rho} \hat{U}_\alpha \left|\Psi_\alpha\right\rangle$$

- Investigate **convergence** of NCSM calculations and **α-dependence of two-body densities**

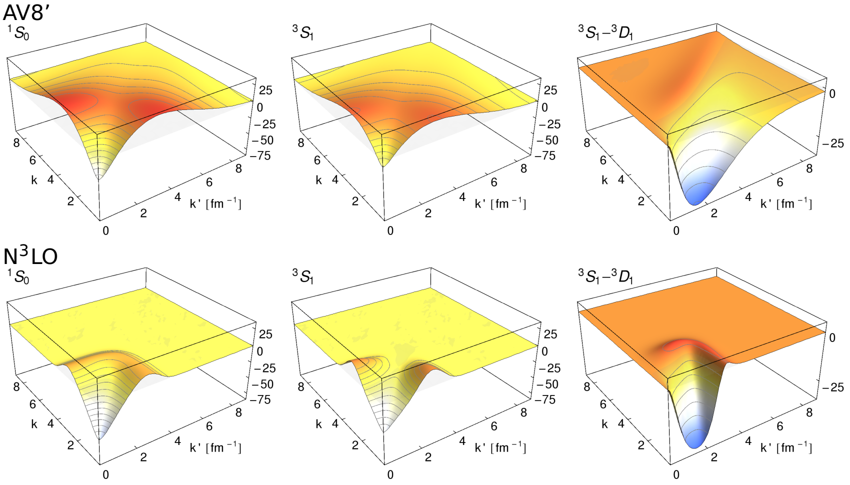
Similarity Renormalization Group

Implementation Details

- SRG evolution for $\hat{H}_\alpha$ and $\hat{U}_\alpha$ in momentum space, $k_{\max} = 15\mathrm{fm}^{-1}$
- Unitary transformation only acts on relative coordinates and not on the center-of-mass of the pairs
- (SRG transformed) momentum space matrix elements are expanded in HO basis
- *jj*-coupled matrix elements are calculated using the Talmi-Moshinski prescription
- *jj*-coupling slightly different than usual for two-body densities that depend on the pair momentum

Similarity Renormalization Group

Hamiltonian Flow

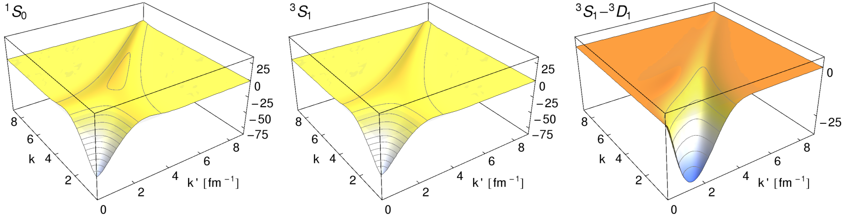


$\alpha = 0.00$ (bare)

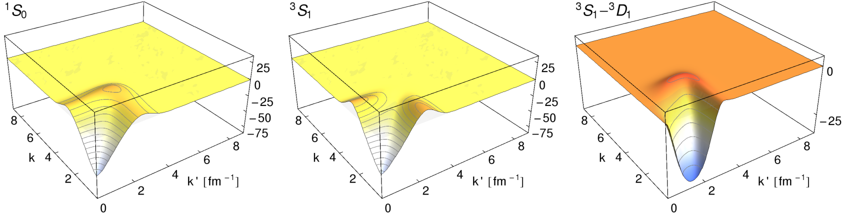
Similarity Renormalization Group

Hamiltonian Flow

AV8'



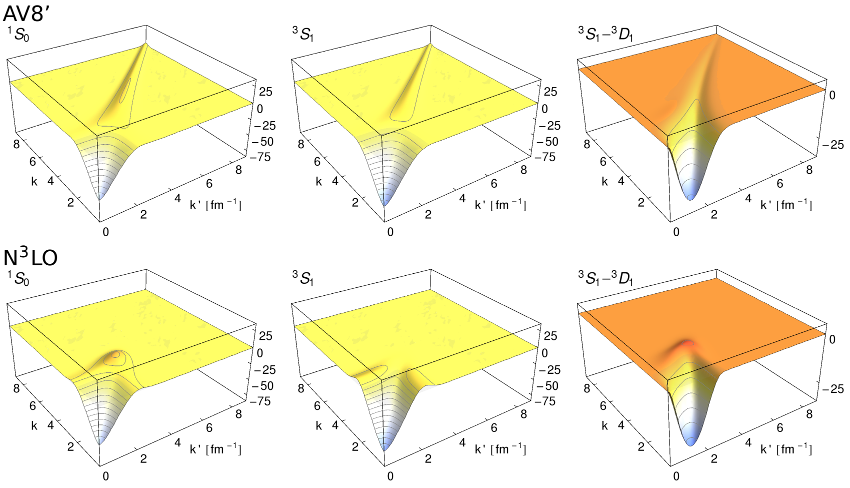
N^{3}LO



$\alpha = 0.01 \text{fm}^4$

Similarity Renormalization Group

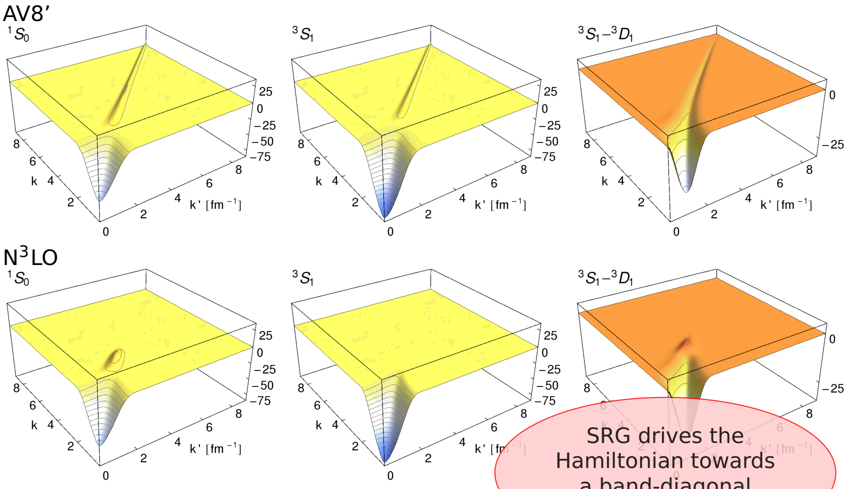
Hamiltonian Flow



$\alpha = 0.04\text{fm}^4$

Similarity Renormalization Group

Hamiltonian Flow



$\alpha = 0.20\text{fm}^4$

SRG drives the Hamiltonian towards a band-diagonal structure

^{4}He Two-body densities

^{4}He advantages

- two-body densities available for "bare" AV8' interaction
- "bare" N^{3}LO can be converged in NCSM

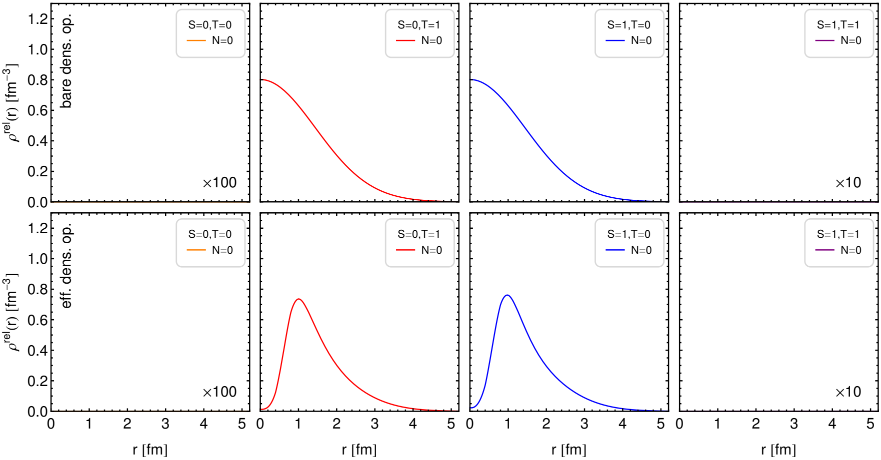
Objectives

- Compare AV8' and N^{3}LO results
- Check for NCSM convergence
- Check flow dependence $\alpha = 0, 0.01, 0.04, 0.20 \mathrm{fm}^4$ $(\Lambda = \infty, 3.16, 2.24, 1.50 \mathrm{fm}^{-1})$
- Can we see many-body effects ?

Convergence with the model space size

AV8' – α = 0.04 fm^4

Two-body Density in Coordinate Space



$\hbar\Omega = 20$MeV corresponds roughly to the size of ^{4}He

Convergence with the model space size

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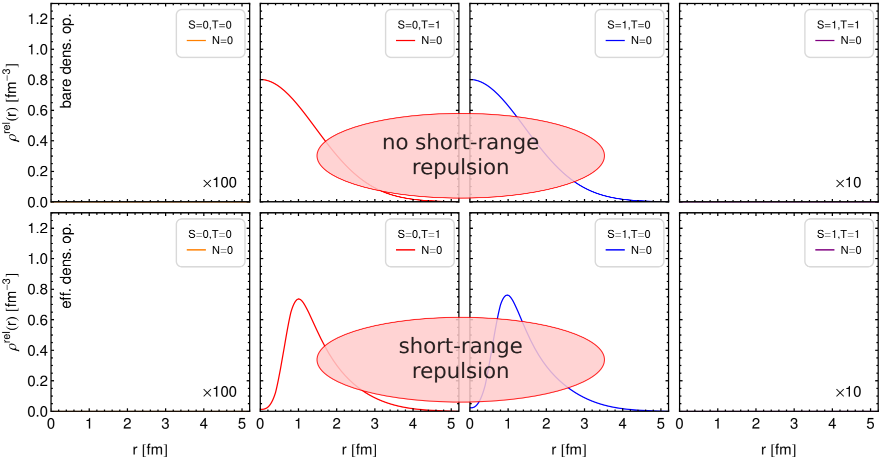

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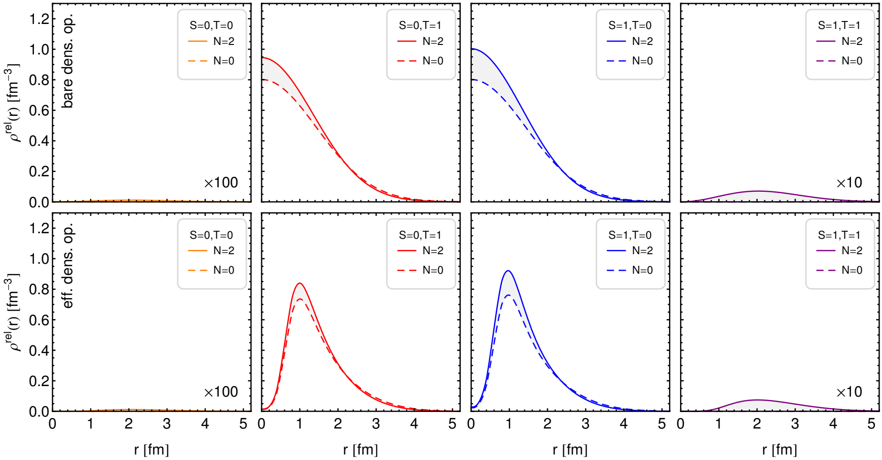

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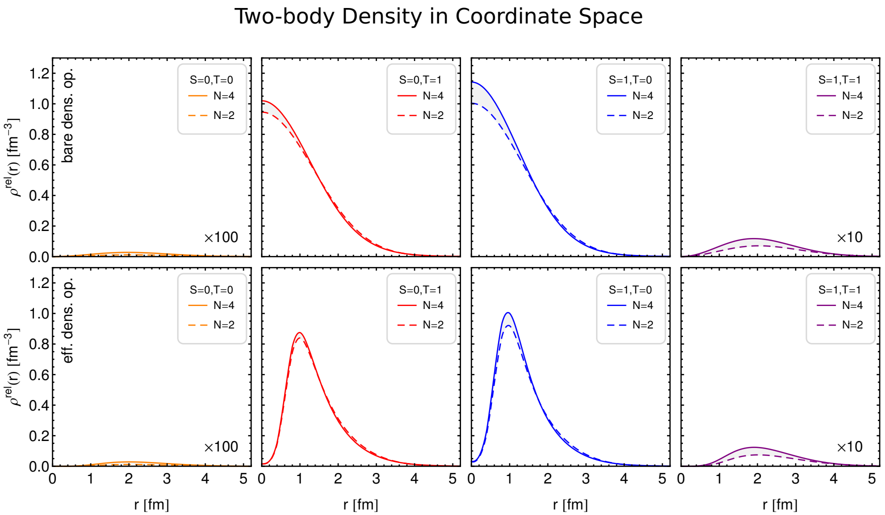

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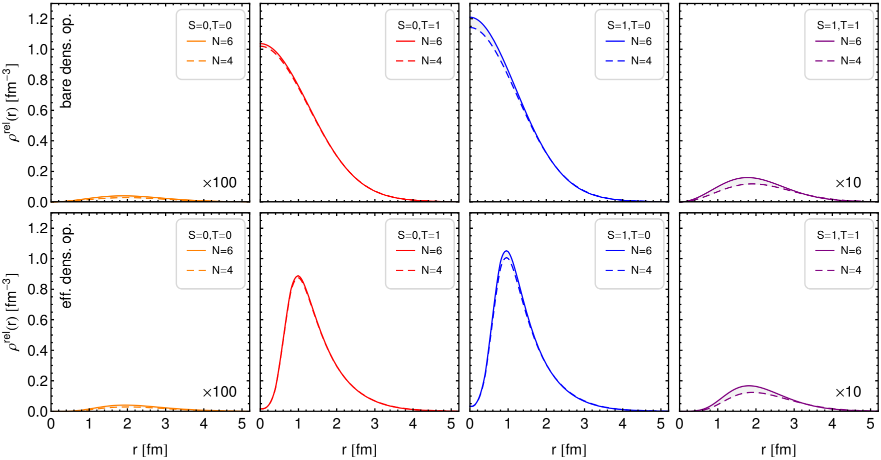

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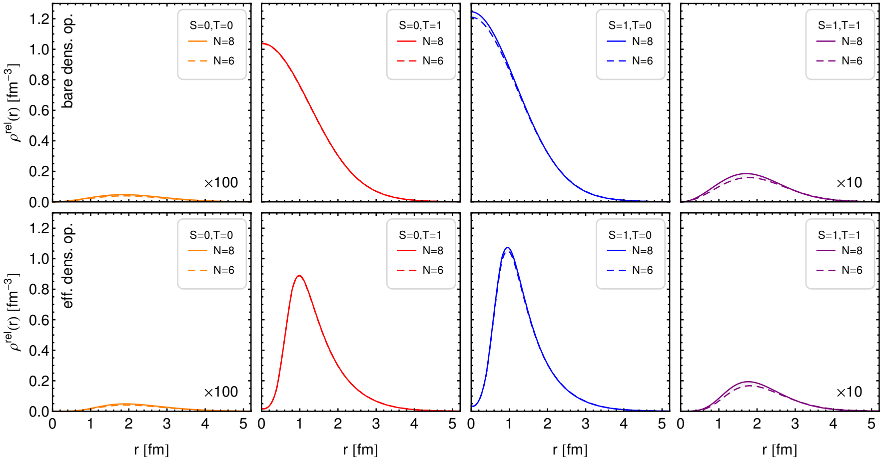

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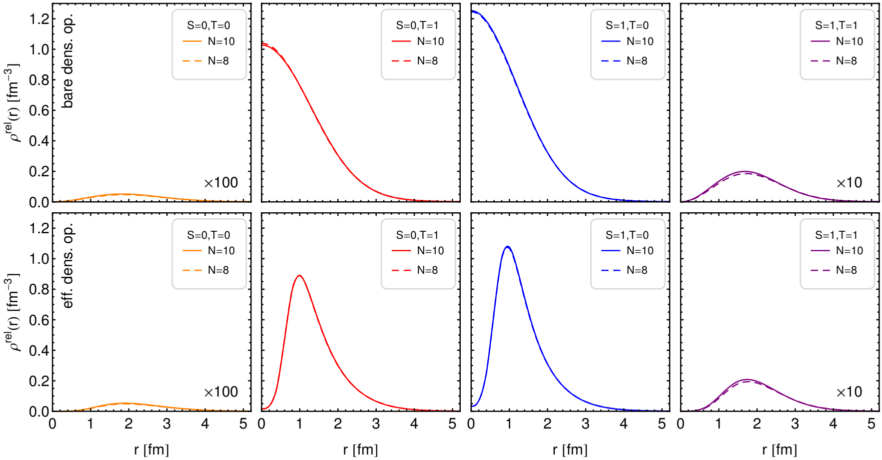

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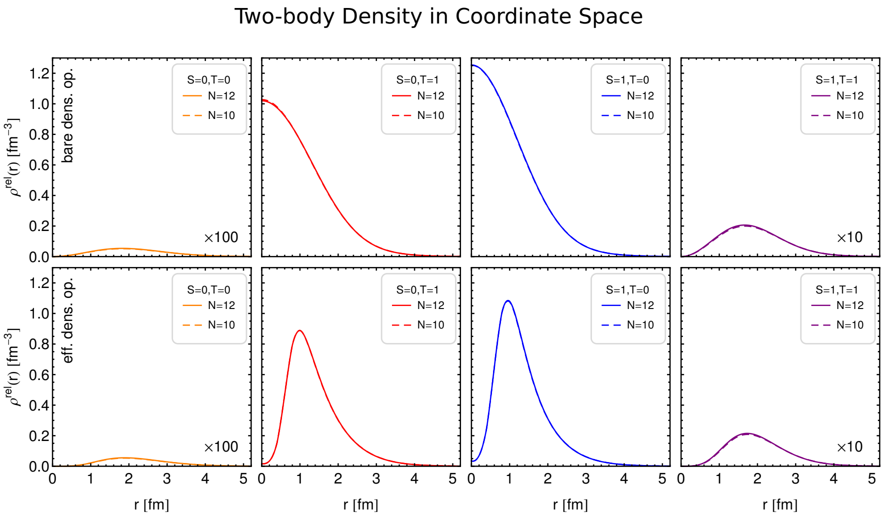

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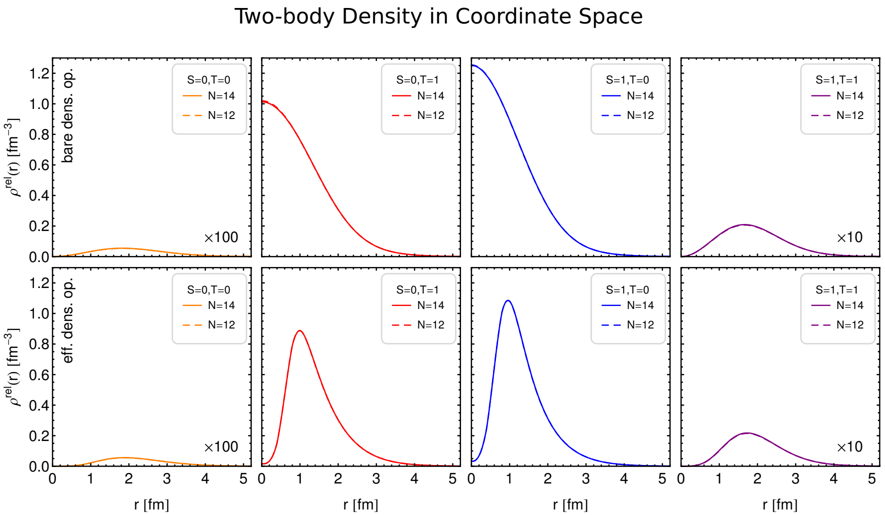

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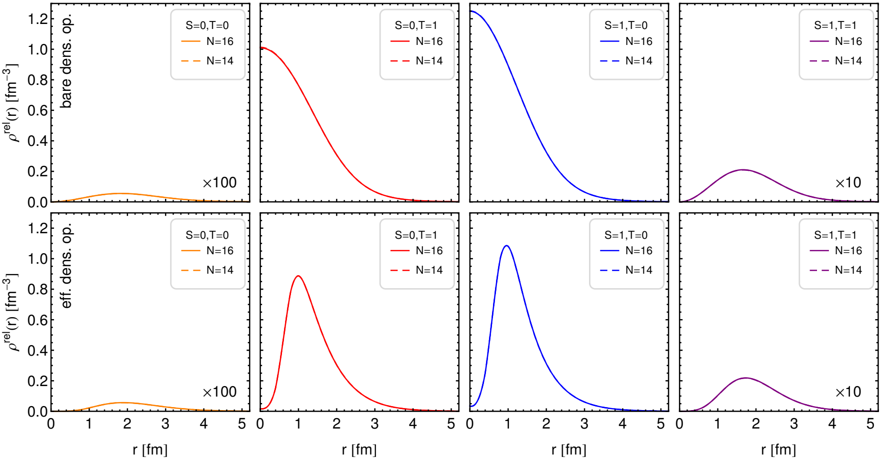

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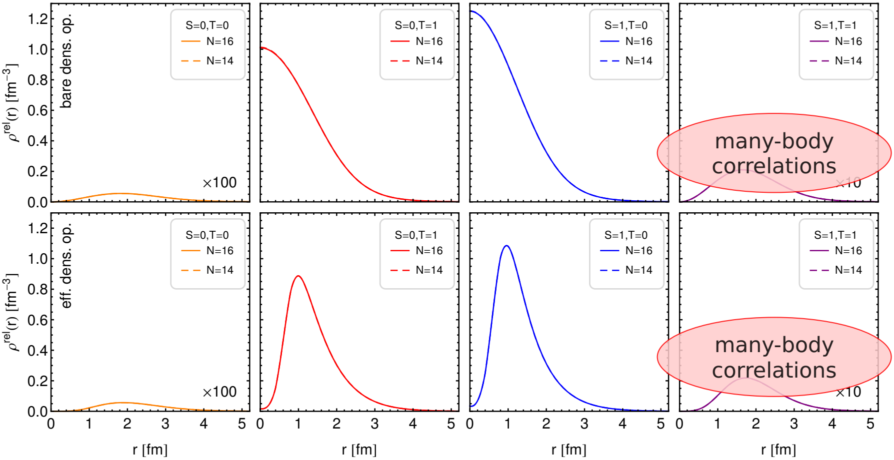

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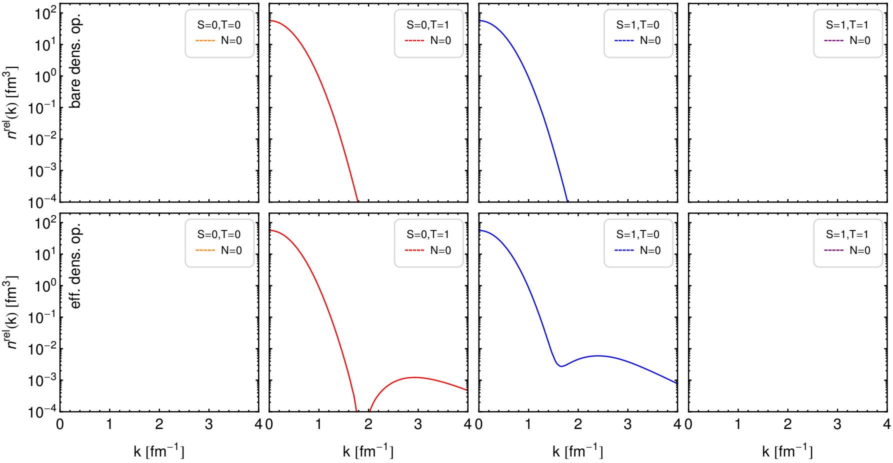

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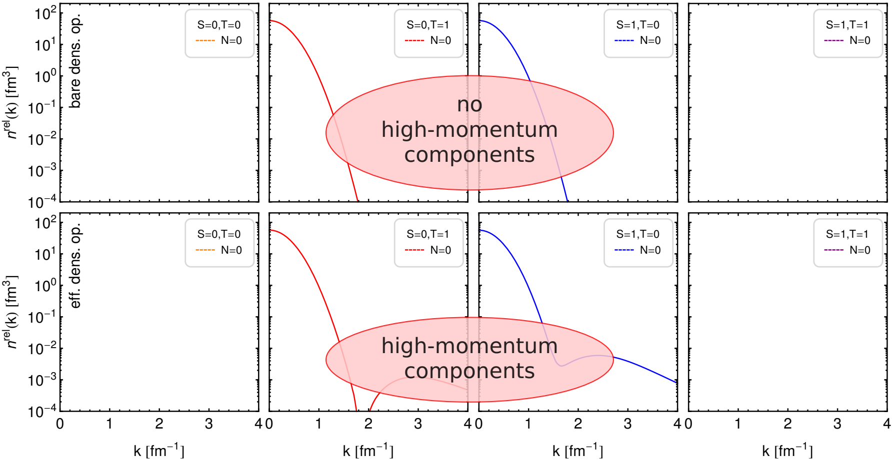

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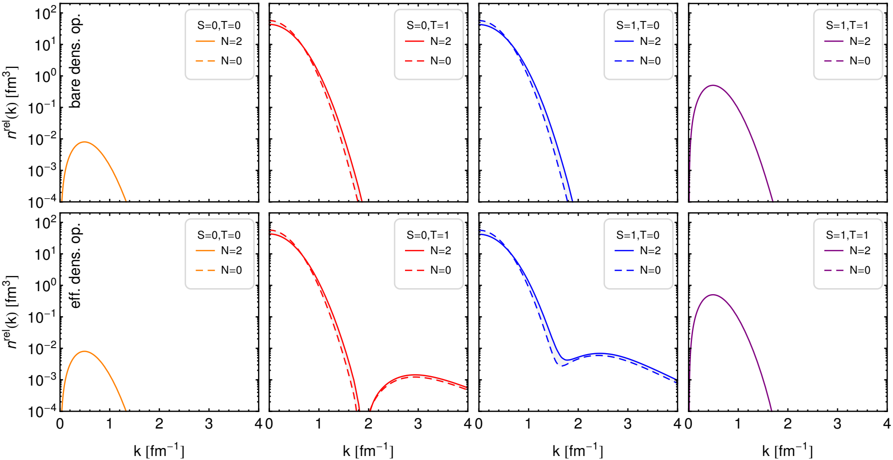

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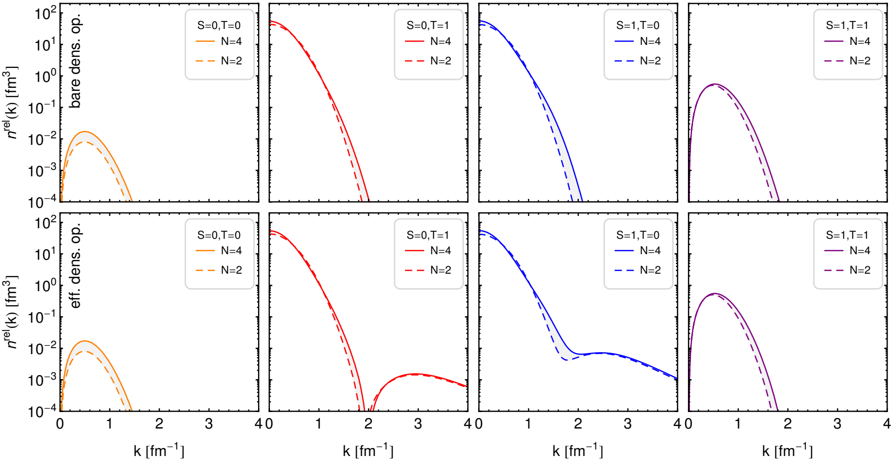

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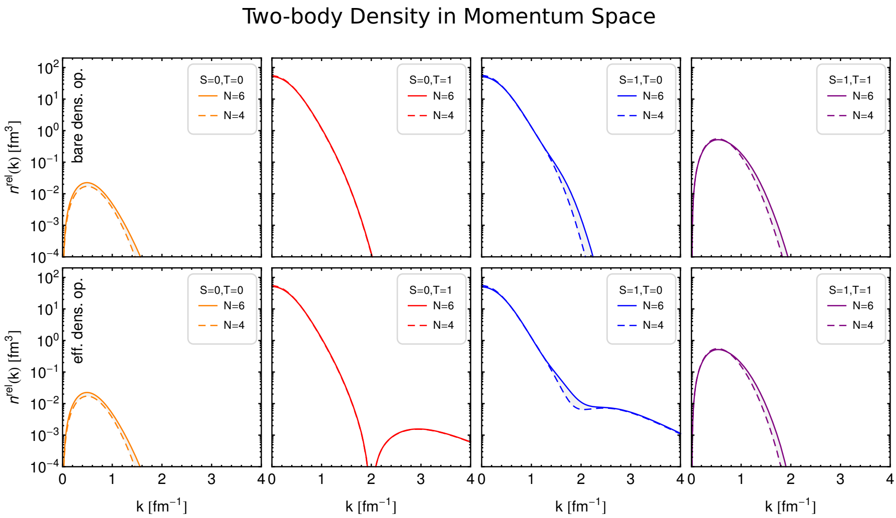

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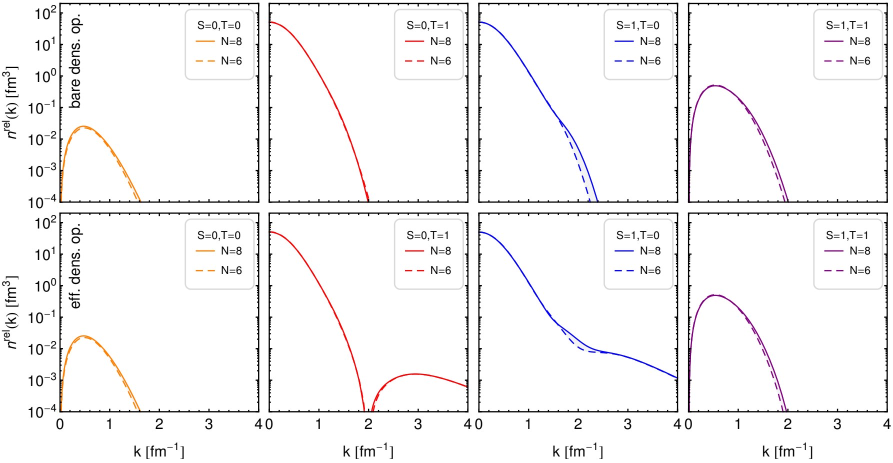

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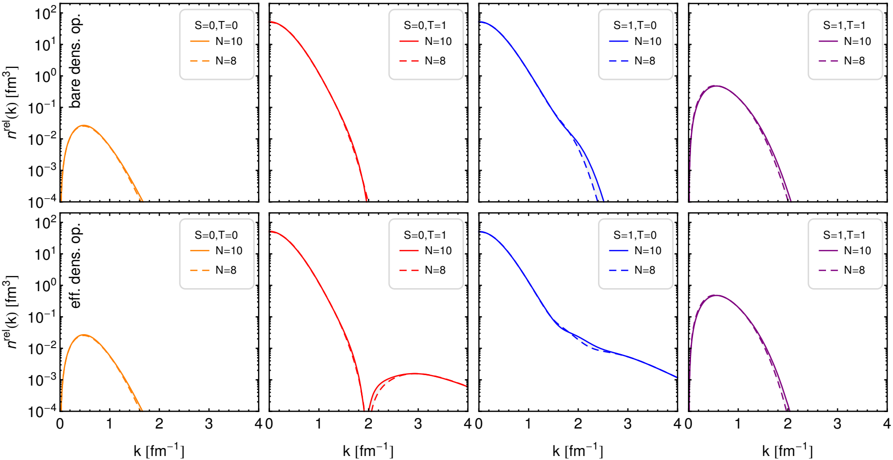

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AV8' – $\alpha = 0.04\ \mathrm{fm}^4$

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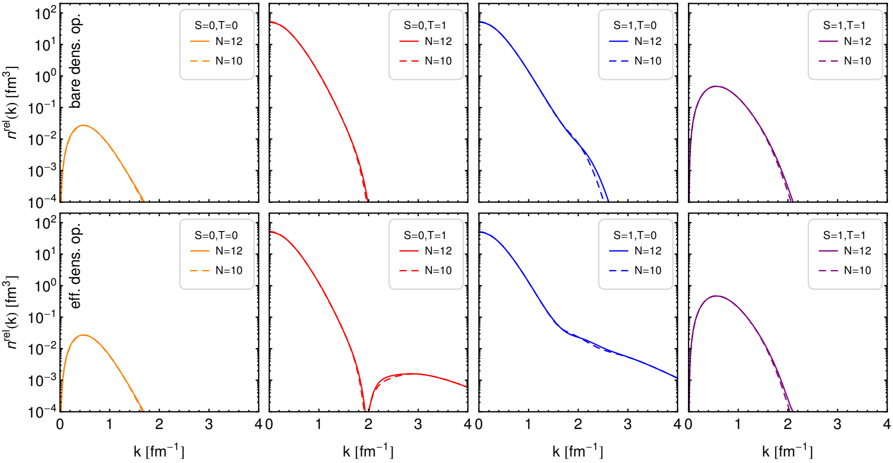

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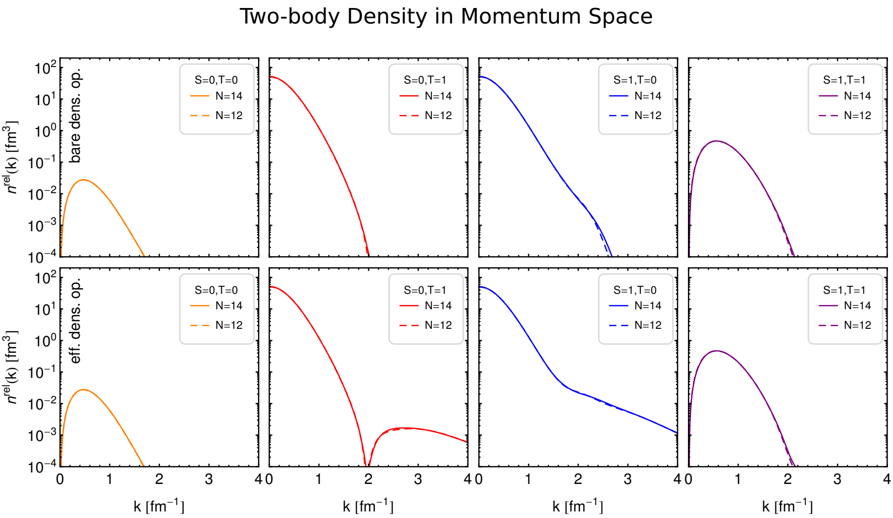

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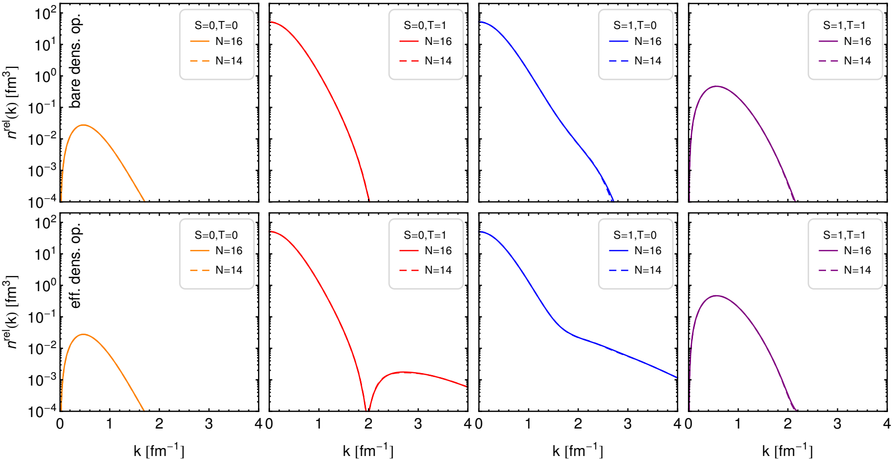

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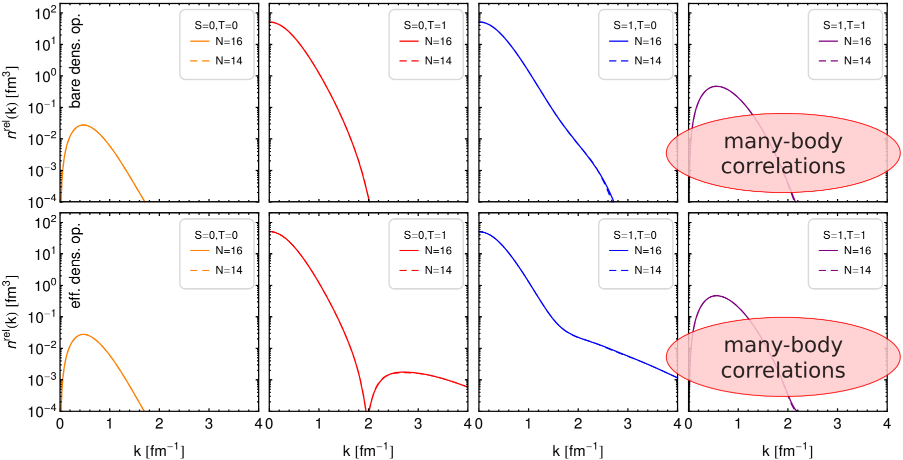

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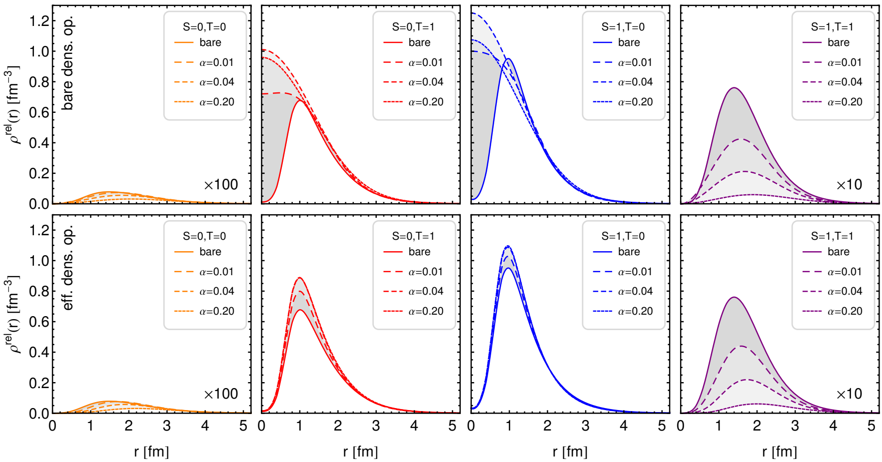
Flow dependence

AV8' Interaction

Two-body Density in Coordinate Space



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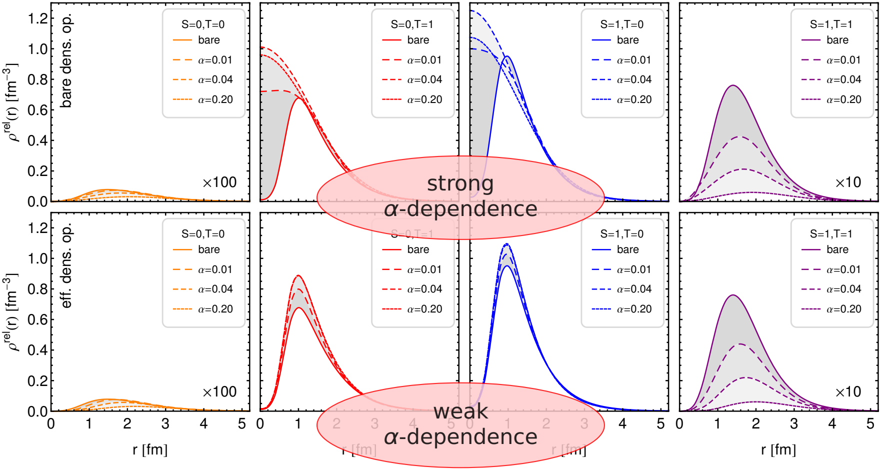
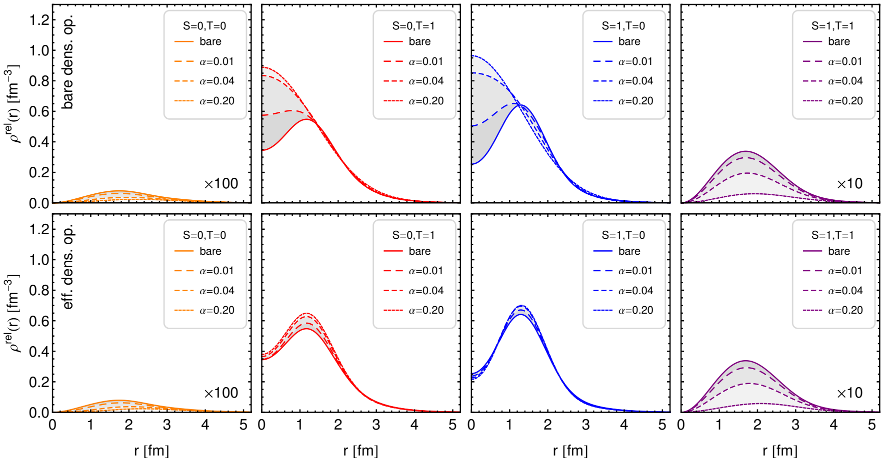


Flow dependence

N^{3}LO Interaction

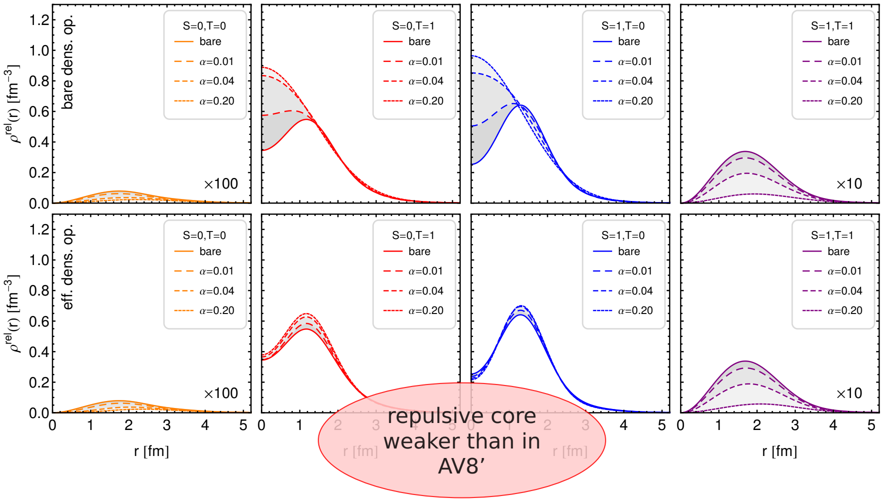
Two-body Density in Coordinate Space



Flow dependence

N^3LO Interaction

Two-body Density in Coordinate Space



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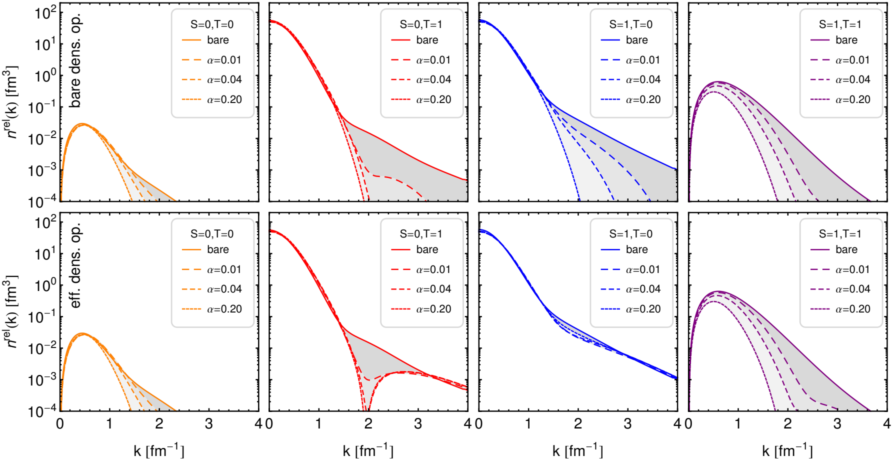

Flow dependence

AV8’ Interaction

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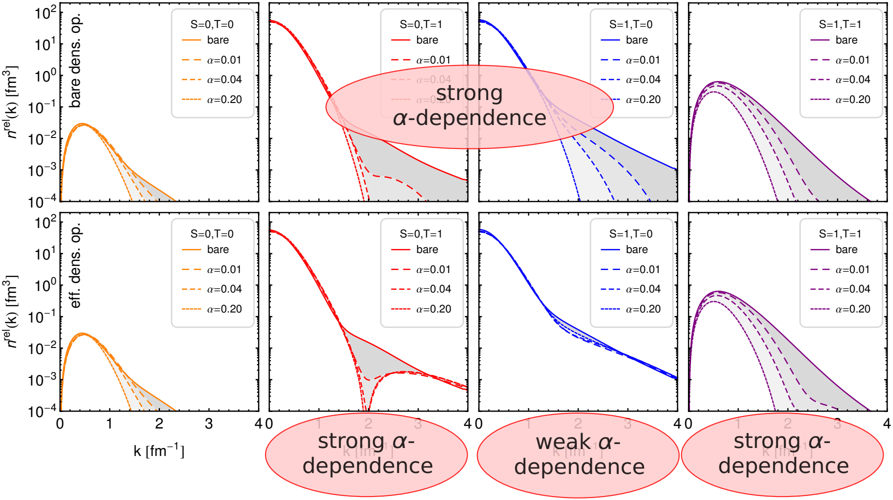
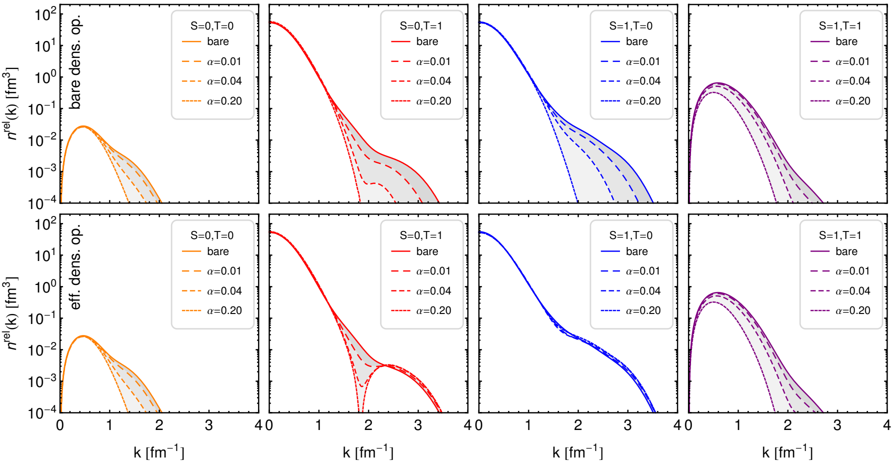


Flow dependence

N^{3}LO Interaction

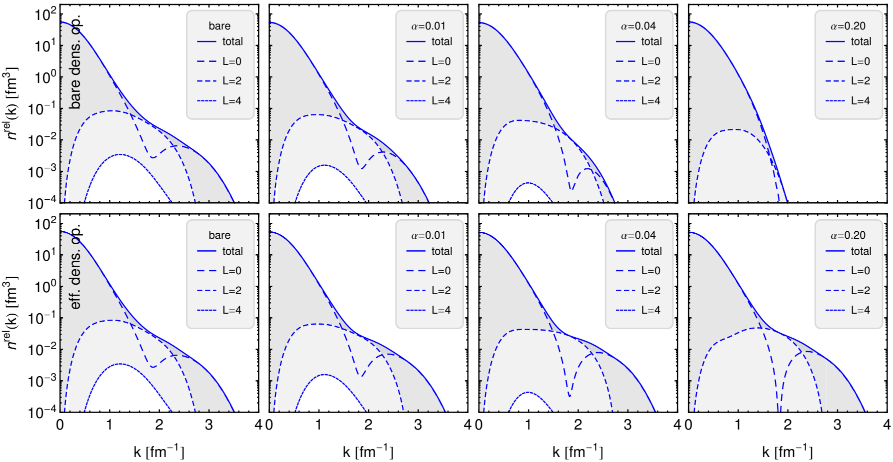
Two-body Density in Momentum Space



Contributions from different angular momenta

N^{3}LO Interaction

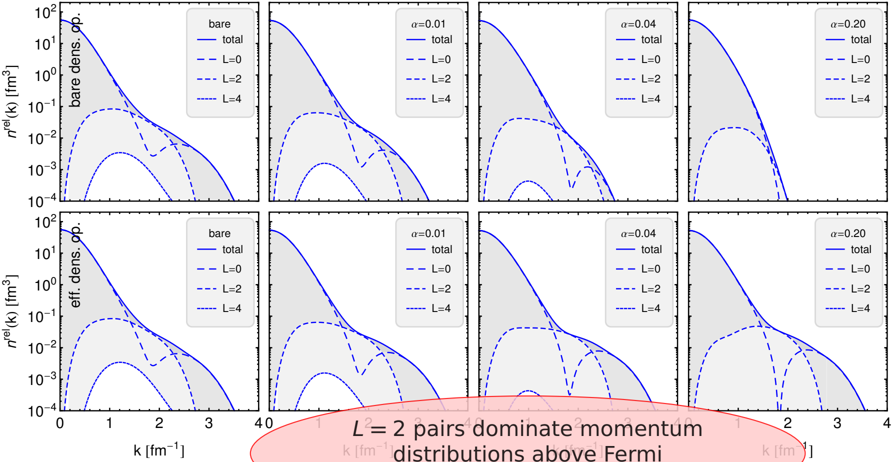
Two-body Density in Momentum Space $S = 1, T = 0$



Contributions from different angular momenta

N^{3}LO Interaction

Two-body Density in Momentum Space $S = 1, T = 0$

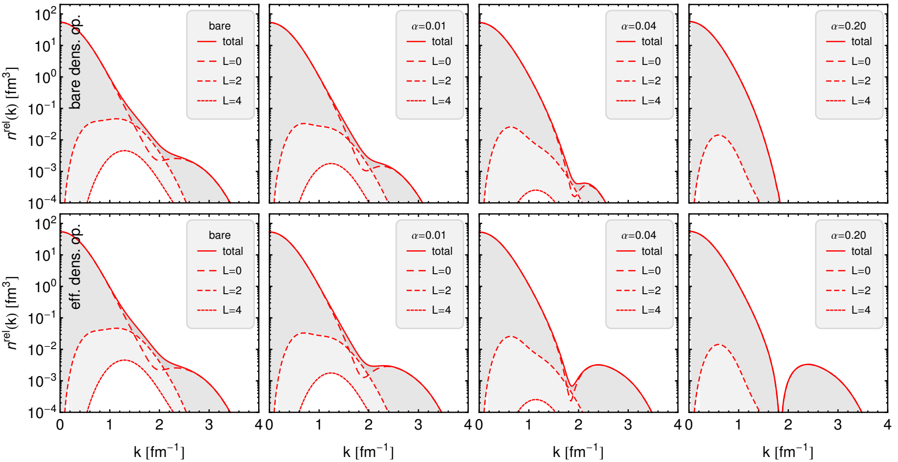


$L = 2$ pairs dominate momentum distributions above Fermi momentum

Contributions from different angular momenta

N^3LO Interaction

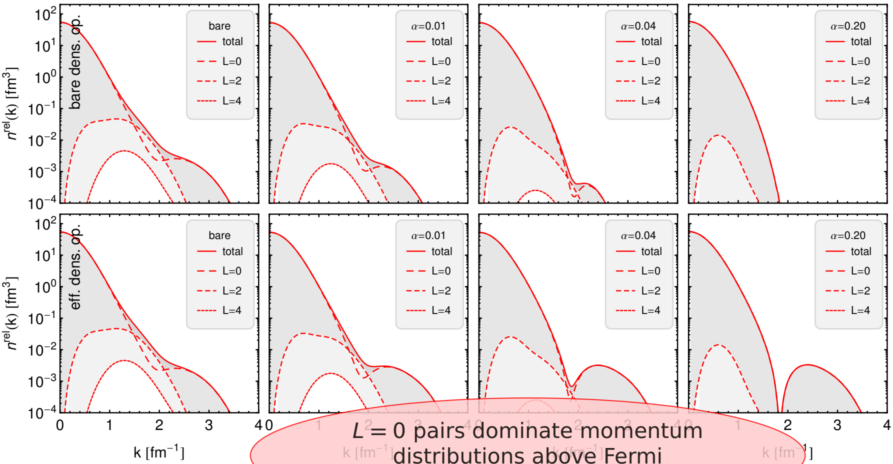
Two-body Density in Momentum Space $S = 0, T = 1$



Contributions from different angular momenta

N^{3}LO Interaction

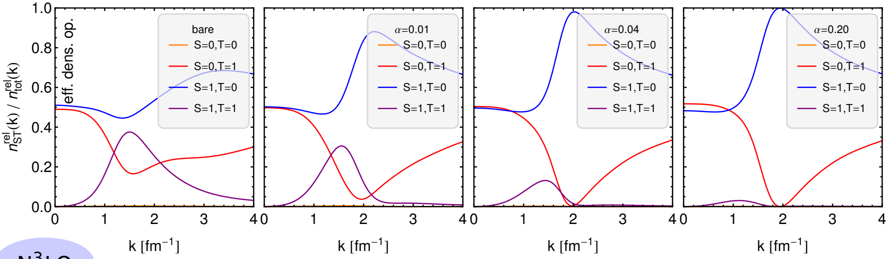
Two-body Density in Momentum Space $S = 0, T = 1$



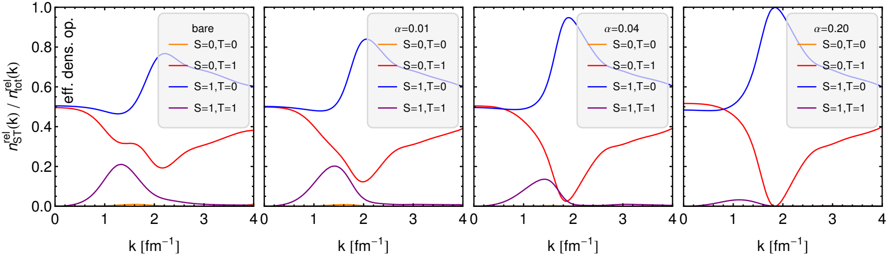
$L = 0$ pairs dominate momentum distributions above Fermi momentum

Relative contributions of ST channels

AV8'

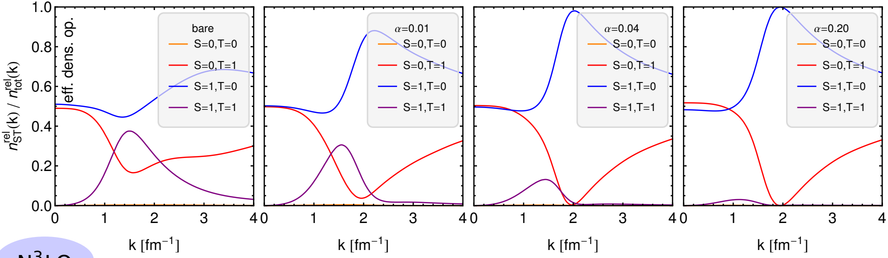


N^{3}LO

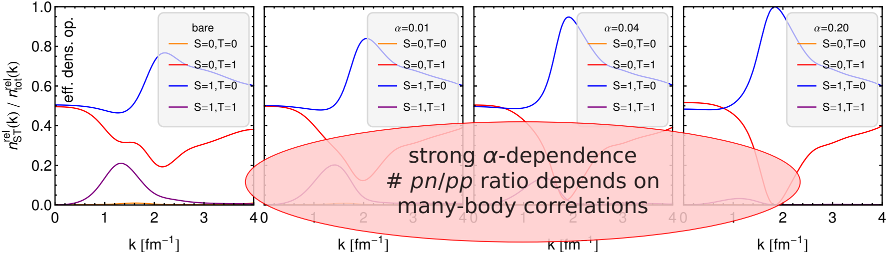


Relative contributions of ST channels

AV8'



N^{3}LO



strong α-dependence
pn/*pp* ratio depends on many-body correlations

^{4}He Many-body correlations

of pairs in (S,T) channels

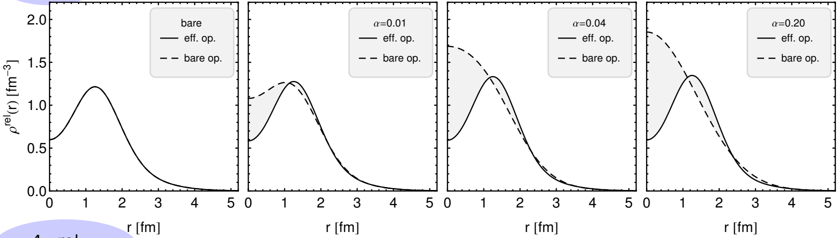
Interaction	(0,0)	(0,1)	(1,0)	(1,1)
AV8’	0.008	2.572	2.992	0.428
AV8’ $\alpha = 0.01\,\mathrm{fm}^4$	0.008	2.708	2.992	0.292
AV8’ $\alpha = 0.04\,\mathrm{fm}^4$	0.007	2.821	2.993	0.179
AV8’ $\alpha = 0.20\,\mathrm{fm}^4$	0.005	2.925	2.995	0.075
N^{3}LO	0.009	2.710	2.991	0.290
N^{3}LO $\alpha = 0.01\,\mathrm{fm}^4$	0.007	2.745	2.992	0.255
N^{3}LO $\alpha = 0.04\,\mathrm{fm}^4$	0.006	2.817	2.994	0.183
N^{3}LO $\alpha = 0.20\,\mathrm{fm}^4$	0.004	2.921	2.995	0.079

- “bare” AV8’ induces stronger many-body correlations than “bare” N^{3}LO
- with increasing flow parameter many-body correlations become weaker

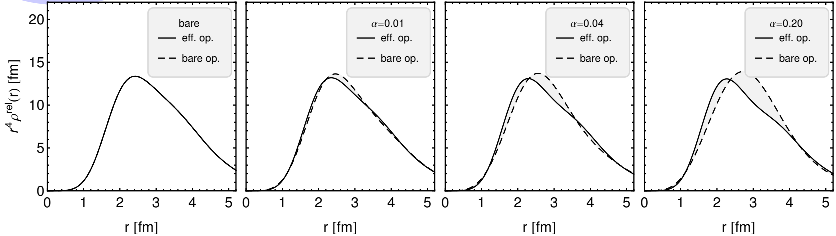
^{4}He Radii

N^3LO Interaction

$\rho^{\mathrm{rel}}(r)$



$r^4\rho^{\mathrm{rel}}(r)$



^{4}He Radii

N^{3}LO Interaction

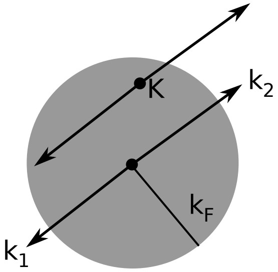
- point matter radius:

$$r_{\text{rms}}^2 = \frac{1}{A^2} \int dr\, r^4 \rho^{\text{rel}}(r) \tag{1}$$

- radii calculated from $N = 16$, $\hbar\Omega = 36$MeV model space wave functions

	"bare" radius [fm]	"effective" radius [fm]
N^{3}LO	1.503	
N^{3}LO $\alpha = 0.01\,\text{fm}^4$	1.478	1.477
N^{3}LO $\alpha = 0.04\,\text{fm}^4$	1.458	1.453
N^{3}LO $\alpha = 0.20\,\text{fm}^4$	1.482	1.451

Pair momentum $K \approx 0$

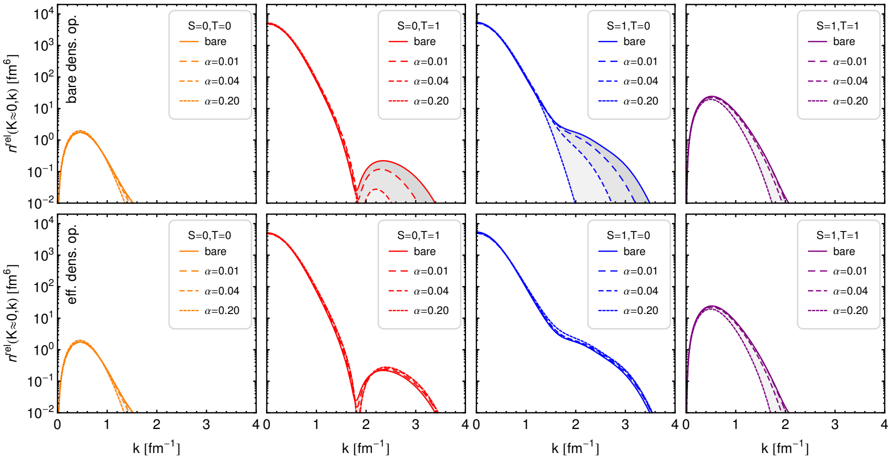


- for vanishing pair momentum and $k > k_F$ only high-momentum nucleons are sampled
- measured in $(e, e'NN)$ experiments
- not (yet) corrected for CM motion in NCSM wave function

Flow dependence – $K \approx 0$

N^3LO Interaction

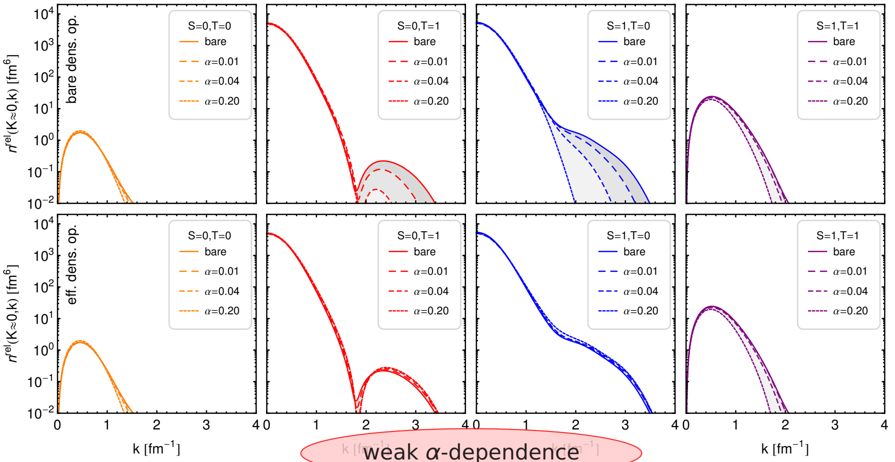
Two-body Density in Momentum Space



Flow dependence – $K \approx 0$

N^{3}LO Interaction

Two-body Density in Momentum Space

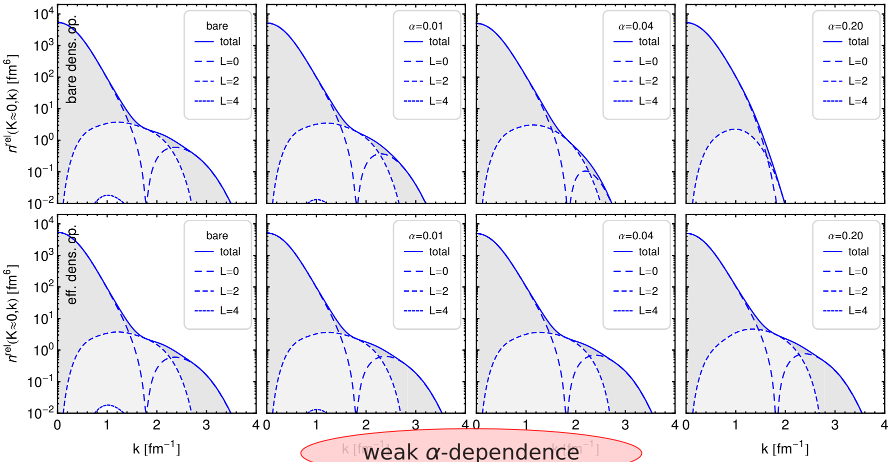


weak α-dependence

Contributions from different angular momenta – $K \approx 0$

N^3LO Interaction

Two-body Density in Momentum Space $S = 1, T = 0$

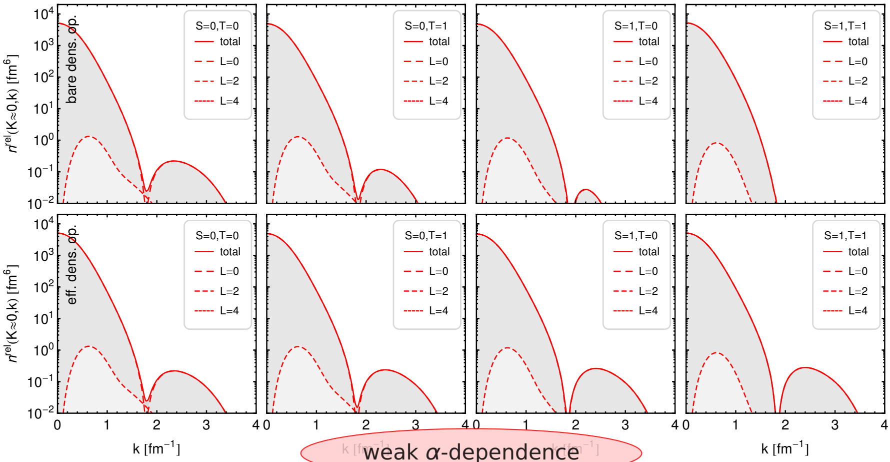


weak α-dependence

Contributions from different angular momenta – $K \approx 0$

N^3LO Interaction

Two-body Density in Momentum Space $S = 0, T = 1$



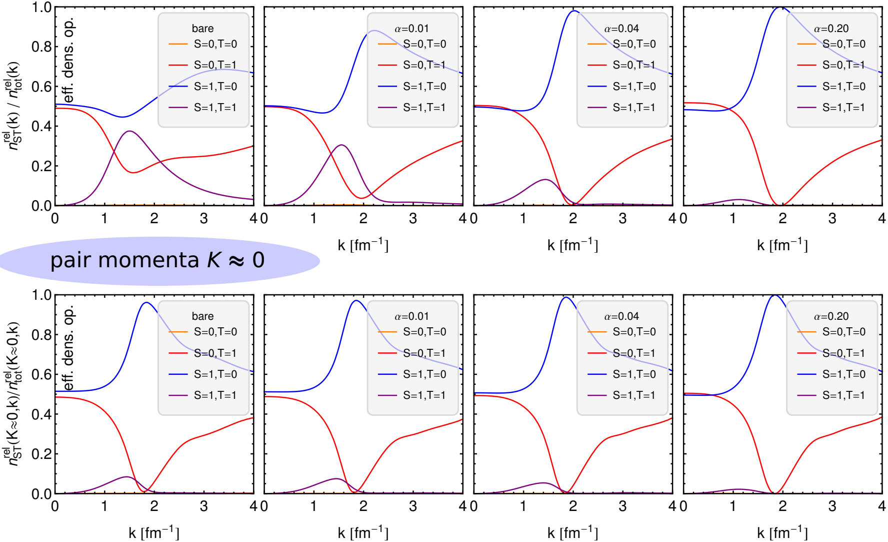
weak α-dependence

Relative contributions of ST channels

N^3LO interaction

all pair momenta

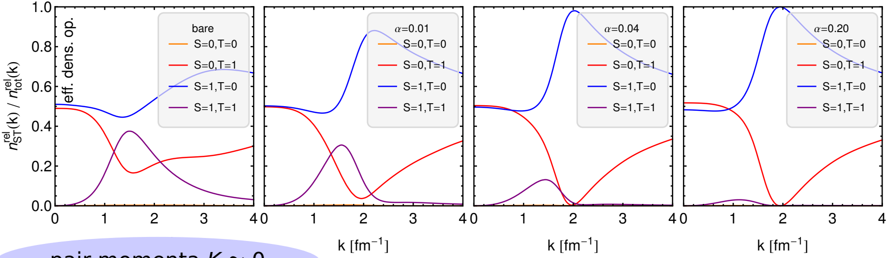
pair momenta $K \approx 0$



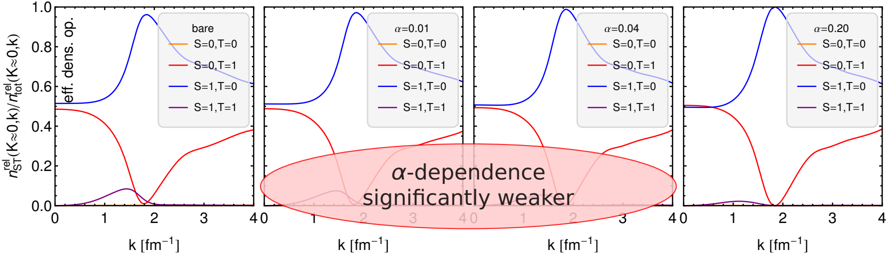
Relative contributions of ST channels

N^{3}LO interaction

all pair momenta



pair momenta $K \approx 0$



α-dependence significantly weaker

^{4}He,^{6}Li,^{10}B,^{12}C Results

Calculation

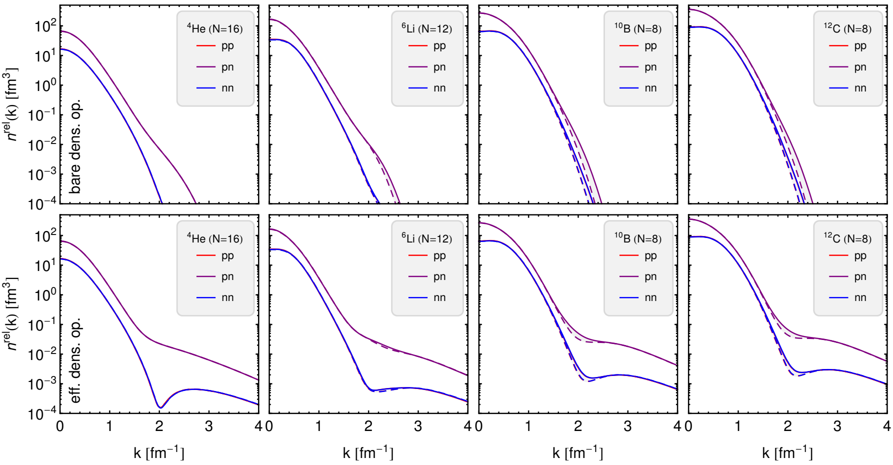
- “bare” AV18 and N^{3}LO can not be converged
- NCSM converges only for larger flow parameters

Objectives

- Compare AV18 and N^{3}LO results
- Check for NCSM convergence
- Check flow dependence $\alpha = 0.04, 0.20\text{fm}^4$ $(\Lambda = 2.24, 1.50\text{fm}^{-1})$
- What is different from ^{4}He ?

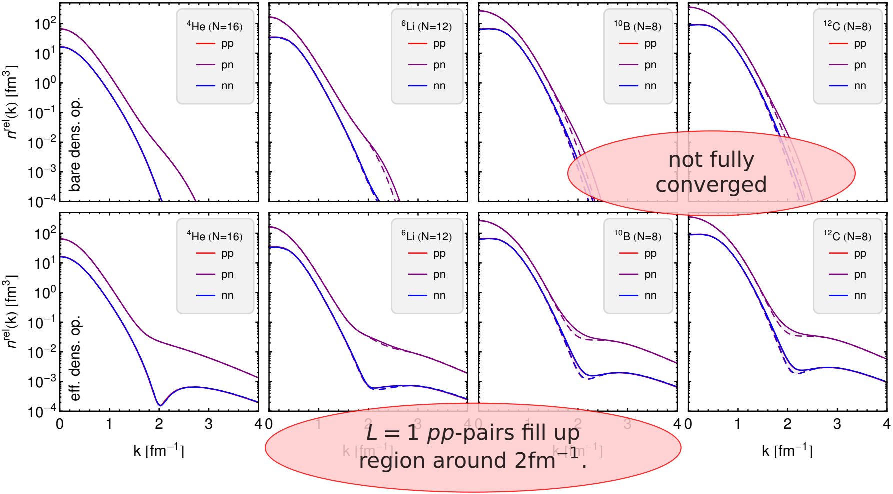
AV18, α=0.04 fm^4

Two-body Density in Momentum Space



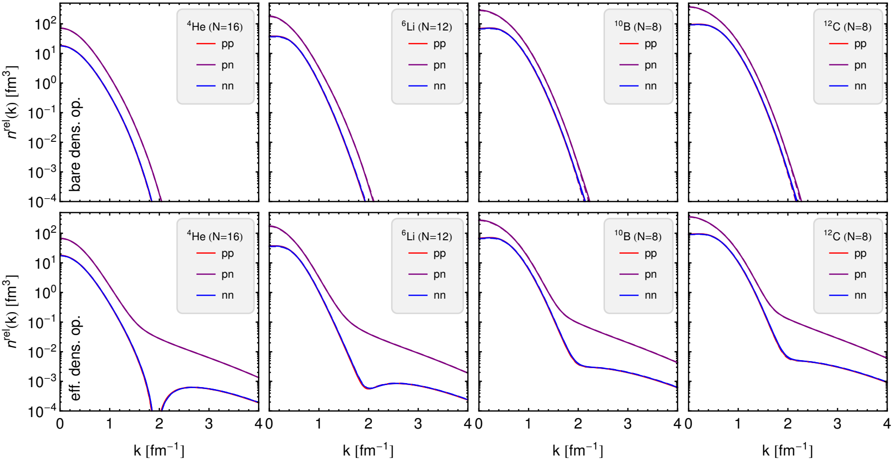
AV18, α=0.04 fm^4

Two-body Density in Momentum Space



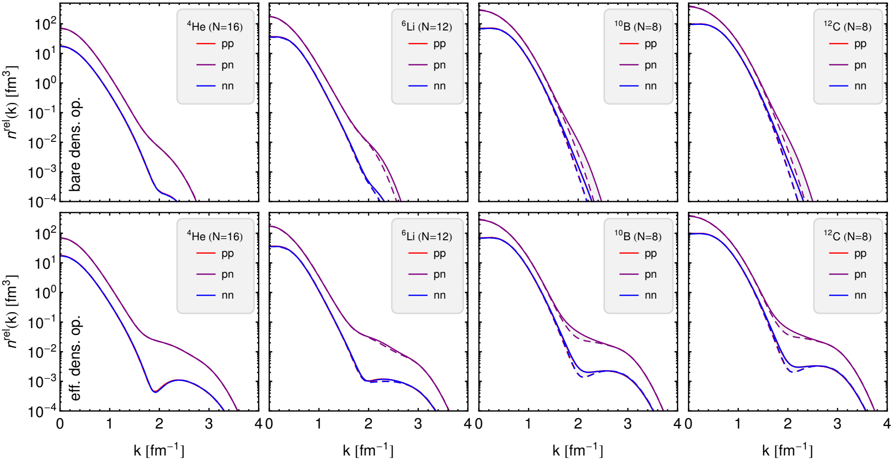
AV18, α=0.20 fm^4

Two-body Density in Momentum Space



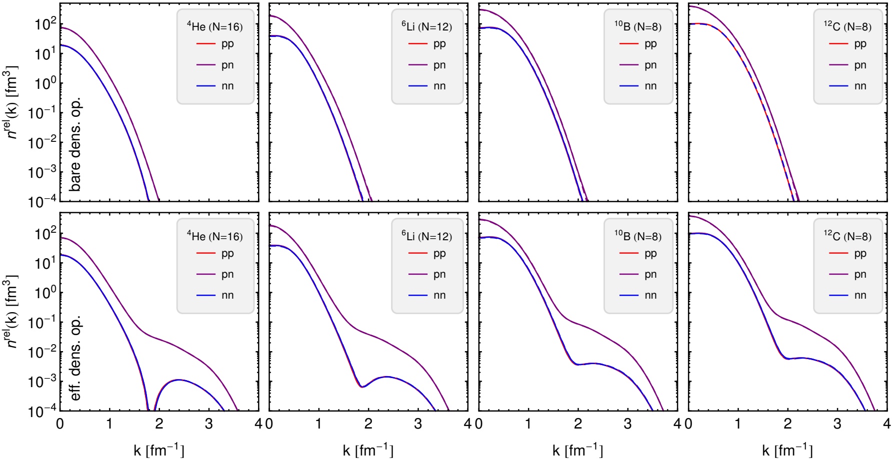
N^{3}LO, α=0.04 fm^4

Two-body Density in Momentum Space



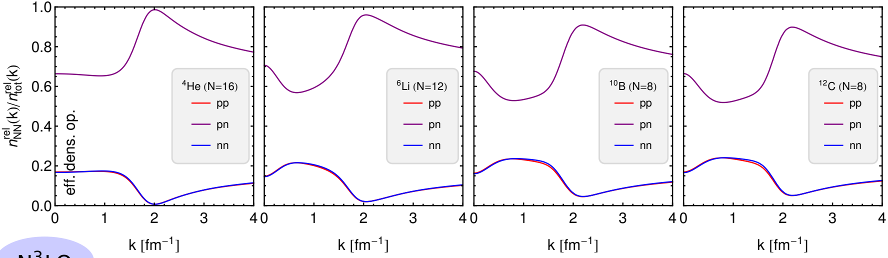
N^{3}LO, α=0.20 fm^4

Two-body Density in Momentum Space

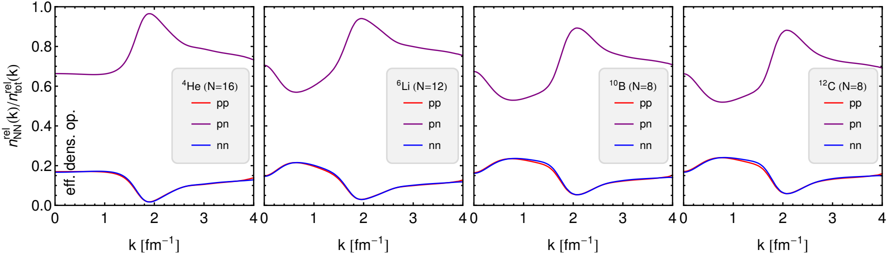


pp, pn, nn contributions

AV18

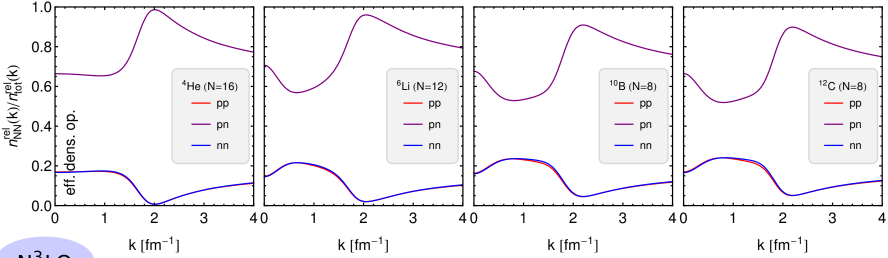


N^{3}LO

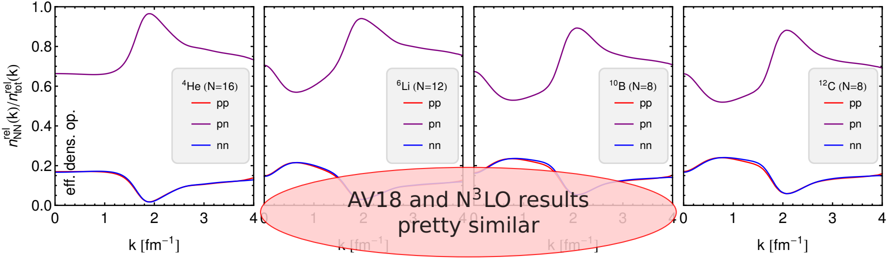


pp, pn, nn contributions

AV18



N^{3}LO



AV18 and N^{3}LO results pretty similar

Summary

Similarity Renormalization Group

- SRG evolved Hamiltonian and transformation matrix
- “bare” and “effective” density operators

^{4}He Two-body densities

- AV8’ and N^{3}LO interactions
- short-range and high-momentum components described by effective operators
- high-momentum components above the Fermi momentum dominated by $L=2$ pairs
- weak α-dependence in the $S=1, T=0$ channel
- strong α-dependence in the $S=0, T=1$ channel due to many-body correlations
- AV8’ and N^{3}LO interaction results differ mainly in the $S=0, T=1$ channel due to different many-body correlations

^{6}Li, ^{10}B, ^{12}C Two-body densities

- $T=1$ pairs with $L=1$ fill up the *pp*/*nn* momentum distributions above the Fermi momentum
- AV18 and N^{3}LO provide very similar results up to $k \approx 3\text{fm}^{-1}$