The automatized partial wave decomposition and its applications

R.Skibiński (JU), J.Golak (JU), D.Rozpędzik (JU),
K.Topolnicki (JU), H.Witała (JU),
E.Epelbaum (RUB), H.Krebs (RUB), W.Glöckle (RUB)
A.Nogga (FZJ), H.Kamada (KIT)

JAGIELLONIAN UNIVERSITY

TRIUMF

Progress in Ab Initio Techniques
in Nuclear Physics
February 21-23, 2013

Outline

Method:

- Partial wave decomposition (PWD) and automatized partial wave decomposition (aPWD)
- Simple case: NN potential

Applications:

- 3NF at N^2LO and N^3LO
- Results on aPWD of 3NF at N^3LO
- Numerical tests
- LECs values at N^3LO
- Some details of ^3H at N^3LO
- Analyzing power $A_Y(N)$
- Electromagnetic current (in the deuteron photodisintegration)
- Comments and Outlook

Introduction – 2N and 3N systems

- Nonrelativistic formalism
- 2N:

  Schrödinger equation,

  Lippmann-Schwinger equation for the t-matrix

  (interaction + free propagation)

$$t(E) = V + VG_0(E)V + VG_0VG_0(E)V + \ldots$$

$$G_0(E) \equiv \lim_{\varepsilon \to 0^+} \frac{1}{E - H_0 + i\varepsilon}$$

- 3N: Faddeev equation

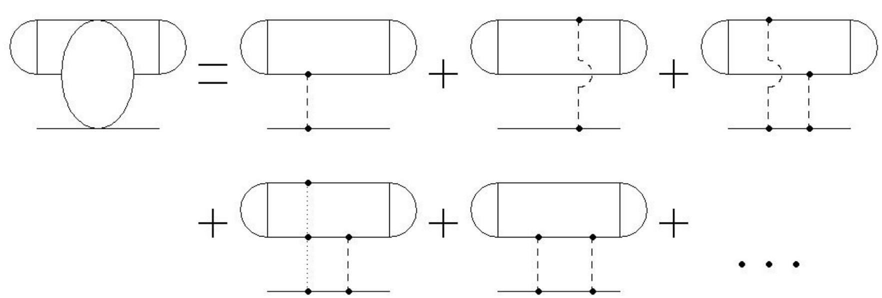
$$T = tP\phi + (1 + tG_0)V_{123}^{(1)}(1 + P)\phi + tPG_0T + (1 + tG_0)V_{123}^{(1)}(1 + P)G_0T$$

Transition amplitudes

$$U = PG_0^{-1} + V_{123}^{(1)}(1 + P)\phi +$$

$$+ PT + V_{123}^{(1)}(1 + P)G_0T$$

$$U_0 = (1 + P)T$$



Introduction – 2N and 3N systems

- The input to the above equations is:

  - the nucleon-nucleon potential V (CD Bonn, AV18, chiral)
  - the three nucleon force V_{123} (TM, Urbana IX, chiral)
  - the nuclear electromagnetic/weak currents

    (in the case of processes with electroweak probes (e, μ, γ))

    (single nucleon current + meson exchange currents (π- and ρ-like or currents from χEFT)

- Solutions of the above mentioned equations allows us to calculate the ^{2}H, ^{3}H, ^{3}He properties and observables in elastic NN and Nd scattering or in deuteron breakup.

2N states

- Two particles with momenta p_1 and p_2 and spin ½ and izospin ½

$$| \vec{p}_1 m_1 \nu_1 \rangle | \vec{p}_2 m_2 \nu_2 \rangle$$

- It is more convenient to work with states $| \vec{p}\, \vec{P}\, m_1 \nu_1\; m_2 \nu_2 \rangle$ where

$$\vec{p} = \frac{1}{2}\left(\vec{p}_2 - \vec{p}_1\right), \quad \vec{P} = \vec{p}_2 + \vec{p}_1$$

Coupling of spins and isospins of both nucleons and using the orbital angular momentum operator leads (in the 2N c.m. system) to

$$\left| p(l\, s) j m_j \right\rangle \left| t m_t \right\rangle \equiv \left| p(l\, s) j m_j ; t m_t \right\rangle \equiv \left| p\, \alpha_2 \right\rangle$$

$$\left| p(l\, s) j m_j \right\rangle \equiv \sum_{m_l, m_s} c\left(l, s, j; m_l, m_s, m_j\right) \left| p l m_l \right\rangle \left| s m_s \right\rangle \qquad (-1)^{l+s+t} = -1$$

$$\left| s m_s \right\rangle \equiv c\left(1/2, 1/2, s; m_1, m_2, m_s\right) \left| 1/2\, m_1 \right\rangle \left| 1/2\, m_2 \right\rangle$$

$$\left\langle \vec{p'} \middle| p l m_l \right\rangle \equiv \frac{\delta(p - p')}{p p'} Y_{l, m_l}(\theta', \varphi')$$

How to calculate the matrix element of the potential?

I-st method (the standard PWD)

- Analyticaly: using the properties of the spherical harmonics, Clebsch-Gordan coefficients, Legendre' a polynomials, making decouplings of spin and momentum spaces
- This method is tedious and (real) time-consuming

- Example: one-pion exchange at N^{2}LO

$$V(\vec{p}\,', \vec{p}) = \overbrace{-\frac{1}{(2\pi)^3}\left(\frac{g_A}{2F_\pi}\right)^2 \frac{\vec{\sigma}_1\cdot\vec{q}\;\vec{\sigma}_2\cdot\vec{q}}{M_\pi^2+\vec{q}^{\,2}}\,\vec{\tau}_1\cdot\vec{\tau}_2}^{OPE}$$

$$\underbrace{+\frac{1}{(2\pi)^3}C_S + \frac{1}{(2\pi)^3}C_T\,\vec{\sigma}_1\cdot\vec{\sigma}_2\,,}_{contact}$$

$$\vec{q} = \vec{p}\,' - \vec{p}\,.$$

Standard PWD – one pion exchange

$$\left\langle p'(l's')j'm';t'm_{t'}\left|V^{OPE}\right|p(ls)jm;tm_t\right\rangle =$$

$$= -\frac{1}{(2\pi)^3}\left(\frac{g_A}{2F_\pi}\right)^2 \delta_{j'j}\,\delta_{m'm}\,\delta_{s's}\,\delta_{t't}\,\delta_{m_{t'}m_t}\,12\pi\sqrt{(2s+1)(2s'+1)}\,(-1)^{j+s}\left[2t(t+1)-3\right]$$

$$\sum_{a=0,2}\sqrt{2a+1}\,c(1,1,a,0,0,0)\sqrt{(2a+1)!}\begin{Bmatrix} l' & l & a \\ s & s' & j \end{Bmatrix}\begin{Bmatrix} 1 & 1 & a \\ 1/2 & 1/2 & s \\ 1/2 & 1/2 & s' \end{Bmatrix}$$

$$\sum_{a_1+a_2=a} p^{a_1}(p')^{a_2}(-1)^{a_2}\frac{1}{\sqrt{(2a_1)!(2a_2)!}}\sum_k (2k+1)(-1)^k g_{ka}\begin{Bmatrix} l' & l & a \\ a_1 & a_2 & k \end{Bmatrix}$$

$$c(k,a_1,l;000)\,c(k,a_2,l';0,0,0),$$

where $g_{ka} = \int_{-1}^{1} dx\, P_k(x)\,\frac{\left(\sqrt{p^2+p'^2-2pp'x}\right)^{2-a}}{M_\pi^2+p^2+p'^2-2pp'x}$

The PWD of NN potential

- Any two-nucleon potential (invariant under rotations,parity and time reversal) can be written as

$$\langle \vec{p}\,' | V^{tm_t} | \vec{p} \rangle = \sum_{j=1}^{6} v_j^{tm_t}(\vec{p}\,',\vec{p})\, w_j(\vec{\sigma}_1,\vec{\sigma}_2,\vec{p}\,',\vec{p}),$$

$$\langle t' m_{t'} | V | t\, m_t \rangle = \delta_{t't}\, \delta_{m_{t'} m_t}\, V^{tm_t}$$

$$w_1(\vec{\sigma}_1,\vec{\sigma}_2,\vec{p}\,',\vec{p}) = 1$$
$$w_2(\vec{\sigma}_1,\vec{\sigma}_2,\vec{p}\,',\vec{p}) = \vec{\sigma}_1 \cdot \vec{\sigma}_2$$
$$w_3(\vec{\sigma}_1,\vec{\sigma}_2,\vec{p}\,',\vec{p}) = i\,(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot(\vec{p} \times \vec{p}\,')$$
$$w_4(\vec{\sigma}_1,\vec{\sigma}_2,\vec{p}\,',\vec{p}) = \vec{\sigma}_1 \cdot (\vec{p} \times \vec{p}\,')\, \vec{\sigma}_2 \cdot (\vec{p} \times \vec{p}\,')$$
$$w_5(\vec{\sigma}_1,\vec{\sigma}_2,\vec{p}\,',\vec{p}) = \vec{\sigma}_1 \cdot (\vec{p} + \vec{p}\,')\, \vec{\sigma}_2 \cdot (\vec{p} + \vec{p}\,')$$
$$w_6(\vec{\sigma}_1,\vec{\sigma}_2,\vec{p}\,',\vec{p}) = \vec{\sigma}_1 \cdot (\vec{p}\,' - \vec{p})\, \vec{\sigma}_2 \cdot (\vec{p}\,' - \vec{p})$$

How to do that simpler (aPWD)

$$M \equiv \langle p'(l's')j'm';t'm_{t'}|\hat{O}|p(ls)jm;tm_t\rangle =$$

$$= \int_0^{\pi} d\theta' \sin\theta' \int_0^{2\pi} d\phi' \int_0^{\pi} d\theta \sin\theta \int_0^{2\pi} d\phi$$

$$\sum_{m_{l'}=-l'}^{l'} c(l',s',j',m_{l'},m'-m_{l'},m') \sum_{m_l=-l}^{l} c(l,s,j,m_l,m-m_l,m)$$

$$Y^*_{l'm_{l'}}(\theta',\phi')\, Y_{lm_l}(\theta,\phi)\ \langle t'm_{t'}|\langle s'm'-m_{l'}|\hat{O}(\vec{p}',\vec{p})|s\,m-m_l\rangle|tm_t\rangle$$

Thus, we face four-dimensional inegration
(and have to know the matrix element in the integrand).

How to do that simpler (aPWD)

$$M_{RINV} \equiv \langle p'(l's')j'm';t'm_{t'} | \hat{O}_{RINV} | p(ls)jm;tm_t \rangle =$$

$$= \frac{1}{2j+1} \sum_{m=-j}^{j} \langle p'(l's')j'm';t'm_{t'} | \hat{O}_{RINV} | p(ls)jm;tm_t \rangle =$$

$$= \int_0^{\pi} d\theta' \sin\theta' \int_0^{2\pi} d\phi' \int_0^{\pi} d\theta \sin\theta \int_0^{2\pi} d\phi \, \frac{1}{2j+1}$$

$$\sum_{m=-j}^{j} \delta_{mm'} \sum_{m_{l'}=-l'}^{l'} c(l',s',j',m_{l'},m'-m_{l'},m') \sum_{m_l=-l}^{l} c(l,s,j,m_l,m-m_l,m)$$

$$Y^*_{l'm_{l'}}(\theta',\phi') \, Y_{lm_l}(\theta,\phi)$$

$$\langle t'm_{t'} | \langle s'm'-m_{l'} | \hat{O}_{RINV}(\vec{p}',\vec{p}) | s\, m-m_l \rangle | tm_t \rangle$$

The integrand depends only on $x \equiv \hat{p}' \cdot \hat{p}$

How to do that simpler (aPWD)

We choose $\hat{p} = (0,0,1)$,

$\hat{p}' = (\sin\theta', 0, \cos\theta')$,

$$M_{RINV} = 8\pi^2 \int_0^\pi d\theta' \sin\theta' \frac{1}{2j+1} \sum_{m=-j}^{j} \delta_{mm'}$$

$$\sum_{m_{l'}=-l'}^{l'} c(l', s', j', m_{l'}, m'-m_{l'}, m') \sum_{m_l=-l}^{l} c(l, s, j, m_l, m-m_l, m)$$

$$Y^*_{l'm_{l'}}(\theta', 0)\, Y_{lm_l}(0,0) \left\langle t' m_{t'} \right| \left\langle s' m'-m_{l'} \right| \hat{O}_{RINV}\left(\vec{p}', \vec{p}\right) \left| s\, m - m_l \right\rangle \left| t m_t \right\rangle$$

1-dimensional integration !

One only needs to know the matix element in the integrand.
O_{RINV} is the matrix element in the momentum space and an operator in the spin and isospin space.

Automatized PWD

- The action of the spin and isospin operators in

$$\langle t' m_{t'} | \langle s' m' - m_{l'} | \hat{O}_{RINV}(\vec{p}', \vec{p}) | s\, m - m_l \rangle | t m_t \rangle$$

can be calculated analyticaly by means of software for the symbolic algebra, for example *Mathematica®*

$$\sum_{j=1}^{6} v_j(\vec{p}', \vec{p}) \langle s\, m_j - m_{l'} | w_j(\vec{\sigma}_1, \vec{\sigma}_2, \vec{p}', \vec{p}) | s\, m_j - m_l \rangle$$

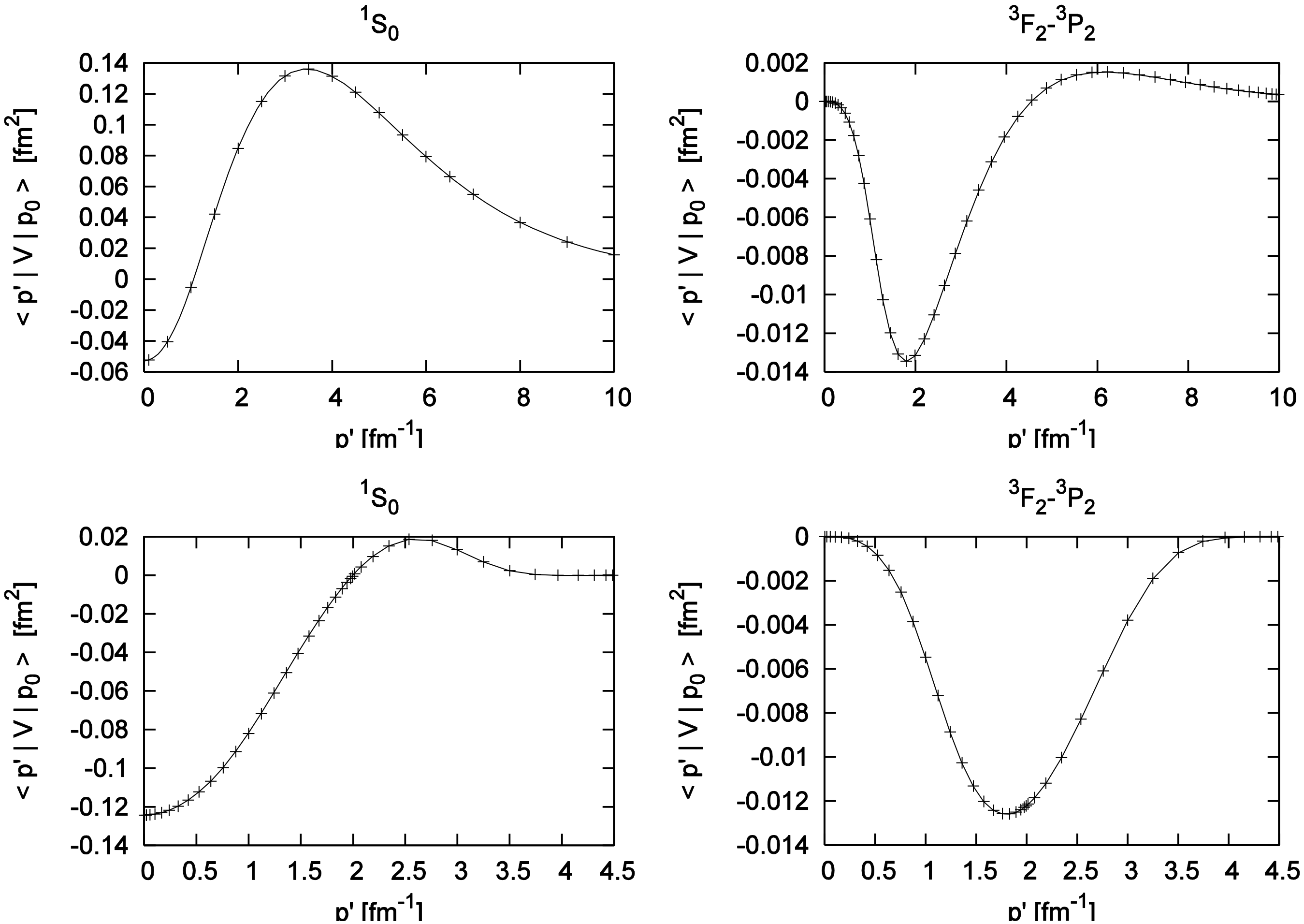
$$H(l', l, s, j) \equiv \frac{1}{2j+1} \sum_{m_j=-j}^{j} \langle p'(l's) j m_j | \mathrm{V} | p(ls) j m_j \rangle$$

$$H(2,0,1,1) =$$

$$= \frac{2\pi\sqrt{2}}{3} \int_{-1}^{1} dx \{ v_4(p', p, x) p'^2\, p^2 (x^2 - 1) + v_5(p', p, x) [(3x^2 - 1) p'^2 + 2p^2 + 4p' px] +$$

$$+ v_6(p', p, x) [(3x^2 - 1) p'^2 + 2p^2 - 4p' px] \}$$

Example: $^1S_0$ and $^3F_2$-$^3F_2$ waves for the BonnB and the chiral N^2LO potentials



Bonn B

+ PWD

— aPWD

P_0=1 fm^{-1}

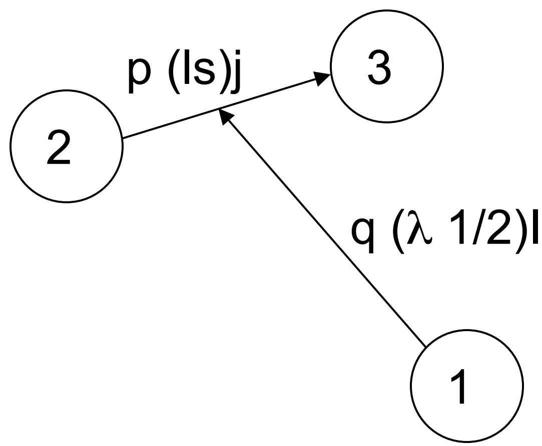
Chiral N^2LO

3N basis states

jI-coupling (used during ^{3}H and scattering states calculations)

$$\left\langle p'q'(l's')j'(\lambda'\frac{1}{2})I'(j'I')J'M_{J'} \middle| V^{3N} \middle| pq(ls)j(\lambda\frac{1}{2})I(jI)JM_J \right\rangle$$

← without isospin



$$\vec{p} = \frac{1}{2}\left(\vec{p}_2 - \vec{p}_3\right)$$

$$\vec{q} = \frac{1}{3}\left(2\vec{p}_1 - \vec{p}_2 - \vec{p}_3\right)$$

LS-coupling (more convenient due to the form of 3NF)

$$\left\langle p'q'(l'\lambda')L'(s'\frac{1}{2})S'(L'S')J'M_{J'} \middle| V^{3N} \middle| pq(l\lambda)L(s\frac{1}{2})S(LS)JM_J \right\rangle$$

aPWD of 3NF

$$M \equiv \left\langle p'q'(l'\lambda')L'(s'\frac{1}{2})S'(L'S')JM_J \left| \hat{O} \right| pq(l\lambda)L(s\frac{1}{2})S(LS)JM_J \right\rangle =$$

$$= \int d\hat{p} \int d\hat{q} \int d\hat{p}' \int d\hat{q}' \sum_{m_{L'}=-L'}^{L'} c(L',S',J,m_{L'},M_J - m_{L'},M_J)$$

$$\sum_{m_L=-L}^{L} c(L,S,J,m_L,M_J - m_L,M_J) \sum_{m_{l'}=-l'}^{l'} c(l',\lambda',L',m_{l'},m_{L'} - m_{l'},m_{L'})$$

$$\sum_{m_l=-l}^{l} c(l,\lambda,L,m_l,m_L - m_l,m_L) Y_{lm_l}(\hat{p}) Y^*_{l'm_{l'}}(\hat{p}') Y_{\lambda m_L - m_l}(\hat{q}) Y^*_{\lambda' m_{L'} - m_{l'}}(\hat{q}')$$

$$\left\langle (s'\frac{1}{2})S' \, M_J - m_{L'} \left| \hat{O}(\vec{p}',\vec{q}',\vec{p},\vec{q}) \right| (s\frac{1}{2})S \, M_J - m_L \right\rangle$$

Traditional PWD:

Decouple momentum and spin spaces, **use** properties of the spherical harmonics, Clebsh-Gordan coefficients, 6j and 9j symbols to reduce the number of integrations, **program** (summations, integrals)

In aPWD one needs to perform:
- 8-dimensional integration for each p’ ,q’ ,p,q
- calculation of the spin-space (isospin-space) element

$$\left\langle (s'\frac{1}{2})S' \, M_J - m_{L'} \left| \hat{O}(\vec{p}',\vec{q}',\vec{p},\vec{q}) \right| (s\frac{1}{2})S \, M_J - m_L \right\rangle$$

aPWD of 3NF

$$M \equiv \left\langle p'q'(l'\lambda')L'(s'\frac{1}{2})S'(L'S')JM_J \left| \hat{O} \right| pq(l\lambda)L(s\frac{1}{2})S(LS)JM_J \right\rangle =$$

$$= \frac{1}{2J+1} \sum_{M_J=-J}^{J} \left\langle p'q'(l'\lambda')L'(s'\frac{1}{2})S'(L'S')JM_J \left| \hat{O} \right| pq(l\lambda)L(s\frac{1}{2})S(LS)JM_J \right\rangle$$

Since *M* is a scalar quantity, taking

$$\hat{p} = (0,0,1),$$

$$\hat{q} = (\sin\theta_q, 0, \cos\theta_q)$$

reduces the number of integrations to 5.
The isospin matrix elements can be easily calculated analyticaly.
The spin matrix elements can be calculated using a software for symbolic algebra (for example *Mathematica*®).

The remainig task is still hard numerically (10^7 5-dim integrations).

3NF at N^{2}LO

- N^{2}LO (E.Epelbaum, Prog.Part.Nucl.Phys. 57, 654(2006)):

$$V_{123} = V_{2\pi}^{(3)} + V_{1\pi,cont}^{(3)} + V_{cont}^{(3)}$$

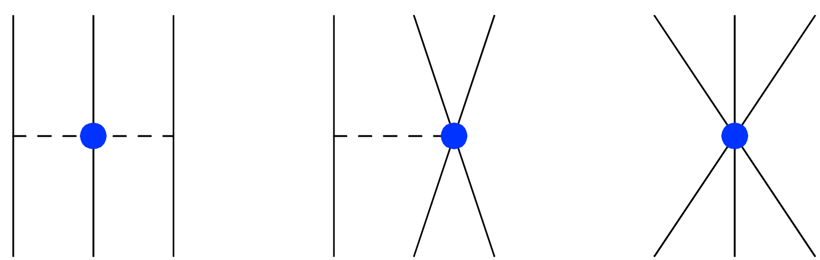
$$V_{2\pi}^{(3)} = \sum_{i\neq j\neq k} \frac{1}{2}\left(\frac{g_A}{2F_\pi}\right)^2 \frac{(\vec{\sigma}_i \cdot \vec{q}_i)(\vec{\sigma}_j \cdot \vec{q}_j)}{(\vec{q}_i^{\,2} + M_\pi^2)(\vec{q}_j^{\,2} + M_\pi^2)} F_{ijk}^{\alpha\beta} \tau_i^\alpha \tau_j^\beta$$

$$\vec{q}_i \equiv \vec{p}_i\,' - \vec{p}_i$$

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta}\left[-\frac{4c_1 M_\pi^2}{F_\pi^2} + \frac{2c_3}{F_\pi^2}\vec{q}_i \cdot \vec{q}_j\right] + \sum_\gamma \frac{c_4}{F_\pi^2}\varepsilon^{\alpha\beta\gamma}\tau_k^\gamma \vec{\sigma}_k \cdot \left[\vec{q}_i \times \vec{q}_j\right]$$

$$V_{1\pi,cont}^{(3)} = -\sum_{i\neq j\neq k} \frac{g_A}{8F_\pi^2} D \frac{\vec{\sigma}_j \cdot \vec{q}_j}{\vec{q}_j^{\,2} + M_\pi^2}\left(\vec{\tau}_i \cdot \vec{\tau}_j\right)\left(\vec{\sigma}_i \cdot \vec{q}_j\right)$$

$$V_{cont}^{(3)} = \frac{1}{2}\sum_{j\neq k} E\left(\vec{\tau}_j \cdot \vec{\tau}_k\right)$$



Two free parameters: D and E

Example: Two-pion exchange potential at N^{2}LO

$$V^{3N} = F_1 \vec{\sigma}_2 \cdot \vec{q}_2 \, \vec{\sigma}_3 \cdot \vec{q}_3 \, \vec{\tau}_2 \cdot \vec{\tau}_3 + F_2 \vec{\sigma}_1 \cdot (\vec{q}_2 \times \vec{q}_3) \, \vec{\sigma}_2 \cdot \vec{q}_2 \, \vec{\sigma}_3 \cdot \vec{q}_3 \, \vec{\tau}_1 \cdot (\vec{\tau}_2 \times \vec{\tau}_3)$$

where

$$\vec{q}_1 = \vec{q}\,' - \vec{q} \qquad \vec{q}_2 = \vec{p}\,' - \frac{1}{2}\vec{q}\,' - \left(\vec{p} - \frac{1}{2}\vec{q}\right)$$

$$\vec{q}_4 = \vec{q}_2 \times \vec{q}_3 \qquad \vec{q}_3 = -\vec{p}\,' - \frac{1}{2}\vec{q}\,' - \left(-\vec{p} - \frac{1}{2}\vec{q}\right)$$

Examples of integrals resulting from symbolic calculations:

$$G(0,0,0,1,\frac{1}{2};0,0,0,0,\frac{1}{2};\frac{1}{2}) = \int d\hat{p}' \int d\hat{q}' \int d\theta_q \, \frac{i}{16\pi^2\sqrt{3}} F_2((\vec{q}_2 \cdot \vec{q}_3)^2 - q_2^2 q_3^2)$$

$$G(1,1,1,0,\frac{1}{2};2,2,0,0,\frac{1}{2};\frac{1}{2}) = \int d\hat{p}' \int d\hat{q}' \int d\theta_q \, \frac{1}{2\sqrt{3}} F_2 \, \vec{q}_2 \cdot \vec{q}_3 \, Y_{2,2}^{0,0}(\hat{p},\hat{q}) \times$$

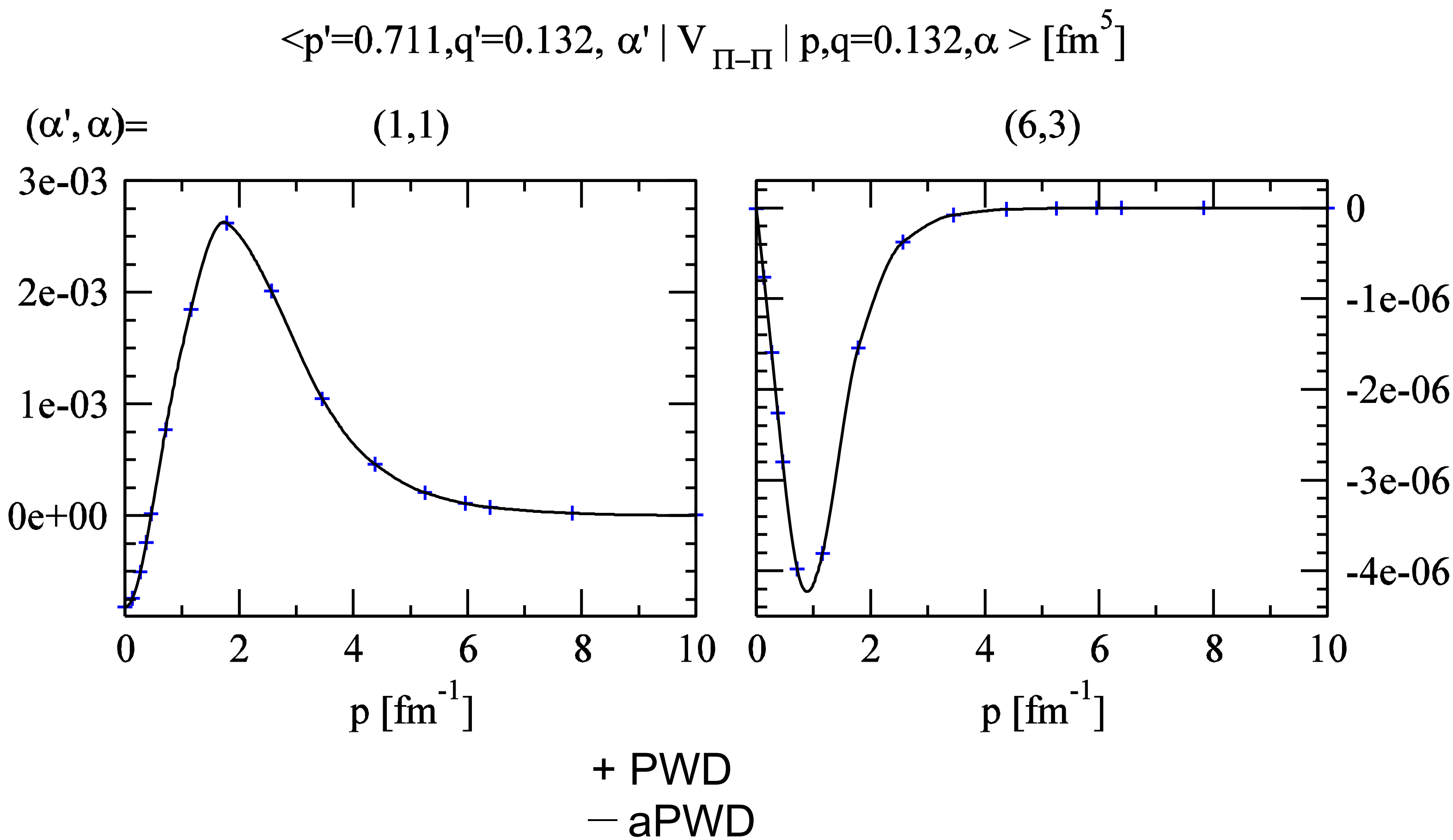
$$\times \left\{ \sqrt{2}\left(q_{4x} - i q_{4y}\right) Y_{1,1}^{1,-1\,*}(\hat{p}',\hat{q}') + 2 q_{4z} \, Y_{1,1}^{1,0\,*}(\hat{p}',\hat{q}') - \sqrt{2}\left(q_{4x} + i q_{4y}\right) Y_{1,1}^{1,1\,*}(\hat{p}',\hat{q}') \right\}$$

$$Y_{l,\lambda}^{L,m_L}(\hat{p},\hat{q}) \equiv \sum_{m_l=-l}^{l} c(l,\lambda,L;m_l,m_L - m_l,m_L) Y_{l,m_l}(\hat{p}) Y_{\lambda,m_L-m_l}(\hat{q})$$

Simple matrix elements of isospin operators give additional factors to G.

Test: aPWD vs PWD for 3NF

Example: 2π-exchange potential for the Tucson-Melbourne 3NF

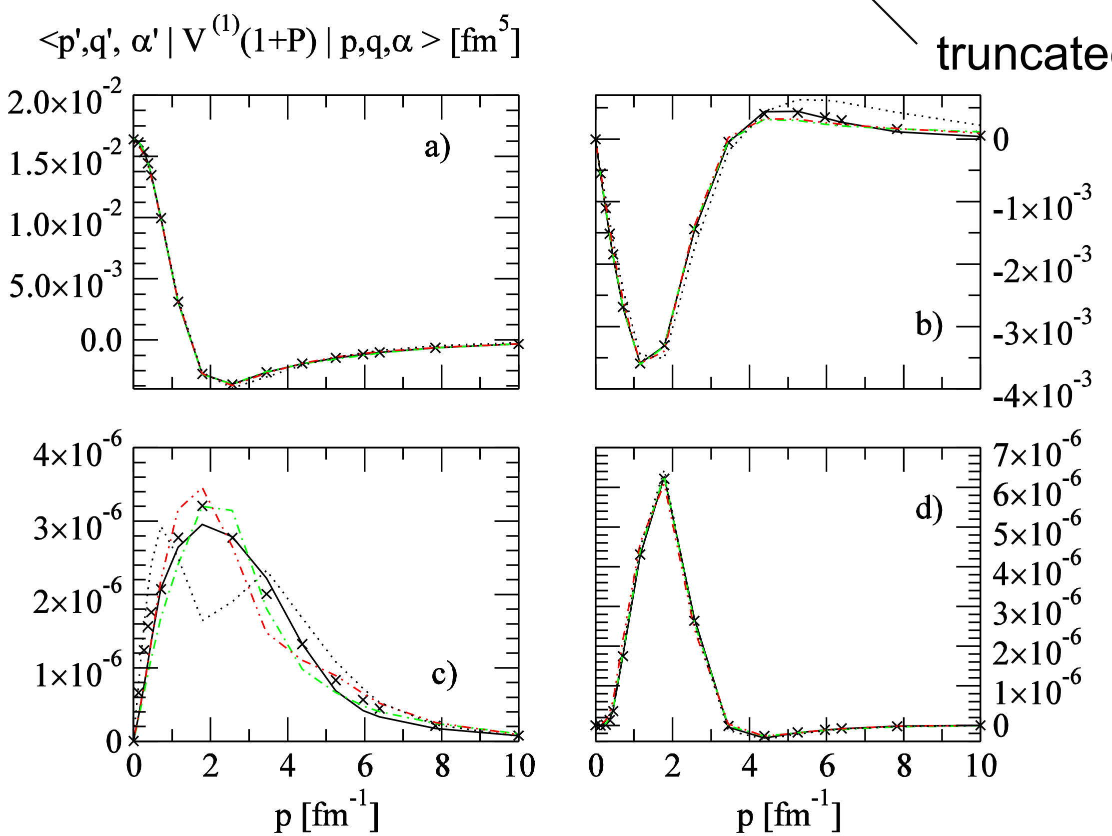


The convergence of the standard PWD for the V(1+P) operator

Standard PWD:

$$\langle p'q'\alpha'|VP|pq\alpha\rangle = \int dp''\,p''^2 \int dq''\,q''^2 \sum_{\alpha''} \langle p'q'\alpha'|V|p''q''\alpha''\rangle\langle p''q''\alpha''|P|pq\alpha\rangle$$

truncated at eg j_{max}=6



The TM 3NF

p' =0.711 fm^{-1}

q' =0.132 fm^{-1}

q =2.842 fm^{-1}

(α' ,α)=

a) (1,1)

b) (1,4)

c) (6,3)

d) (6,8)

Test: symmetries of the (1+P)V(1+P) operator

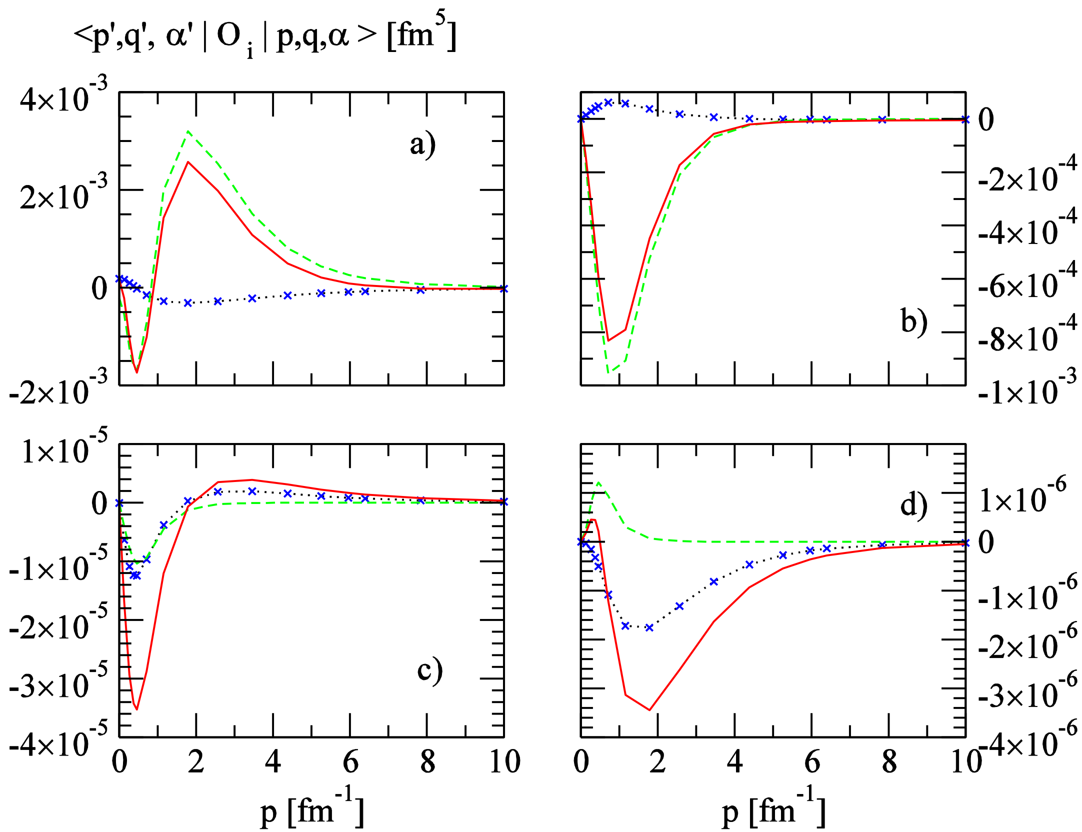
For V symmetrical under the exchange of particles 2 and 3, in (antysymmetric in 2-3 exchange) basis $|pq\alpha\rangle$ following symmetries are valid:

$$VP_{12}P_{23} = VP_{13}P_{23}$$

$$P_{12}P_{23}VP_{12}P_{23} = P_{13}P_{23}VP_{13}P_{23}$$

$$P_{12}P_{23}VP_{13}P_{23} = P_{13}P_{23}VP_{12}P_{23}$$

$$P_{12}P_{23}V = P_{13}P_{23}V$$



3NF at N^{3}LO long range part

N^{3}LO V.Bernard, E.Epelbaum, H.Krebs, U-G.Meißner, Phys Rev C77 (2008) 064004.

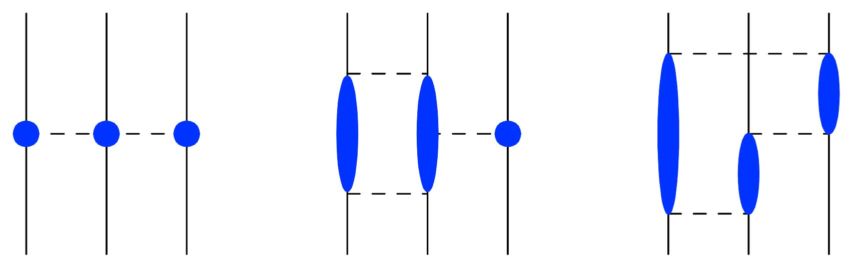
- $V_{2\pi}$ – already at N^{2}LO, at N^{3}LO the same operator structure but new values of C_1,C_3,C_4 and momentum dependence in formfactors

Two new topologies:

- Two pion – one pion exchange $V_{2\pi\text{-}1\pi}$
- The ring term V_{ring}

No new free parameters

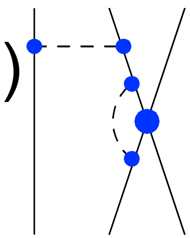
More operator structures and more complicated momentum dependece



3NF at N^3LO short range part

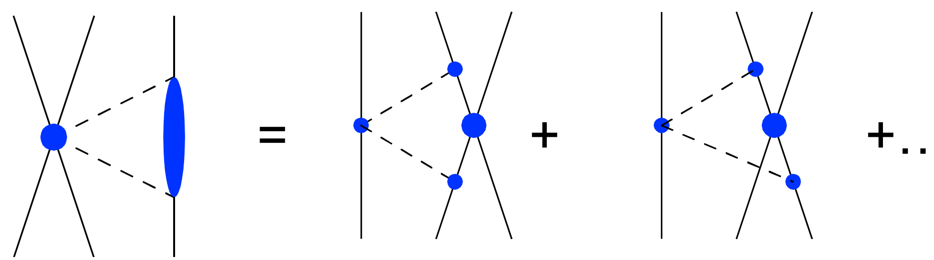
N^3LO: V.Bernard, E.Epelbaum, H.Krebs, U-G.Meißner,
Phys Rev C84 (2011) 054001.

- 1π-contact – already at N^2LO (one free parameter D)
  at N^3LO all terms cancel thus no new contributions at this order

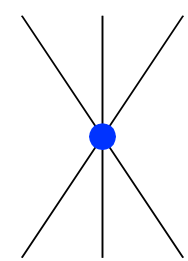


- 2π-contact

  No new free parameters



- Three nucleon contact term – already at N^2LO
  One free parameter E

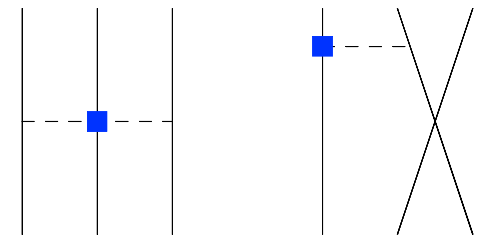


3NF at N^{3}LO short range part

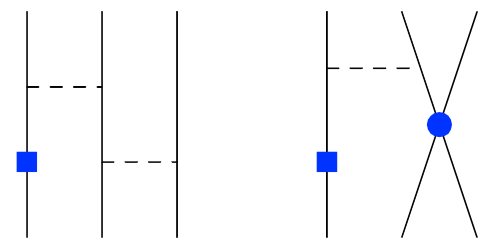
N^{3}LO: V.Bernard, E.Epelbaum, H.Krebs, U-G.Meißner,
Phys Rev C84 (2011) 054001.

- Relativistic 1/m corrections to 2π and 1π-contact terms origins in:

corrections to πNN
and $\pi\pi$NN vertices



retardation effects



No new free parameters

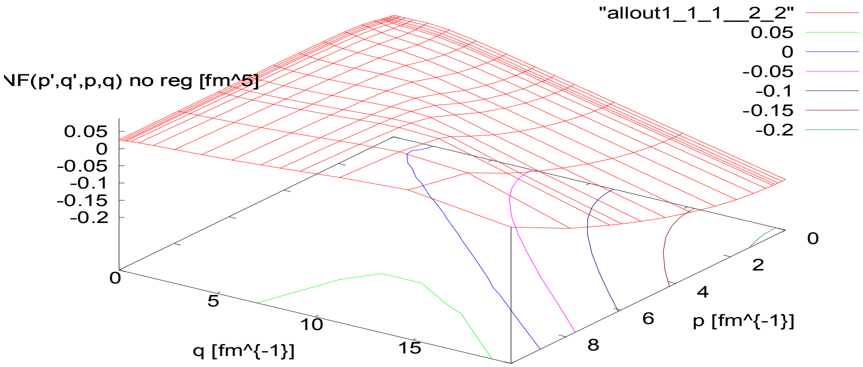
Chiral 3NF at N^3LO

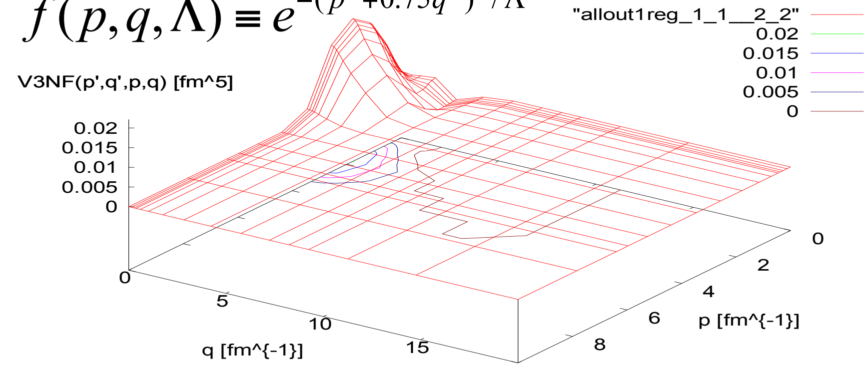
3NF matrix elements depend on p' q' α' pqα. Here is example for

$p' = 0.132 fm^{-1}$ $q' = 0.132 fm^{-1}$ $\alpha' = \alpha = 1$ as a function of p and q

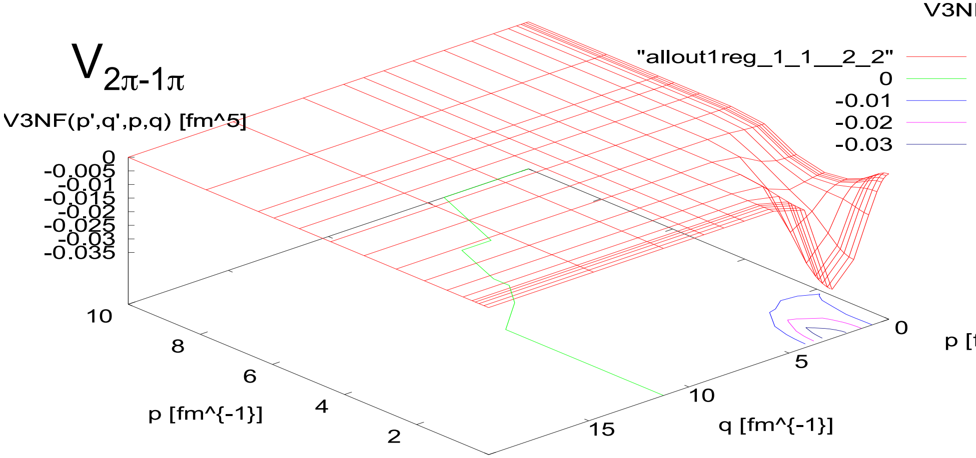
before regularization $V_{\pi\pi}$ after regularization (Λ=600 MeV)

$$f(p,q,\Lambda) \equiv e^{-(p^2+0.75q^2)^3/\Lambda^6}$$

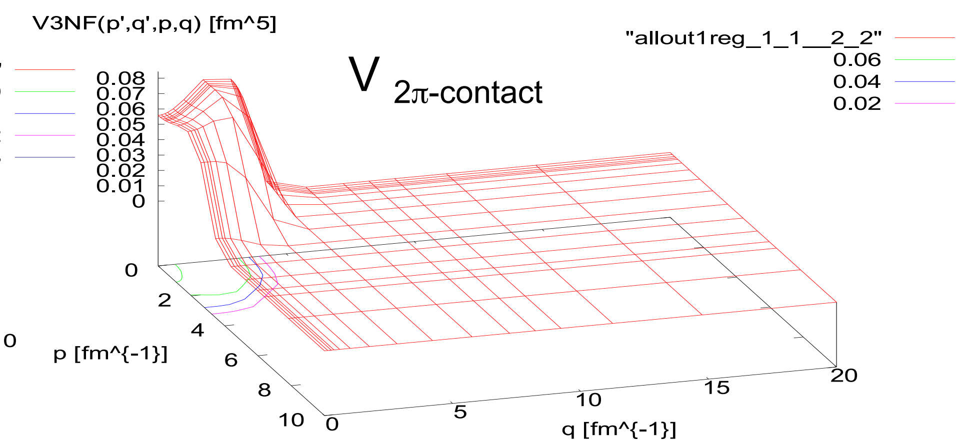




$V_{2\pi\text{-}1\pi}$

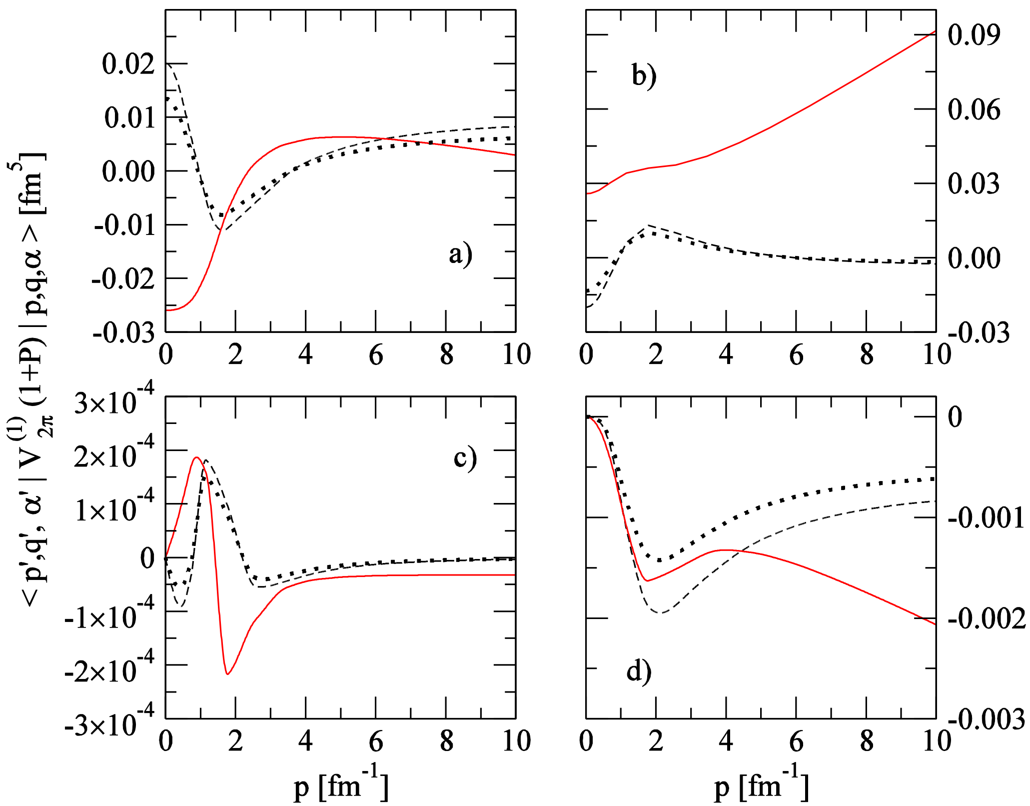


$V_{2\pi\text{-contact}}$



2π-exchange force at N^{2}LO and N^{3}LO

- At N^{3}LO different values of c1,c3,c4 parameters
  + additional terms in formfactors with a new momentum dependence



$V_{\pi\pi}(1+P)$ before regularization at
p' =0.268 fm^{-1}
q' =2.842 fm^{-1}
q =0.132 fm^{-1}

- - - - N^{2}LO
—— N^{3}LO
...... N^{3}LO old momentum dependence only

Values of free parameters d and e

- Values of the d and e constants are obtained from the ^3H binding energy and the $^2a_{nd}$ scattering lenght. Only long range terms of 3NF supplemented by d- and e- short range terms are taken into account.

Cut-off	Λ [MeV]	d	e
1	450	11.4	0.56
2	600	12.03	2.196
3	550	11.85	3.04
4	450	7.59	-0.063
5	600	14.1	2.649

$$d = D \cdot F_\pi^2 \cdot \Lambda_\chi$$

$$e = E \cdot F_\pi^4 \cdot \Lambda_\chi$$

F_π= 92.4 MeV
Λ_χ= 700 MeV

- Big compared to N^2LO:

  e.g cut-off=3: d=-0.45 e=-0.798 but

  $^2a_{nd}$: exp: 0.645 fm, for pure NN: N^2LO: 0.794 fm, N^3LO: 1.5873 fm.

^3H at N^3LO with relativistic corrections to 3NF (cut-off=1)

- New values of d and e

	d	e
$V_{\pi\pi}+V_{2\pi\text{-}1\pi}+V_{ring}+V_d+V_e$	**11.4**	**0.56**
$V_{\pi\pi}+V_{2\pi\text{-}1\pi}+V_{ring}+V_d+V_e+V_{2\pi\text{-cont}}$	**13.442**	**0.206**
$V_{\pi\pi}+V_{2\pi\text{-}1\pi}+V_{ring}+V_d+V_e+V_{2\pi\text{-cont}}+V_{1/m}$	**13.78**	**0.372**

- Expectation values [MeV]

	E_{NN}	E_{3NF}
$V_{\pi\pi}+V_{2\pi\text{-}1\pi}+V_{ring}+V_d+V_e$	**-43.449**	**-0.996**
$V_{\pi\pi}+V_{2\pi\text{-}1\pi}+V_{ring}+V_d+V_e+V_{2\pi\text{-cont}}$	**-43.399**	**-1.024**
$V_{\pi\pi}+V_{2\pi\text{-}1\pi}+V_{ring}+V_d+V_e+V_{2\pi\text{-cont}}+V_{1/m}$	**-43.382**	**-1.017**

	$V_{\pi\pi}$	$V_{2\pi\text{-}1\pi}$	V_{ring}	$V_{2\pi\text{-cont}}$	$V_{d\text{-term}}$	$V_{e\text{-term}}$	$V_{1/m}$
$V_{\pi\pi}+V_{2\pi\text{-}1\pi}+V_{ring}+V_d+V_e$	**-0.648**	**0.470**	**0.015**	---------	**-0.746**	**-0.087**	--------
$V_{\pi\pi}+V_{2\pi\text{-}1\pi}+V_{ring}+V_d+V_e+V_{2\pi\text{-cont}}$	**-0.661**	**0.485**	**0.014**	**0.082**	**-0.912**	**-0.032**	--------
$V_{\pi\pi}+V_{2\pi\text{-}1\pi}+V_{ring}+V_d+V_e+V_{2\pi\text{-cont}}+V_{1/m}$	**-0.655 (100%)**	**0.481 (73.4%)**	**0.014 (2.1%)**	**0.082 (12.5%)**	**-0.930 (142%)**	**-0.057 (8.7%)**	**0.048 (7.3%)**

^3H at N^3LO with NN potential by R.Machleidt cut=500 (600)

- New values of d and e

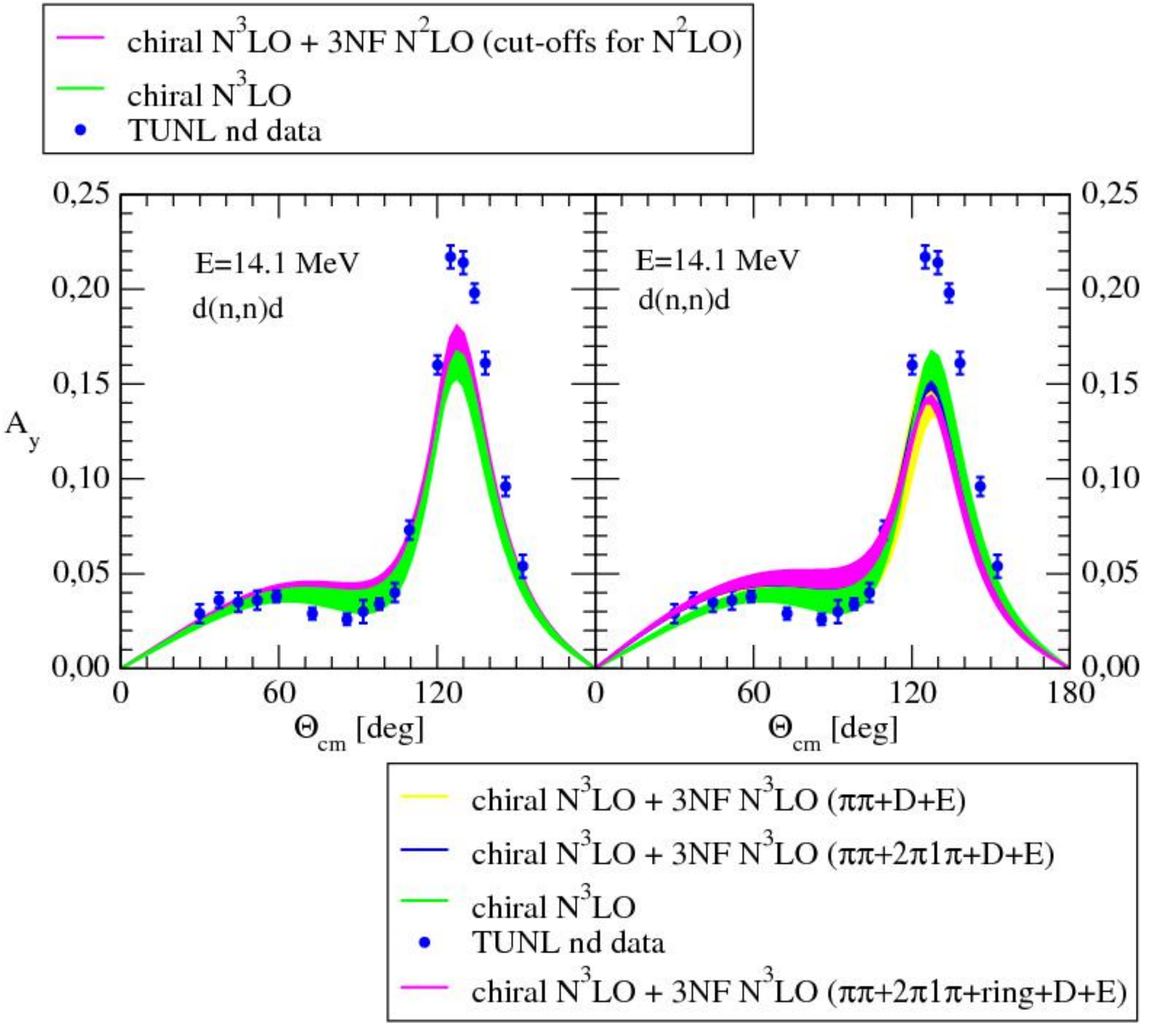
	d	e
$V_{\pi\pi}+V_{2\pi\text{-}1\pi}+V_{ring}+V_d+V_e$	**5.96 (6.3)**	**-0.43 (-0.3222)**
$V_{\pi\pi}+V_{2\pi\text{-}1\pi}+V_{ring}+V_d+V_e+V_{2\pi\text{-}cont}+V_{1/m}$	**7.25 (7.53)**	**-0.5625 (-0.499)**

- Expectation values [MeV]

	E_{NN}	E_{3NF}
$V_{\pi\pi}+V_{2\pi\text{-}1\pi}+V_{ring}+V_d+V_e$	**-44.382 (-44.572)**	**-0.768 (-0.869)**
$V_{\pi\pi}+V_{2\pi\text{-}1\pi}+V_{ring}+V_d+V_e+V_{2\pi\text{-}cont}+V_{1/m}$	**-44.381 (-44.617)**	**-0.768 (-0.870)**

	$V_{\pi\pi}$	$V_{2\pi\text{-}1\pi}$	V_{ring}	$V_{2\pi\text{-}cont}$	$V_{d\text{-}term}$	$V_{e\text{-}term}$	$V_{1/m}$
$V_{\pi\pi}+V_{2\pi\text{-}1\pi}+V_{ring}+V_d+V_e$	**-1.466** **-1.629**	**0.787** **0.494**	**-1.150** **-2.233**	**----** **----**	**0.518** **1.790**	**0.545** **0.709**	**----** **----**
$V_{\pi\pi}+V_{2\pi\text{-}1\pi}+V_{ring}+V_d+V_e+V_{2\pi\text{-}cont}+V_{1/m}$	**-1.463** **-1.620**	**0.782** **0.478**	**-1.175** **-2.273**	**-0.495** **-1.049**	**0.653** **2.186**	**0.724** **1.114**	**0.206** **0.294**

A_y puzzle



chiral N3LO + 3NF N2LO (cut-offs for N2LO)
chiral N3LO
TUNL nd data
E=14.1 MeV
d(n,n)d
E=14.1 MeV
d(n,n)d
Ay
0,25
0,20
0,15
0,10
0,05
0,00
0
60
120
180
Θcm [deg]
Θcm [deg]
chiral N3LO + 3NF N3LO (ππ+D+E)
chiral N3LO + 3NF N3LO (ππ+2π1π+D+E)
chiral N3LO
TUNL nd data
chiral N3LO + 3NF N3LO (ππ+2π1π+ring+D+E)

Electromagnetic processes

Example: the deuteron photodisintegration

$$N_\tau^{np} = \left\langle \phi_{np} \left| (1 + tG_0) j_\tau(\vec{Q}) \right| \Psi_{deuteron} \right\rangle$$

Example: the 3N bound state photodisintegration

$$N_\tau^{Nd} = \left\langle \phi_{Nd} \left| (1 + P) j_\tau(\vec{Q}) \right| \Psi_{bound} \right\rangle + \left\langle \phi_{Nd} \left| P \right| U \right\rangle$$

$$N_\tau^{3N} = \left\langle \phi_0 \left| (1 + P) j_\tau(\vec{Q}) \right| \Psi_{bound} \right\rangle + \left\langle \phi_0 \left| tG_0 (1 + P) j_\tau(\vec{Q}) \right| \Psi_{bound} \right\rangle +$$

$$+ \left\langle \phi_0 \left| P \right| U \right\rangle + \left\langle \phi_0 \left| tG_0 P \right| U \right\rangle$$

$$\left| U \right\rangle = (tG_0 + 0.5(1 + P) V_4^{(1)} G_0 (tG_0 + 1))(1 + P) j_\tau(\vec{Q}) \left| \Psi_{bound} \right\rangle +$$

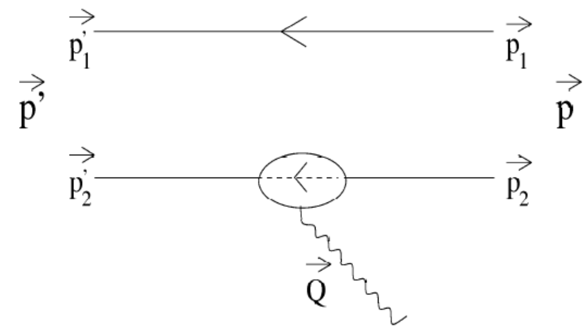
$$+ (tG_0 P + 0.5(1 + P) V_4^{(1)} G_0 (tG_0 + 1) P) \left| U \right\rangle$$

New component: electromagnetic current

Matrix elements of the EM current operator

We deal with the (relatively simple) single nucleon current, where

$$j^{\mu}\left(\vec{Q}\right) = j_1^{\mu}\left(\vec{Q}\right) + j_2^{\mu}\left(\vec{Q}\right) \longrightarrow$$



Examples:

single nucleon momenta

charge density

$$\left\langle \vec{p}' \left| \frac{1}{e} J_1^0(0) \right| \vec{p} \right\rangle = \left(G_E^p \Pi^p + G_E^n \Pi^n\right),$$

$$\left\langle \vec{p}' \left| \frac{1}{e} \vec{J}_1(0) \right| \vec{p} \right\rangle = \frac{\vec{p} + \vec{p}'}{2M_N}\left(G_E^p \Pi^p + G_E^n \Pi^n\right) + \frac{i}{2M_N}\left(G_M^p \Pi^p + G_M^n \Pi^n\right) \vec{\sigma} \times (\vec{p}' - \vec{p}).$$

convection current

spin current

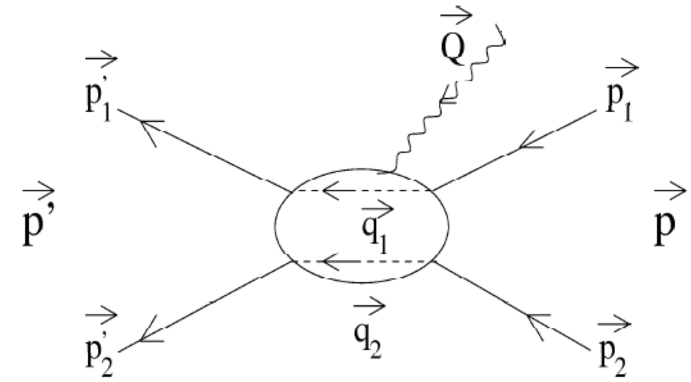
single nucleon momenta

$$\Pi^p \equiv \frac{1}{2}\left(1 + (\tau)_3\right) \quad \Pi^n \equiv \frac{1}{2}\left(1 - (\tau)_3\right)$$

Matrix elements of the EM current operator

We have also (more complicated) two-nucleon current

$$j^{\mu}\left(\vec{Q}\right) = j^{\mu}_{12}\left(\vec{Q}\right)$$



Example: one-pion-exchange current $\vec{J}_{ope} = \vec{J}^{\,seagull}_{ope} + \vec{J}^{\,pionic}_{ope}$

$$\vec{J}^{\,seagull}_{ope} = -i\left(\frac{g_A}{2F_\pi}\right)^2 \vec{\sigma}_1\,[\vec{\tau}_1 \times \vec{\tau}_2]_3\,\frac{\vec{\sigma}_2\cdot\vec{q}_2}{\vec{q}_2^{\;2} + M_\pi^2} + (1 \leftrightarrow 2)$$

$$\vec{J}^{\,pionic}_{ope} = i\left(\frac{g_A}{2F_\pi}\right)^2 \frac{\vec{\sigma}_1\cdot\vec{q}_1}{\vec{q}_1^{\,2} + M_\pi^2}\,\frac{\vec{\sigma}_2\cdot\vec{q}_2}{\vec{q}_2^{\,2} + M_\pi^2}\,[\vec{\tau}_1 \times \vec{\tau}_2]_3\,(\vec{q}_1 - \vec{q}_2)\,.$$

$$
\begin{aligned}
&\tilde{I}_2^{\text{seagull}}(p',p,Q;(l's')j'\mu',(ls)j\mu)\\
&= -6\sqrt{3}\,\pi\sqrt{\hat{l}\hat{s}\hat{j}\hat{l}'\hat{s}'\hat{j}'}(-1)^{l'+s'}\delta_{\mu',\mu+\zeta}\\
&\quad\times\sum_{\alpha_1}(-1)^{\alpha_1}\hat{\alpha}_1\begin{Bmatrix}1 & 1 & \alpha_1\\ \frac{1}{2} & \frac{1}{2} & s\\ \frac{1}{2} & \frac{1}{2} & s'\end{Bmatrix}\\
&\quad\times\sum_{\alpha_2}(-1)^{\alpha_2}C(1j\alpha_2;\zeta\mu\mu')\\
&\quad\times\sum_{\alpha_3}(-1)^{\alpha_3}\hat{\alpha}_3\begin{Bmatrix}1 & \alpha_1 & 1\\ l & s & j\\ \alpha_3 & s' & \alpha_2\end{Bmatrix}\\
&\quad\times\sum_{h_1+h_2=1}(-1)^{h_2}\left(\tfrac{1}{2}Q\right)^{h_1}\\
&\quad\times\sum_{r}\hat{r}[1-(-1)^{\alpha_1+h_2+r}]\\
&\quad\times\sum_{f_1}\sqrt{\hat{f}_1}\,C(rh_1f_1;000)C(f_1j'\alpha_2;0\mu'\mu')\begin{Bmatrix}f_1 & l' & \alpha_3\\ s' & \alpha_2 & j'\end{Bmatrix}\\
&\quad\times\sum_{f_2}\sqrt{\hat{f}_2}\,C(rh_2f_2;000)\sqrt{(2f_2+1)!}\begin{Bmatrix}f_1 & f_2 & 1\\ l & \alpha_3 & l'\end{Bmatrix}\begin{Bmatrix}f_2 & f_1 & 1\\ h_1 & h_2 & r\end{Bmatrix}\\
&\quad\times\sum_{u_1+u_2=f_2}(p')^{u_1}(p)^{u_2}\frac{1}{\sqrt{(2u_1+1)!(2u_2)!}}\\
&\quad\times\sum_{z}\sqrt{\hat{z}}\,C(u_2lz;000)C(l'zu_1;000)\begin{Bmatrix}u_1 & u_2 & f_2\\ l & l' & z\end{Bmatrix}G_{zr}^{h_2f_2},
\end{aligned}
$$

PWD already done for this operator

(see V.V. Kotlyar *et al*., Few-Body Systems 28, 35 (2000))

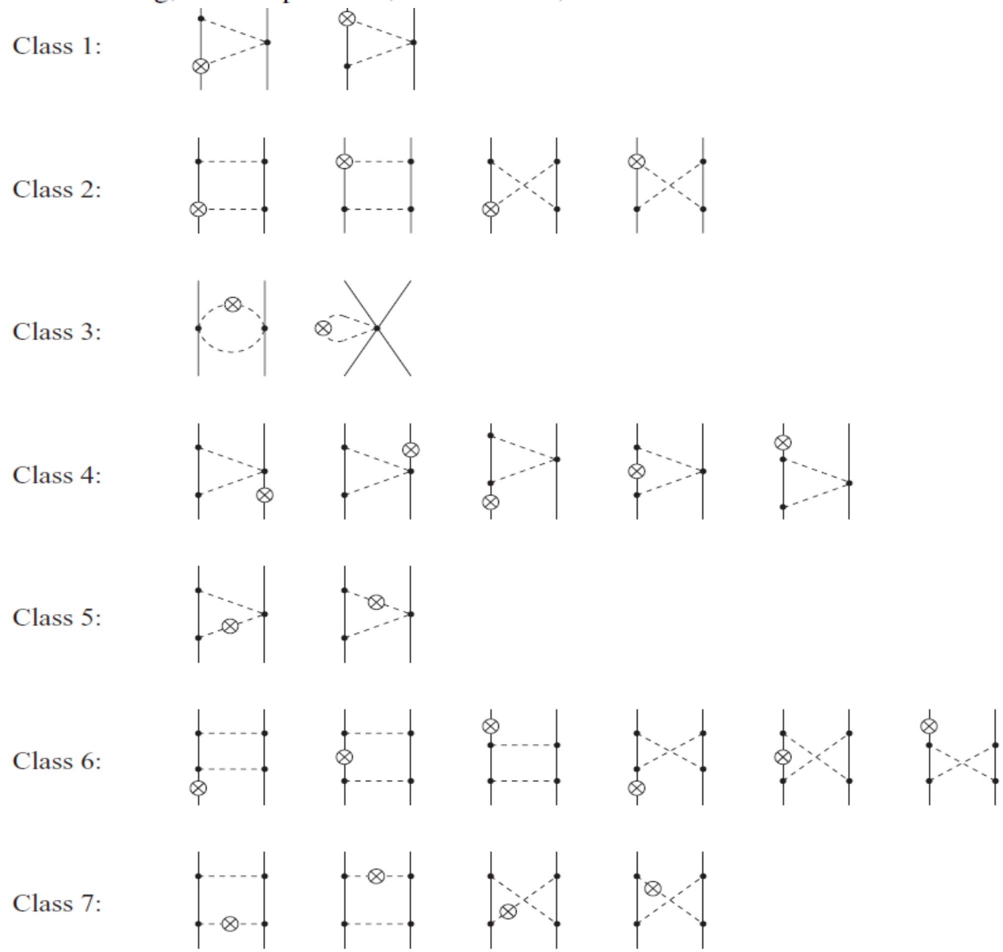
Obviously PWD of that kind can be carried out for small number of operators.

BUT WE HAVE TO EXPECT VERY MANY OPERATORS

PHYSICAL REVIEW C **80**, 045502 (2009)

Two-pion exchange electromagnetic current in chiral effective field theory using the method of unitary transformation

S. Kölling,[1,2,*] E. Epelbaum,[1,2,†] H. Krebs,[2,‡] and U.-G. Meißner[2,1,3,§]



Electromagnetic current at NLO

$$\vec{J} = \sum_{i=1}^{5} \sum_{j=1}^{24} f_i^j(\vec{q}_1, \vec{q}_2) T_i \vec{O}_j,$$

$$J^0 = \sum_{i=1}^{5} \sum_{j=1}^{8} f_i^{jS}(\vec{q}_1, \vec{q}_2) T_i O_j^S,$$

One can expect 24 spin operators for the vector components

No new free parameters

$$\begin{aligned}
\vec{O}_1 &= \vec{q}_1 + \vec{q}_2, \\
\vec{O}_2 &= \vec{q}_1 - \vec{q}_2, \\
\vec{O}_3 &= [\vec{q}_1 \times \vec{\sigma}_2] + [\vec{q}_2 \times \vec{\sigma}_1], \\
\vec{O}_4 &= [\vec{q}_1 \times \vec{\sigma}_2] - [\vec{q}_2 \times \vec{\sigma}_1], \\
\vec{O}_5 &= [\vec{q}_1 \times \vec{\sigma}_1] + [\vec{q}_2 \times \vec{\sigma}_2], \\
\vec{O}_6 &= [\vec{q}_1 \times \vec{\sigma}_1] - [\vec{q}_2 \times \vec{\sigma}_2], \\
\vec{O}_7 &= \vec{q}_1(\vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_2]) + \vec{q}_2(\vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_1]), \\
\vec{O}_8 &= \vec{q}_1(\vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_2]) - \vec{q}_2(\vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_1]), \\
\vec{O}_9 &= \vec{q}_2(\vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_2]) + \vec{q}_1(\vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_1]), \\
\vec{O}_{10} &= \vec{q}_2(\vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_2]) - \vec{q}_1(\vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_1]),
\end{aligned}$$

Electromagnetic current at NLO

$$\vec{O}_{11} = (\vec{q}_1 + \vec{q}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2),$$
$$\vec{O}_{12} = (\vec{q}_1 - \vec{q}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2),$$
$$\vec{O}_{13} = \vec{q}_1(\vec{q}_1 \cdot \vec{\sigma}_1)(\vec{q}_1 \cdot \vec{\sigma}_2) + \vec{q}_2(\vec{q}_2 \cdot \vec{\sigma}_1)(\vec{q}_2 \cdot \vec{\sigma}_2),$$
$$\vec{O}_{14} = \vec{q}_1(\vec{q}_1 \cdot \vec{\sigma}_1)(\vec{q}_1 \cdot \vec{\sigma}_2) - \vec{q}_2(\vec{q}_2 \cdot \vec{\sigma}_1)(\vec{q}_2 \cdot \vec{\sigma}_2),$$
$$\vec{O}_{15} = (\vec{q}_1 + \vec{q}_2)(\vec{q}_2 \cdot \vec{\sigma}_1)(\vec{q}_1 \cdot \vec{\sigma}_2),$$
$$\vec{O}_{16} = (\vec{q}_1 - \vec{q}_2)(\vec{q}_2 \cdot \vec{\sigma}_1)(\vec{q}_1 \cdot \vec{\sigma}_2),$$
$$\vec{O}_{17} = (\vec{q}_1 + \vec{q}_2)(\vec{q}_1 \cdot \vec{\sigma}_1)(\vec{q}_2 \cdot \vec{\sigma}_2),$$
$$\vec{O}_{18} = (\vec{q}_1 - \vec{q}_2)(\vec{q}_1 \cdot \vec{\sigma}_1)(\vec{q}_2 \cdot \vec{\sigma}_2),$$
$$\vec{O}_{19} = \vec{\sigma}_1(\vec{q}_1 \cdot \vec{\sigma}_2) + \vec{\sigma}_2(\vec{q}_2 \cdot \vec{\sigma}_1),$$
$$\vec{O}_{20} = \vec{\sigma}_1(\vec{q}_1 \cdot \vec{\sigma}_2) - \vec{\sigma}_2(\vec{q}_2 \cdot \vec{\sigma}_1),$$
$$\vec{O}_{21} = \vec{\sigma}_1(\vec{q}_2 \cdot \vec{\sigma}_2) + \vec{\sigma}_2(\vec{q}_1 \cdot \vec{\sigma}_1),$$
$$\vec{O}_{22} = \vec{\sigma}_1(\vec{q}_2 \cdot \vec{\sigma}_2) - \vec{\sigma}_2(\vec{q}_1 \cdot \vec{\sigma}_1),$$
$$\vec{O}_{23} = \vec{q}_1(\vec{q}_2 \cdot \vec{\sigma}_1)(\vec{q}_2 \cdot \vec{\sigma}_2) + \vec{q}_2(\vec{q}_1 \cdot \vec{\sigma}_1)(\vec{q}_1 \cdot \vec{\sigma}_2),$$
$$\vec{O}_{24} = \vec{q}_1(\vec{q}_2 \cdot \vec{\sigma}_1)(\vec{q}_2 \cdot \vec{\sigma}_2) - \vec{q}_2(\vec{q}_1 \cdot \vec{\sigma}_1)(\vec{q}_1 \cdot \vec{\sigma}_2),$$

Electromagnetic current at NLO

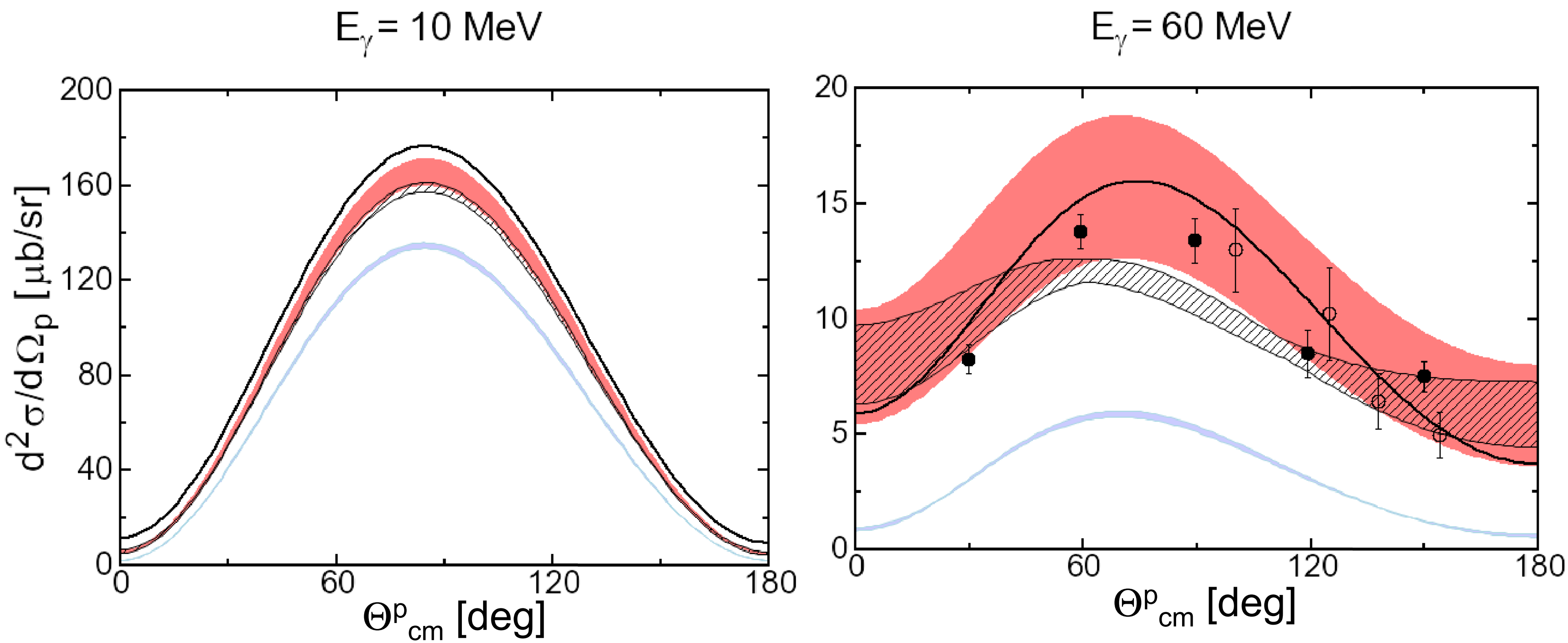
Additionally 8 spin operators for the charge density !

$$O_1^S = \mathbb{1},$$
$$O_2^S = \vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_2] + \vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_1],$$
$$O_3^S = \vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_2] - \vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_1],$$
$$O_4^S = \vec{\sigma}_1 \cdot \vec{\sigma}_2,$$
$$O_5^S = (\vec{q}_1 \cdot \vec{\sigma}_2)(\vec{q}_2 \cdot \vec{\sigma}_1),$$
$$O_6^S = (\vec{q}_1 \cdot \vec{\sigma}_1)(\vec{q}_2 \cdot \vec{\sigma}_2),$$
$$O_7^S = (\vec{q}_2 \cdot \vec{\sigma}_1)(\vec{q}_2 \cdot \vec{\sigma}_2) + (\vec{q}_1 \cdot \vec{\sigma}_1)(\vec{q}_1 \cdot \vec{\sigma}_2),$$
$$O_8^S = (\vec{q}_2 \cdot \vec{\sigma}_1)(\vec{q}_2 \cdot \vec{\sigma}_2) - (\vec{q}_1 \cdot \vec{\sigma}_1)(\vec{q}_1 \cdot \vec{\sigma}_2).$$

Isospin operators are chosen as

$$T_1 = \tau_1^3 + \tau_2^3,$$
$$T_2 = \tau_1^3 - \tau_2^3,$$
$$T_3 = [\vec{\tau}_1 \times \vec{\tau}_2]^3,$$
$$T_4 = \vec{\tau}_1 \cdot \vec{\tau}_2,$$
$$T_5 = \mathbb{1}.$$

The deuteron photodisintegration



chiral SNC

chiral SNC+OPE

chiral SNC+OPE+long-range TPE

AV18

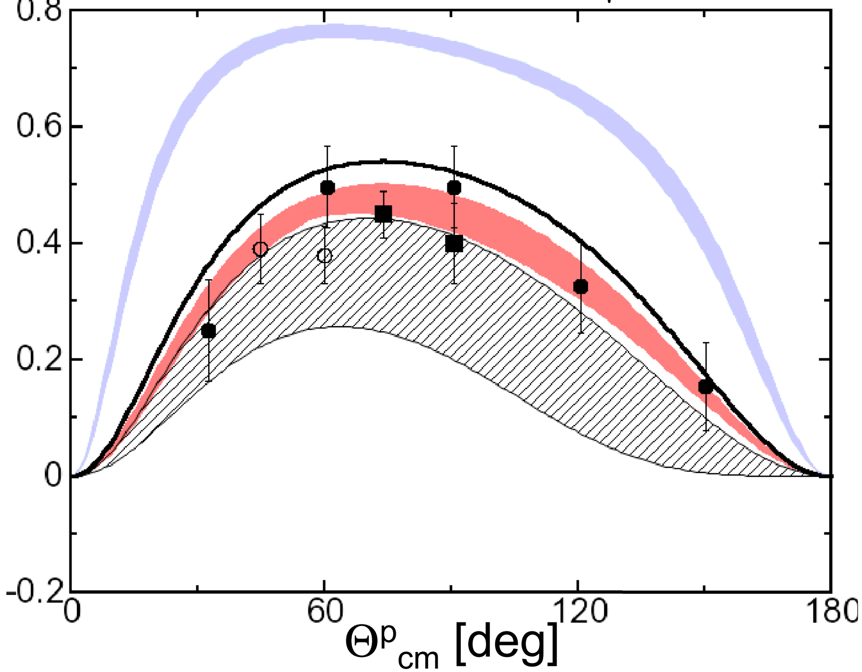
Exp Ying et al. PRC38(1988)1584

NN interaction at N^2LO

electromagnetic current at NLO

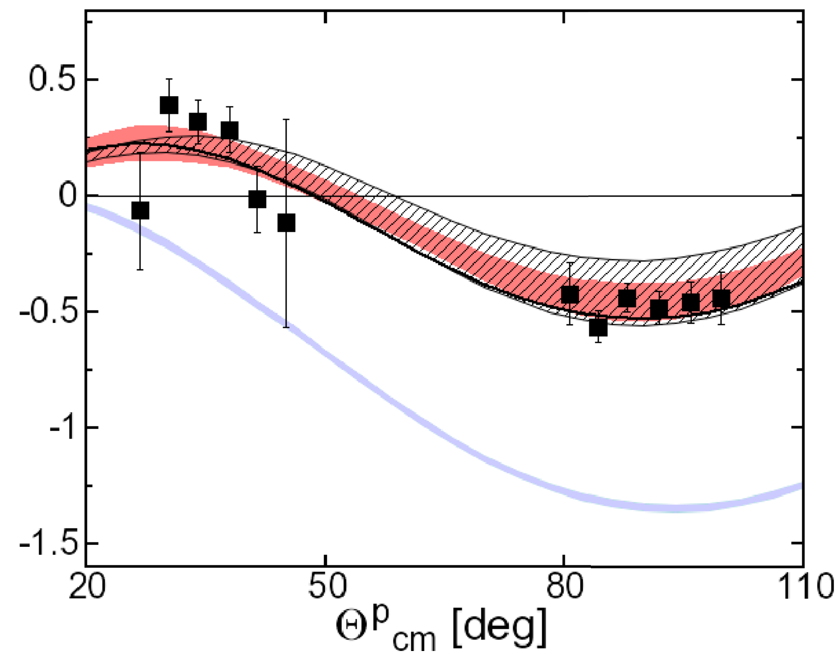
The deuteron photodisintegration – processes with polarization

Photon analyzing power at E_γ=60 MeV



Exp. Ying et al. PRC38(1988)1584

T_{22} at E_γ=45-70 MeV



Exp. Rachek et al. PRL98(2007)182303

More results: D.Rozpędzik et al. PRC83 (2011) 064004

The similar picture for ^{3}He photodisintegration

Comments on N^3LO 3NF calculations

- Project: 3NF at N^3LO matrix elements
  CPU – many terms, huge number of integrations (one integration is not so expensive, Monte-Carlo will not help much)
  thanks to J.Vary, K.Heberle

- More integration points required for higher partial waves?

- Which terms at N^3LO are the most important for light nuclei ?

- Fixing free paramaters
  -currently we use $E_{bound}(^3H)$, $^2a_{nd}$
  -future:
  the cross section in e.g. elastic nd (pd) scattering
  or
  weak process (^3H beta decay: effects of MECs are expected to be small)

NN at N^{3}LO – needed revision?

- **To describe 2N system it is necessary to go to N3LO in chiral expansion:**

- E. Epelbaum, H. -W. Hammer, U.-G. Meißner, Rev. Mod. Phys. 81, 1773 (2009)
- R. Machleidt, D. R. Entem, Phys. Rept. 503, 1 (2011)

Potential	LS cut-off [MeV]	SFR cut-off [MeV]	E_d [MeV]	P_d [%]
N2LO 101	450	500	-2.1922	3.536
N2LO 102	600	500	-2.1842	4.566
N2LO 103	550	600	-2.1887	4.383
N2LO 104	450	700	-2.2019	3.613
N2LO 105	600	700	-2.1997	4.709
N3LO 201	450	500	-2.2161	2.727
N3LO 202	600	600	-2.2212	3.545
N3LO 203	550	600	-2.2193	3.283
N3LO 204	450	700	-2.2187	2.844
N3LO 205	600	700	-2.2232	3.634

TABLE I: The cut-off's for Lippmann-Schwinger eq. (LS) regularization and spectral function regularization (SFR) together with the deuteron properties (E. Epelbaum Prog. Part. Nucl. Phys. **57**, 654 (2006)).

Summary and Outlook

- 3NF at N^3LO:
  $V_{\pi\pi}$ and $V_{2\pi\text{-}1\pi}$ dominate
  V_{ring}, $V_{2\pi\text{-contact}}$ and $V_{1/m}$ play a smaller role
  Contributions of $V_{d\text{-term}}$ and $V_{e\text{-term}}$ strongly depend on cut-offs

- NN and 3NF at N^4LO or from explicit Δ approach
- Revision of NN at N^3LO (?)

- The preparation of the matrix elements of 3NF already started.
  $$V_{NN}, \quad V^{(3)}(1+P), \quad (1+P)V^{(3)}(1+P), \quad \dots$$

- aPWD is a usefull tool not only for 3NF forces !
- aPWD technically is similar to the new 3-dimensional approach for the two- and three-body systems. Up to now we calculated the deuteron electrodisintegration, triton and the NN scattering including the first calculations of pp scattering without partial wave decomposition (Golak et al. Few-Body Syst. 53 (2012) 237).

Thank you for your attention and …