

The exotic He-9 nucleus studied with NCSM/RGM and the NCSMC

Progress in ab-initio Techniques in Nuclear Physics

23 February 2013

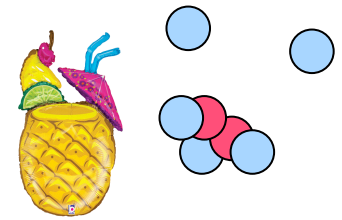
Michael Kruse, LLNL

 Lawrence Livermore
National Laboratory

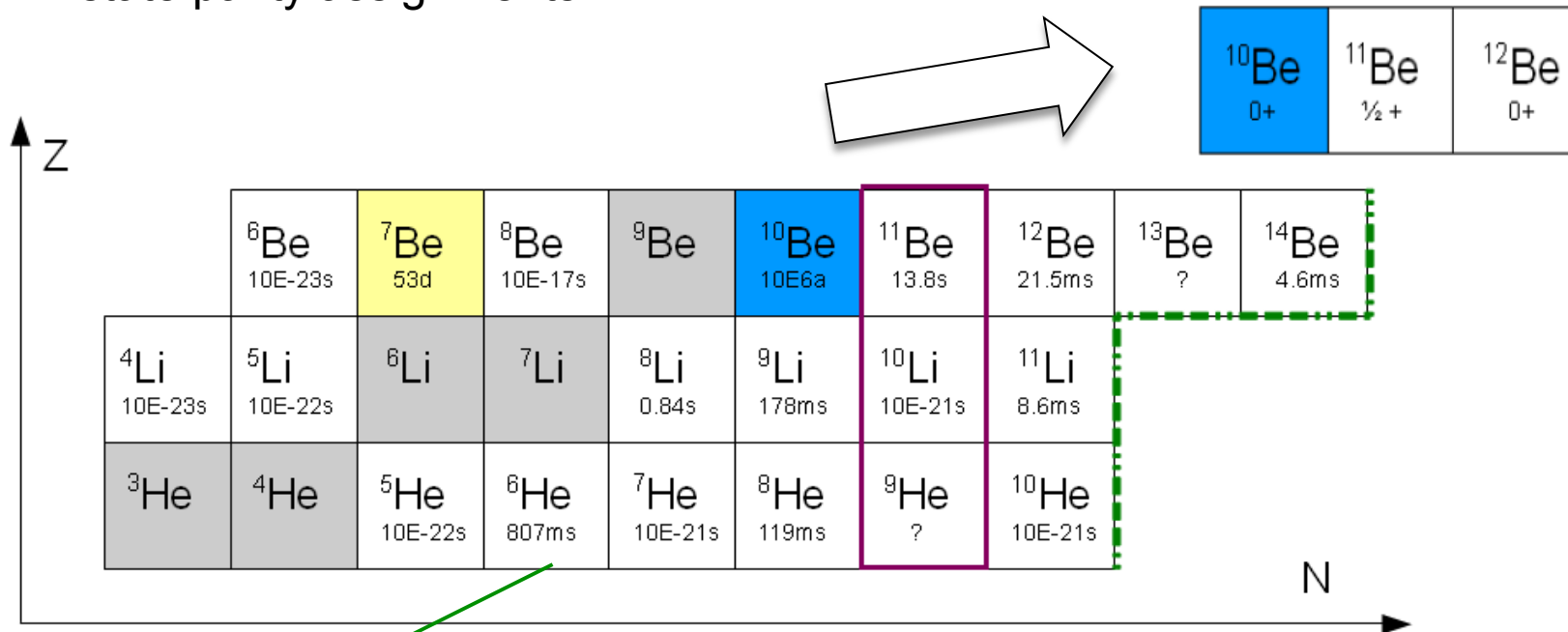


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Exotic nuclei



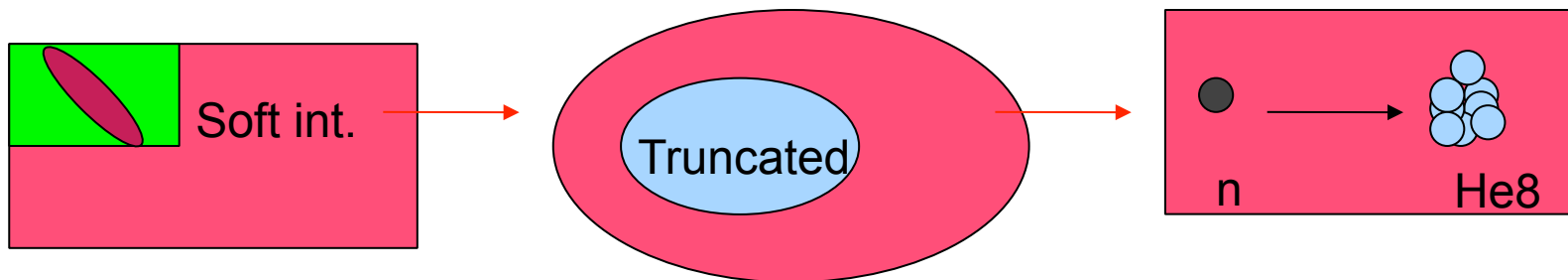
- Nuclei away from the valley of stability exhibit a rich set of phenomena, such as relatively long half-lives or unnatural ground state parity assignments.



- NCSM is good at calculating bound-states,**
- But has problems with weakly bound states**
- Weakly bound states → scattering observables.**

Is ${}^9\text{He}$ bound?

- Interesting physics question: One of the few nuclei that can be studied theoretically and experimentally that lies near the neutron drip-line.
- Be-11 and Li-10, both $N=7$ isotones, have un-natural parity assignments. What about He-9?
- Theory possibilities: Intruder states from the sd shell. Sensitive test of theory that bridges p-sd shell.

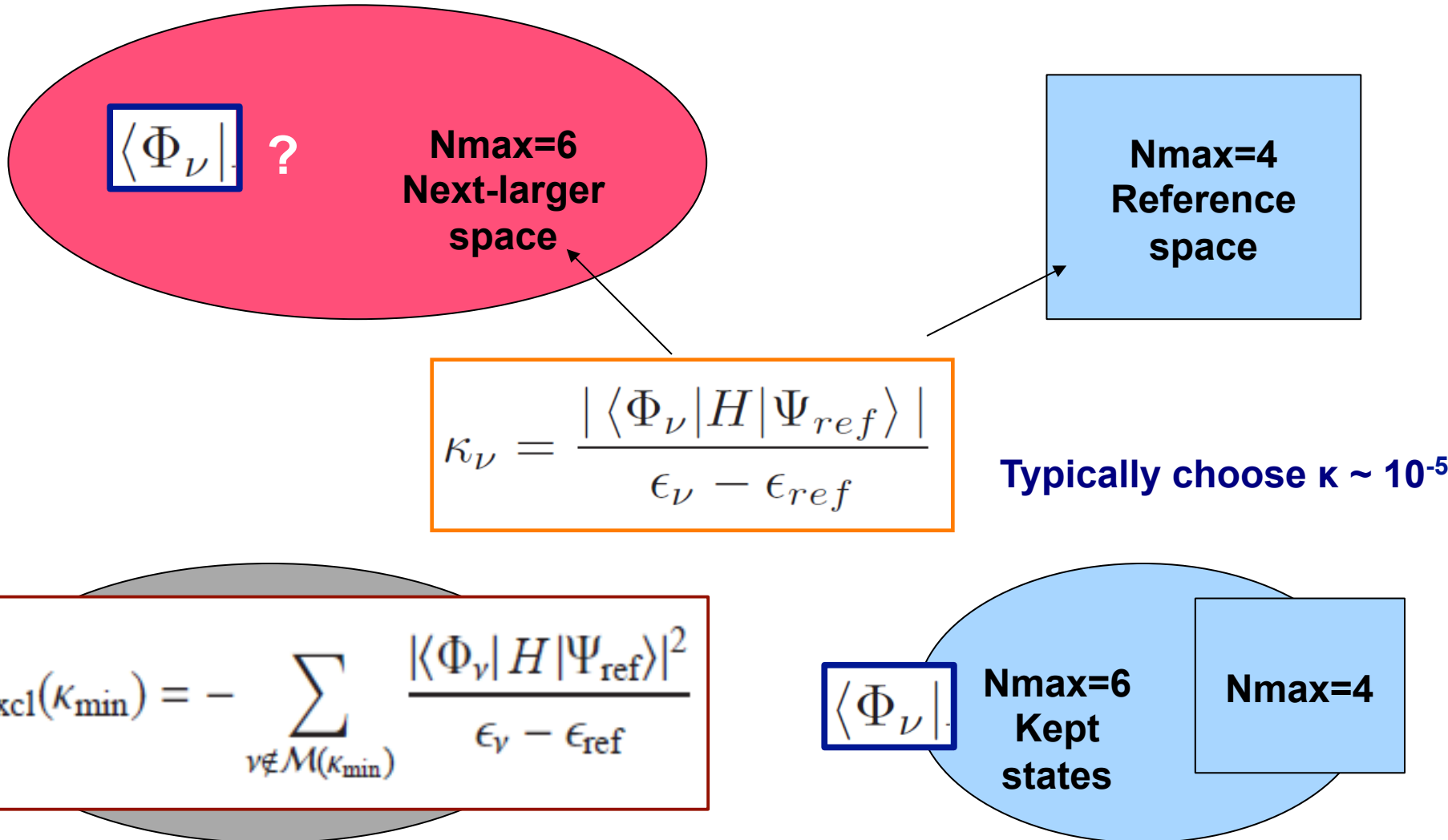


Use „soft“ interactions as input. Note: NN only SRG potentials.

Truncate the full model space to a smaller feasible space.
Importance Truncation.

Do a scattering calculation of a neutron on He8 – look for bound states in He9.
NCSM/RGM calculation

Importance truncation schematically



New results in IT-NCSM.

Extrapolation uncertainties in the importance-truncated No-Core Shell Model

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Careful investigation of IT-NCSM results as a function of N_{\max} , HO energy, SRG momentum decoupling (λ) and multiple reference states.

Full NCSM Li-6 calculations up to $N_{\max} = 14$ are used to benchmark IT-NCSM calculations of Li-6 varying the quantities above.

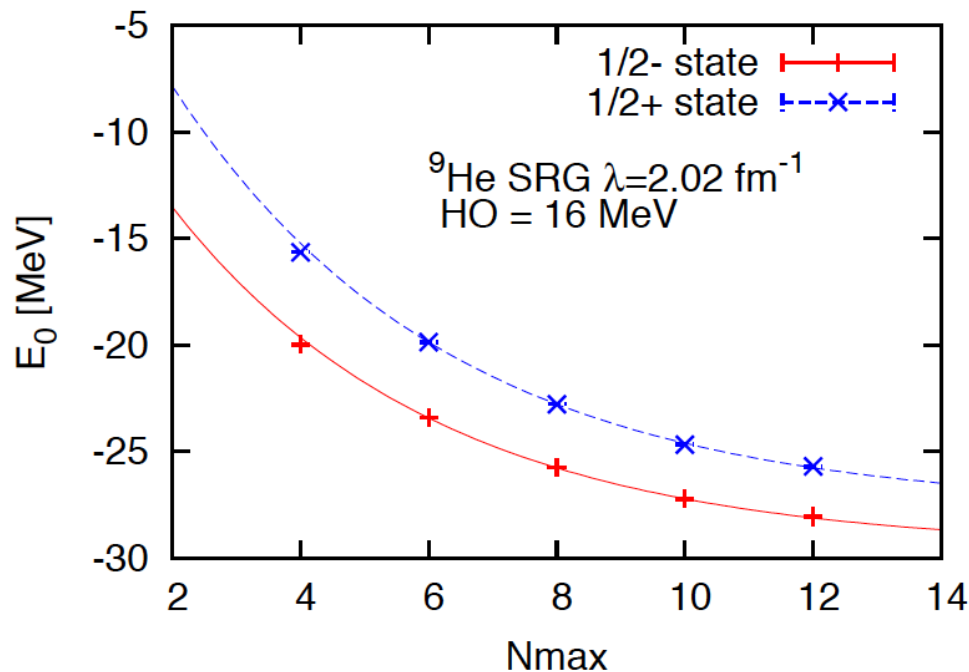
Extrapolations to N_{\max} infinity are handled very carefully taking into account uncertainties of extrapolating to $\kappa = 0$ in a given N_{\max} space.

[arXiv:1302.1226 \[nucl-th\]](https://arxiv.org/abs/1302.1226)

NCSM calculation of He-9

To demonstrate the advantages of the NCSM/RGM and NCSMC techniques I will briefly show a NCSM calculation of He-9. This is to be compared to later results.

All calculations use **HO = 16 MeV** and SRG NN chiral interaction at $\lambda = 2.02 \text{ fm}^{-1}$.



IT-NCSM calculations with $k \sim 1.0 \times 10^{-5}$. Extrapolated to $k = 0$.

Predict $1/2^-$ state to be ground-state.

Extrapolated to $N_{\text{max}} = \text{infinity}$.

	IT-NCSM	
J^π	$\frac{1}{2}^-$	$\frac{1}{2}^+$
E [MeV]	-28.040	-25.694

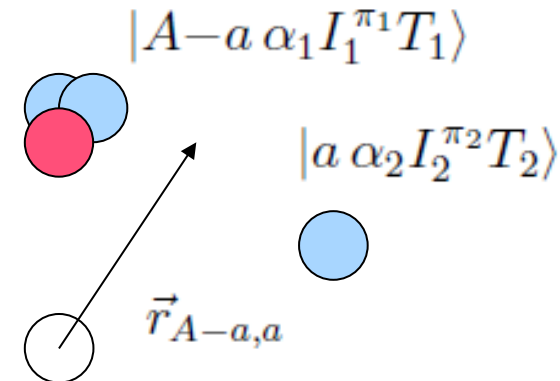
NCSM/RGM key ideas

- Expand wavefunction on a basis of binary clusters.
- Clusters themselves are anti-symmetric, but not anti-symmetric with each respect to each other.

$$|\Phi_{\nu r}^{J^{\pi T}}\rangle = \left[(|A-a \alpha_1 I_1^{\pi_1 T_1}\rangle |a \alpha_2 I_2^{\pi_2 T_2}\rangle)^{(sT)} \times Y_\ell(\hat{r}_{A-a,a}) \right]^{(J^{\pi T})} \frac{\delta(r - r_{A-a,a})}{r r_{A-a,a}}.$$

$$|\Psi^{J^{\pi T}}\rangle = \sum_{\nu} \int dr r^2 \frac{g_{\nu}^{J^{\pi T}}(r)}{r} \hat{\mathcal{A}}_{\nu} |\Phi_{\nu r}^{J^{\pi T}}\rangle$$

$$\sum_{\nu} \int dr r^2 [\mathcal{H}_{\nu'\nu}^{J^{\pi T}}(r', r) - E \mathcal{N}_{\nu'\nu}^{J^{\pi T}}(r', r)] \frac{g_{\nu}^{J^{\pi T}}(r)}{r} = 0$$



- **Clusters determined from NCSM calculation**
- **Calculate matrix elements for kernels.**

Norm and Hamiltonian kernel

- The Hamiltonian



$$H = T_{\text{rel}}(r) + \mathcal{V}_{\text{rel}} + \bar{V}_C(r) + H_{(A-a)} + H_{(a)}$$

Relative kinetic
between
clusters

Relative NN int
between
clusters

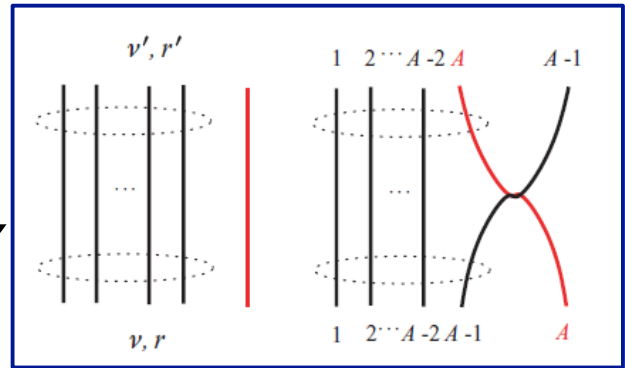
Avg.
Coulomb

Determined in same
(Nmax,hΩ) space

$$\sum_{\nu} \int dr r^2 [\mathcal{H}_{\nu'\nu}^{J^{\pi}T}(r', r) - E \mathcal{N}_{\nu'\nu}^{J^{\pi}T}(r', r)] \frac{g_{\nu}^{J^{\pi}T}(r)}{r} = 0$$

$$\mathcal{H}_{\nu'\nu}^{J^{\pi}T}(r', r) = \langle \Phi_{\nu'r'}^{J^{\pi}T} | \hat{A}_{\nu'} H \hat{A}_{\nu} | \Phi_{\nu r}^{J^{\pi}T} \rangle$$

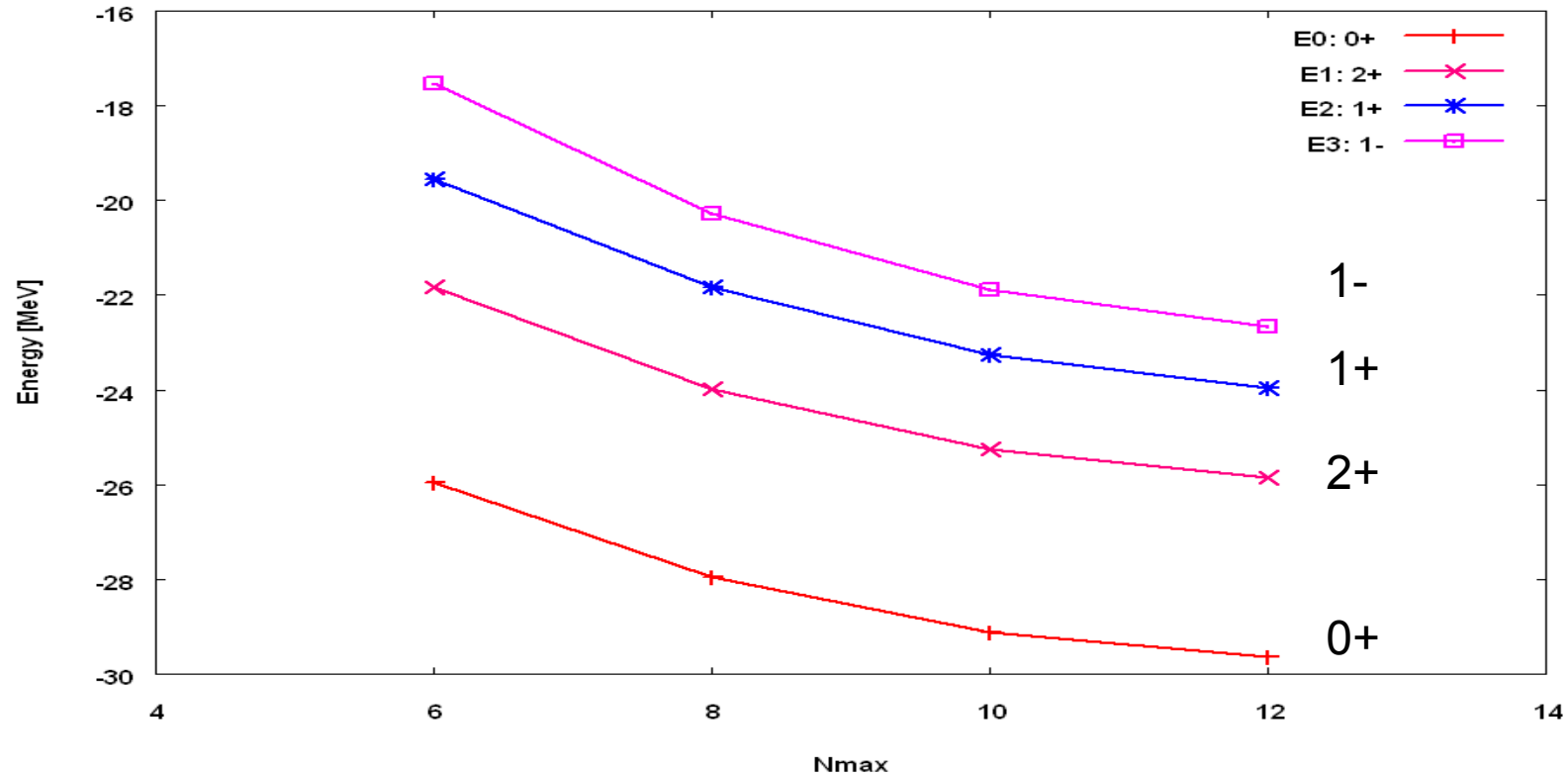
$$\mathcal{N}_{\nu'\nu}^{J^{\pi}T}(r', r) = \langle \Phi_{\nu'r'}^{J^{\pi}T} | \hat{A}_{\nu'} \hat{A}_{\nu} | \Phi_{\nu r}^{J^{\pi}T} \rangle$$



Direct

Exchange

He 8: Nmax=12 (Importance Truncated)

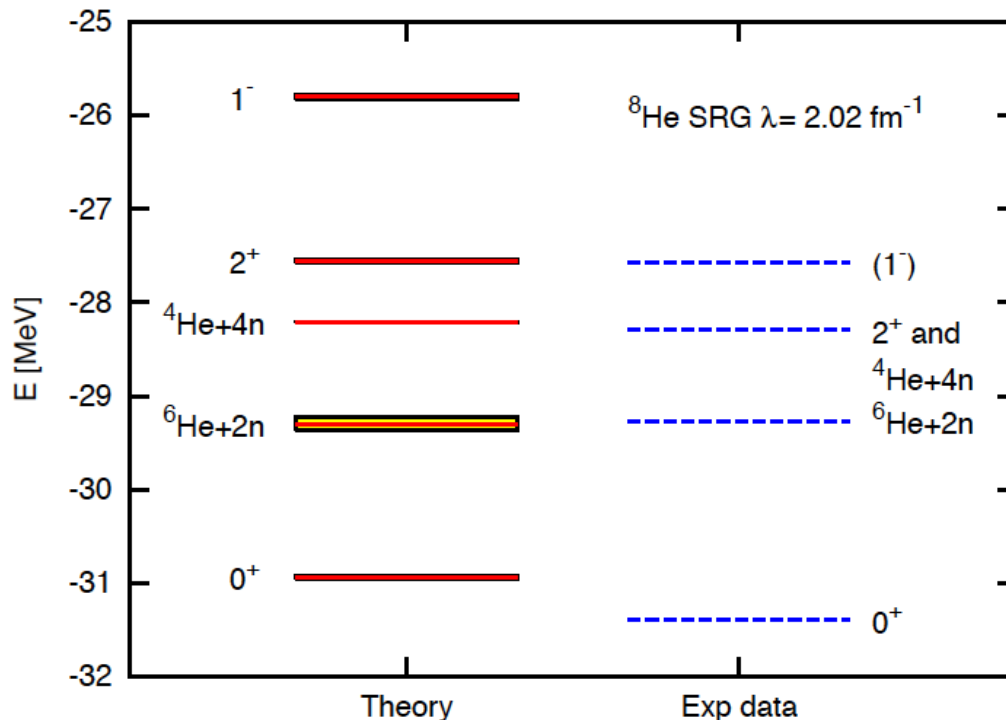


Nmax	E0 (MeV)	E1 (MeV)	E2 (MeV)	E3 (MeV)
12	-29.604	-25.854	-23.951	-22.664

Nmax	Full space (+) Parity	IT space (k=1E-5)
12	~ 428 million	~ 13.65 million

SRG-tuning of NN interaction

- We use the chiral N3LO NN interaction (500 MeV/c)
- SRG transformed to $\lambda = 2.02 \text{ fm}^{-1}$.
- How well do you describe other He isotopes then?



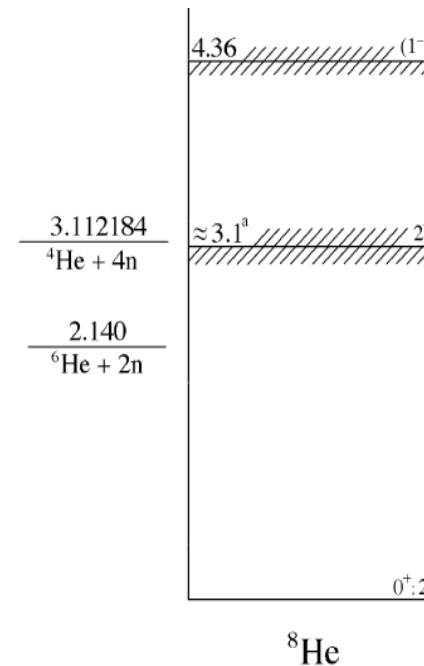
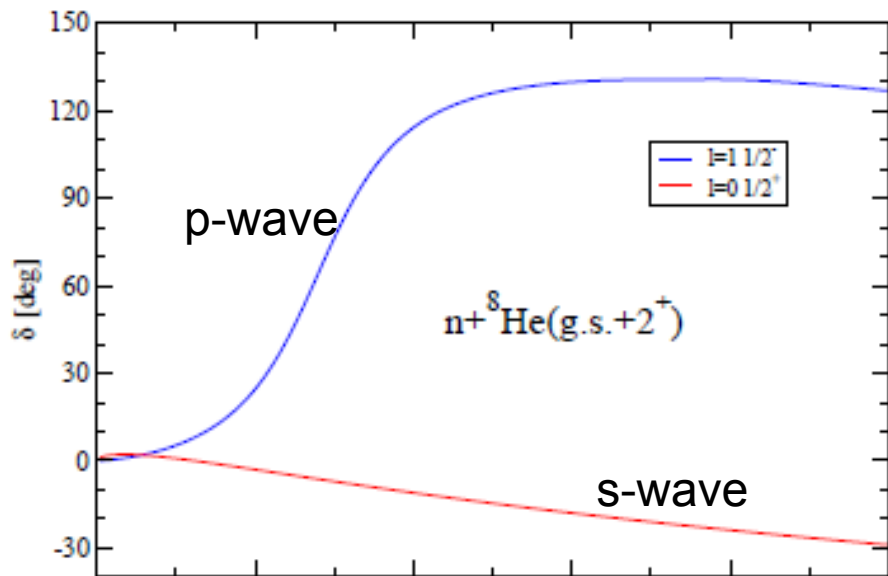
Performed NCSM calculations for He-6 and He-4 using same interaction and HO = 16 MeV.

Note: These results are extrapolations to N_{max} infinity.

We find that $\lambda = 2.02 \text{ fm}^{-1}$ tunes the NN interaction in such a way as to accurately reproduce exp.

Compares well to S. Bacca Phys. Rev. C 86, 034321 (2012)

NCSM/RGM: Inclusion of various states

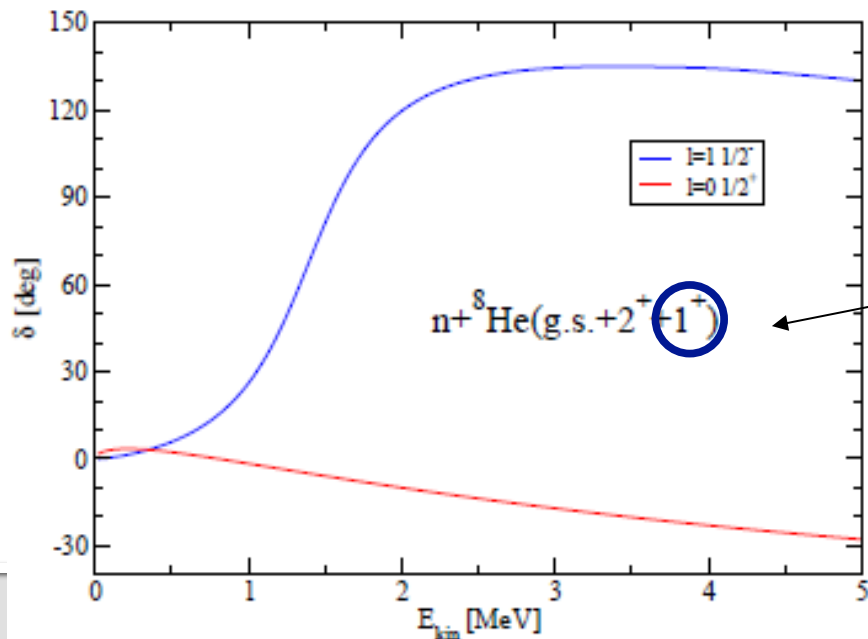


(1-) state: 4.36 MeV

2+ state: 3.1 MeV

$$\frac{2.574}{{}^7\text{He} + n}$$

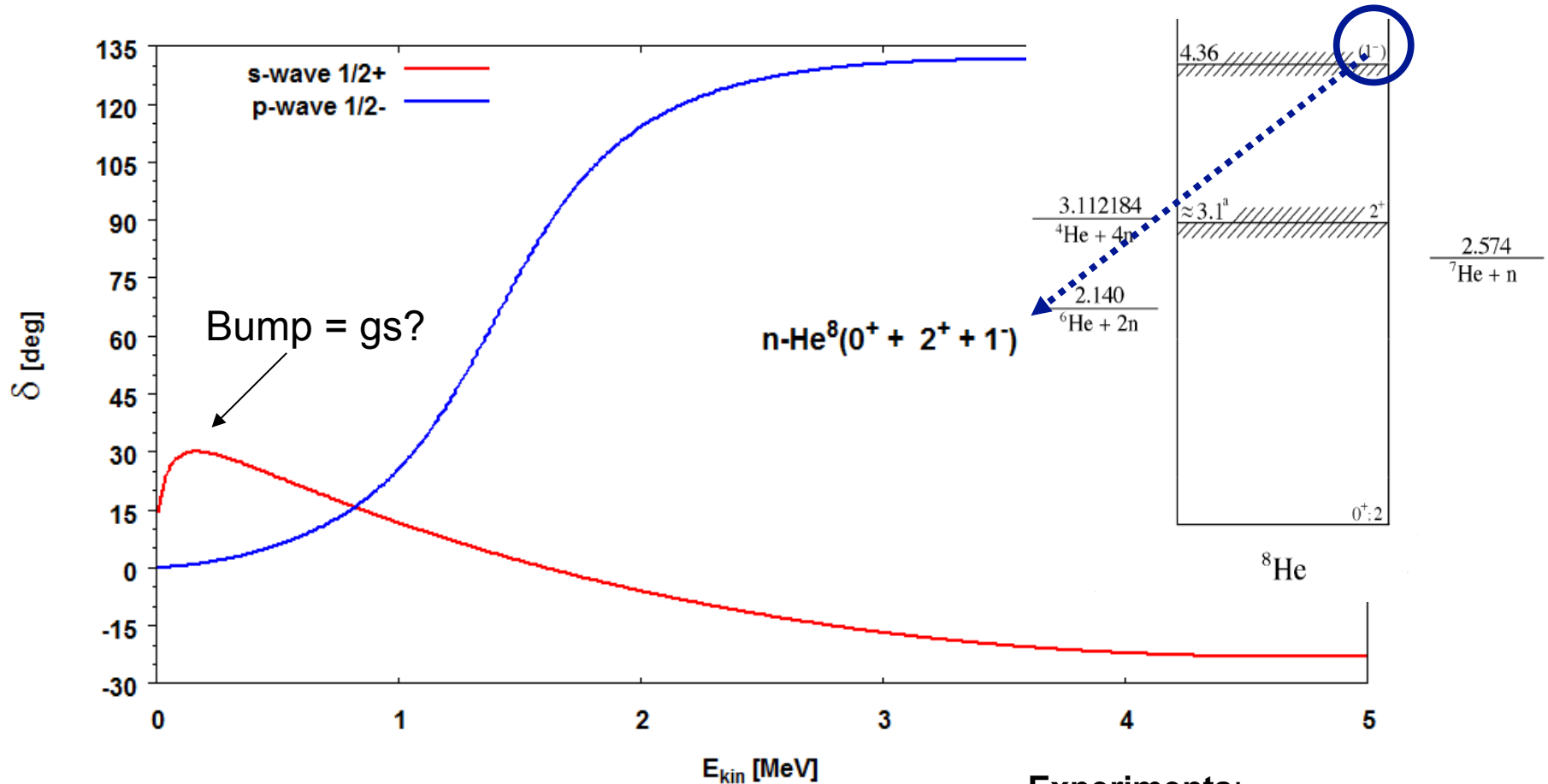
0+ state



There is almost no difference in the calculated phase shifts for including the 1+ state as well. Scattering length $a_0 = -1$ fm.

No bound state.

NCSM/RGM: Inclusion of negative parity state

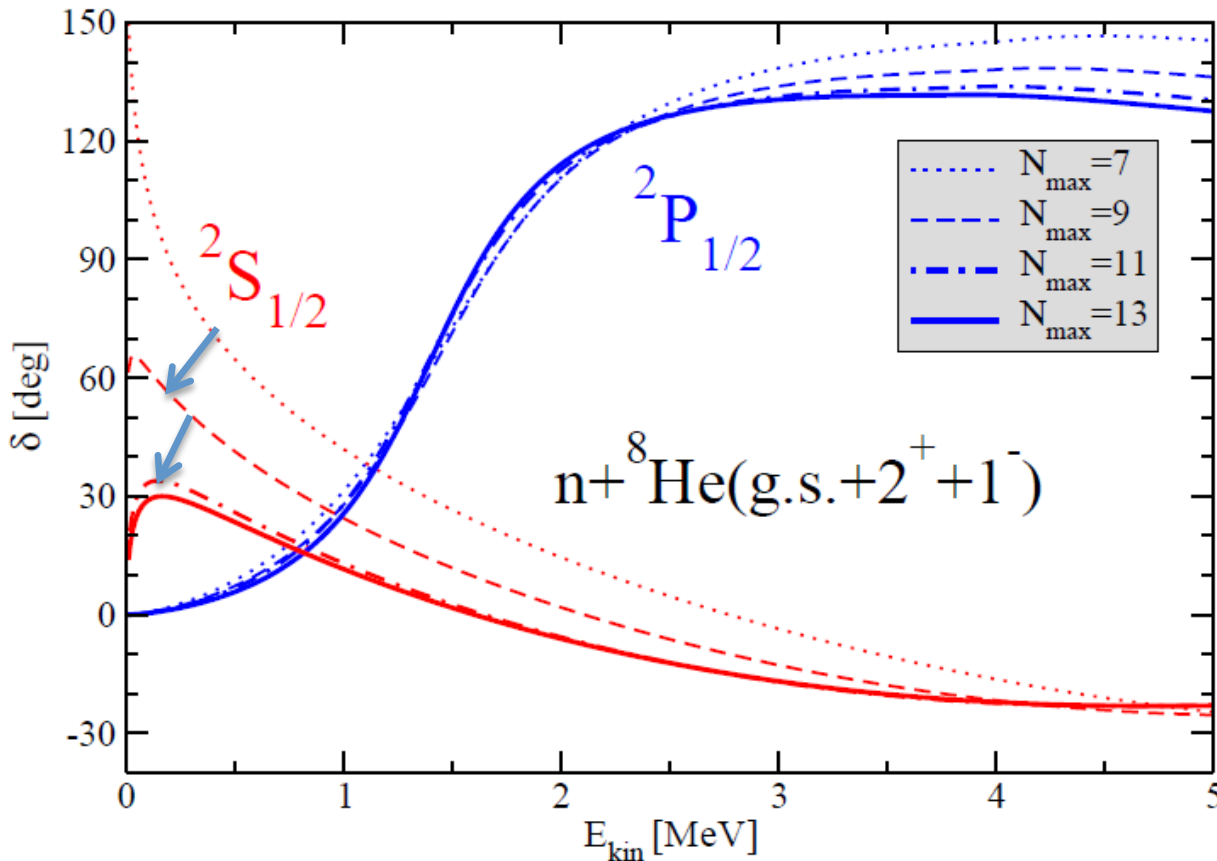


The inclusion of the negative parity state has a large effect! Now the predicted scattering length is $a_0 = -12.59$ fm.

Experiments:

- $a_0 < -10$ fm (Chen et al.) [PLB 505, 2001]
- $a_0 \sim -3$ fm (Al Falou, et al.) [arxiv:nucl-ex: 1008:0543]

Convergence check: Nmax



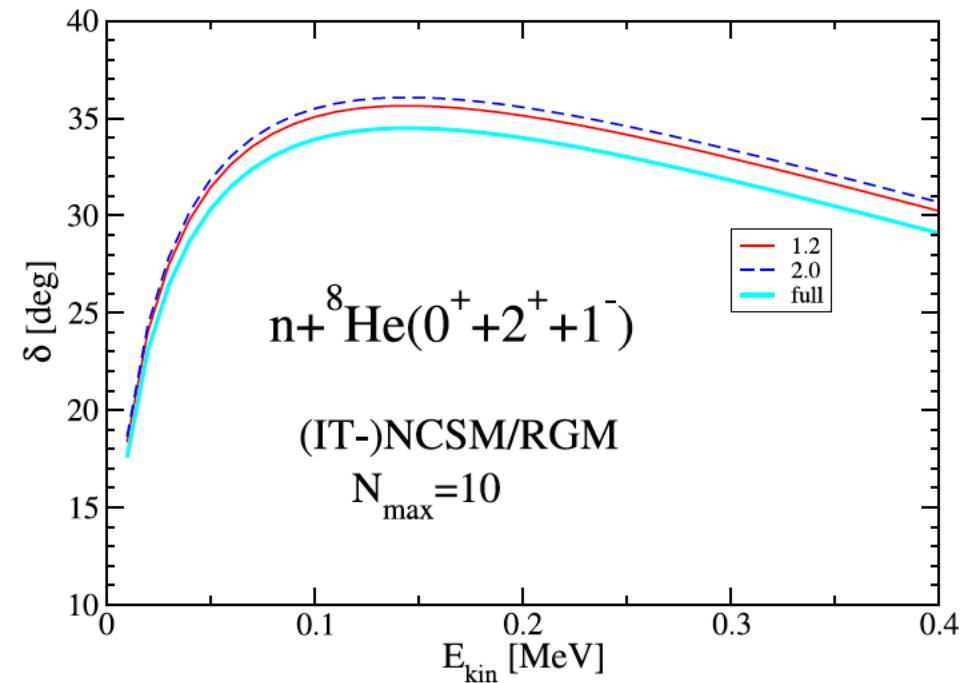
Use the full NCSM basis for $N_{\text{max}} = 7 - 11$.

$N_{\text{max}} = 13$ has IT-NCSM wavefunctions.

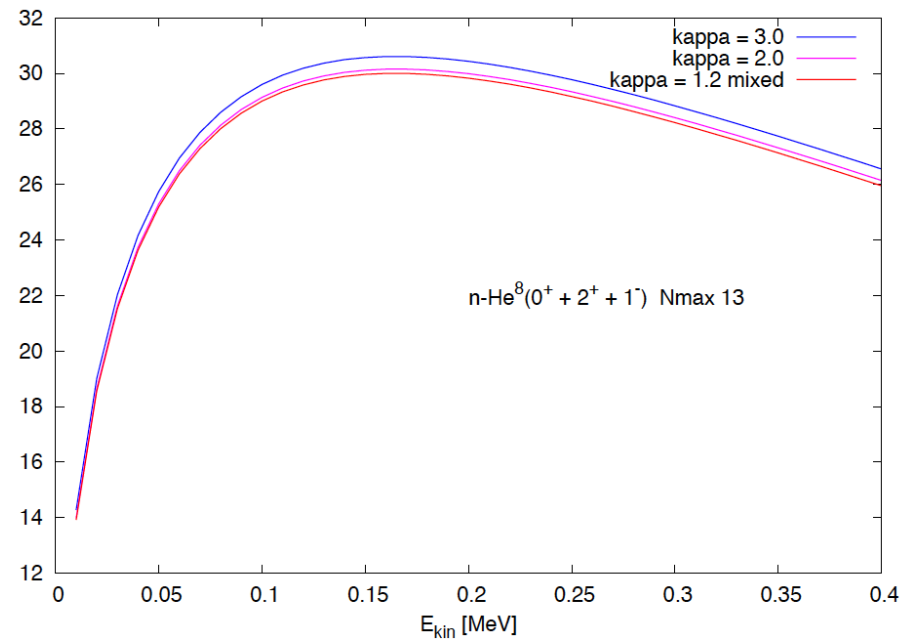
Convergence of s-wave seems ok.

Convergence of p-wave is very good up to 2.5 MeV.

Convergence check: IT-NCSM “kappa” s-wave phase-shifts at $N_{\max} = 11 - 13$.



At $N_{\max} = 10$ we can still compare IT-NCSM wavefunctions to full NCSM. Note that we underestimate the peak about 3 degrees. Overall trend is reproduced.



$N_{\max} = 12$ uses only IT-NCSM wavefunctions. Convergence in kappa seems reasonable. True peak is probably a little lower than shown.

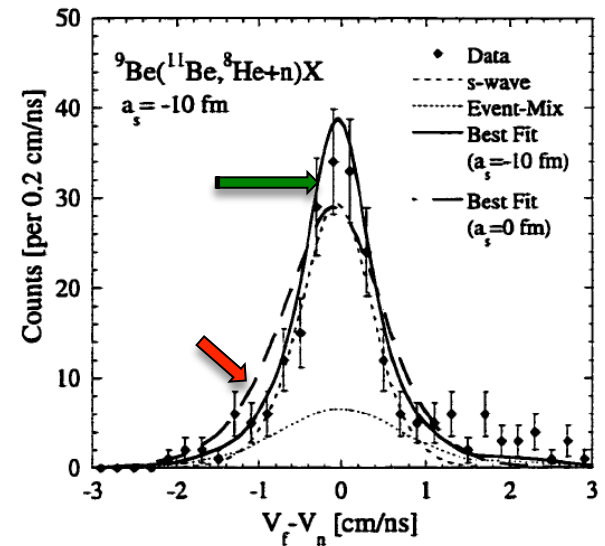
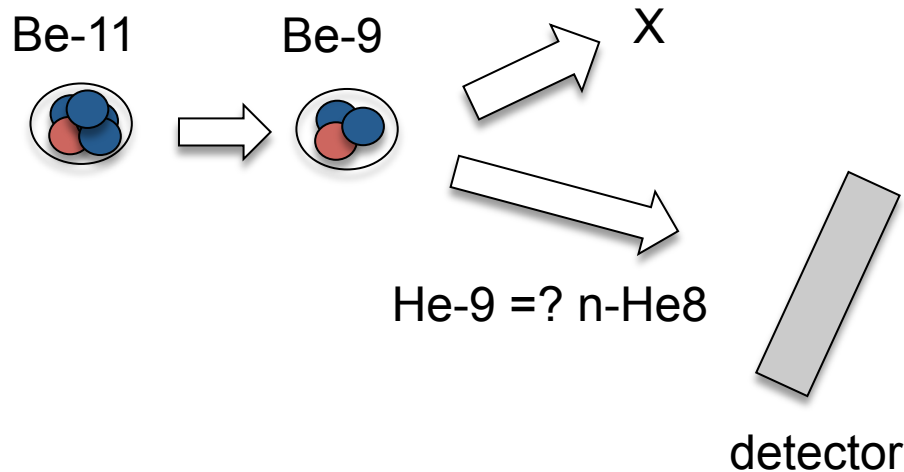
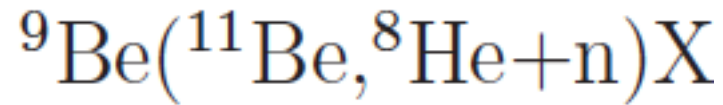
Status of He-9 calculation (NCSM/RGM)

- Predict a resonance in the $\frac{1}{2}^+$ channel, but no bound state.
- $\frac{1}{2}^-$ resonance agrees with experiments.
- Agrees with other theory calculations, but only agrees with the 2001 MSU experiment.
- Experiments are tough. Can't just get some He-8 and fire neutrons at it! (Well, not easily).
- Need to do a NCSM He-9 calculation and use the NCSM/ (NCSM/RGM) technique to study the missing many-body correlations in the n-He8 calculation.
- Bench-mark calculation of He-8 with Roth in $N_{\text{max}}=14$. Agreement seems pretty good, even though he uses a different IT-NCSM scheme.

MSU experiment: Chen et al (2001)

The half-life of He-8 is 120 ms, which is typically too small to perform experiments on. Thus, one can't study the reaction n-He8 directly.

But you can try produce He-9 perhaps in some loosely bound n-He8 configuration by other means.



Potential-model fit to differences in velocities between neutron and He-8 fragment favor a Scattering length of $< -10 \text{ fm}$.

The s-wave is determined by selection rule arguments from Be-11 and from the Shape of the velocity distribution above.

NCSM/RGM coupled with NCSM = NCSMC

Why do we need yet another technique when we already have NCSM and NCSM/RGM?

NCSM: excellent technique for light nuclei that are tightly bound (e.g. He-4, Li-6).
Poor asymptotic behavior of wavefunctions = poor description of halo-nuclei.

NCSM/RGM: keep the power of many-body correlations from the NCSM and improve the tail of intrinsic wavefunctions from the RGM part (e.g. A=5).
But, there are limitations. If you want to include dynamic effects in the target projectile you need to include many excited states (e.g. d-alpha).
The more excited states needed the tougher the calculation.

NCSMC: Couple the NCSM basis with the NCSM/RGM.
Introduces dynamic properties of the target/projectile nucleus (i.e. less excited states needed in calculations). Particularly useful for d-t reactions.

Ab Initio Description of the Exotic Unbound ${}^7\text{He}$ Nucleus

Simone Baroni,^{1,2,*} Petr Navrátil,^{2,3,†} and Sofia Quaglioni^{3,‡}

PRL **110**, 022505 (2013)

Unified *ab initio* approach to bound and unbound states:
no-core shell model with continuum and its application to ${}^7\text{He}$

Simone Baroni,^{1,2,*} Petr Navrátil,^{2,3,†} and Sofia Quaglioni^{3,‡}

Arxiv:1301.3450

NCSMC formalism

$$|\Psi_A^{J^\pi T}\rangle = \sum_\lambda c_\lambda |A\lambda J^\pi T\rangle + \sum_\nu \int dr r^2 \frac{\gamma_\nu(r)}{r} \hat{A}_\nu |\Phi_{\nu r}^{J^\pi T}\rangle.$$

NCSM eigenstates.

NCSM/RGM part

Expansion coefficient is unknown.

Relative motion wavefunction unknown.

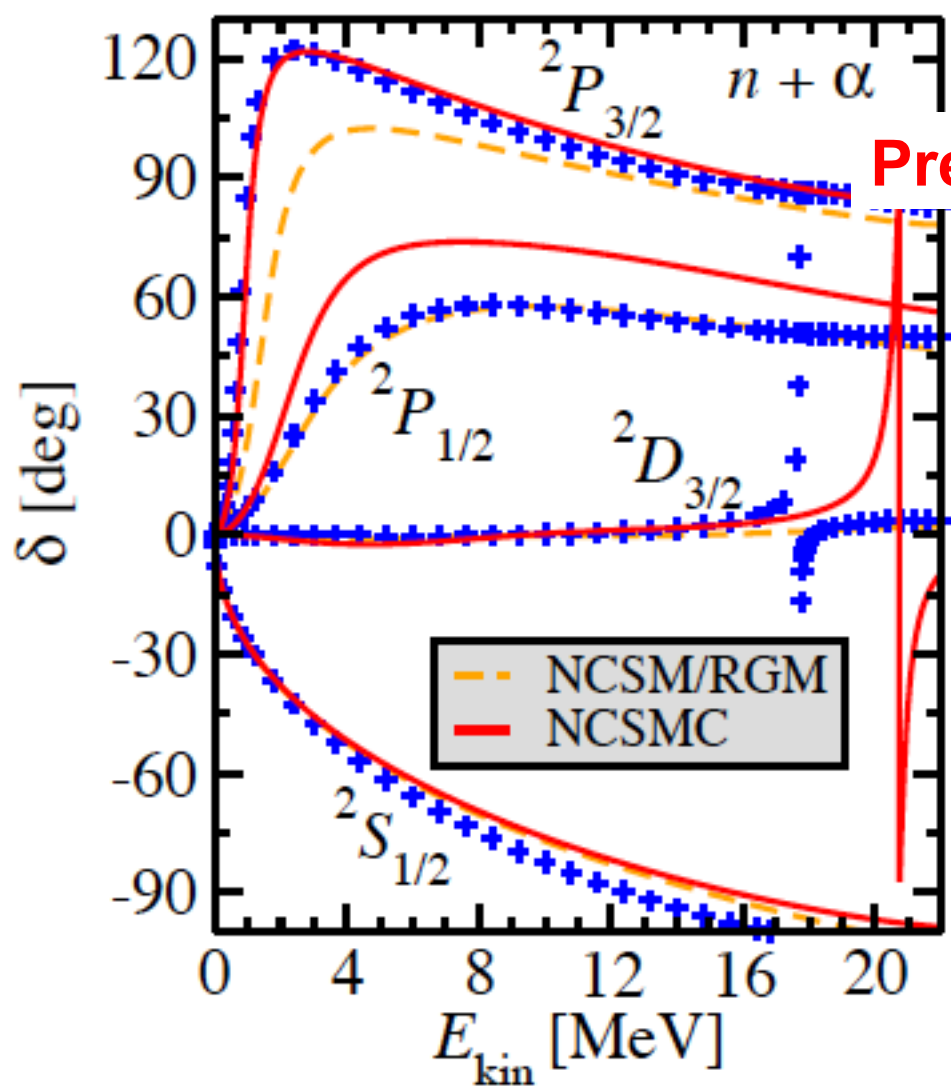
We then have to solve the following matrix equation

$$\begin{pmatrix} H_{NCSM} & \bar{h} \\ \bar{h} & \bar{\mathcal{H}} \end{pmatrix} \begin{pmatrix} c \\ \chi \end{pmatrix} = E \begin{pmatrix} 1 & \bar{g} \\ \bar{g} & 1 \end{pmatrix} \begin{pmatrix} c \\ \chi \end{pmatrix}$$

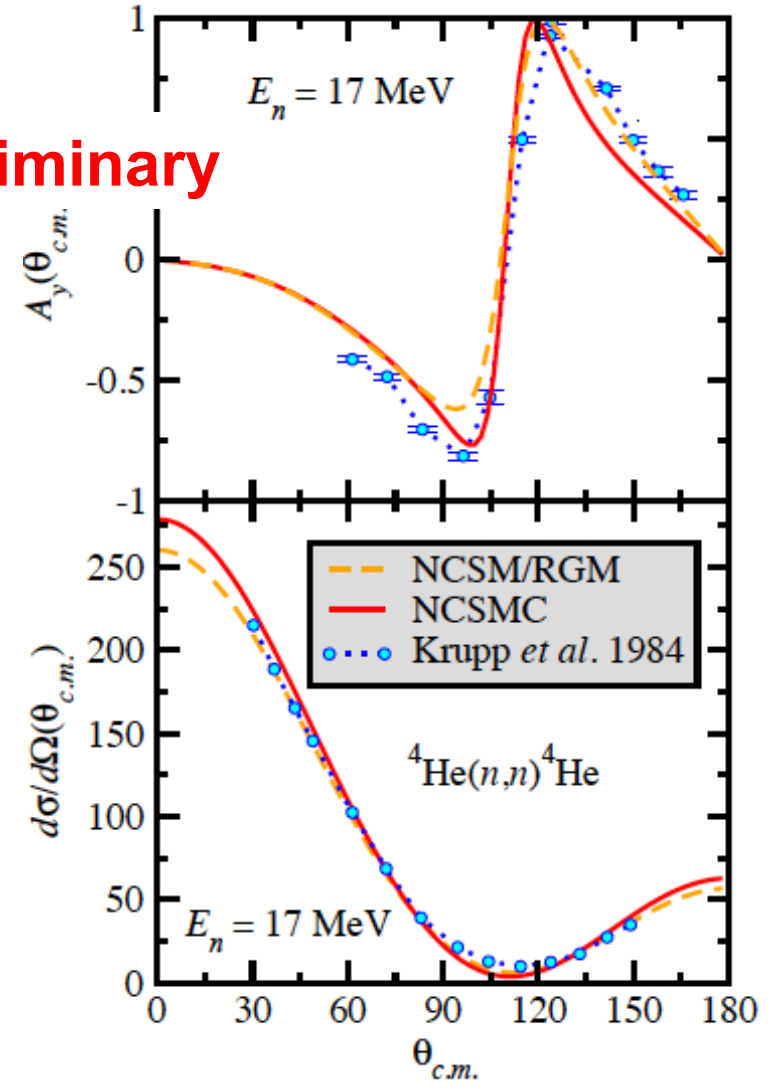
The norm kernel is of course diagonal
For the NCSM and NCSM/RGM parts.
The off-diagonal elements are the
cluster form factors.

$$N_{\nu r \nu' r'}^{\lambda \lambda'} = \begin{pmatrix} \delta_{\lambda \lambda'} & \bar{g}_{\lambda \nu'}(r') \\ \bar{g}_{\lambda' \nu}(r) & \delta_{\nu \nu'} \frac{\delta(r-r')}{rr'} \end{pmatrix}$$

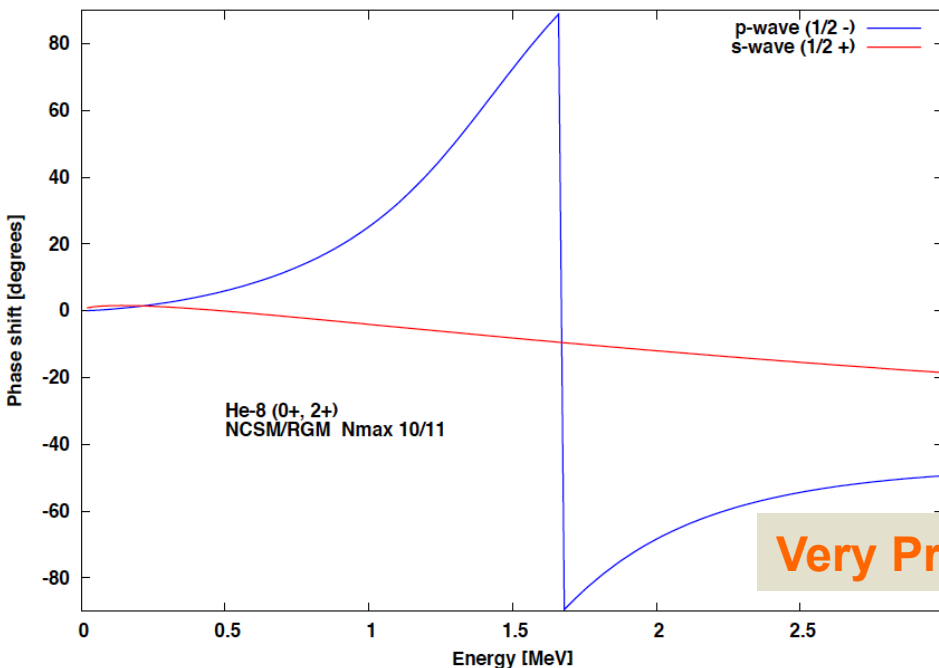
Interlude: n-He4 NCSMC



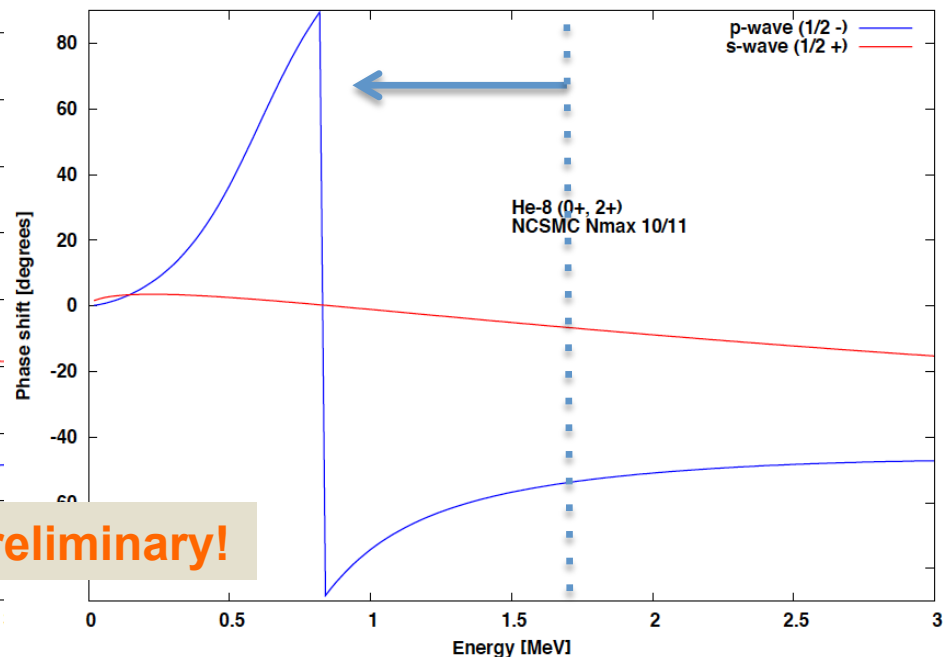
Preliminary



NCSM/RGM vs. NCSMC calculations



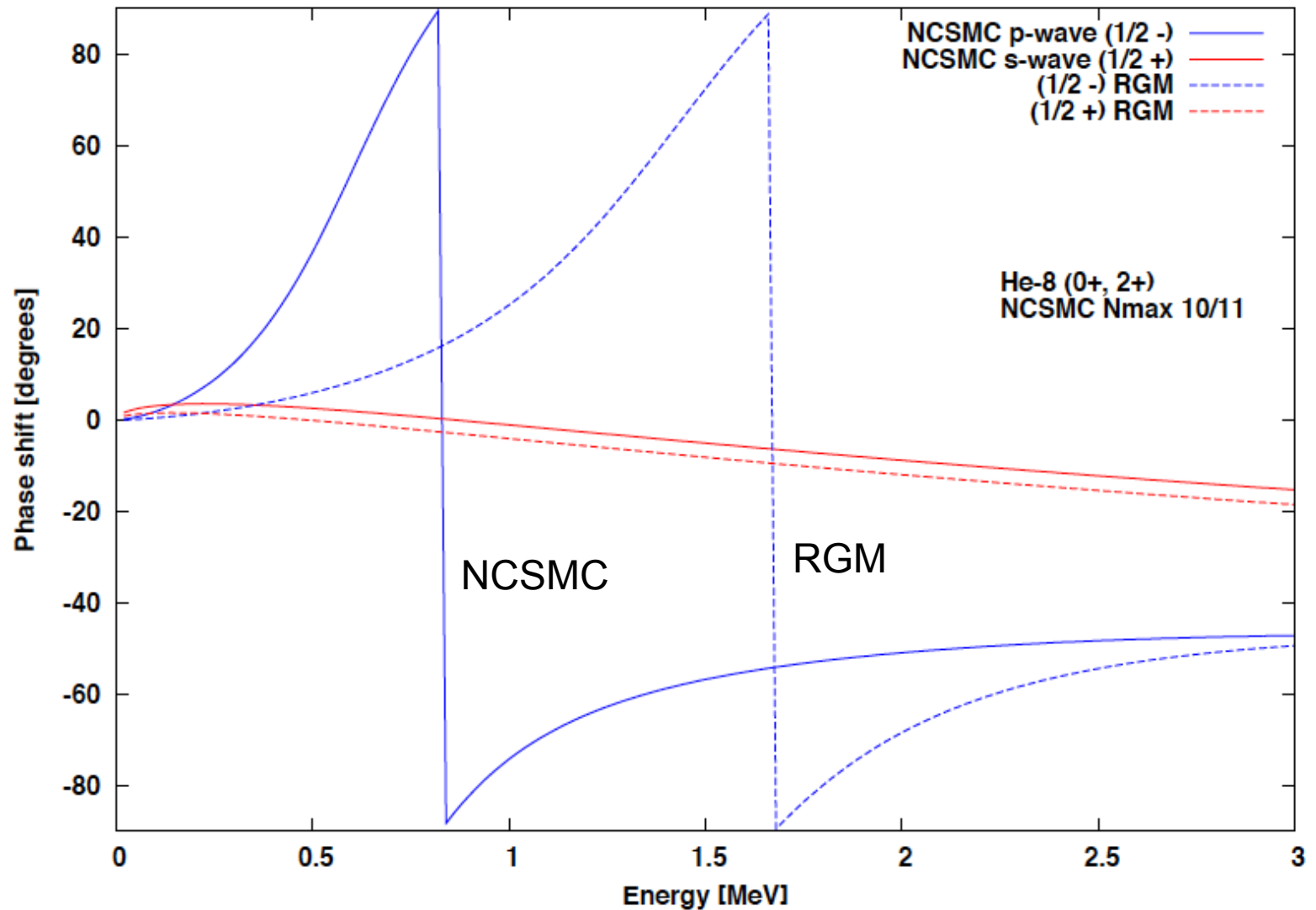
Very Preliminary!



NCSM/RGM:
 Full NCSM wavefunctions used!
 He-8: 0+ and 2+
 S-wave: no bump (missing 1-).
 P-wave resonance is located at 1.7 MeV

NCSMC:
 He-9: 1/2-, 3/2-, 1/2+, 5/2+
 The p-wave resonance is now at 0.8 MeV.
 Will need more excited states in He-8 and larger Nmax values.

NCSMC vs. NCSM/RGM



Conclusions

- We have begun to investigate the results that the NCSMC gives.
- May need to include the 1- state in He-8 NCSM.
- Also need to check higher N_{\max} which will require IT-NCSM.
- Ongoing work...

Collaborations

- Petr Navrátil (TRIUMF)
 - Sofia Quaglioni (LLNL)
- 
- n-He8 project
- Erich Ormand (LLNL)
 - Bruce Barrett (Arizona)
 - Sid Coon (Arizona)
- 
- Other projects