The Role of the Delta Resonance in Chiral Three-Nucleon Forces

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Progress in Ab Initio Techniques in Nuclear Physics
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LENPIC

Low Energy Nuclear Physics International Collaboration

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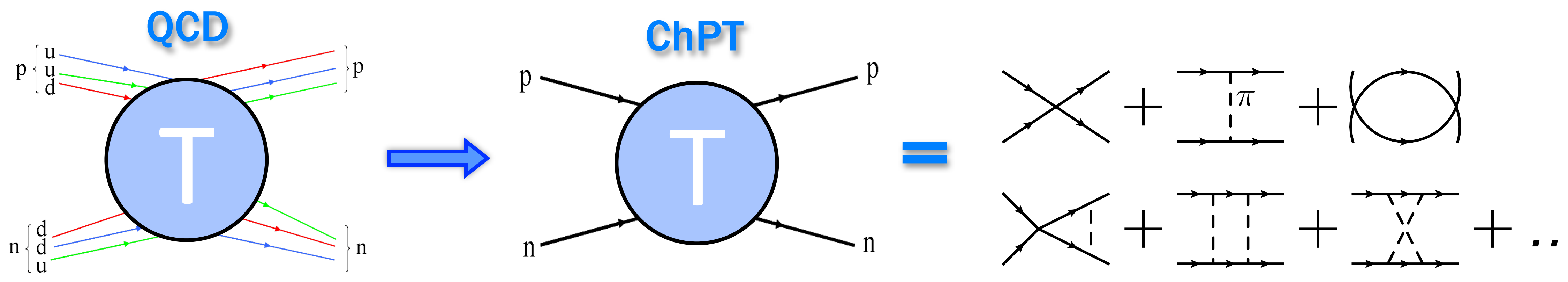
Outline

- From QCD to nuclear physics
- Nuclear forces in chiral EFT
- Three-nucleon forces up to N^3LO
- Long-range part of three-nucleon forces up to N^4LO
- Summary & Outlook

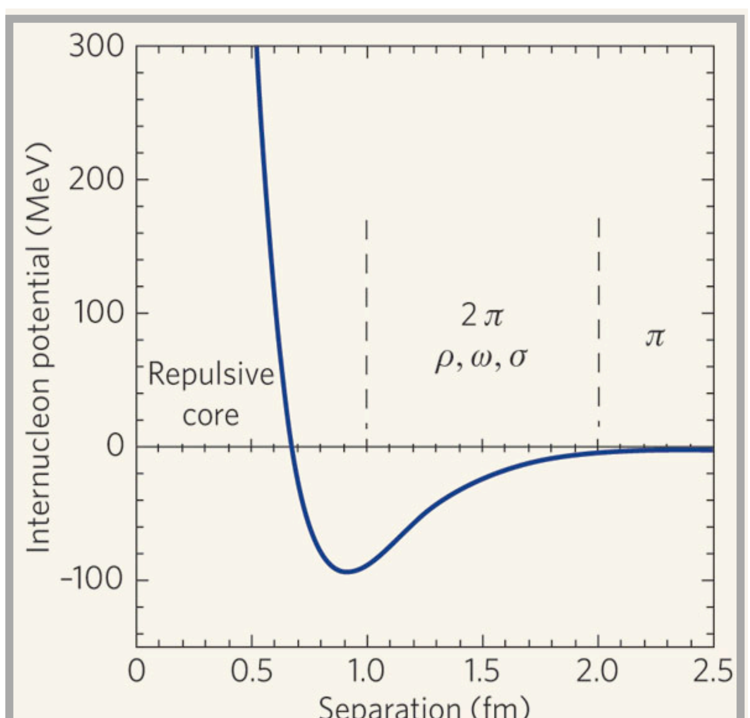
From QCD to nuclear physics

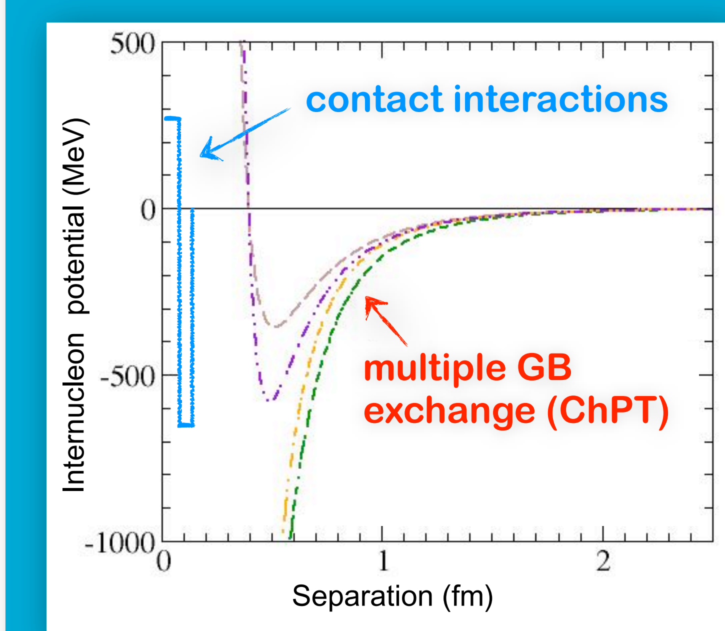


- **NN interaction is strong**: resummations/nonperturbative methods needed
- $1/m_N$ - expansion: nonrelativistic problem ($|\vec{p}_i| \sim M_\pi \ll m_N$) ⟹ the QM A-body problem

$$\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\textit{derived within ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

Weinberg '91





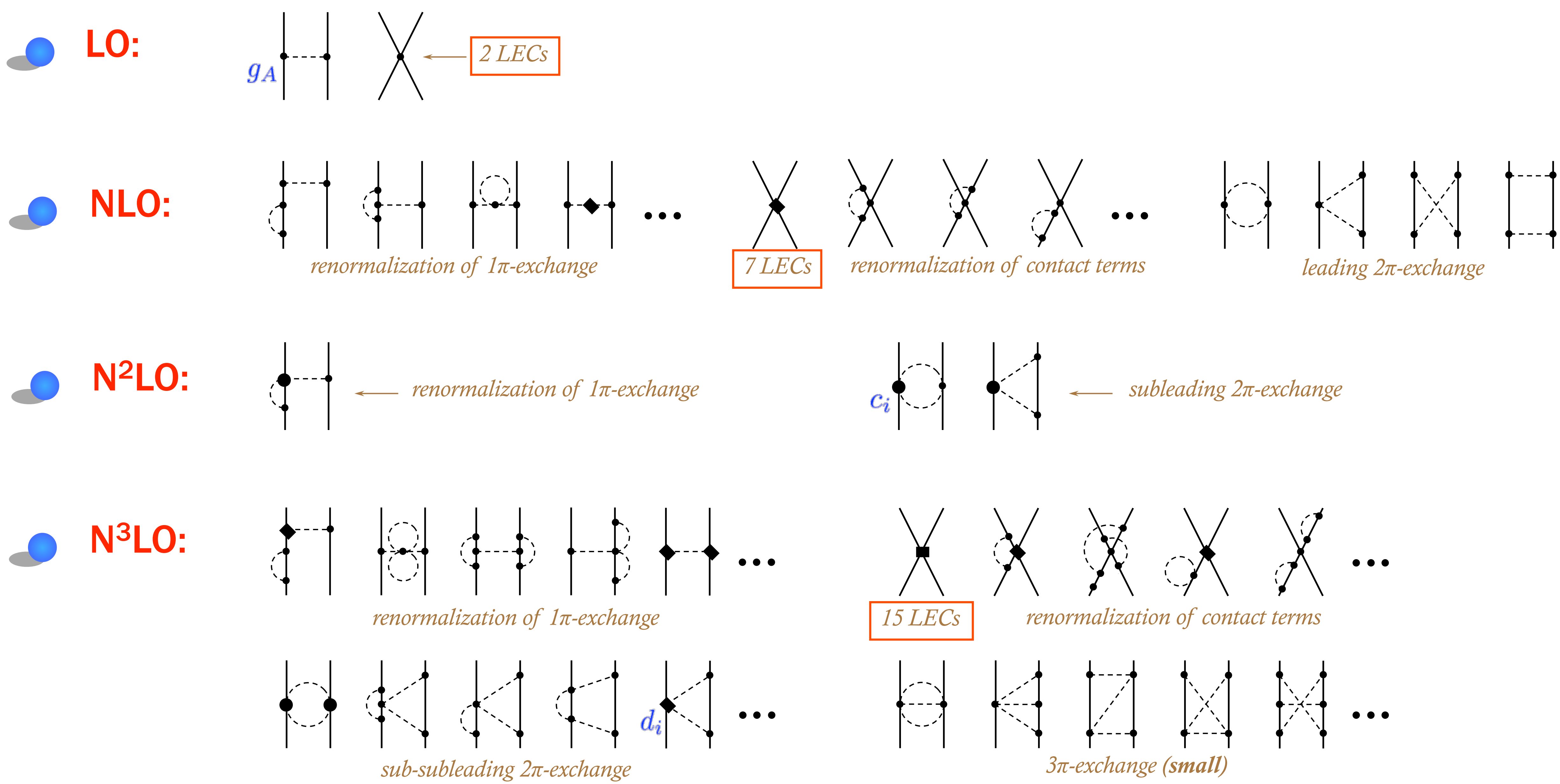
- unified description of $\pi\pi$, πN and NN
- consistent many-body forces and currents
- systematically improvable
- bridging different reactions (electroweak, π-prod., ...)
- precision physics with/from light nuclei

Nucleon-nucleon force up to N^{3}LO

Ordonez et al. '94; Friar & Coon '94; Kaiser et al. '97; Epelbaum et al. '98,'03; Kaiser '99-'01; Higa et al. '03; ...

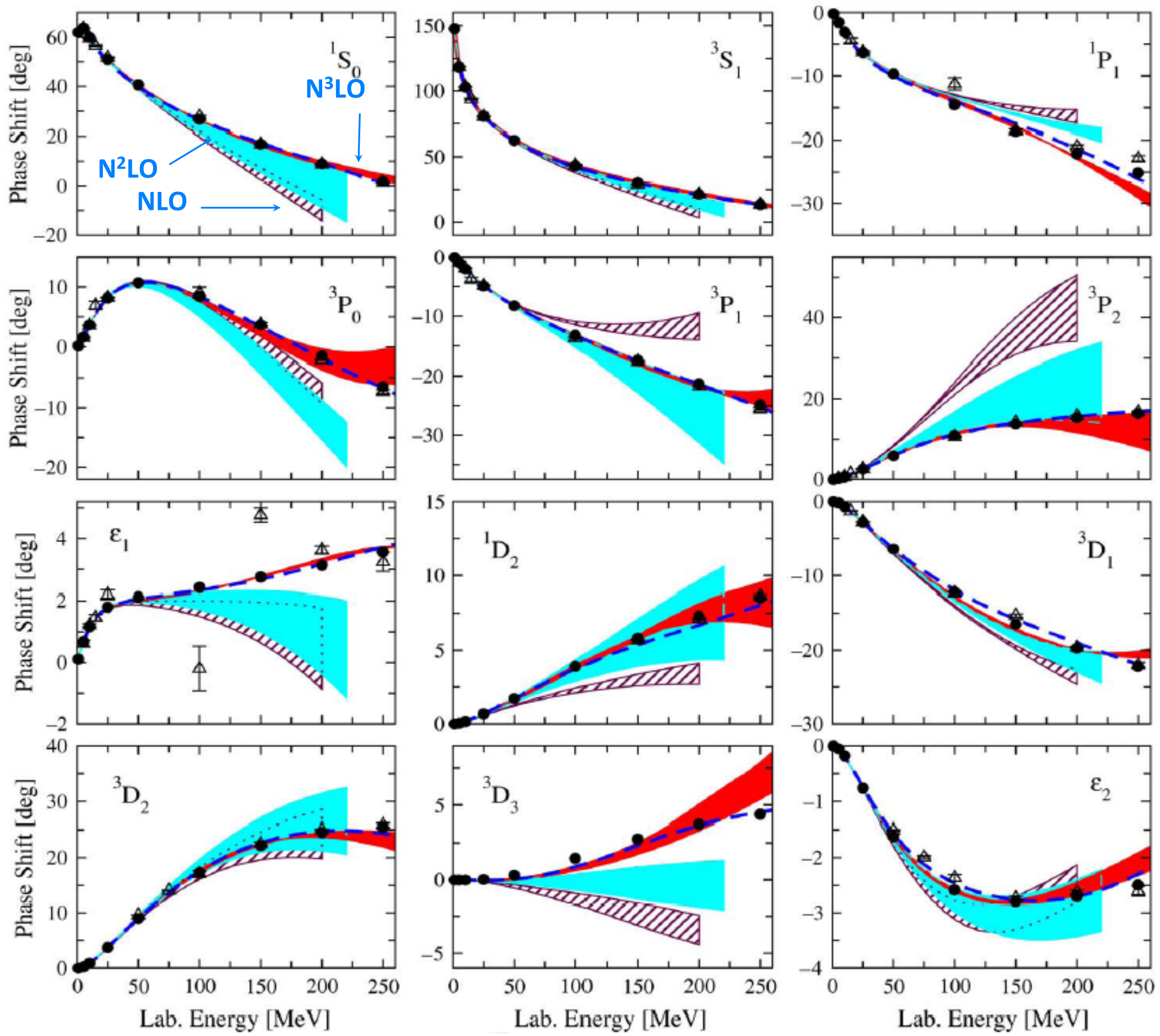
Chiral expansion for the 2N force:

$$V_{2N} = V_{2N}^{(0)} + V_{2N}^{(2)} + V_{2N}^{(3)} + V_{2N}^{(4)} + \ldots$$

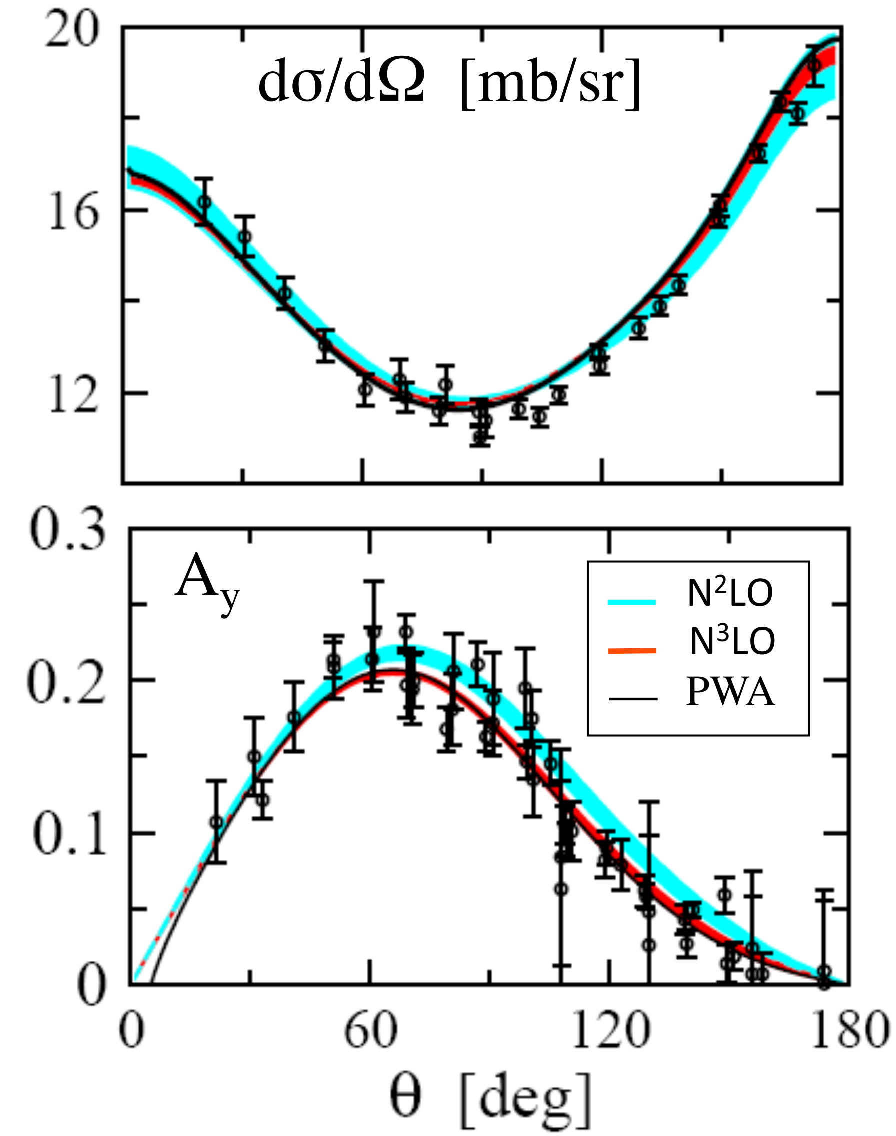


+ 1/m and isospin-breaking corrections…

Neutron-proton phase shifts up to N³LO



np scattering at 50 MeV



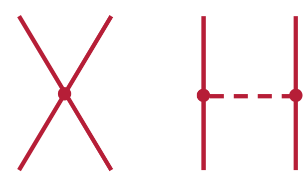
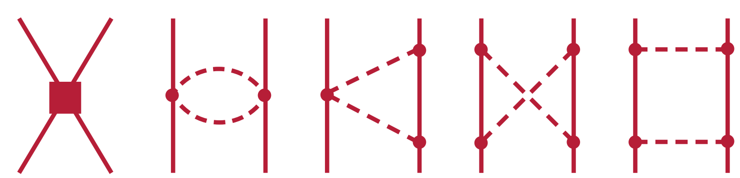
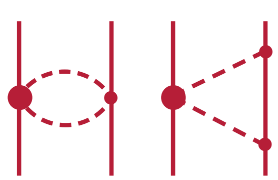
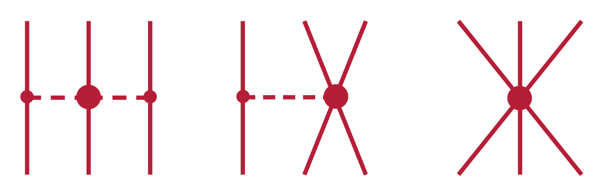
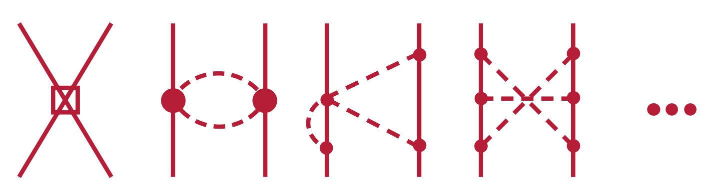
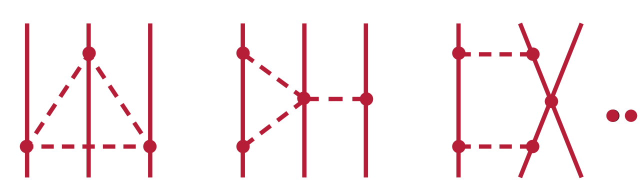
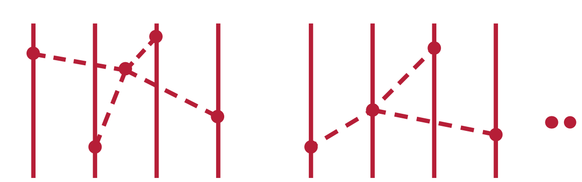
Deuteron binding energy & asymptotic normalizations A_s and η_d

	NLO	N²LO	N³LO	Exp
E_d [MeV]	$-2.171 \ldots -2.186$	$-2.189 \ldots -2.202$	$-2.216 \ldots -2.223$	$-2.224575(9)$
A_S [fm$^{-1/2}$]	$0.868 \ldots 0.873$	$0.874 \ldots 0.879$	$0.882 \ldots 0.883$	$0.8846(9)$
η_d	$0.0256 \ldots 0.0257$	$0.0255 \ldots 0.0256$	$0.0254 \ldots 0.0255$	$0.0256(4)$

Entem & Machleidt '03; Epelbaum, Glöckle & Meißner '05

Nuclear forces up to N^3LO

dimensional analysis counting

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)		—	—
NLO (Q^2)		—	—
N^2LO (Q^3)			—
N^3LO (Q^4)			

Two-nucleon force:
- converged
- accurate description of NN at least up to $E_{lab} \sim 200$ MeV

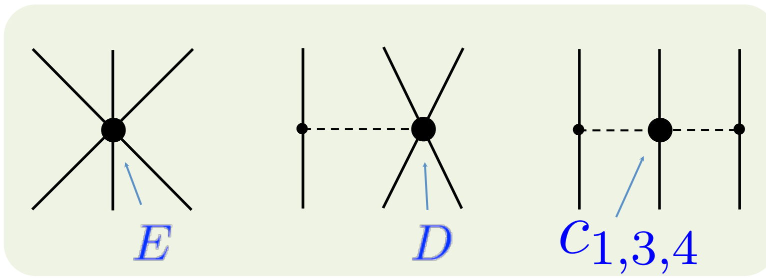
Three-nucleon force:
- not yet converged
- higher orders in progress
- impact on few- & many-N systems?

Four-nucleon force:
- converged ??
- presently out of reach for few- & many-N studies

Three-nucleon forces

- Three-nucleon forces in chiral EFT start to contribute at NNLO

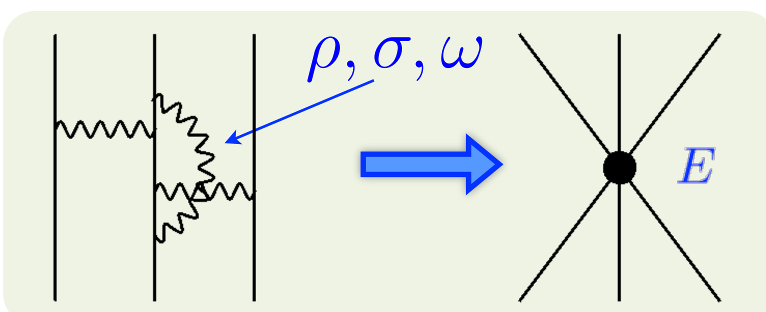
(U. van Kolck ´94; Epelbaum et al. ´02; Nogga et al. ´05; Navratil et al. ´07)



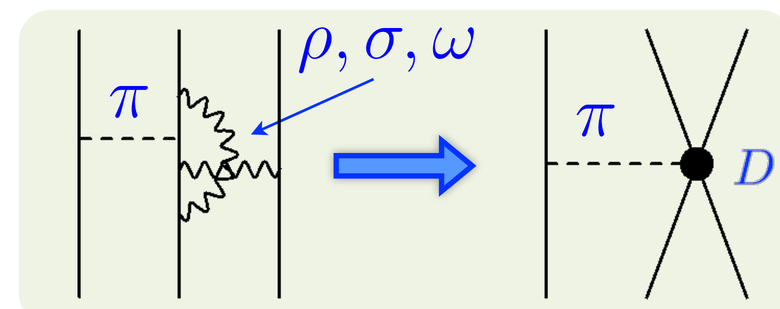
$c_{1,3,4}$ from the fit to πN-scattering data

D, E from ${}^3\mathrm{H}, {}^4\mathrm{He}, {}^{10}\mathrm{B}$ binding energy + coherent nd - scattering length

- LECs D and E incorporate short-range contr.



Resonance saturation interpretation of LECs



- Delta contributions encoded in LECs

(Bernard, Kaiser & Meißner '97)

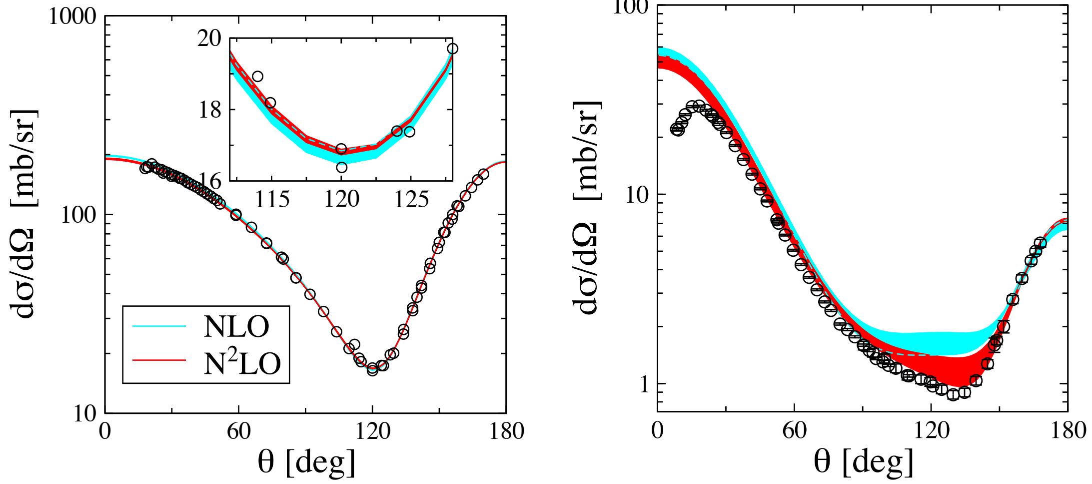
Delta-resonance saturation

$$c_3 = -2c_4 = c_3(\not\Delta) - \frac{4h_A^2}{9\Delta}$$

Enlargement due to Delta contribution

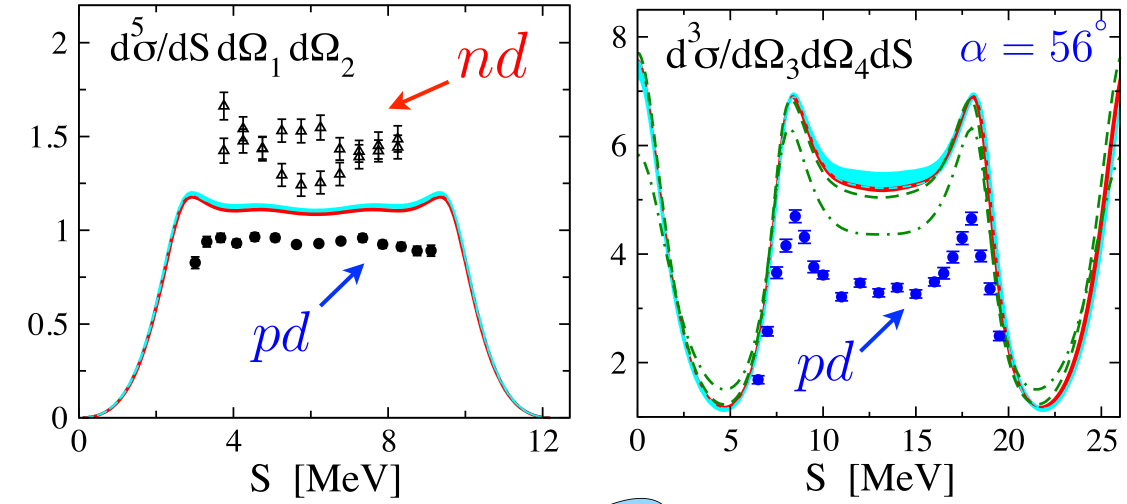
nd elastic scattering

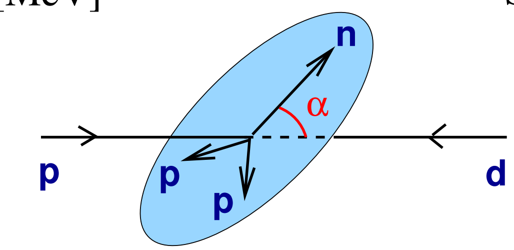
$E_{\mathrm{Lab}} = 10\,\mathrm{MeV}$ $E_{\mathrm{Lab}} = 65\,\mathrm{MeV}$



nd break-up $[\mathrm{mb\,MeV^{-1}sr^{-2}}]$

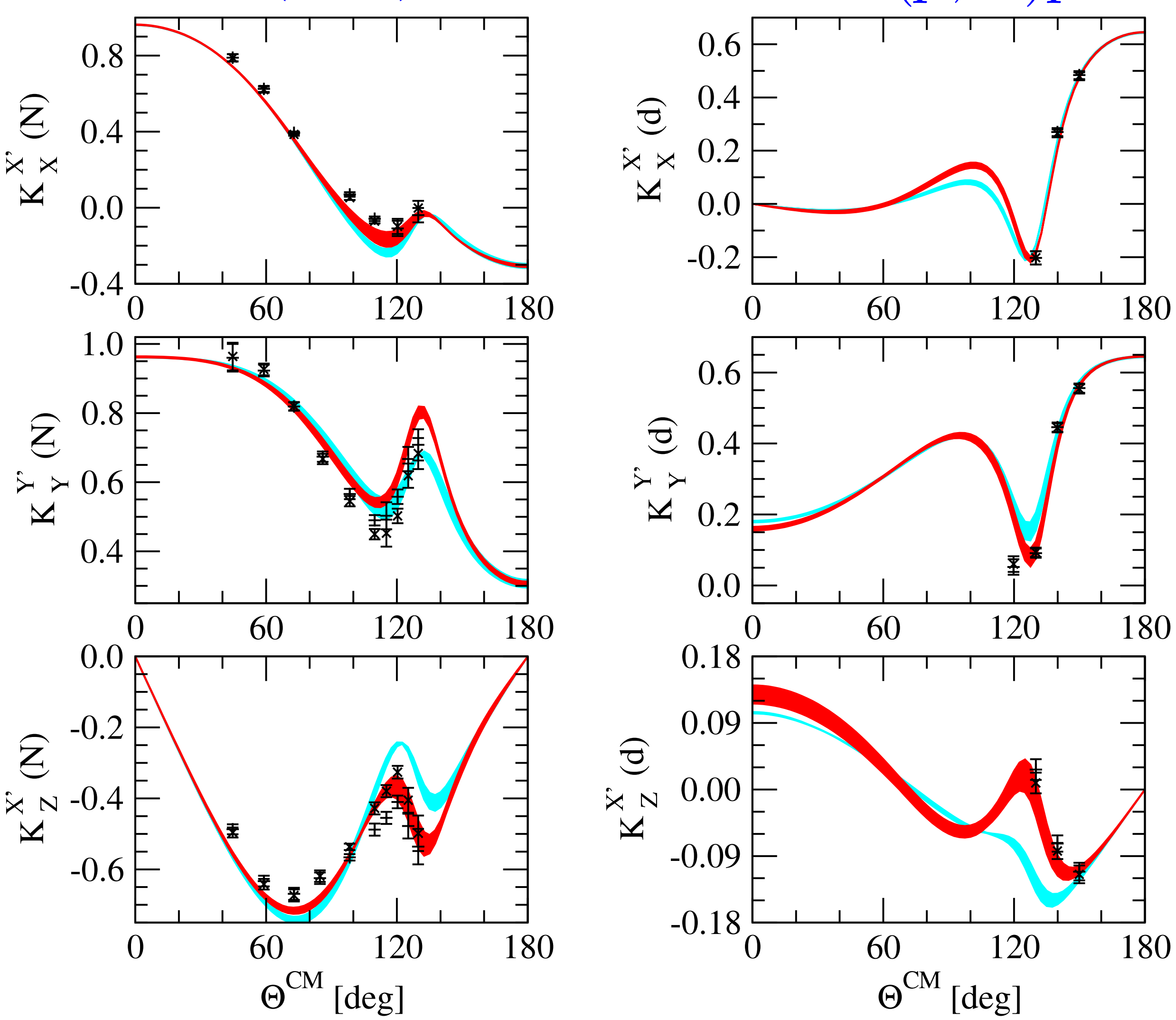
SST at $E_N = 12\,\mathrm{MeV}$ SCRE at $E_N = 19\,\mathrm{MeV}$





polarization transfer: $E_p^{\mathrm{Lab}} = 22.7\,\mathrm{MeV}$

$d(\vec{p},\vec{p})d$ $d(\vec{p},\vec{d})p$



For references see recent reviews:

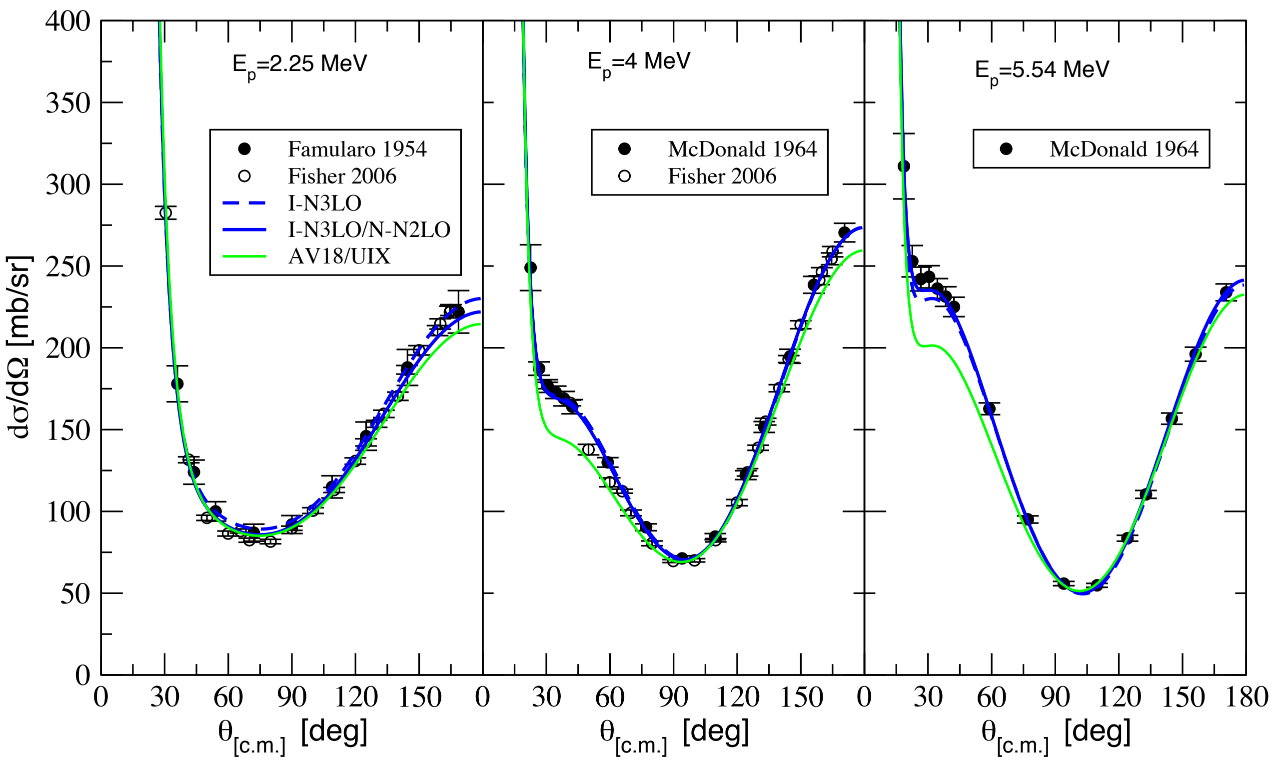
- Epelbaum, Prog. Part Nucl. Phys. 57 (06) 654
- Epelbaum, Hammer, Meißner, Rev. Mod. Phys. 81 (09) 1773
- Entem, Machleidt, Phys. Rept. 503 (11) 1
- Epelbaum, Meißner, Ann. Rev. Nucl. Part. Sci. 62 (12) 159
- Kalantar et al. Rep. Prog. Phys. 75 (12) 016301
- Hammer, Schwenk, Nogga Rev. Mod. Phys. 85 (13) 197

- Generally good description of data. But some discrepancies arise. E.g. break-up observables for SCRE/SST configuration at low energy
- Hope for improvement at N^3LO

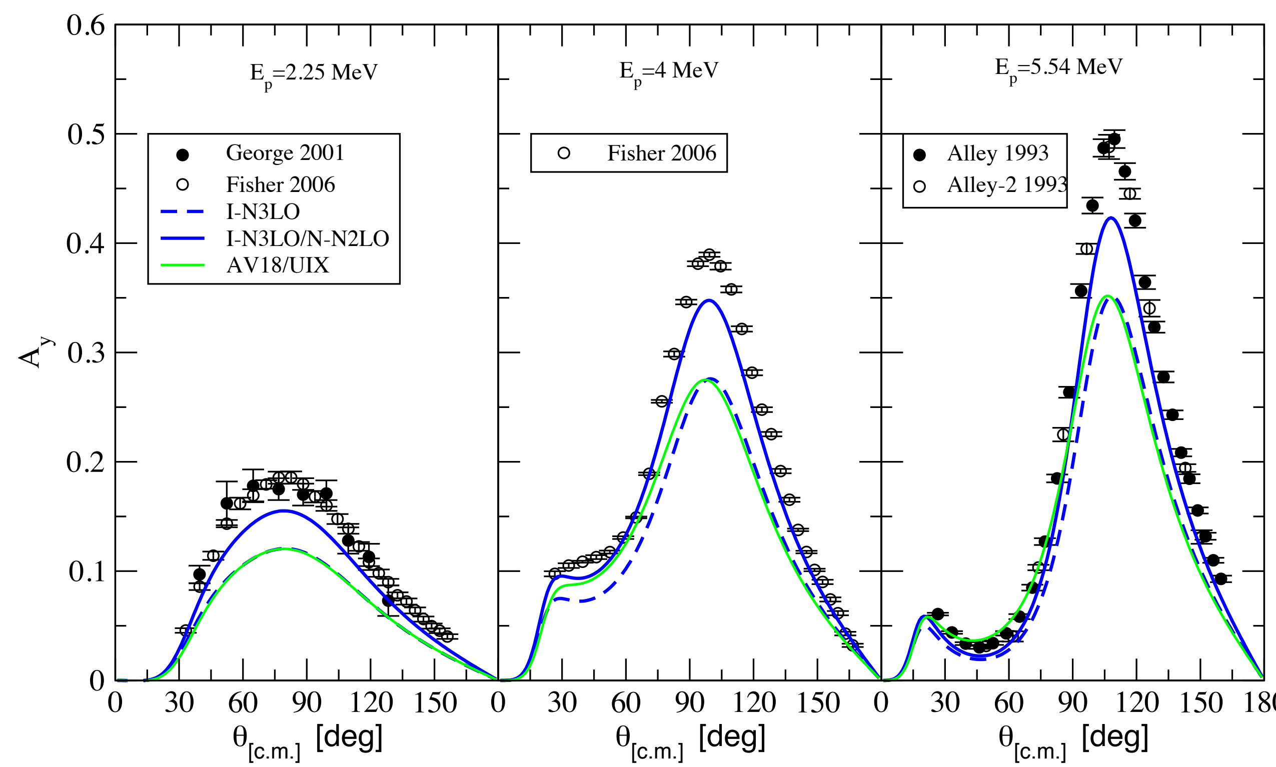
Proton-^{3}He elastic scattering

Viviani, Girlanda, Kievsky, Marcucci, Rosati arXiv: 1004.1306

p-^{3}He differential cross section at low energies



proton vector analyzing power A_y-puzzle



As in n-d scattering case N^{2}LO 3NF's are not enough to resolve underprediction of A_y

→

Hope for improvement at higher orders

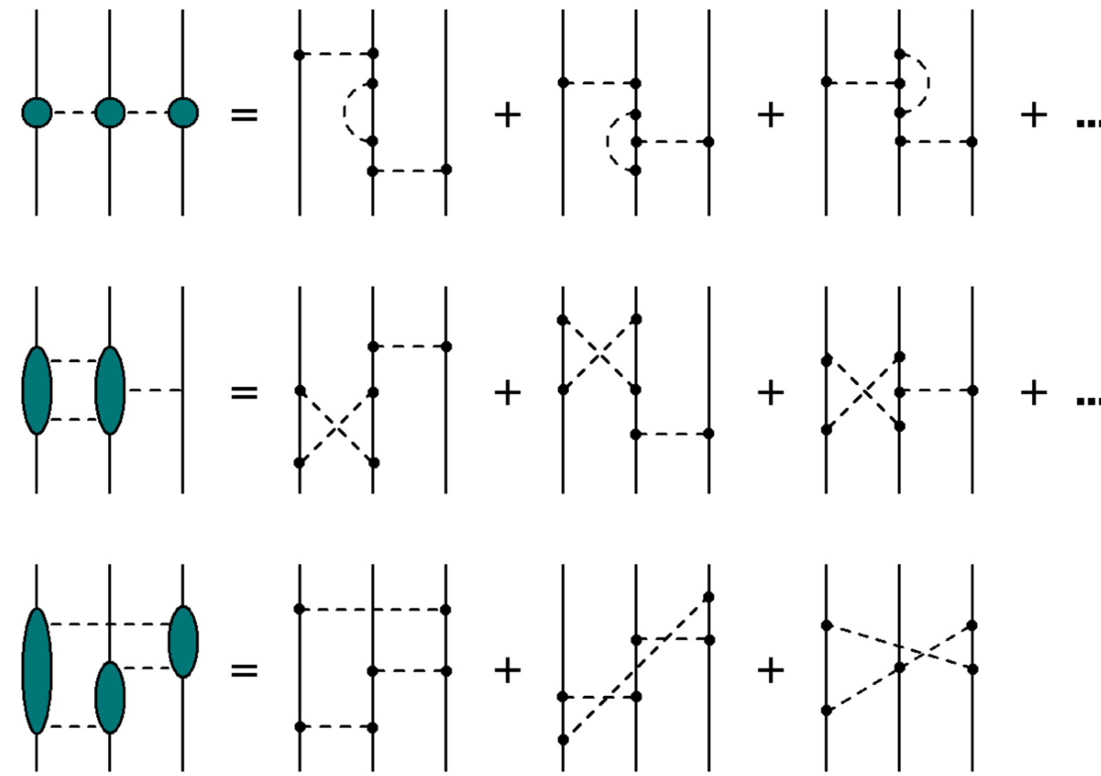
Three-nucleon forces

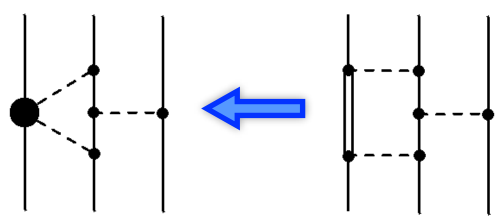
- Three-nucleon forces at N^{3}LO

Long range contributions

Bernard, Epelbaum, H.K., Meißner ´08; Ishikawa, Robilotta ´07

- No additional free parameters
- Expressed in terms of g_A, F_π, M_π
- Rich isospin-spin-orbit structure
- Δ(1232)-contr. are important

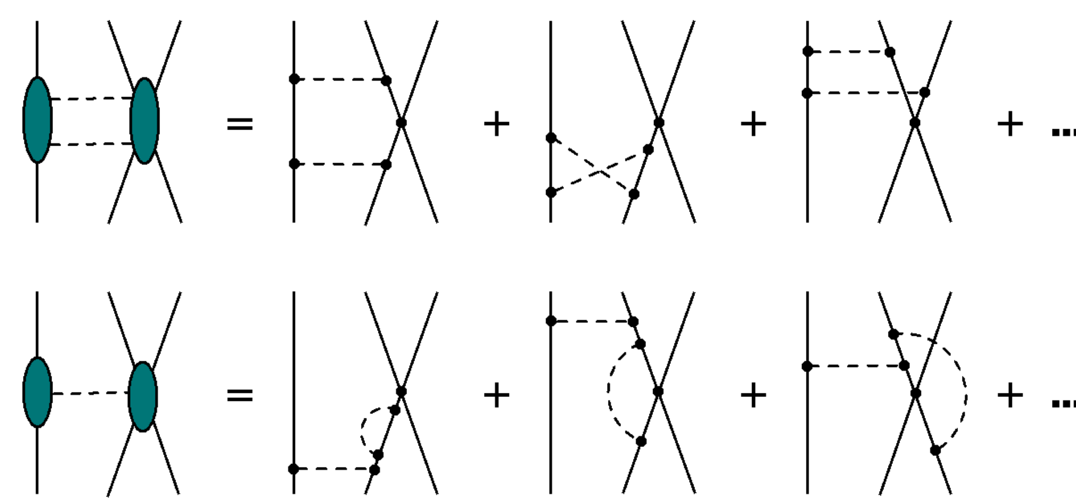


Large c_i $\sim$ 

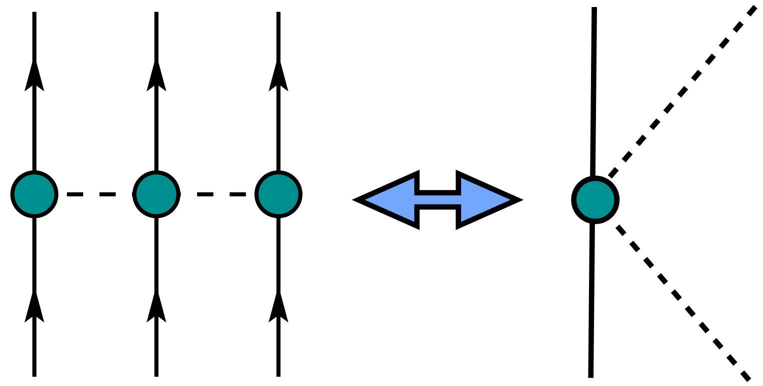
Shorter range contributions

Bernard, Epelbaum, H.K., Meißner ´11

- LECs needed for shorter range contr.
  g_A, F_π, M_π, C_T
- Central NN contact interaction does not contribute
- Unique expressions in the static limit for a renormalizable 3NF



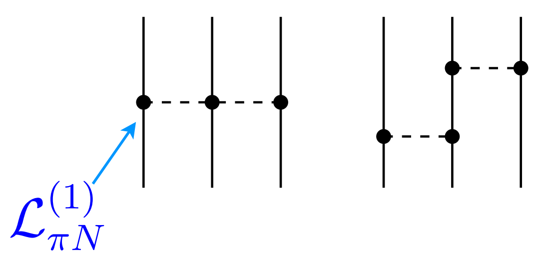
Two-pion-exchange 3NF



- Two-pion-exchange 3NF is connected to pion-nucleon scattering amplitude
  Ishikawa, Robilotta ´07
  - The same linear combinations of LECs
  - The same renormalization

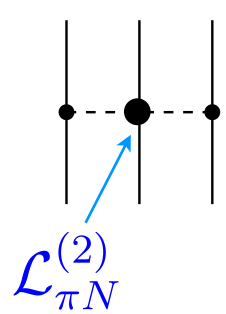
$$V_{2\pi} \;=\; \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$

NLO - contr.

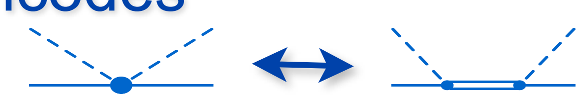


⟵ yield vanishing 3NF contributions

N²LO - contr.



⟵ first nonvanishing 3NF, encodes information about the Δ:

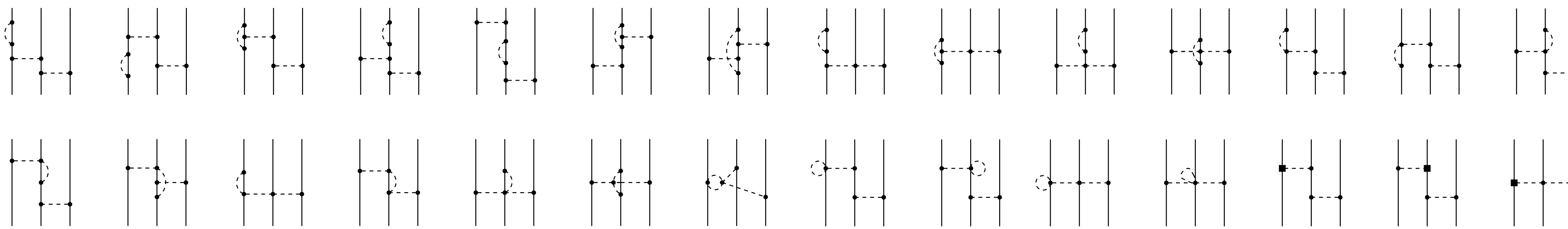


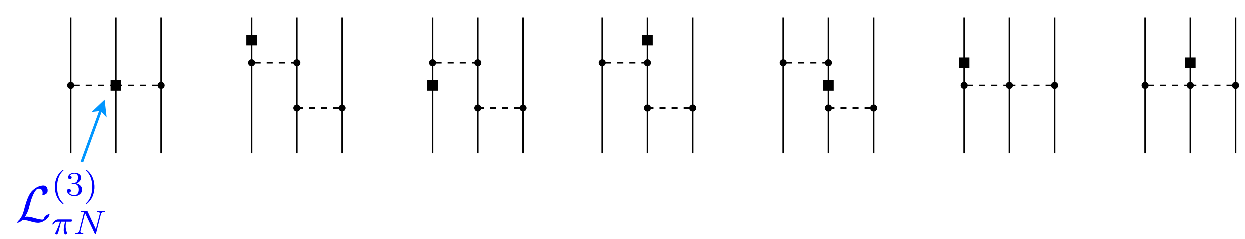
$$\mathcal{A}^{(3)}(q_2) = \frac{g_A^2}{8F_\pi^4} \Big((2c_3 - 4c_1) M_\pi^2 + c_3 q_2^2 \Big) \,, \qquad \mathcal{B}^{(3)}(q_2) = \frac{g_A^2 c_4}{8F_\pi^4}$$

van Kolck ´94

Two-pion-exchange 3NF

N^3LO - contr. (leading 1 loop)





renormalization nontrivial !

$$\mathcal{A}^{(4)}(q_2) = \frac{g_A^4}{256\pi F_\pi^6}\left[A(q_2)\left(2M_\pi^4 + 5M_\pi^2 q_2^2 + 2q_2^4\right) + \left(4g_A^2+1\right)M_\pi^3 + 2\left(g_A^2+1\right)M_\pi q_2^2\right],$$

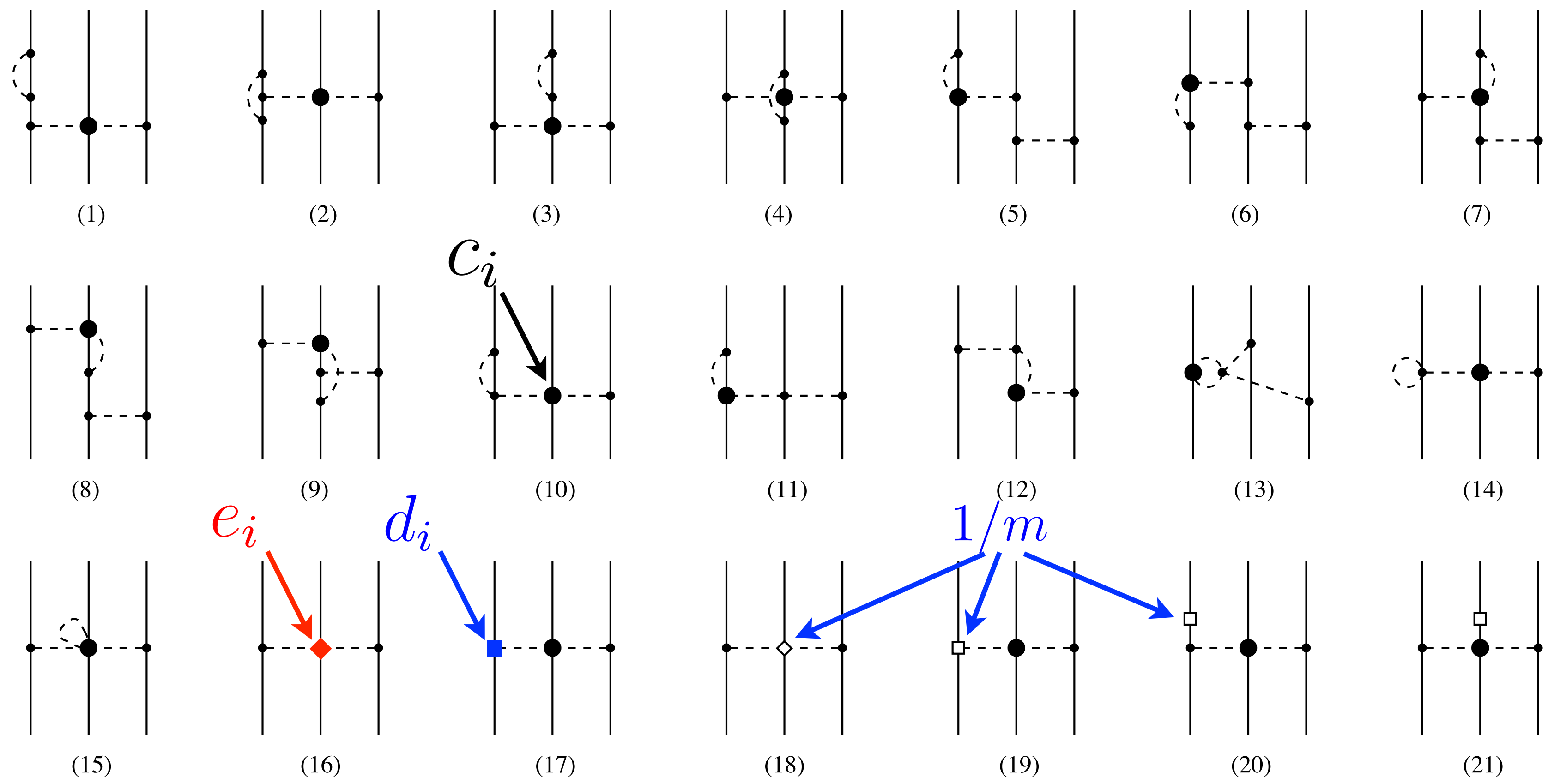
$$\mathcal{B}^{(4)}(q_2) = -\frac{g_A^4}{256\pi F_\pi^6}\left[A(q_2)\left(4M_\pi^2 + q_2^2\right) + (2g_A^2+1)M_\pi\right]$$

Ishikawa, Robilotta ´07,
Bernard, Epelbaum, HK, Meißner ´07

- No unknown parameters at this order
- Everything is expressed in terms of loop function $A(q) = \frac{1}{2q}\arctan\frac{q}{2M_\pi}$
- Additional unitarity transformations required for proper renormalization

Two-pion-exchange 3NF

N[4]LO - contr. (subleading 1 loop) *Epelbaum, Gasparyan, H.K., PRC85 (2012) 054006*



c_i´s LECs from $\mathcal{L}_{\pi N}^{(2)}$, d_i´s LECs from $\mathcal{L}_{\pi N}^{(3)}$, e_i´s LECs from $\mathcal{L}_{\pi N}^{(4)}$: fitted to πN - scattering data

- Leading Δ - contributions are taken into account through c_i´s
- Vanishing $1/m$ - contributions at this order

Two-pion-exchange 3NF at N^{4}LO

$$
\begin{aligned}
\mathcal{A}^{(5)}(q_2) &= \frac{g_A}{4608\pi^2 F_\pi^6}\Big[M_\pi^2 q_2^2 \big(F_\pi^2 \left(2304\pi^2 g_A (4\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36}) - 2304\pi^2 \bar{d}_{18} c_3\right) \\
&+ g_A (144 c_1 - 53 c_2 - 90 c_3)\big) + M_\pi^4 \left(F_\pi^2 \left(4608\pi^2 \bar{d}_{18}(2c_1 - c_3) + 4608\pi^2 g_A (2\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{36} - 4\bar{e}_{38})\right)\right. \\
&+ \left. g_A \left(72 \left(64\pi^2 \bar{l}_3 + 1\right) c_1 - 24 c_2 - 36 c_3\right)\right) + q_2^4 \left(2304\pi^2 \bar{e}_{14} F_\pi^2 g_A - 2 g_A (5c_2 + 18 c_3)\right)\Big] \\
&- \frac{g_A^2}{768\pi^2 F_\pi^6} L(q_2) \left(M_\pi^2 + 2q_2^2\right) \left(4M_\pi^2 (6c_1 - c_2 - 3c_3) + q_2^2(-c_2 - 6c_3)\right) \\
\mathcal{B}^{(5)}(q_2) &= -\frac{g_A}{2304\pi^2 F_\pi^6}\Big[M_\pi^2 \left(F_\pi^2 \left(1152\pi^2 \bar{d}_{18} c_4 - 1152\pi^2 g_A (2\bar{e}_{17} + 2\bar{e}_{21} - \bar{e}_{37})\right) + 108 g_A^3 c_4 + 24 g_A c_4\right) \\
&+ q_2^2 \left(5 g_A c_4 - 1152\pi^2 \bar{e}_{17} F_\pi^2 g_A\right)\Big] + \frac{g_A^2 c_4}{384\pi^2 F_\pi^6} L(q_2) \left(4M_\pi^2 + q_2^2\right)
\end{aligned}
$$

Some LECs can be absorbed by shifting c_i´s

$$
\begin{aligned}
c_1 &\to c_1 - 2M_\pi^2 \left(\bar{e}_{22} - 4\bar{e}_{38} - \frac{\bar{l}_3 c_1}{F_\pi^2}\right), \\
c_3 &\to c_3 + 4M_\pi^2 \left(2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36} + 2\frac{\bar{l}_3 c_1}{F_\pi^2}\right), \\
c_4 &\to c_4 + 4M_\pi^2 (2\bar{e}_{21} - \bar{e}_{37}),
\end{aligned}
$$

$$
L(q) = \frac{\sqrt{q^2 + 4M_\pi^2}}{q} \log \frac{\sqrt{q^2 + 4M_\pi^2} + q}{2M_\pi}
$$

- No d_i dependence of TPE-contr. besides d_{18}
- Pion-nucleon scattering does strongly depend on d_i´s

$$
g_{\pi NN} = \frac{g_A\, m}{F_\pi}\left(1 - \frac{2M_\pi^2 \bar{d}_{18}}{g_A}\right)
$$

⇐ Violation of Goldberger-Treiman relation

Pion-nucleon scattering

Heavy baryon calculation up to order q^4 *Fettes, Meißner Nucl. Phys. A676 (2000) 311*

1/m power counting used in FM work $\Rightarrow$ $\frac{p}{m} \sim \frac{q}{\Lambda_\chi}$

- Difference in Weinberg's power counting for NN $\Rightarrow$ $\frac{p}{m} \sim \left(\frac{q}{\Lambda_\chi}\right)^2$

Refit of d_i and e_i LECs is needed

$$\pi^a(q_1) + N(p_1) \to \pi^b(q_2) + N(p_2)$$

$$T^{ba}_{\pi N} = \frac{E+m}{2m}\left(\delta^{ba}\left[g^+(\omega,t) + i\,\vec\sigma\cdot\vec q_2\times\vec q_1\,h^+(\omega,t)\right] + i\,\epsilon^{bac}\tau^c\left[g^-(\omega,t) + i\vec\sigma\cdot\vec q_2\times\vec q_1\,h^-(\omega,t)\right]\right)$$

CMS kinematics: $\omega = q_1^0 = q_2^0, \quad E = E_1 = E_2 = \sqrt{\vec q^{\,2} + m^2}, \quad \vec q_1^{\,2} = \vec q_2^{\,2} = \vec q^{\,2}, \quad t = (q_1 - q_2)^2$

Partial wave amplitudes: $f^{\pm}_{l\pm}(s) = \frac{E+m}{16\pi\sqrt{s}}\int_{-1}^{1} dz\left[g^{\pm}P_l(z) + \vec q^{\,2}h^{\pm}\left(P_{l\pm1}(z) - zP_l(z)\right)\right]$

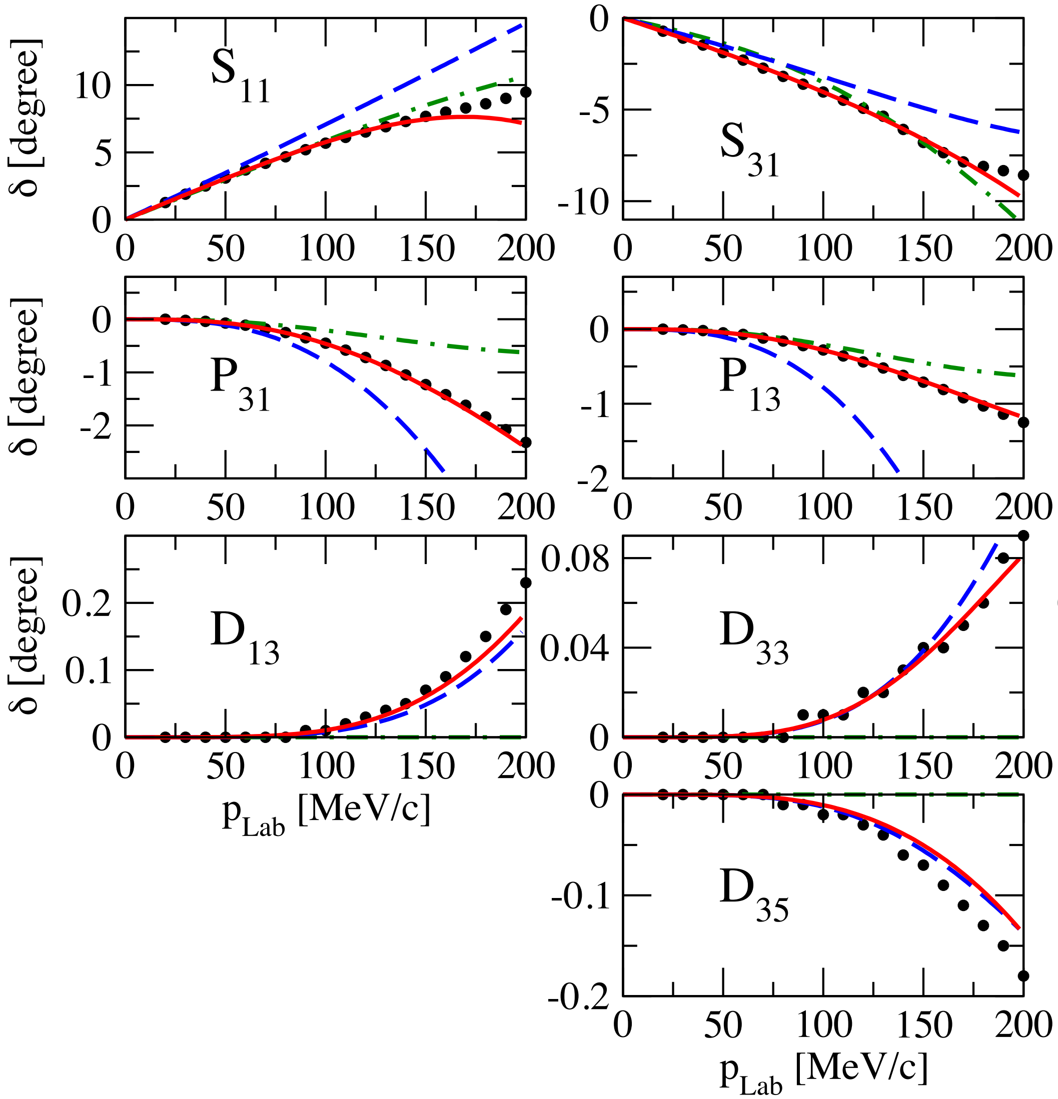
In the isospin basis: $f^{1/2}_{l\pm} = f^{+}_{l\pm} + 2f^{-}_{l\pm}, \quad f^{3/2}_{l\pm} = f^{+}_{l\pm} - f^{-}_{l\pm}$

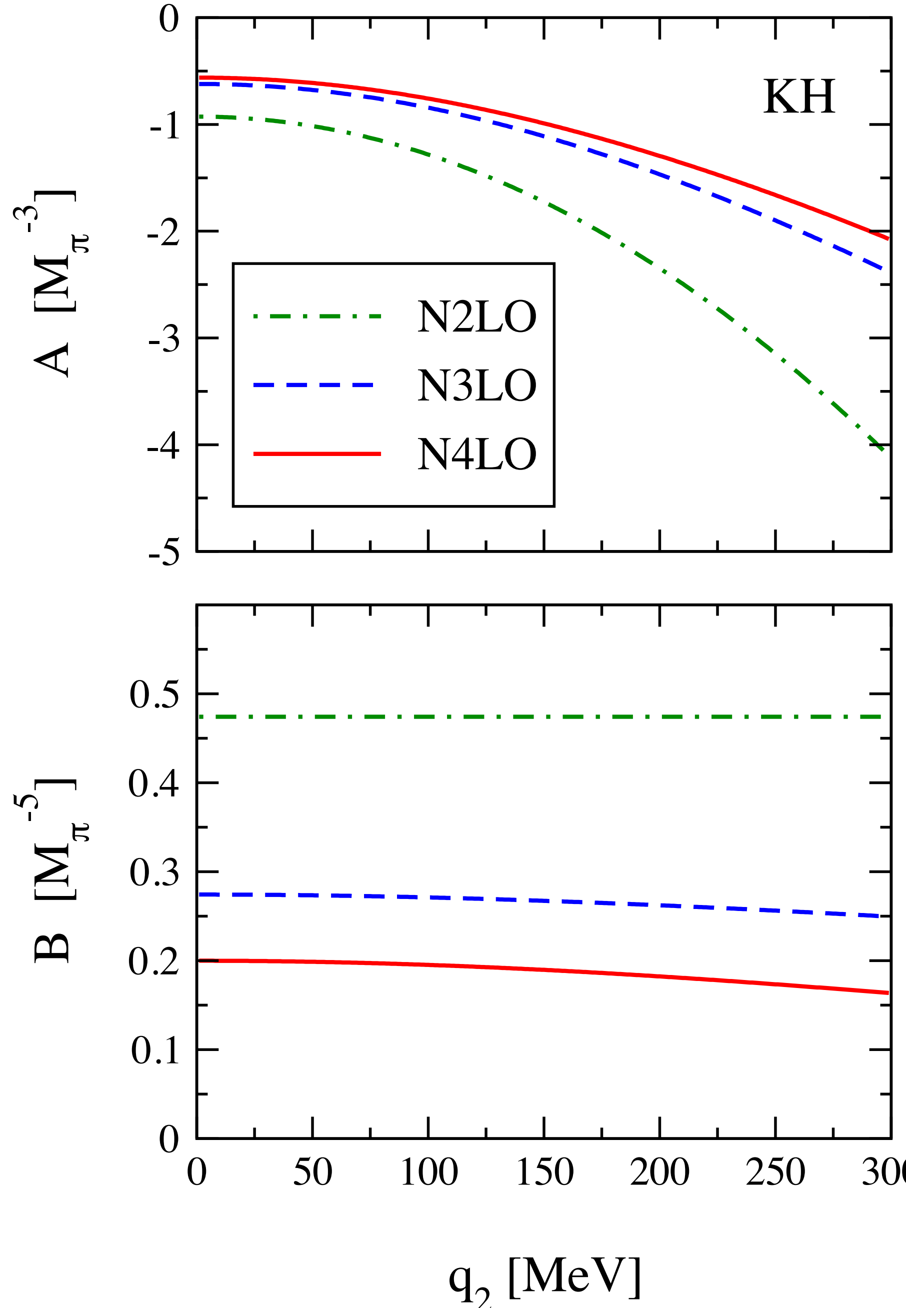
Absence of inelasticity below the two-pion production threshold

$$\delta^I_{l\pm}(s) = \arctan\left(|\vec q|\,\mathcal{R}e\, f^I_{l\pm}(s)\right)$$

Two-pion-exchange at N^4LO

Data fitted for $p_{Lab} < 150\,\mathrm{MeV}$





Karlsruhe-Helsinki (KH) PWA: R. Koch Nucl. Phys. A 448 (1986) 707

Similar fit to George-Washington (GW) PWA: Arndt et al. Phys. Rev. C 74 (2006) 045205

	c_1	c_2	c_3	c_4	$\bar{d}_1 + \bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14} - \bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
GW-fit	-1.13	3.69	-5.51	3.71	5.57	-5.35	0.02	-10.26	1.75	-5.80	1.76	-0.58	0.96
KH-fit	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-10.41	6.08	-0.37	3.26

- No dependence on d_i´s
- e_i´s are of natural size
- Good convergence of TPE 3NF

Most general structure of a local 3NF

Epelbaum, Gasparyan, H.K., arXiv: 1302.2872

Up to N^{4}LO, the computed contributions are local → it is natural to switch to r-space.

A meaningful comparison requires a complete set of independent operators

Generators $\mathcal{G}$ of 89 independent operators	S	A	G_{12}	G_{22}	G_{11}	G_{21}
$\mathcal{G}_1 = 1$	O_1	0	0	0	0	0
$\mathcal{G}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3$	O_2	0	O_3	O_4	0	0
$\mathcal{G}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3$	O_5	0	O_6	O_7	0	0
$\mathcal{G}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_3$	O_8	0	O_9	O_{10}	0	0
$\mathcal{G}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2$	O_{11}	O_{12}	O_{13}	O_{14}	O_{15}	O_{16}
$\mathcal{G}_6 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$	O_{17}	0	0	0	0	0
$\mathcal{G}_7 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	O_{18}	0	O_{19}	O_{20}	0	0
$\mathcal{G}_8 = \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_3$	O_{21}	O_{22}	O_{23}	O_{24}	O_{25}	O_{26}
$\mathcal{G}_9 = \vec{q}_1 \cdot \vec{\sigma}_3 \vec{q}_3 \cdot \vec{\sigma}_1$	O_{27}	0	O_{28}	O_{29}	0	0
$\mathcal{G}_{10} = \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	O_{30}	0	O_{31}	O_{32}	0	0
$\mathcal{G}_{11} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	O_{33}	O_{34}	O_{35}	O_{36}	O_{37}	O_{38}
$\mathcal{G}_{12} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	O_{39}	O_{40}	O_{41}	O_{42}	O_{43}	O_{44}
$\mathcal{G}_{13} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	O_{45}	O_{46}	O_{47}	O_{48}	O_{49}	O_{50}
$\mathcal{G}_{14} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	O_{51}	O_{52}	O_{53}	O_{54}	O_{55}	O_{56}
$\mathcal{G}_{15} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{q}_2 \cdot \vec{\sigma}_1 \vec{q}_2 \cdot \vec{\sigma}_3$	O_{57}	0	O_{58}	O_{59}	0	0
$\mathcal{G}_{16} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_2 \vec{q}_3 \cdot \vec{\sigma}_3$	O_{60}	O_{61}	O_{62}	O_{63}	O_{64}	O_{65}
$\mathcal{G}_{17} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	O_{66}	0	O_{67}	O_{68}	0	0
$\mathcal{G}_{18} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{\sigma}_3 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	O_{69}	0	O_{70}	O_{71}	0	0
$\mathcal{G}_{19} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_3 \cdot \vec{q}_1 \vec{q}_1 \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$	O_{72}	O_{73}	O_{74}	O_{75}	O_{76}	O_{77}
$\mathcal{G}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_2 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot (\vec{q}_1 \times \vec{q}_3)$	O_{78}	O_{79}	O_{80}	O_{81}	O_{82}	O_{83}
$\mathcal{G}_{21} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{q}_2 \vec{\sigma}_3 \cdot \vec{q}_2 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	O_{84}	0	O_{85}	O_{86}	0	0
$\mathcal{G}_{22} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	O_{87}	0	O_{88}	O_{89}	0	0

Most general, local 3NF involves 89 operators, can be generated (by permutations) from 22 structures:

$$V_{3N}^{\rm loc} = \sum_{i=1}^{22} \mathcal{G}_i F_i(r_{12}, r_{23}, r_{31}) + 5\,\text{perm.}$$

The structures $\mathcal{O}_i$ are defined as:

$$S(\mathcal{G}) := \frac{1}{6} \sum_{P \in S_3} P\mathcal{G}$$

$$A(\mathcal{G}) := \frac{1}{6} \sum_{P \in S_3} (-1)^P P\mathcal{G}$$

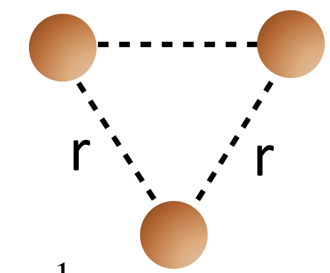
$$G_{ij}(\mathcal{G}) := \frac{1}{3} \sum_{P \in S_3} \mathcal{D}_{ij}(P) P\mathcal{G}, \quad i,j = 1,2$$

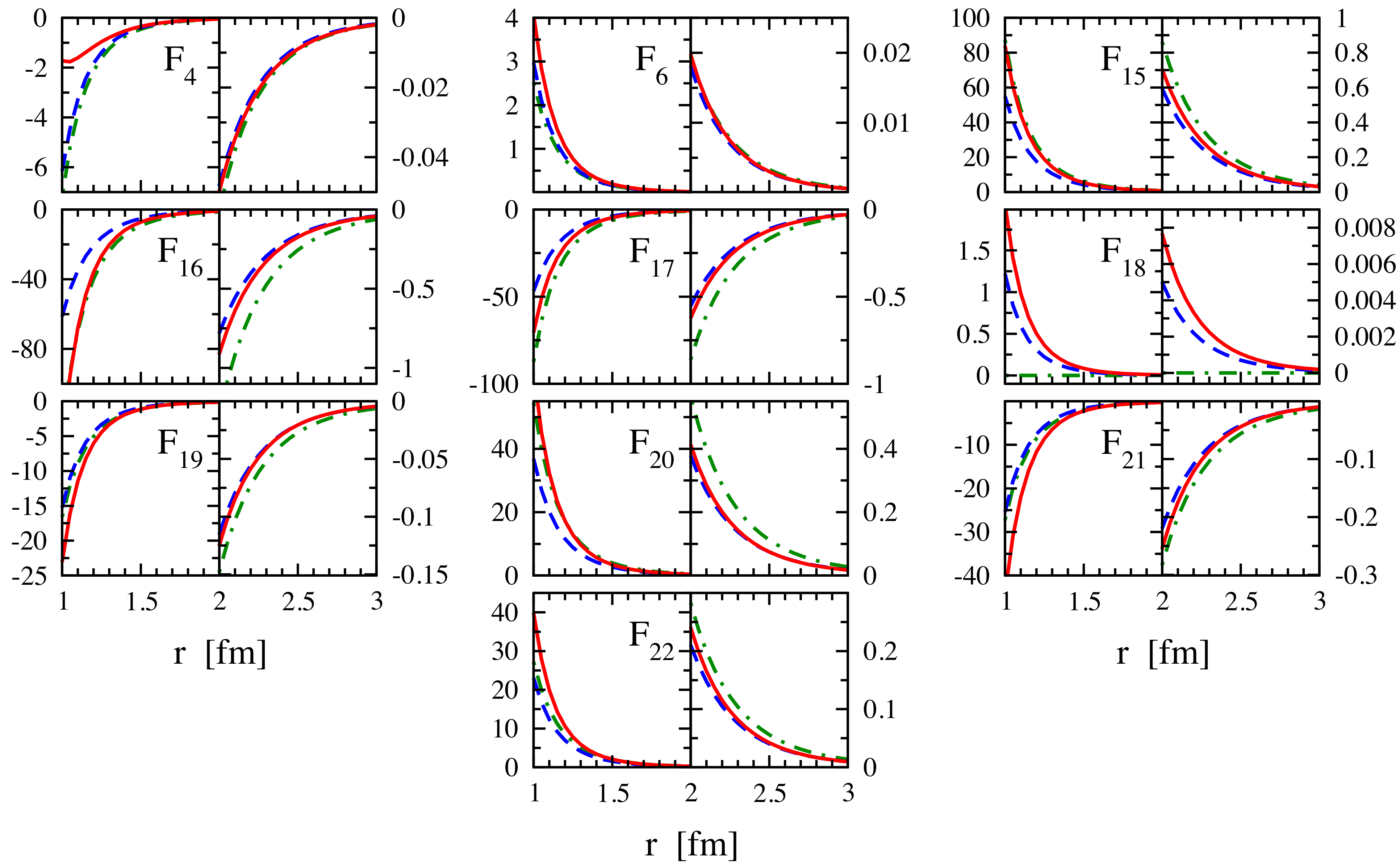
2-dim. irred. repr. of S_3

Two-pion-exchange up to N^{4}LO

Epelbaum, Gasparyan, H.K., arXiv: 1302.2872

Chiral expansion of TPE „structure functions“ F_i (in MeV) in the equilateral-triangle configuration



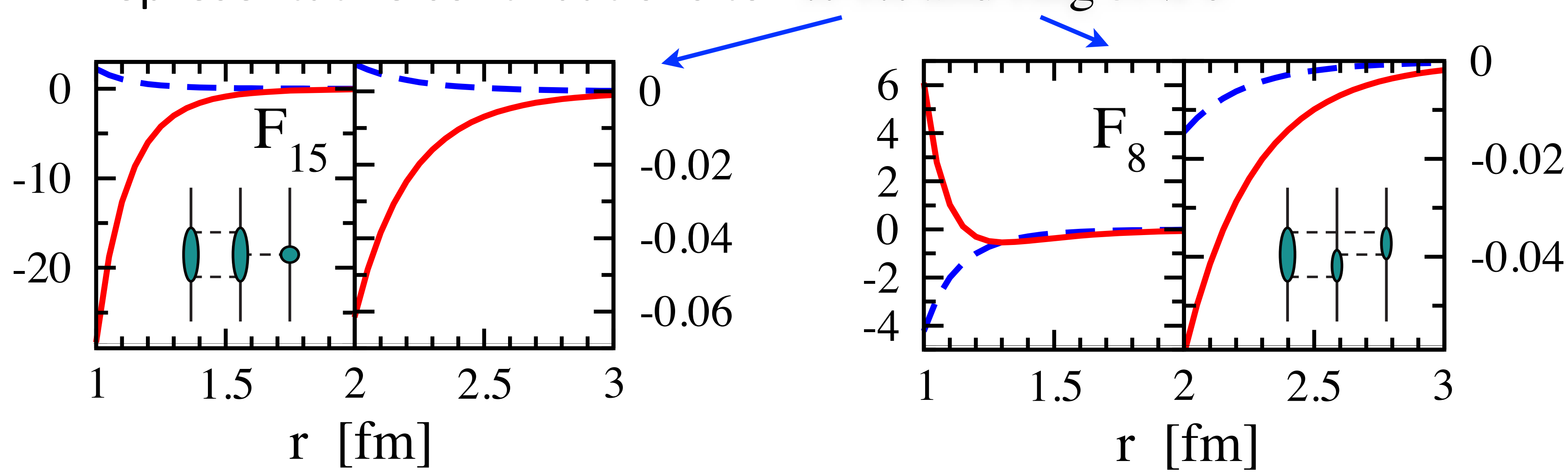


Excellent convergence of TPE-force at distance $r \geq 2\,\mathrm{fm}$

2π - 1π and ring 3NFs up to N^{4}LO

Epelbaum, Gasparyan, H.K., arXiv: 1302.2872

Representative contributions to 2π-1π and ring 3NFs

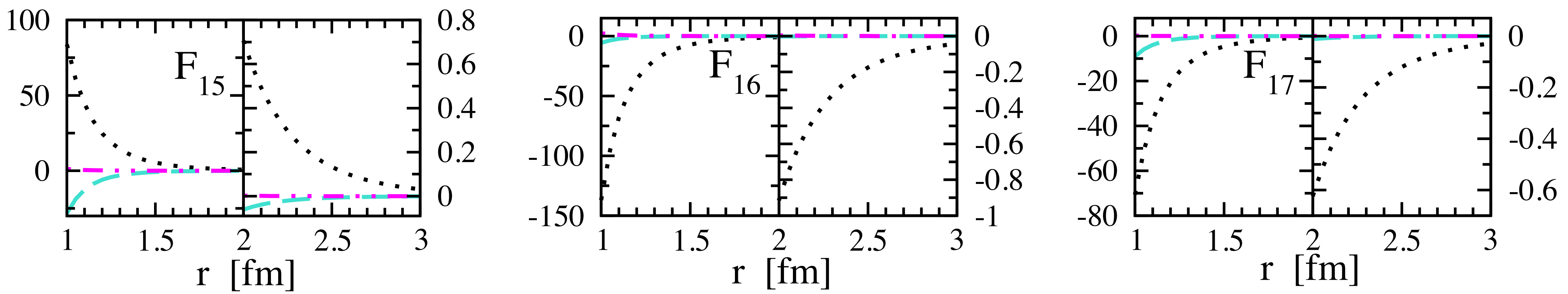


- Convergence of chiral expansion of 2π-1π and ring 3NFs is much worse
- In nearly all cases subleading N^{4}LO dominate leading N^{3}LO contributions
- Leading Δ-contributions first at N^{4}LO ⟹ N^{4}LO > N^{3}LO
- Considerably shorter range as compared with 2π-exchange contributions

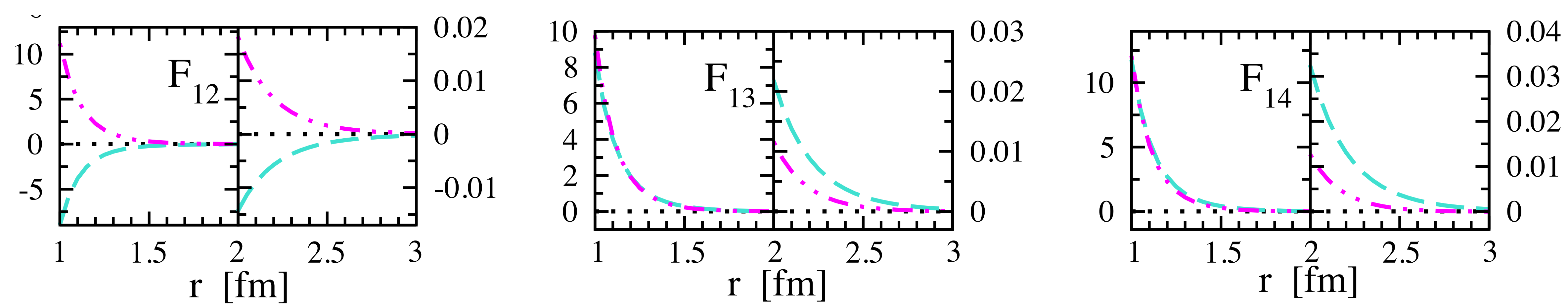
Not clear whether the lack of convergence will have any significant phenomenological effect.

Individual contr. to 3NF up to N^{4}LO

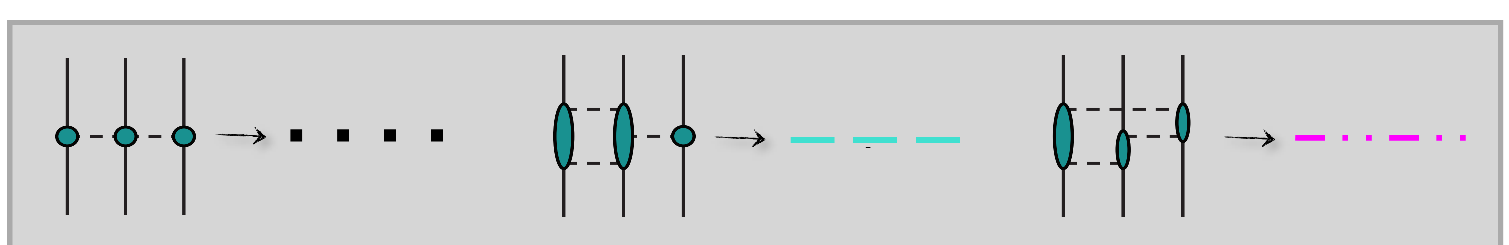
Representative contributions from individual topologies



- Clear dominance of 2π-exch. 3NF (if contributes) over two other topologies at $r \geq 2$ fm
- At shorter distances $r \sim 1$ fm 2π-1π and ring 3NFs become more significant

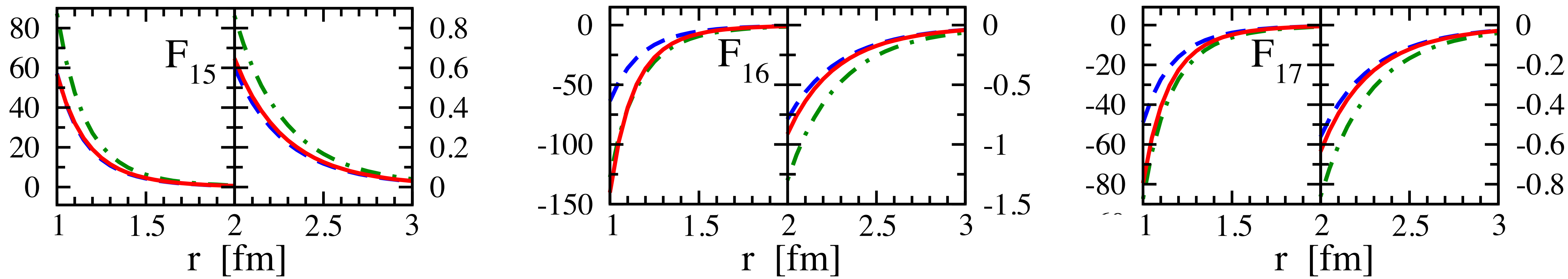


- 2π-1π and ring 3NFs are in the most cases of comparable size
- No conclusion about phenomenological impact due to still missing short-range contr.



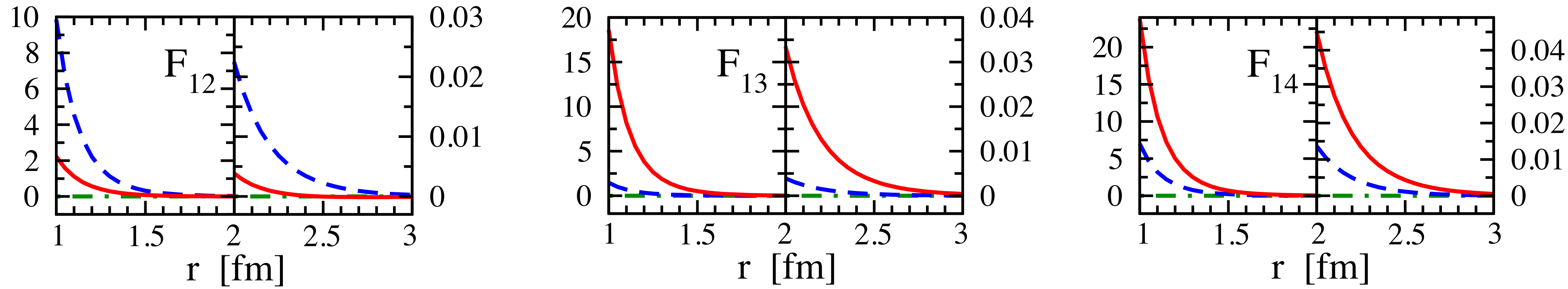
Long-range 3NF up to N^4LO

Epelbaum, Gasparyan, H.K., arXiv: 1302.2872



- Good convergence at long distances $r \geq 2\,\mathrm{fm}$ for profile functions which are dominated by 2π-exch. 3NF

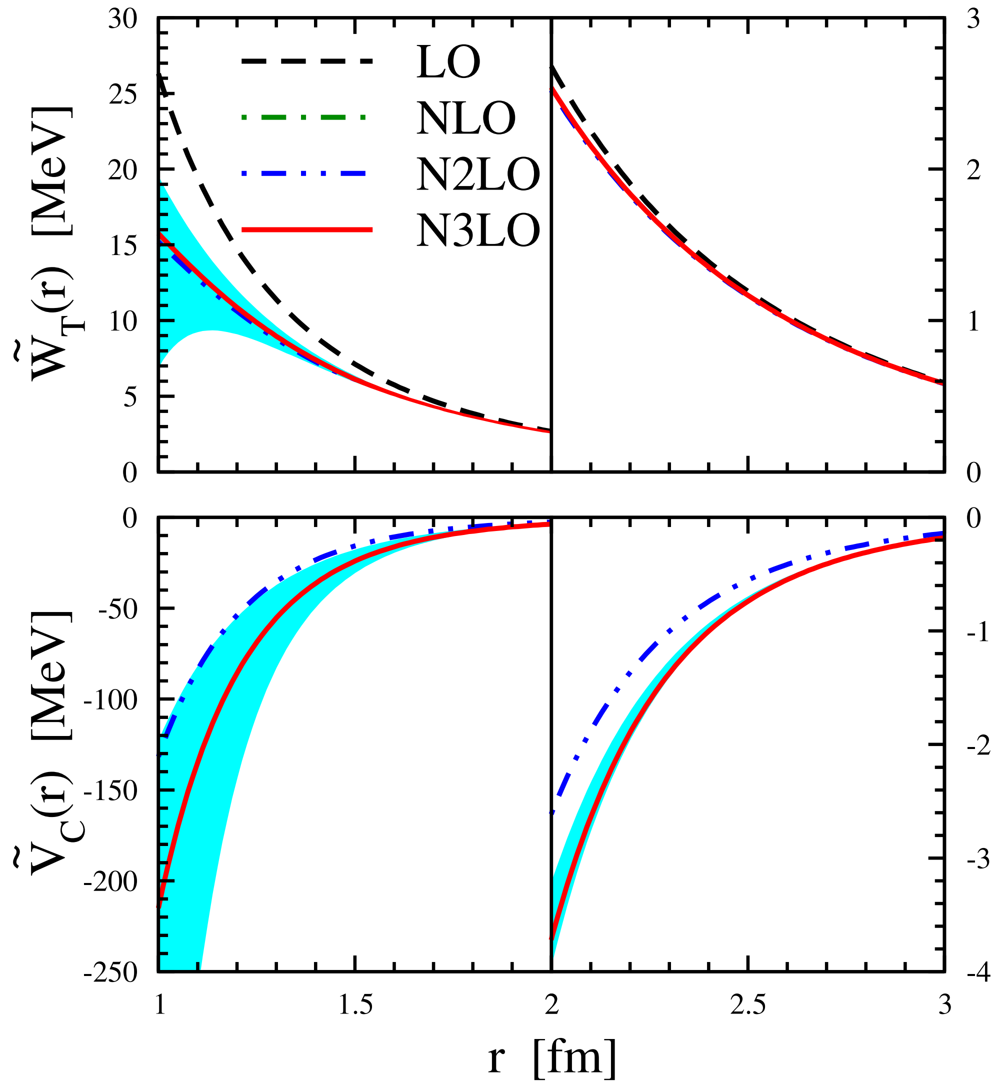
- At shorter distances $r \sim 1\,\mathrm{fm}$ 2π-1π and ring 3NFs start becoming more important



- Profile functions which are not affected by 2π-exch. 3NF are typically dominated by N^4LO contributions and might still not be converged at this order.

⇨ Supports assumption about important role of Δ-excitation which is partially taken into account at N^4LO through resonance saturation of c_i's

Comparison with NN force



Epelbaum, Meißner Ann. Rev. Nucl. Part. Sci 62 (12) 159

$$\tilde{V}(\vec{r}) = \tilde{V}_C + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tilde{W}_C + \left[\tilde{V}_S + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tilde{W}_S\right] \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \left[\tilde{V}_T + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tilde{W}_T\right] (3\,\vec{\sigma}_1 \cdot \hat{r}\,\vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

Bands ($800 \text{ MeV} \leq \tilde{\Lambda}$) visualize estimated scheme-dependence for separation between short- and long-range contributions

Long-range behavior at $r \geq 2 \text{ fm}$ of

- $\tilde{W}_T$ is governed by 1π-exchange
- $\tilde{V}_C$ is governed by subleading 2π-exchange

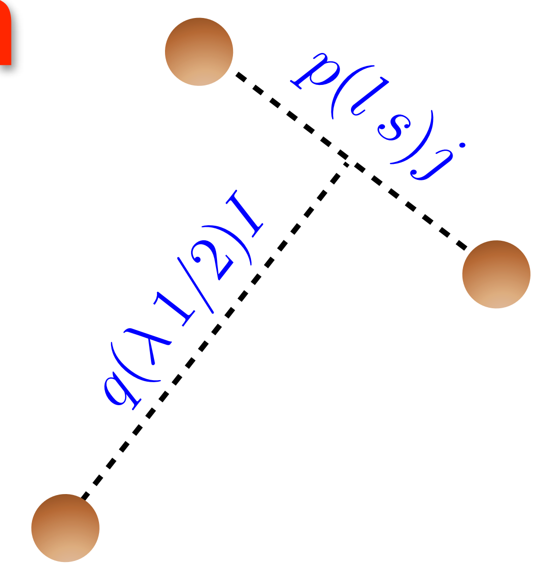
Size of various dominant contributions at $r = 2 \text{ fm}$

NN	2π–3NF	2π – 1π–3NF	ring-3NF
$\sim 3 \ldots 4$ MeV	$\sim 0.7 \ldots 1$ MeV	~ 50 keV	~ 70 keV

Long-range 3NFs are considerably weaker than NN forces, but not negligible!

Partial wave decomposition

Golak et al. Eur. Phys. J. A 43 (2010) 241



- Faddeev equation is solved in the partial wave basis

$$|p, q, \alpha\rangle \equiv |pq(ls)j(\lambda\tfrac{1}{2})I(jI)JM_J\rangle\,|(t\tfrac{1}{2})TM_T\rangle$$

- Too many terms for doing PWD by hand ⟹ Automatization

$$\underbrace{\langle p'q'\alpha'|V|pq\alpha\rangle}_{\text{matrix} \sim 10^5 \times 10^5} = \int \underbrace{d\hat{p}'\, d\hat{q}'\, d\hat{p}\, d\hat{q}}_{\text{can be reduced to 5 dim. integral}} \sum_{m_l,\ldots} \Big(\text{CG coeffs.}\Big)\Big(Y_{l,m_l}(\hat{p})\, Y_{l',m_l'}(\hat{p}') \ldots\Big)\underbrace{\langle m'_{s_1} m'_{s_2} m'_{s_3}|V|m_{s_1} m_{s_2} m_{s_3}\rangle}_{\text{depends on spin \& isospin}}$$

- Ring-diagram-contr. expensive to calculate on the fly

We prestore ring-contr. to 3nf's on a fine momentum grid ⟹ Numerical interpolation of ring terms

see talk by Roman Skibinski

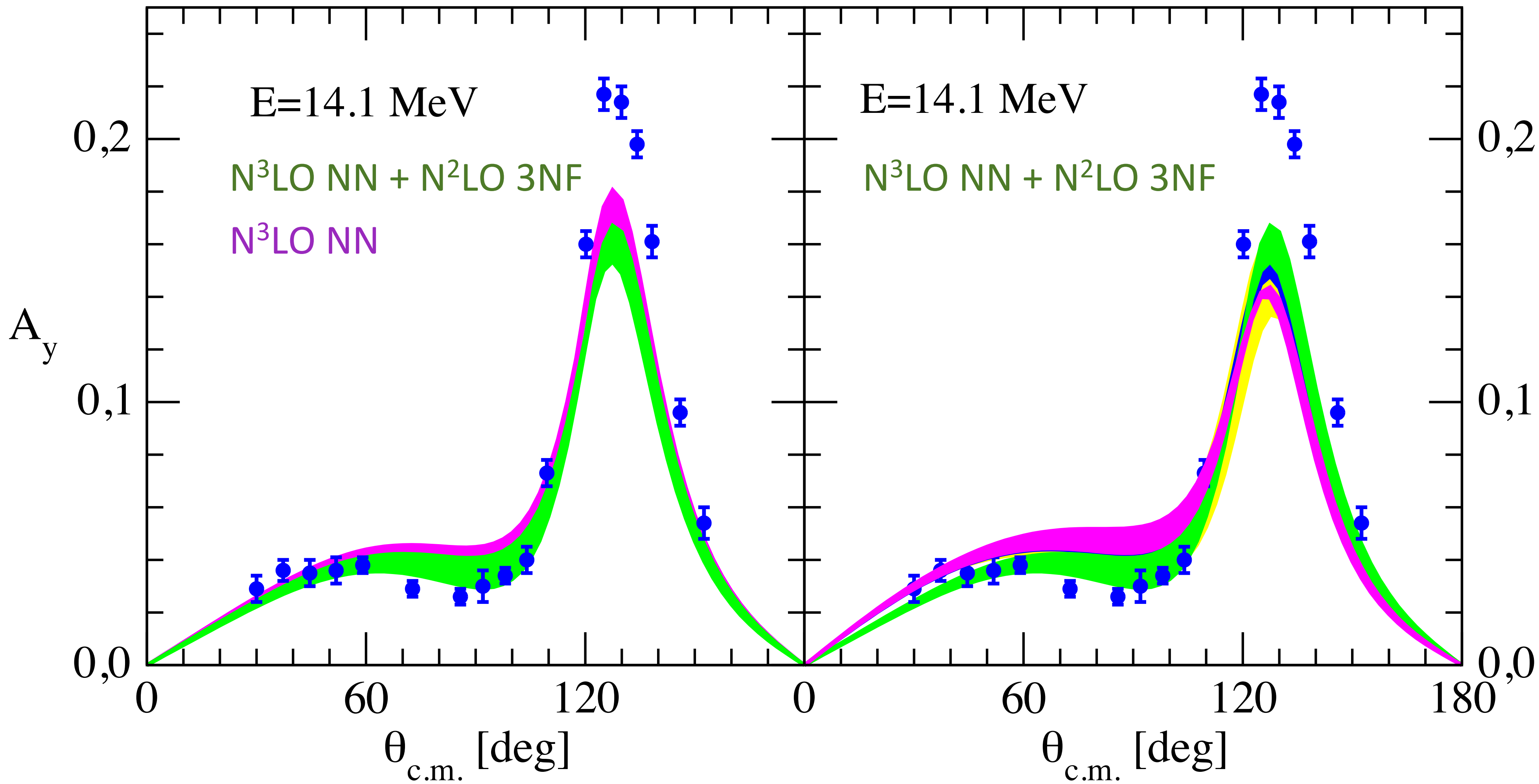
- PWD matrix-elements can be used to produce matrix-elements in harmonic oscillator basis

see talk by Robert Roth & Kai Hebeler

Straightforward implementation of high order 3nf's in many-body calc.
within No-Core Shell Model

A_y-puzzle in elastic nd scattering

Witala et al. Proceedings of Few Body 20



Right panel: X = N^{3}LO NN + N^{2}LO 3NF + N^{3}LO 3NF (1π-cont.) + N^{3}LO 3NF (cont.)

- (yellow) = X + N^{3}LO 3NF (2π-exch.)
- (blue) = X + N^{3}LO 3NF (2π-exch. & 2π-1π-exch.)
- (magenta) = X + N^{3}LO 3NF (2π-exch. & 2π-1π-exch. & ring)

Incomplete results: N^{3}LO 3NF (2π-cont. & 1/m-corr.) are missing

For neutron-matter analysis with N3LO 3NF see talk by Achim Schwenk.

Summary

- Chiral nuclear forces are analyzed up to N^3LO
- Long-range part of chiral three-nucleon forces is analyzed up to N^4LO
- In general there are 89 spin-isospin structures in local 3NF's built out of 22 + perm.
- Two-pion-exchange part dominates 3NF but does not fill all 22 structures
- With two-pion-one-pion-exchange and ring diagrams all 22 structures are filled
- First (incomplete) results for A_y in nd elastic scattering with N^3LO 3NF's

Outlook

- Partial wave decomposition of N^3LO three-nucleon forces
- Complete study of 3NF and 4NF up to N^4LO with explicit delta-isobar
- Implementations in Nd, light nuclei & nuclear matter