# Three-Body Forces and the Structure of Exotic Nuclei

## Jason D. Holt

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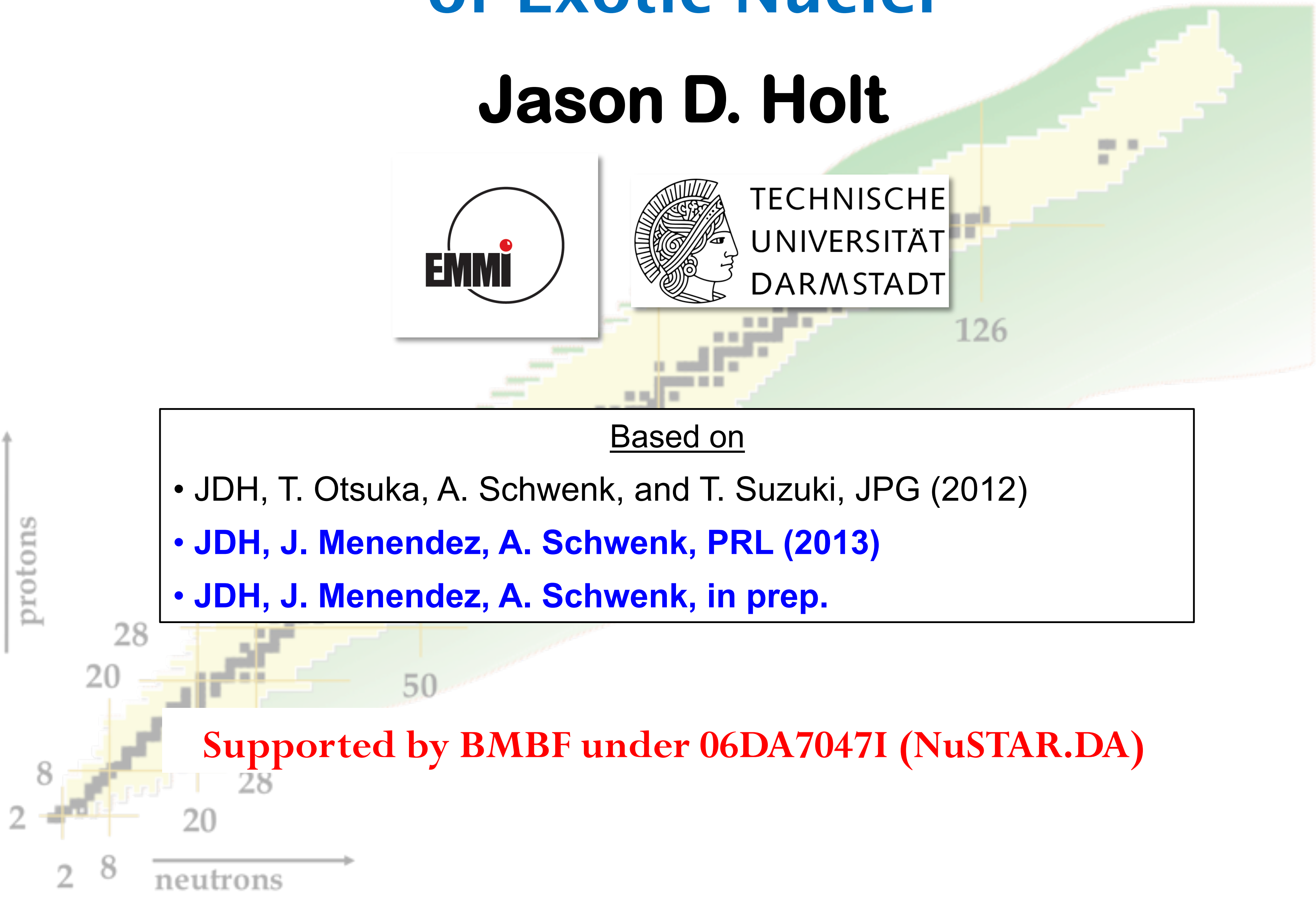


Based on

- JDH, T. Otsuka, A. Schwenk, and T. Suzuki, JPG (2012)
- **JDH, J. Menendez, A. Schwenk, PRL (2013)**
- **JDH, J. Menendez, A. Schwenk, in prep.**

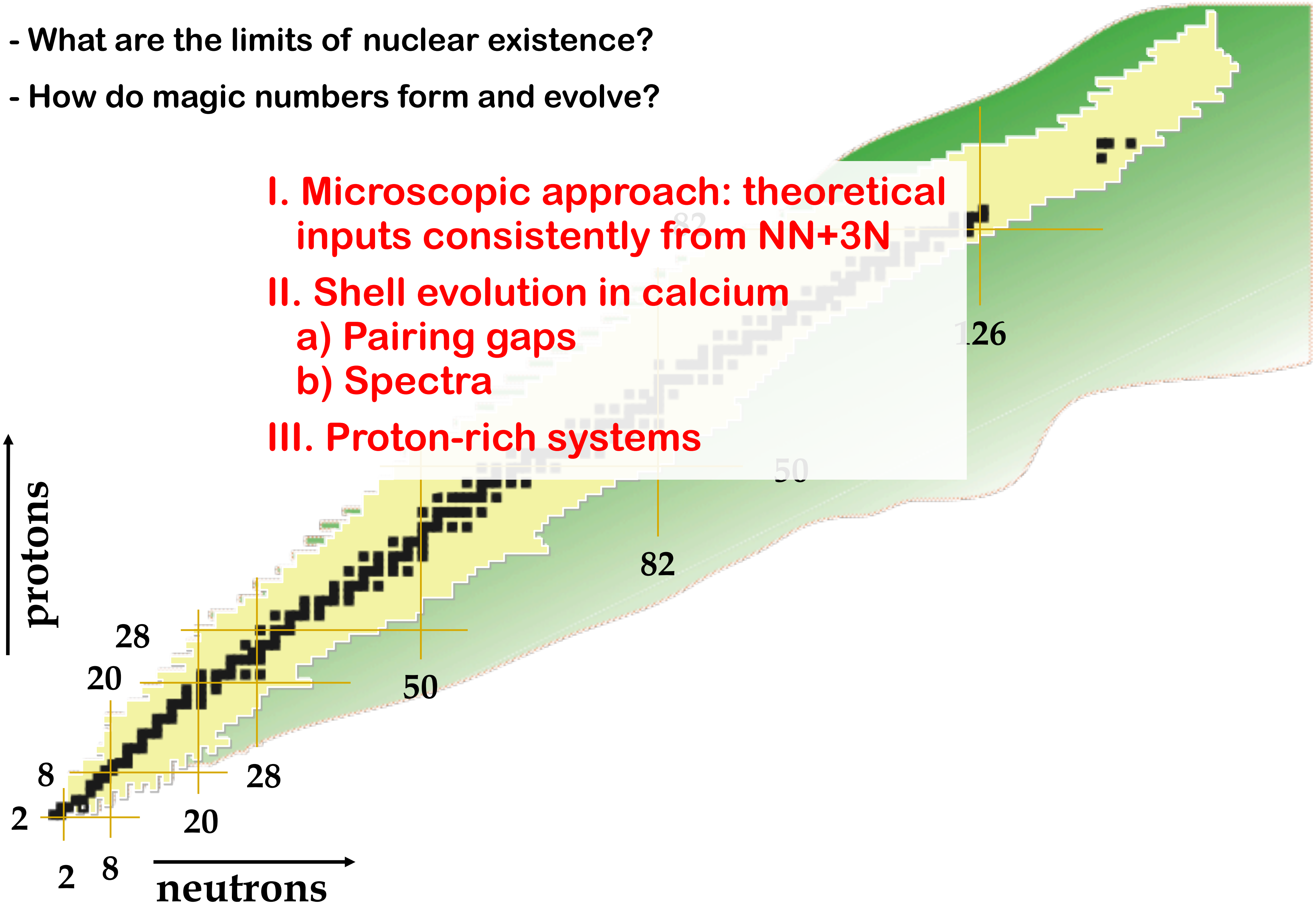
Supported by BMBF under 06DA7047I (NuSTAR.DA)

# Outline

Goal: understand the role of 3N forces for structure of medium-mass exotic nuclei

- What are the limits of nuclear existence?

- How do magic numbers form and evolve?



I. Microscopic approach: theoretical inputs consistently from NN+3N

II. Shell evolution in calcium

- a) Pairing gaps
- b) Spectra

III. Proton-rich systems

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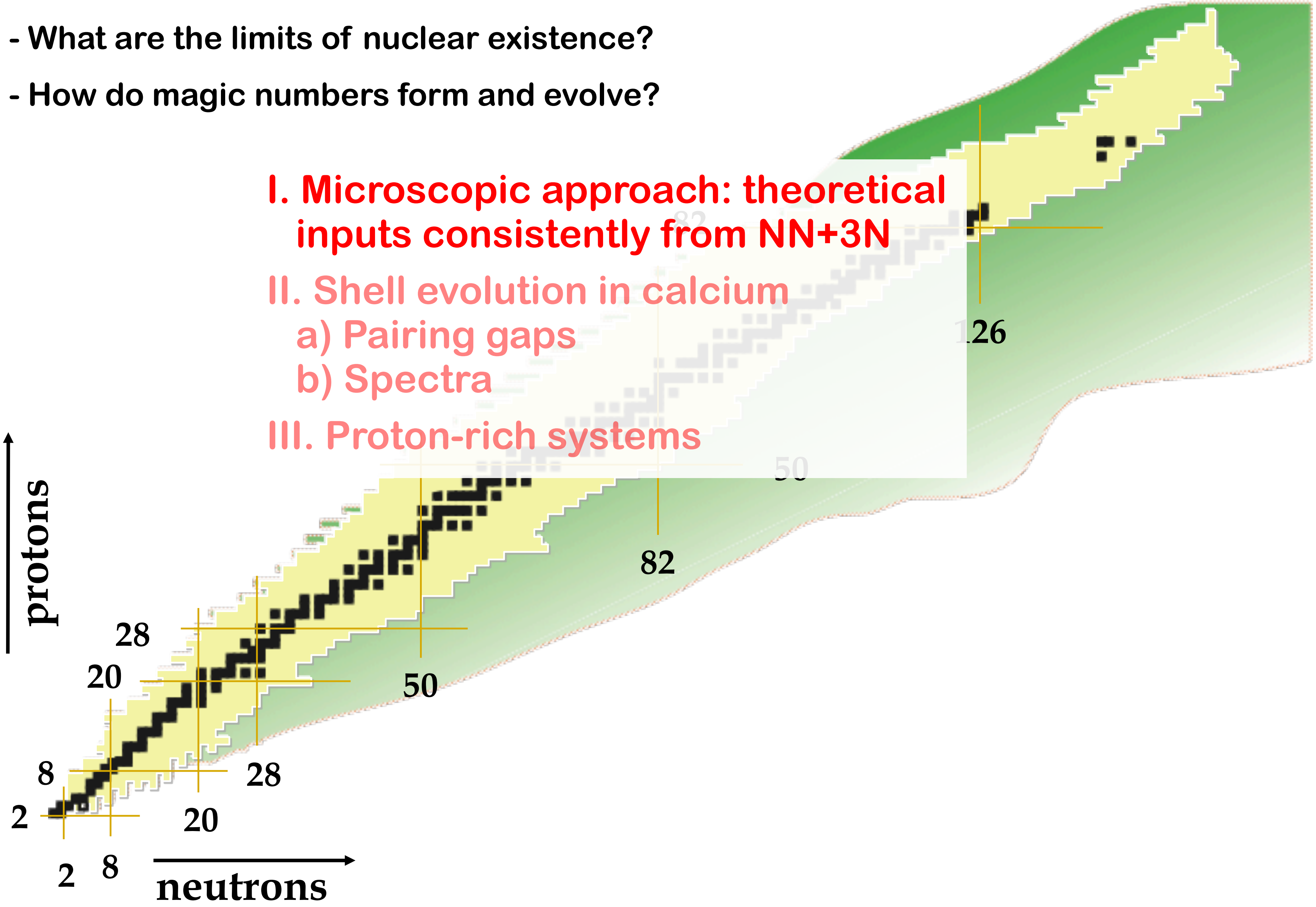

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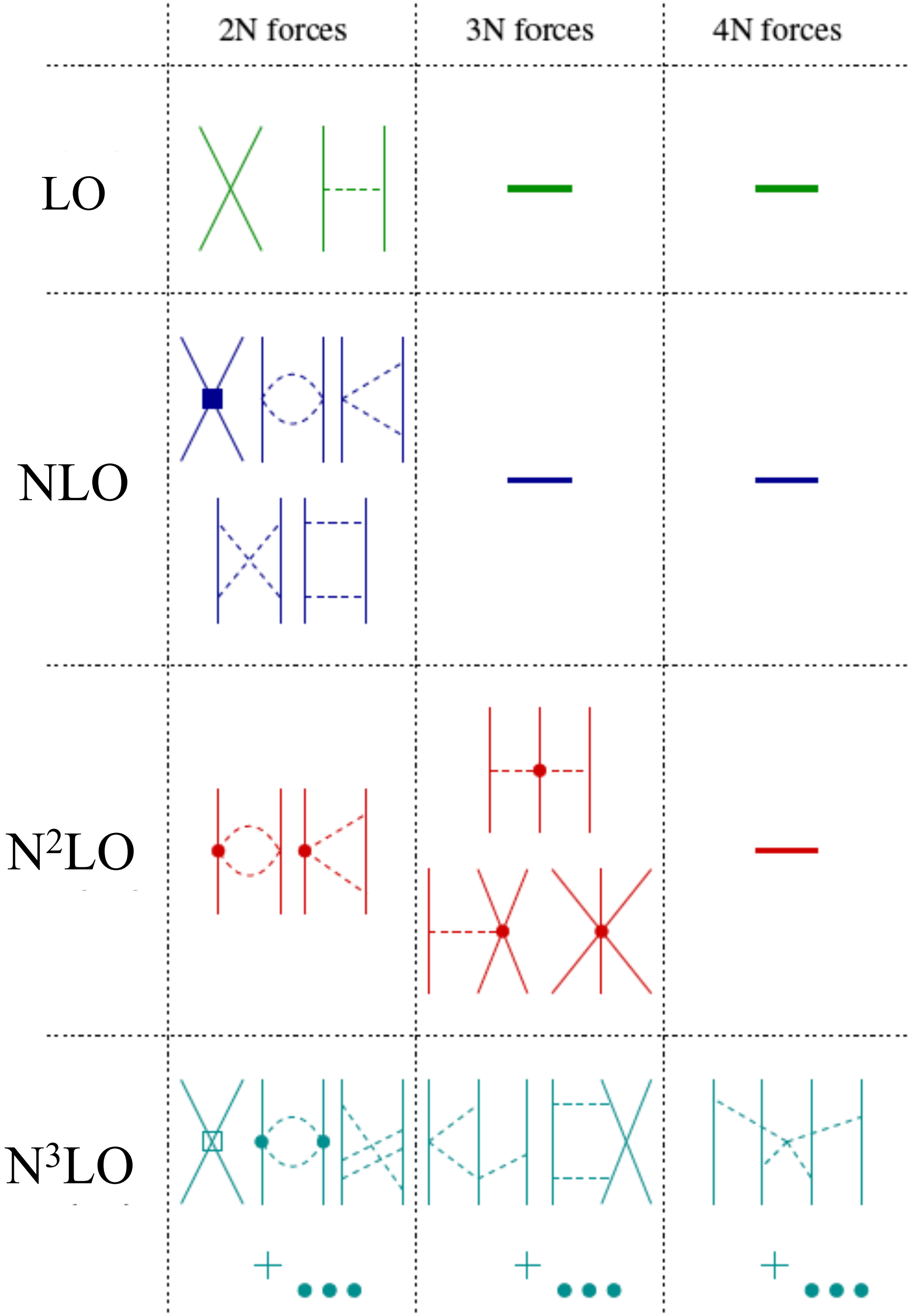
# Chiral Effective Field Theory: Nuclear Forces



Nucleons interact via pion exchanges and contact interactions

Hierarchy: $V_{\mathrm{NN}} > V_{\mathrm{3N}} > \ldots$

Consistent treatment of NN, 3N, … electroweak operators

Couplings fit to experiment once

Evolve to **low-momentum** $V_{\text{low } k}$

3N constants fit to properties of light nuclei at low momentum

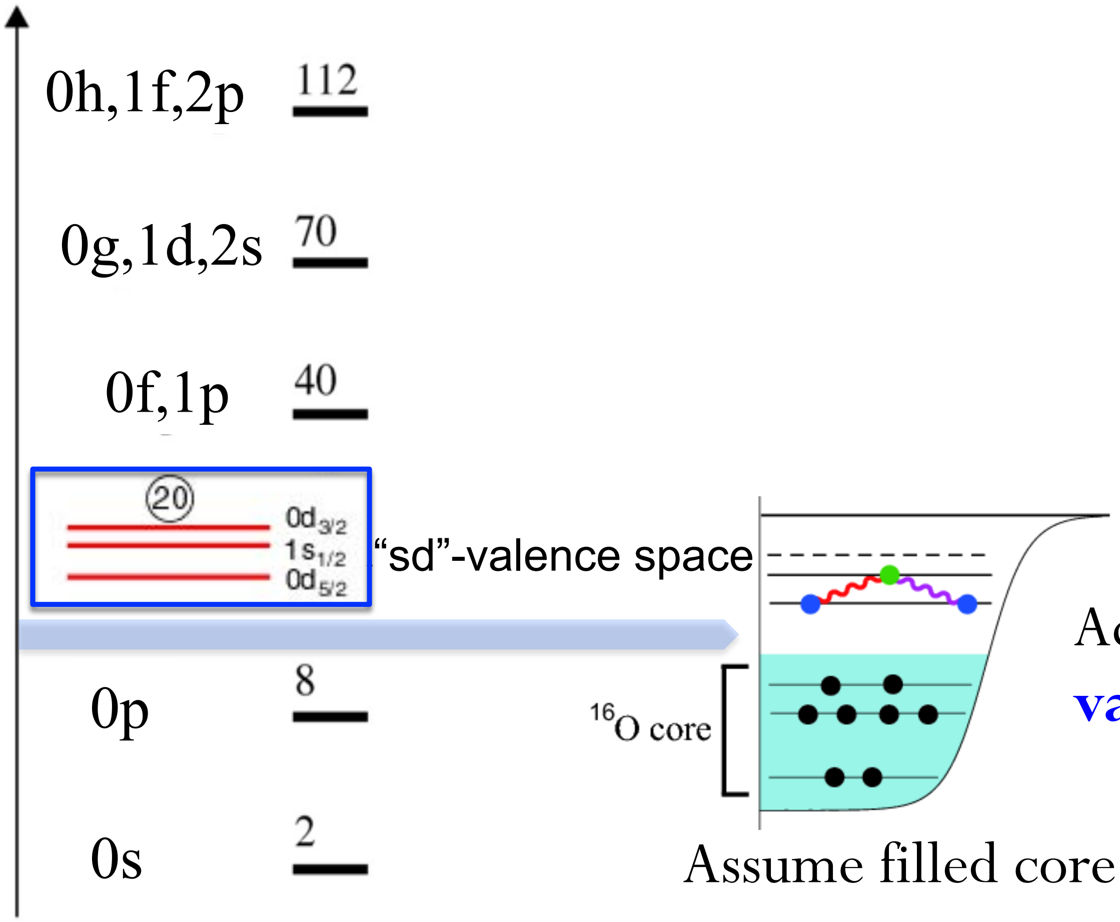
Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Meissner,…

# Solving the Nuclear Many-Body Problem

Nuclei understood as many-body system starting from closed shell, add nucleons

Interaction and energies of valence space orbitals from $V_{\mathrm{low}\ k}$

**Does not reproduce experimental data**



Active nucleons occupy **valence space**

Assume filled core

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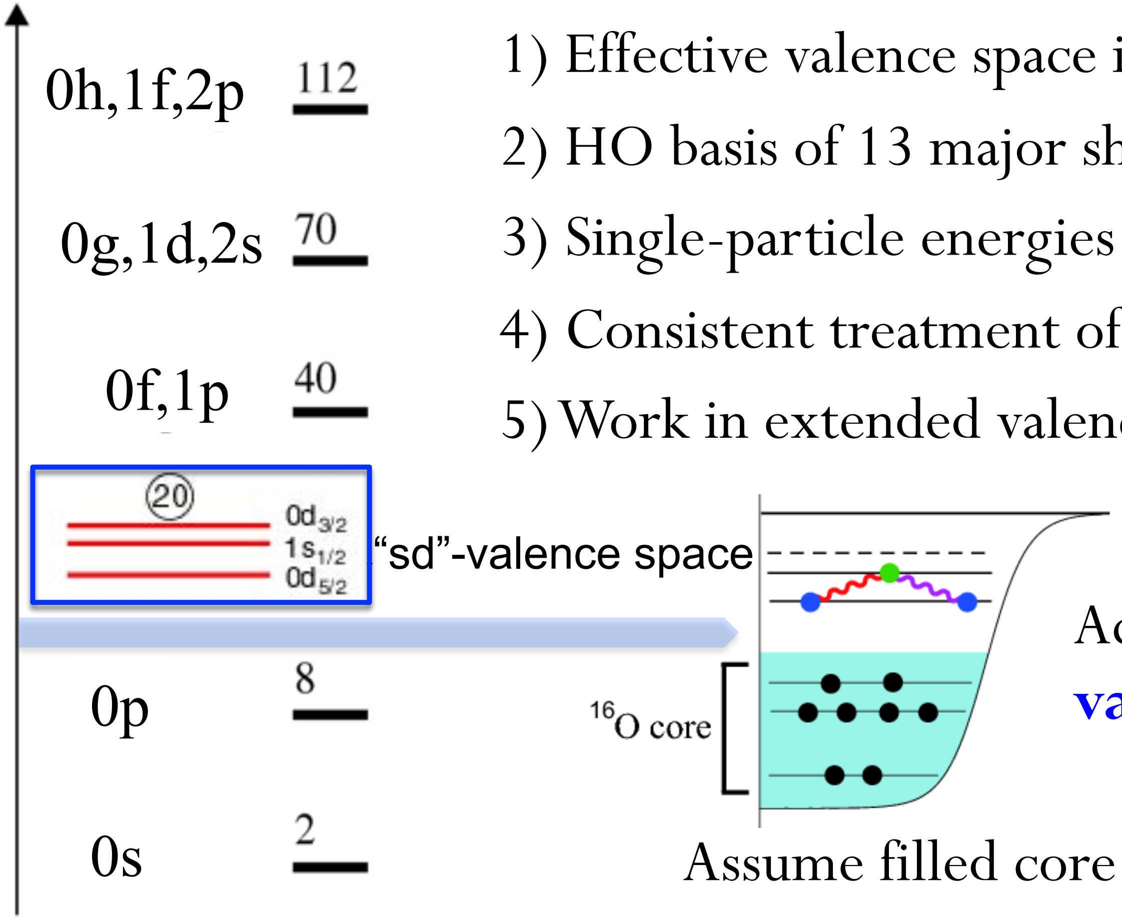
## Strategy

1) Effective valence space interaction – 3rd order MBPT
2) HO basis of 13 major shells
3) Single-particle energies calculated self-consistently
4) Consistent treatment of chiral NN and 3N forces
5) Work in extended valence spaces



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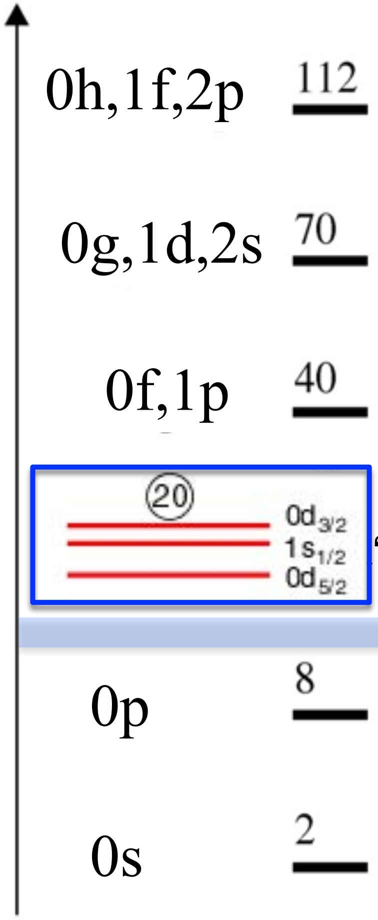
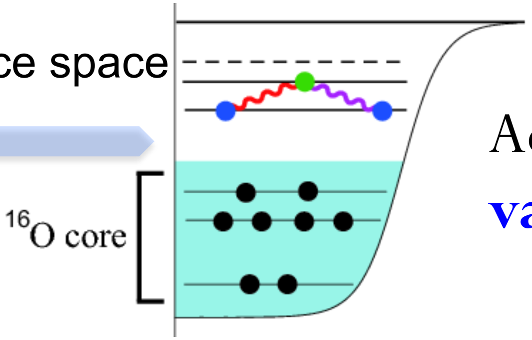


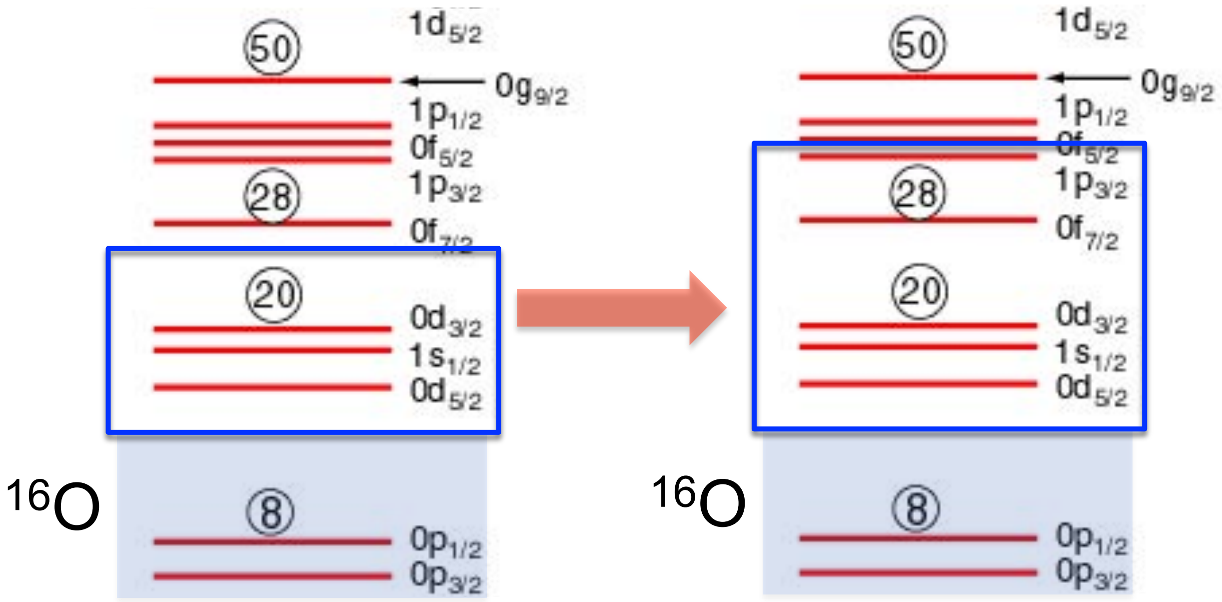


Active nucleons occupy **valence space**

Assume filled core

# Extended Valence Spaces

**Philosophy**: diagonalize in largest possible valence space (where orbits relevant)



Explore calculations in
**extended valence spaces**
(more than one major shell)

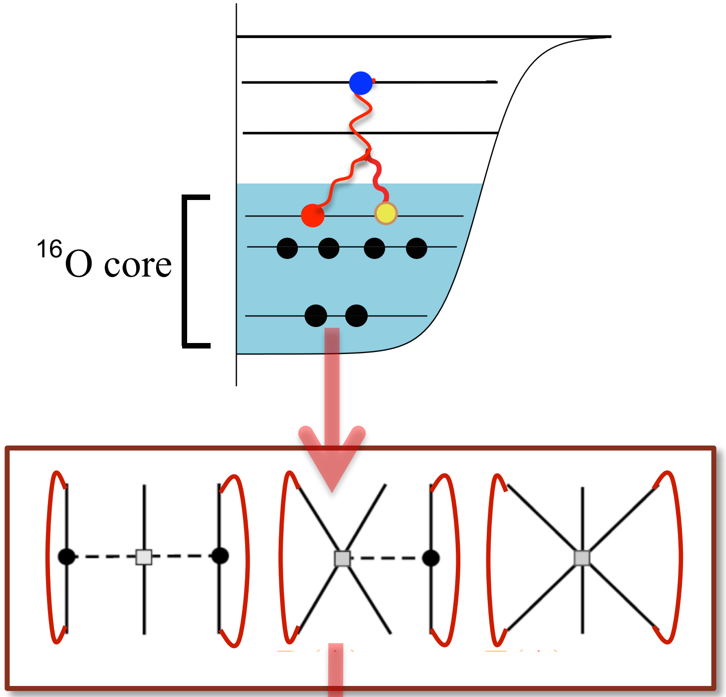
When do extended-space orbits impact exotic nuclei

**Caution: possible center-of-mass contamination**

# 3N Forces for Valence-Shell Theories

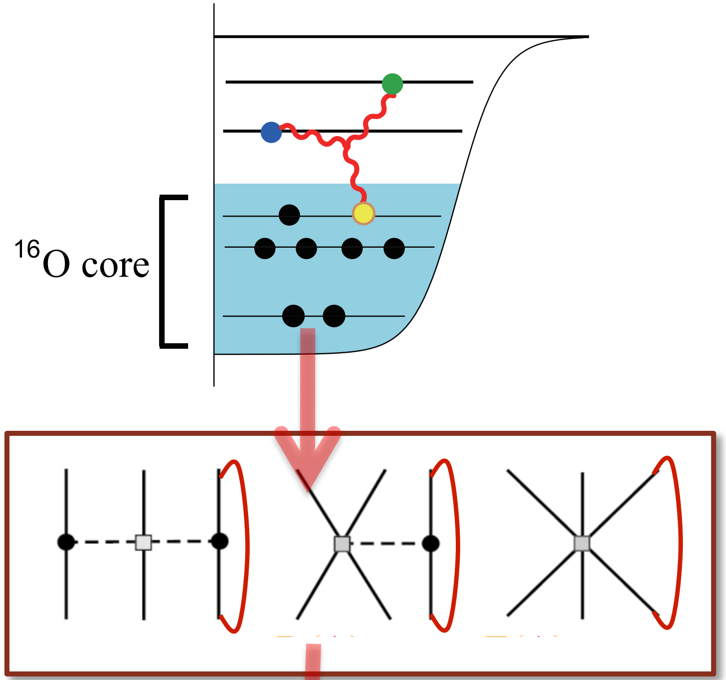
**Normal-ordered 3N**: contribution to valence nucleon interactions

## Effective one-body



$$\langle a | \, V_{3N,\mathrm{eff}} \, | a' \rangle = \frac{1}{2} \sum_{\alpha\beta = core} \langle \alpha\beta a | \, V_{3N} \, | \alpha\beta a' \rangle$$

## Effective two-body



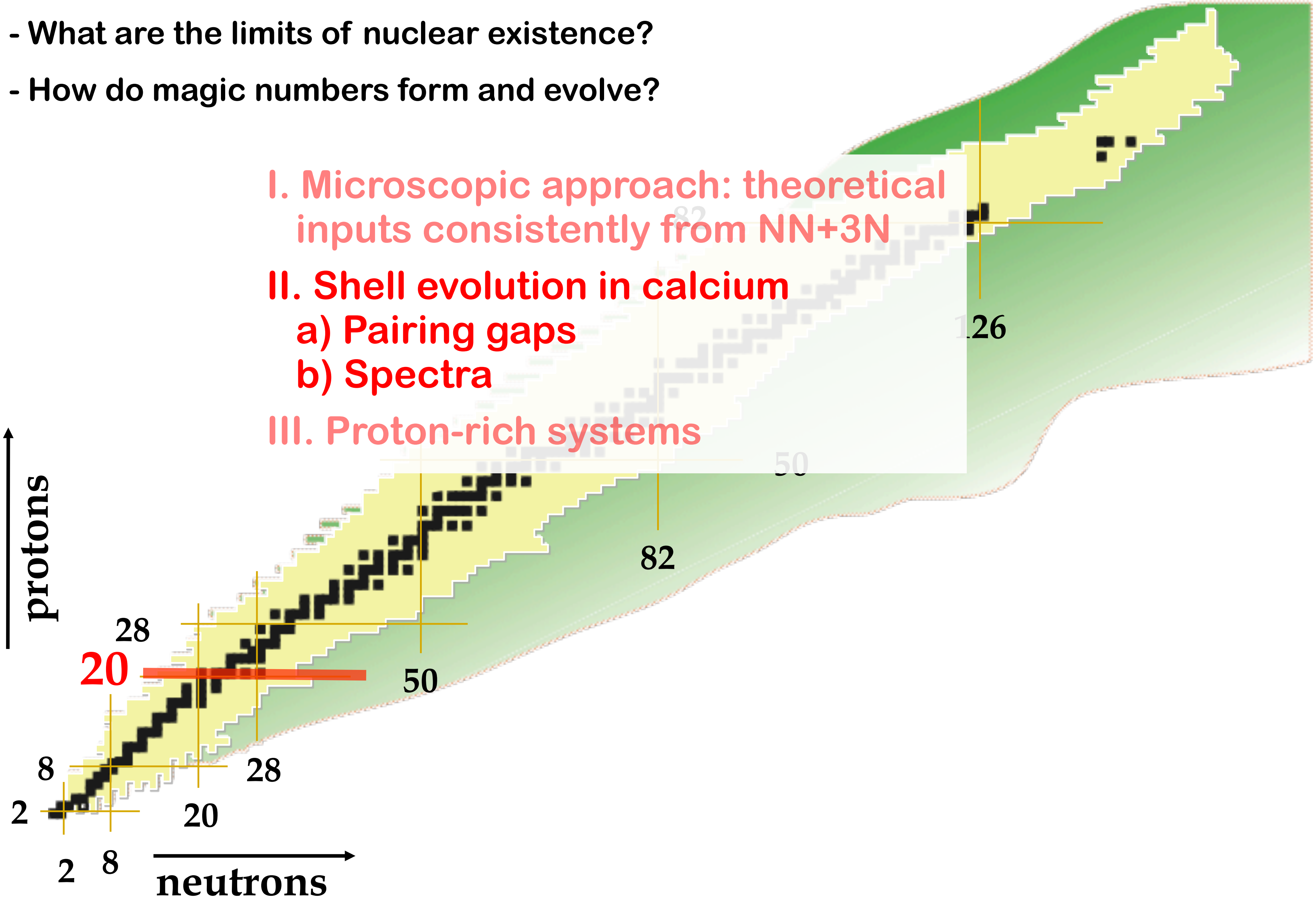
$$\langle ab | \, V_{3N,\mathrm{eff}} \, | a'b' \rangle = \sum_{\alpha = core} \langle \alpha ab | \, V_{3N} \, | \alpha a'b' \rangle$$

Combine with microscopic NN (**Third Order**): no empirical adjustments

# Shell Formation/Evolution in Calcium Isotopes

**Goal: understand the role of 3N forces for structure of medium-mass exotic nuclei**

- What are the limits of nuclear existence?
- How do magic numbers form and evolve?



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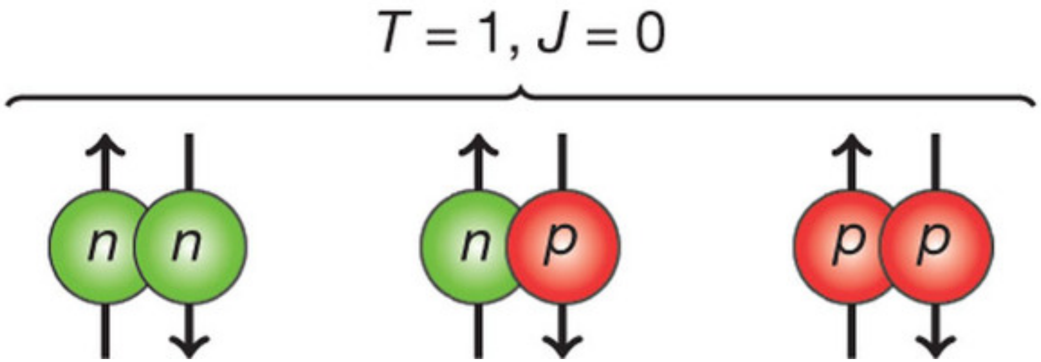
# Nuclear Pairing



Pairing of even number of nucleons – even/odd staggering

Pairing gaps deduced from **3-point mass difference**:

$$\Delta_n^{(3)} = \frac{(-1)^N}{2}\left[BE(N+1,Z) + BE(N-1,Z) - 2BE(N,Z)\right]$$

Allows comparison with experiment

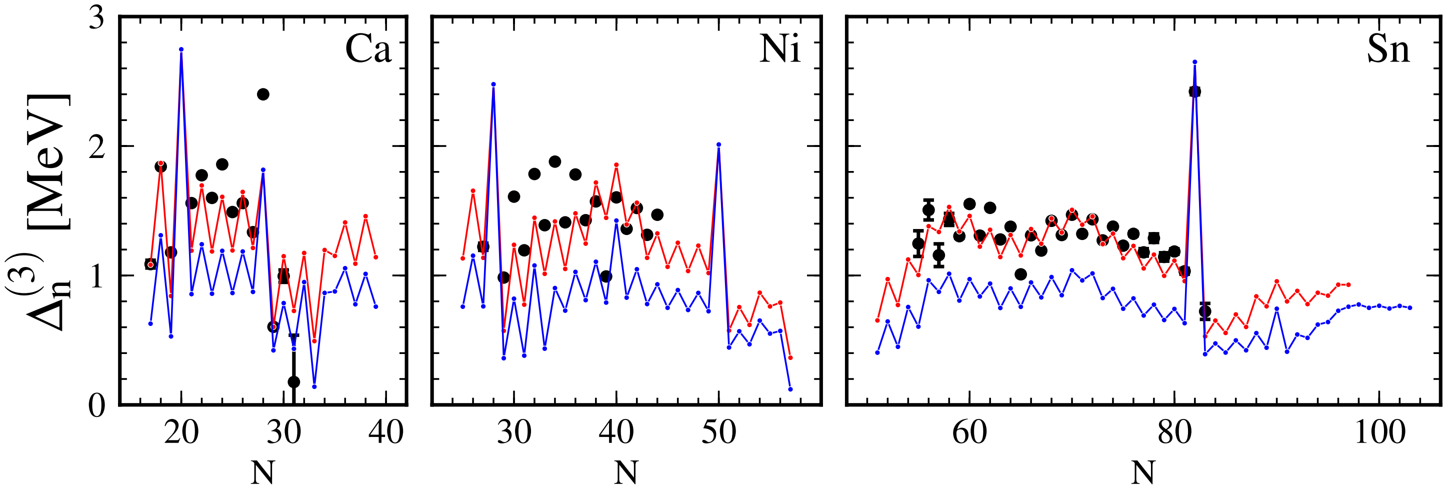
Relative peak in $\Delta_n^{(3)}$ indicates **shell closure**

- **additional tool to evaluate shell evolution**

# Pairing in EDF with 3N Forces

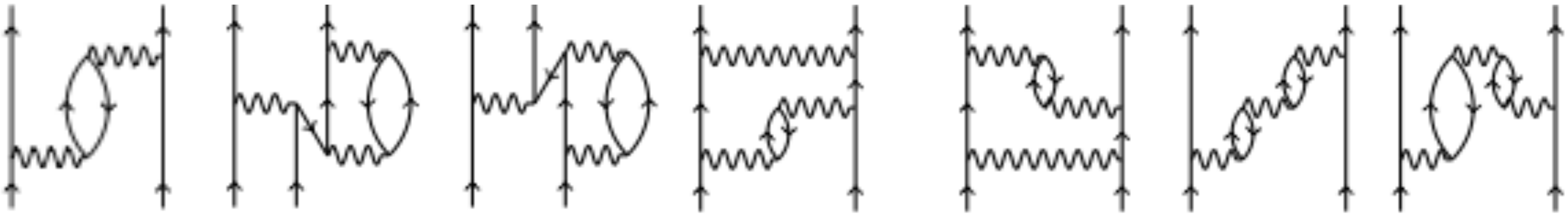
Energy Density Functional calculations: 3N lowers gaps systematically ~30%

Lesinski, Hebeler, Duguet, Schwenk, JPG (2012)



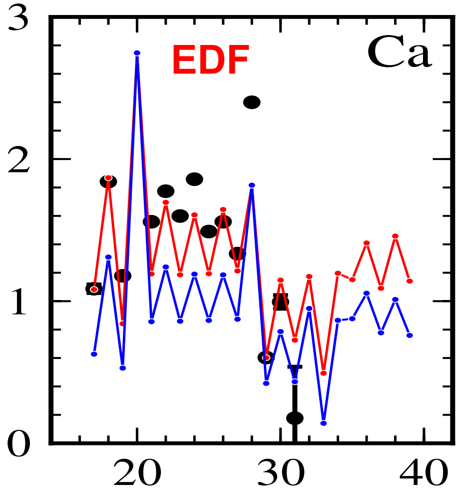
**What are the contributions from neglected many-body effects?**

**(Core polarization)**



# Pairing in Calcium Isotopes: Ladders

Compare with $\Delta_n^{(3)}$ calculated from microscopic NN+3N in calcium



HFB iterates ladders microscopically in pairing channel

Compare with ***pp*, *hh* ladders to 3$^{rd}$ order**

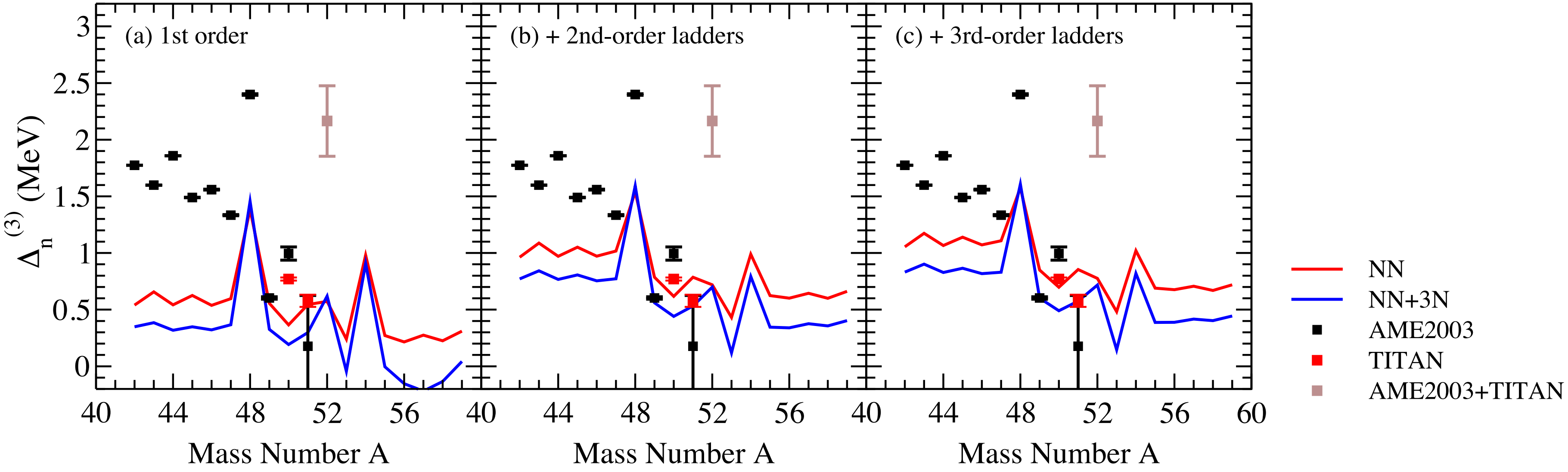
Improved agreement with experiment

Convergence in order-by-order ladders

Suppression from 3N forces as in EDF

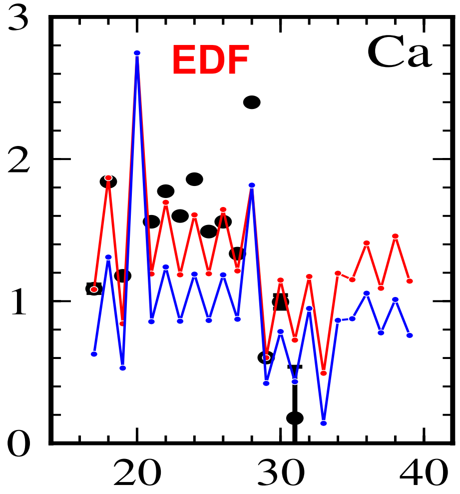
Incorrect odd/even staggering

JDH, Menendez, Schwenk, in prep



# Pairing in Calcium Isotopes: Full 3$^{rd}$ order

Compare with $\Delta_n^{(3)}$ calculated from microscopic NN+3N in calcium



**Full 3$^{rd}$-order MBPT**

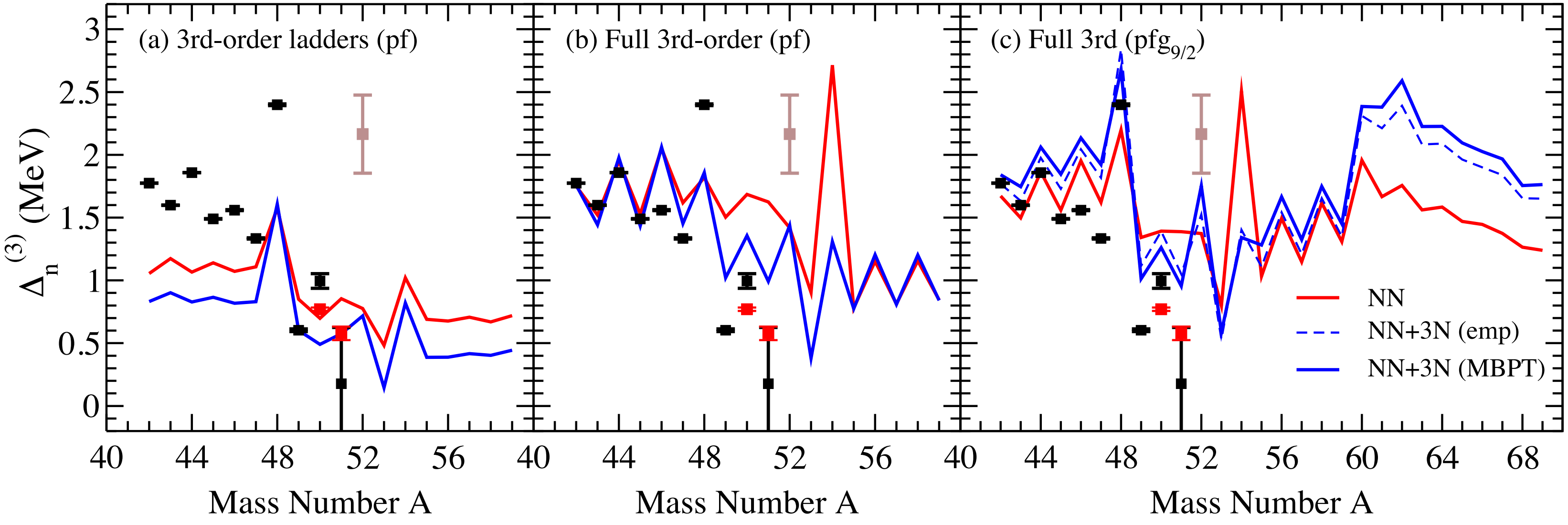
Further increases gaps

Correct odd/even staggering; more pronounced

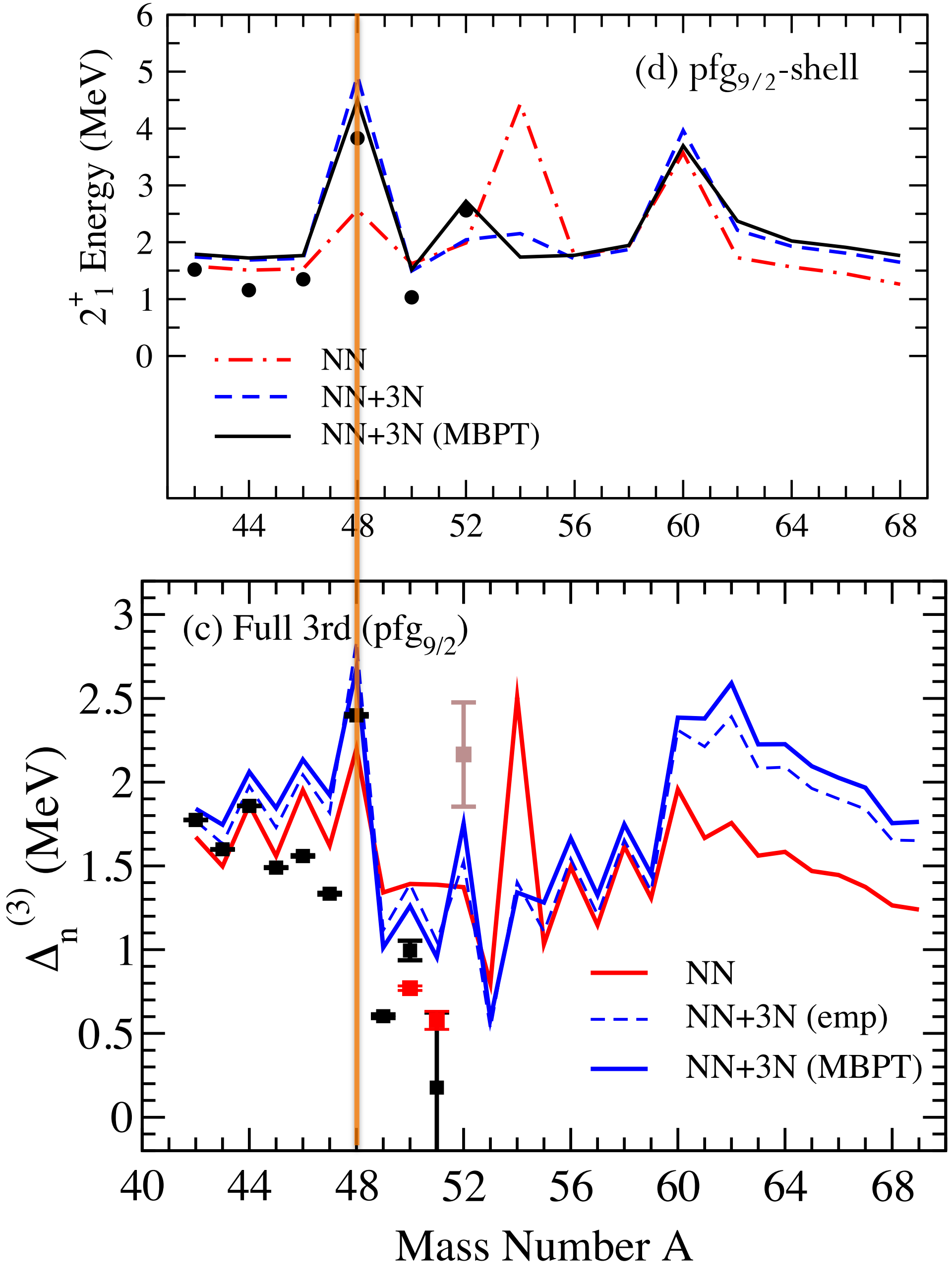
Good experimental reproduction with 3$^{rd}$-order NN+3N

Can account for missing physics in EDF calculations

JDH, Menendez, Schwenk, in prep



# Pairing for Shell Evolution N=28



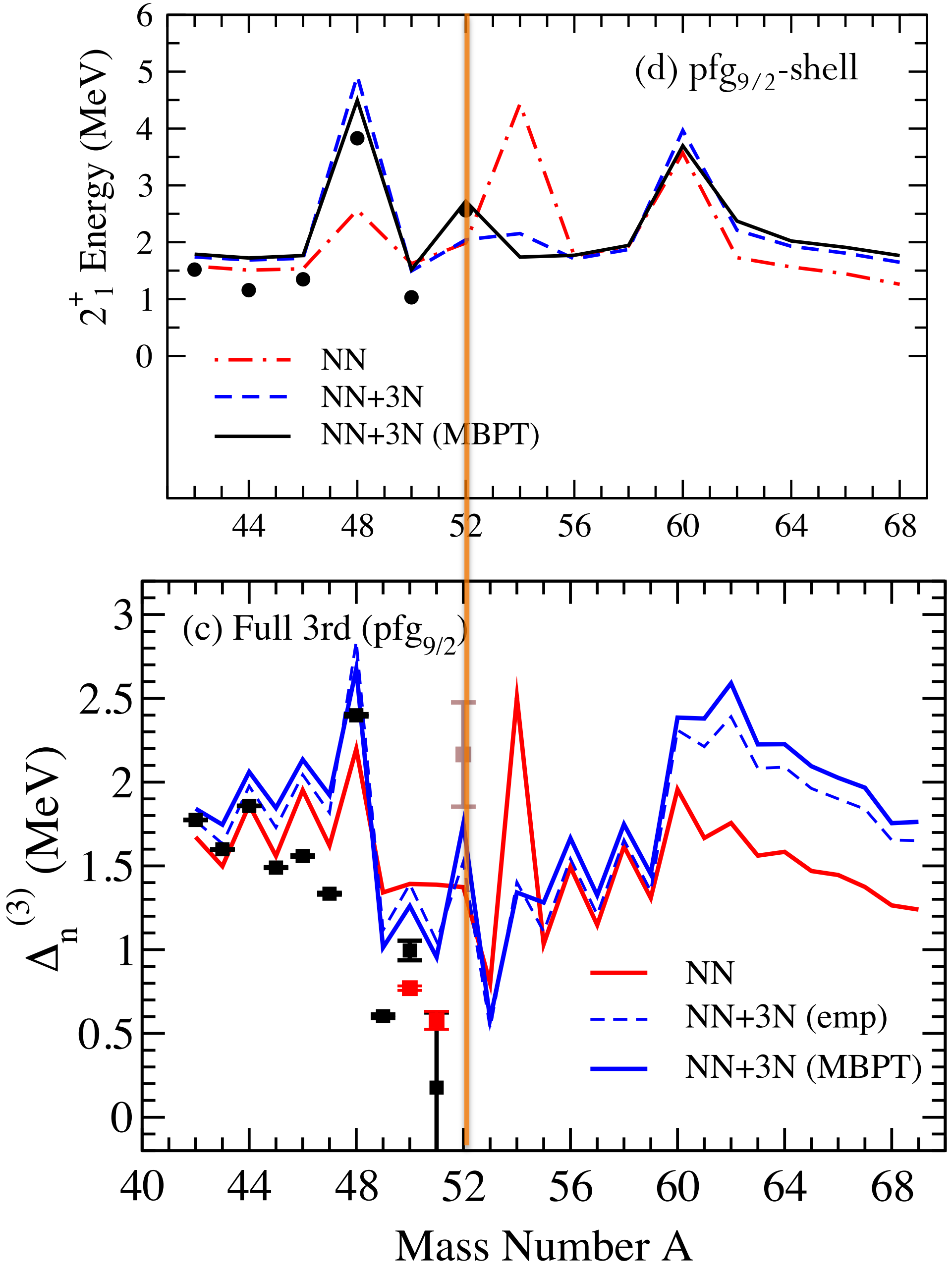
Peak in pairing gaps: complementary signature for shell closure

Compare with $2^+$ energies for Ca

General agreement with CC predictions Hagen et al PRL (2012)

**N=28**: strong peak, strength overpredicted in both cases

# Pairing for Shell Evolution N=32



Peak in pairing gaps: complementary signature for shell closure

Compare with $2^+$ energies for Ca

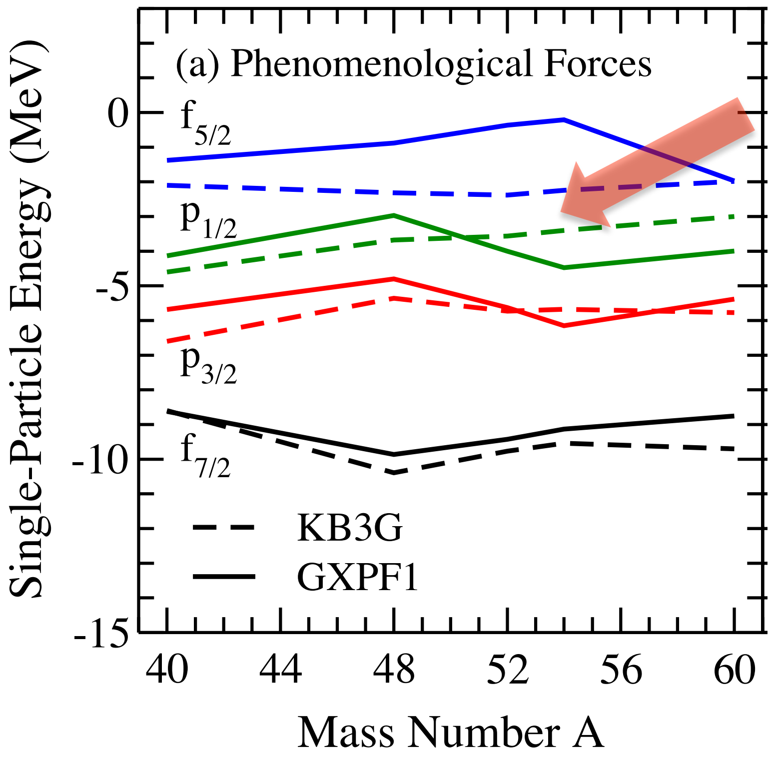
**N=32**: moderate peak

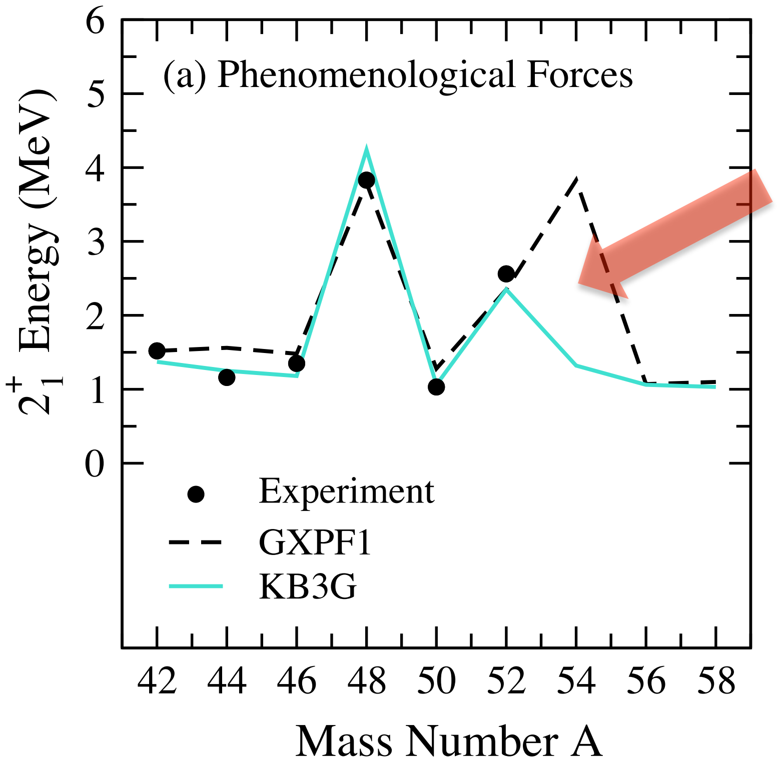
Close to experimental value with new TITAN data

Experimental measurement of $^{53}$Ca mass needed to reduce uncertainty

# Evolution of Magic Numbers: N=34

**N=34 magic number in calcium?**

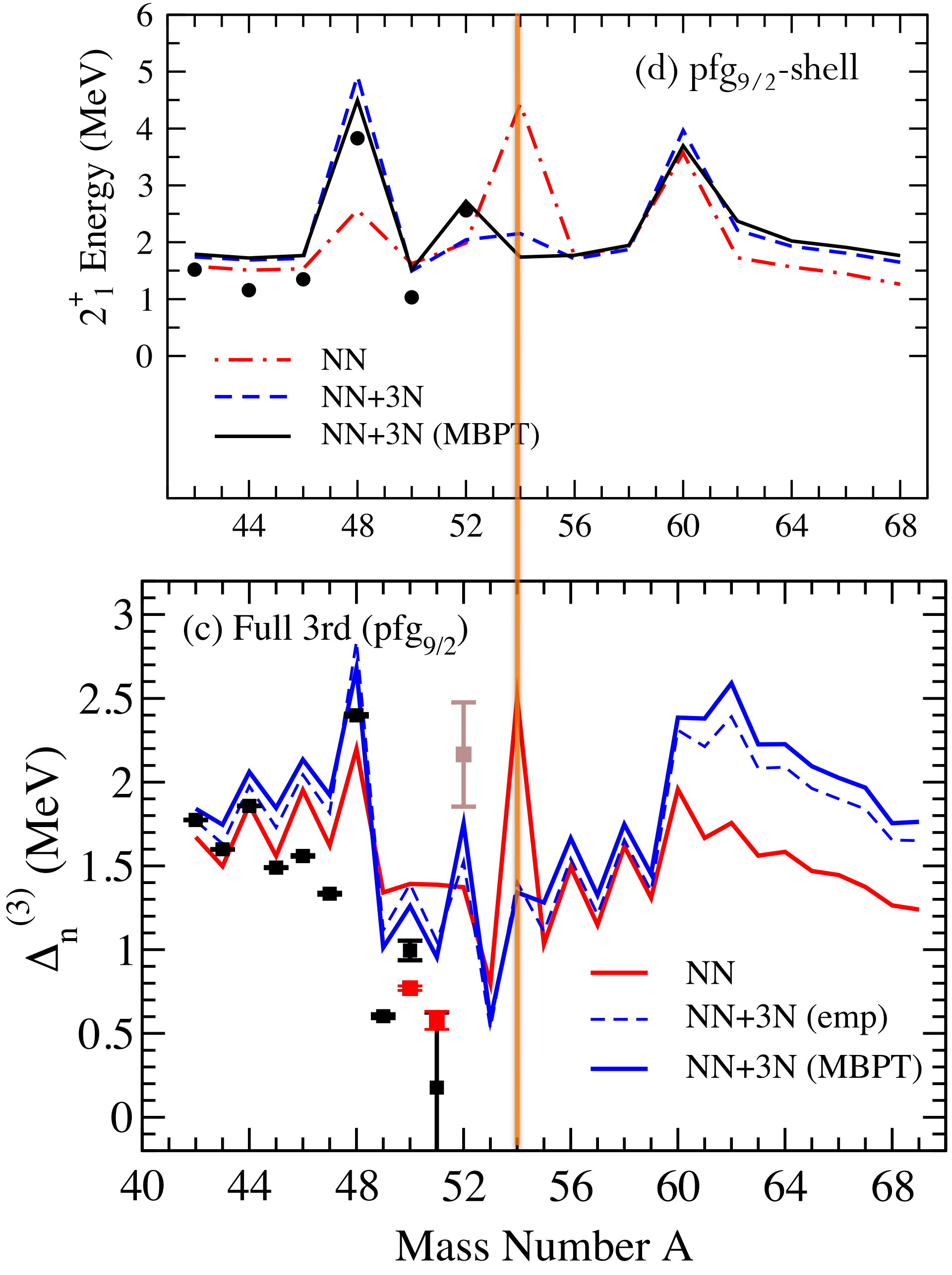




GXPF1: Honma, Otsuka, Brown, Mizusaki (2004)
KB3G: Poves, Sanchez-Solano, Caurier, Nowacki (2001)

Significant phenomenological disagreement for neutron-rich calcium

# Pairing for Shell Evolution N=34



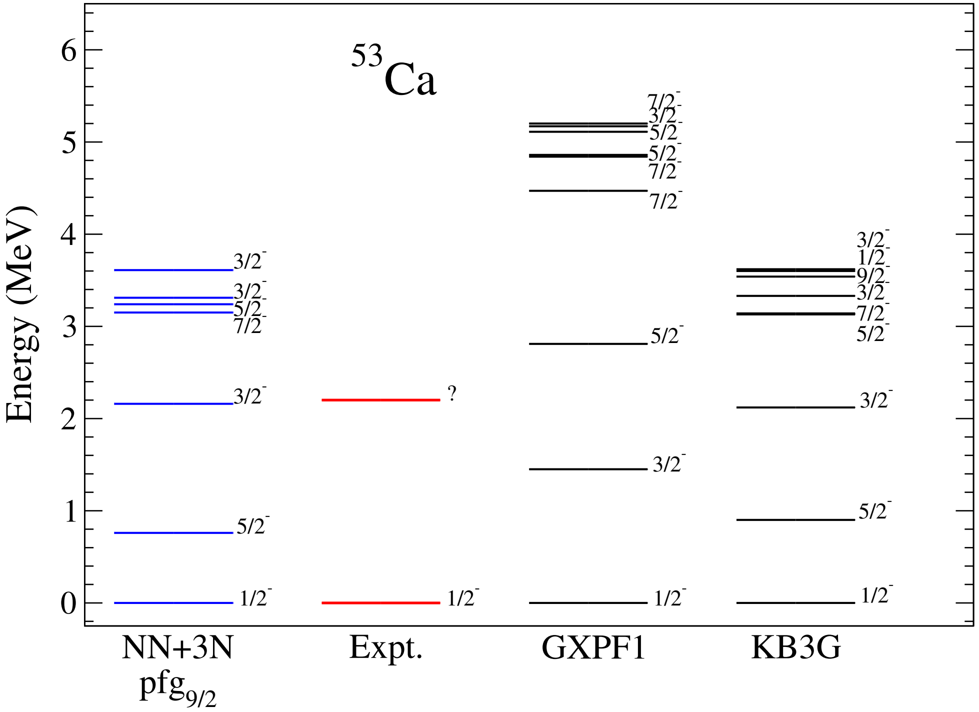
Peak in pairing gaps: complementary signature for shell closure

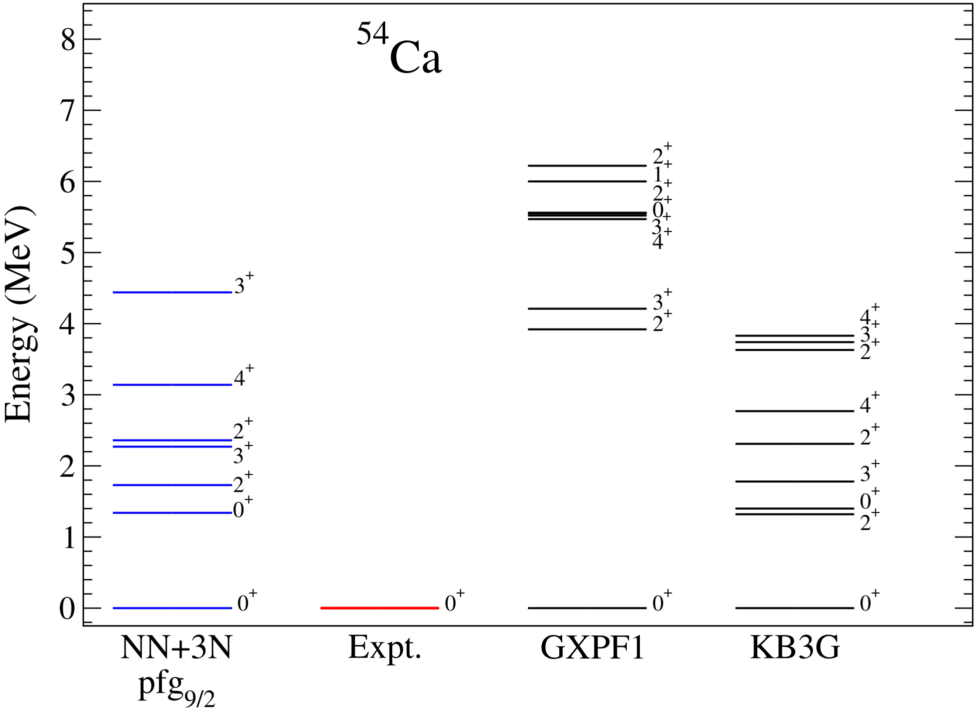
Compare with $2^+$ energies for Ca

**N=34**: weak signature – suppression from 3N forces

# Neutron-Rich Ca Spectra Near N=34

**Neutron-rich calcium** spectra with NN+3N





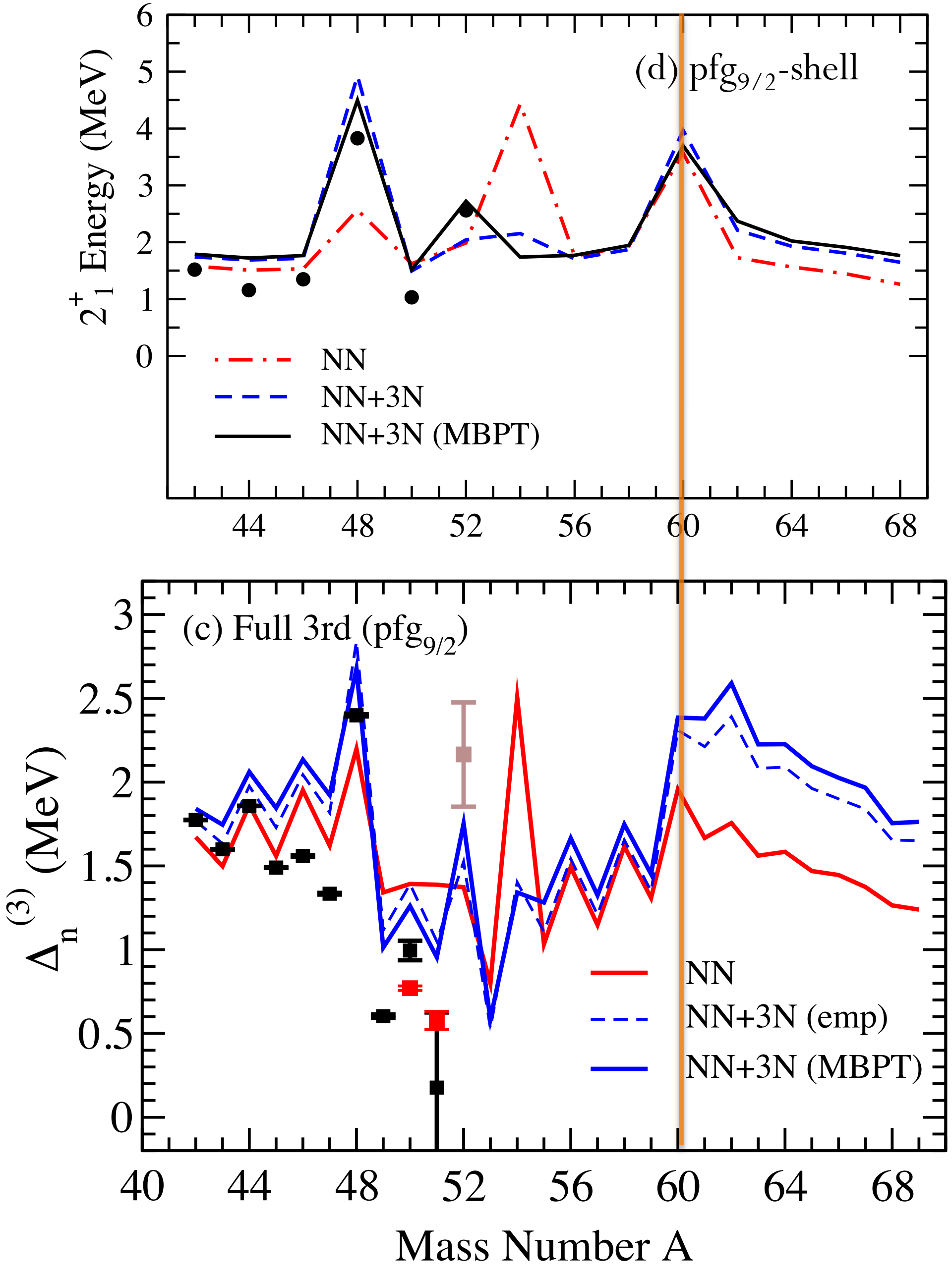
JDH, Menendez, Schwenk, in prep.

Different predictions from phenomenology

NN+3N similar to KB3G – no indication of *N=34* magic number

**Consistent with predictions from Coupled-Cluster theory**

# Pairing for Shell Evolution N=40



Peak in pairing gaps: complementary signature for shell closure

Compare with $2^+$ energies for Ca

**N=40**: robust signature of shell closure

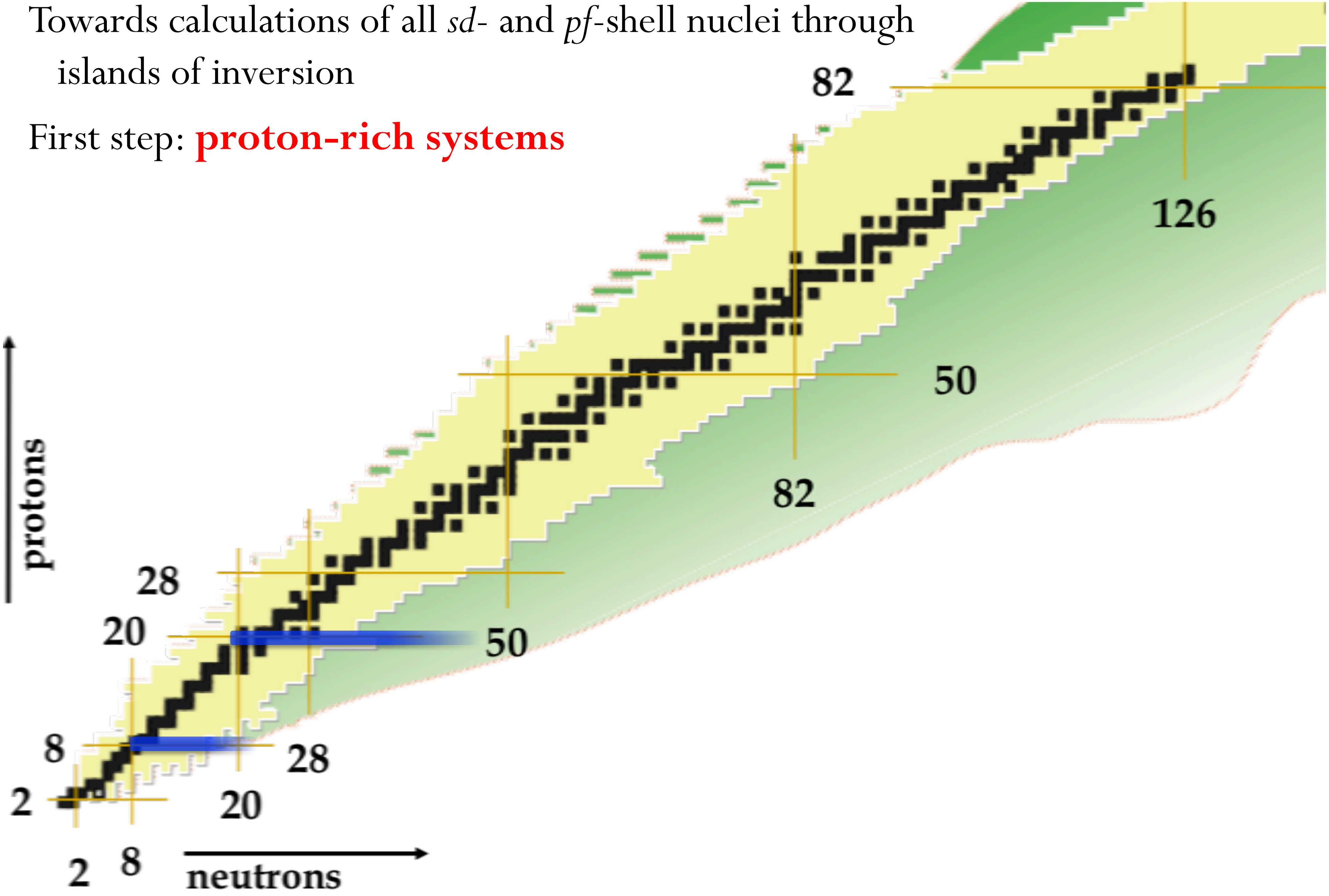
CC: continuum lowers $2s_{1/2}$, $1d_{5/2}$

Inclusion will affect N=40 prediction

# Proton-Rich Systems

Towards calculations of all *sd*- and *pf*-shell nuclei through islands of inversion

First step: **proton-rich systems**



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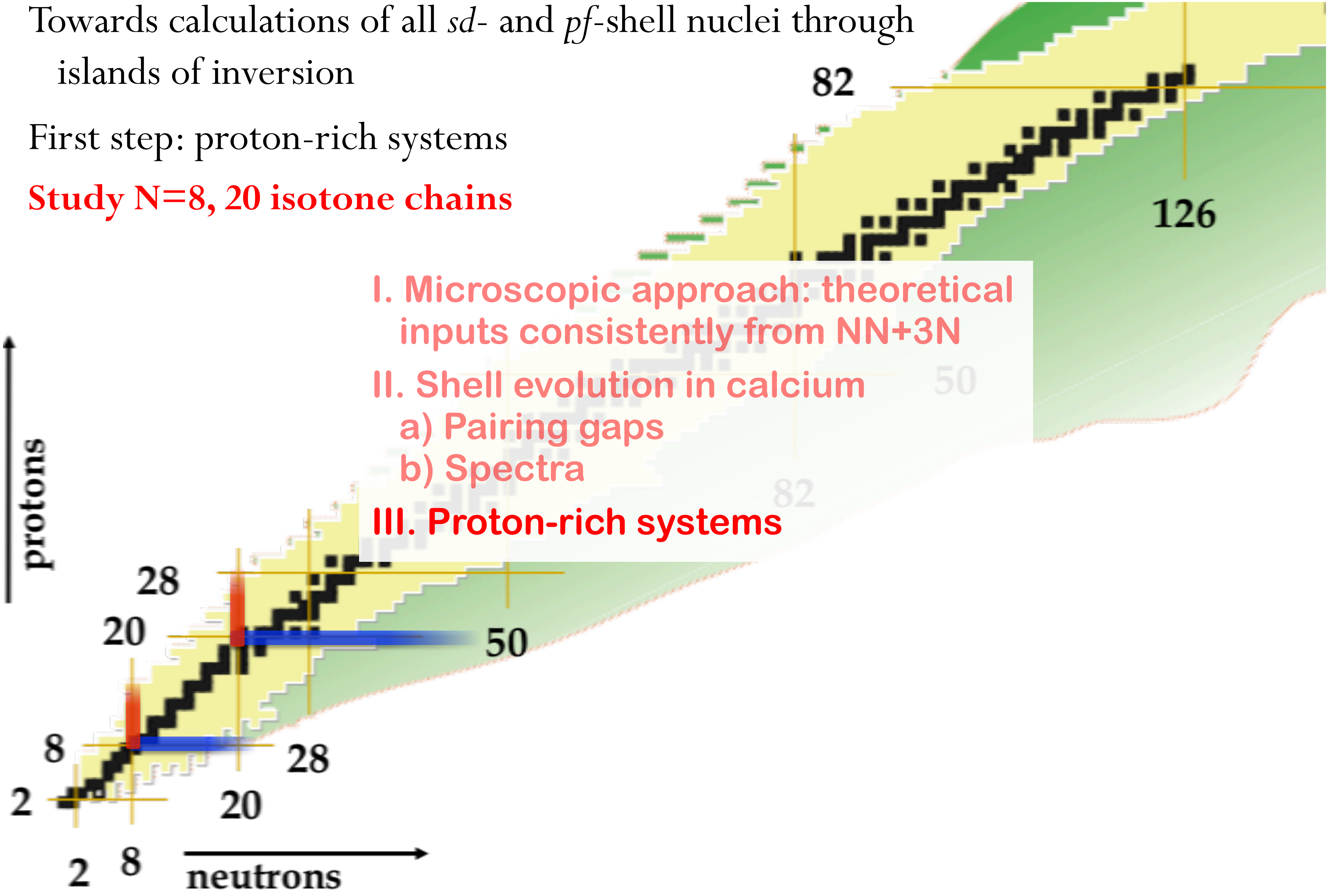
First step: proton-rich systems

**Study N=8, 20 isotone chains**



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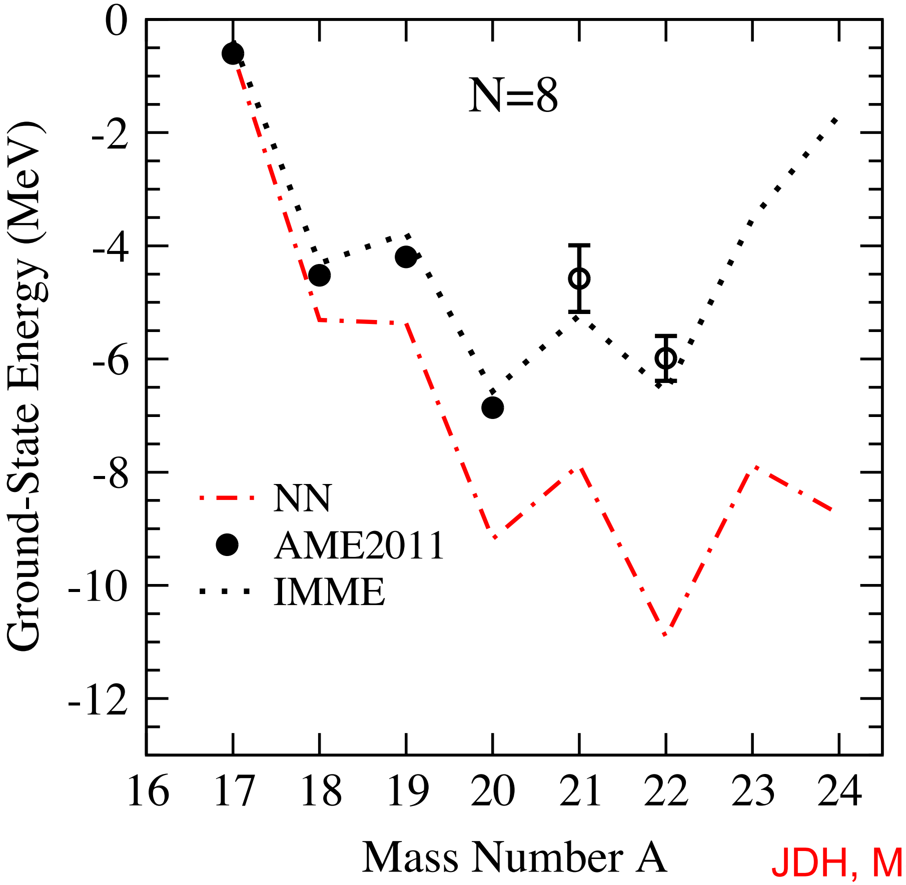

Data limited – use phenomenological isobaric multiplet mass equation (IMME)

$$E(A,T,T_z) = E(A,T,-T_z) + 2b(A,T)T_z$$

$$b = 0.7068A^{2/3} - 0.9133$$

**NN-only**: overbound

JDH, Menendez, Schwenk, PRL (2013)

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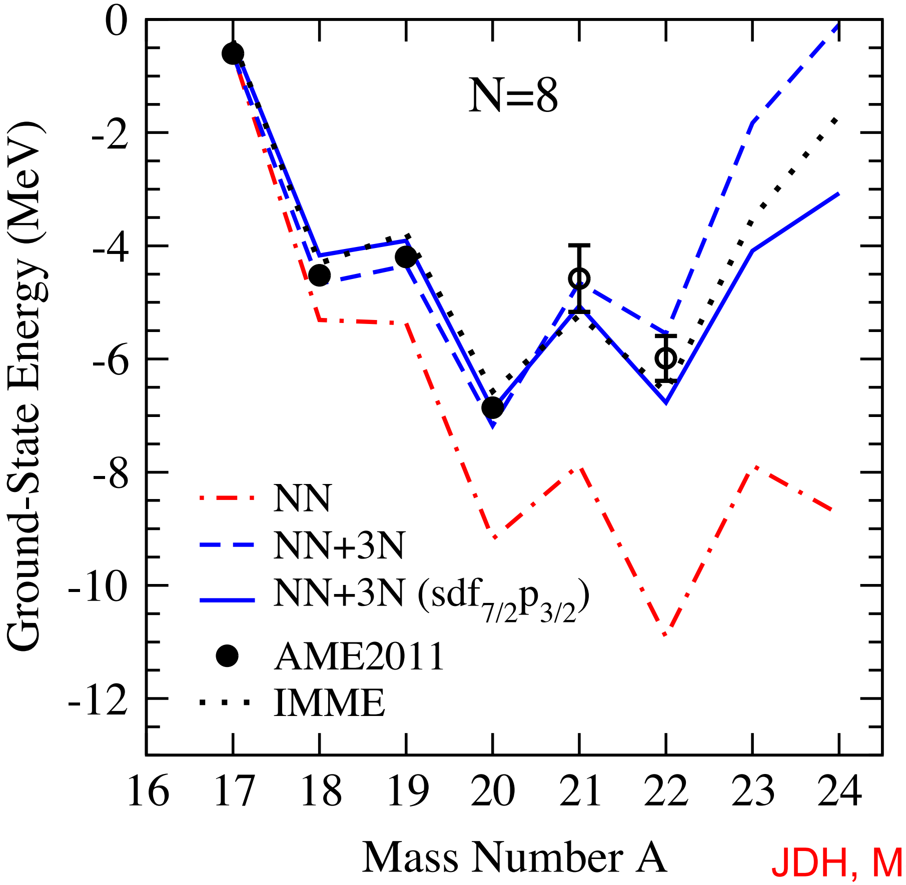

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**NN-only**: overbound

**NN+3N**: improved agreement with experiment/IMME

Extended space important ~ $A = 21$

JDH, Menendez, Schwenk, PRL (2013)

**Dripline unclear**: $^{22}$Si unbound in AME, NN+3N; bound in IMME

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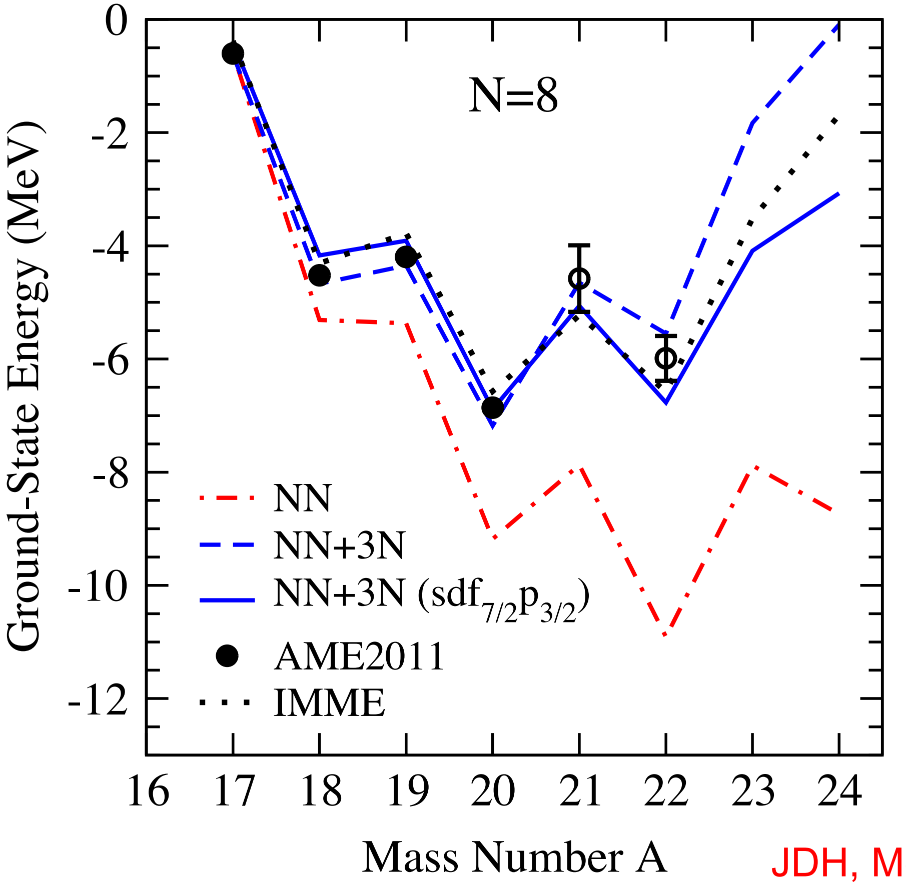

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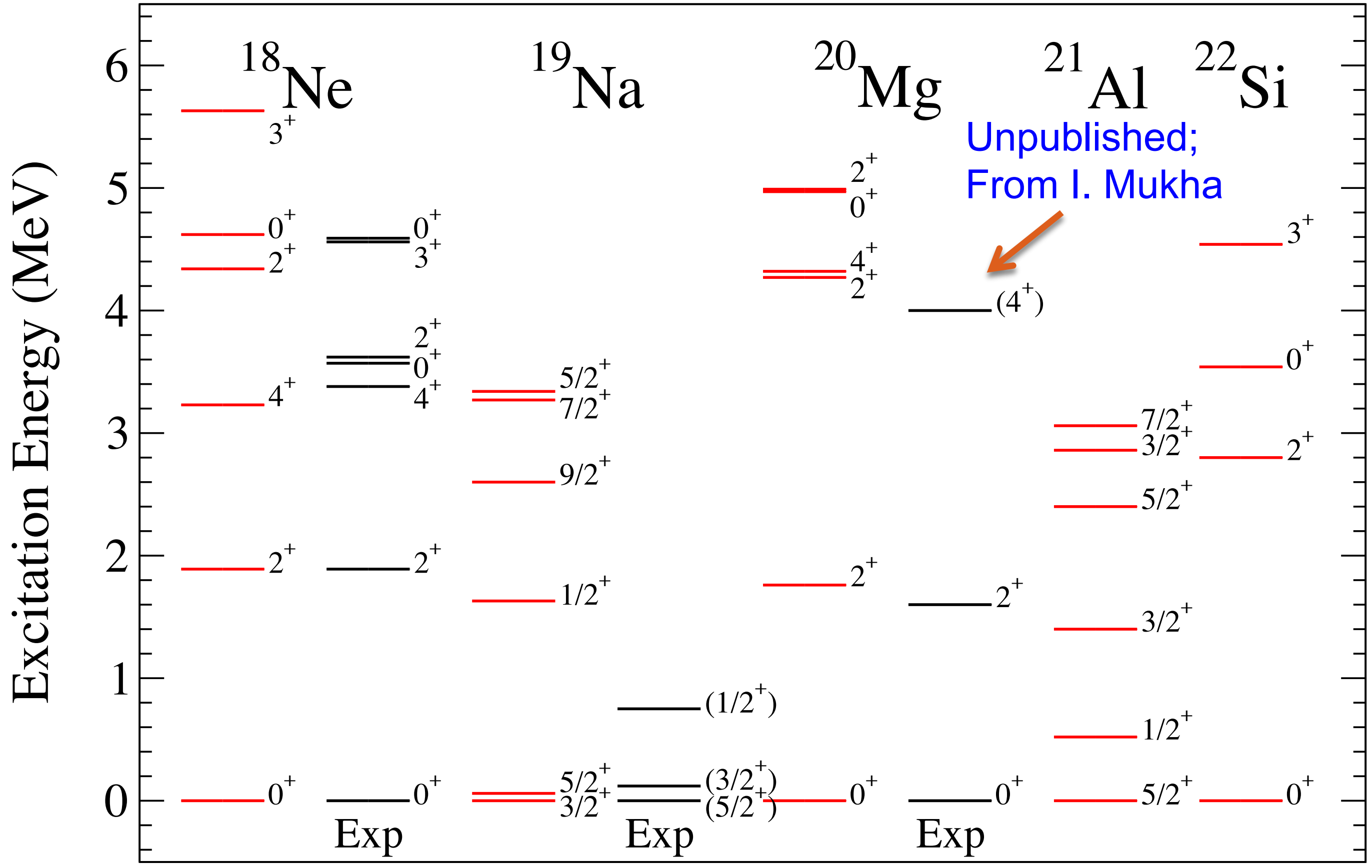
$^{22}$Si possible two-proton emitter

Measurement needed

|  | **IMME** | **NN+3N (*sd*)** | **NN+3N (*sdf*$_{7/2}$*p*$_{3/2}$)** |
|---|---|---|---|
| **$S_{2p}$** | 0.01 MeV | -1.63 MeV | -0.12 MeV |

# Spectra of N=8 Isotones



JDH, Menendez, Schwenk, PRL (2012)

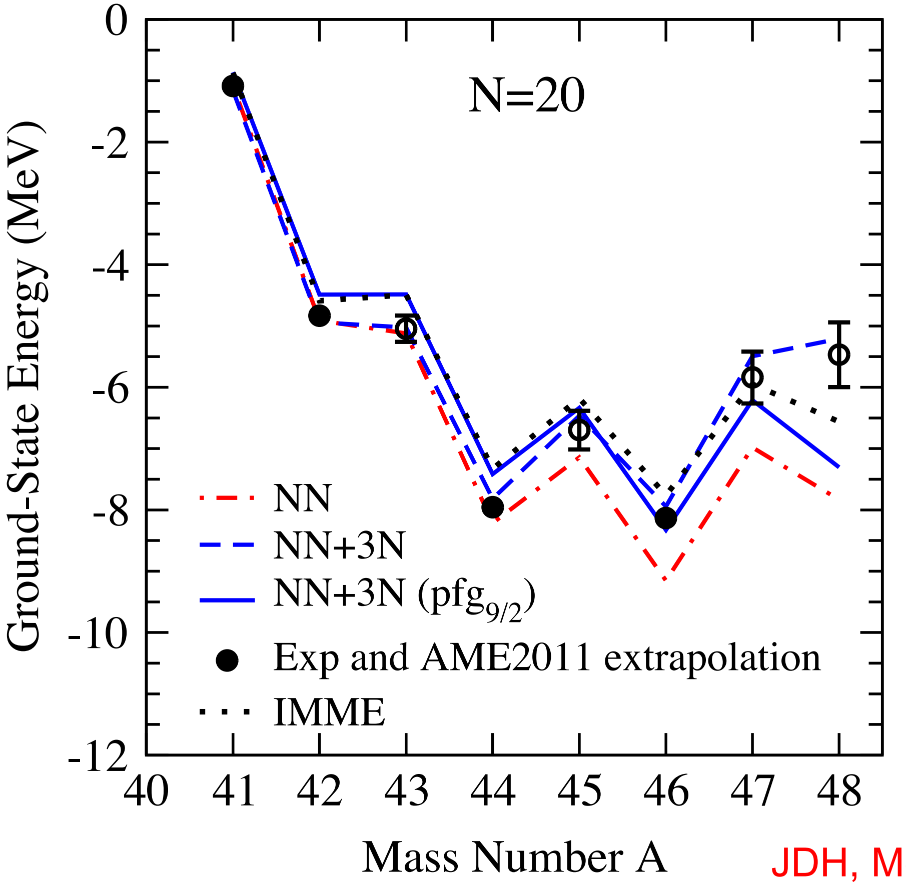
**NN+3N**: reasonable agreement with experiment

**New measurement**: excited state in $^{20}$Mg close to predicted $4^+$-$2^+$ doublet

Predictions for proton-rich $^{21}$Al, $^{22}$Si spectra

Closed sub-shell signature in $^{22}$Si

# Ground-State Energies of N=20 Isotones



**NN-only**: overbound beyond $^{45}$Mn

**NN+3N**: close to experiment/IMME

JDH, Menendez, Schwenk, PRL (2013)

**Dripline**: Predicted to be $^{46}$Fe in all calculations

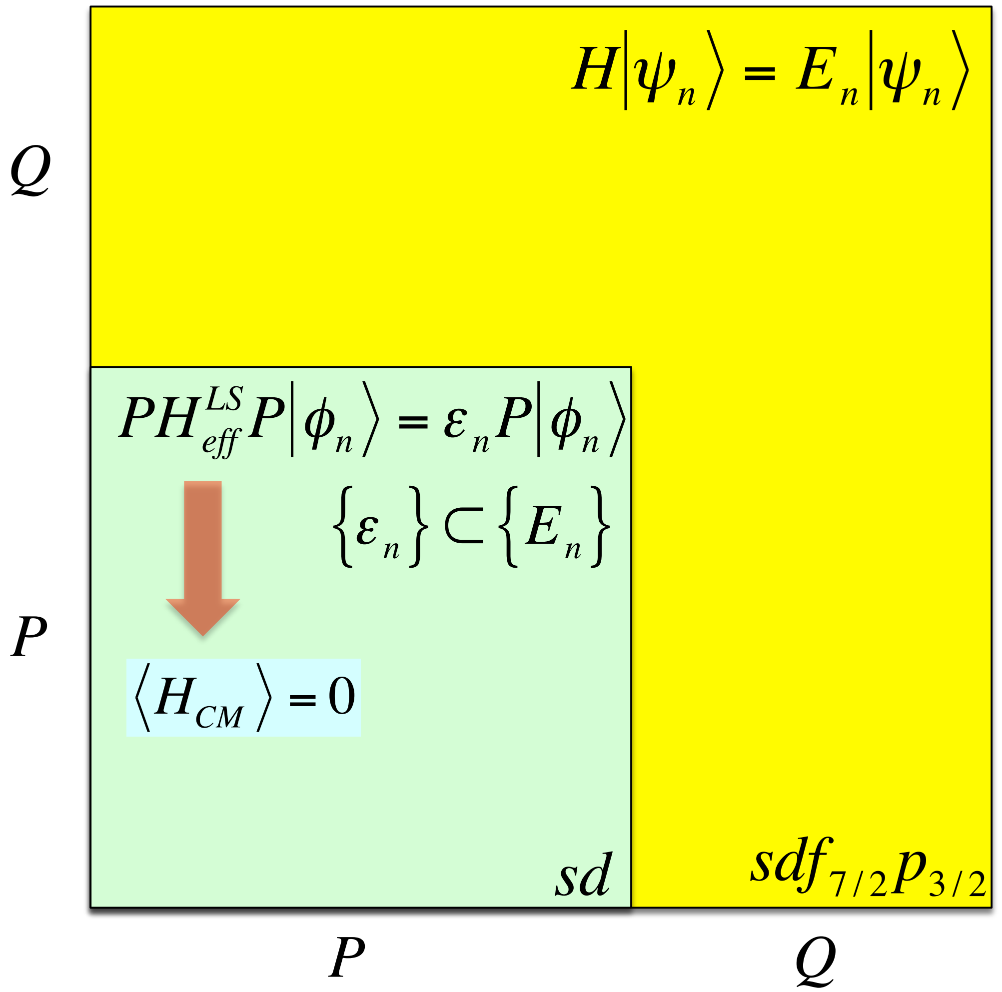
| $S_{2p}$ | **Expt.** | **NN+3N (*pf*)** | **NN+3N ($pfg_{9/2}$)** |
|---|---|---|---|
| | -1.28(6) MeV | -2.73 MeV | -1.02 MeV |

Prediction for $^{48}$Ni within 300keV of experiment

Dossat et al (2005); Pomorski et al (2012)

# Evaluating Center-of-Mass Contamination

Nonperturbative Lee-Suzuki (LS) transformation from extended space



$$H|\psi_n\rangle = E_n|\psi_n\rangle$$

$$PH_{eff}^{LS}P|\phi_n\rangle = \varepsilon_n P|\phi_n\rangle$$

$$\{\varepsilon_n\} \subset \{E_n\}$$

$$\langle H_{CM}\rangle = 0$$

Diagonalize **two-body** system (*e.g.*, $^{18}$O, $^{42}$Ca)

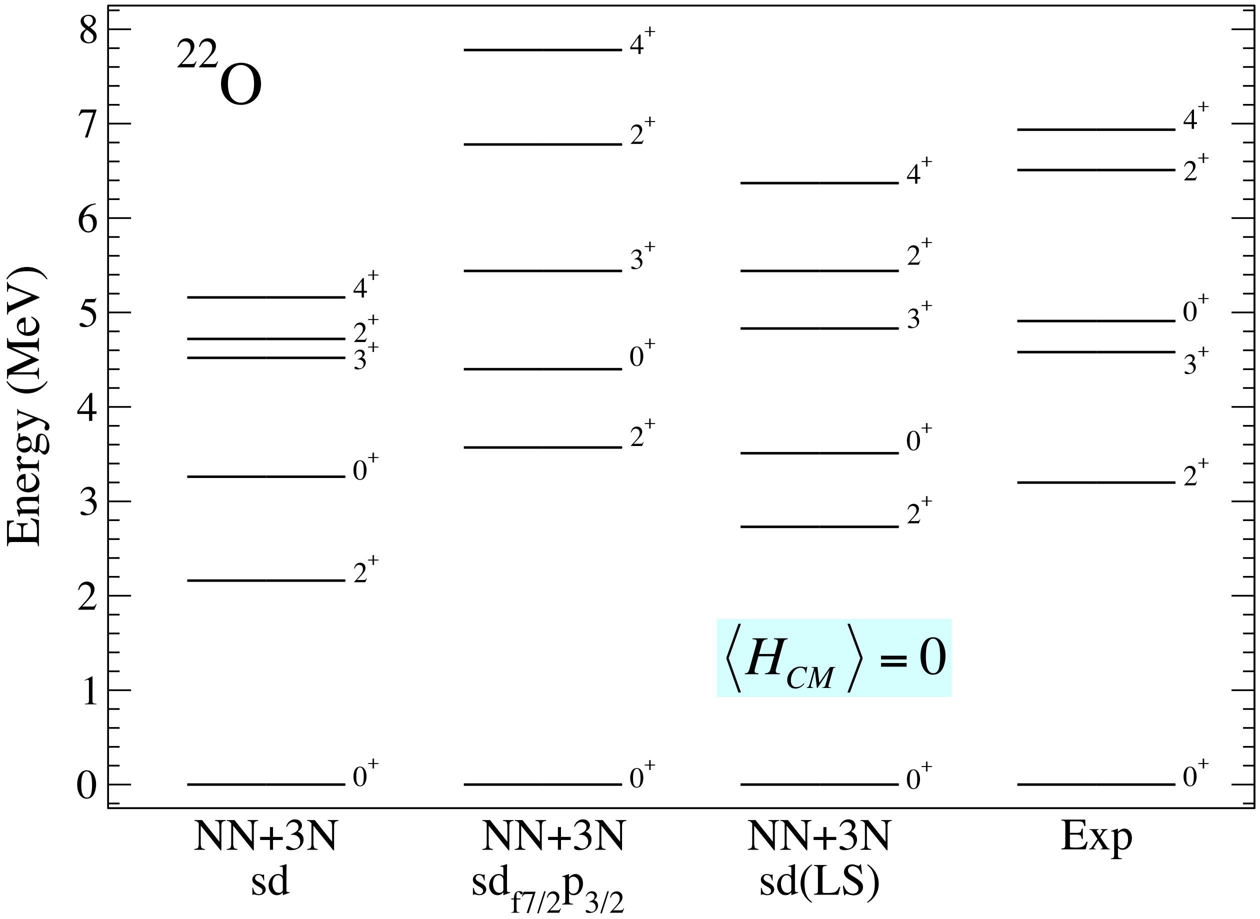
**Extended-space spectrum free of CM contamination**

Preserve eigenenergies from extended space calculation via LS

Use $H_{eff}^{LS}$ as new two-body Hamiltonian in *sd*-shell valence-space calculations

# Evaluating Center-of-Mass Contamination

Apply new $H_{eff}^{LS}$ to calculate spectra in neutron-rich oxygen



$\langle H_{CM} \rangle = 0$

Improvements from standard *sd*-shell – not due to center of mass

**Work in progress**: involving $N > 2$ neutrons in extended space

# Conclusion

- Nuclear structure theory of medium-mass nuclei with 3N forces, extended spaces
- Robust repulsive 3N mechanism for *T=1* neutron/proton-rich nuclei

- **Oxygen isotopes**
  - Cures NN-only failings: dripline, shell evolution, spectra
- **Calcium isotopes**
  - Shell evolution towards the dripline from $2^+$ energies and $\Delta_n^{(3)}$
  - Weak *N=34* closure predicted

- **Proton-rich N=8, 20 isotones**: similar improvements in g.s. energies/spectra

# Acknowledgments

## Collaborators

EMMI

TECHNISCHE UNIVERSITÄT DARMSTADT

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T. Suzuki (Nihon U.)

MICHIGAN STATE UNIVERSITY

S. Bogner

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UNEDF SciDAC Collaboration
Universal Nuclear Energy Density Functional

NICS

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