Polarizability Effects to Lamb Shifts in Muonic Atoms

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in collaboration with
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How small is the proton?

- **electron-proton**
  1. e-p scattering: $r_p = 0.875(10)$ fm
  2. eH atomic spectroscopy: $r_p = 0.8768(69)$ fm
  3. CODATA-2010: $r_p = 0.8775(51)$ fm

     Mohr *et al.*, Rev. Mod. Phys. (2012)

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- **muonic hydrogen Lamb shift (2S-2P)**
  1. μH $2S_{1/2}^{F=1}$-$2P_{3/2}^{F=2}$: $r_p = 0.84184(67)$ fm (4σ)

     Pohl *et al.*, Nature (2010)
  2. Combine μH $2S_{1/2}^{F=0}$-$2P_{3/2}^{F=1}$: $r_p = 0.84087(39)$ fm (7σ)

     Antognini *et al.*, Science (2013)

μH Lamb Shift Experiment

Lamb Shift:

2S-2P splitting in atomic spectrum

a. prompt X-ray ($t \sim 0$)

- μ^- stopped in H_2 gases
- 99% $\rightarrow$ 1S
- 1% $\rightarrow$ 2S ($\tau_{2S} \approx 1\mu s$)

b. delayed X-ray ($t \sim 1\mu s$)

- laser induced 2S$\rightarrow$2P
- measure $K_\alpha^{\text{delayed}} / K_\alpha^{\text{prompt}}$
- $f_{res} = \Delta E_{LS}$

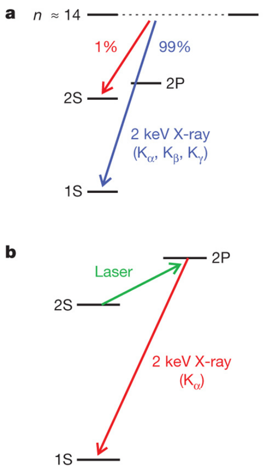


Figure from Pohl *et al.* Nature (2010)

μH Lamb Shift Experiment

r_p from μH experiment disagrees with eH (ep) by 7σ!

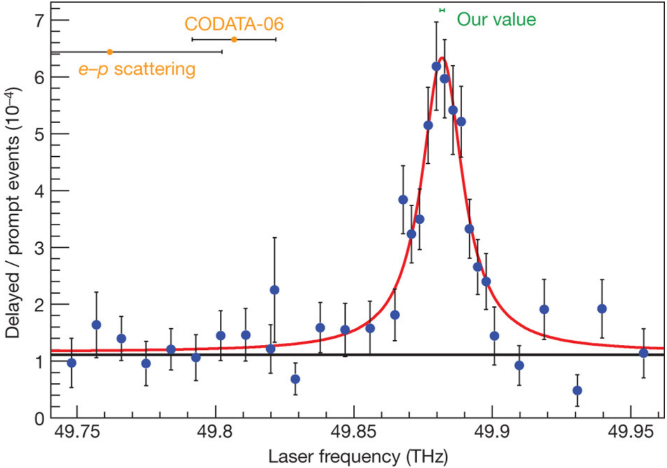
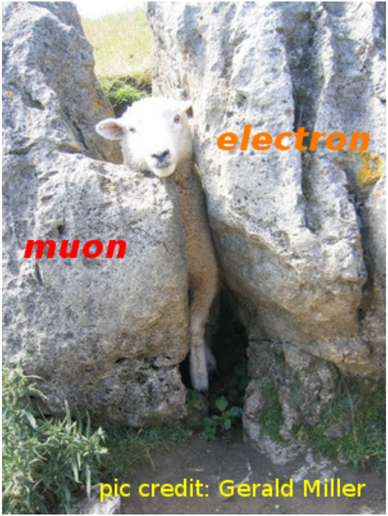


Figure from Pohl *et al.* Nature (2010)

Origins of the Discrepancy?

- **study r_p's discrepancies between μp and ep experiments**
  - systematic errors in ep scattering
  - new physics that distinguishes μp and ep interactions
  - high-precision experiments need high-accuracy theoretical results
- **new muonic atom experiment**
  - Lamb shift in μD
    CREMA collaboration, ongoing
  - Lamb shift in muonic helium
    CREMA collaboration, planned in 2013



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- **new muonic atom experiment**
  - Lamb shift in μD
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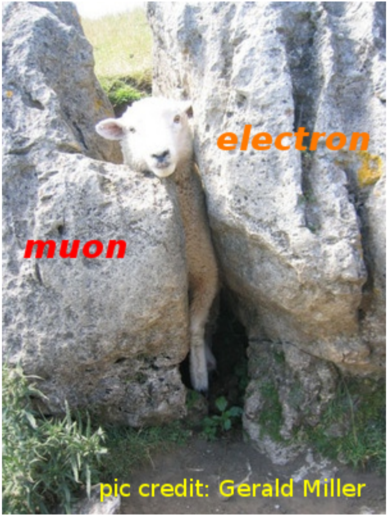
- **Aim:**
  provide ***Ab-initio*** calculations of **nuclear-polarizability** shifts in μHe



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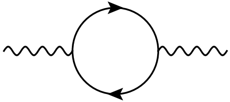
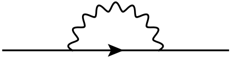
Lamb Shift in Theory

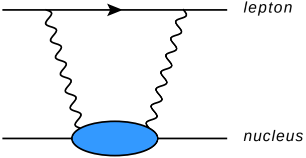
- **QED corrections:**
  - vacuum polarization
  - lepton self energy
  - recoil effects





- **nuclear corrections:**

  1. finite-size corrections (elastic):
     - leading term $\sim (Z\alpha)^4$:
       $-\frac{2m_r^3}{3n^3}(Z\alpha)^4\langle r^2\rangle$
     - Zemach moment $\sim (Z\alpha)^5$:
       $\frac{m_r^4}{3n^3}(Z\alpha)^5\langle r^3\rangle_{(2)}$
  2. nuclear polarizability (inelastic):
     - dominant contribution $\sim (Z\alpha)^5$



$$E_{LS} = E_{QED} + \boldsymbol{E_{pol}} - \frac{2m_r^3}{3n^3}(Z\alpha)^4\langle r^2\rangle + \frac{m_r^4}{3n^3}(Z\alpha)^5\langle r^3\rangle_{(2)}$$

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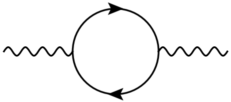

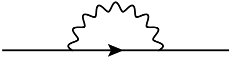

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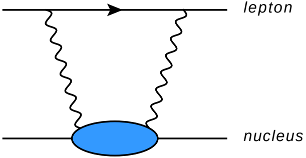

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Calculate Nuclear Polarizability

- In $A > 1$ muonic atoms
  $< 5\%$ accuracy is needed to shed light on the radius puzzle
- nuclear response functions are needed for calculating nuclear polarizabilities

- **polarizability in Deuterium atom**
  - response function from realistic NN potentials
    - eD (AV14) Leidemann & Rosenfelder, PRC 1995
    - μD (AV18) Pachucki, PRL 2011
- **polarizability in ^{4}He atom**
  - response functions from photoabsorption cross sections
    - e-^{4}He: Pachucki & Moro, PRA 2007
    - μ-^{4}He: Friar, PRC 1977 $\boldsymbol{E_{pol} = -3.1 meV \pm 20\%}$

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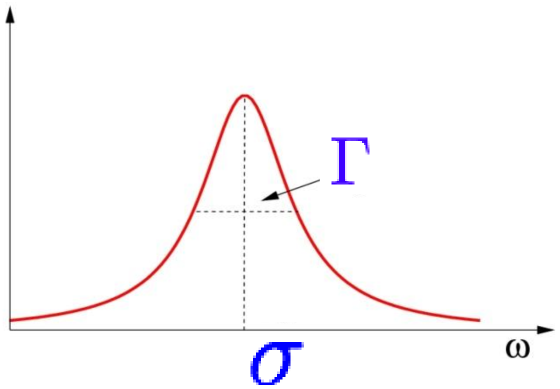
  - **no systematic calculations with response functions from *ab-initio* methods have been done in muonic atoms!**

Ab-initio Response Functions

Response in the continuum ⟶ Use the Lorentz Integral Transform method

Efros *et al.*, J. Phys. G: Nucl. Part. Phys. 34 (2007)

$$R(\omega) = \sum\!\!\!\!\!\!\int_f \left| \left\langle \psi_f \middle| \hat{O} \middle| \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$$



$$L(\sigma, \Gamma) = \int d\omega \frac{R(\omega)}{(\omega - \sigma)^2 + \Gamma^2} = \langle \tilde{\psi} | \tilde{\psi} \rangle$$

$$(H - E_0 - \sigma + i\Gamma) \mid \tilde{\psi} \rangle = \hat{O} \mid \psi_0 \rangle$$

- Due to imaginary part Γ the solution $|\tilde{\psi}\rangle$ is unique
- Since the r.h.s. is finite, then $|\tilde{\psi}\rangle$ has bound state asymptotic behavior

TRIUMF *Ab-initio* Response Functions

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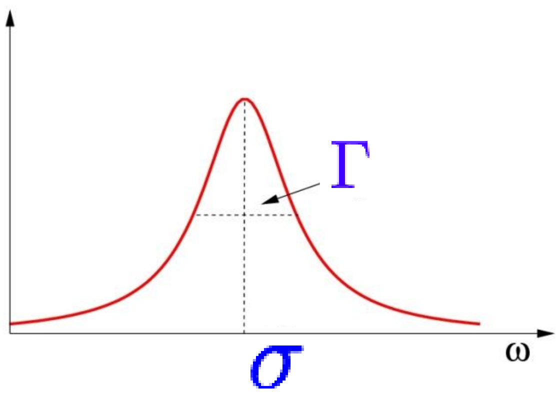

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$L(\sigma, \Gamma) \xleftrightarrow{\text{inversion}} R(\omega)$ with the exact final state interaction

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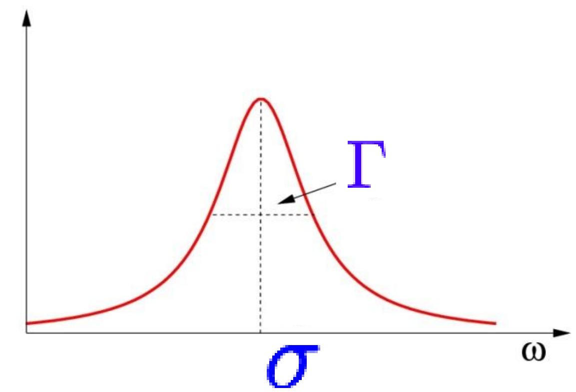

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$L(\sigma,\Gamma)$ ⟵ inversion ⟶ $R(\omega)$ with the exact final state interaction

You can use any good bound state method! We use Hyperspherical Harmonics expansions

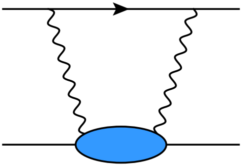
Nuclear Polarizability: Basic Idea

- Hamiltonian for muonic atoms

$$H = H_N + T_\mu - \sum_i^Z \frac{\alpha}{|\boldsymbol{r} - \boldsymbol{R}_i|}$$

- Corrections to the point Coulomb

$$\Delta V_{N\mu} = \alpha \sum_i^Z \left(\frac{1}{r} - \frac{1}{|\boldsymbol{r} - \boldsymbol{R}_i|} \right)$$



- Evaluate $\Delta V_{N\mu}$'s inelastic effects to the muonic atom spectrum by 2nd-order perturbation theory
  - **results are expanded in powers of $Z\alpha$ and M_μ/M_N**
    1. non-relativistic approximation (multipole expansion)
    2. relativistic dipole corrections
    3. Coulomb dipole corrections

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Non-Relativistic Approximation

- Coulomb interactions in the (virtual) intermediate state are neglected
- Results are expressed in an multipole expansion

$\Delta E_{NR} = \Delta E_{NR}^{(2)} + \Delta E_{NR}^{(3)} + \Delta E_{NR}^{(4)}$

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$$\Delta E_{NR} = \Delta E_{NR}^{(2)} + \Delta E_{NR}^{(3)} + \Delta E_{NR}^{(4)}$$

1. $\sim R^2$ **term:**

- $\Delta E_{NR}^{(2)}$ **is the dominant polarizability contribution**

$$\Delta E_{NR}^{(2)} = -\frac{16\pi m_r^3}{9n^3}(Z\alpha)^5 \int_{\omega_{\text{th}}}^{\infty} d\omega \sqrt{\frac{2m_r}{\omega}} S^{D_1}(\omega)$$

- $S^{D_1}(\omega) = \frac{1}{2J_0+1} \sum_{N \neq N_0, J} |\langle N_0 J_0 || \hat{D}_1 || N J \rangle|^2 \delta(\omega - E_N + E_{N_0})$
- $\hat{D}_1 = \frac{1}{Z} \sum_i^Z R_i Y_1(\hat{R}_i)$

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$$\Delta E_{NR} = \Delta E_{NR}^{(2)} + \Delta E_{NR}^{(3)} + \Delta E_{NR}^{(4)}$$

2. $\sim R^3$ **term:**

$$\Delta E_{NR}^{(3)} = -\frac{m_r^4}{3n^3}(Z\alpha)^5 \langle r^3 \rangle_{(2)}$$

- $\langle r^3 \rangle_{(2)} = \iint d^3R d^3R' |\boldsymbol{R} - \boldsymbol{R}'|^3 \rho(\boldsymbol{R}) \rho(\boldsymbol{R}')$
- $\Delta E_{NR}^{(3)}$ cancels exactly the **Zemach term** in (elastic) finite-size corrections *c.f.* Pachucki PRL 2011 (μD)
- such cancellation applies for all muonic atoms with $A > 1$

3. $\sim R^4$ **term:**

- $\Delta E_{NR}^{(4)}$ **corresponds to higher-multipole corrections**

$$\Delta E_{NR}^{(4)} = \frac{2m_r^5}{15n^3}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} \sqrt{\frac{\omega}{2m_r}} \left[\frac{10}{3} S^{R^2}(\omega) + \frac{32\pi}{15} S^{Q_2}(\omega) + \frac{32\pi}{3} S^{D_{13}}(\omega)\right]$$

- $S^{R^2}(\omega) = \frac{1}{2J_0+1} \sum_{N\neq N_0,J} |\langle N_0 J_0||\hat{R}^2||NJ\rangle|^2 \delta(\omega - E_N + E_{N_0})$

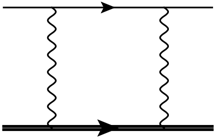
  $S^{Q_2}(\omega) = \frac{1}{2J_0+1} \sum_{N\neq N_0,J} |\langle N_0 J_0||\hat{Q}_2||NJ\rangle|^2 \delta(\omega - E_N + E_{N_0})$

  $$S^{D_{13}}(\omega) = \frac{1}{2J_0+1} \sum_{N\neq N_0,J} (-1)^{J_0-J+1} \times \langle N_0 J_0||\hat{D}_3||NJ\rangle \langle NJ||\hat{D}_1||N_0 J_0\rangle\, \delta(\omega - E_N + E_{N_0})$$

- $\hat{R}^2 = \frac{1}{Z}\sum_i^Z R_i^2$ $\hat{D}_1 = \frac{1}{Z}\sum_i^Z R_i Y_1(\hat{R}_i)$

  $\hat{Q}_2 = \frac{1}{Z}\sum_i^Z R_i^2 Y_2(\hat{R}_i)$ $\hat{D}_3 = \frac{1}{Z}\sum_i^Z R^3 Y_1(\hat{R}_i)$

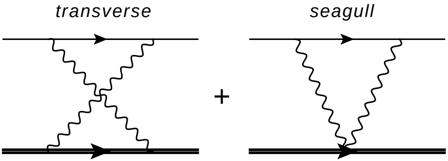
- **Longitudinal contributions**
  - exchange Coulomb photon
  - intermediate-state propagator changes to relativistic one



$$\Delta E_R^{(l)} = \frac{8\pi m_r^3}{9n^3}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega\, S^{D_1}(\omega)\; K^{(l)}\left(\frac{\omega}{m_r}\right)$$

- $K^{(l)} \approx \sqrt{\frac{\omega}{2m_r}} - \frac{4}{3\pi}\frac{\omega}{m_r} + \cdots$

- **Transverse contributions**

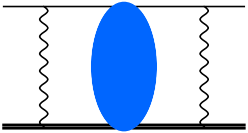


  - convection current & spin current
  - seagull term cancels infrared divergence

$$\Delta E_R^{(t)} = \frac{16\pi m_r^3}{9n^3}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega\, S^{D_1}(\omega)\; K^{(t)}\left(\frac{\omega}{m_r}\right)$$

  - $K^{(t)} \approx \frac{\omega}{m_r} + \frac{\omega}{m_r}\ln\frac{4\omega}{m_r} + \cdots$

- **Nonperturbative Coulomb interactions in the virtual intermediate state**
  - Coulomb effects contribute to both 2S and 2P atomic states
  - depends on hyperfine state F
  - in an expansion of $Z\alpha\sqrt{\frac{m_r}{\omega}}$
    - *a.* μD: $Z\alpha\sqrt{\frac{m_r}{\omega_{th}}} \sim 0.05$
    - *b.* μ^4He: $Z\alpha\sqrt{\frac{m_r}{\omega_{th}}} \sim 0.03$



- **for μ^4He:**

$$\Delta_{C1}^{2S-2P} = -\frac{16\pi m_r^3}{9n^3}(Z\alpha)^6 \int_{\omega_{\rm th}}^{\infty} d\omega\, S^{D_1}(\omega)\frac{m_r}{\omega}\left(\frac{1}{6} + \ln\frac{2Z^2\alpha^2 m_r}{\omega}\right)$$

Friar, PRC 1977

Nuclear Polarizability in μ-D

- **Test Run:**
  compare with existing calculations on μ-D's polarizability

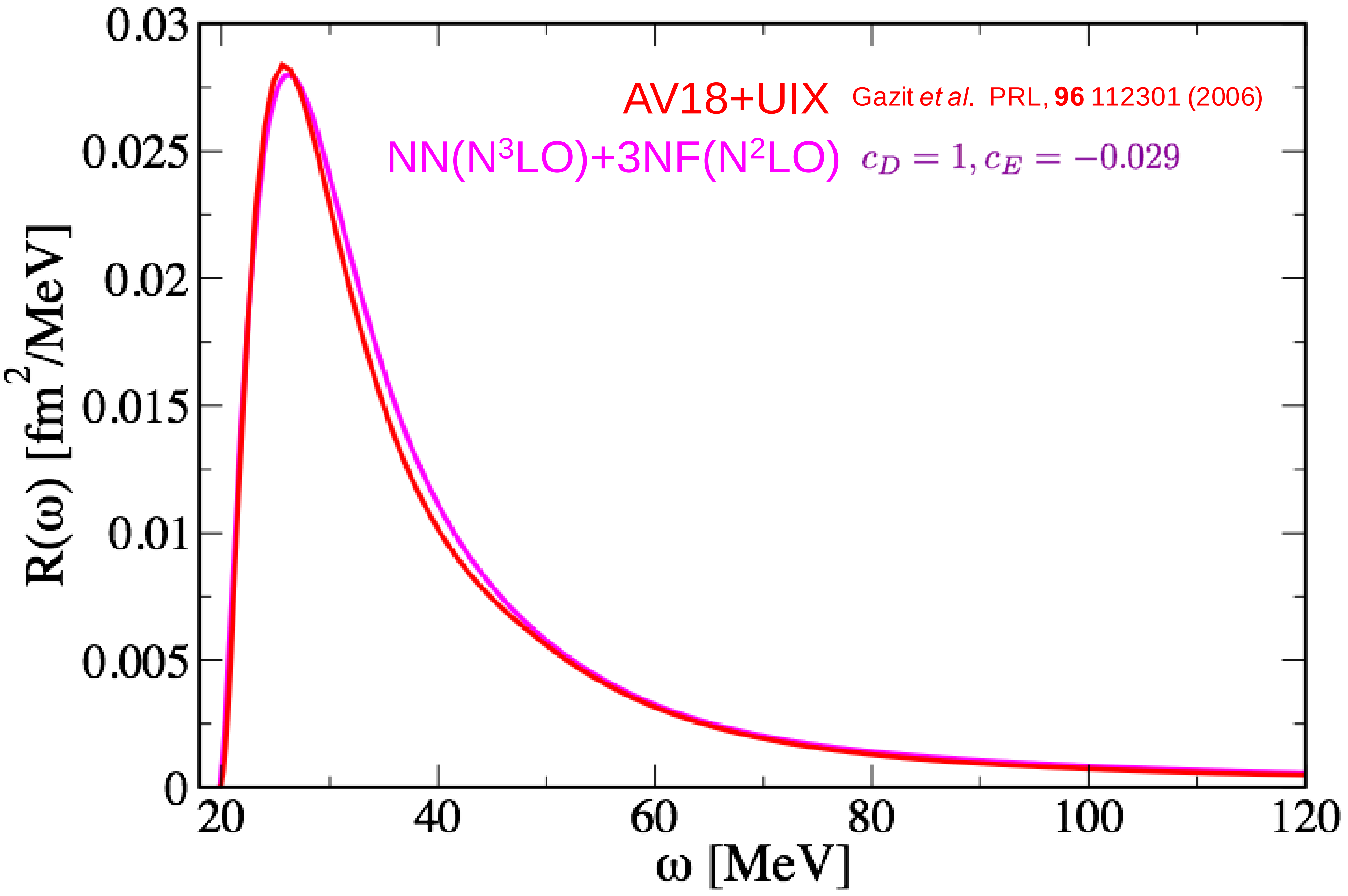
  (1) μ-D's polarizability (AV18): Pachucki, PRL 2011

  (2) we calculate terms containing D dipole-response functions (AV18):
  Leidemann, *private comm*

μ-D		*(1)*	*(2)*
non-relativistic dipole	$\delta_0 E$ [meV]	-1.910	-1.907
relativistic	$\delta_{L1} E$ [meV]	0.035	0.036
	$\delta_L^{\mathrm{full}} E$ [meV]		0.029
	$\delta_T E$ [meV]		-0.012
Coulomb	$\delta_{c1} E$ [meV]	0.255	0.255
	$\delta_{c2} E$ [meV]	0.006	0.004
higher multipoles	$\delta_Q E$ [meV]	0.040	
magnetic dipole	$\delta_M E$ [meV]	-0.016	
proton polarizability	$\delta_p E$ [meV]	0.043	

Nuclear Polarizability in μ-^{4}He

Example: ^{4}He Dipole Response Function



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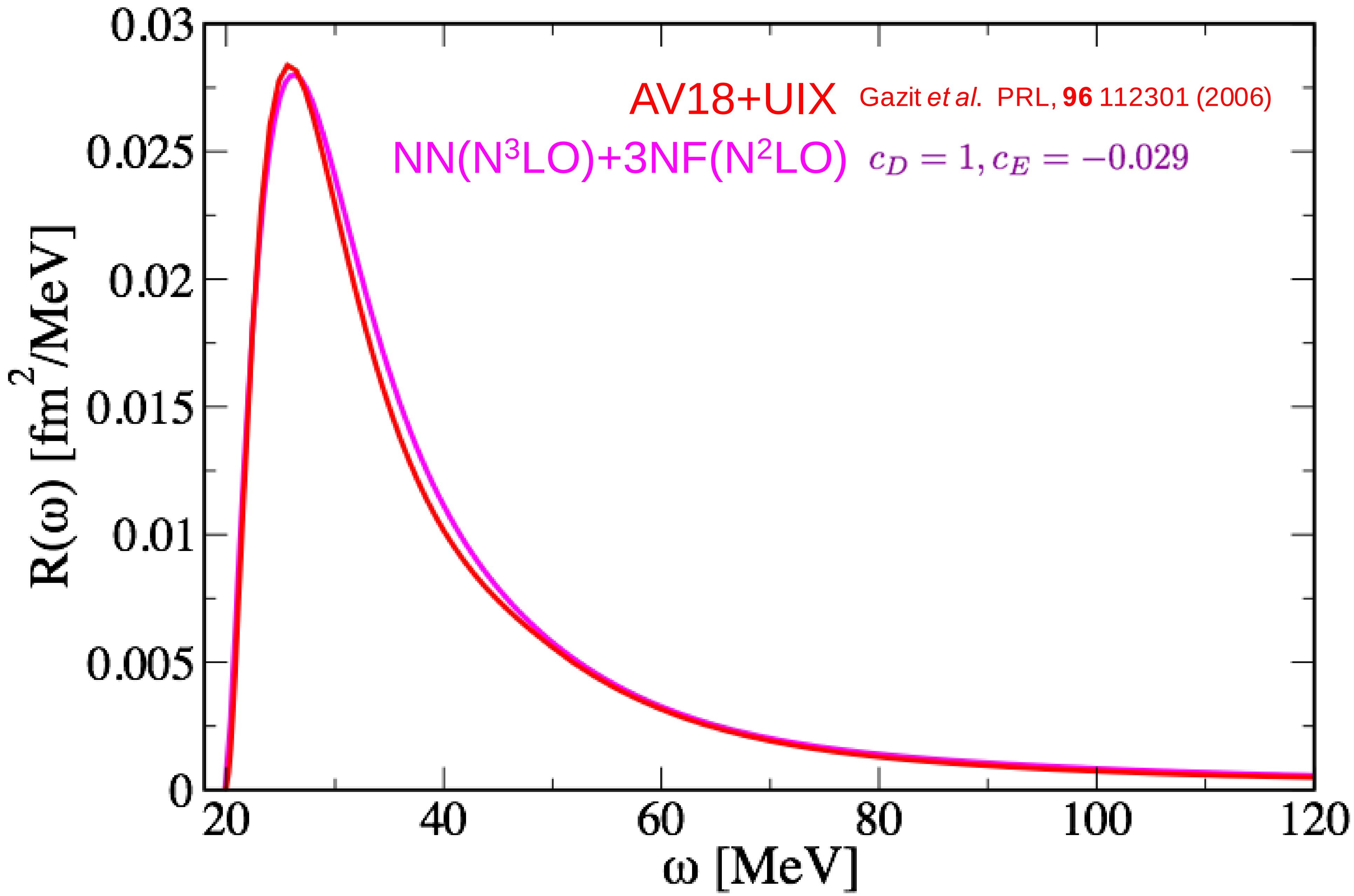


Mild dependence on the three-nucleon Hamiltonian.
Use this difference as a way to assess the error coming from the “unknown” nuclear force

Nuclear Polarizability in μ-^{4}He

- **μ-^{4}He polarizability from dipole response functions**
  - *(1)* response functions from AV18/UIX
  - *(2)* response functions from χEFT [NN(N^{3}LO) & $3N$(N^{2}LO)]

μ-^{4}He		AV18/UIX	χEFT
non-relativistic dipole	$\delta_0 E$ [meV]	-4.414	-4.652
relativistic	$\delta_L E$ [meV]	0.272	0.295
	$\delta_T E$ [meV]	-0.058	-0.062
Coulomb	$\delta_{c1} E$ [meV]	0.513	0.537
	$\delta_{c2} E$ [meV]	0.006	0.007
total	[meV]	-3.681	-3.875(1)

PRELIMINARY

- average AV18/UIX & χEFT: -3.778 meV $\pm 5.1\%$
- *c.f.* Friar '77: -3.1 meV $\pm 20\%$

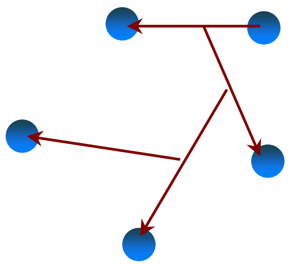
Summary and Outlook

- Lamb shifts in muonic atoms connect nuclear and atomic physics
- We calculate nuclear-polarizability corrections to Lamb shift in μ-^{4}He
- Nuclear response functions are calculated with realistic / chiral potentials
  - contributions related to dipole response function have been calculated
  - calculations from higher-multipole response functions are ongoing
- Solid error estimation from all physics needs to be done

BACK UP

Hyperspherical Harmonics Expansions

Starts from relative coordinates



$\vec{\eta}_0 = \sqrt{A}\vec{R}_{CM} \quad \vec{\eta}_1, ..., \vec{\eta}_{A-1}$

Recursive definition of hyper-spherical coordinates

$$\rho, \Omega \qquad \rho^2 = \sum_{i=1}^{A} r_i^2 = \sum_{i=1}^{A-1} \eta_i^2$$

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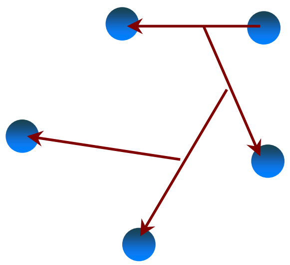

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Kinetic energy: $H_0(\rho, \Omega) = T_\rho + \frac{K^2(\Omega)}{\rho^2}$

$$K^2 \mathcal{Y}_{[K]}(\Omega) = K(K + 3A - 5)\mathcal{Y}_{[K]}(\Omega)$$

Expansion: $\Psi = \sum_{[K],\nu}^{K_{max},\nu_{max}} c_\nu^{[K]} e^{-\rho/2b} \rho^{n/2} L_\nu^n(\frac{\rho}{b}) [\mathcal{Y}_{[K]}(\Omega)\chi_{ST}]_{JT}^a$

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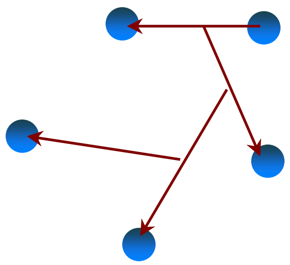

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Kinetic energy: $H_0(\rho, \Omega) = T_\rho + \frac{K^2(\Omega)}{\rho^2}$

$$K^2 \mathcal{Y}_{[K]}(\Omega) = K(K + 3A - 5)\mathcal{Y}_{[K]}(\Omega)$$

Expansion: $\Psi = \sum_{[K],\nu}^{K_{max},\nu_{max}} c_\nu^{[K]} e^{-\rho/2b} \rho^{n/2} L_\nu^n\left(\frac{\rho}{b}\right)\left[\mathcal{Y}_{[K]}(\Omega)\chi_{ST}\right]_{JT}^a$

- Exact method applicable for $3 \leq A \lesssim 8$
- Can accommodate local and non-local two and three-body interactions

AV18+UIX

NN(N^{3}LO)+3NF(N^{2}LO)