

News from the Importance Truncated NCSM

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■ **Importance Truncated No-Core Shell Model**

- Concept & Implementation
- Uncertainty Quantification
- Applications: Carbon & Oxygen Isotopes

■ **New Directions**

- Multi-Reference Normal-Ordering Approximation
- Importance Truncated Shell Model
- Ab Initio Description of Hypernuclei

Importance Truncated No-Core Shell Model

Roth, Calci, Langhammer, Binder — in preparation (2013)

Roth, Langhammer, Calci et al. — Phys. Rev. Lett. 107, 072501 (2011)

Navrátil, Roth, Quaglioni — Phys. Rev. C 82, 034609 (2010)

Roth — Phys. Rev. C 79, 064324 (2009)

Roth, Gour & Piecuch — Phys. Lett. B 679, 334 (2009)

Roth, Gour & Piecuch — Phys. Rev. C 79, 054325 (2009)

Roth, Navrátil — Phys. Rev. Lett. 99, 092501 (2007)

No-Core Shell Model

Barrett, Vary, Navratil, Maris, Nogga, Roth,...

NCSM is one of the most powerful and universal exact ab-initio methods

- construct matrix representation of Hamiltonian using a **basis of HO Slater determinants** truncated w.r.t. HO excitation energy $N_{\max}\hbar\Omega$
- solve **large-scale eigenvalue problem** for a few extremal eigenvalues
- **all relevant observables** can be computed from the eigenstates
- range of applicability limited by **factorial growth** of basis with N_{\max} & A
- adaptive **importance truncation** extends the range of NCSM by reducing the model space to physically relevant states

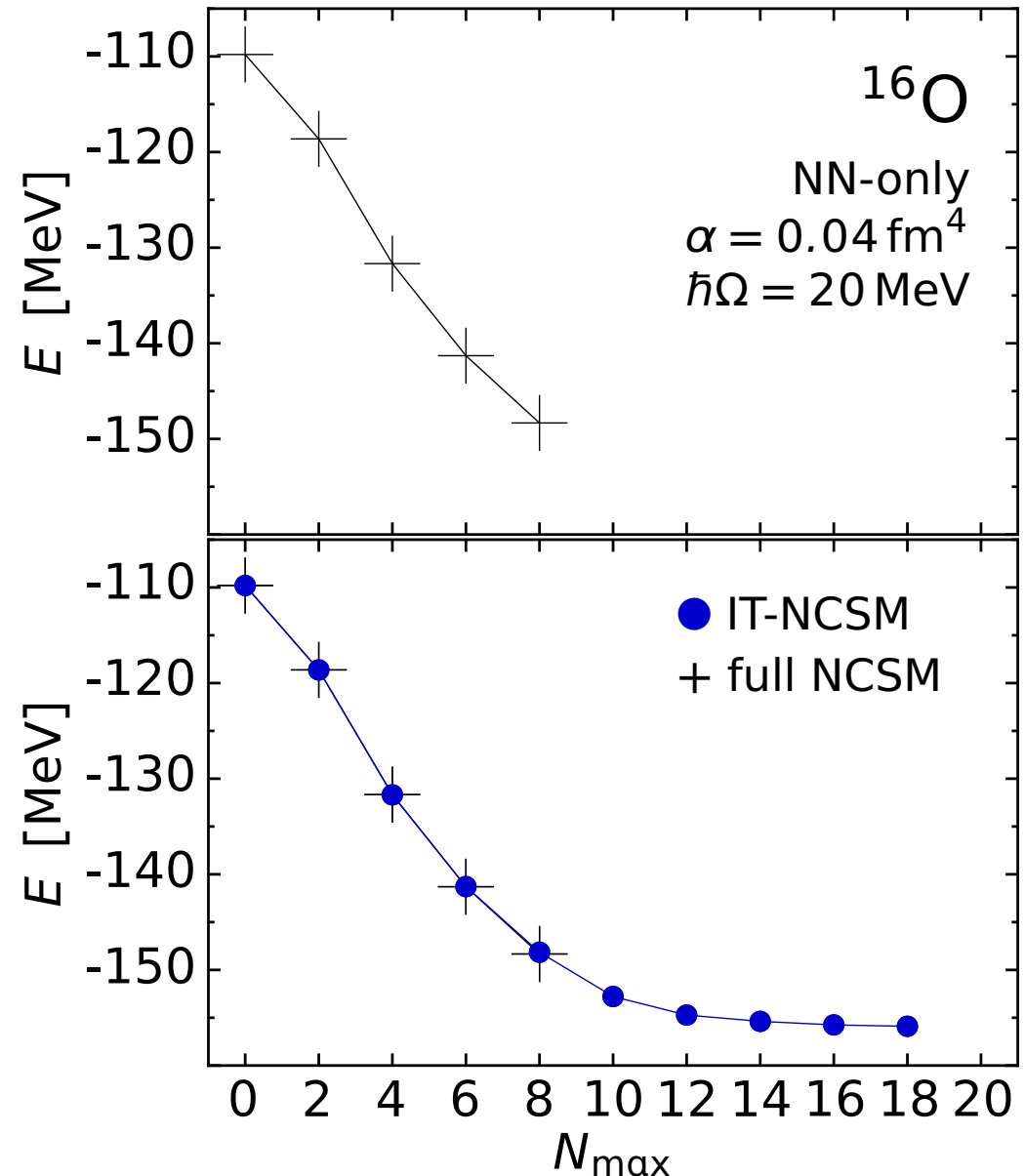
Importance Truncated NCSM

Roth, PRC 79, 064324 (2009); PRL 99, 092501 (2007)

- converged NCSM calculations essentially restricted to lower/mid p-shell
- full $10\hbar\Omega$ calculation for ^{16}O getting very difficult (basis dimension $> 10^{10}$)

Importance Truncation

reduce model space to the relevant basis states using an **a priori importance measure** derived from MBPT



Importance Truncation: Basic Idea

- **starting point**: approximation $|\Psi_{\text{ref},m}\rangle$ for the **target states** within a limited reference space \mathcal{M}_{ref}

$$|\Psi_{\text{ref},m}\rangle = \sum_{\nu \in \mathcal{M}_{\text{ref}}} C_{\nu}^{(\text{ref},m)} |\Phi_{\nu}\rangle$$

- **measure the importance** of individual basis state $|\Phi_{\nu}\rangle \notin \mathcal{M}_{\text{ref}}$ via first-order multiconfigurational perturbation theory

$$\frac{\langle \Phi_{\nu} | H | \Psi_{\text{ref},m} \rangle}{\epsilon_{\nu}}$$

importance measure only probes 2p2h excitations on top of \mathcal{M}_{ref} for a two-body Hamiltonian

- collect $C_{\nu}^{(\text{ref},m)}$ from all basis states

embed into iterative scheme to access full model space

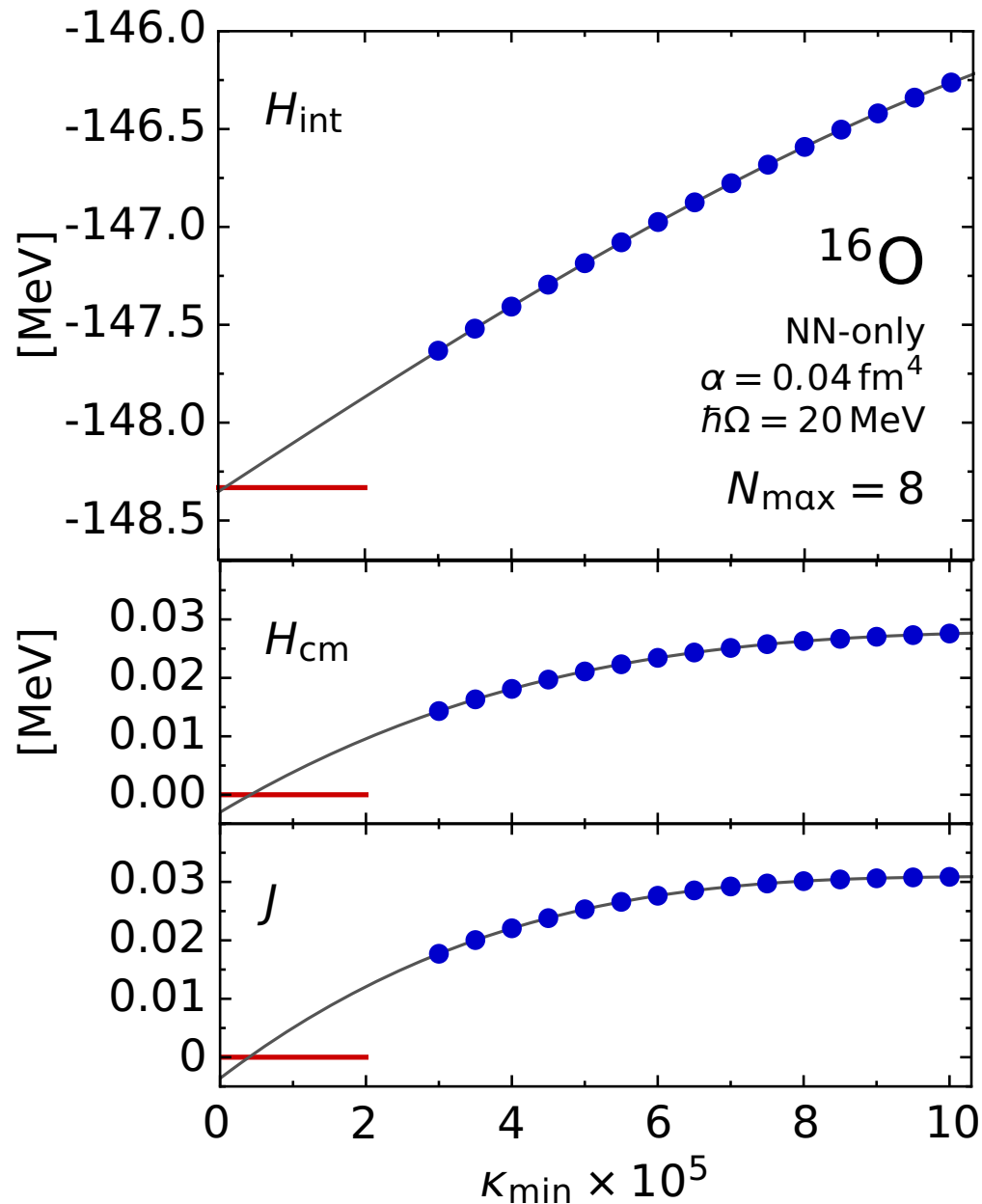
- **solve eigenvalue problem** in $\mathcal{M}_{\text{IT}}(K_{\text{min}})$ and obtain improved approximation of target state

Importance Truncation: Iterative Scheme

- **property of N_{\max} -truncated space**: step from N_{\max} to $N_{\max} + 2$ requires 2p2h excitations at most
- **sequential calculation** for a range of $N_{\max} \hbar \Omega$ spaces:
 - do full NCSM calculations up to a convenient N_{\max}
 - ★ use components of eigenstates with $|C_{\nu}^{(m)}| \geq C_{\min}$ as initial $|\Psi_{\text{ref},m}\rangle$
 - ① consider all states $|\Phi_{\nu}\rangle \notin \mathcal{M}_{\text{ref}}$ from an $N_{\max} + 2$ space and add those with $|K_{\nu}^{(m)}| \geq K_{\min}$ to importance-truncated space \mathcal{M}_{IT}
 - ② solve eigenvalue problem in \mathcal{M}_{IT}
 - ③ use components of eigenstates as initial state
 - ④ goto ①

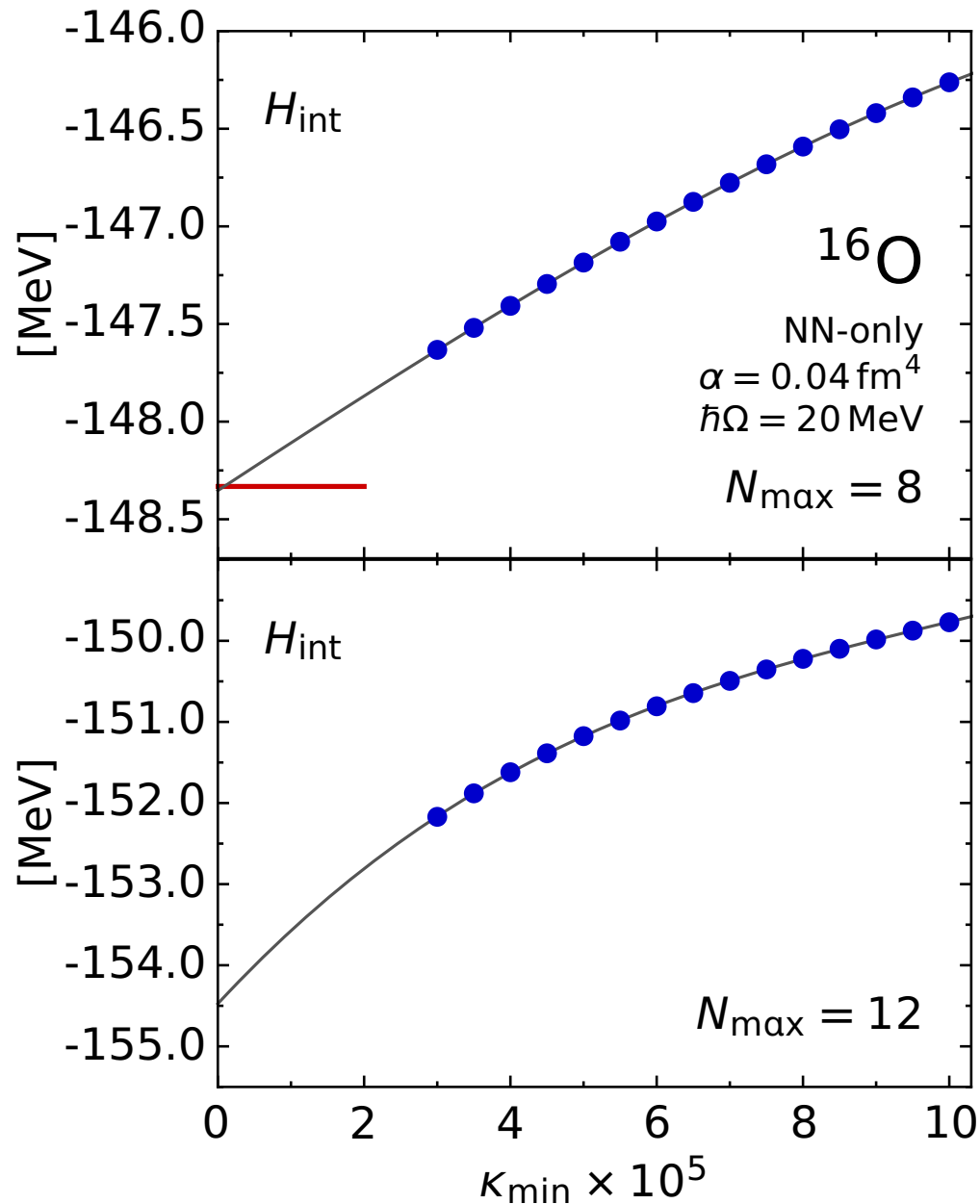
full NCSM space is recovered in the limit
 $(K_{\min}, C_{\min}) \rightarrow 0$

Threshold Extrapolation



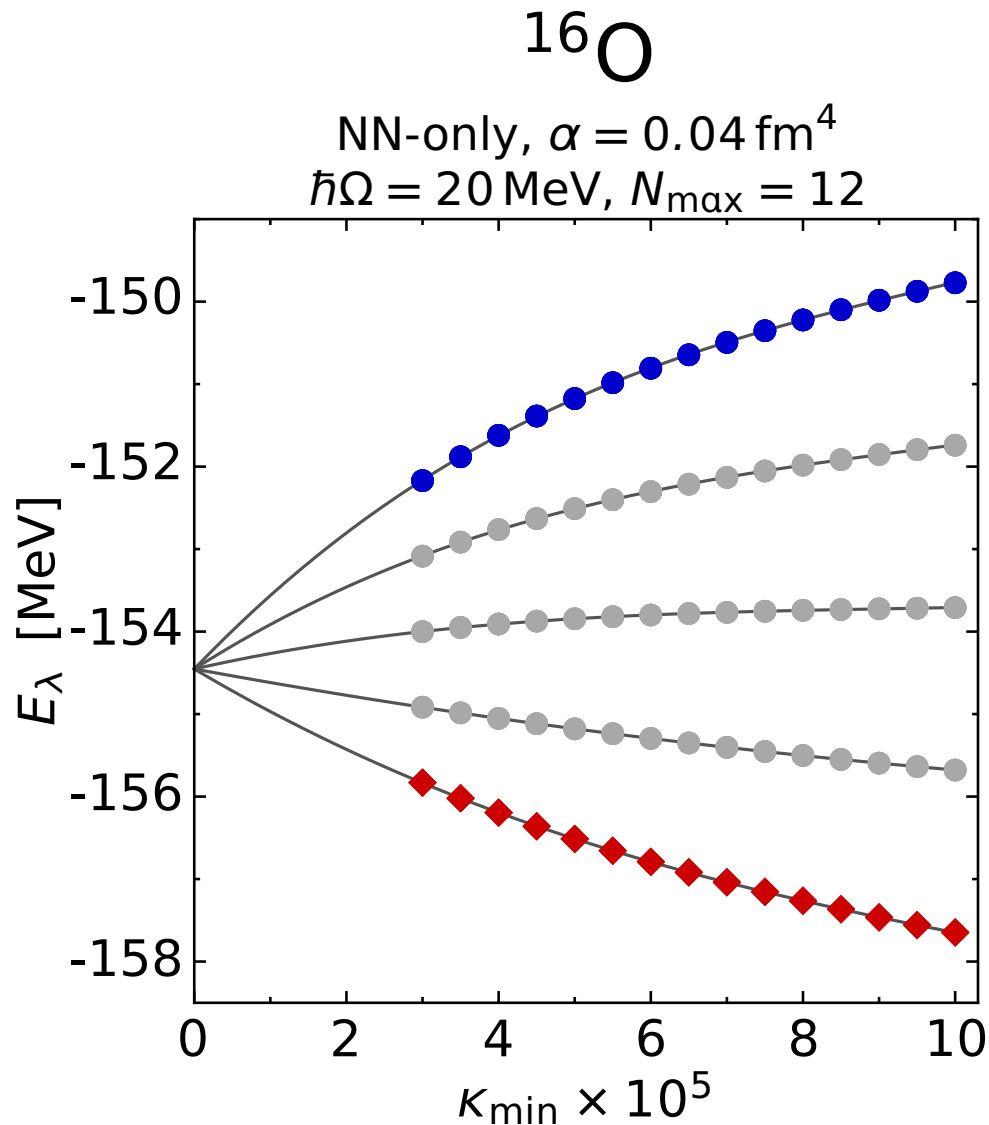
- repeat calculations for a **sequence of importance thresholds** K_{\min}
- observables show **smooth threshold dependence** and systematically approach the full NCSM limit
- use **a posteriori extrapolation** $K_{\min} \rightarrow 0$ of observables to account for effect of excluded configurations

Threshold Extrapolation



- repeat calculations for a **sequence of importance thresholds** K_{min}
- observables show **smooth threshold dependence** and systematically approach the full NCSM limit
- use **a posteriori extrapolation** $K_{\text{min}} \rightarrow 0$ of observables to account for effect of excluded configurations

Constrained Threshold Extrapolation



- for free: importance selection gives perturbative energy correction $\Delta_{\text{excl}}(K_{\text{min}})$ accounting for **excluded states**
- formal property
 $\Delta_{\text{excl}}(K_{\text{min}}) \rightarrow 0$ for $K_{\text{min}} \rightarrow 0$
- auxiliary parameter λ defining a family of energy sequences
$$E_{\lambda}(K_{\text{min}}) = E(K_{\text{min}}) + \lambda \Delta_{\text{excl}}(K_{\text{min}})$$
- **simultaneous extrapolation** for family of λ -values with constraint $E_{\lambda}(0) = E_{\text{extrap}}$

Uncertainty Quantification

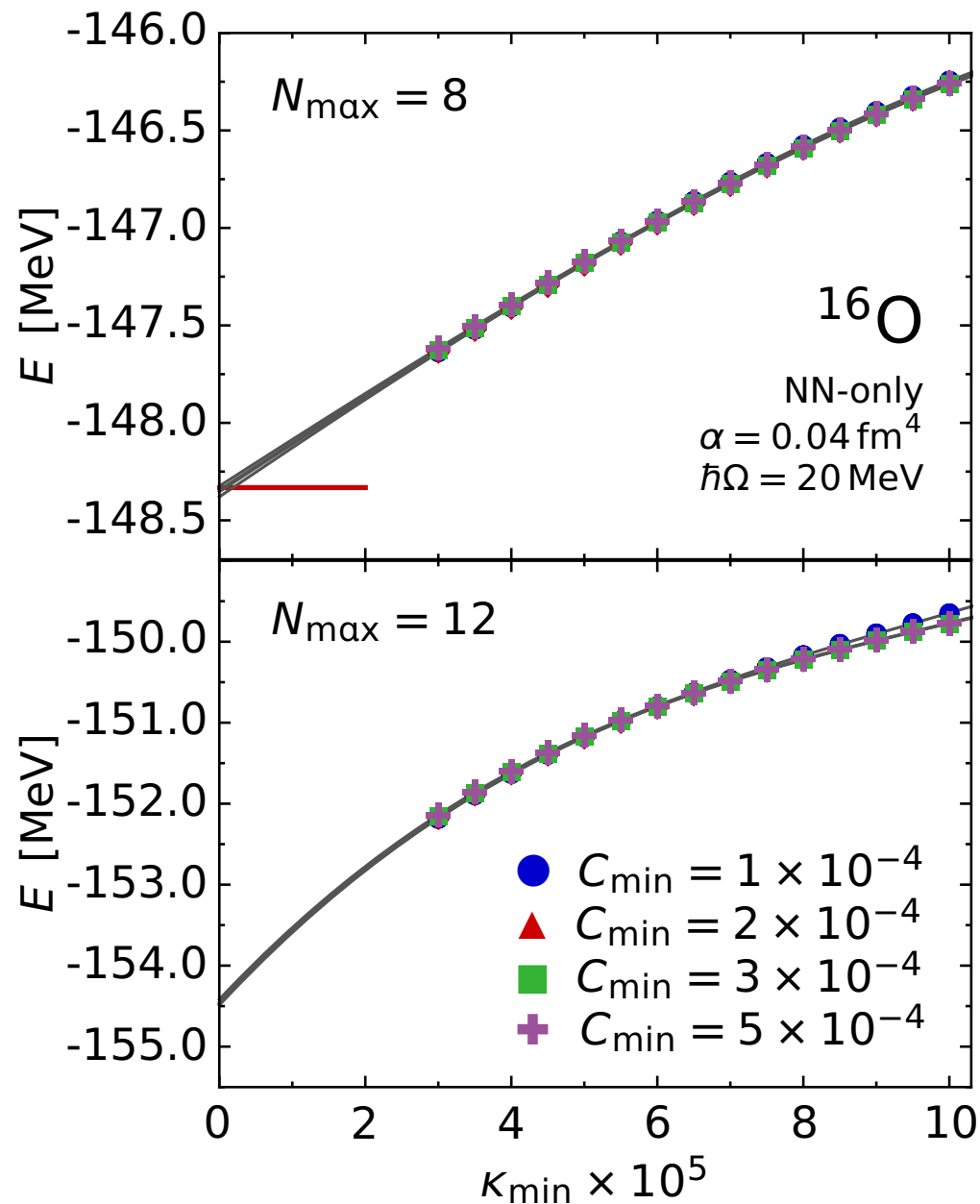
Importance Truncation

- use sequence of (C_{\min}, K_{\min}) -truncated model spaces
- extrapolate to $K_{\min} \rightarrow 0$ using polynomial ansatz or more refined constrained extrapolation scheme
- uncertainty estimate derived from extrapolation protocol
- **systematic uncertainty** absent in full NCSM

Model-Space Truncation

- use sequence of N_{\max} -truncated model spaces
- extrapolate to $N_{\max} \rightarrow \infty$ using exponential ansatz or more elaborate extrapolation schemes
- uncertainty estimate derived from extrapolation protocol
- same **extrapolation uncertainties** as in full NCSM

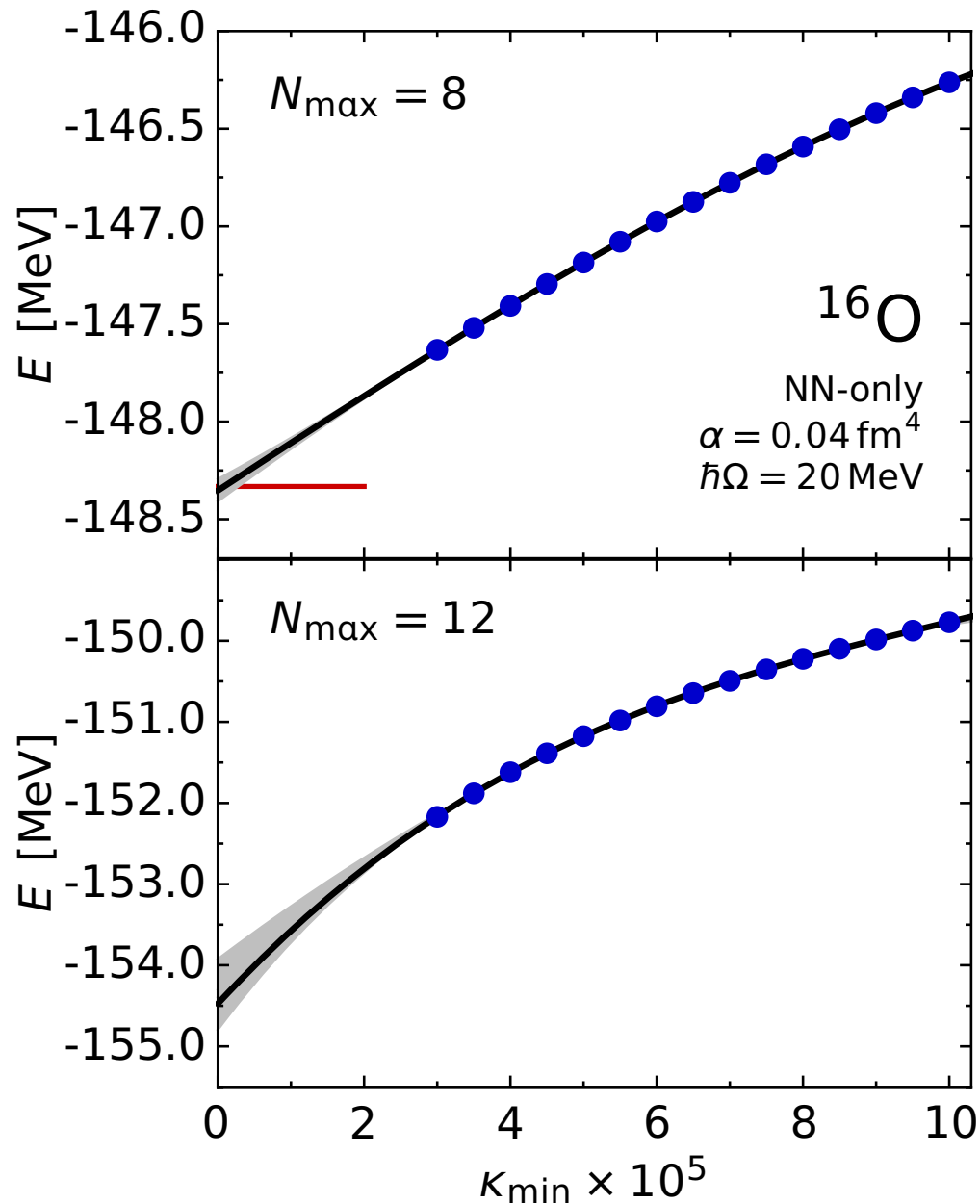
Comment on C_{\min} Truncation



- truncation of reference state to components with $|C_{\nu}| \geq C_{\min}$
- technical reason: importance selection phase scales with $(\dim \mathcal{M}_{\text{ref}})^2$
- typically $C_{\min} = 2 \times 10^{-4}$

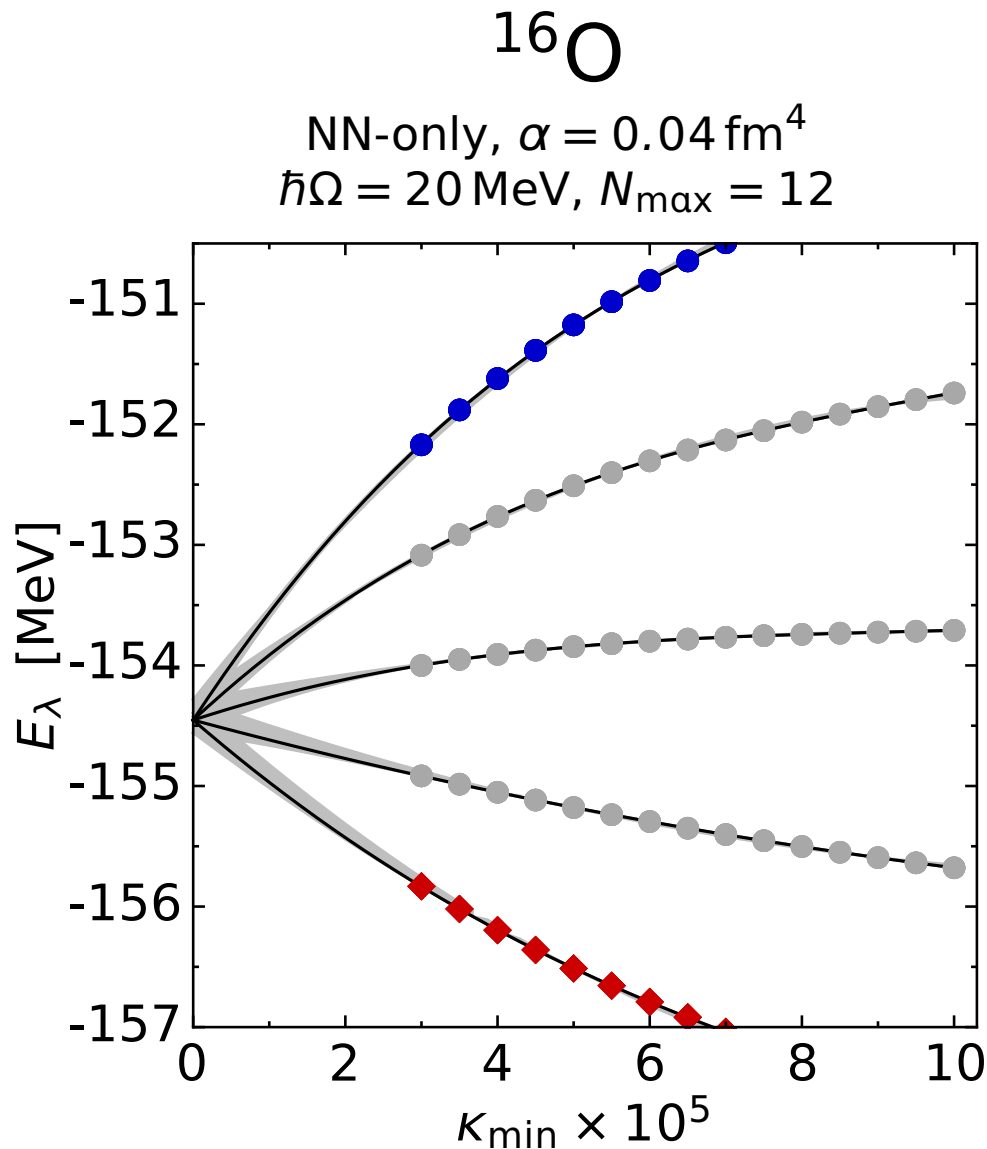
practically no influence on threshold extrapolated energies

Protocol: Simple κ_{\min} Extrapolation



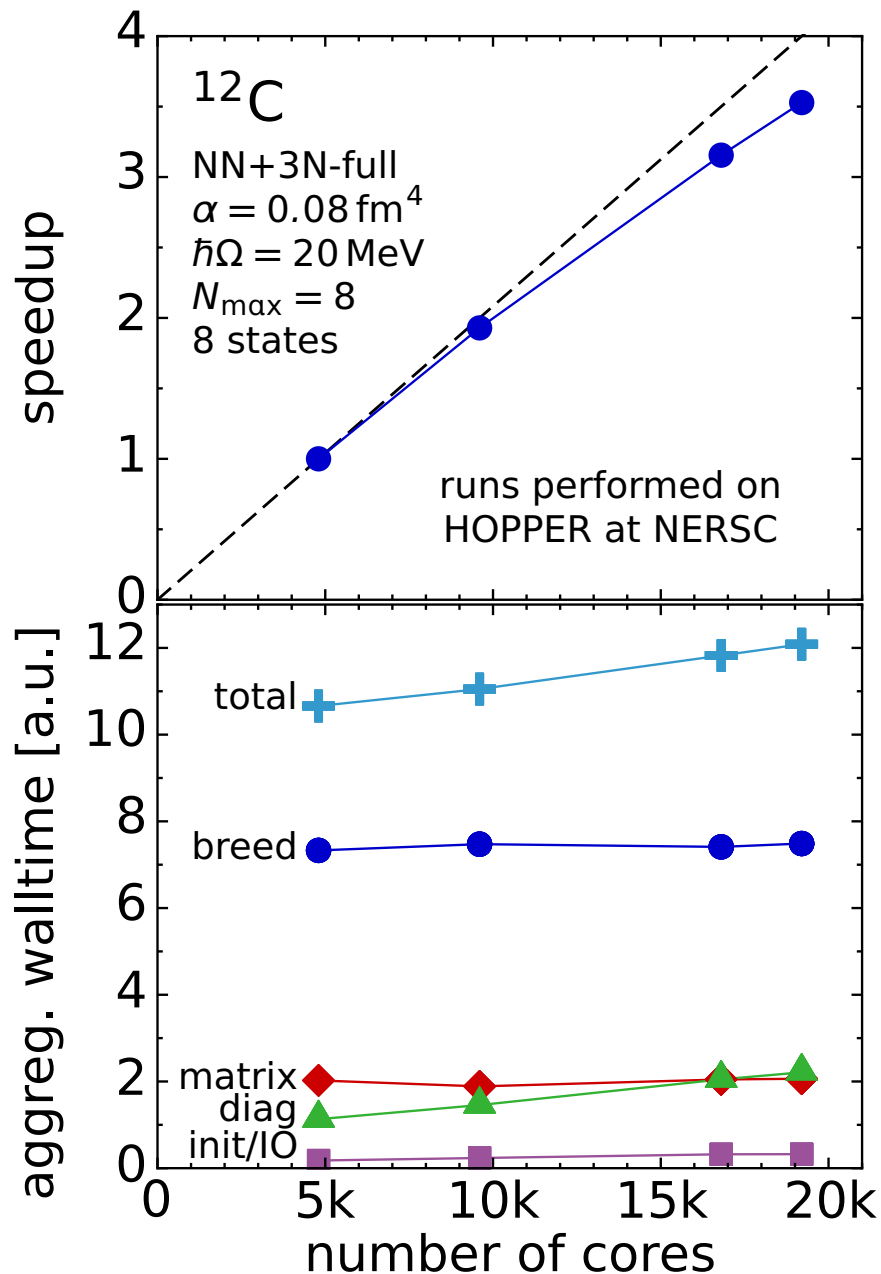
- perform IT-NCSM calculations for range of κ_{\min} -values, typically $\kappa_{\min} = 3, 3.5, \dots, 10 \times 10^{-5}$
- extrapolation $\kappa_{\min} \rightarrow 0$ using polynomial $P_p(\kappa_{\min})$ fit to full κ_{\min} -set, typically of order $p = 3$
- generate uncertainty band from set of alternative extrapolations
 - P_{p-1} and P_{p+1} extrapolations using full κ_{\min} -range
 - P_p extrapolations with lowest and lowest two κ_{\min} -points dropped
- quote standard deviation as nominal uncertainty

Protocol: Constrained κ_{\min} Extrapolation



- select a few λ -values to get symmetrical approach towards common $E_{\text{extrap}} = E_\lambda(\kappa_{\min} = 0)$
- constrained simultaneous extrapolation $\kappa_{\min} \rightarrow 0$ using polynomial $P_p(\kappa_{\min})$, typically of order $p = 3$
- generate uncertainty band from set of constrained extrapolations
 - P_{p-1} and P_{p+1} extrapolations using full κ_{\min} -range
 - P_p extrapolations with lowest and lowest two κ_{\min} -points dropped
 - P_p extrapolations with smallest and largest λ -set dropped
- std. deviation gives uncertainty

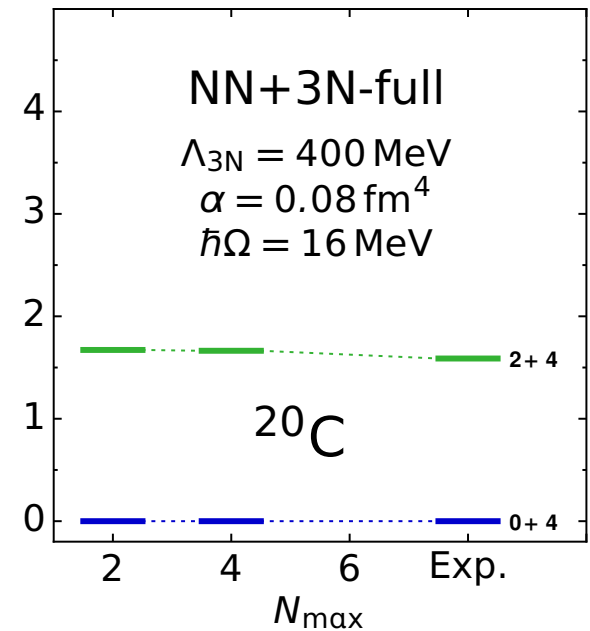
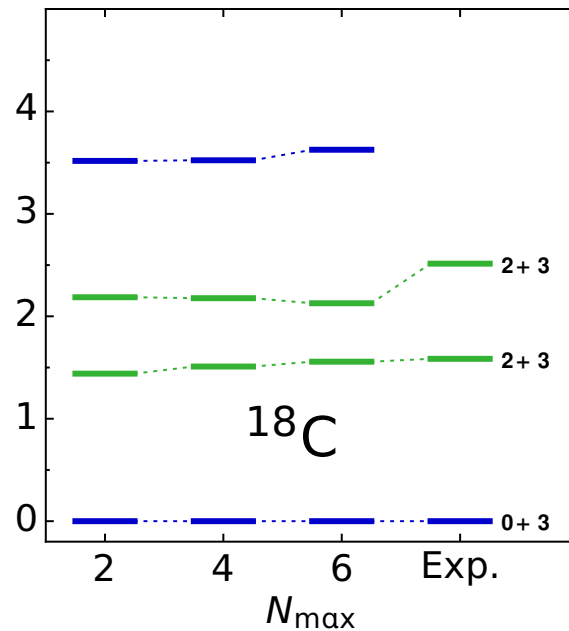
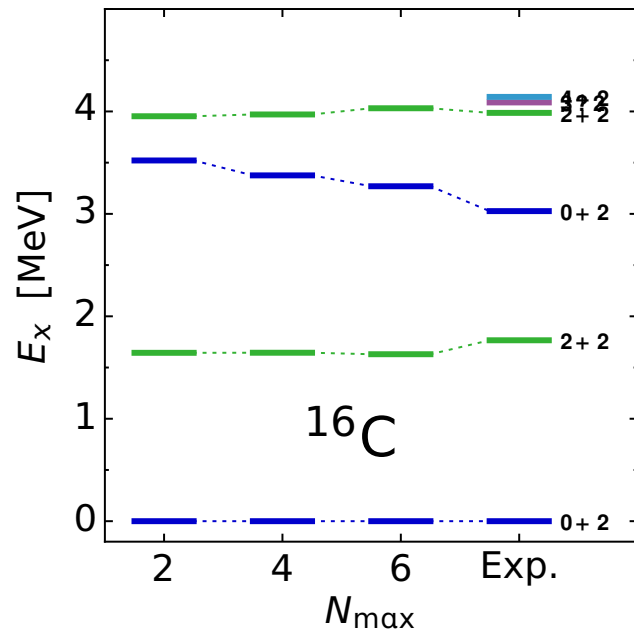
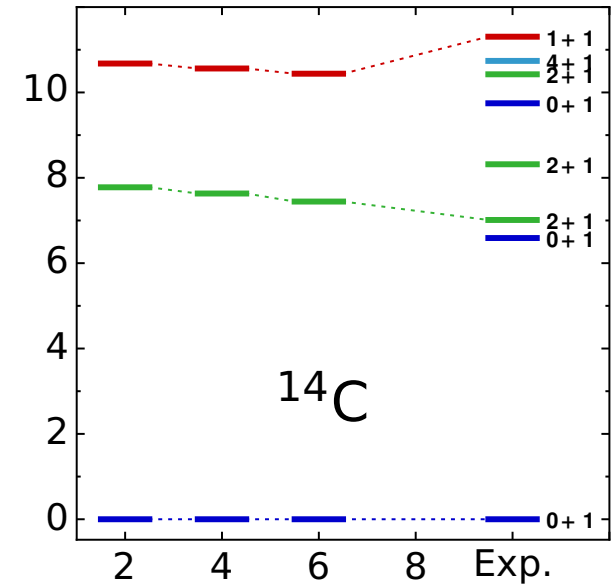
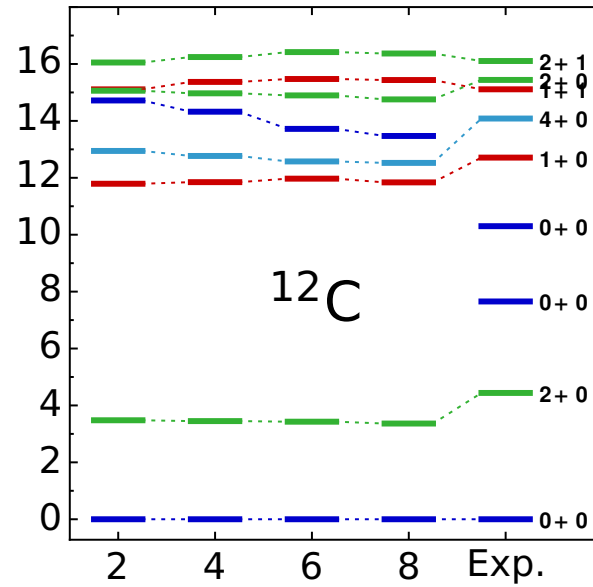
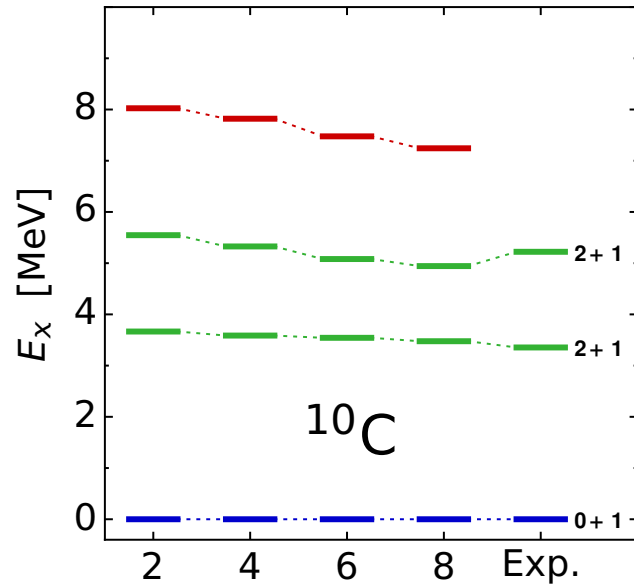
Computational Aspects



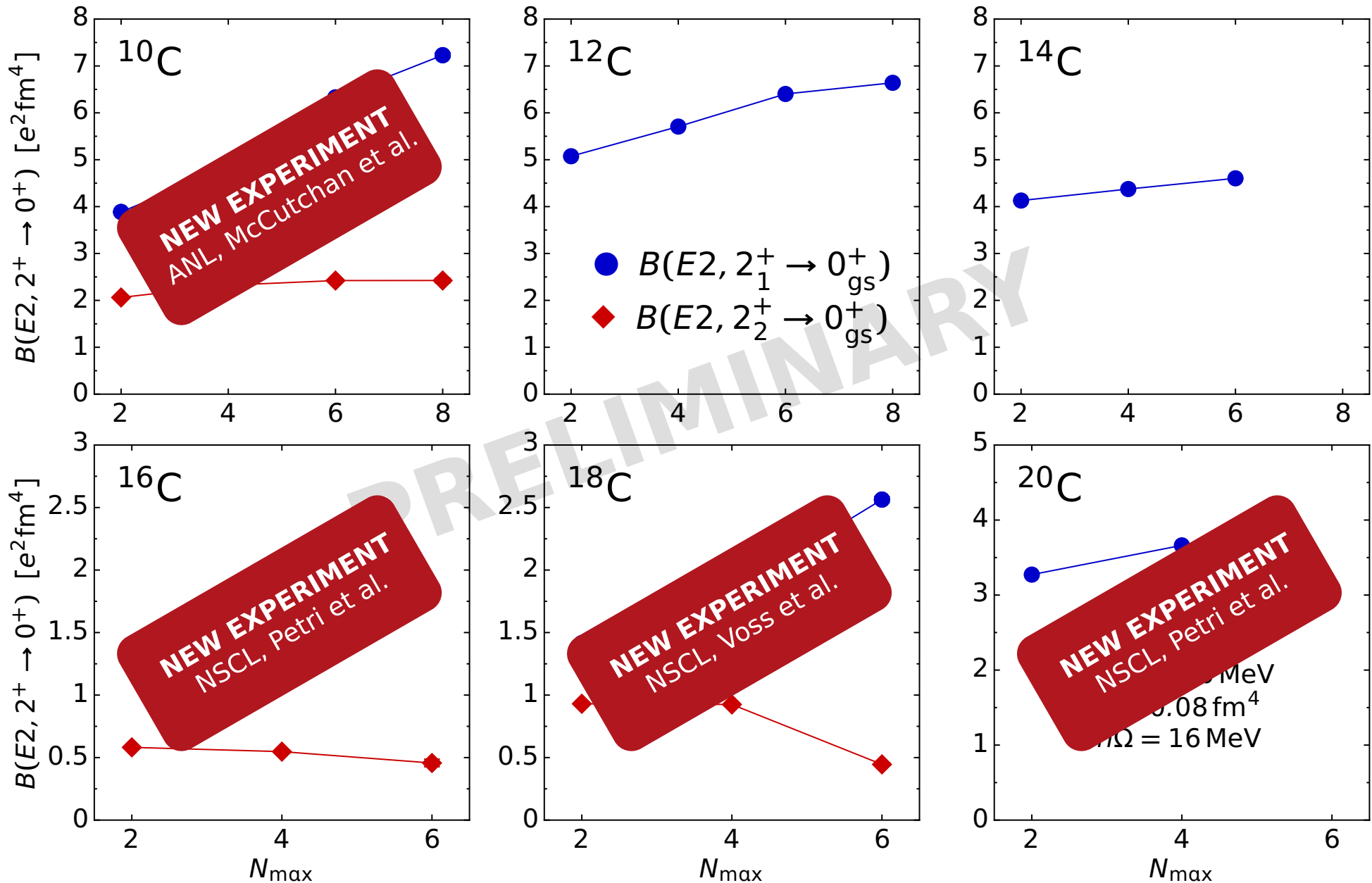
- hybrid OpenMP/MPI implementation of IT-NCSM developed from scratch (no 'vintage' code pieces)
- capable of using large sets of JT-coupled 3N matrix elements
- efficient on-the-fly decoupling of NN,3N,4N matrix elements
 - first GPU version looks extremely promising (with Vary et al.)
- dynamic load balancing and automatic checkpointing/restart capabilities
- very good scaling behavior

Ab Initio Calculations
for p- and sd-Shell Nuclei

Spectroscopy of Carbon Isotopes

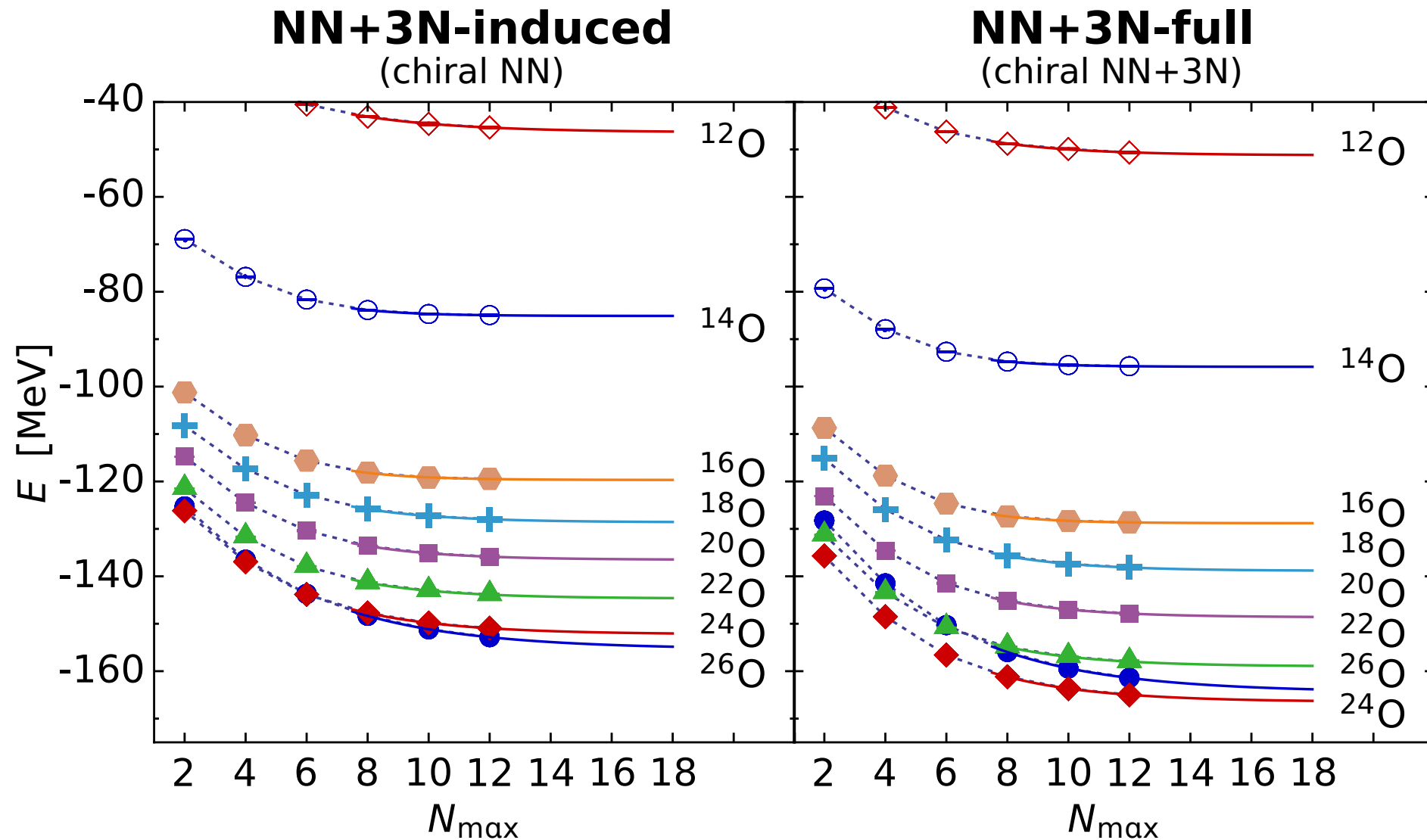


Spectroscopy of Carbon Isotopes



Ground States of Oxygen Isotopes

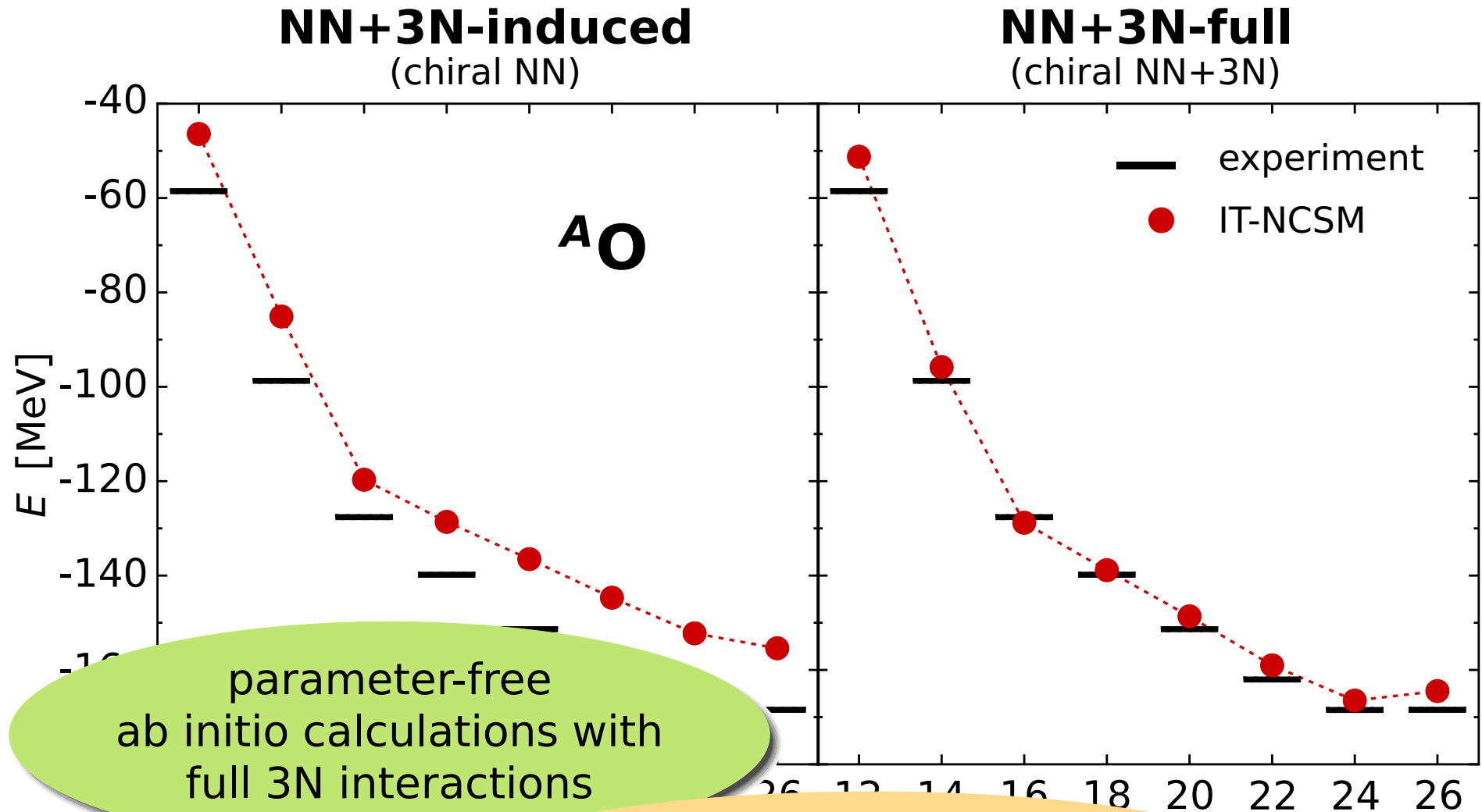
Hergert, Binder, Calci, Langhammer, Roth; in prep.



$\Lambda_{3N} = 400$ MeV, $\alpha = 0.08$ fm⁴, $E_{3\max} = 14$, optimal $\hbar\Omega$

Ground States of Oxygen Isotopes

Hergert, Binder, Calci, Langhammer, Roth; in prep.



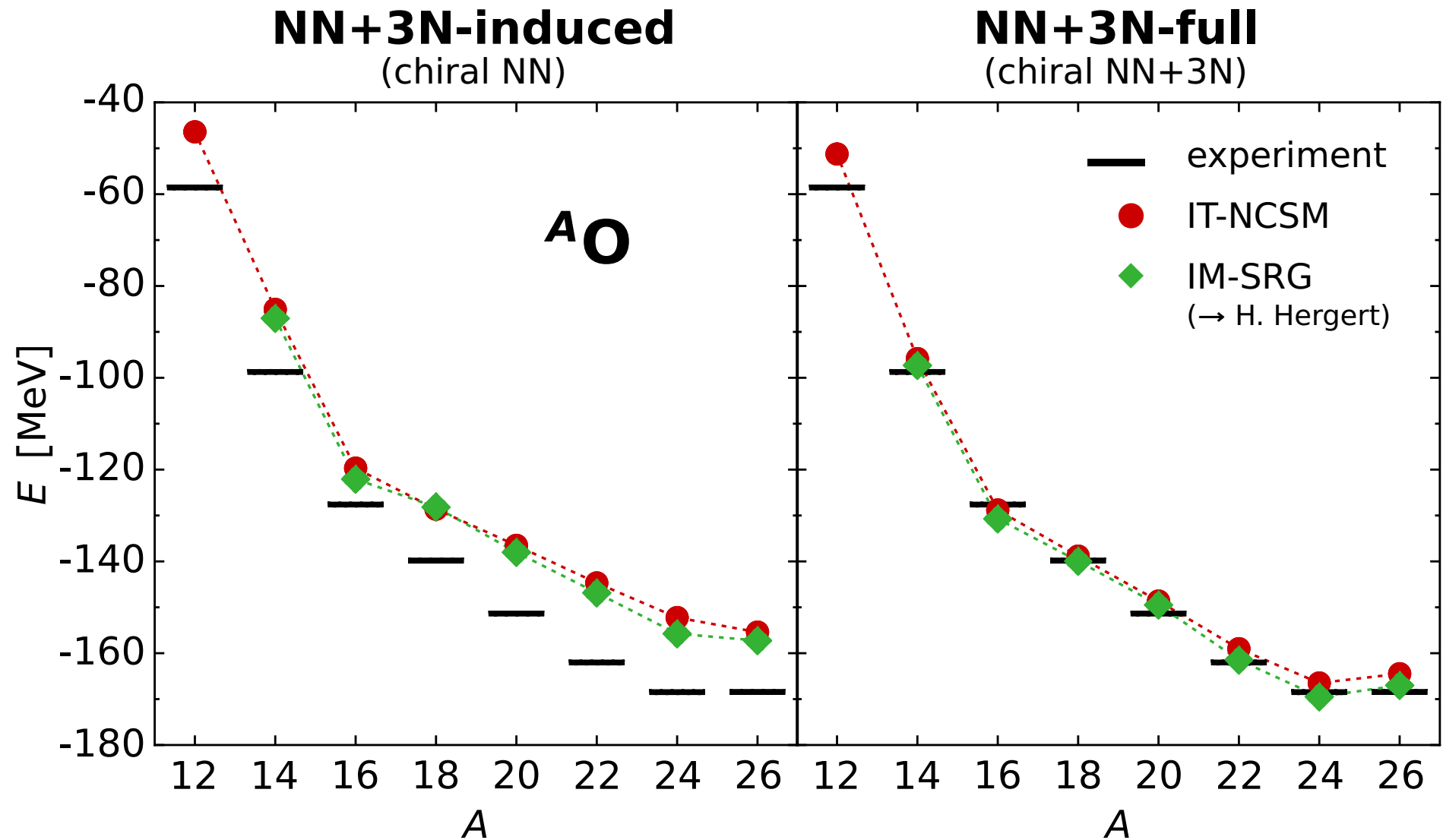
parameter-free
ab initio calculations with
full 3N interactions

highlights predictive power
of chiral NN+3N Hamiltonians

$\Lambda_{3N} = 400$ MeV, $\mu = 0.045$ fm⁻¹, $\beta = 0.16$, $\gamma = 0.01$, $\hbar\Omega = 10$ MeV

Ground States of Oxygen Isotopes

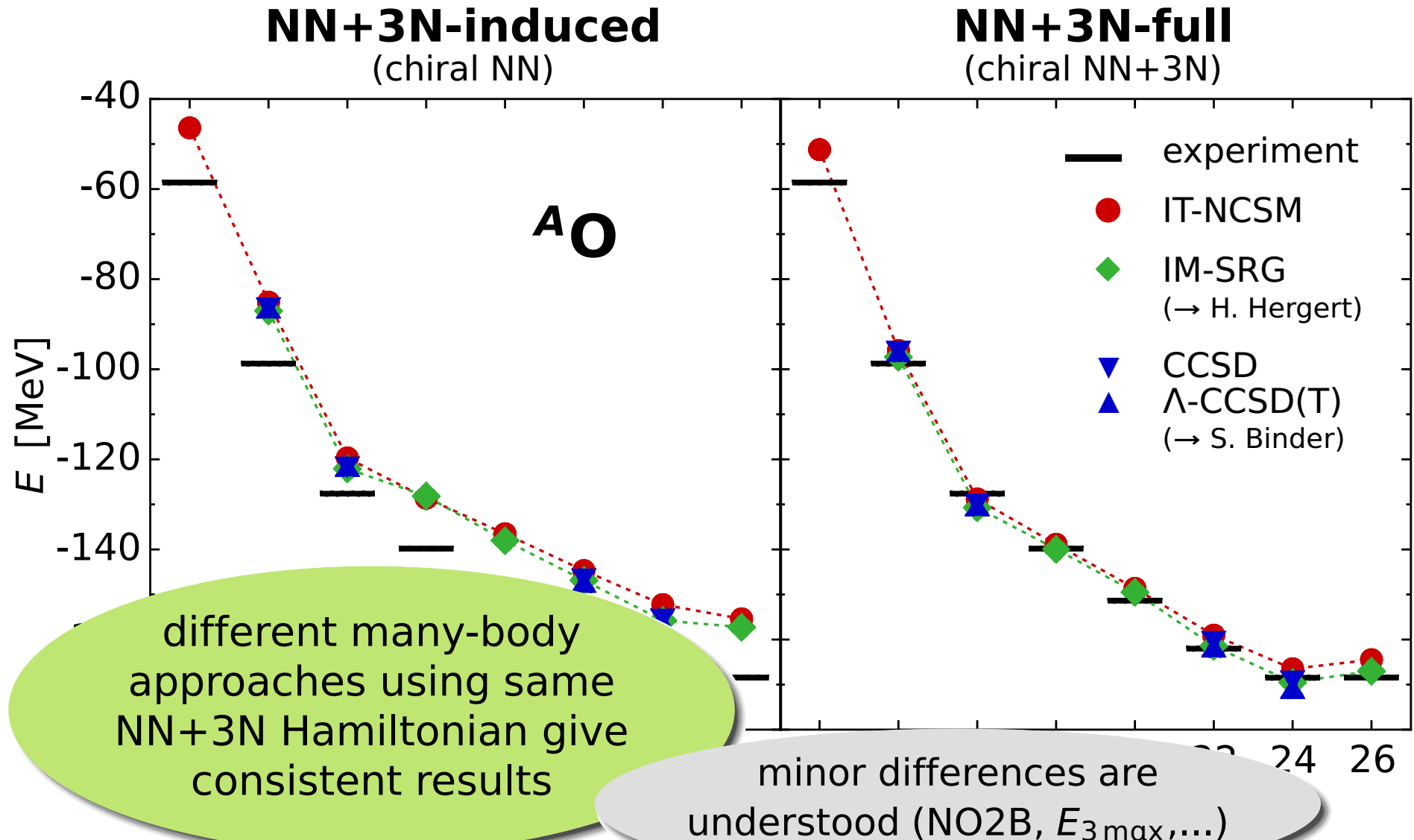
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Ground States of Oxygen Isotopes

Hergert, Binder, Calci, Langhammer, Roth; in prep.



$\Lambda_{3N} = 400$ MeV, $\alpha = 0.08$ fm $^{-1}$, $E_{3\max} = 14$, $\Omega = 0.1$ spinorial $\hbar\Omega$

Multi-Reference Normal Ordering

with
Eskendr Gebrerufael

Motivation: Normal Ordering

avoid formal and computational challenges of including explicit 3N terms in many-body calculations

- circumvent **formal extension of many-body method** to include explicit 3N interactions
- avoid the **increase of computational cost** caused by inclusion of explicit 3N interactions
- **normal-ordered two-body approximation** works very well for closed-shell systems (→ S. Binder)
- can we do the same for **open-shell nuclei**?

Normal Ordering of 3N Interaction

- **starting point**: three-body operator in second-quantized form with respect to the zero-body vacuum $|0\rangle$

$$V_{3N} = \frac{1}{36} \sum_{abc, \bar{a}\bar{b}\bar{c}} V_{\bar{a}\bar{b}\bar{c}}^{abc} A_{\bar{a}\bar{b}\bar{c}}^{abc}$$

$$V_{\bar{a}\bar{b}\bar{c}}^{abc} = \langle abc | V_{3N} | \bar{a}\bar{b}\bar{c} \rangle \quad A_{\bar{a}\bar{b}\bar{c}}^{abc} = a_a^\dagger a_b^\dagger a_c^\dagger a_{\bar{c}} a_{\bar{b}} a_{\bar{a}}$$

- **single-reference normal ordering**: assume reference state $|\Phi_{SR}\rangle$ given by a single Slater determinant
 - standard toolbox: Wick theorem, contractions, etc.
- **multi-reference normal ordering**: assume reference state $|\Phi_{MR}\rangle$ given by a superposition of Slater determinants
 - generalized Wick theorem and n-tupel contractions proposed by Mukherjee & Kutzelnigg (1997)

Multi-Reference Normal Ordering

- **three-body operator in normal-ordered form** with respect to multi-reference state $|\Phi_{\text{MR}}\rangle$

$$V_{3\text{N}} = W + \sum_{c, \bar{c}} W_{\bar{c}}^c \tilde{A}_{\bar{c}}^c + \frac{1}{4} \sum_{bc, \bar{b}\bar{c}} W_{\bar{b}\bar{c}}^{bc} \tilde{A}_{\bar{b}\bar{c}}^{bc} + \frac{1}{36} \sum_{abc, \bar{a}\bar{b}\bar{c}} W_{\bar{a}\bar{b}\bar{c}}^{abc} \tilde{A}_{\bar{a}\bar{b}\bar{c}}^{abc}$$

where $\tilde{A}_{\circ\circ\circ}^{\circ\circ\circ}$ indicates multi-reference normal ordered string of creation and annihilation operators (abstract concept)

- matrix elements of **normal-ordered n -body contributions** involve one-, two- and three-body density matrices for $|\Phi_{\text{MR}}\rangle$

$$W = \frac{1}{36} \sum_{abc, \bar{a}\bar{b}\bar{c}} V_{\bar{a}\bar{b}\bar{c}}^{abc} \rho_{\bar{a}\bar{b}\bar{c}}^{abc}$$

$$W_{\bar{c}}^c = \frac{1}{4} \sum_{ab, \bar{a}\bar{b}} V_{\bar{a}\bar{b}\bar{c}}^{abc} \rho_{\bar{a}\bar{b}}^{ab}$$

$$W_{\bar{b}\bar{c}}^{bc} = \sum_{a, \bar{a}} V_{\bar{a}\bar{b}\bar{c}}^{abc} \rho_{\bar{a}}^a$$

$$W_{\bar{a}\bar{b}\bar{c}}^{abc} = V_{\bar{a}\bar{b}\bar{c}}^{abc}$$

Multi-Reference Normal Ordering

- discard normal-ordered three-body contribution to define the **normal-ordered two-body (NO2B) approximation**

$$V_{\text{NO2B}} = W + \sum_{c, \bar{c}} W_{\bar{c}}^c \tilde{A}_{\bar{c}}^c + \frac{1}{4} \sum_{bc, \bar{b}\bar{c}} W_{\bar{b}\bar{c}}^{bc} \tilde{A}_{\bar{b}\bar{c}}^{bc}$$

- converted back into **vacuum normal order** with respect to $|0\rangle$

$$V_{\text{NO2B}} = \bar{V} + \sum_{c, \bar{c}} \bar{V}_{\bar{c}}^c A_{\bar{c}}^c + \frac{1}{4} \sum_{bc, \bar{b}\bar{c}} \bar{V}_{\bar{b}\bar{c}}^{bc} A_{\bar{b}\bar{c}}^{bc}$$

with new matrix elements

$$\bar{V} = \frac{1}{36} \sum_{abc, \bar{a}\bar{b}\bar{c}} V_{\bar{a}\bar{b}\bar{c}}^{abc} \left(\rho_{\bar{a}\bar{b}\bar{c}}^{abc} - 18 \rho_{\bar{a}}^a \rho_{\bar{b}\bar{c}}^{bc} + 36 \rho_{\bar{a}}^a \rho_{\bar{b}}^b \rho_{\bar{c}}^c \right)$$

$$\bar{V}_{\bar{c}}^c = \frac{1}{4} \sum_{ab, \bar{a}\bar{b}} V_{\bar{a}\bar{b}\bar{c}}^{abc} \left(\rho_{\bar{a}\bar{b}}^{ab} - 4 \rho_{\bar{a}}^a \rho_{\bar{b}}^b \right)$$

$$\bar{V}_{\bar{b}\bar{c}}^{bc} = \sum_{a, \bar{a}} V_{\bar{a}\bar{b}\bar{c}}^{abc} \rho_{\bar{a}}^a$$

Single-Reference Normal Ordering

- **single-reference normal ordering** is recovered by pugging in density matrices for a single Slater-determinant

$$\begin{aligned}\rho_{\bar{a}}^a &= n_a \delta_{\bar{a}}^a \\ \rho_{\bar{a}\bar{b}}^{ab} &= \rho_{\bar{a}}^a \rho_{\bar{b}}^b - \rho_{\bar{a}}^b \rho_{\bar{b}}^a \\ \rho_{\bar{a}\bar{b}\bar{c}}^{abc} &= \dots\end{aligned}$$

- three-body operator in **single-reference NO2B approximation** converted back into vacuum representation

$$V_{\text{NO2B}} = \bar{V} + \sum_{c, \bar{c}} \bar{V}_{\bar{c}}^c A_{\bar{c}}^c + \frac{1}{4} \sum_{bc, \bar{b}\bar{c}} \bar{V}_{\bar{b}\bar{c}}^{bc} A_{\bar{b}\bar{c}}^{bc}$$

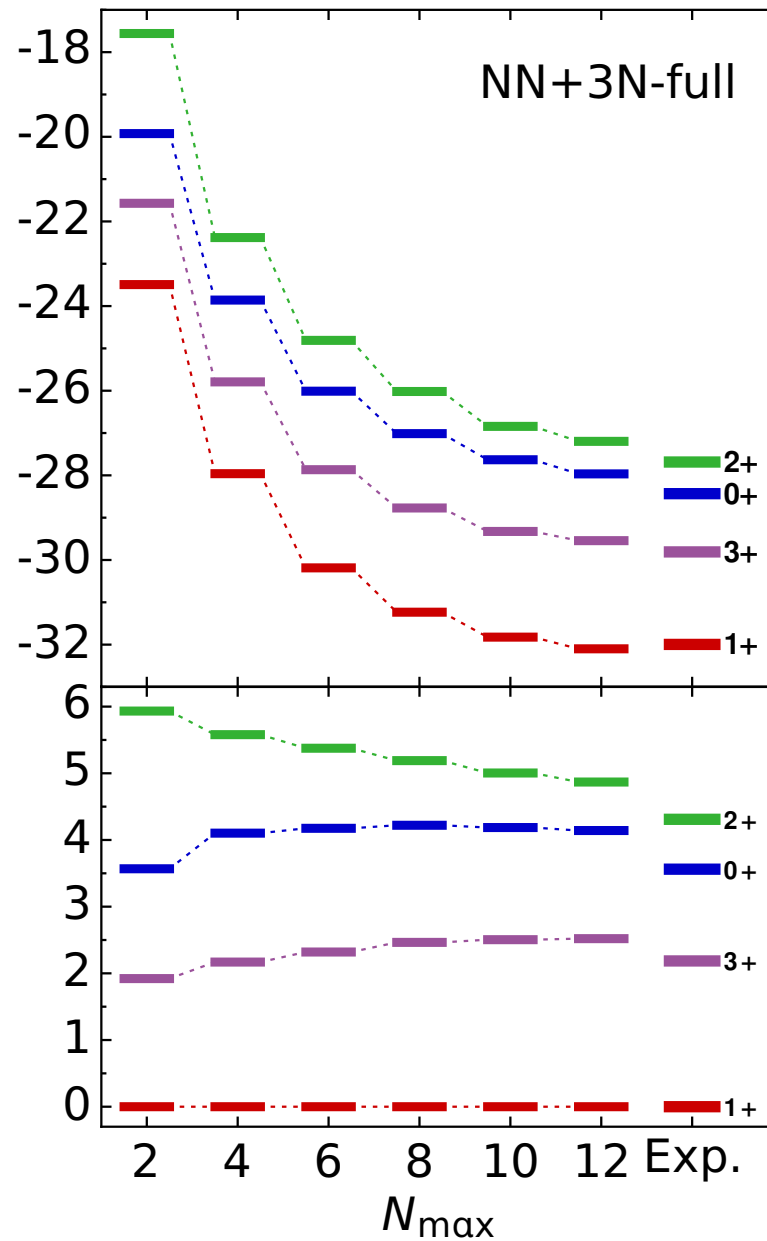
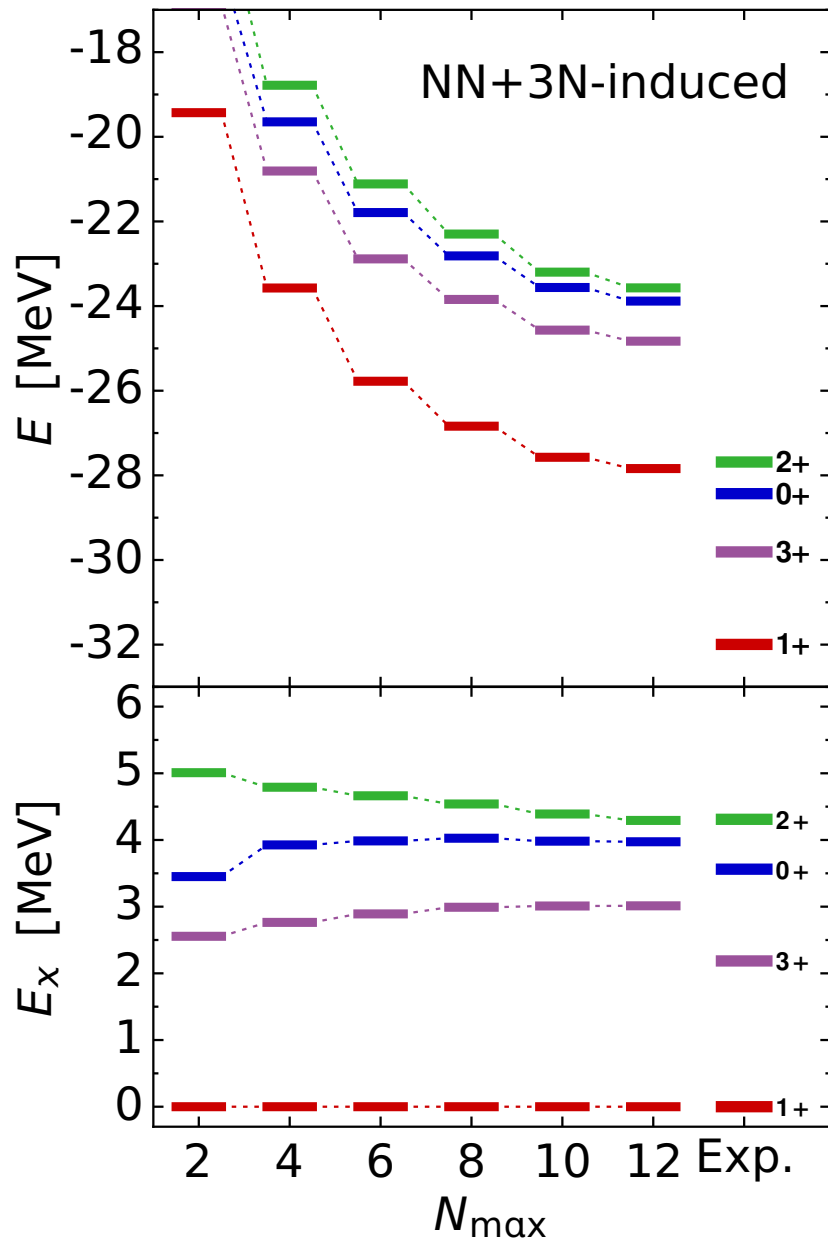
with simplified matrix elements

$$\bar{V} = \frac{1}{6} \sum_{abc} V_{abc}^{abc} n_a n_b n_c \quad \bar{V}_{\bar{c}}^c = -\frac{1}{2} \sum_{ab} V_{ab\bar{c}}^{abc} n_a n_b \quad \bar{V}_{\bar{b}\bar{c}}^{bc} = \sum_a V_{a\bar{b}\bar{c}}^{abc} n_a$$

IT-NCSM with MR-NO2B Approximation

- ① perform NCSM with explicit $3N$ interaction for small N_{\max}
 - ground state defines the reference state $|\Phi_{\text{MR}}\rangle$
 - no explicit information on excited states enters
- ② compute zero-, one- and two-body matrix elements of MR-NO2B approximation
 - density matrices for $|\Phi_{\text{MR}}\rangle$ can be precomputed and stored
 - three-body density matrix is not need explicitly
- ③ perform NCSM or IT-NCSM calculation up to large N_{\max} using MR-NO2B approximation
 - same computational cost as a simple NN-only calculation
 - larger model spaces become accessible

Benchmark: ${}^6\text{Li}$

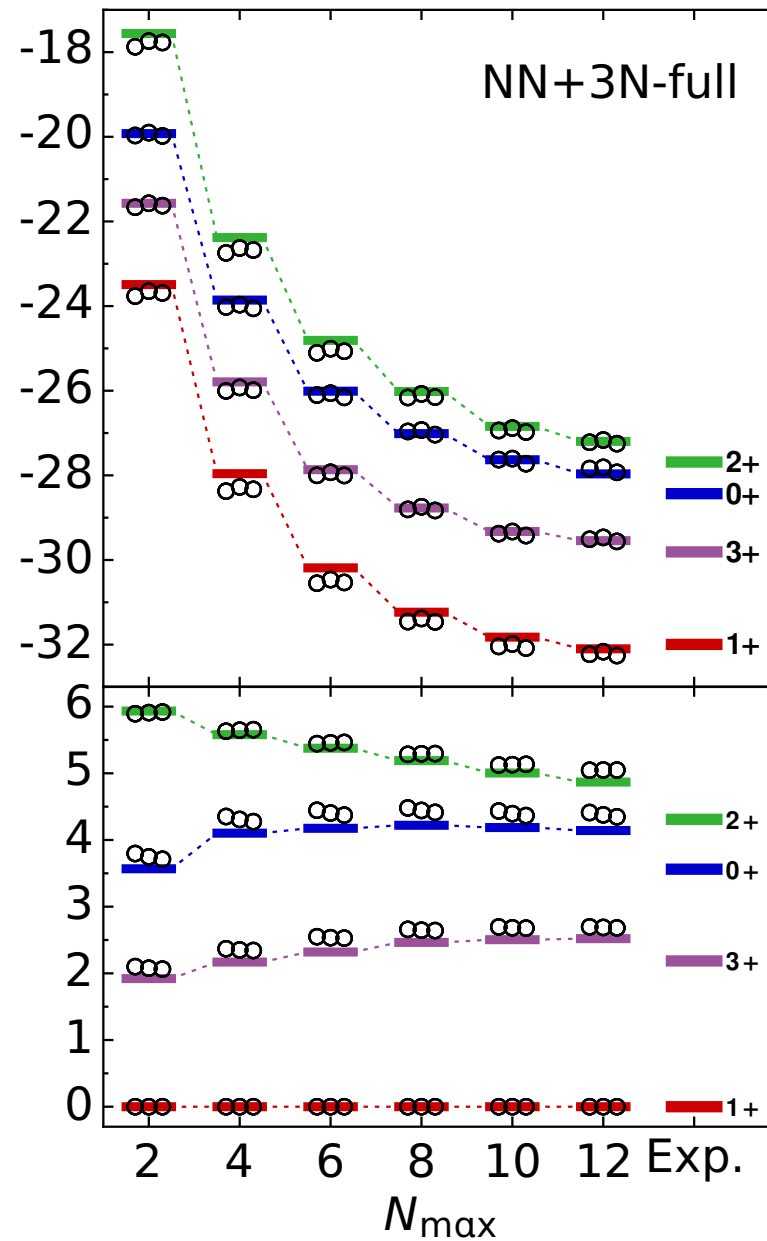
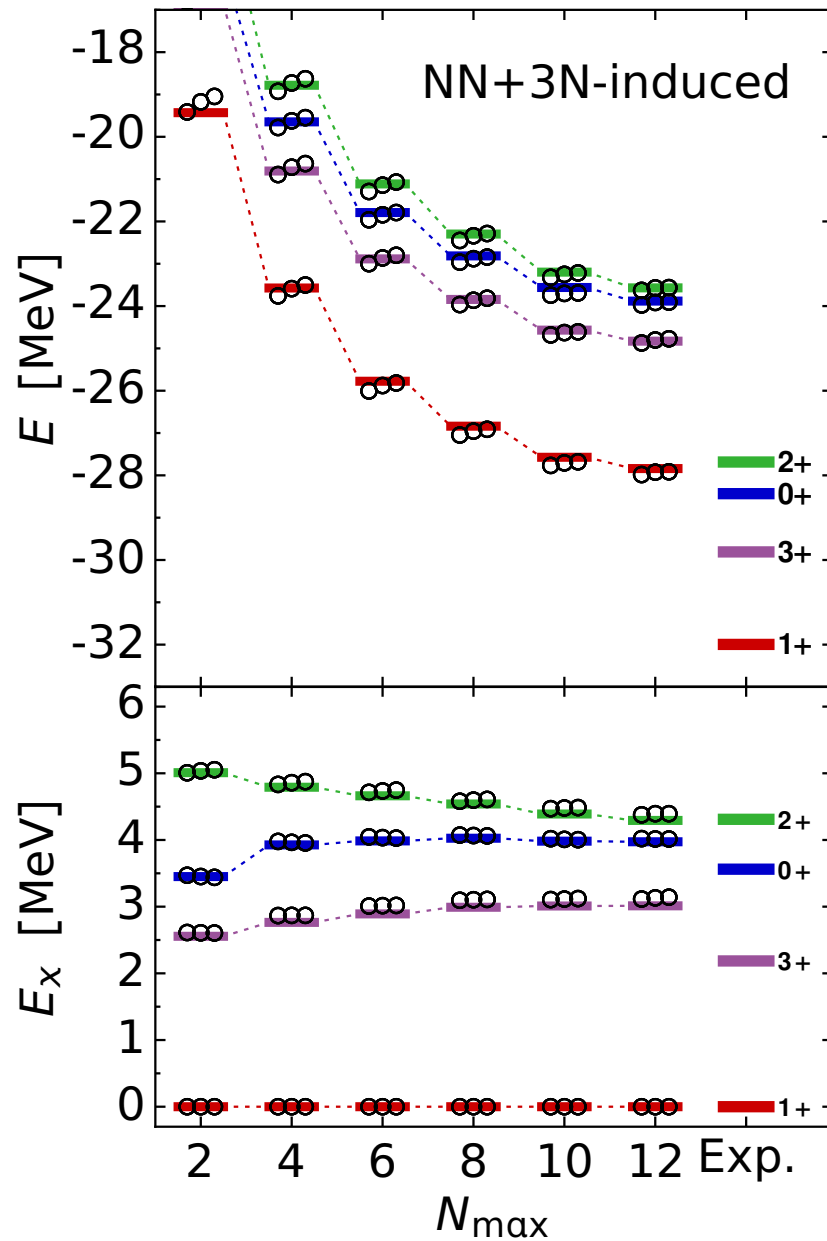


${}^6\text{Li}$

$\Lambda_{3N} = 500 \text{ MeV}$
 $\alpha = 0.08 \text{ fm}^4$
 $\hbar\Omega = 20 \text{ MeV}$

— explicit 3N


Benchmark: ${}^6\text{Li}$



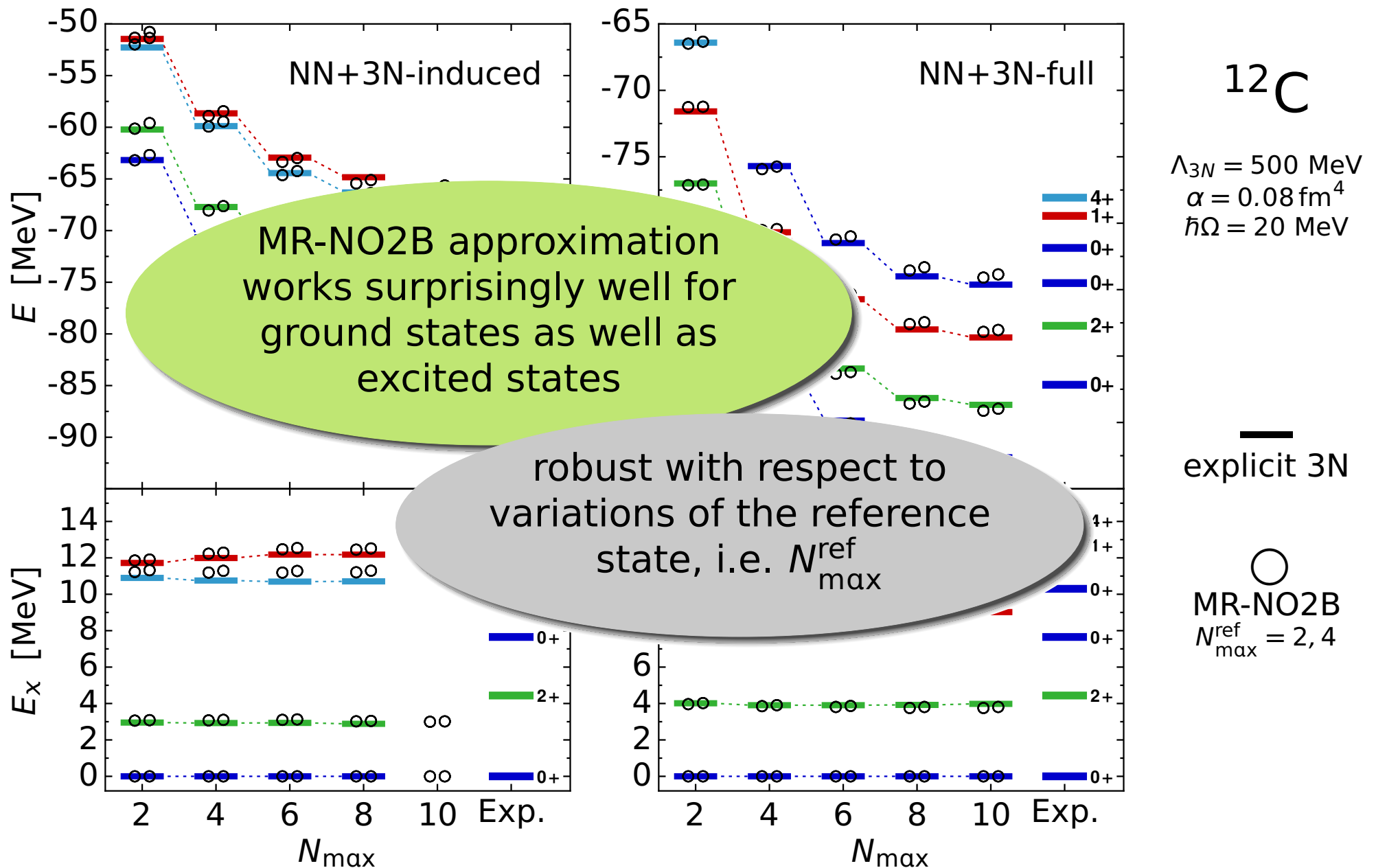
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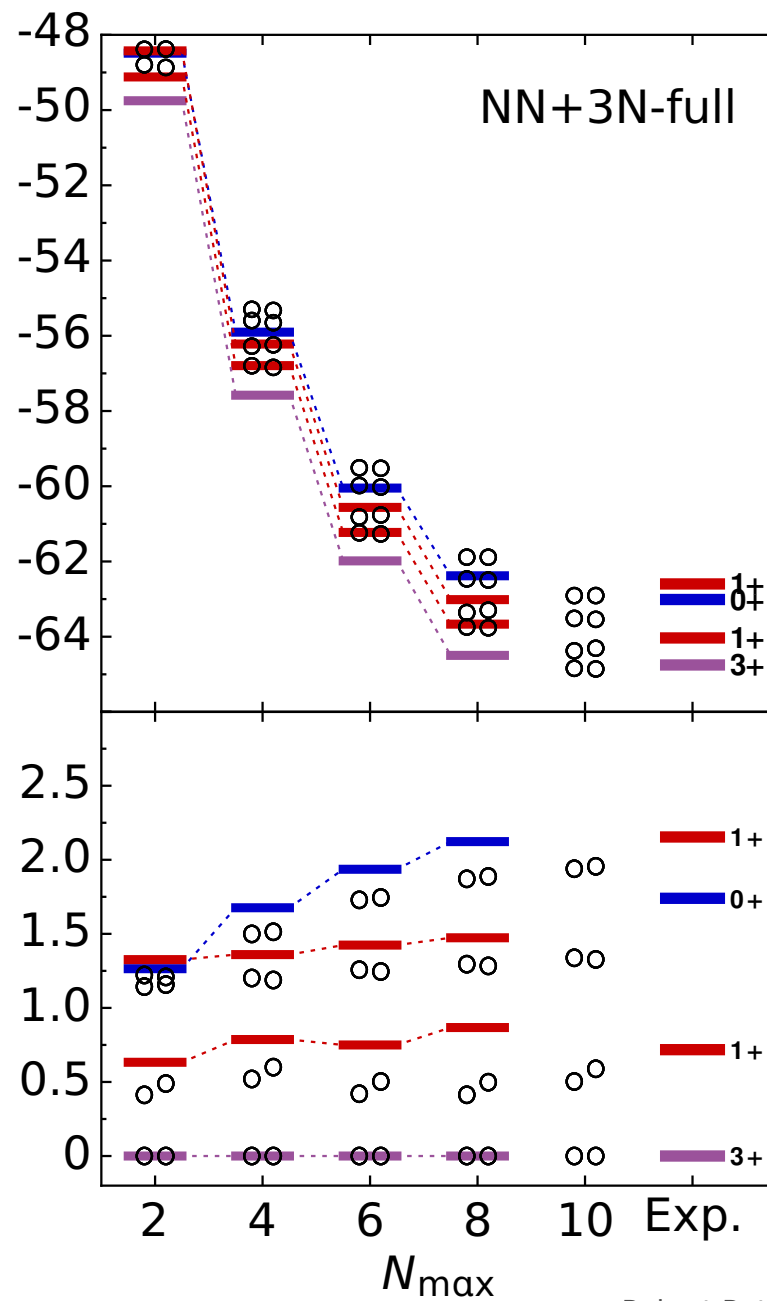
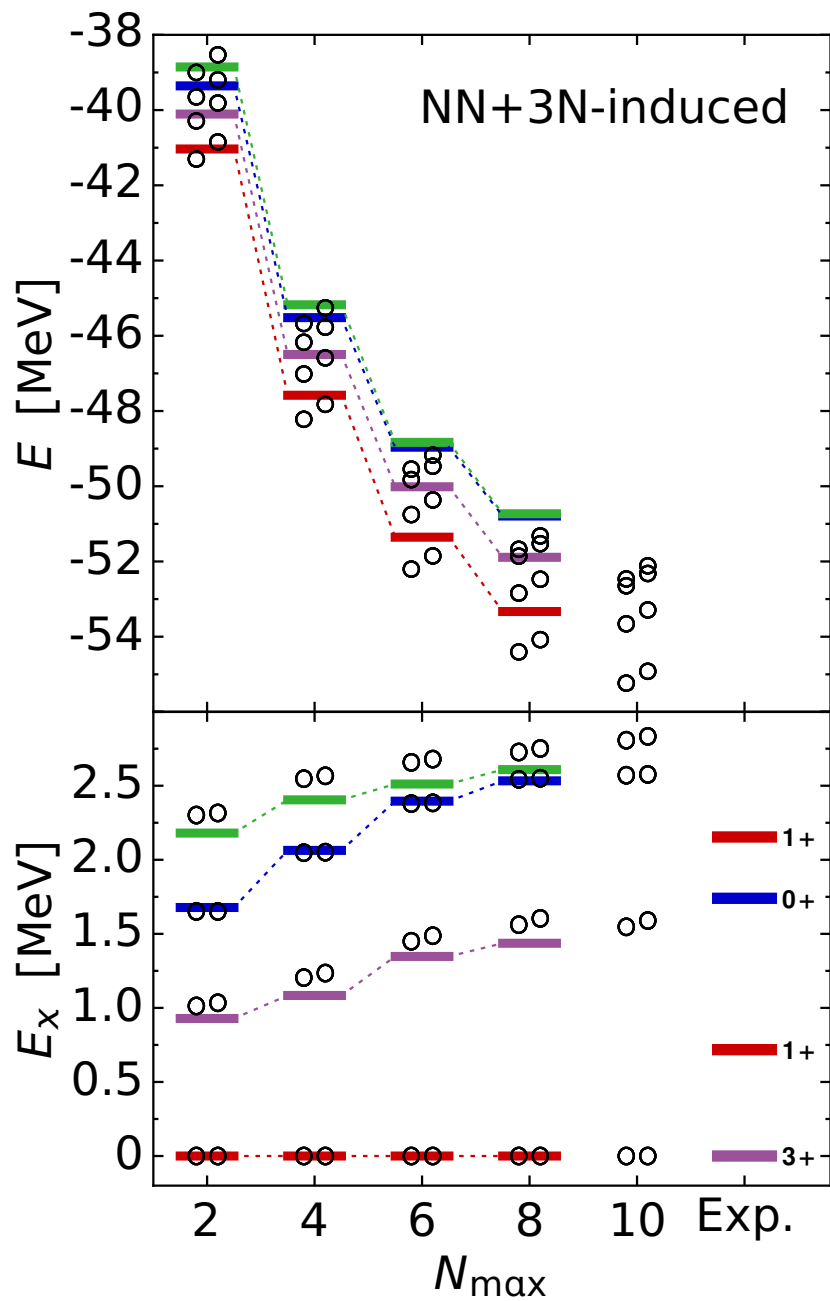

 explicit 3N


 MR-NO2B
 $N_{\text{max}}^{\text{ref}} = 2, 4, 6$

Benchmark: ^{12}C




Challenge: ^{10}B



^{10}B

$\Lambda_{3N} = 500 \text{ MeV}$
 $\alpha = 0.08 \text{ fm}^4$
 $\hbar\Omega = 16 \text{ MeV}$


 explicit 3N


 MR-NO2B
 $N_{\text{max}}^{\text{ref}} = 2, 4$

Importance-Truncated Shell Model

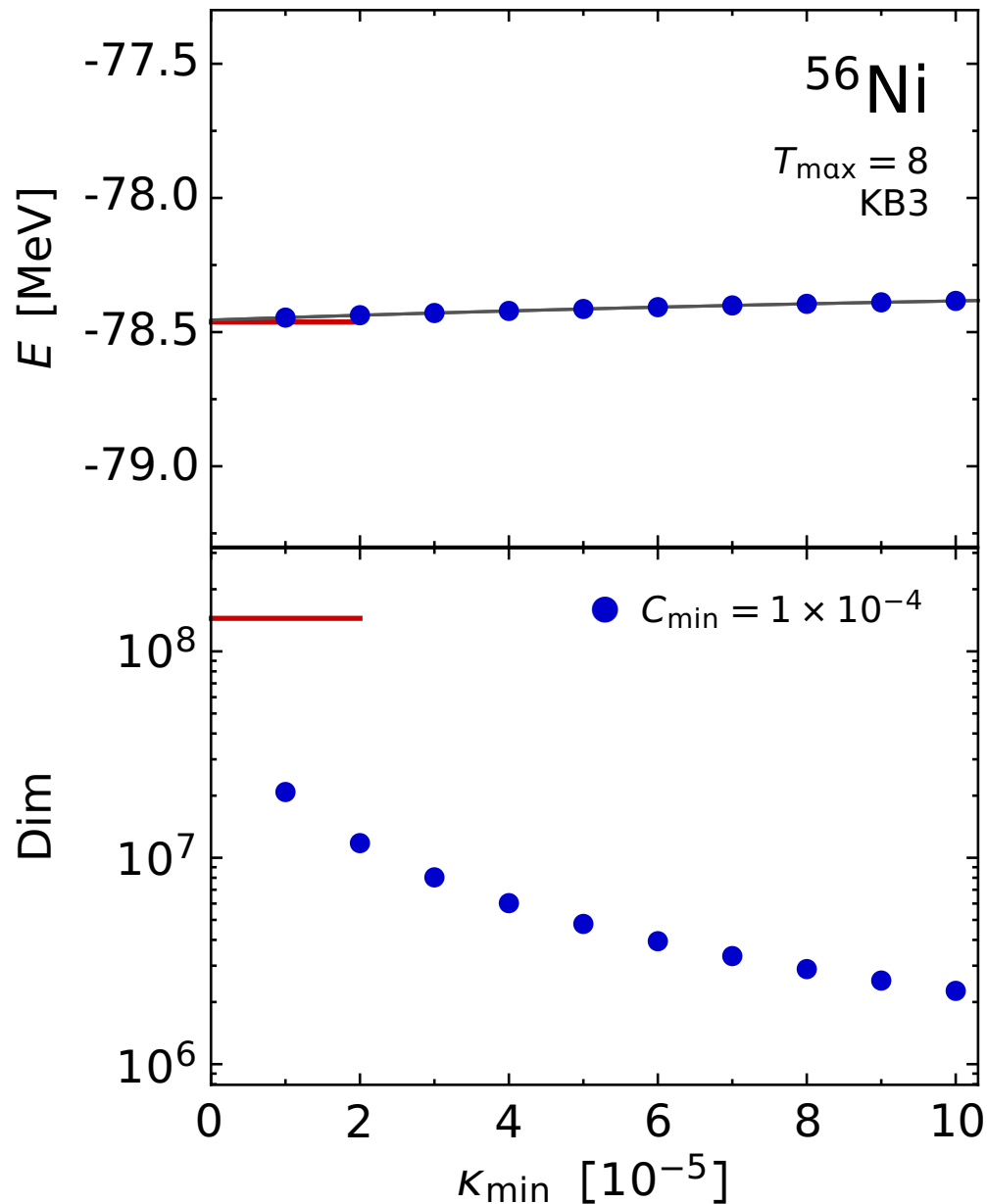
with
Christina Stumpf

Motivation: Importance Truncated SM

valence-space shell model suffers from the same limitation as the NCSM: model-space dimension

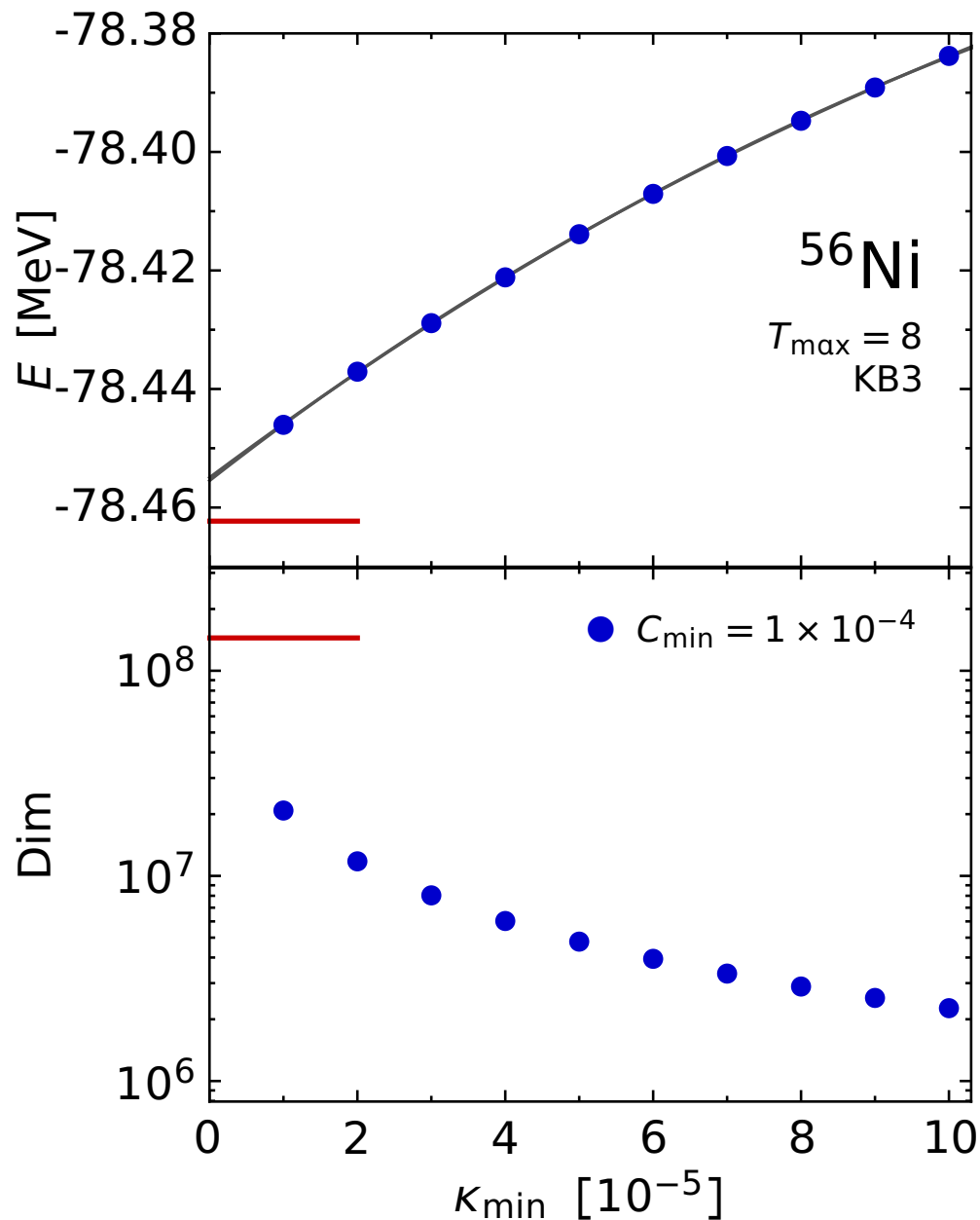
- concept of importance truncation can be **transferred directly to valence-space shell model**
- **new IT-SM code** based on our existing optimized IT-NCSM code
- use **T_{\max} truncation** instead of N_{\max} to set up iterative scheme
- start with full calculation at small T_{\max} to define initial reference state and iteratively increase T_{\max} in steps of 2

Test Case: ^{56}Ni in pf Valence-Space



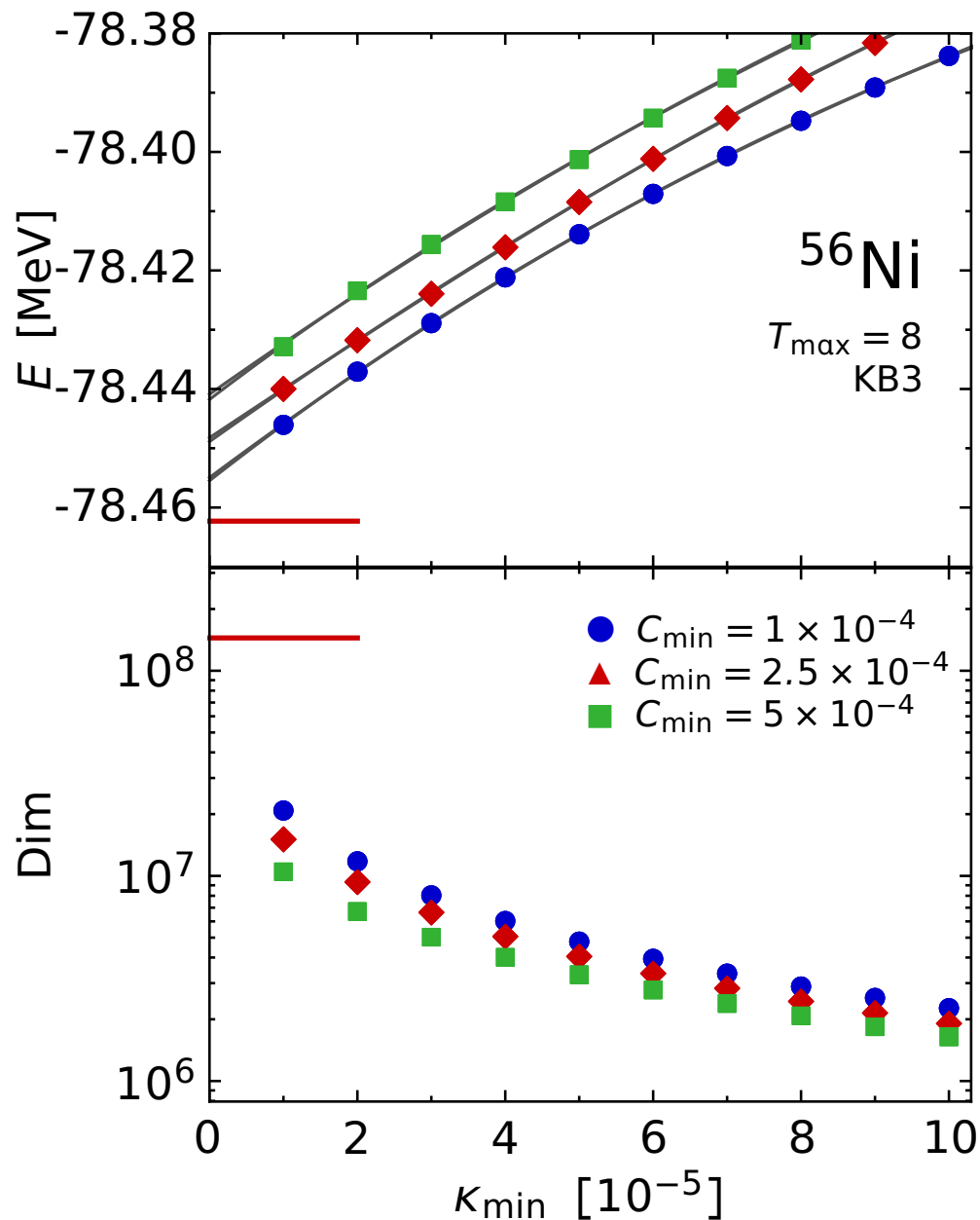
- very weak K_{min} -dependence of energies
- dimension reduced by one order of magnitude

Test Case: ^{56}Ni in pf Valence-Space



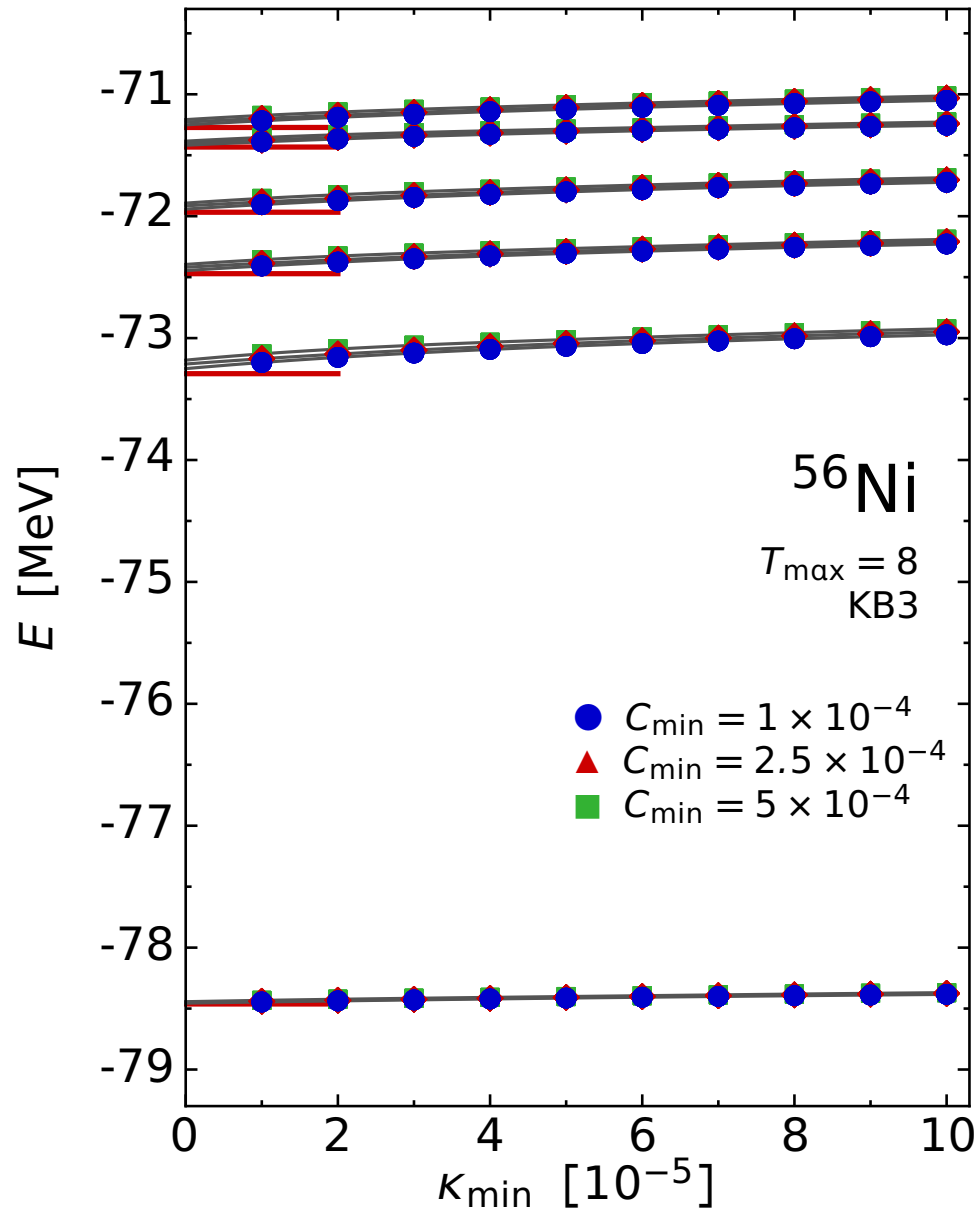
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Test Case: ^{56}Ni in pf Valence-Space



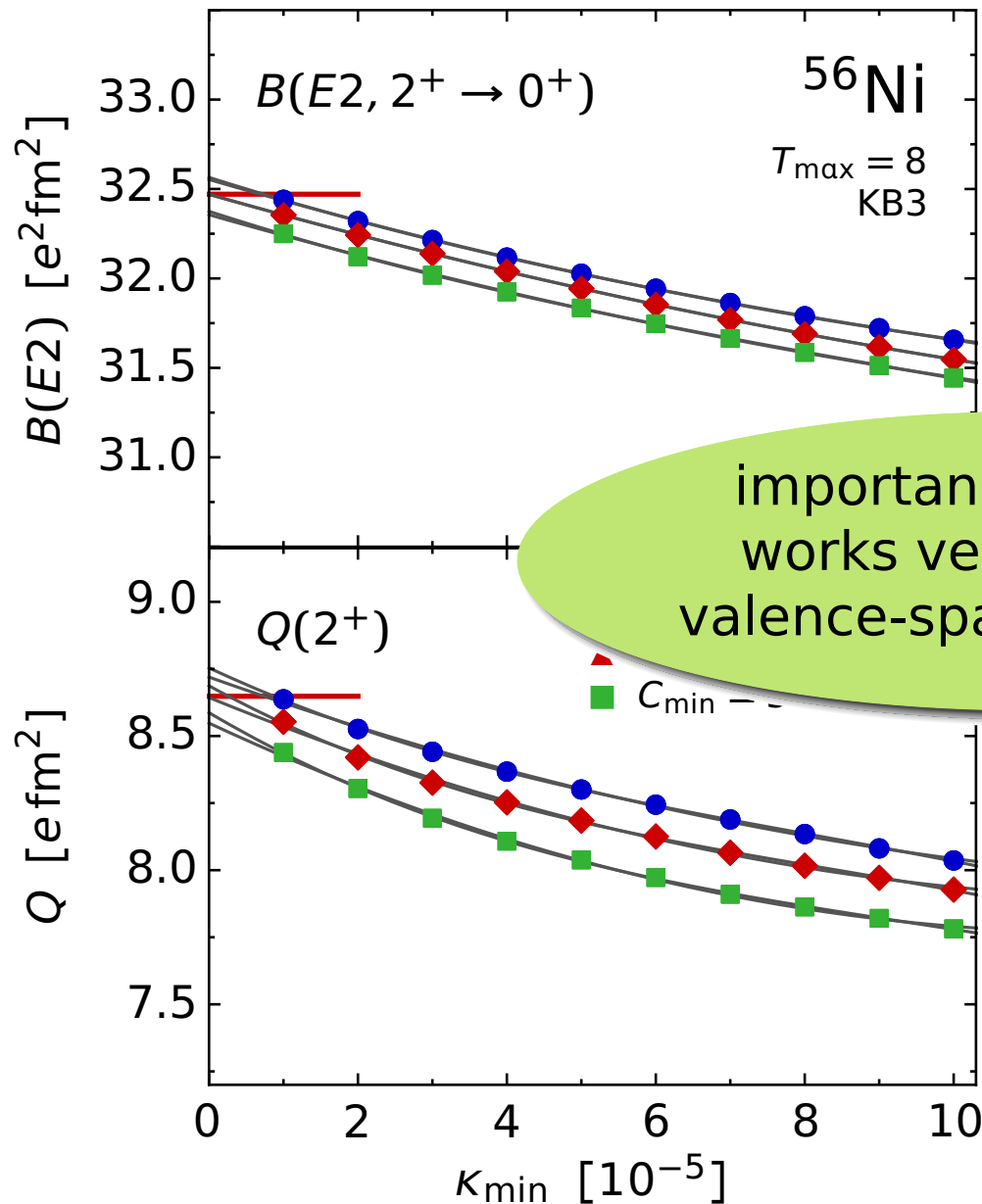
- very weak K_{min} -dependence of energies
- dimension reduced by one order of magnitude
- reference-space truncation induces ~ 5 keV deviation

Test Case: ^{56}Ni in pf Valence-Space



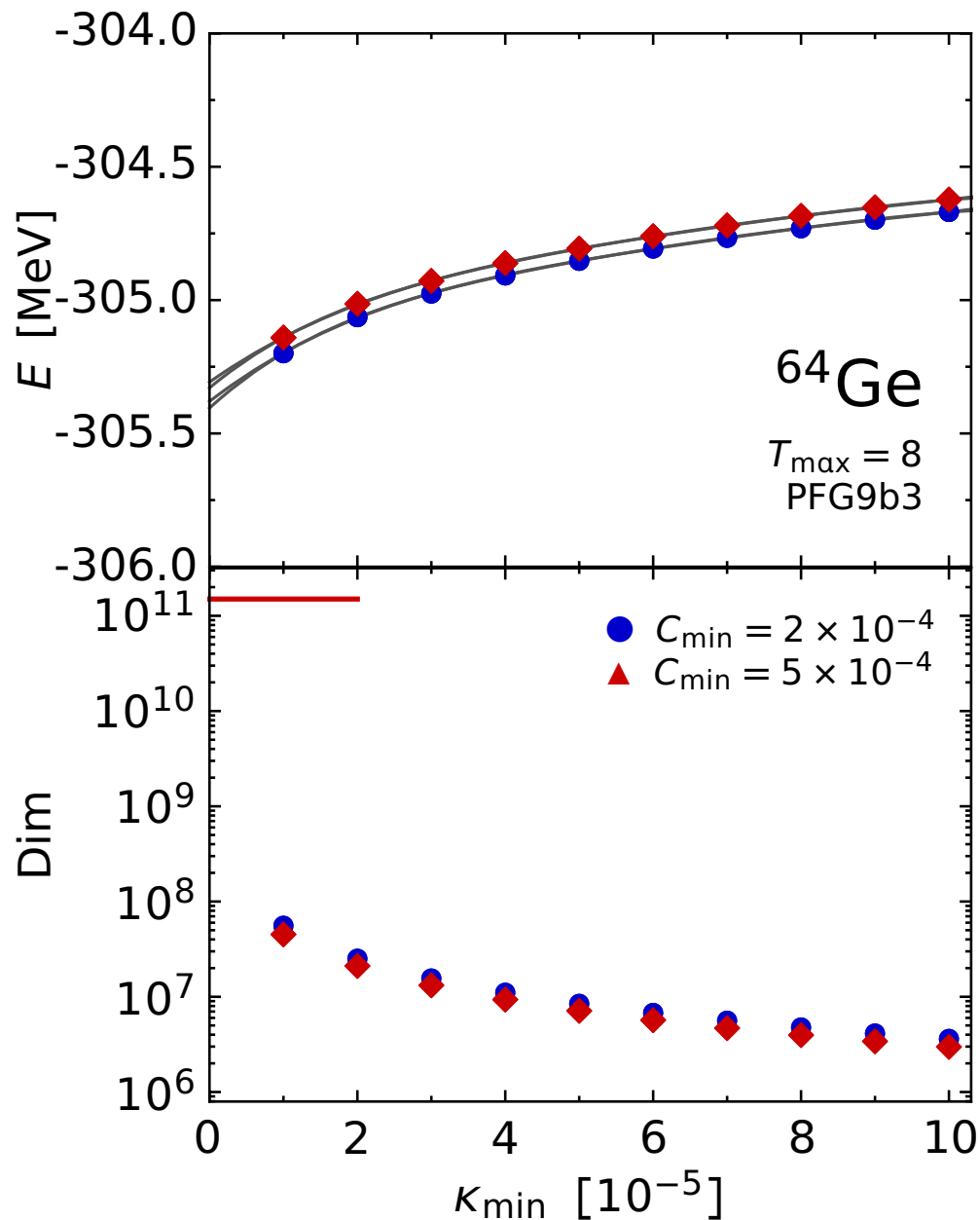
- very weak K_{\min} -dependence of energies
- dimension reduced by one order of magnitude
- reference-space truncation induces ~ 5 keV deviation
- excitation spectrum is accessible with the same accuracy

Test Case: ^{56}Ni in pf Valence-Space



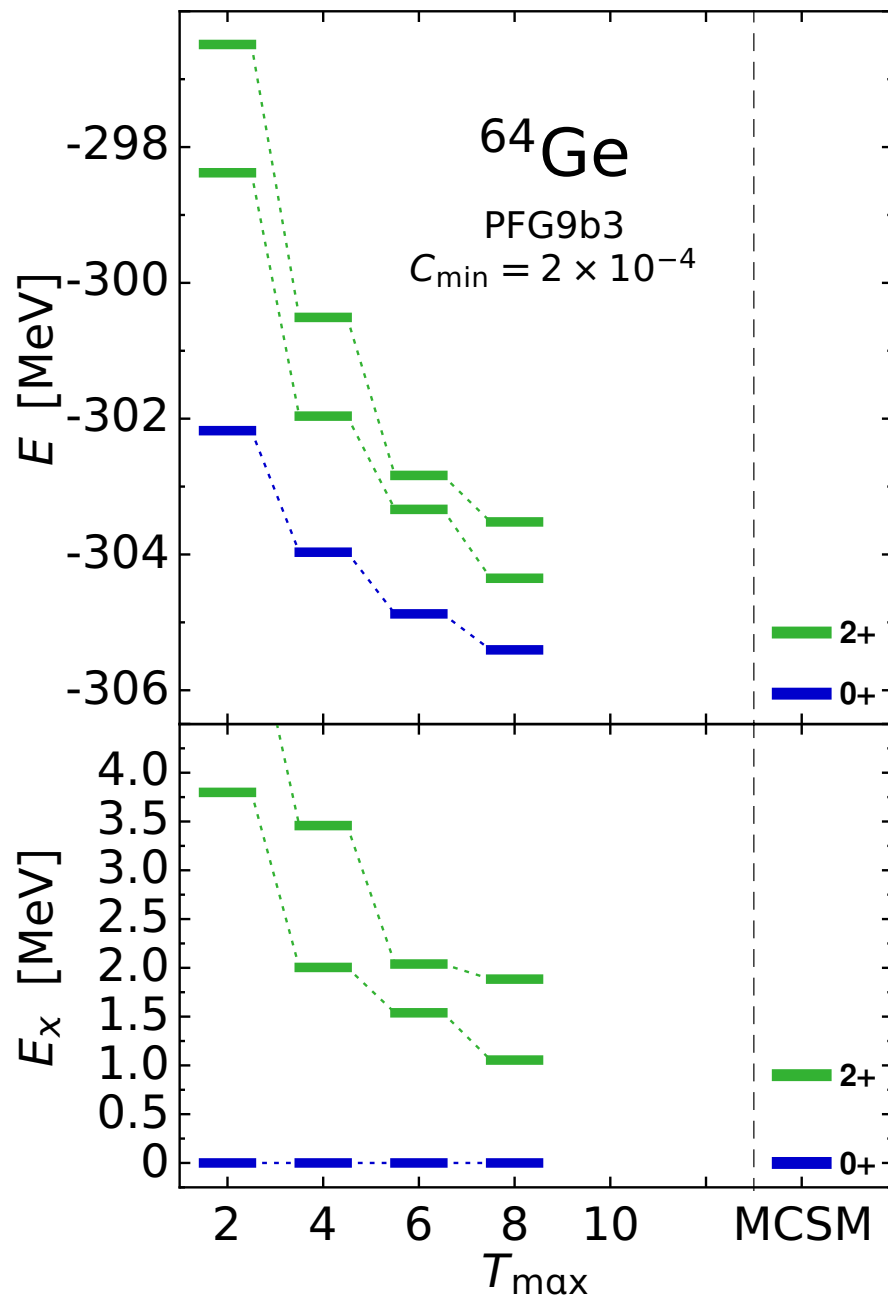
- very weak K_{\min} -dependence of energies
- dimension reduced by one order of magnitude
- importance truncation works very well in the valence-space shell model
- importance truncation in valence-space shell model is more accurate than full configuration spectrum is accessible with the same accuracy
- spectroscopic observables are equally well described

Challenge: ^{64}Ge in $pfg_{9/2}$ Valence-Space



- beyond the reach of the full shell model for $T_{\max} > 4$
- moderate K_{\min} -dependence of energies
- dimension reduced by several orders of magnitude

Challenge: ^{64}Ge in $pf g_{9/2}$ Valence-Space

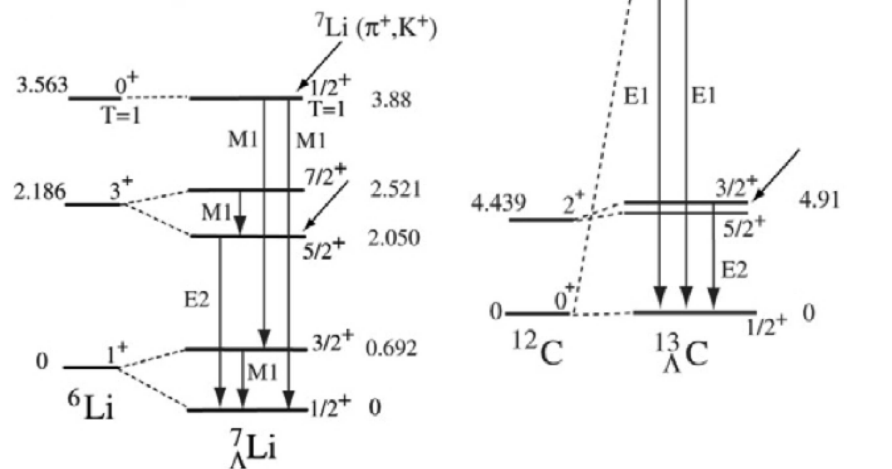
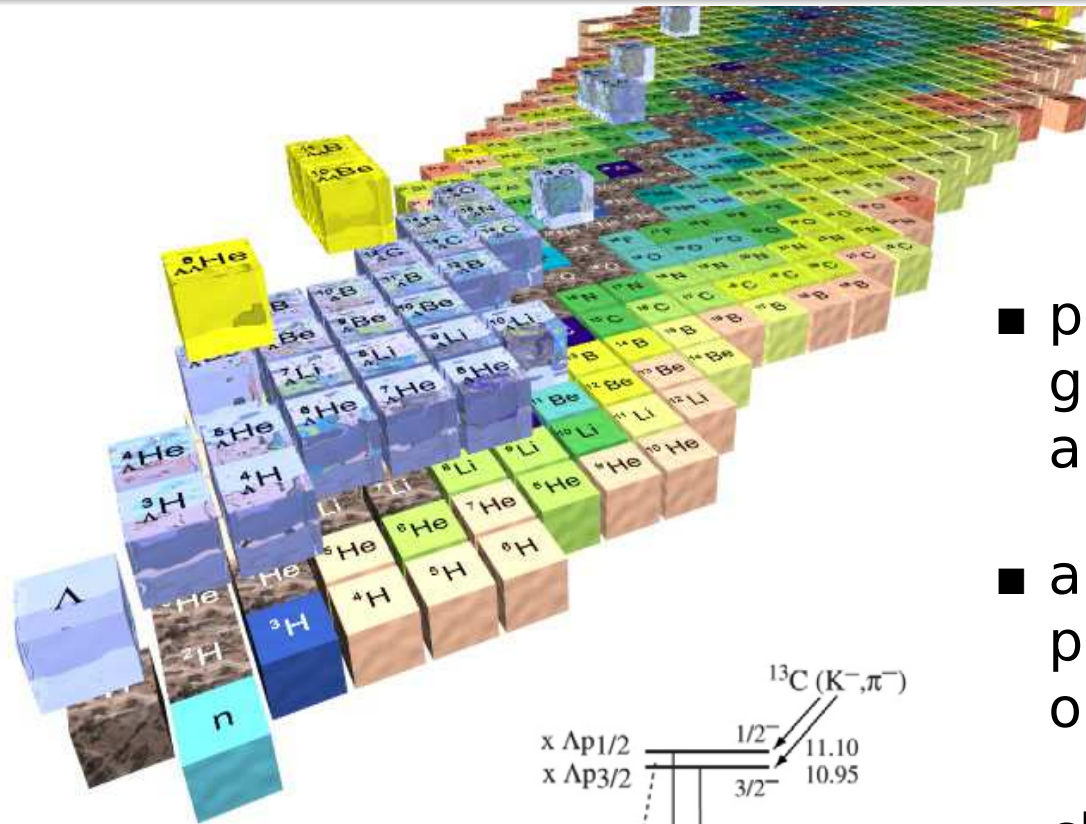


- beyond the reach of the full shell model for $T_{\max} > 4$
- moderate K_{\min} -dependence of energies
- dimension reduced by several orders of magnitude
- results consistent with Monte Carlo Shell Model (MCSM) by Otsuka et al.
- IT-SM calculations for larger T_{\max} are on their way...

Ab Initio Hypernuclear Structure

with
Roland Wirth

Motivation: Hypernuclear Structure



- precision data on hypernuclear ground states and spectroscopy are available
- ab initio few-body ($A \lesssim 4$) and phenomenological shell model or cluster calculations so far
- chiral EFT interactions including hyperons are being constructed
- constrain YN & YY interaction by ab initio hypernuclear structure calculations

Ab Initio Toolbox

■ Hamiltonian from chiral EFT

- NN+3N: standard chiral Hamiltonian (Entem&Machleidt, Navrátil)
- YN: LO chiral interaction (Haidenbauer et al.), NLO in progress

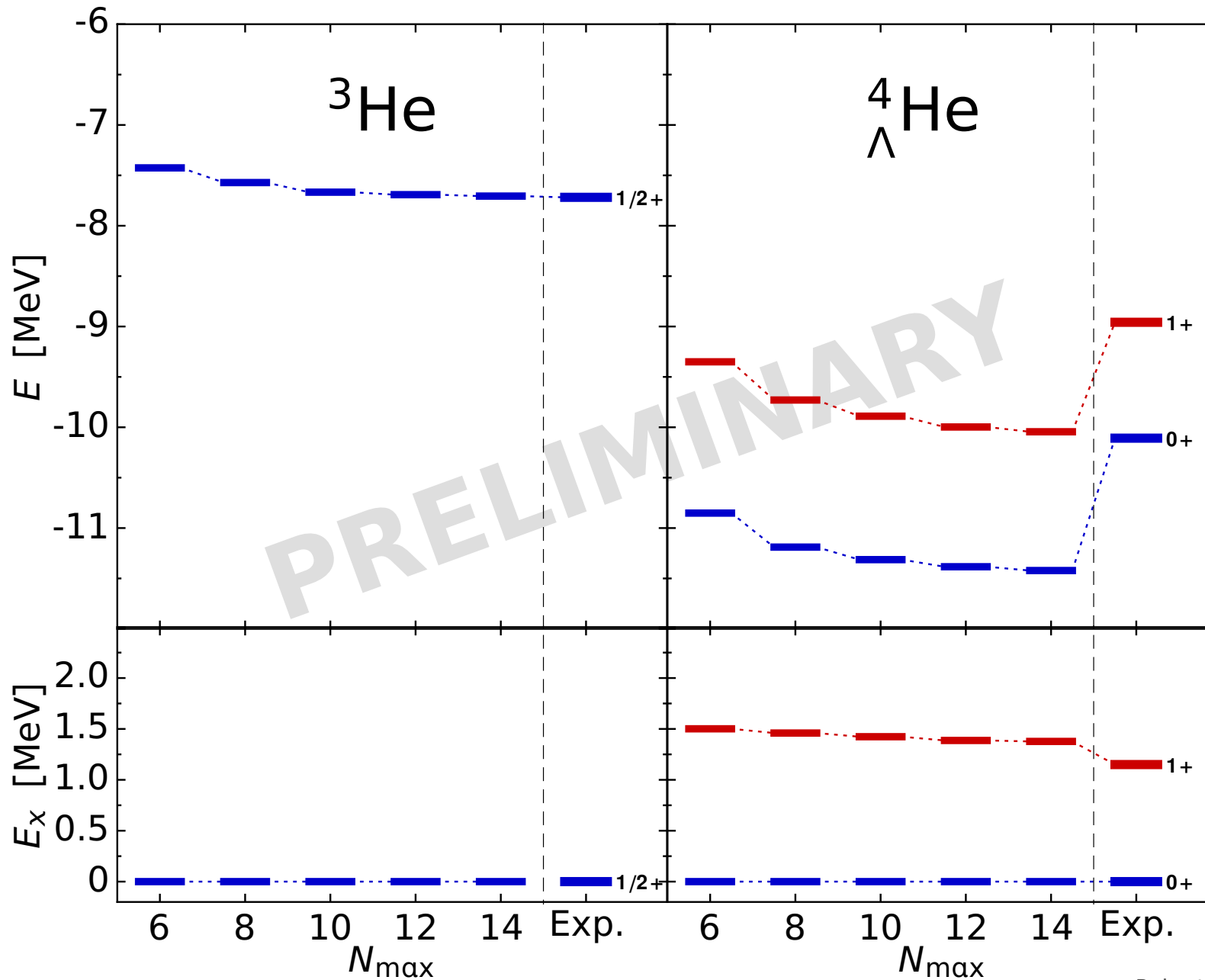
■ Similarity Renormalization Group

- consistent SRG-evolution of NN, 3N, YN interactions
- using particle basis and including $\Lambda\Sigma$ -coupling (larger matrices)
- Λ - Σ mass difference and $p\Sigma^\pm$ Coulomb included consistently

■ Importance Truncated No-Core Shell Model

- include explicit $(p, n, \Lambda, \Sigma^+, \Sigma^0, \Sigma^-)$ with physical masses
- larger model spaces easily tractable with importance truncation
- all p-shell single- Λ hypernuclei are accessible

Application: ${}^4_{\Lambda}\text{He}$



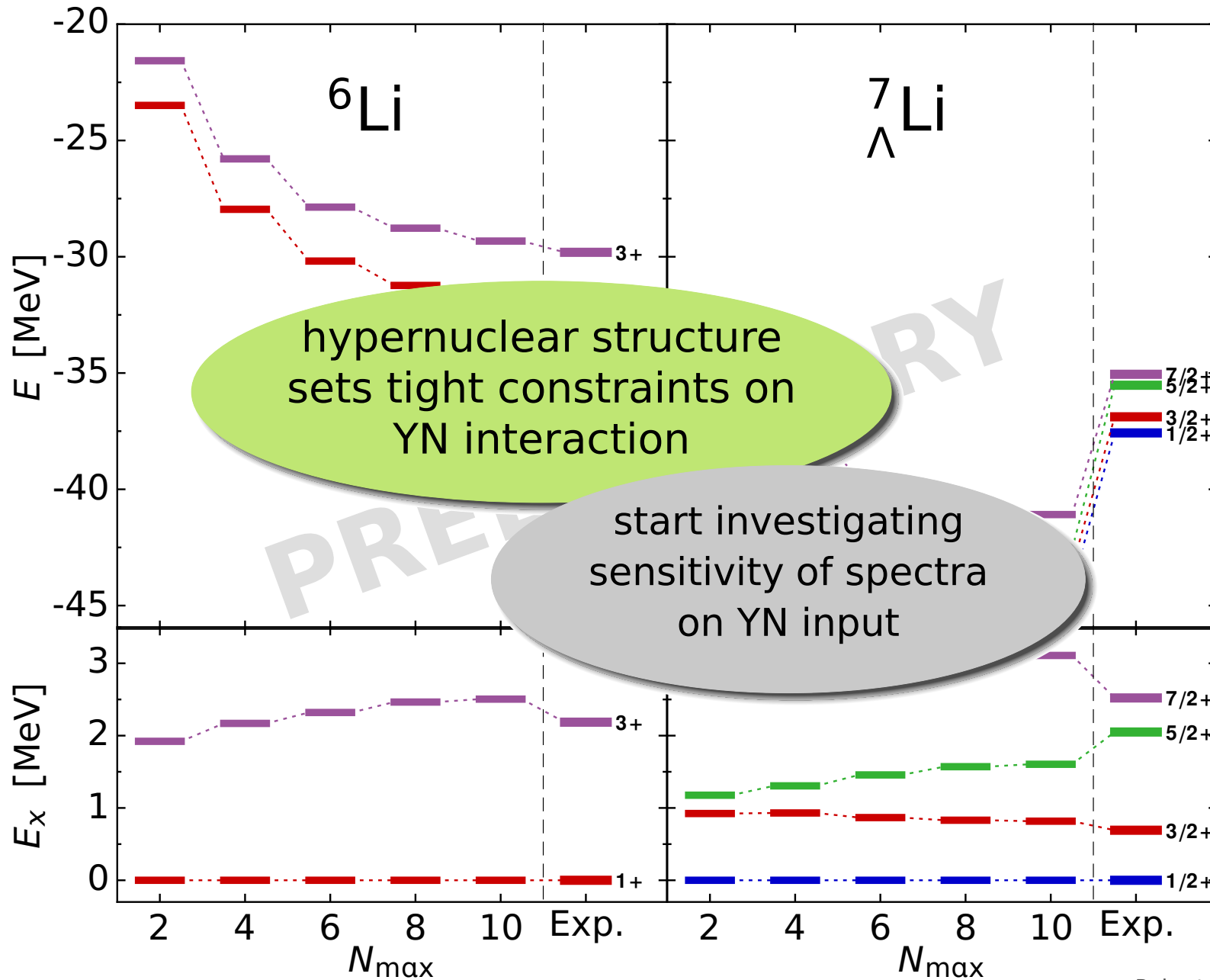
NN @ N3LO
Entem&Machleidt
 $\Lambda_{NN} = 500$ MeV

3N @ N2LO
Navratil
 $\Lambda_{3N} = 500$ MeV
triton fit

YN @ LO
Haidenbauer et al.
 $\Lambda_{YN} = 600$ MeV
scatt. & hypertriton

$\alpha = 0.08 \text{ fm}^4$
 $\hbar\Omega = 20$ MeV

Application: ${}^7_{\Lambda}\text{Li}$



hypernuclear structure sets tight constraints on YN interaction

start investigating sensitivity of spectra on YN input

NN @ N3LO
 Entem&Machleidt
 $\Lambda_{NN} = 500$ MeV

3N @ N2LO
 Navratil
 $\Lambda_{3N} = 500$ MeV
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YN @ LO
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Conclusions

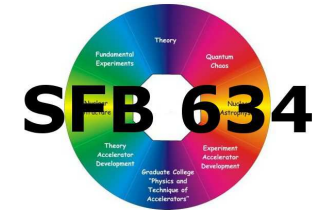
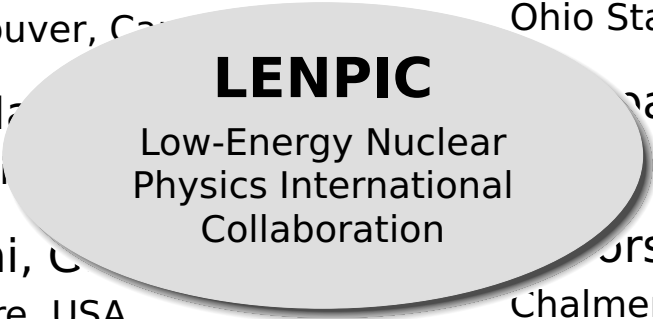
Conclusions

- importance truncation pushes the limit for **ab initio NCSM calculations** into the sd-shell
 - controlled and quantified uncertainties
 - preserves all the goodies of the full NCSM
 - carbon and oxygen isotopic chains up to the drip lines
- several **new directions**
 - multi-reference normal-ordering for approximate inclusion of 3N
 - importance truncation for the valence-space shell model
 - first ab initio calculations for p-shell hypernuclei
- many **exciting applications** ahead...

Epilogue

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