Adequacy of the SU(3)-scheme Basis for No-Core Shell Model Calculations

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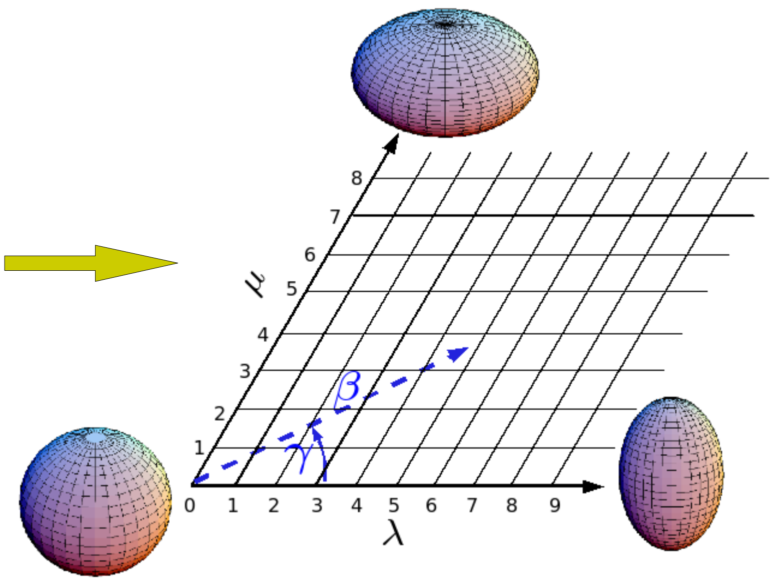
Perspectives of the Ab Initio No-Core Shell Model, February 23–25 2012, Vancouver, Canada

SU(3)-Scheme Basis

- Complete basis
- Relevant for description of spatially deformed nuclei & nuclear collective motion
- SU(3) is a subgroup of the symplectic model of the nuclear collective motion
- $(\lambda\ \mu)$ related to shape variables β and γ of the collective model
- Allows to include correlations important for α - cluster structures



J-coupled proton-neutron basis labeled by intrinsic spins Sp Sn S and deformations

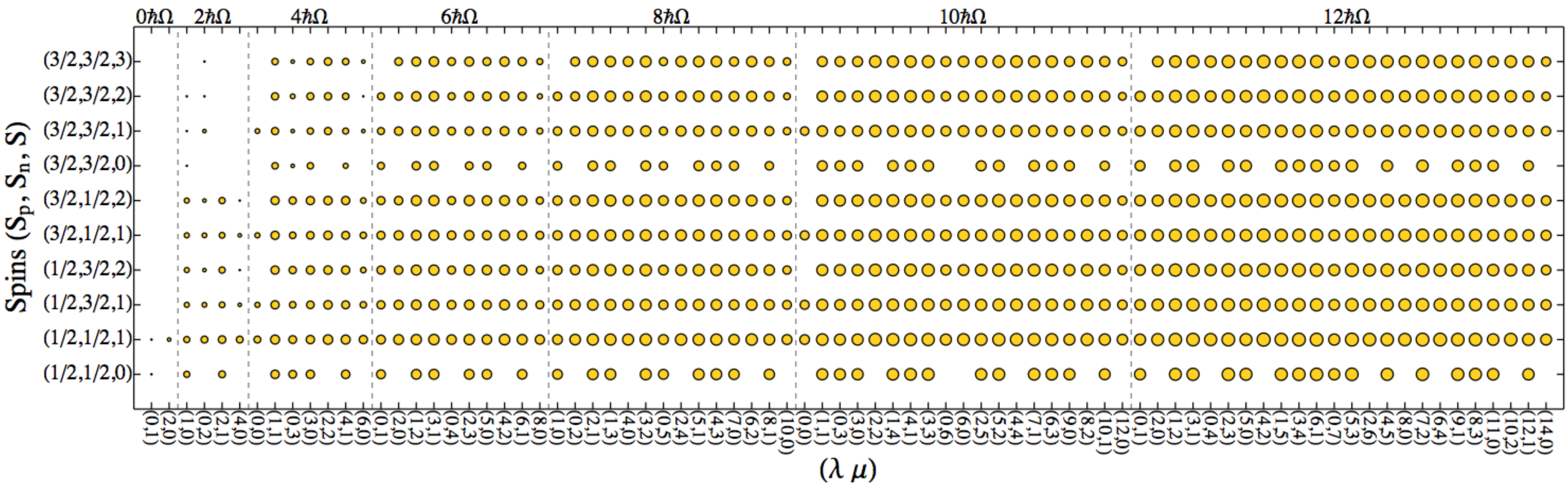
$$|\gamma\ \underbrace{S_p\, S_n S}_{\text{intrinsic spin part}}\ \underbrace{(\lambda\, \mu)\ \kappa\ L}_{\text{spatial part}}\ \ J\, M\rangle$$

- $(\lambda\, \mu)$ → SU(3) quantum numbers
- κ → multiplicity label – needed to distinguish multiple occurrence of L
- L → orbital angular momentum

Nuclear Hilbert Space in SU(3)-scheme Basis

Example: $^{6}\mathrm{Li}$

$N_{\max} = 12$
$J = 1$



- $N\hbar\Omega$ space: direct sum of subspaces [●] of states carrying the same $(\lambda\ \mu)$ and $S_p S_n S$
- c.m. spurious states can be removed from each subspace exactly

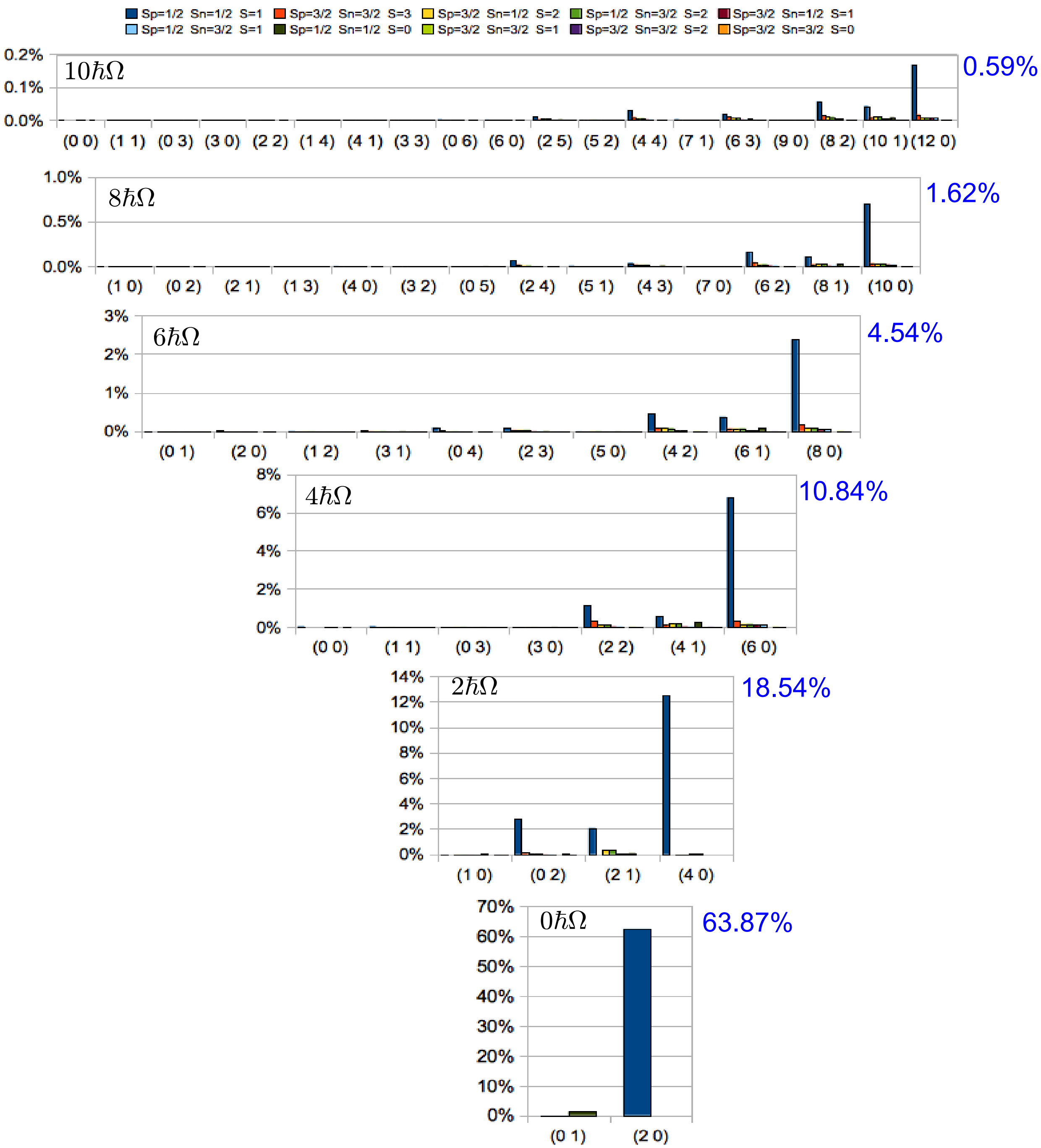
- SU(3)-scheme allows truncations according to (1) maximal number of total HO quanta Nmax
  (2) intrinsic spins $S_p\ S_n\ S$
  (3) deformations $(\lambda\ \mu)$
- Realistic interactions: enormous mixing of different $S_p S_n S\ (\lambda\ \mu)$ subspaces
- Coherent mixing of $N\hbar\Omega\ \ S_p S_n S\ (\lambda\ \mu)$ subspaces due to a persistent Sp(3,R) symmetry

^{6}Li : ground state

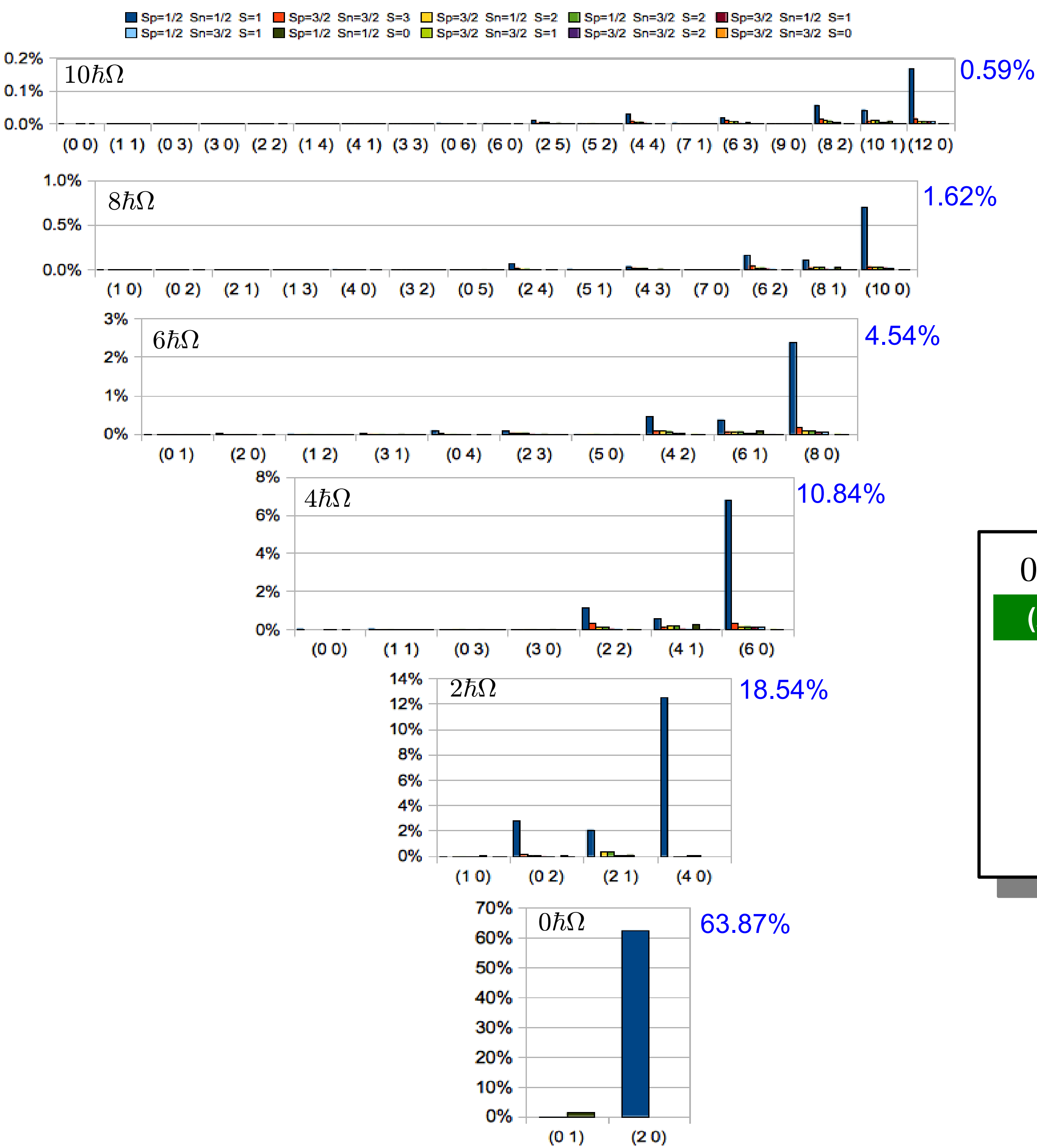
- JISP16 bare + Vcoul interactions

$N_{\max} = 10$

$\hbar\Omega = 20$ MeV



^{6}Li : ground state



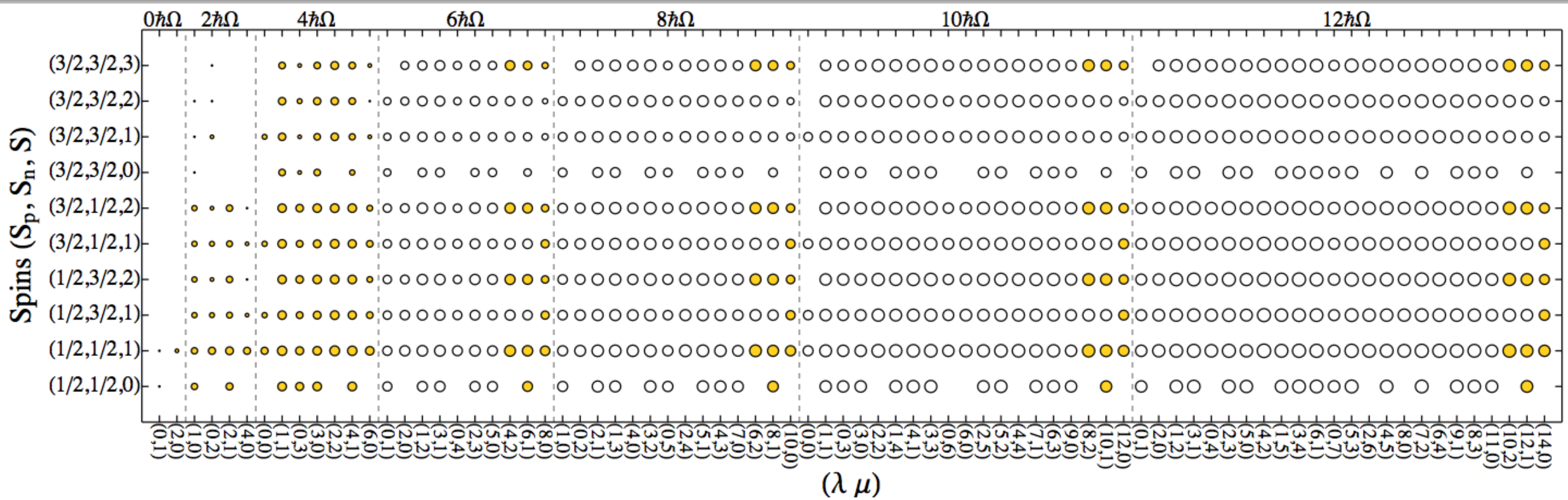
- JISP16 bare + Vcoul interactions
  $N_{\max} = 10$
  $\hbar\Omega = 20$ MeV

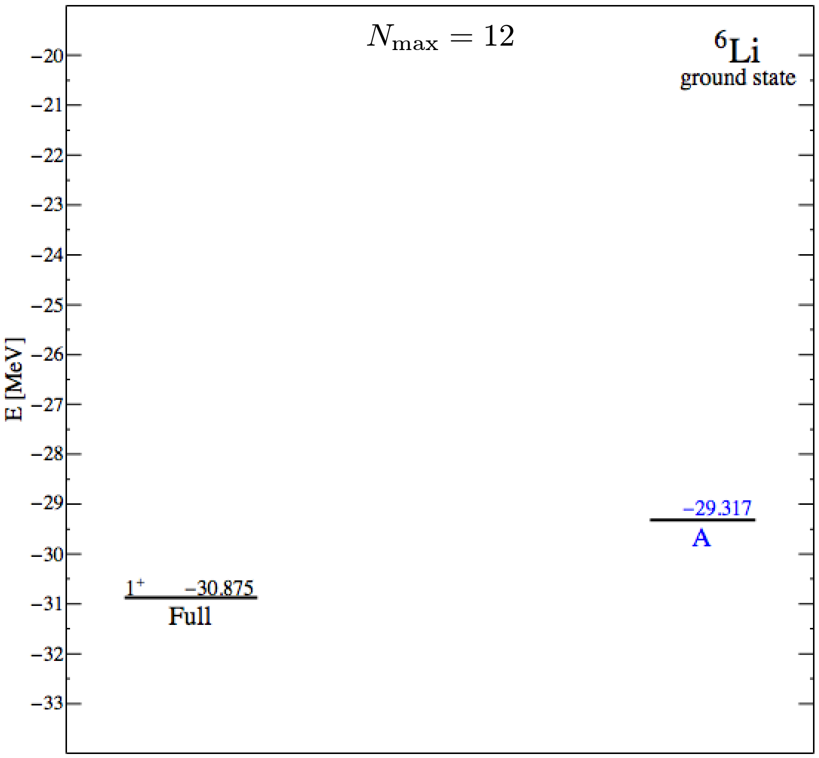
- four Sp Sn S components dominate (over 99%)
- Coherent pattern of important deformations

$0\hbar\Omega$	$2\hbar\Omega$	$4\hbar\Omega$	$6\hbar\Omega$	$8\hbar\Omega$	$10\hbar\Omega$	...
(2 0)	(4 0)	(6 0)	(8 0)	(10 0)	(12 0)	
	(2 1)	(4 1)	(6 1)	(8 1)	(10 1)	
	(0 2)	(2 2)	(4 2)	(6 2)	(8 2)	
			(2 3)	(4 3)	(6 3)	
			(0 4)	(2 4)	(4 4)	

- $(\lambda_0 + k \quad \mu_0)\ k = 0, 2, 4, 6 \ldots$
- indication that Sp(3,R) symmetry is persistent

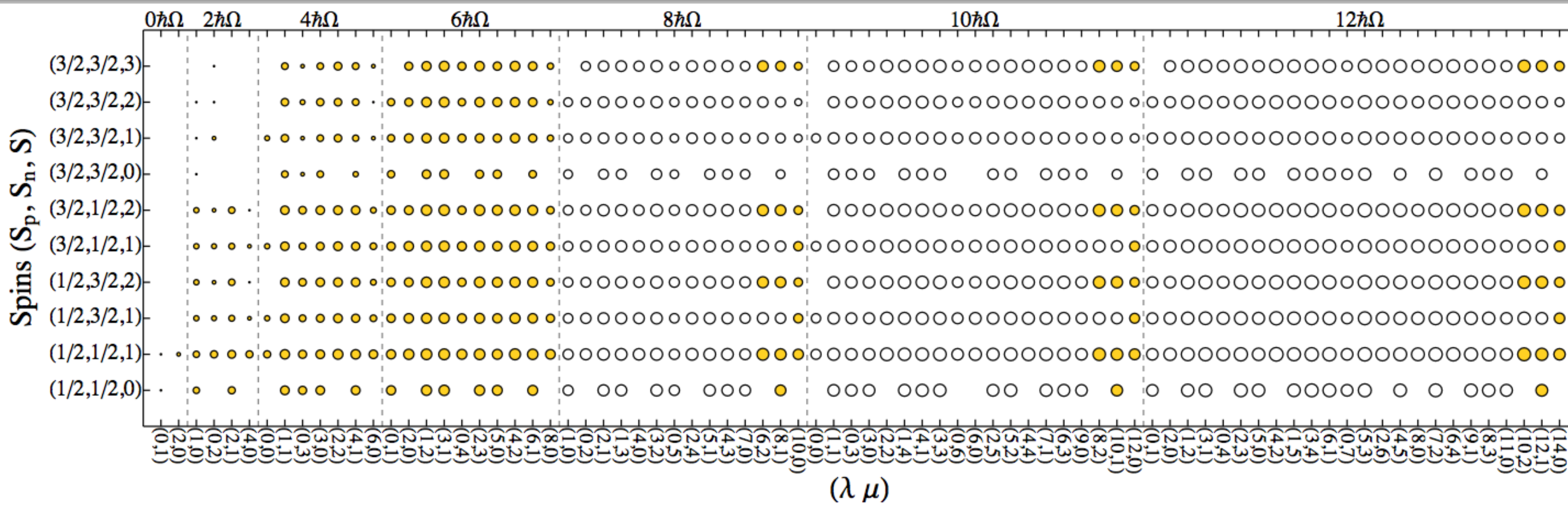
Model Space A

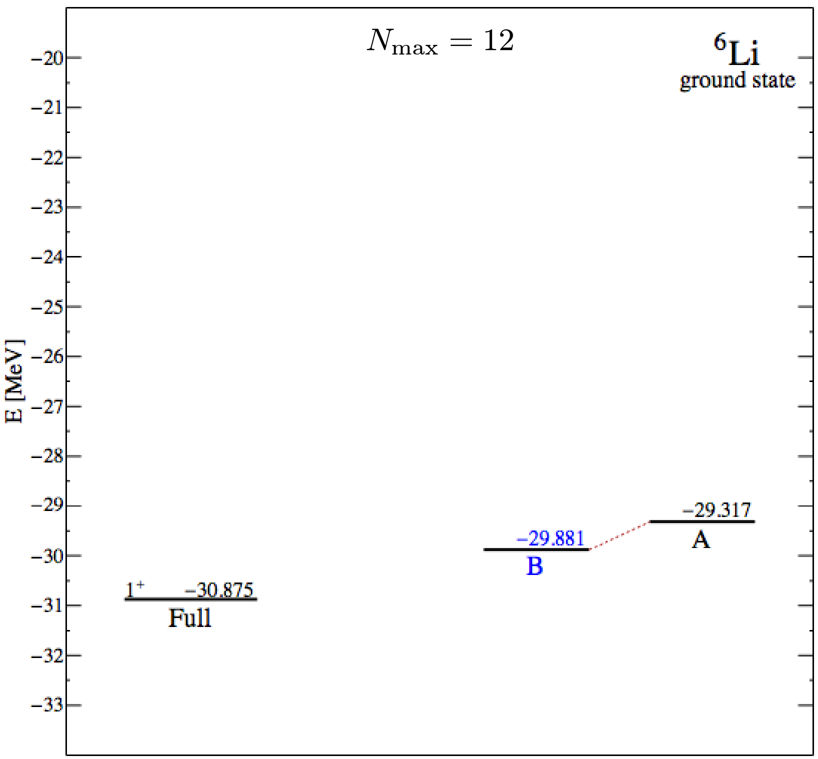




	A	B	C	Full
E	-29.317			-30.875
RMS (mass)	2.035			2.090
E2 moment	-0.062			-0.066
M1 moment	0.839			0.836
dimension	3.7%			100%

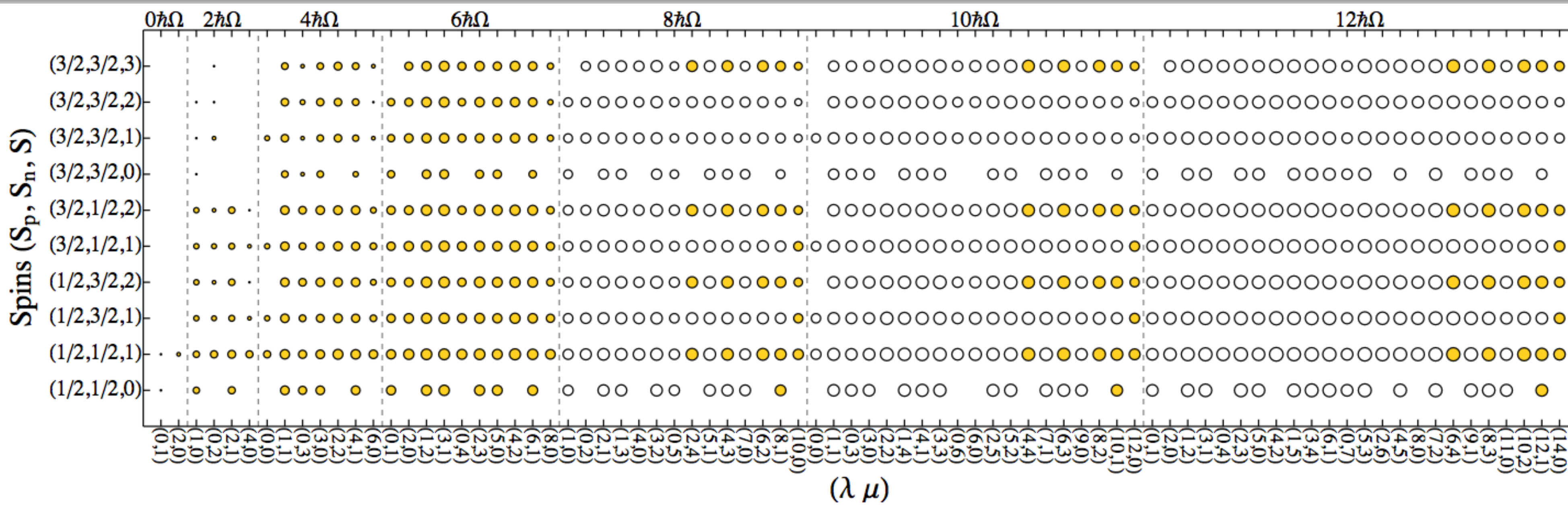
Model Space B

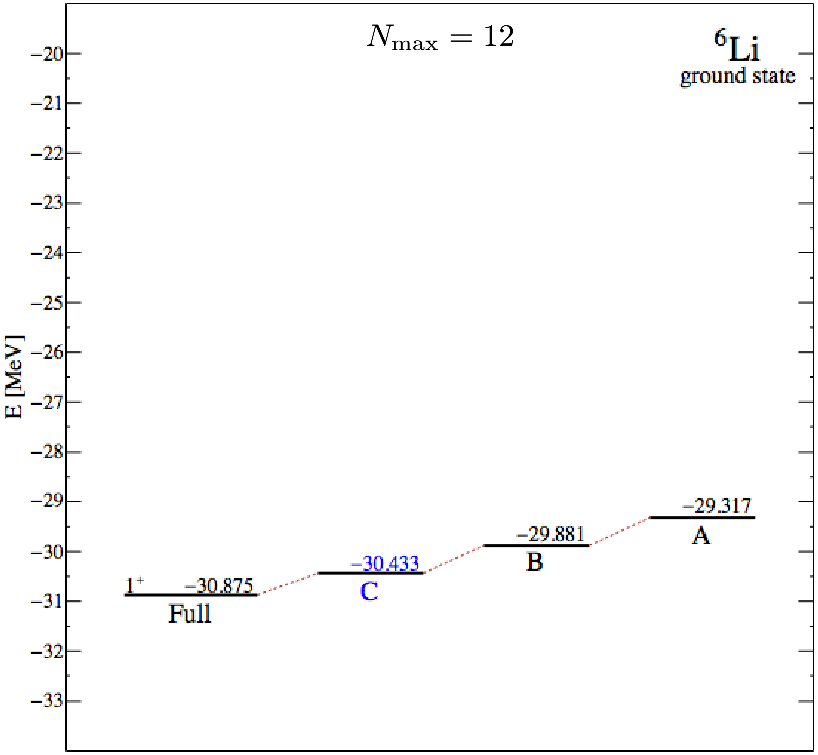




	A	**B**	C	Full
E	-29.317	-29.881		-30.875
RMS (mass)	2.035	2.042		2.090
E2 moment	-0.062	-0.069		-0.066
M1 moment	0.839	0.838		0.836
dimension	3.7%	4.2%		100%

Model Space C





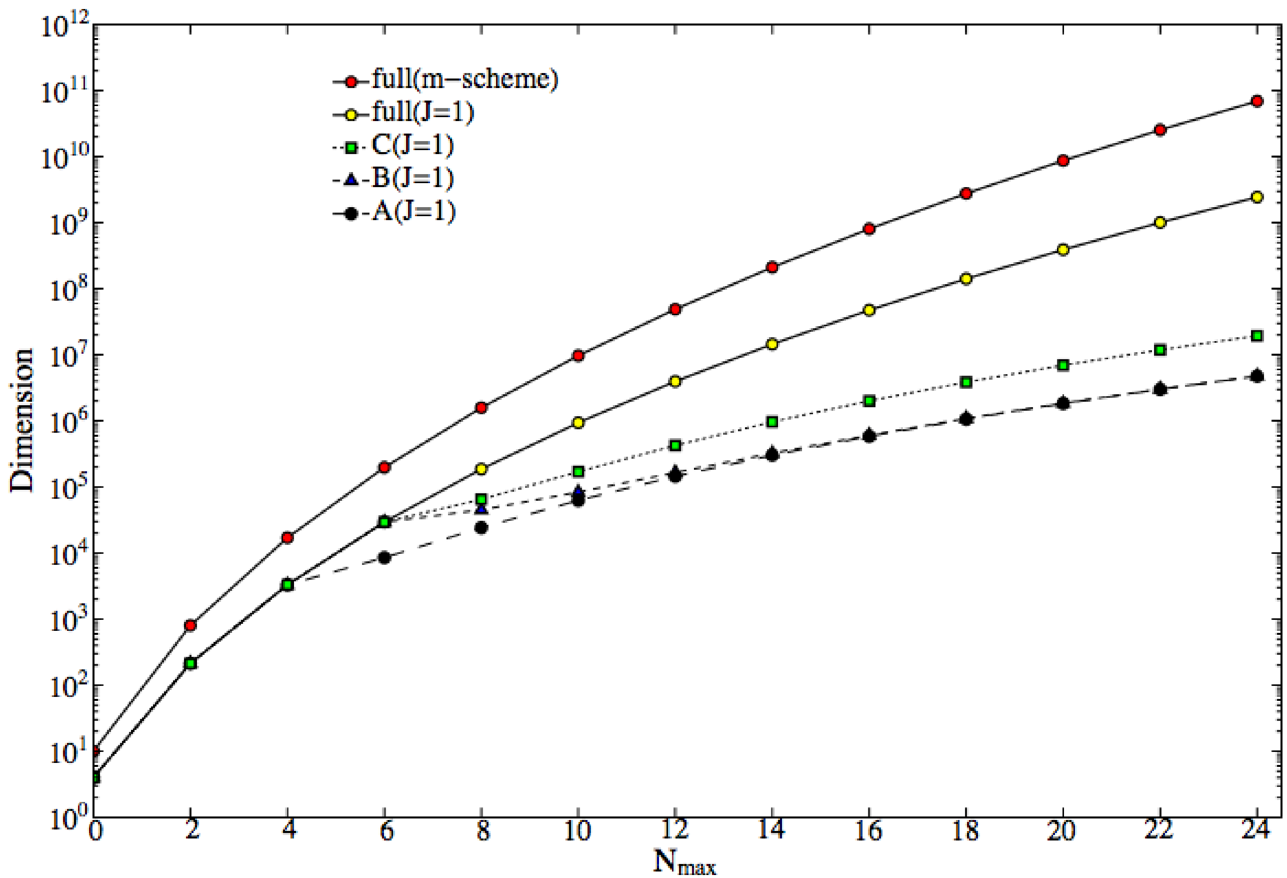
	A	B	C	Full
E	-29.317	-29.881	-30.433	-30.875
RMS (mass)	2.035	2.042	2.075	2.090
E2 moment	-0.062	-0.069	-0.074	-0.066
M1 moment	0.839	0.838	0.837	0.836
dimension	3.7%	4.2%	10.8%	100%

Truncation Efficacy

- Number of non zero matrix elements [millons]

Nmax:	8	10	12
M-scheme M=1	776	8,443	70,381
J-scheme	636	7,249	62,286
SU(3)-scheme	1,945	31,177	~380,000
B	146	325	823
C	276	1,193	4,861

- Model space reduction: two orders of magnitude and even more substantial for higher Nmax and heavier nuclei

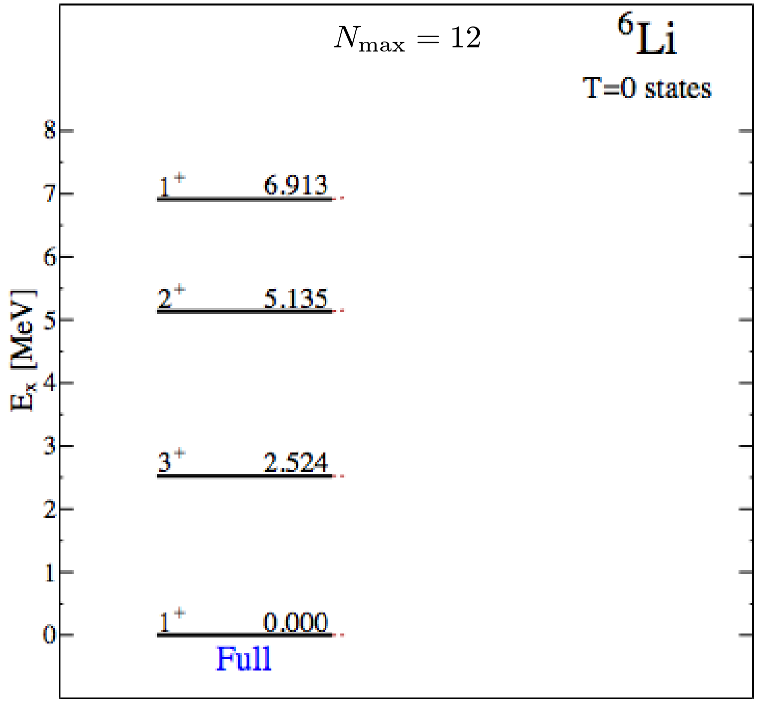


^{6}Li : low-lying T=0 states

- Symmetry-truncated model space C: "tuned" to describe the ground state of 6Li

(2 0) S=1 L=0 x S=1 --> J = 1 → leading configuration of 6Li ground state [62%]

(2 0) S=1 L=2 x S=1 --> J = 3, 2, 1 major components of excited T=0 states?



- Calculate excited T=0 states in model space C

Low-lying T=0 states in 6Li

- Symmetry-truncated model space C: "tuned" to describe the ground state of 6Li

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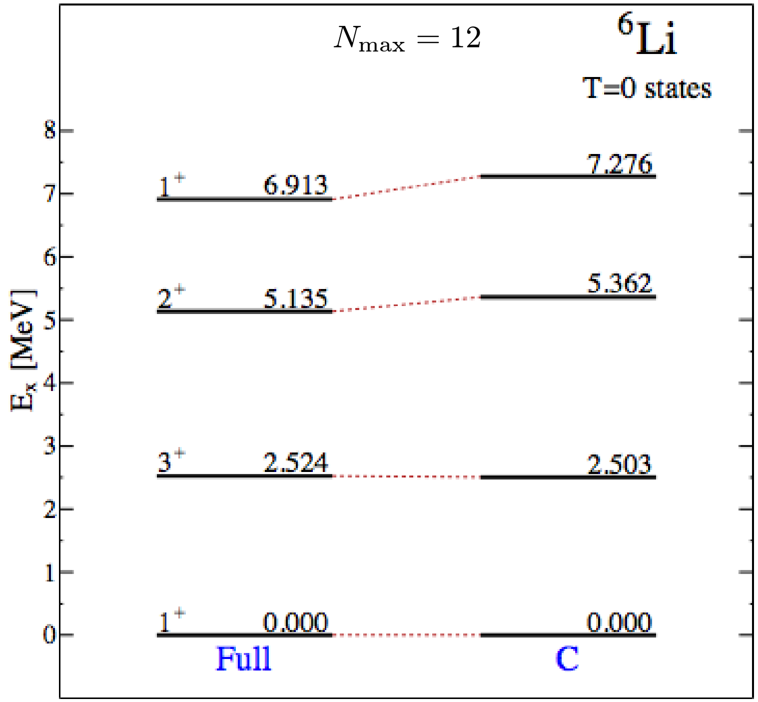
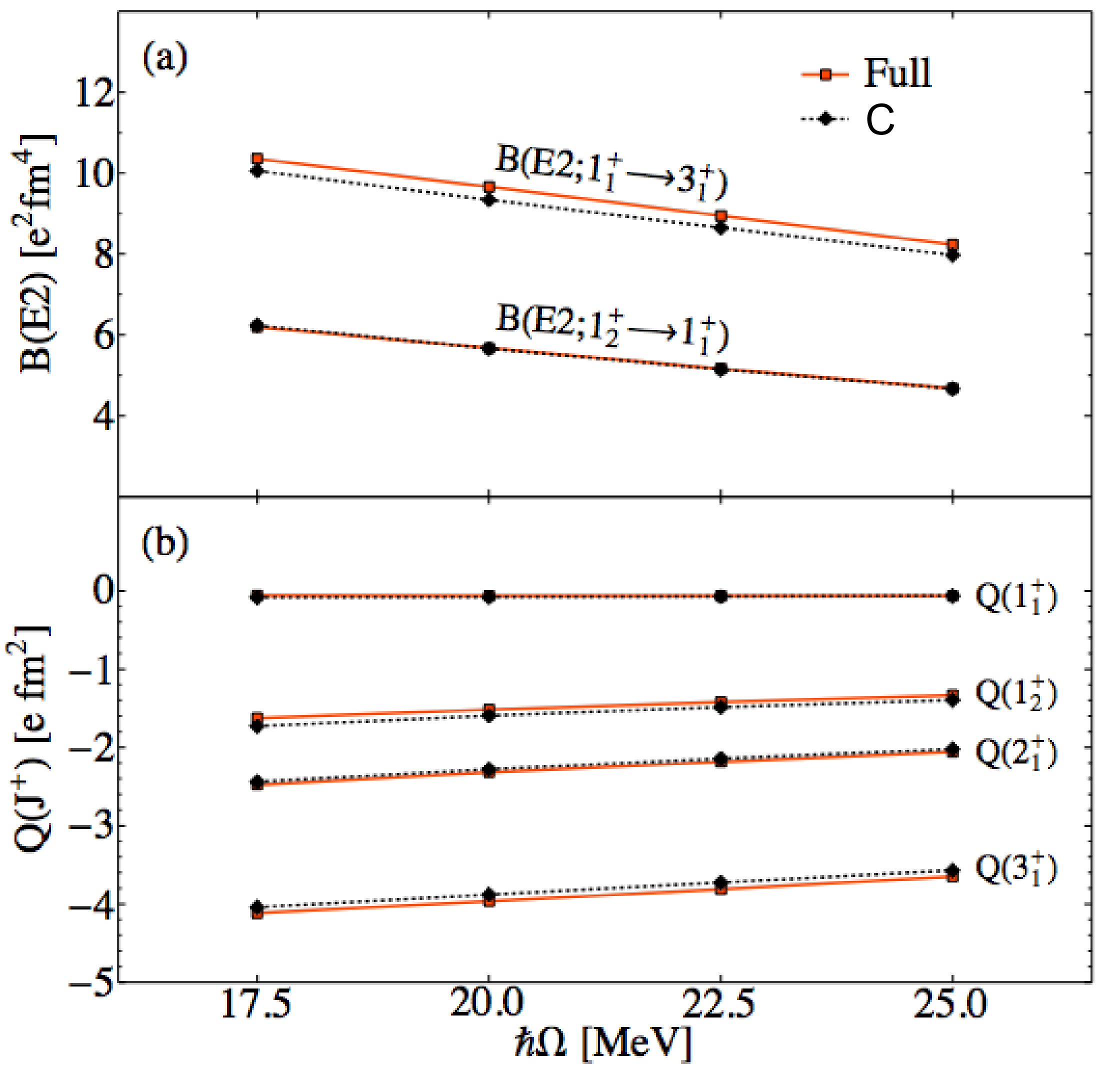


- Calculate excited T=0 states in model space C

- Model space C provides a good approximation to excitation spectra of low-lying T=0 states in 6Li

Physical Observables in Truncated Model Space



- model space C reproduces B(E2) & quadrupole moments independently of harmonic oscillator strength

Physical Observables in Truncated Model Space

$\hbar\Omega = 17.5$ MeV

Magnetic dipole moments $[\mu_N]$

$N_{\max} = 12$	1^+_{gs}	3^+	2^+	1^+_2
full	0.838	1.866	0.960	0.336
C	0.840	1.866	1.015	0.337

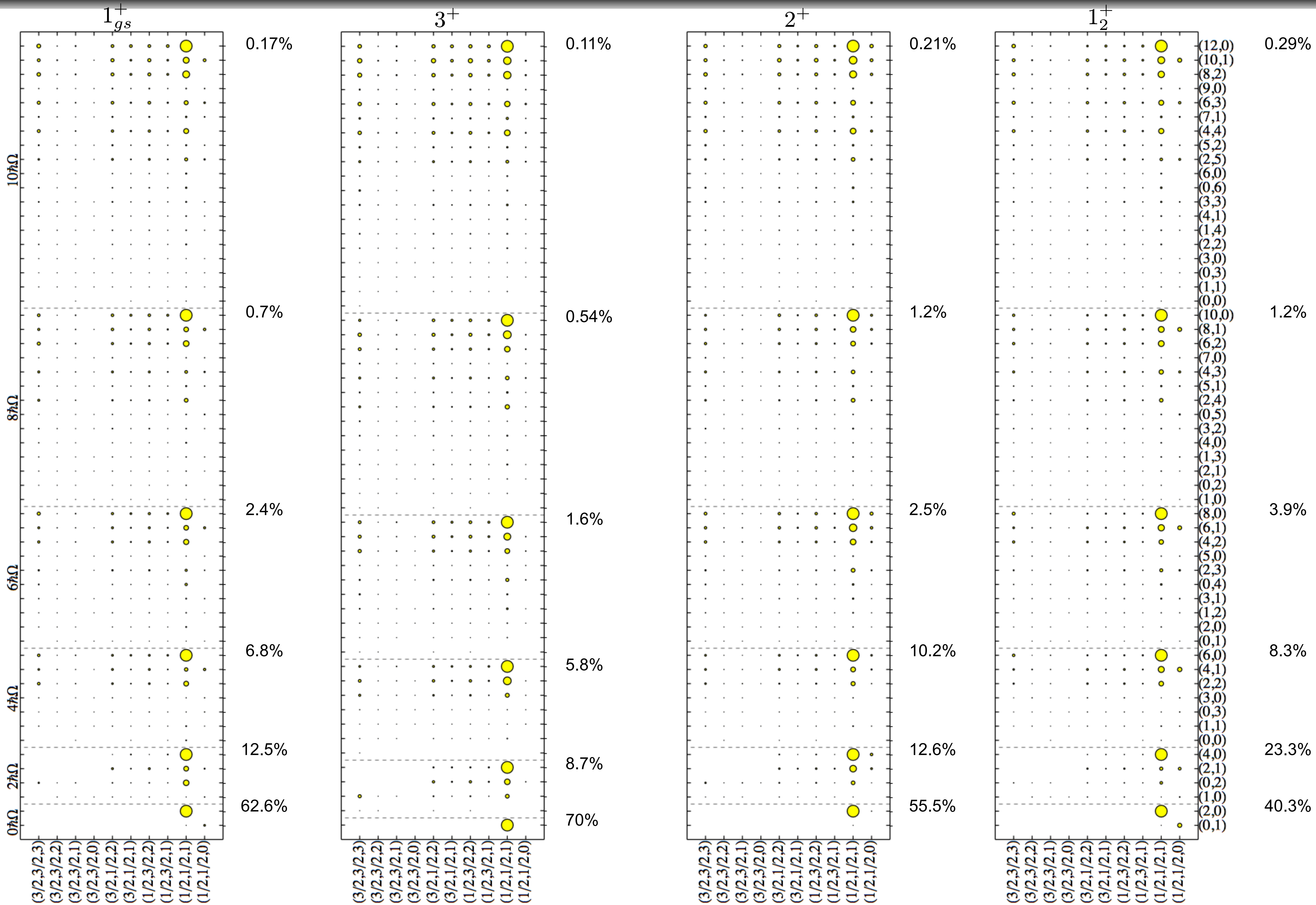
$\hbar\Omega = 17.5$ MeV

Matter rms radii [fm]

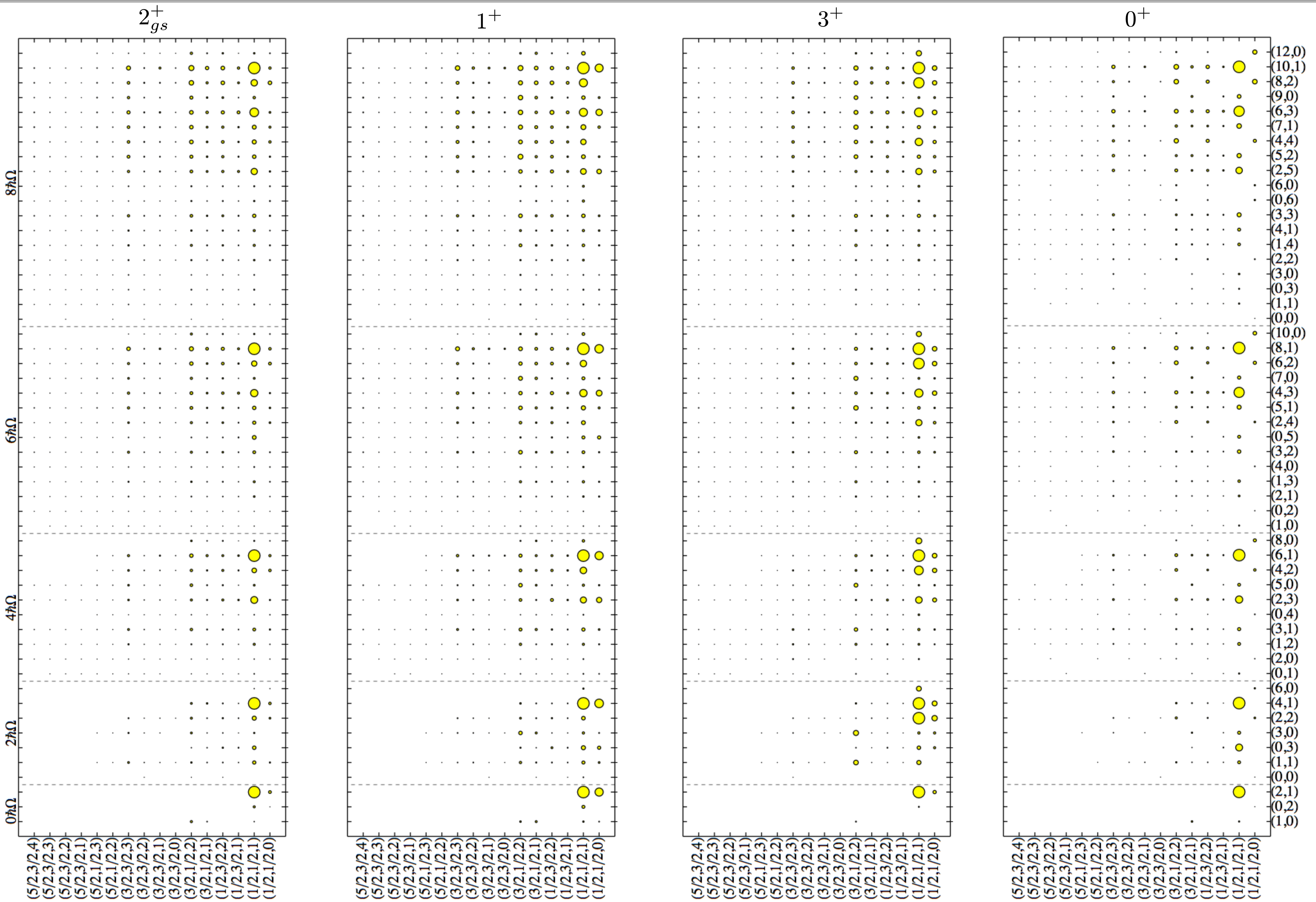
$N_{\max} = 12$	1^+_{gs}	3^+	2^+	1^+_2
full	2.146	2.092	2.257	2.373
C	2.139	2.079	2.236	2.355

- model space C reproduces physical observables independently on HO strength

^{6}Li - coherent structure of T=0 states



^{8}B



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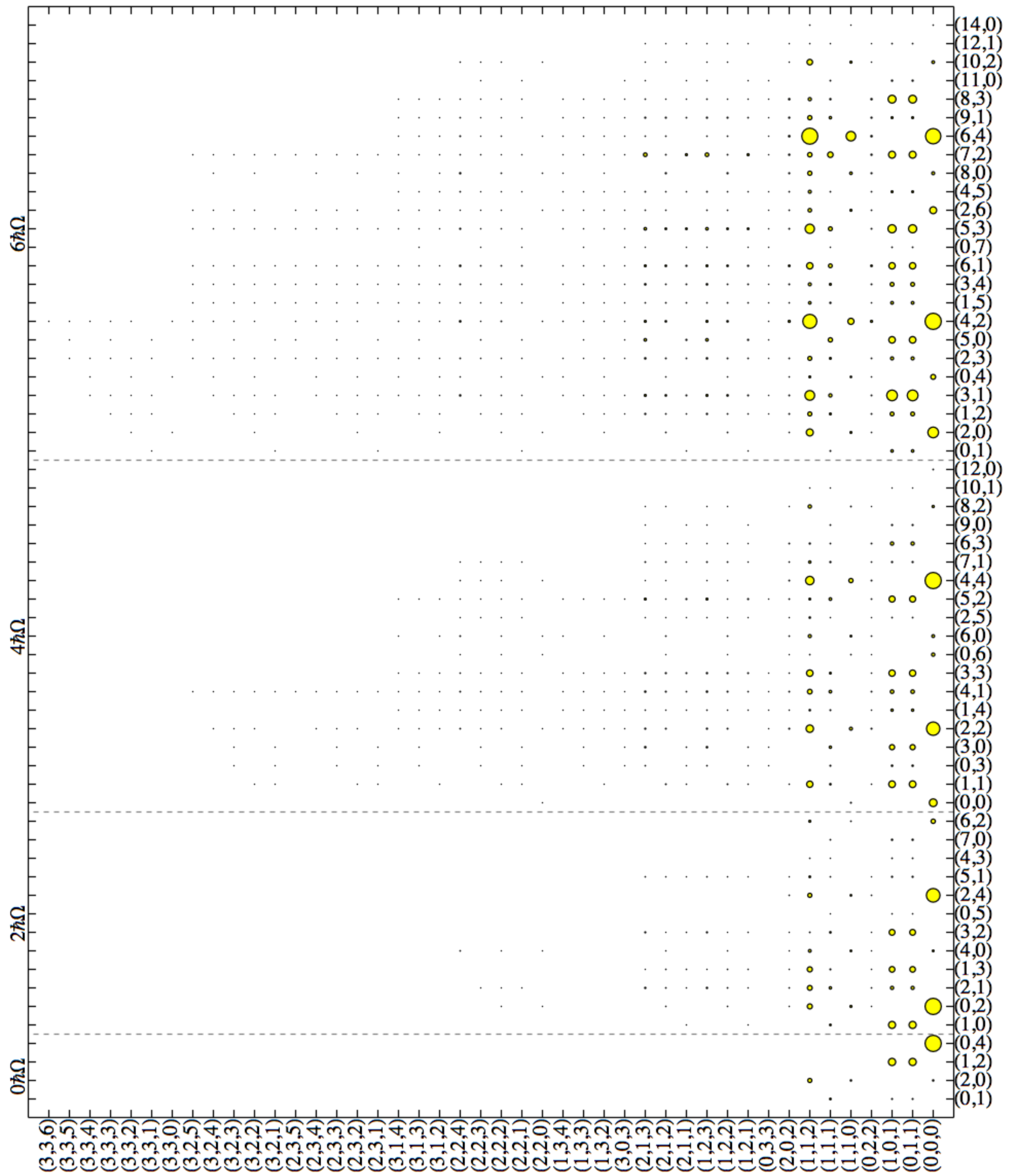
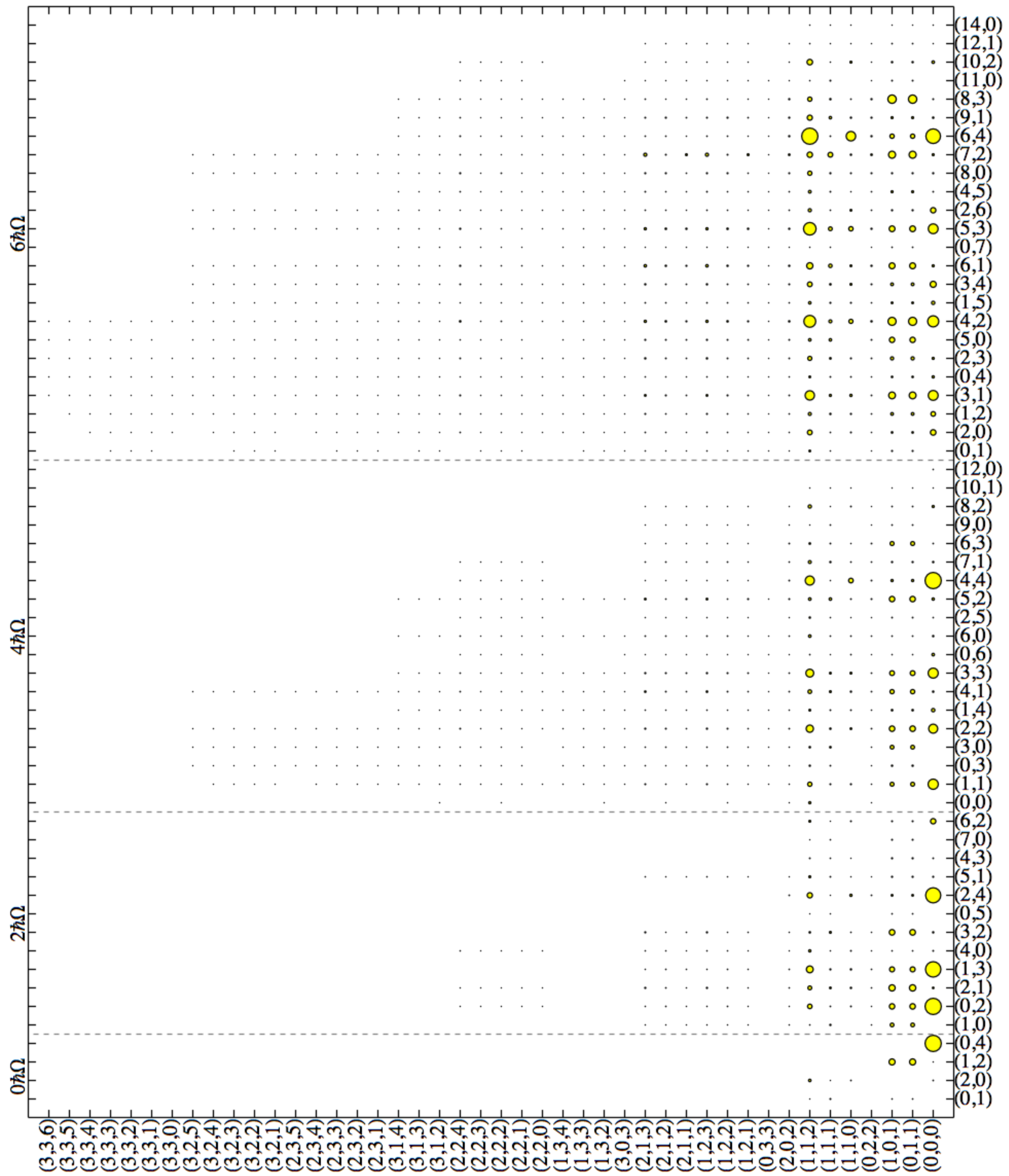
^{12}C : model space decomposition

^{12}C : J=0 ground state



^{12}C : $J=2^+_1$



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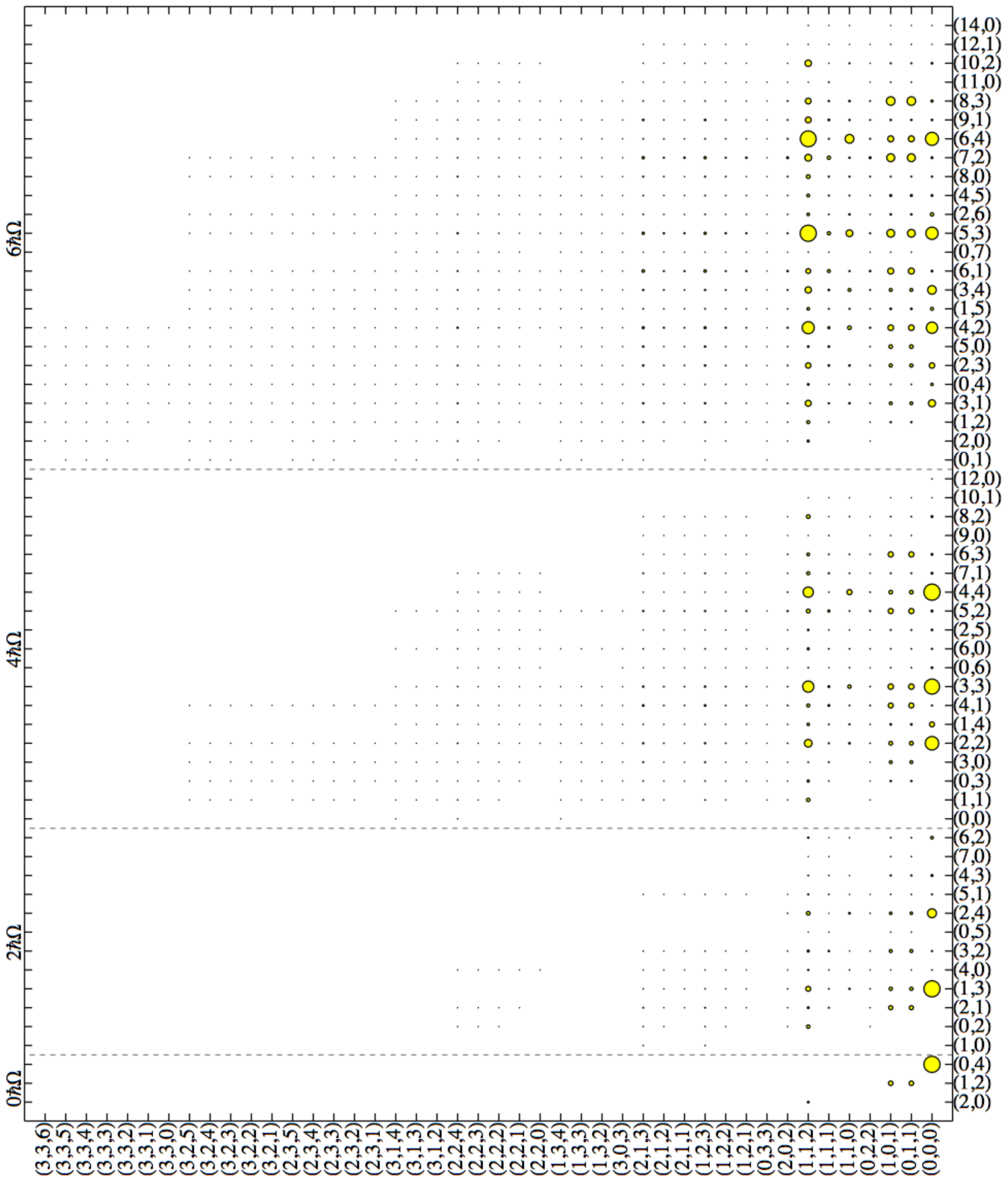
^{12}C : $J=4^+_1$



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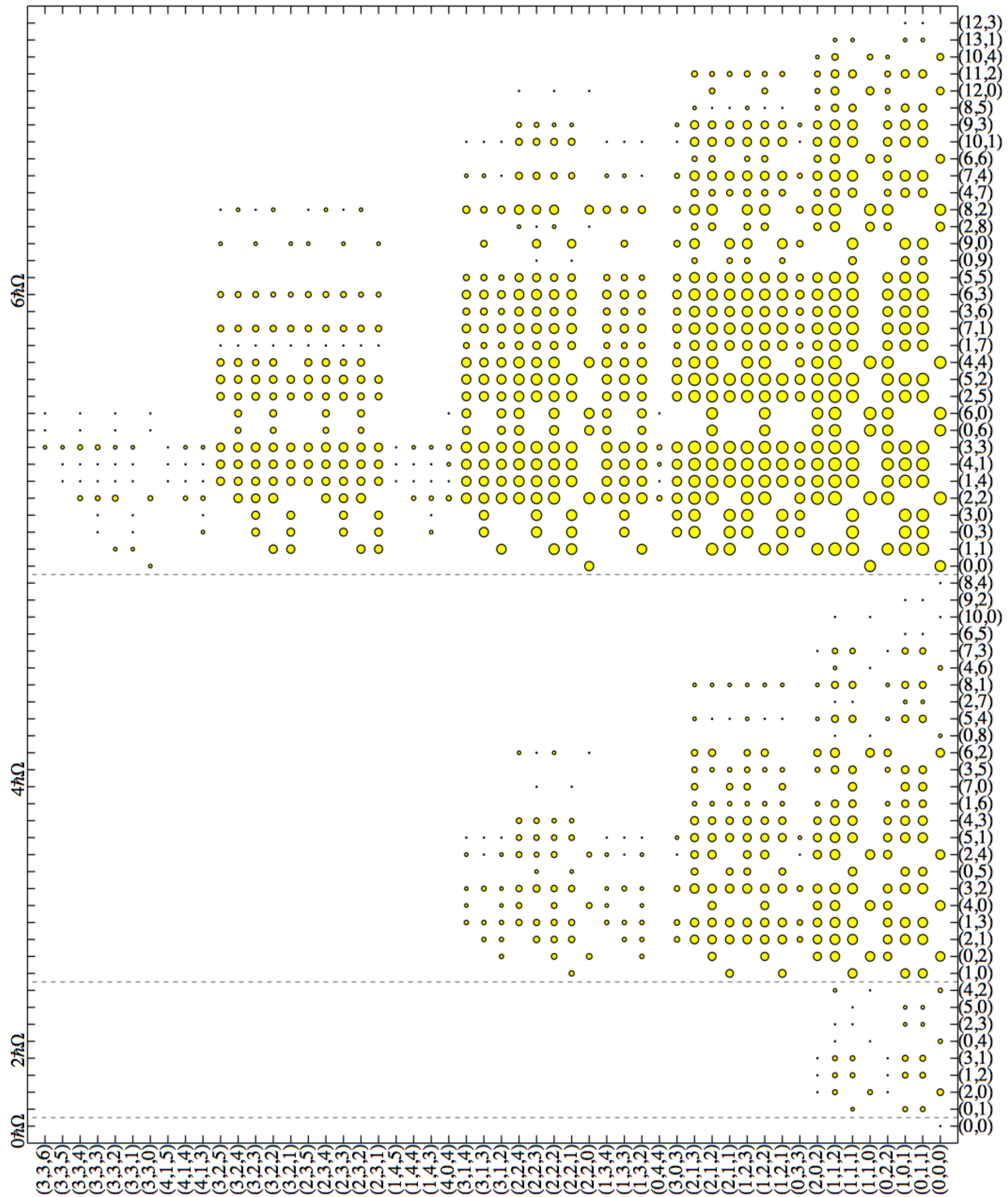
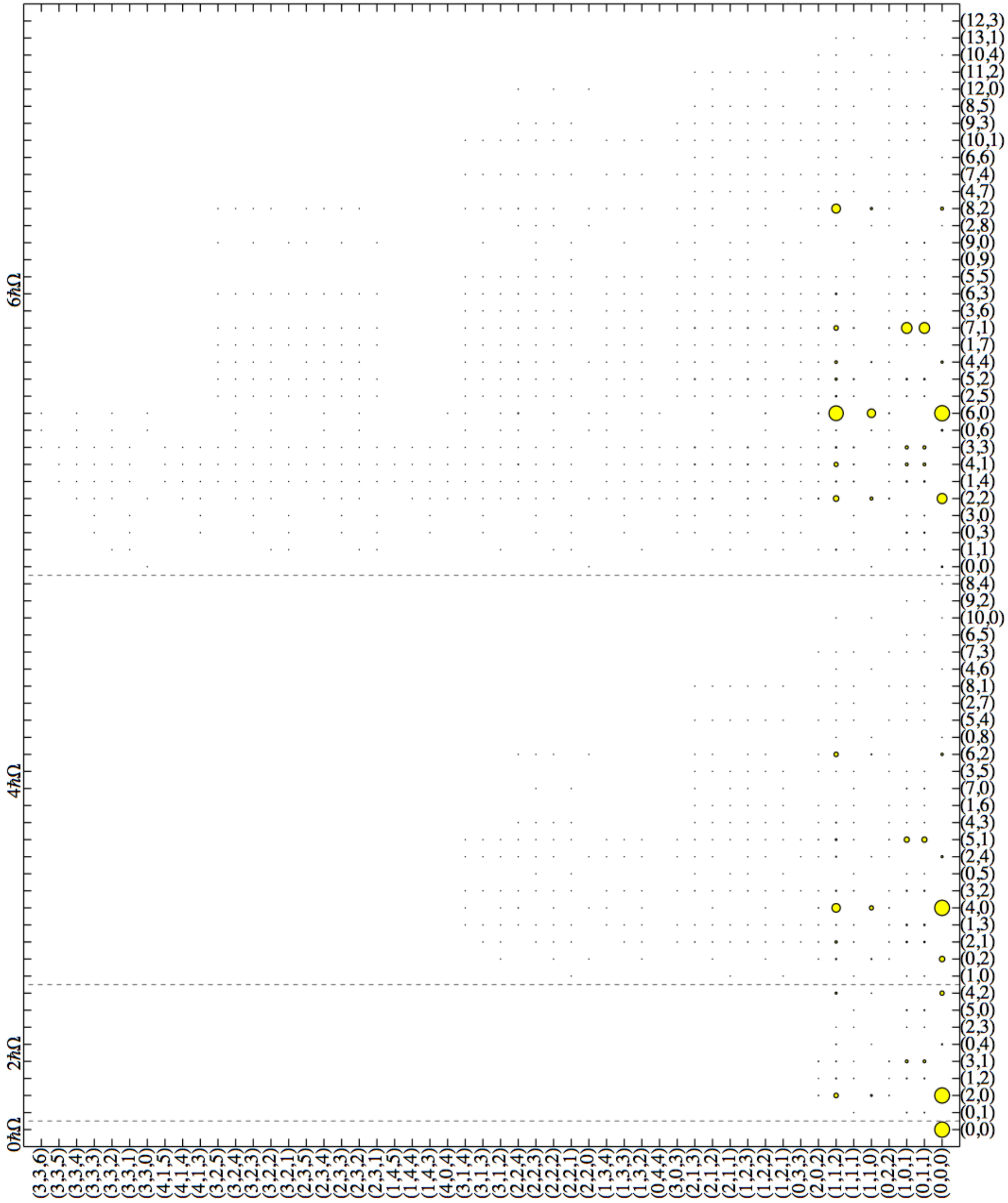
^{16}O : model space decomposition



^{16}O : ground state



Summary & Outlook

- We have tested SU(3) and spin based truncation scheme
- Our results suggest the existence of coherent SU(3) structures and reaffirm the importance of the symplectic symmetry
- Tranform N3LO NN interaction into SU(3) compatible form
- Implement 3N forces in SU(3)-scheme
- Move toward sd-shell nuclei