Chiral three-nucleon forces

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Perspectives of the Ab initio No-Core Shell Model

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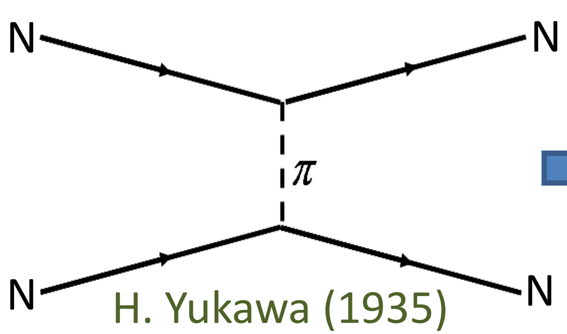
With V. Bernard, E. Epelbaum, A. Gasparyan, U.-G. Meißner

Outline

- Nuclear forces in chiral EFT
- Convergence of nuclear forces and the role of Δ-isobar
- N^3LO three-nucleon forces
- N^3LO with Δ or N^4LO without Δ (better both)
- Longest range three-nucleon force at N^4LO without Δ
- Summary & Perspectives

Nucleon-Nucleon forces

Phenomenological description by Meson-exchange

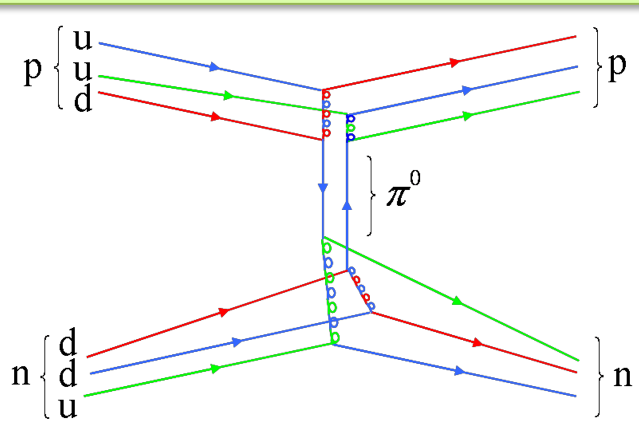


H. Yukawa (1935)

- Boson-Exchange Models as basis for NN-force
- Highly sophisticated phen. NN potentials
- Excellent description of many experimental data
- Connection to QCD is unclear

QCD Interpretation of NN forces

- NN force as residual strong interaction between hadrons



Chiral EFT Interpretation of NN forces

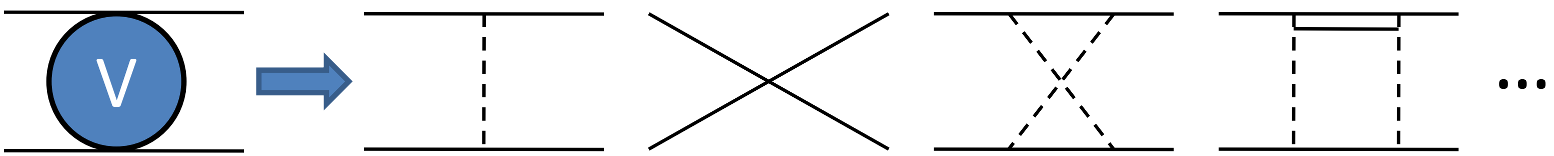
- Model independent treatment
- At low energies NN force dominated by Goldstone Boson dynamics + short range int.
- Systematic perturbative description of few nucleon potentials
- Underlying QCD symmetries implemented by construction

Weinberg's scheme for NN

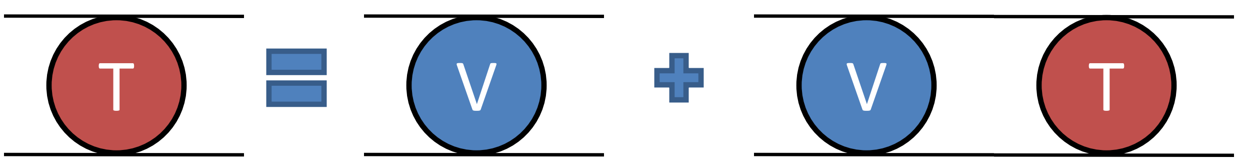
Weinberg, Nucl. Phys. B 363: 3 (1991)

- No perturbative description for bound states

- Construct effective potential perturbatively



- Solve Lippmann-Schwinger equation nonperturbatively



Effective potential

- Decomposition of the Fock space $\mathcal{H}$

Projector operators: $\eta + \lambda = 1$

$$\mathcal{H}_M \overset{\eta}{\leftarrow} \mathcal{H} = \mathcal{H}_M \oplus \mathcal{H}_R \overset{\lambda}{\rightarrow} \mathcal{H}_R$$

Model-space including only pure nucleon states

Remainder-space including states with at least one pion

$$H|\Psi\rangle = (H_0 + H_I)|\Psi\rangle = E|\Psi\rangle \Longleftrightarrow \begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} \eta|\Psi\rangle \\ \lambda|\Psi\rangle \end{pmatrix} = E \begin{pmatrix} \eta|\Psi\rangle \\ \lambda|\Psi\rangle \end{pmatrix}$$

- Block-diagonalization by applying unitary transformation

$$\tilde{H} = U^\dagger H U = \begin{pmatrix} \eta \tilde{H} \eta & 0 \\ 0 & \lambda H \lambda \end{pmatrix}$$

$$V_{\text{eff}} = \eta(\tilde{H} - H_0)\eta$$

V_{eff} is E - indep. ⟹ important for few-nucleon simulations

Possible parametrization by Okubo '54

$$U = \begin{pmatrix} \eta(1 + A^\dagger A)^{-1/2} & -A^\dagger(1 + AA^\dagger)^{-1/2} \\ A(1 + A^\dagger A)^{-1/2} & \lambda(1 + AA^\dagger)^{-1/2} \end{pmatrix}$$

With decoupling eq. $\lambda(H - [A, H] - AHA)\eta = 0$

Can be solved perturbatively within ChPT
Epelbaum et al. '98

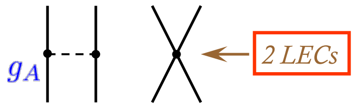
Nucleon-nucleon force up to N³LO

Ordonez et al. '94; Friar & Coon '94; Kaiser et al. '97; Epelbaum et al. '98,'03; Kaiser '99-'01; Higa et al. '03; …

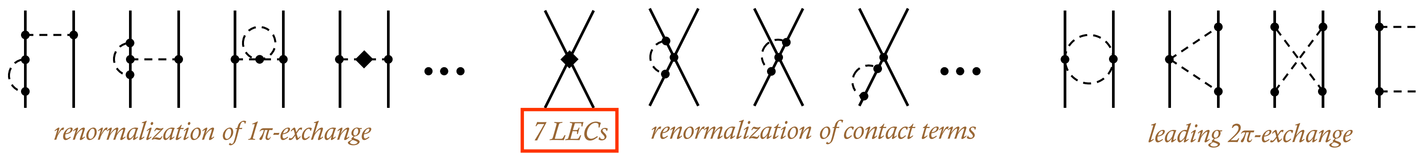
Chiral expansion for the 2N force:

$$V_{2N} = V_{2N}^{(0)} + V_{2N}^{(2)} + V_{2N}^{(3)} + V_{2N}^{(4)} + \dots$$

- **LO:**

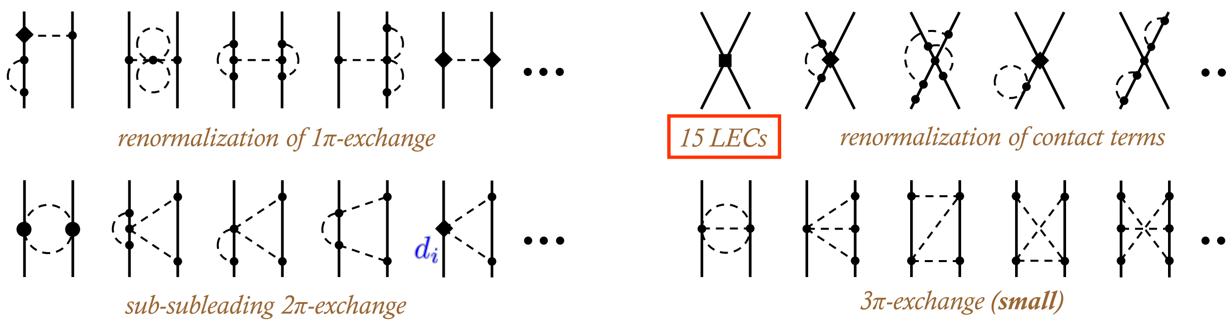


- **NLO:**



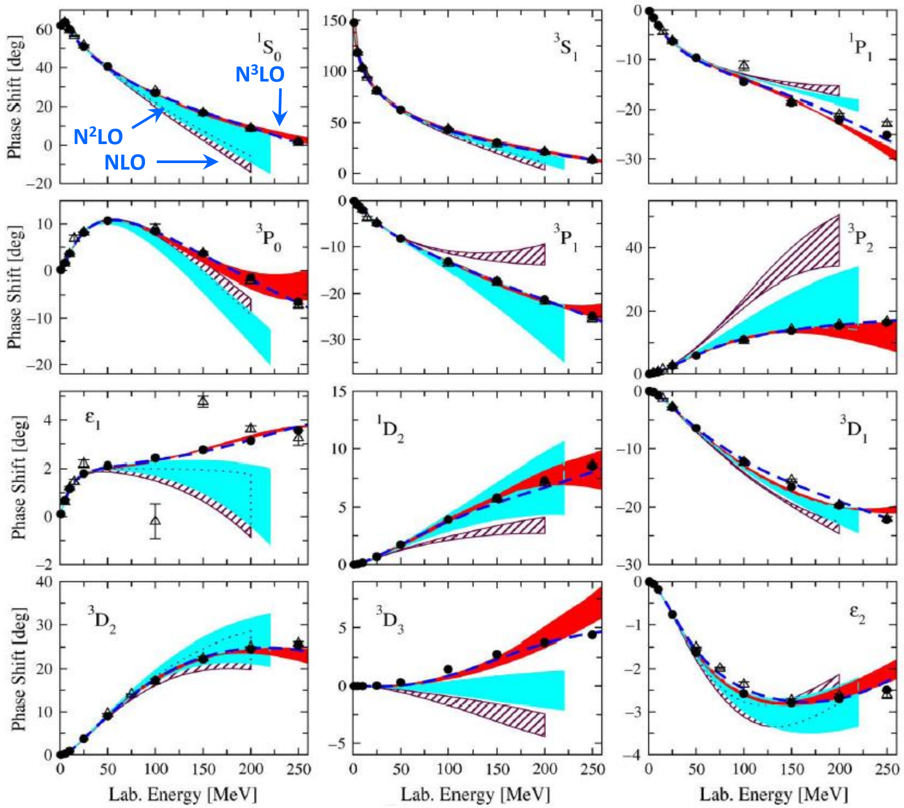
- **N²LO:**

- **N³LO:**

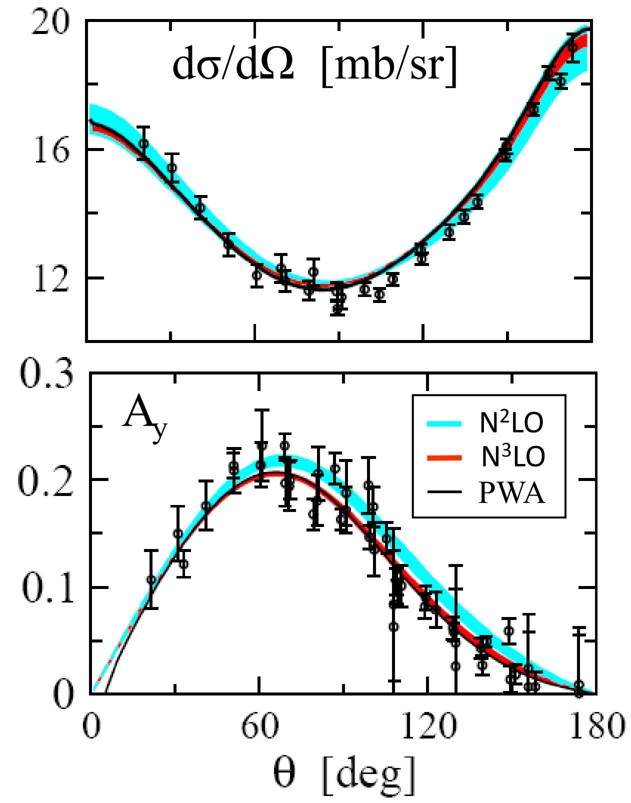


+ 1/m and isospin-breaking corrections…

Neutron-proton phase shifts up to N³LO



np scattering at 50 MeV



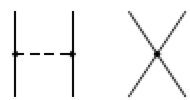
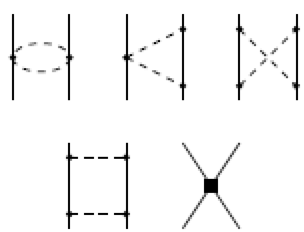
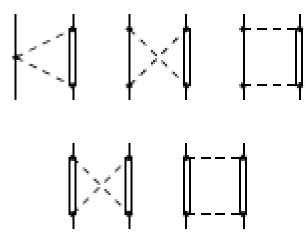
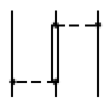
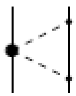
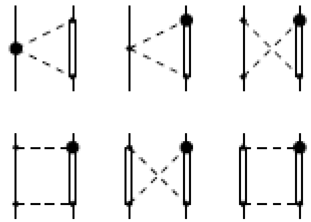
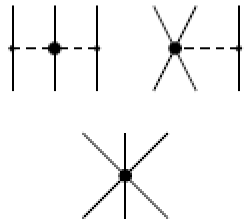
Deuteron binding energy & asymptotic normalizations A_s and η_d

	NLO	N^2LO	N^3LO	Exp
E_d [MeV]	$-2.171\ldots-2.186$	$-2.189\ldots-2.202$	$-2.216\ldots-2.223$	$-2.224575(9)$
A_S [$\mathrm{fm}^{-1/2}$]	$0.868\ldots0.873$	$0.874\ldots0.879$	$0.882\ldots0.883$	$0.8846(9)$
η_d	$0.0256\ldots0.0257$	$0.0255\ldots0.0256$	$0.0254\ldots0.0255$	$0.0256(4)$

Entem & Machleidt '03; Epelbaum, Glöckle & Meißner '05

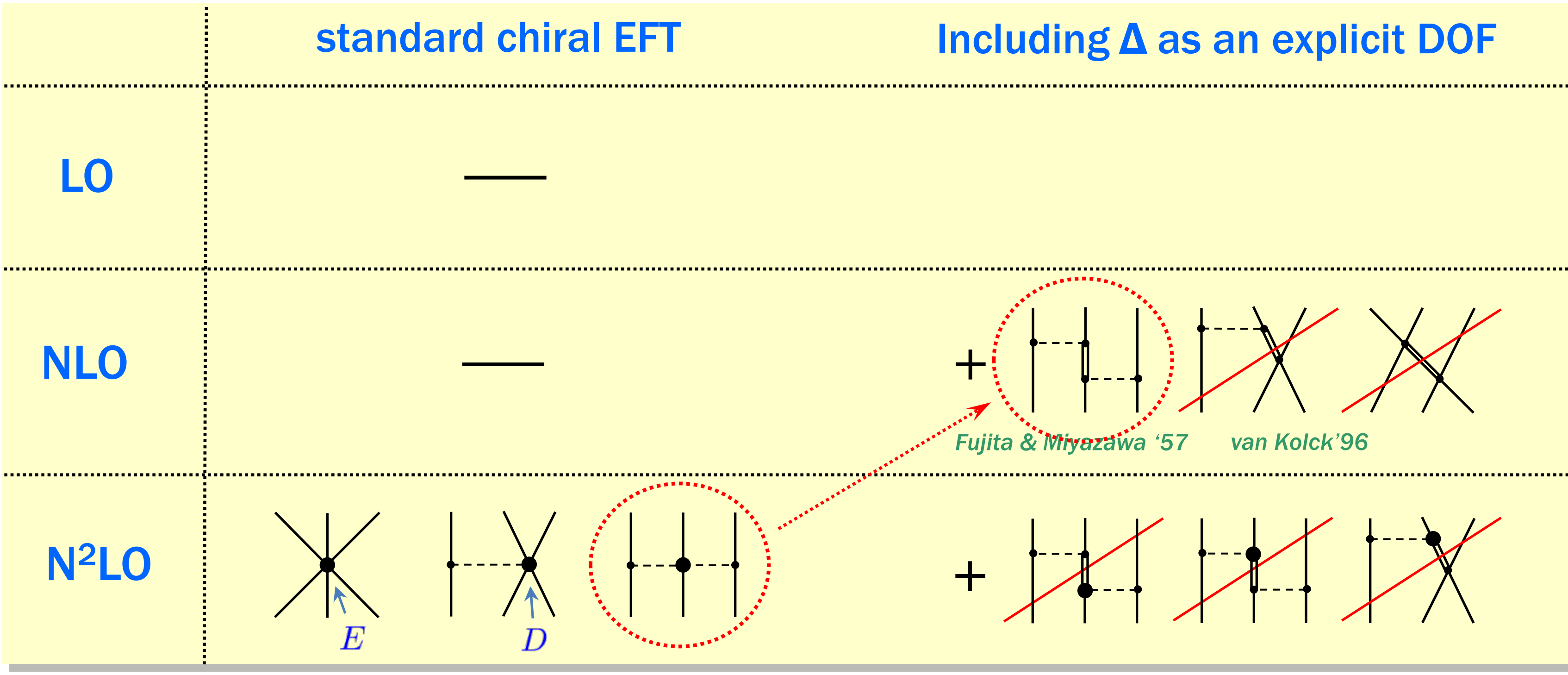
Few-nucleon forces with the Delta

Isospin-symmetric contributions

	Two-nucleon force		Three-nucleon force	
	Δ–less EFT	Δ–contributions	Δ–less EFT	Δ–contributions
LO		—	—	—
NLO		 Ordonez et al.'96, Kaiser et al. '98	—	
NNLO		 H.K., Epelbaum & Meißner '07		—

Delta excitations and the three-nucleon force

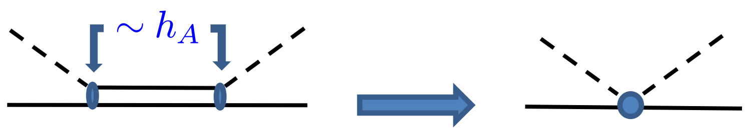
Epelbaum, H.K., Meißner, Nucl. Phys. A806 (2008) 65



- The LO NNNΔ contact interaction $\bar{T}_i^{\mu} N \bar{N} S_{\mu} \tau^i N + \text{ h.c.}$ vanishes due to the Pauli principle
  ⟹ the LECs D and E are not saturated by the delta.
- No contributions from subleading 2π –exchange due to ∂^0 at the $b_3 + b_8$ vertex.
- The entire effect of the Δ is given by a partial shift of the N²LO TPE 3NF to NLO…

Delta-less effective potential

- Standard chiral expansion: $Q \sim M_\pi \ll \Delta \equiv m_\Delta - m_N = 293 \text{ MeV}$
- Small scale expansion: $Q \sim M_\pi \sim \Delta \ll \Lambda_\chi$ *(Hemmert, Holstein & Kambor '98)*
- Delta contributions encoded in LECs
  (Bernard, Kaiser & Meißner '97)



Delta-resonance saturation

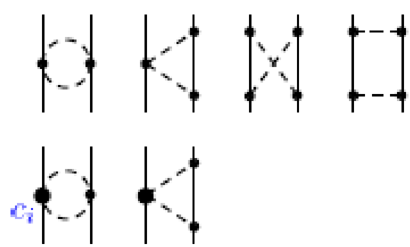
$$c_3 = -2c_4 = c_3(\not\Delta) - \boxed{\frac{4h_A^2}{9\Delta}}$$

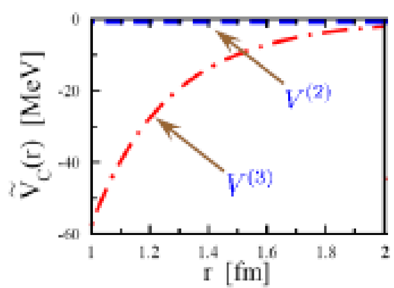
Enlargement due to Delta contribution

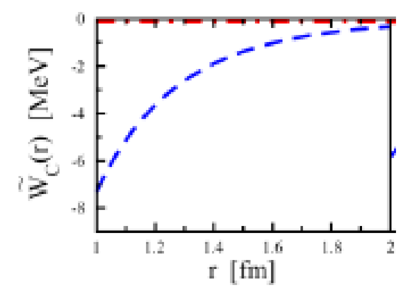
- Convergence of EFT potential

Q^2:

Q^3:







The subleading contribution is bigger than the leading one!

Expectation from inclusion of Δ explicitely

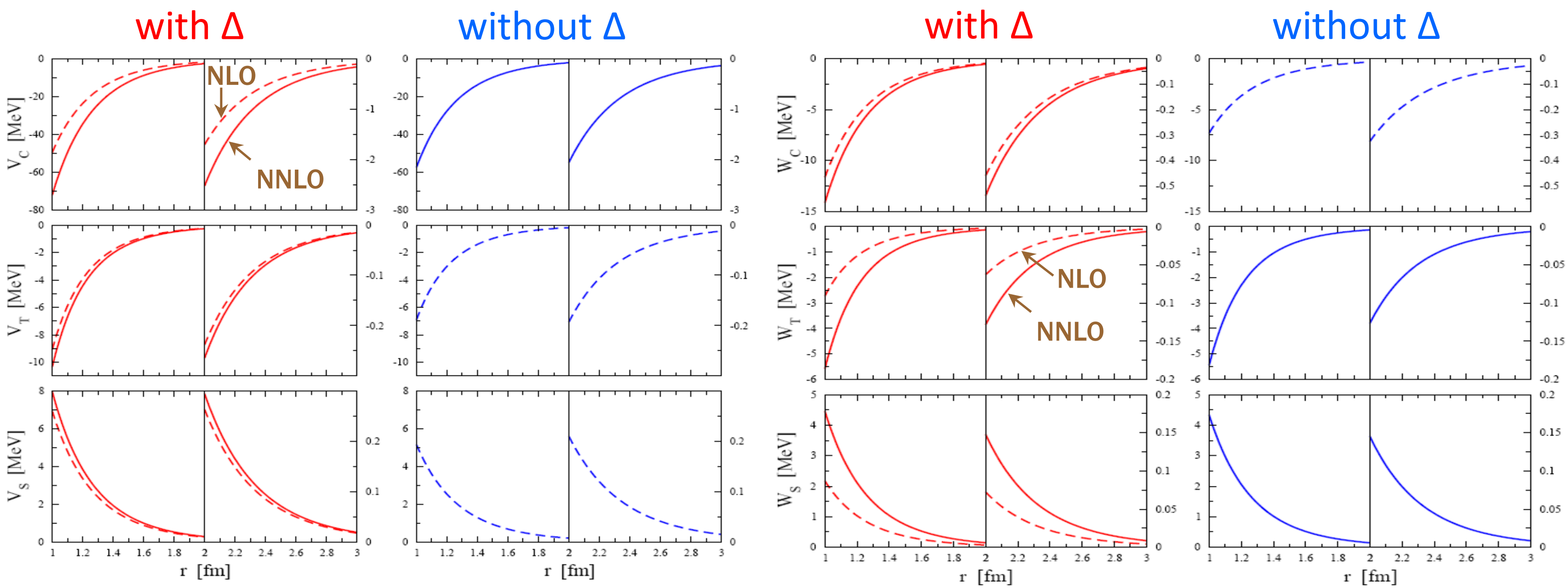
- more natural size of LECs
- better convergence
- applicability at higher energies

NN potential with explicit Δ

Epelbaum, H.K., Meißner, Eur. Phys. J. A32 (2007) 127

$$V_{\rm eff} = V_C + W_C \vec{\tau}_1\cdot\vec{\tau}_2 + [V_S + W_S \vec{\tau}_1\cdot\vec{\tau}_2]\vec{\sigma}_1\cdot\vec{\sigma}_2 + [V_T + W_T \vec{\tau}_1\cdot\vec{\tau}_2](3\,\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2)$$

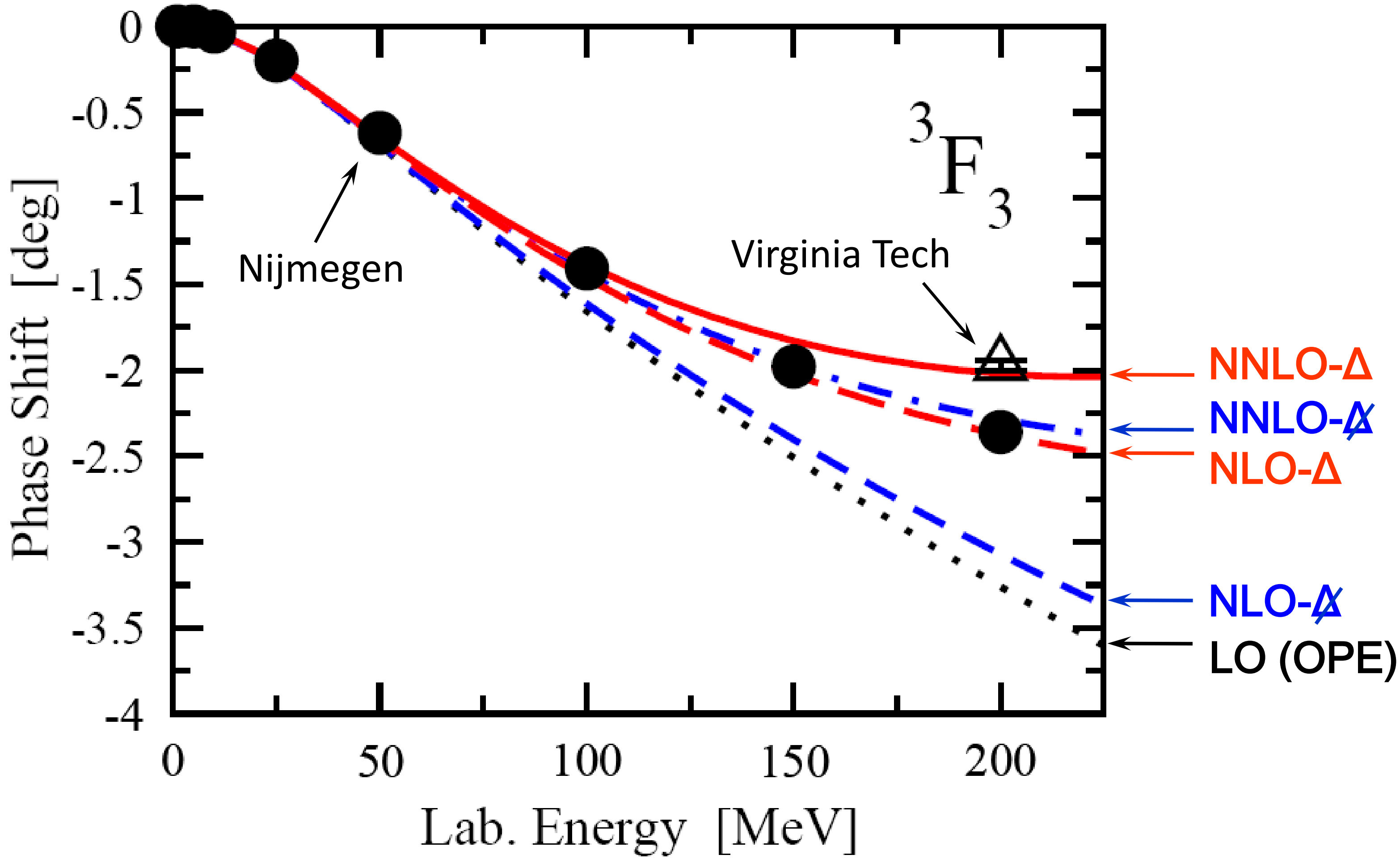
Chiral 2π- exchange potential up to NNLO



Advantages when Δ is included explicitly

- Dominant contributions already at NLO
- Much better convergence in all potentials

$^3F_3$ partial waves up to NNLO with and without Δ

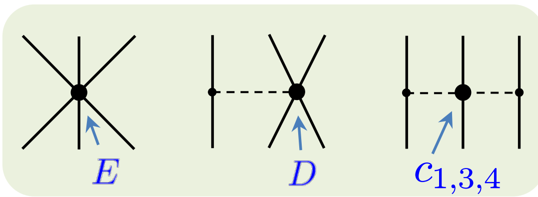


(calculated in the first Born approximation)

Three-nucleon forces

- Three-nucleon forces in chiral EFT start to contribute at NNLO

U. van Kolck ‘94; Epelbaum et al. ’02; Nogga et al. ’05; Navratil et al. ‘07



$c_{1,3,4}$ from the fit to πN-scattering data

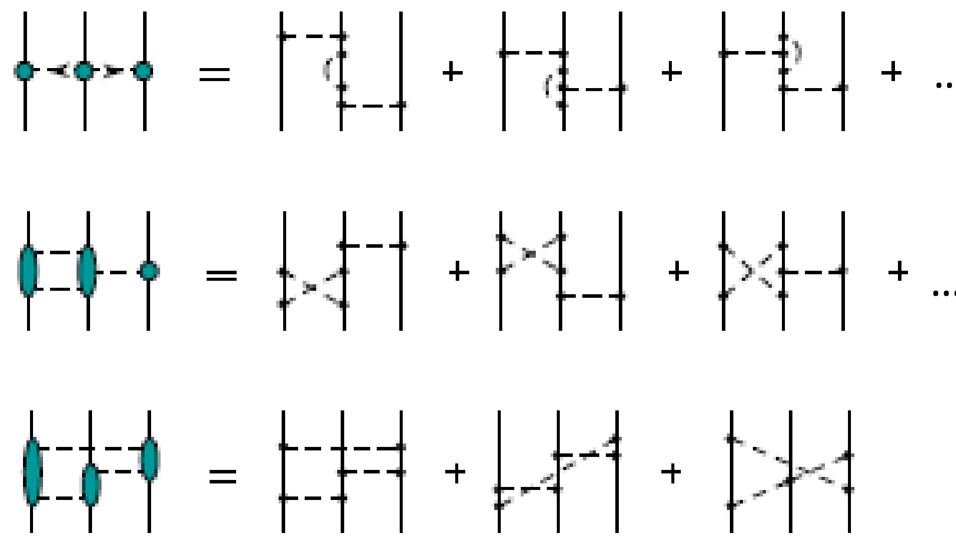
D, E from $^3H, ^4He, ^{10}B$ binding energy + coherent nd scattering length

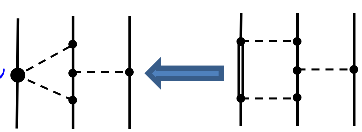
- Three-nucleon forces at N^{3}LO

Long range contributions

Bernard, Epelbaum, H.K. , Meißner ‘08; Ishikawa, Robilotta ‘07

- No additional free parameters
- Expressed in terms of g_A, F_π, M_π
- Rich isospin-spin-orbit structure
- Δ(1232)-contr. may be important



Large → c_i ~ 

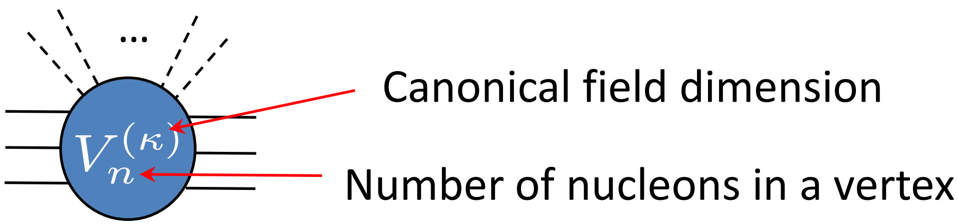
N^{4}LO-contr.

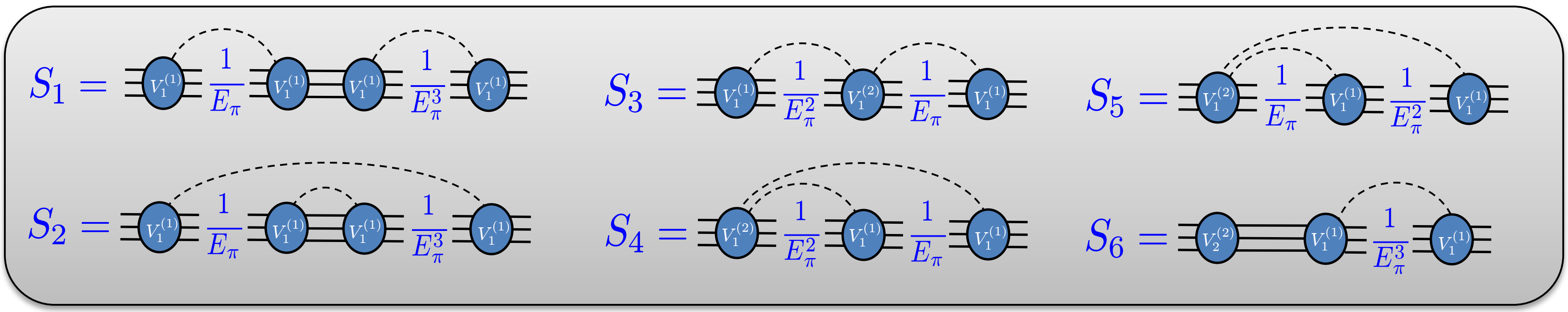
Fixing unitary transformation

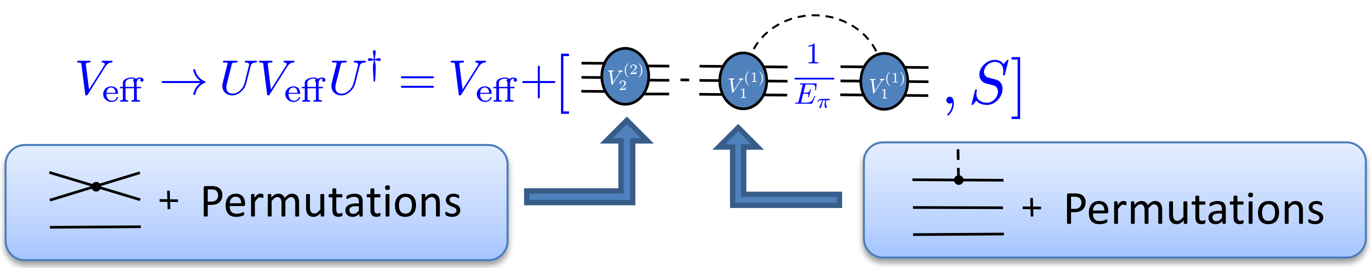
Epelbaum Eur. Phys. J. A34 (2007) 197

- Additional unitary transformations at N³LO : $U = \exp(S - S^\dagger),\, S = \sum_{i=1}^{6} \alpha_i S_i$

Free parameters





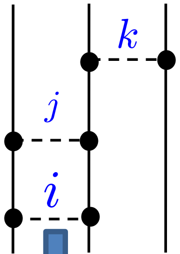


- Requirement of renormalizability of effective potential ⟹ Restrictions on α_i

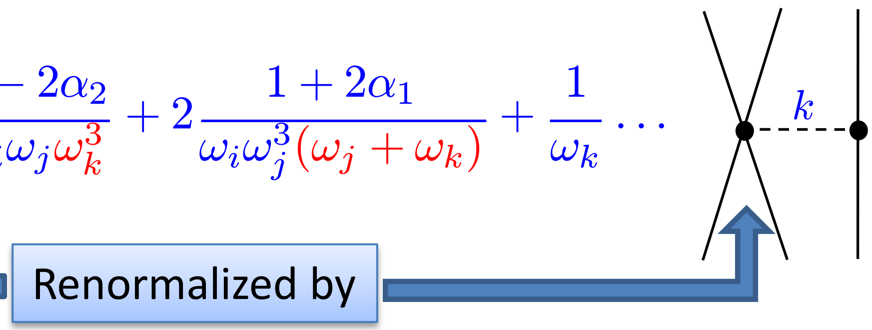
$$\alpha_1 = -\alpha_2 = -\frac{1}{2}, \alpha_3 = -\alpha_5, \alpha_4 = \frac{1}{2} + 2\alpha_5, \alpha_6 = \frac{1}{2}$$

Factorization of one pion exchange

- Renormalization of effective potential requires factorization of OPE

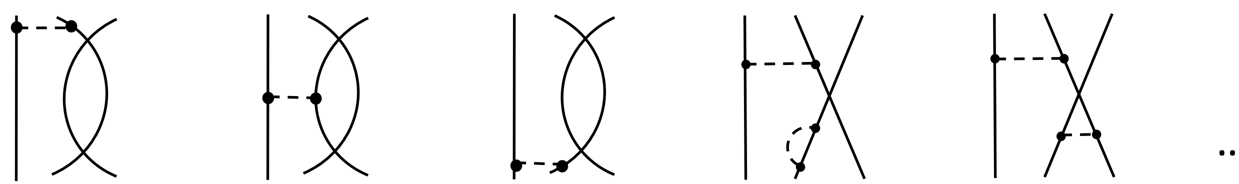


$$\sim \frac{1+2\alpha_1}{\omega_i\omega_j^2\omega_k^2} - 4\frac{1-2\alpha_2}{\omega_i\omega_j\omega_k^3} + 2\frac{1+2\alpha_1}{\omega_i\omega_j^3(\omega_j+\omega_k)} + \frac{1}{\omega_k}\dots$$

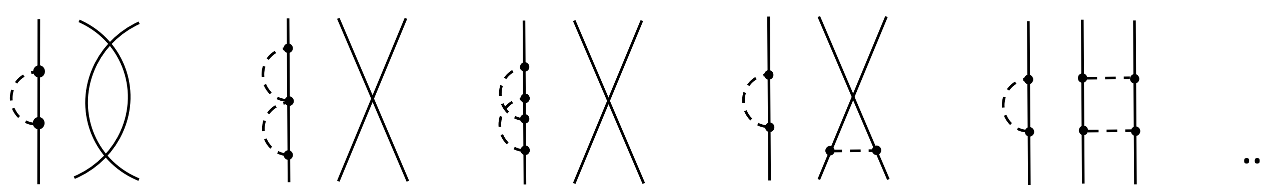


$$\sim \frac{1}{\omega_k}$$

- Shorter range Diagrams $\sim 1-2\alpha_6$ which individually vanish for $\alpha_6 = \frac{1}{2}$



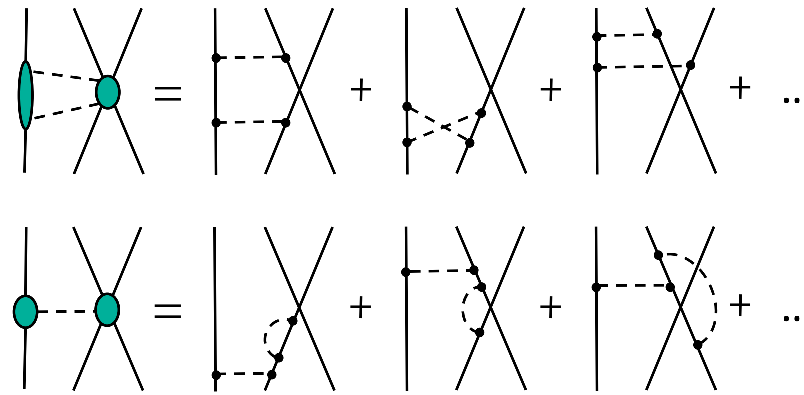
- Disconnected diagrams vanish for all values of α_i



Shorter range contributions

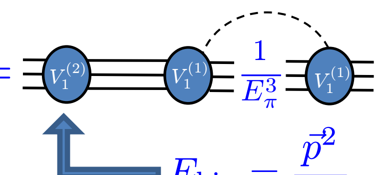
Bernard, Ebelbaum, H.K., Meißner : Phys. Rev. C84 (2011) 054001

- LECs needed for shorter range contr.
  g_A, F_π, M_π, C_T
- Central NN contact interaction $\sim C_S$ does not contribute (note $C_S \gg C_T$)
- Smaller N^{3}LO shorter range contr. expected (approx. Wigner sym.)



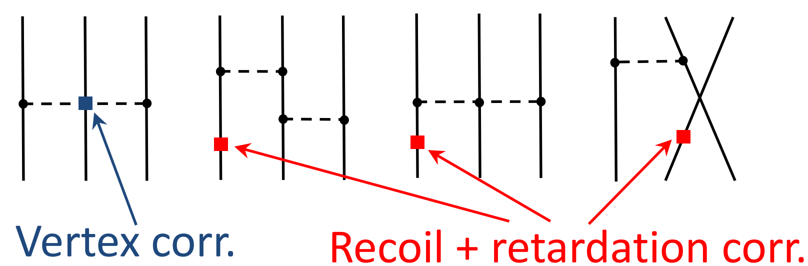
Relativistic 1/m corrections

- Additional unitary transformation: $U = \exp(\alpha_7 (S_7 - S_7^\dagger))$ $S_7 =$
  Adam et al. '92, Friar & Coon '94, Friar'99



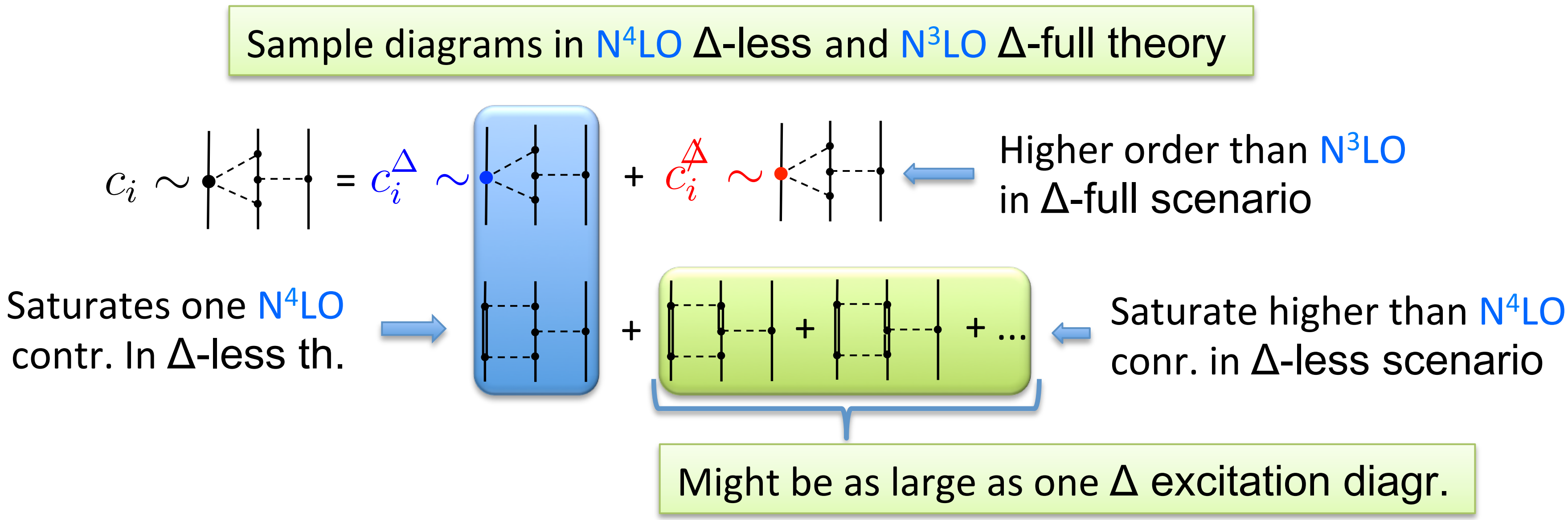
$E_{\rm kin} = \frac{\vec{p}^{\,2}}{2m}.$

- $\alpha_7 = \frac{1}{4}$ consistent with N^{3}LO NN potential
  Epelbaum, Glöckle, Meißner '05
- Slight disagreement with Friar & Coon ´94 on 1/m corr. to TPE. (Presumably due to our additional UT)



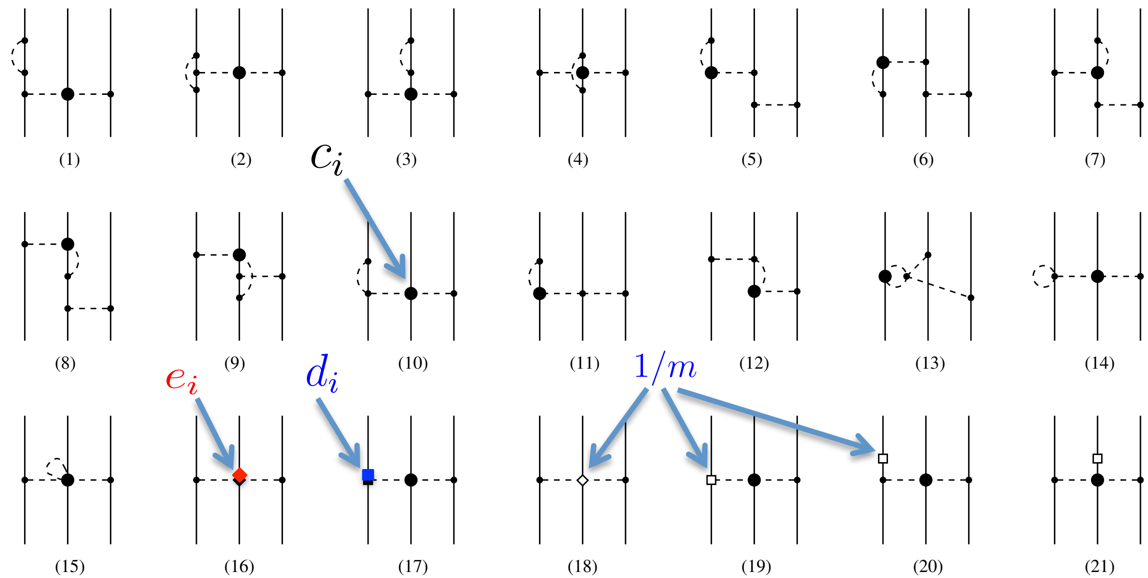
Improvement beyond Δ−less N^3LO

- N^4LO Δ-less or N^3LO Δ-full theory?



- N^4LO Δ-less theory catches up one Δ excitation from N^3LO Δ-full theory
- N^4LO Δ-less theory includes terms e.g. $\sim c_i^{\not\Delta}$ which are absent in N^3LO Δ-full theory
- In N^3LO Δ-full theory two and three Δ excitations appear at the same order as one Δ excitation. Those, however, are not taken in N^4LO Δ-less theory .

Longest range contr. from Δ−less N⁴LO



c_i LECs from $\mathcal{L}_{\pi N}^{(2)}$

d_i LECs from $\mathcal{L}_{\pi N}^{(3)}$

e_i LECs from $\mathcal{L}_{\pi N}^{(4)}$

- Linear combinations of c_i , d_i and e_i LECs are fixed from pion – nucleon scattering

$$d_i = d_i^r(\mu) + \frac{\beta_i}{F^2}\lambda$$

$$e_i = e_i^r(\mu) + \frac{\gamma_i}{F^2}\lambda$$

Beta-functions

3NF has to be renormalized with the same beta functions

Two additional unitary transf. are needed for renormalization

$$\lambda = \frac{\mu^{d-4}}{16\pi^2}\left[\frac{1}{d-4} - \frac{1}{2}\left(\Gamma'(1) + 1 + \log(4\pi)\right)\right]$$

Two-pion-exchange at N^4LO

$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1\, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2\, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2\, \mathcal{B}(q_2) \Big) ,$$

$$\begin{aligned}
\mathcal{A}^{(5)}(q_2) \;=\; & \frac{g_A}{4608\pi^2 F_\pi^6} \Big[M_\pi^2 q_2^2 \big(F_\pi^2 \big(2304\pi^2 g_A (4\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36}) - 2304\pi^2 \bar{d}_{18} c_3\big) \\
& + g_A (144 c_1 - 53 c_2 - 90 c_3)\big) + M_\pi^4 \big(F_\pi^2 \big(4608\pi^2 \bar{d}_{18} (2c_1 - c_3) + 4608\pi^2 g_A (2\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{36} - 4\bar{e}_{38})\big) \\
& + g_A \big(72 \big(64\pi^2 \bar{l}_3 + 1\big) c_1 - 24 c_2 - 36 c_3\big)\big) + q_2^4 \big(2304\pi^2 \bar{e}_{14} F_\pi^2 g_A - 2 g_A (5c_2 + 18 c_3)\big) \Big] \\
& - \frac{g_A^2}{768\pi^2 F_\pi^6} L(q_2) \big(M_\pi^2 + 2q_2^2\big) \big(4M_\pi^2 (6c_1 - c_2 - 3c_3) + q_2^2(-c_2 - 6c_3)\big) \\
\mathcal{B}^{(5)}(q_2) \;=\; & -\frac{g_A}{2304\pi^2 F_\pi^6} \Big[M_\pi^2 \big(F_\pi^2 \big(1152\pi^2 \bar{d}_{18} c_4 - 1152\pi^2 g_A (2\bar{e}_{17} + 2\bar{e}_{21} - \bar{e}_{37})\big) + 108 g_A^3 c_4 + 24 g_A c_4\big) \\
& + q_2^2 \big(5 g_A c_4 - 1152\pi^2 \bar{e}_{17} F_\pi^2 g_A\big) \Big] + \frac{g_A^2 c_4}{384\pi^2 F_\pi^6} L(q_2) \big(4M_\pi^2 + q_2^2\big)
\end{aligned}$$

$$L(q) = \frac{\sqrt{q^2 + 4M_\pi^2}}{q} \log \frac{\sqrt{q^2 + 4M_\pi^2} + q}{2M_\pi} .$$

Some LECs can be absorbed by shifting c_i´s

$$\begin{aligned}
c_1 &\rightarrow c_1 - 2M_\pi^2 \left(\bar{e}_{22} - 4\bar{e}_{38} - \frac{\bar{l}_3 c_1}{F_\pi^2} \right) , \\
c_3 &\rightarrow c_3 + 4M_\pi^2 \left(2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36} + 2\frac{\bar{l}_3 c_1}{F_\pi^2} \right) , \\
c_4 &\rightarrow c_4 + 4M_\pi^2 (2\bar{e}_{21} - \bar{e}_{37}) ,
\end{aligned}$$

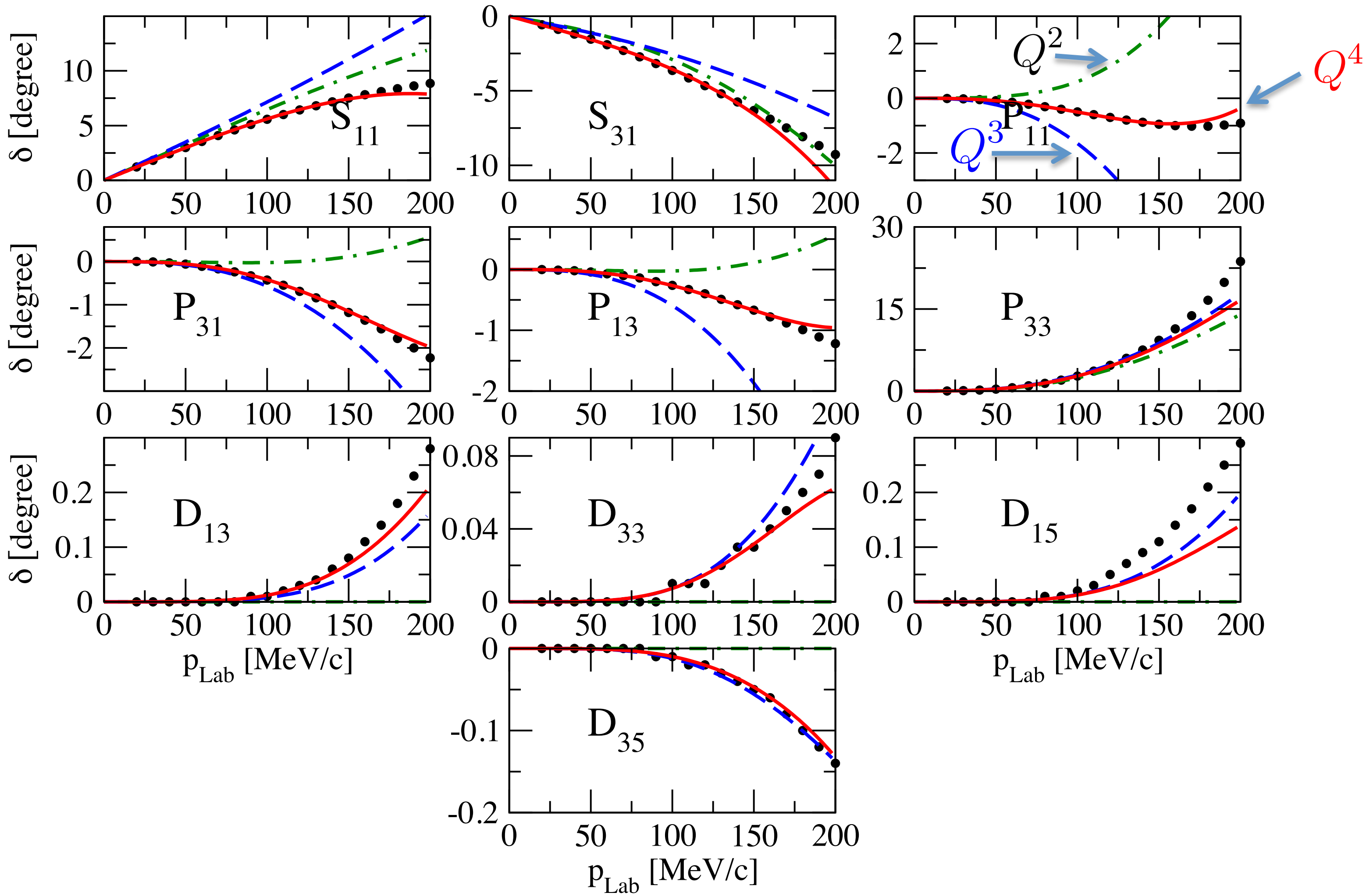
$$g_{\pi NN} = \frac{g_A\, m}{F_\pi} \left(1 - \frac{2M_\pi^2\, \bar{d}_{18}}{g_A} \right)$$

⇐ Violation of Goldberger-Treiman rel.

- No d_i dependence of TPE-contr. beside d_{18}
- Pion-nucleon scattering does Strongly depend on d_i´s

GW-Fit to pion-nucleon scattering

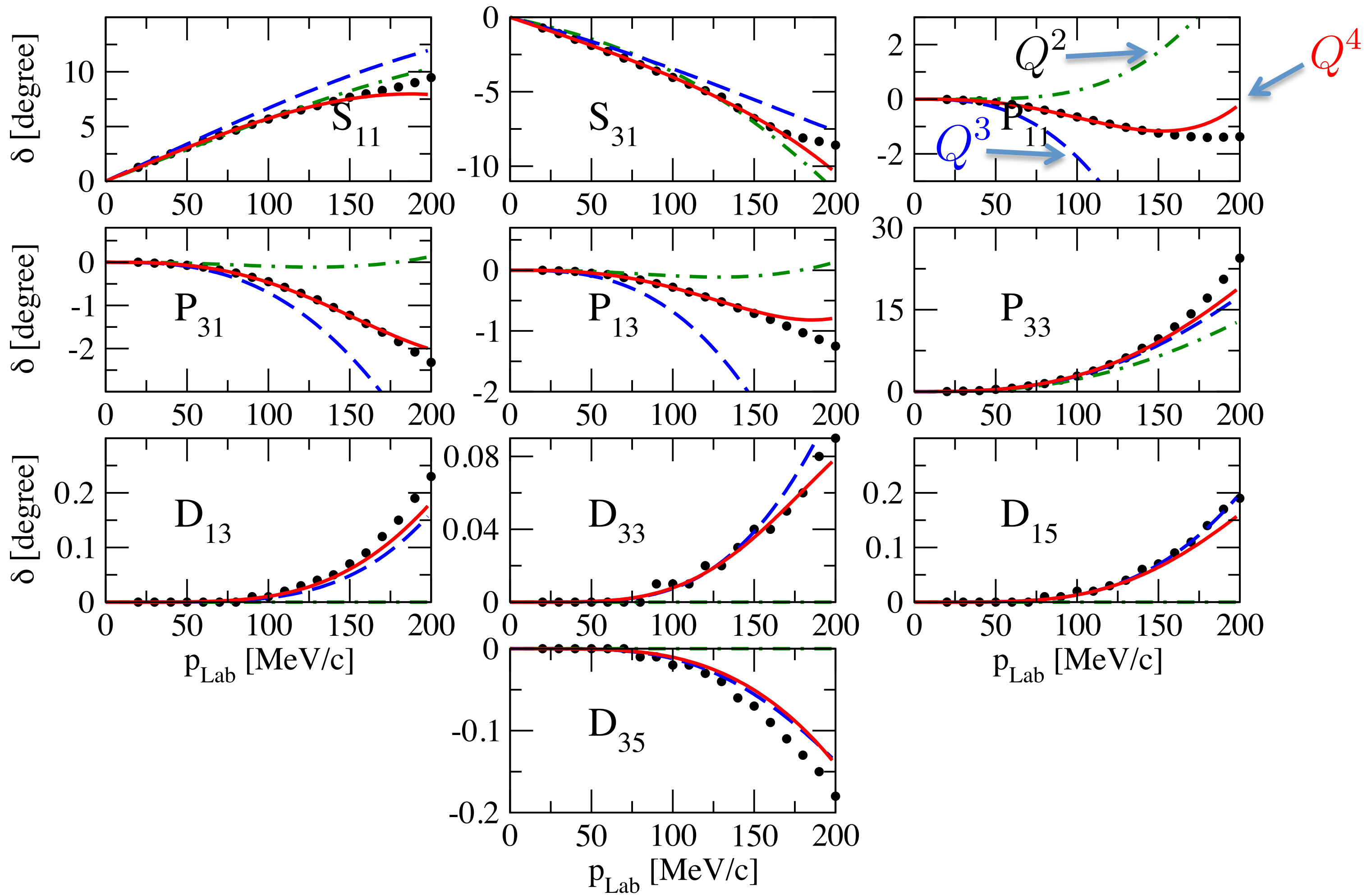
GW-PWA: Arndt et al. Phys. Rev. C 74 (2006) 045205



Data fitted for $p_{\mathrm{Lab}} < 150\,\mathrm{MeV}$

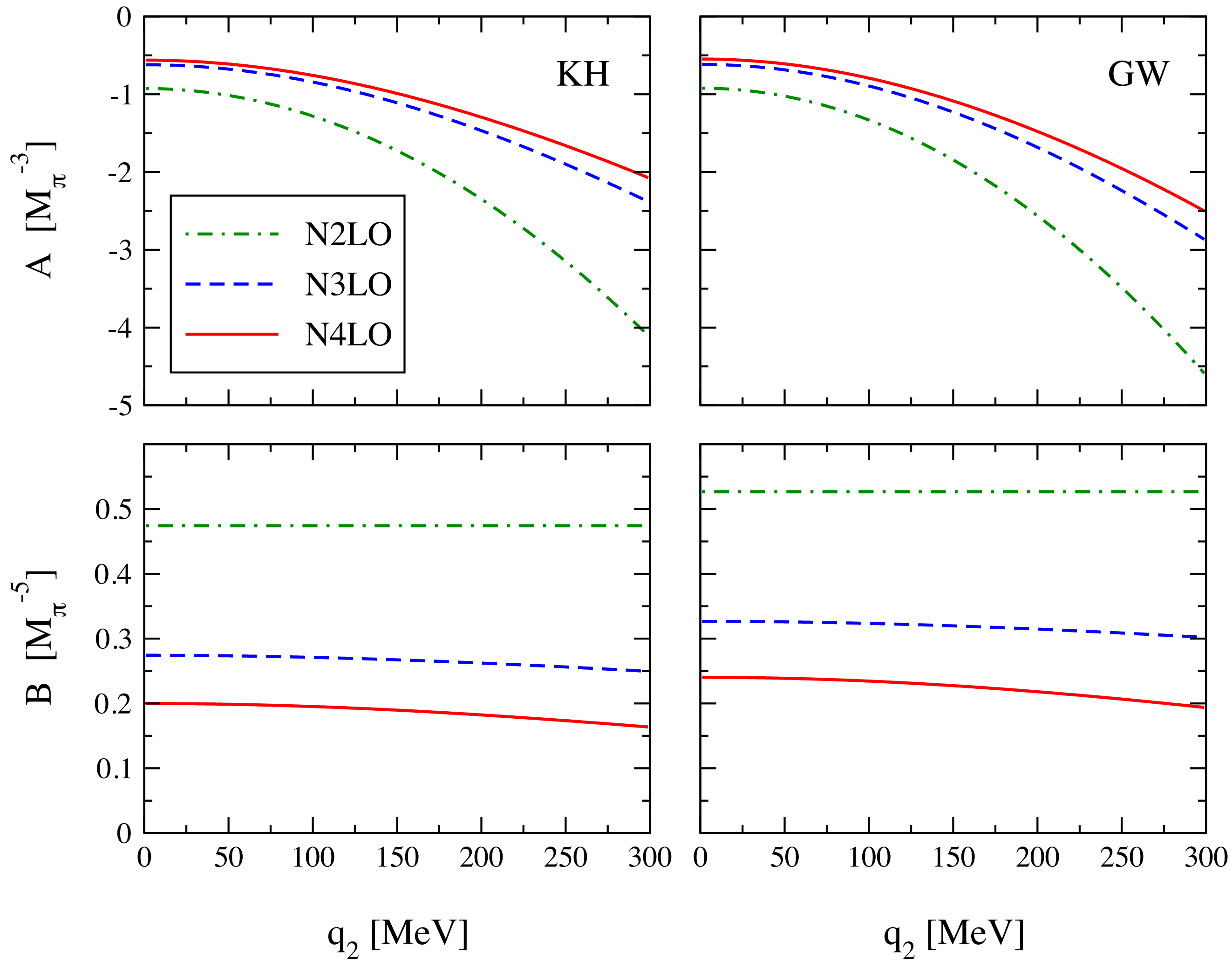
KH-Fit to pion-nucleon scattering

KH-PWA: R. Koch Nucl. Phys. A 448 (1986) 707



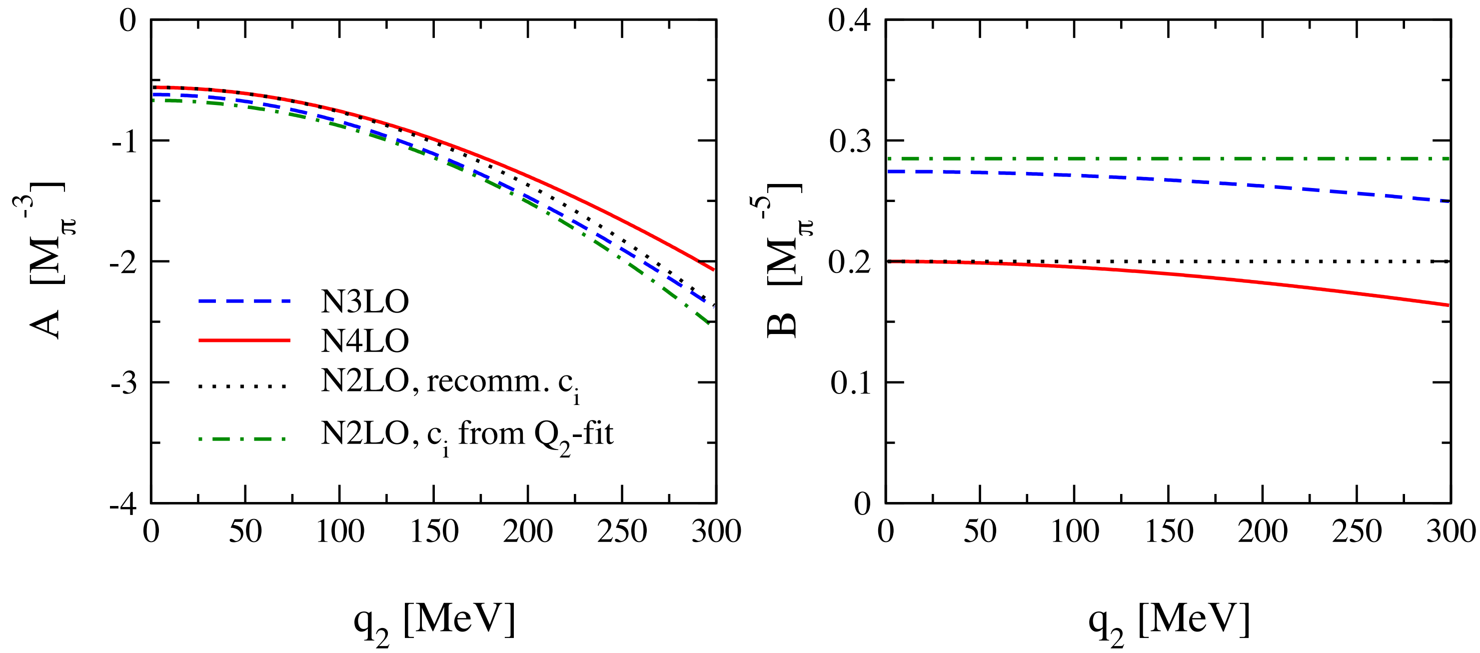
Data fitted for $p_{\text{Lab}} < 150\,\text{MeV}$

Two-pion-exchange at N^4LO



	c_1	c_2	c_3	c_4	$\bar{d}_1 + \bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14} - \bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
GW-fit	-1.13	3.69	-5.51	3.71	5.57	-5.35	0.02	-10.26	1.75	-5.8	1.76	-0.58	0.96
KH-fit	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-10.41	6.08	-0.37	3.26

Working with N^2LO 3NF



Recommended c_i´s by working with N^2LO 3NF

$$c_1 = -0.37\,\text{GeV}^{-1}, \quad c_3 = -2.71\,\text{GeV}^{-1}, \quad c_4 = 1.41\,\text{GeV}^{-1}.$$

- With these parameters we get at $q_2 = 0$ the value and slope of N^4LO results
- c_i´s fitted to pion-nucleon at Q^2 (KH-fit) lead to slightly different results for B-function

$$c_1 = -0.25\,\text{GeV}^{-1}, \quad c_2 = 2.02\,\text{GeV}^{-1}, \quad c_3 = -2.80\,\text{GeV}^{-1}, \quad c_4 = 2.01\,\text{GeV}^{-1}.$$

Summary

- Few-nucleon forces within chiral EFT are analyzed upto N^3LO
- Better convergence of nuclear forces if Δ-isobar is included explicitly
- Good convergence of the longest range 3NF up to N^4LO

Perspectives

- Partial wave analysis of N^3LO three body forces
- Complete studies of 3NF and 4NF upto N^3LO Δ-full / N^4LO Δ-less
- Electroweak reactions with few-nucleon systems