

Shell-model description of the Hoyle state

in Carbon-12

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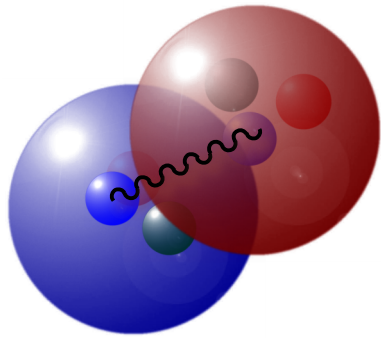
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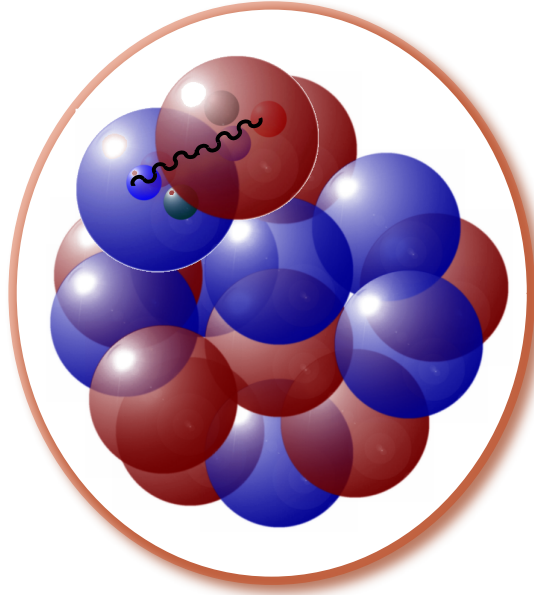
NASA / ISS / Space Shuttle Atlantis

Ab Initio Vision

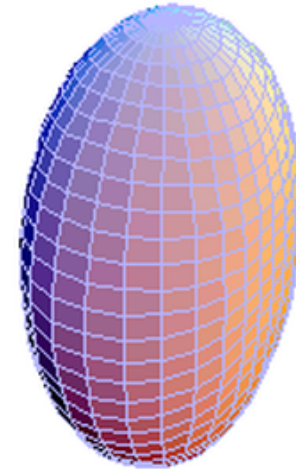
Fundamental Principles



Many-body Dynamics



Collective Properties



**Nucleon-nucleon
interaction**
(NN, NNN, ...)
quarks/gluons



**NCSM
To**

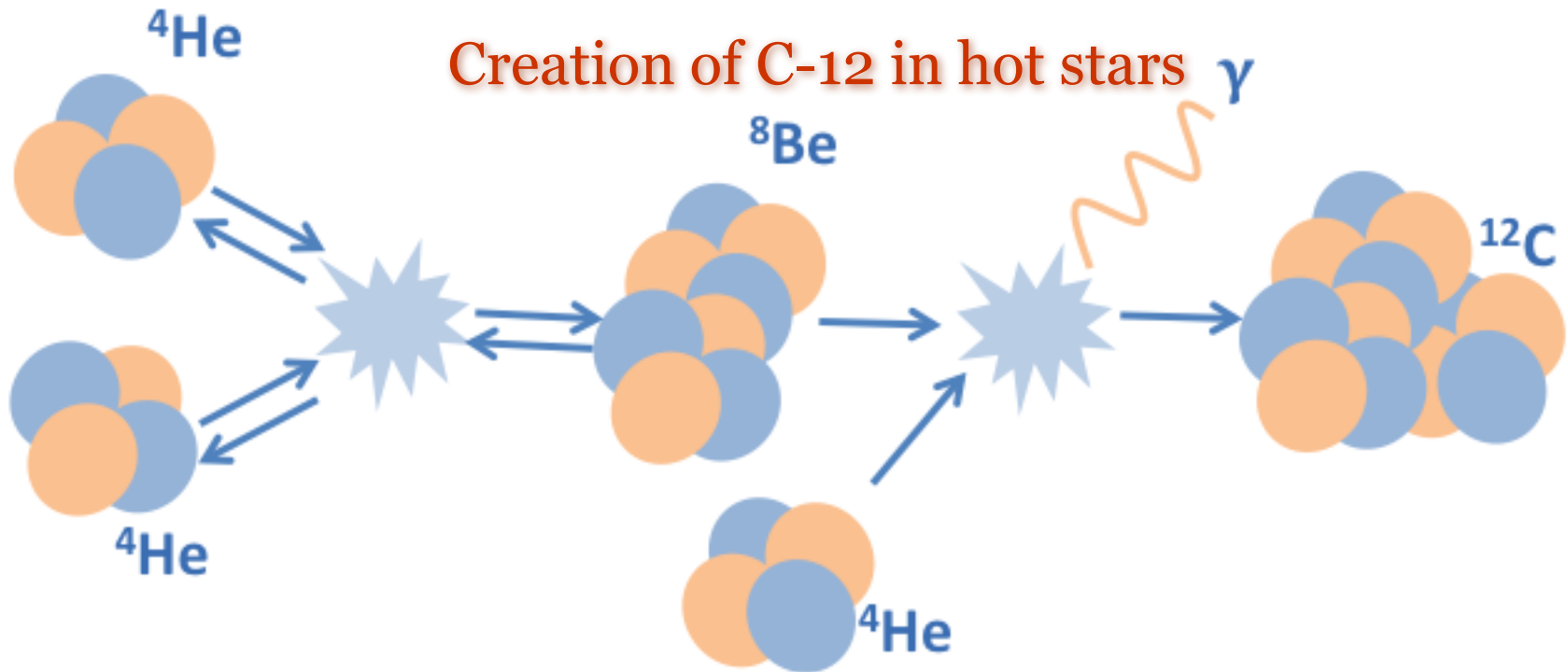
**No-core Symplectic
Model (NCSpM)**



**Structure &
reactions**

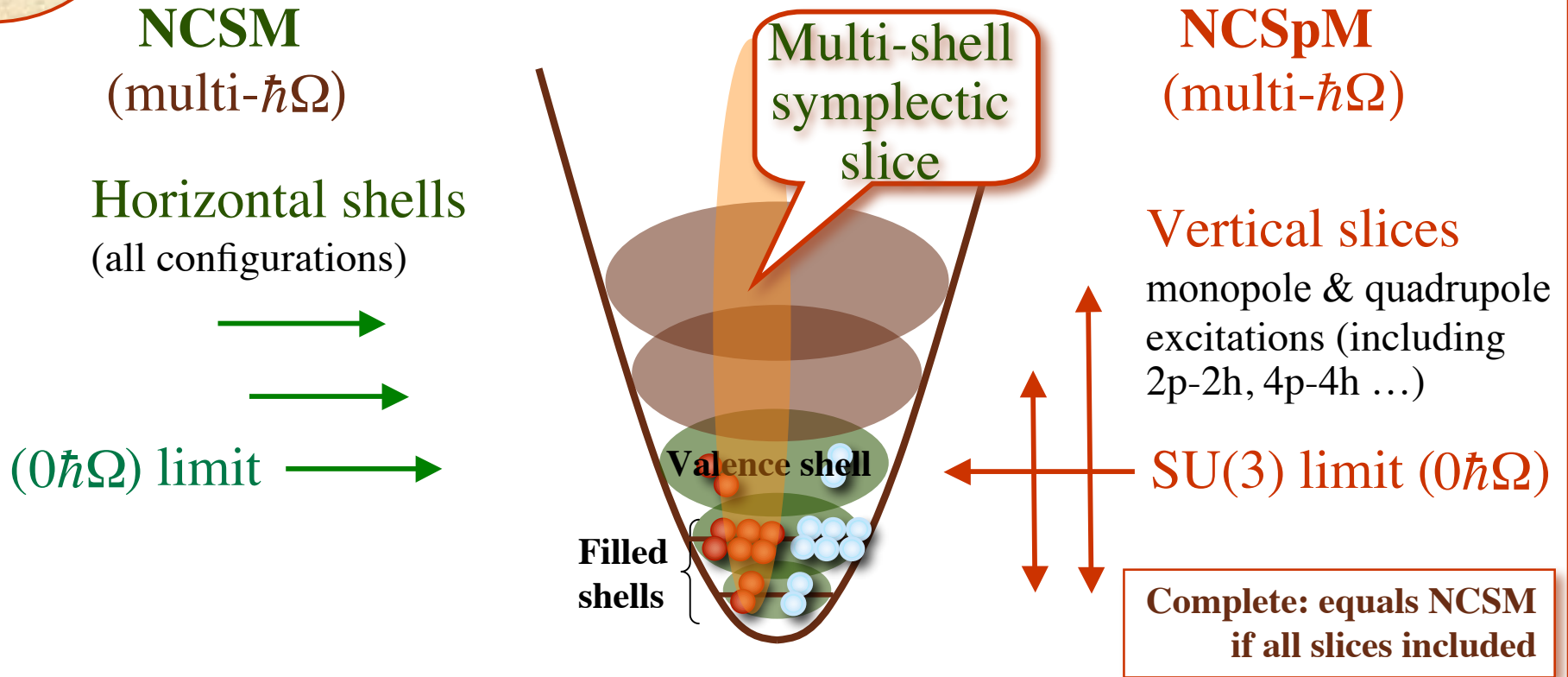
Beyond Reach?

Provide guidance:
no-core shell model with large model spaces



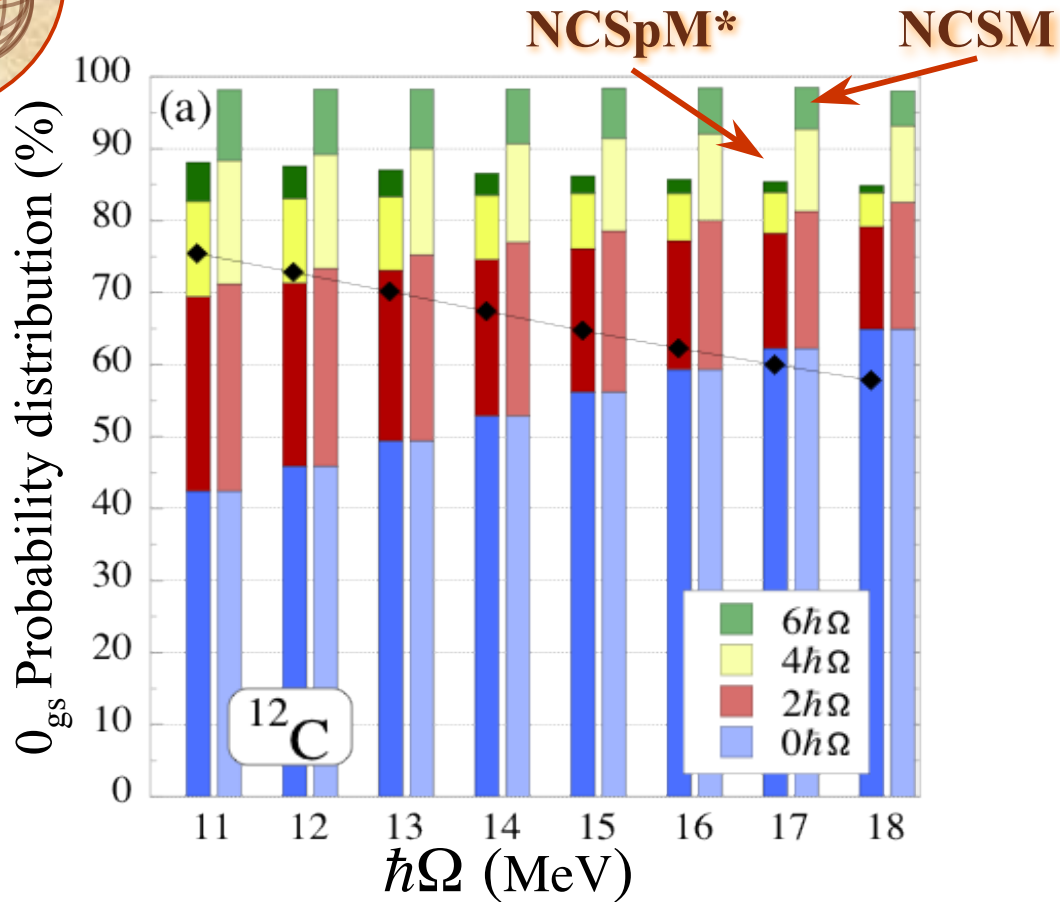
... The elusive Hoyle state ...

Ab initio No-Core Shell Model



- Manage spurious center-of-mass motion
- Relation to the NCSM: fully microscopic!
- Extensible: spaces; 3&4 body enabled, etc.

Probability Distribution: Ground States ^{12}C



*Only 3 0p-0h slices: ~85%

$N_{\sigma}(\lambda\mu)$
 24.5(04) : most deformed
 24.5(12)²: S=1 states

T. Dytrych, K.D. Sviracheva,
 C. Bahri, J.P. Draayer, J.P. Vary,
 Phys. Rev. Lett. 98 (2007) 162503

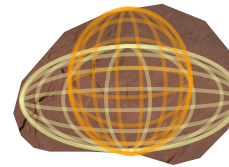
'Microscopic' Symmetries: $Sp(3, \mathbb{R}) \supset SU(3)$

Mars' Deimos



*Intrinsic nucleon dynamics -
reflected in shape deformations*

$SU(3)$



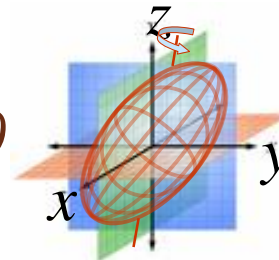
shapes

$SO(3)$



rotations

$SO(2)$



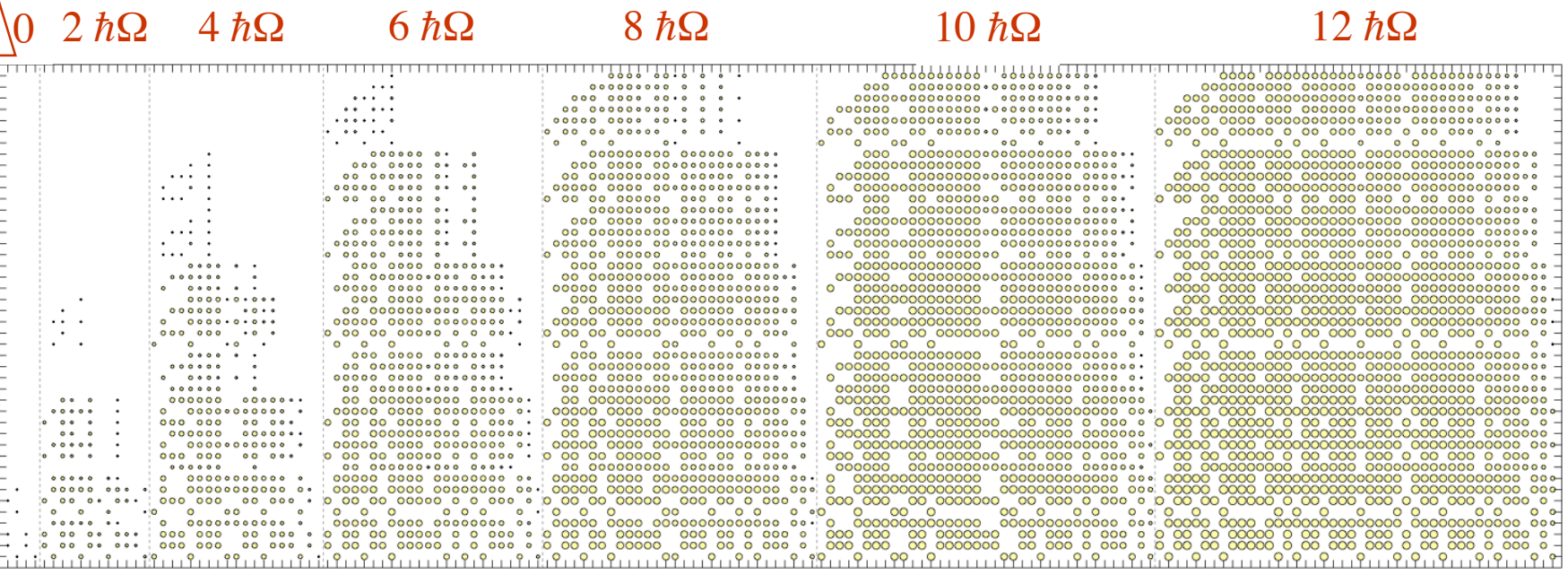
space orientation

Full (NCSM) Model Space

$$^{12}\text{C}; J=0; N_{\text{max}}=12$$

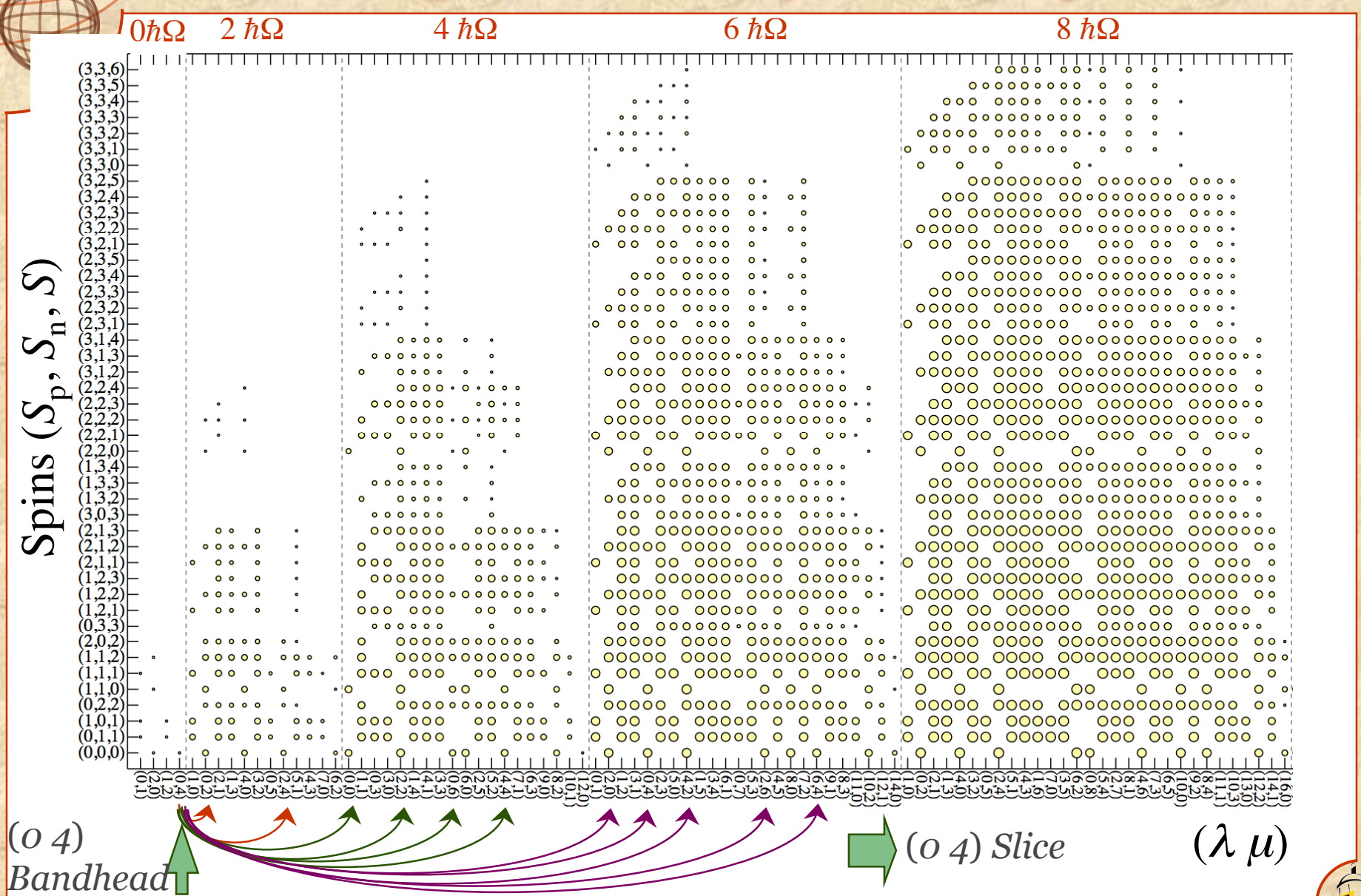
Dimensions of 'space-spin pixels'

Spin part, SU(2): S_p, S_n, S

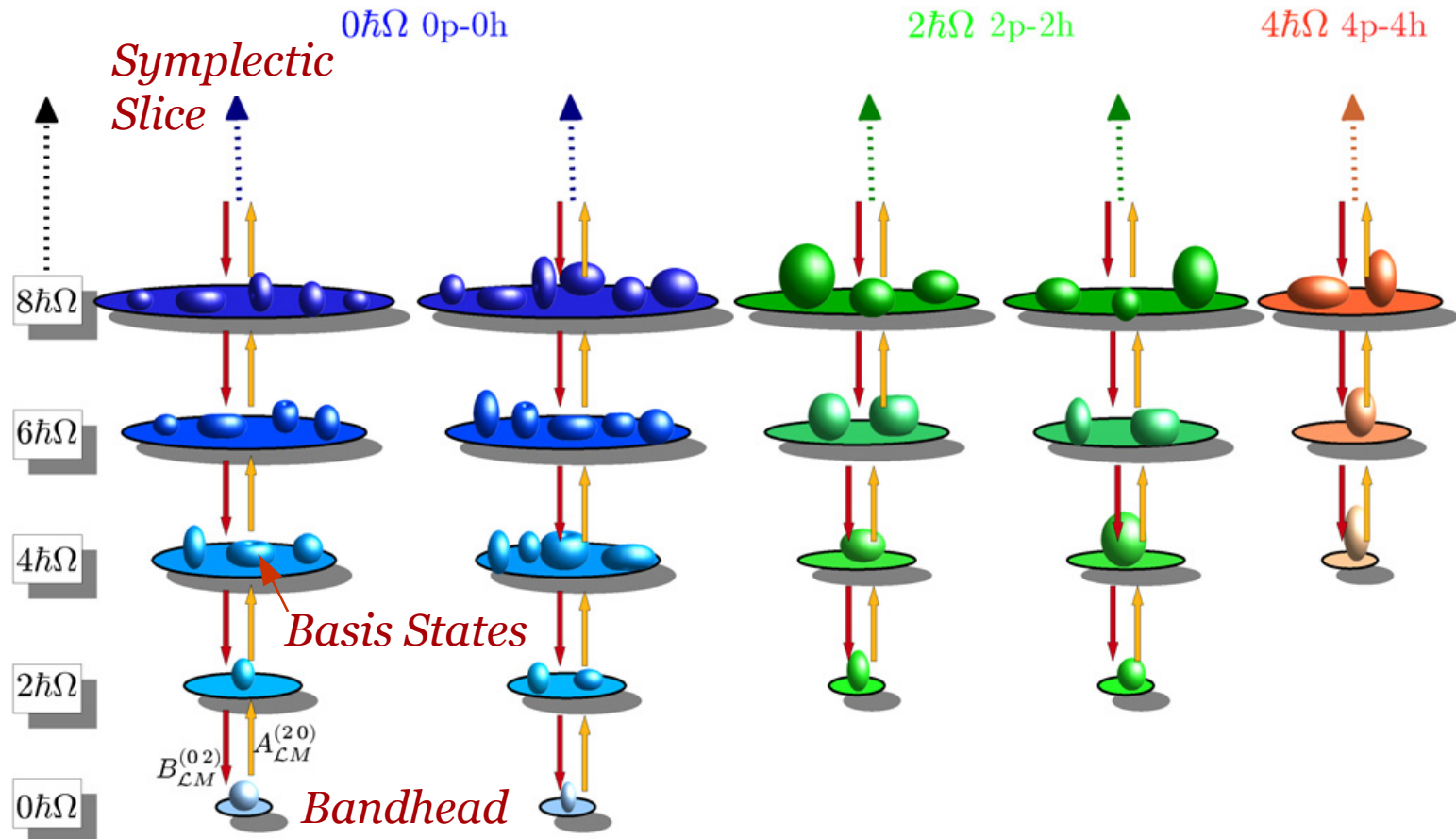


Spatial part, SU(3): $(\lambda \mu)$

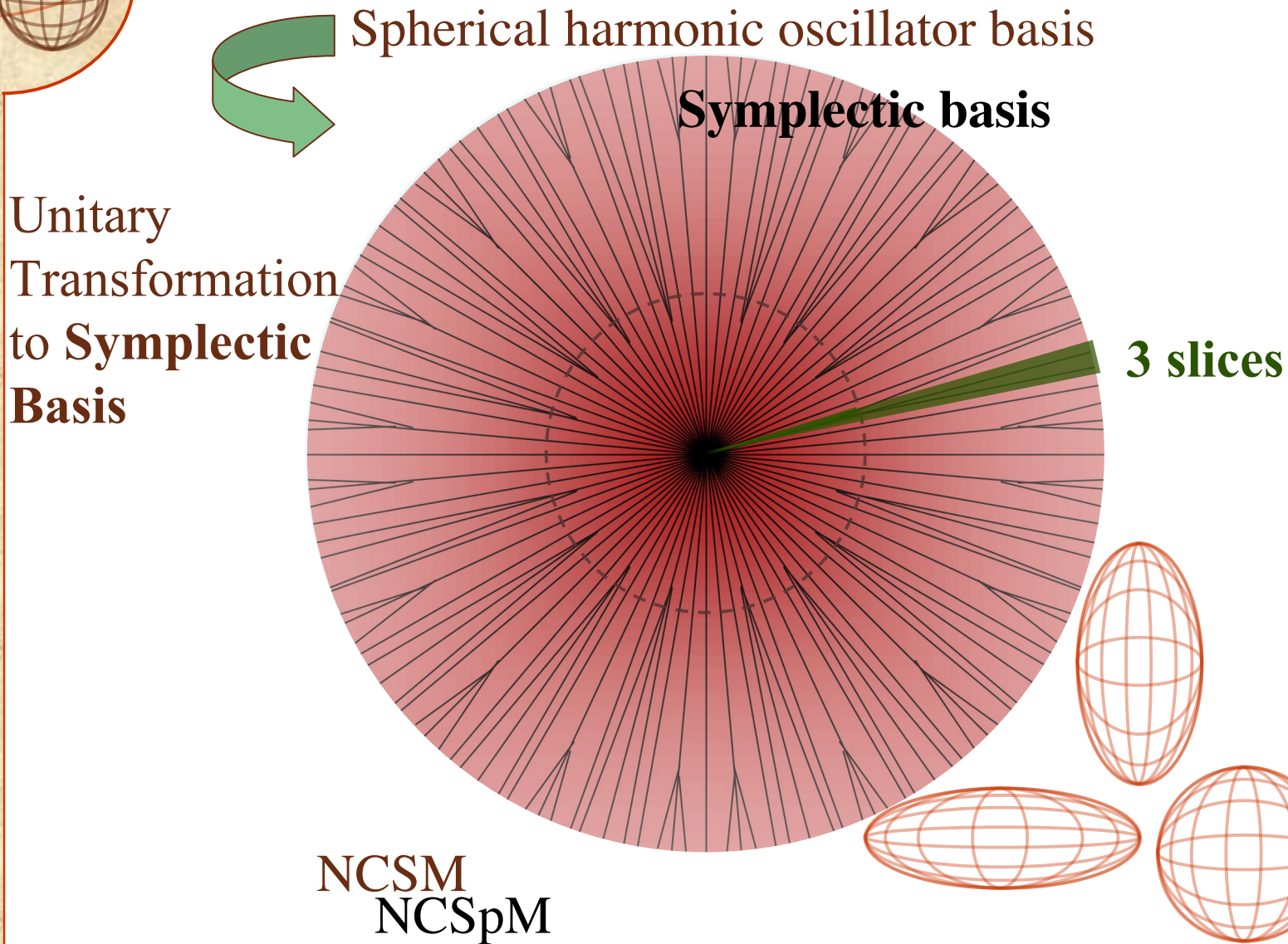
Further organization of space ... symplectic slices



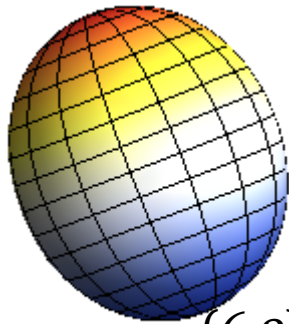
Symplectic (vertical) slices



Selected Model Space

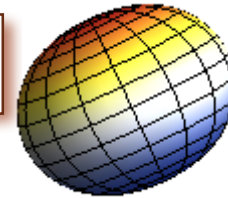


The three slices



*(6 2) Bandhead
(Prolate)*

0p-0h

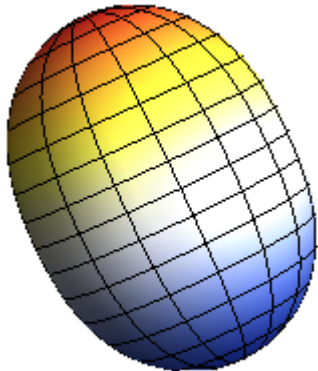


*(0 4) Bandhead
(Oblate)*

2p-2h

Main contribution
to 0_3^+

Main contribution to
ground state
rotational band
($0_{g.st.}^+, 2_1^+, 4_1^+$)



4p-4h

*(12 0) Bandhead
(More Prolate)*

Main contribution to
Hoyle state

Simple Effective Interaction

Long-range expansion of any 2-b central force (range parameter a)

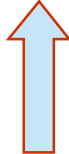
$$\sum_{i < j} V(r_{ij}/a) = \sum_{i < j} \xi_0 + \xi_2 r_{ij}^2 / a^2 + \xi_4 r_{ij}^4 / a^4 + \dots$$

Harmonic oscillator potential 

Valence shell


$$H = H_0 - \frac{\chi}{2} Q \cdot Q$$

Elliott SU(3) model

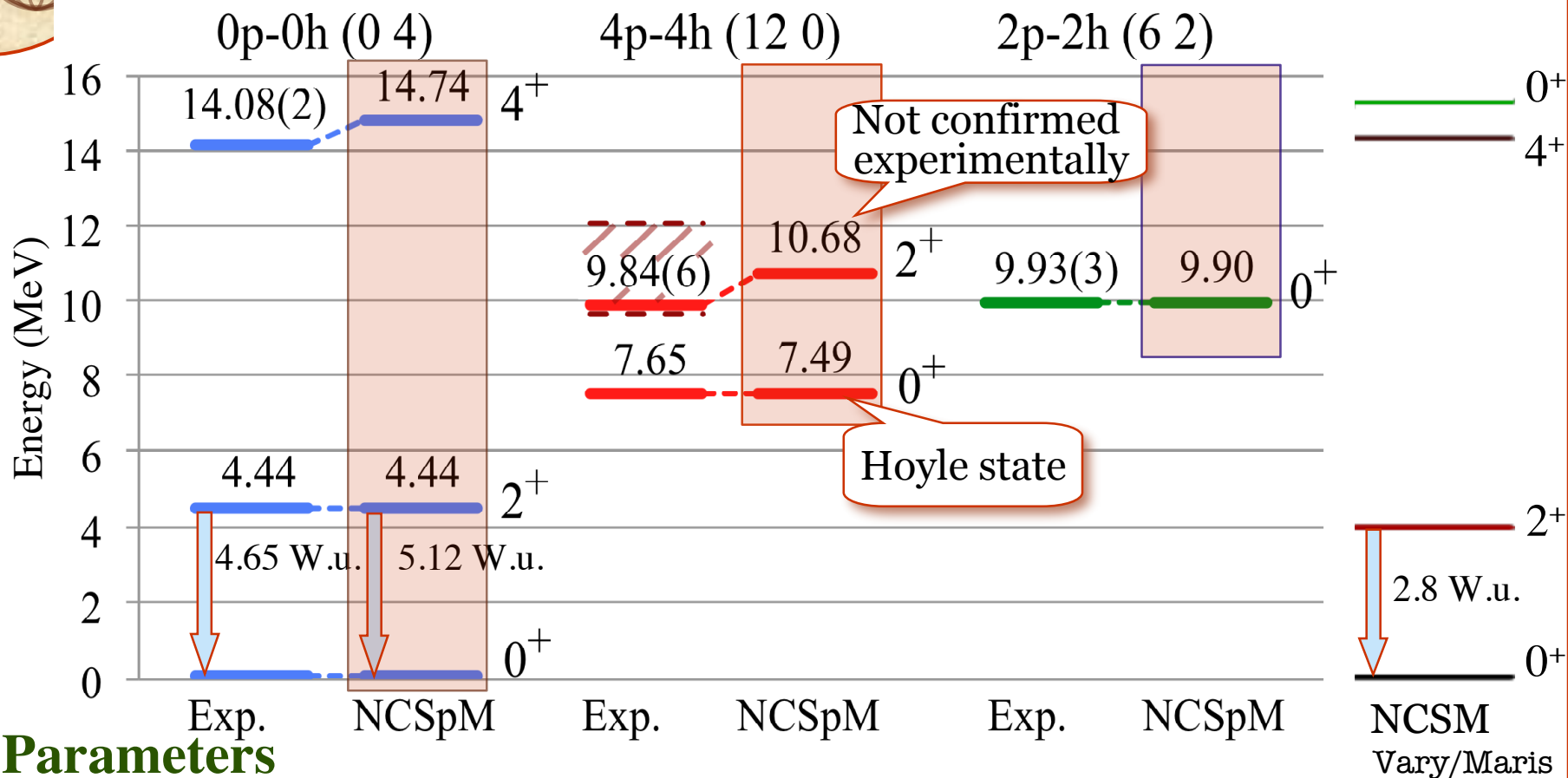
 $\gamma \rightarrow 0$

Multiple shells

$$H = H_0 - \frac{\chi}{2} \frac{1}{\gamma} (e^{\gamma Q \cdot Q} - 1)$$

 Not adjusted... fixed by $\hbar\Omega$

C-12 Energy Spectrum



Parameters

NCSpM:

Fixed:

$$\hbar\Omega = 18 \text{ MeV}, N_{\max} = 20$$

Adjusted:

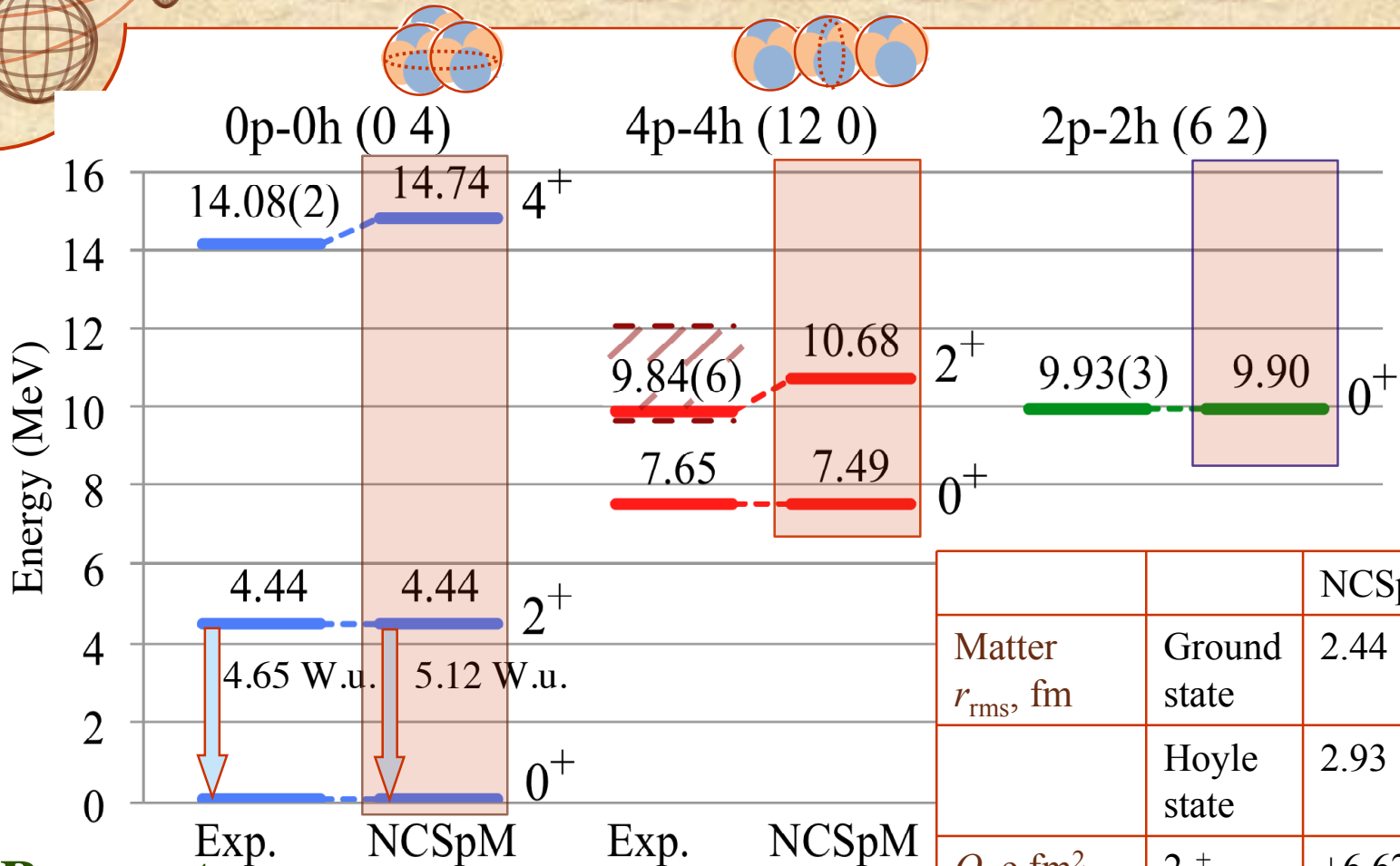
$$\gamma = -1.71 \times 10^{-4}$$

NCSM with JISP16:

Fixed:

$$\hbar\Omega = 15 \text{ MeV}, N_{\max} = 6$$

C-12: Other Observables



Parameters

NCSpM:

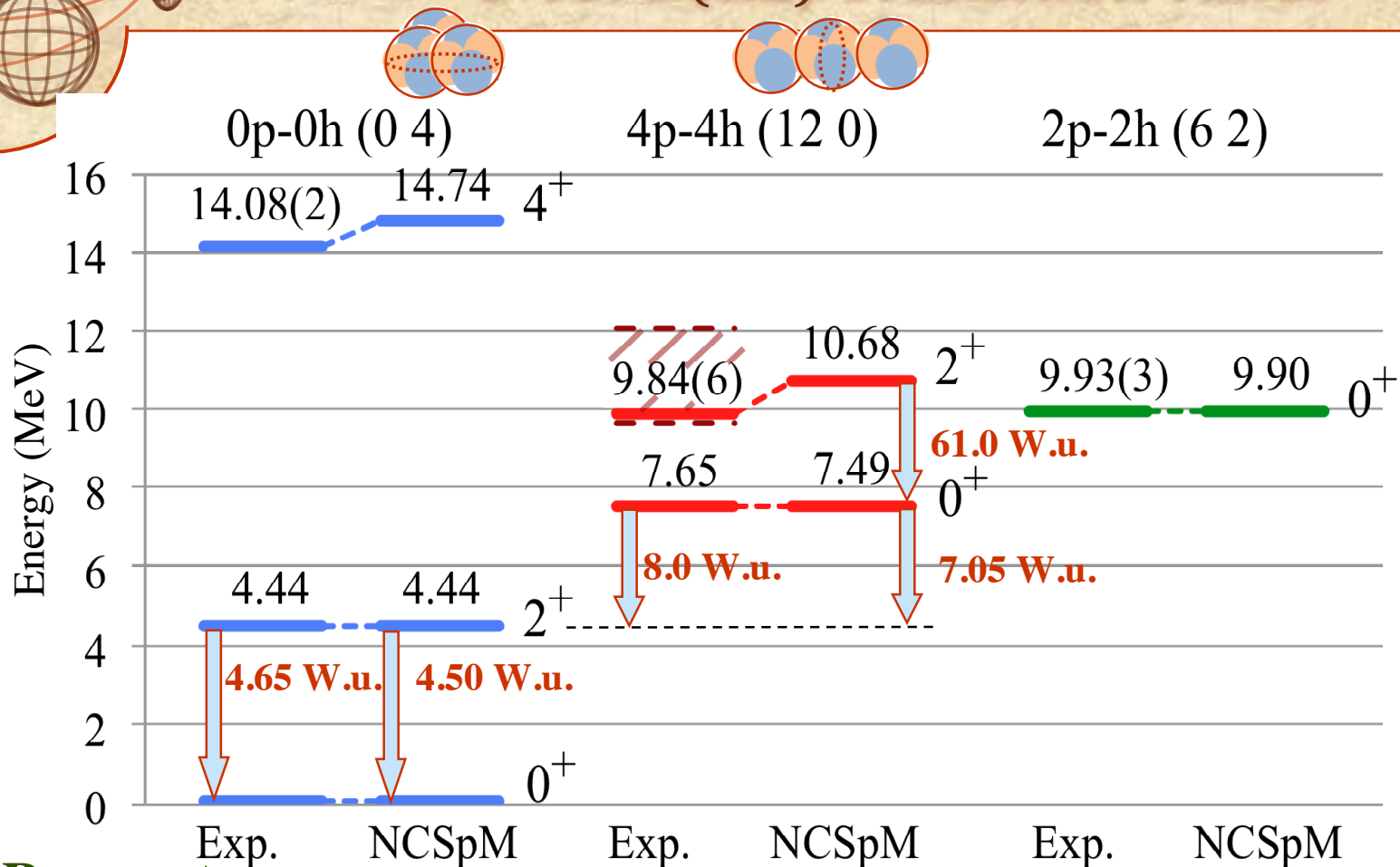
Fixed:

$$\hbar\Omega = 18 \text{ MeV}, N_{\max} = 20$$

Adjusted:

$$\gamma = -1.71 \times 10^{-4}$$

C-12: B(E2) Transition Rates



Parameters

NCSpM:

Fixed:

$$\hbar\Omega = 18 \text{ MeV}, N_{\text{max}} = 20$$

Adjusted:

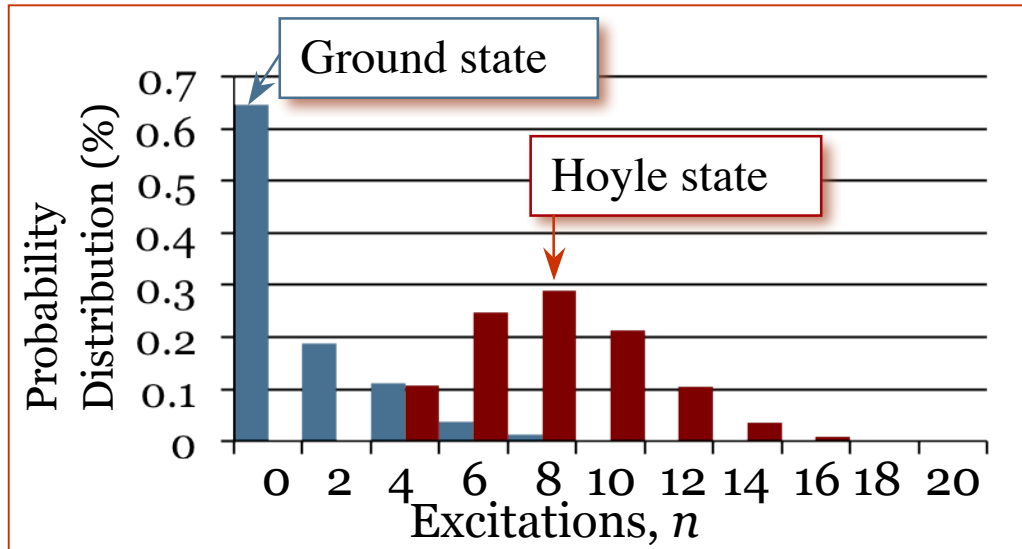
$$\gamma = -1.71 \times 10^{-4}$$

Ground state (small mixing)

0p-0h: 98.63% , 4p-4h: 1.37%

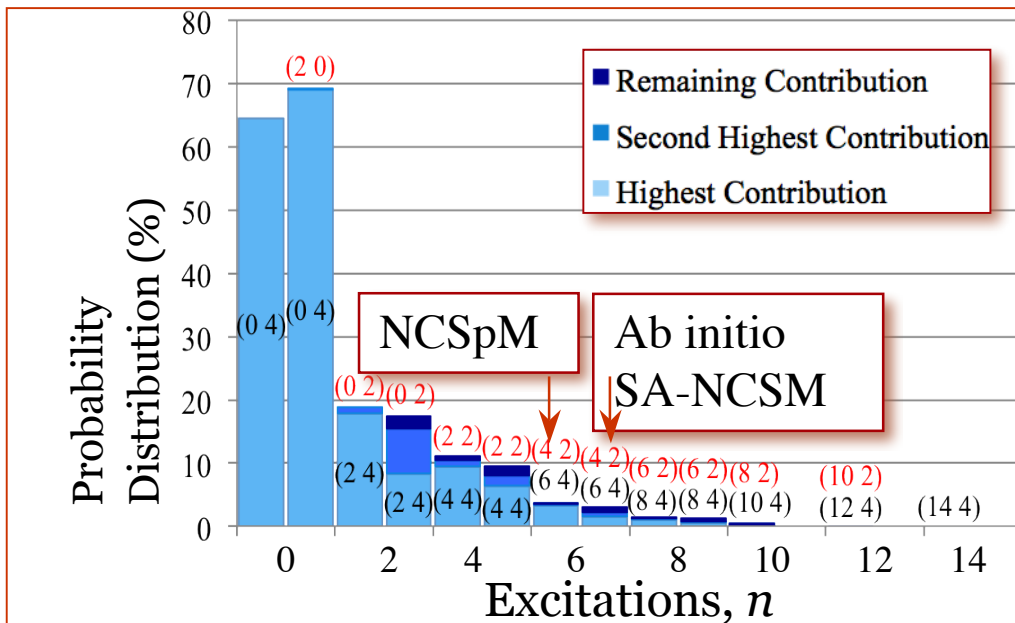
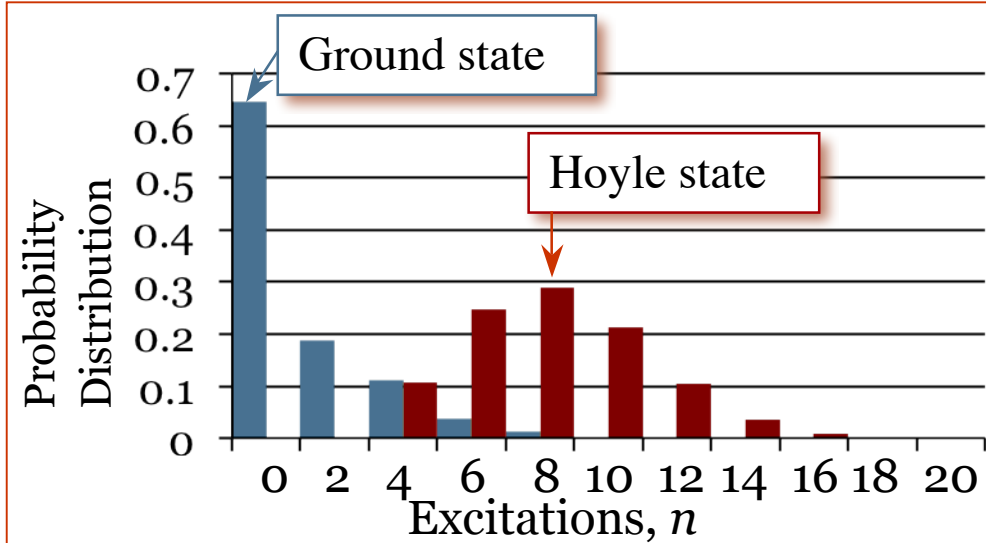
Probability Distribution

- Ground state peaks at $0h\Omega$ and extends to $\sim 10h\Omega$
- Hoyle state peaks at $8h\Omega$ and extends to $\sim 18h\Omega$

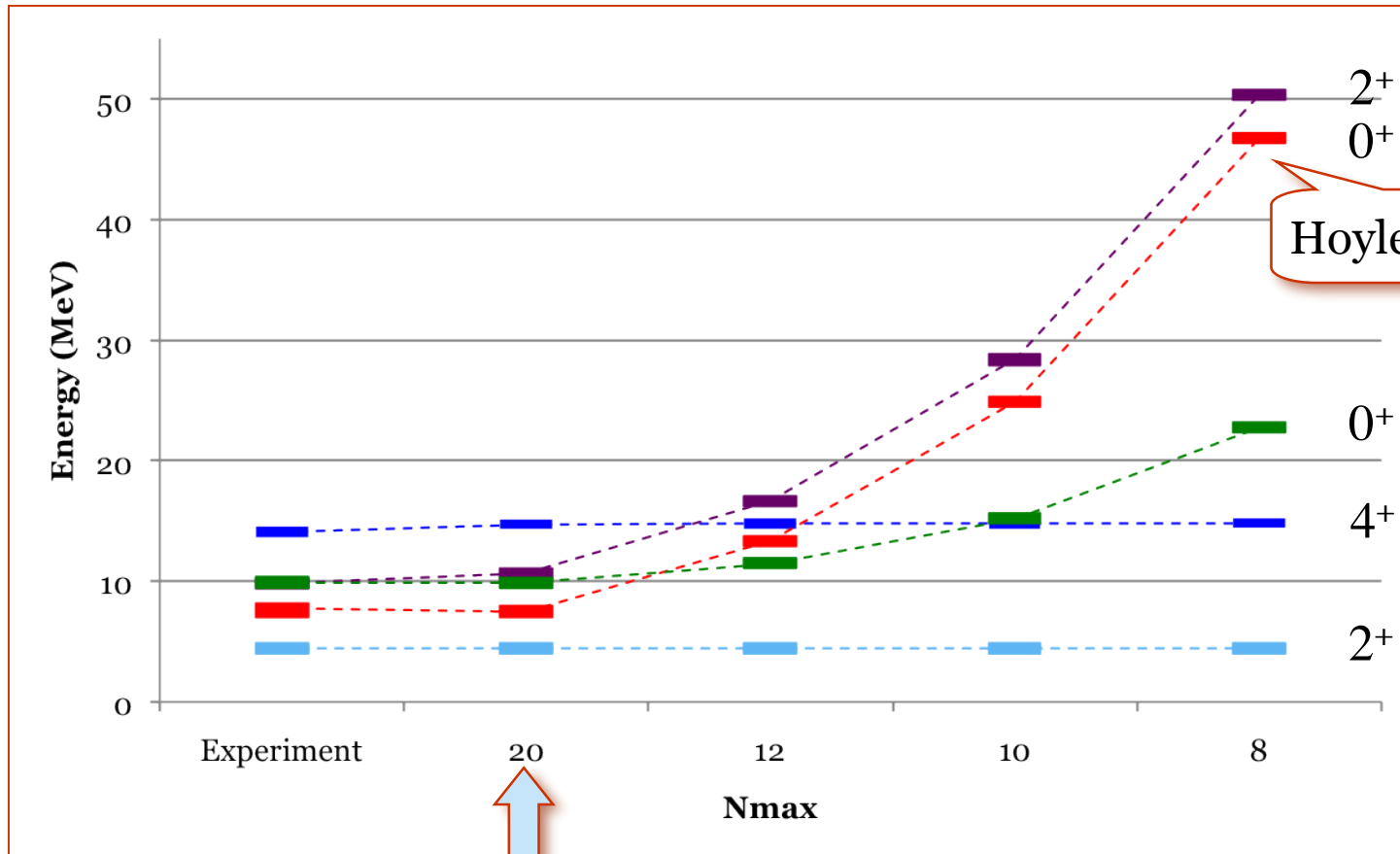


Probability Distribution

- Ground state peaks at $0h\Omega$ and extends to $\sim 10h\Omega$
- Hoyle state peaks at $8h\Omega$ and extends to $\sim 18h\Omega$
- Ground state: compared to ab initio SA-NCSM ($S_p=S_n=0$) with JISP16 ($N_{\max}=8$)

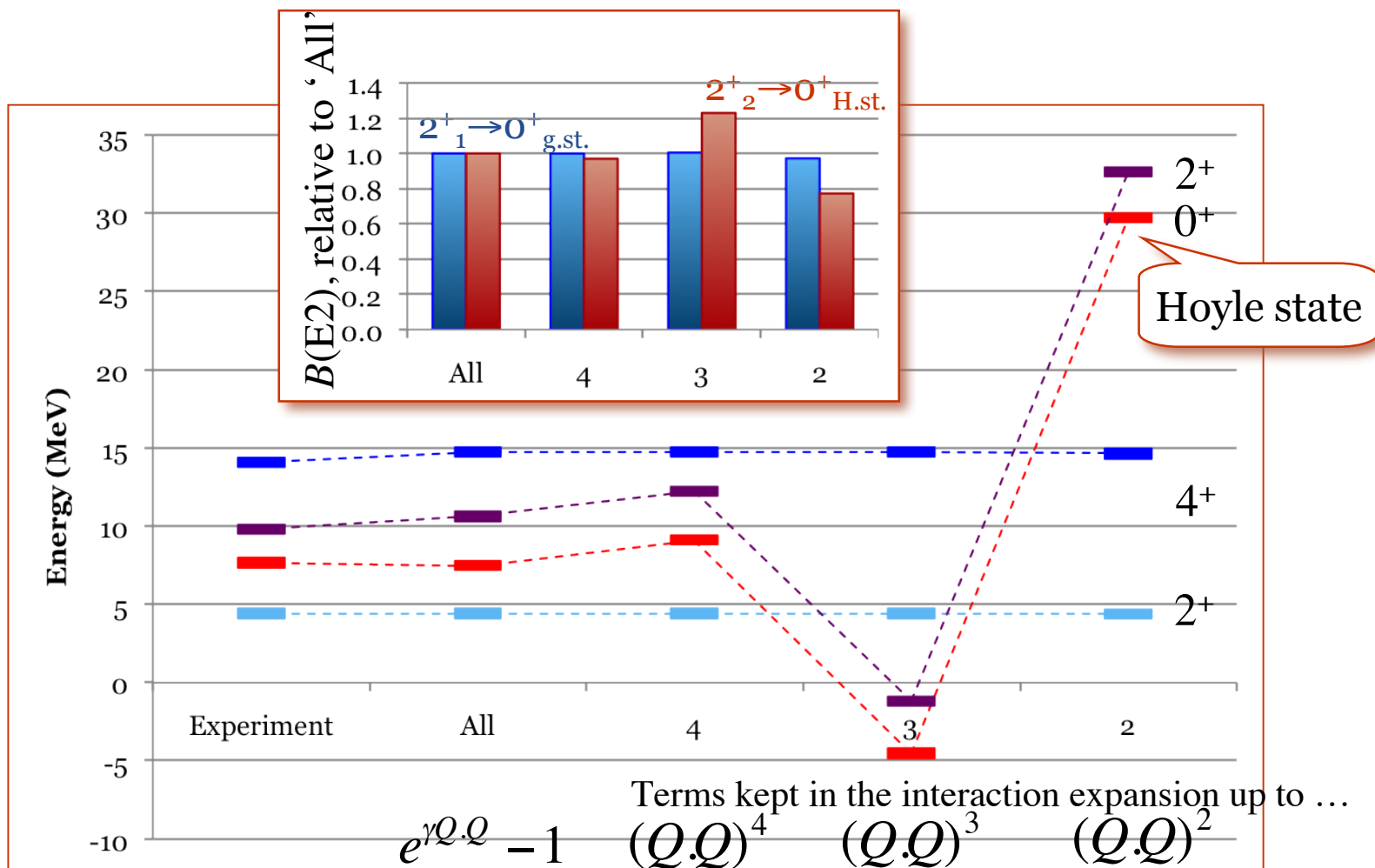


NCSM Current Model Spaces



NCSpM

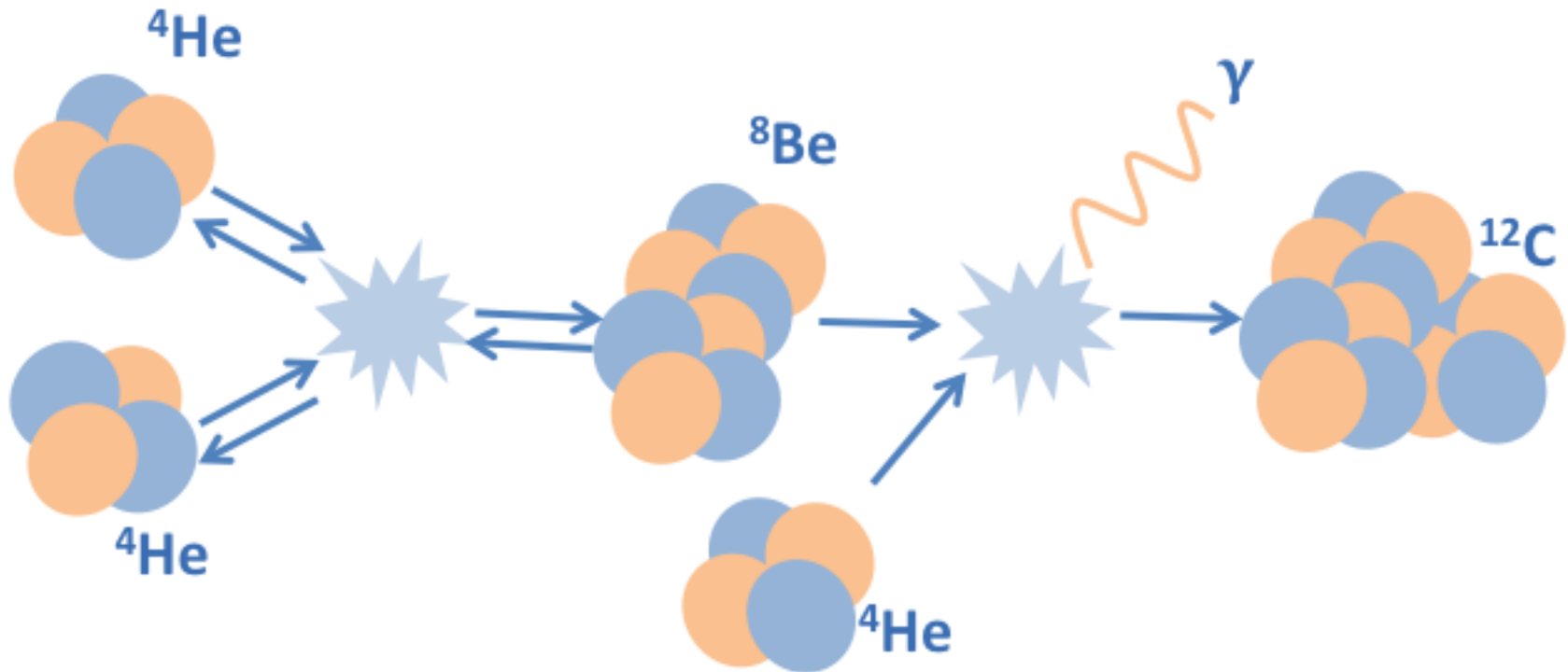
Contribution of (simple) many-body terms



Present results

Conclusions

NCSpM... understanding deformations, rotations, clusters
... O-16, Si-28, Mg-24,...



... The elusive Hoyle state ... within the *ab initio* NCSM reach
... need of many-body interactions
... simple ...



... Not possible without ...

Elliott/Moshinsky ... $SU(3)$

Draayer/Akiyama ... $SU(3)$ (re)coupling coefficients

Rosentsteel/Rowe ... Adding K.E.: $Sp(3, R)$

Rowe ... Vector Coherent State: MEs of $Sp(3, R)$ generators

Bahri ... $Sp(3, R)$ -based code

Vary/Maris ... NCSM

Dytrych ... SA-NCSM

NERSC + LONI (LSU)