The ground state of He-9

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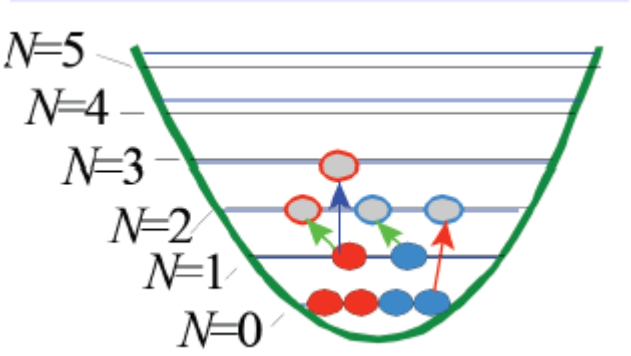
The No-Core Shell Model (NCSM)

Starting Hamiltonian is translationally invariant.

$$H_A = \frac{1}{A}\sum_{i<j}^{A}\frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i<j}^{A} V_{\mathrm{NN},ij}$$

NCSM has two parameters: Nmax and Ω



Provided interaction is "soft" we don't need to do any renormalization of interaction,

It's that "simple".

If we now use a single-particle basis, we have to remove the spurious CM states.

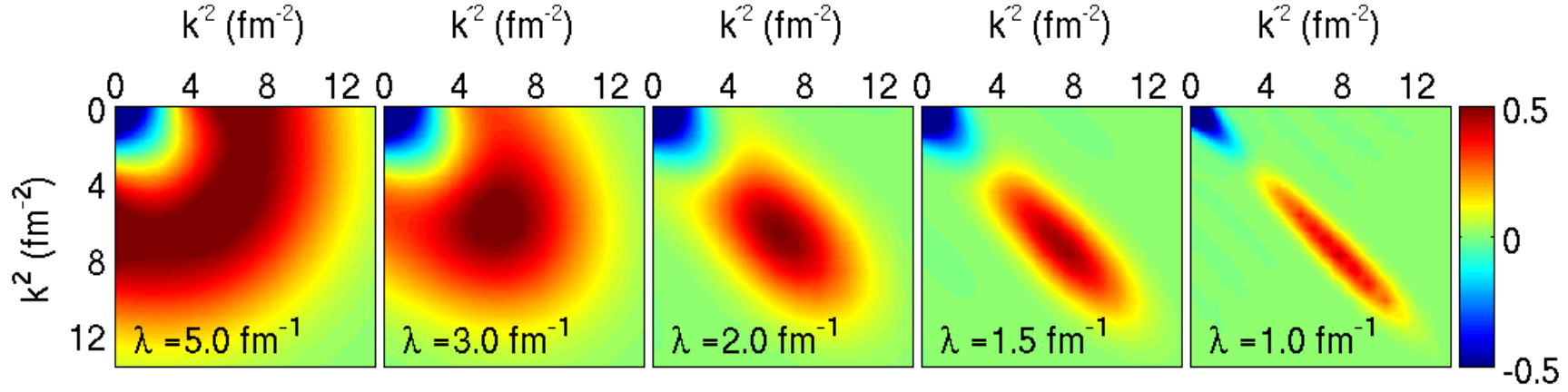
Advantage in m-scheme: Antisymmetry is easy to implement.

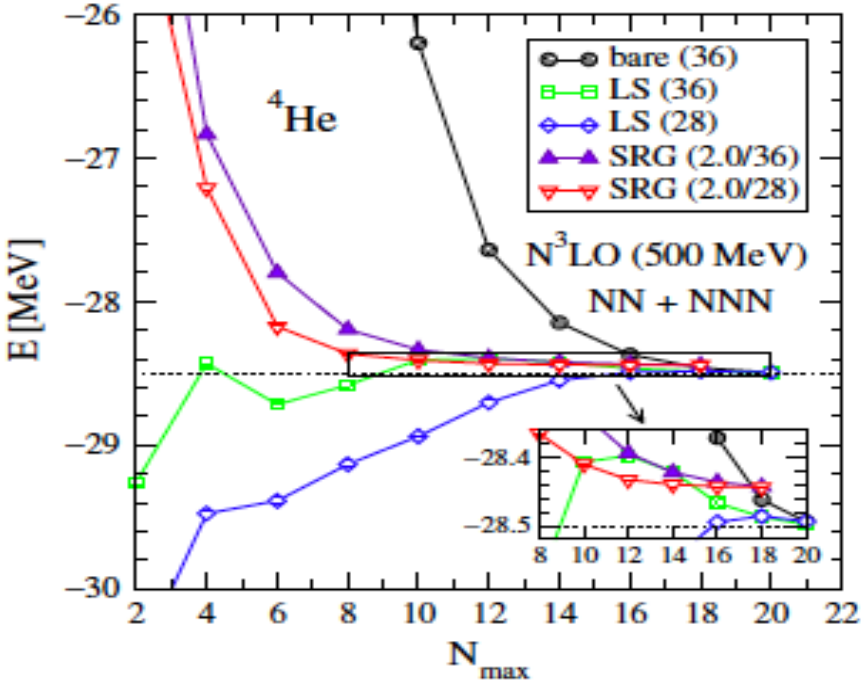
Disadvantage in m-scheme: Number of basis states is much larger than JT basis

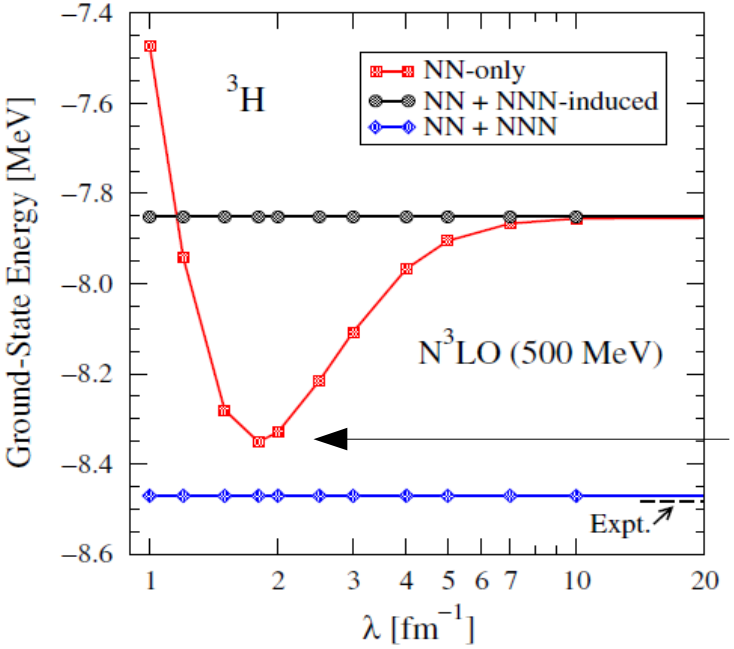
SRG evolved potentials

PRL **103,** 082501 (2009)
Jurgenson et al

$$H_\lambda = U_\lambda H_{\lambda=\infty} U_\lambda^\dagger \qquad \frac{dH_\lambda}{d\lambda} = -\frac{4}{\lambda^5}[[T, H_\lambda], H_\lambda]$$

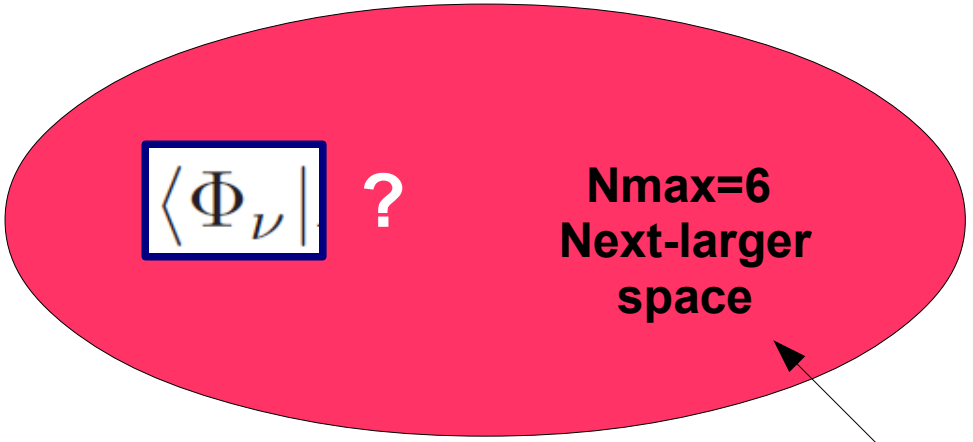






NN force „acts“ as though it is a NNN force. Closest to 3N at about 1.5-2.0/fm

Importance truncation schematically

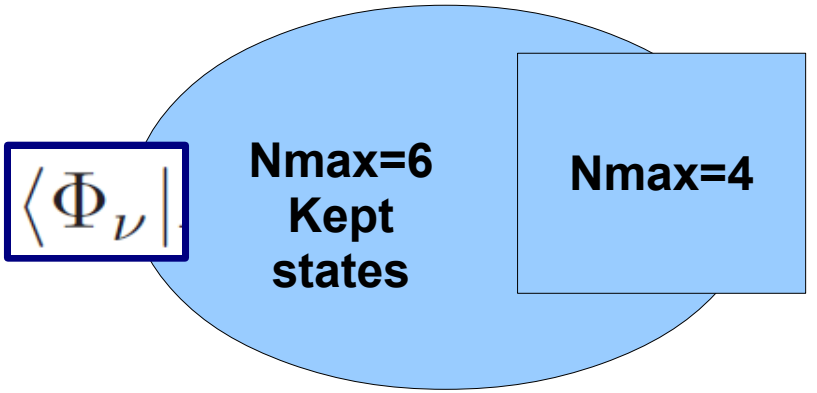


Nmax=4
Reference space

$$\kappa_\nu = \frac{|\langle \Phi_\nu | H | \Psi_{ref} \rangle|}{\epsilon_\nu - \epsilon_{ref}}$$

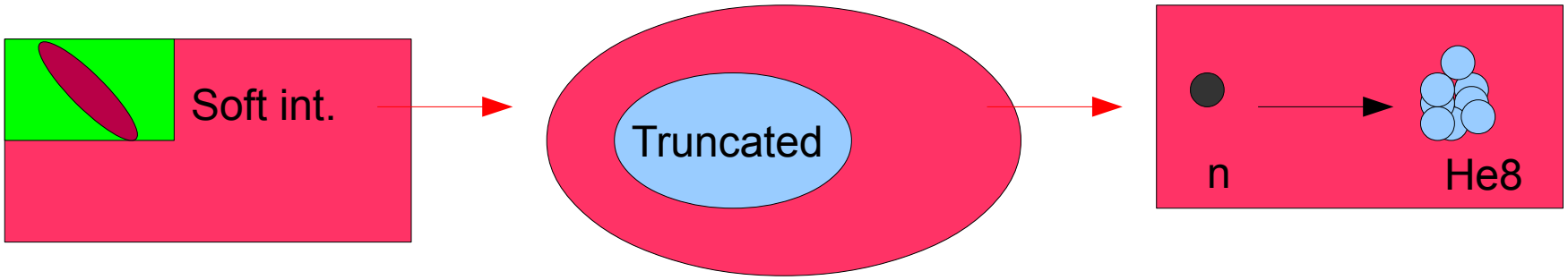
Typically choose κ ~ 10^{-5}

$$\Delta_{\mathrm{excl}}(\kappa_{\min}) = -\sum_{\nu \notin \mathcal{M}(\kappa_{\min})} \frac{|\langle \Phi_\nu | H | \Psi_{\mathrm{ref}} \rangle|^2}{\epsilon_\nu - \epsilon_{\mathrm{ref}}}$$



Physics question: Is ^{9}He bound?

- **Interesting physics question: One of the few nuclei that can be studied theoretically and experimentally, that lies beyond the neutron drip-line.**
- **Example: Intruder states from the sd shell. Be-11 and Li-10 have unnatural parity for gs (N=7 isotones).**



Use „soft“ interactions as input. Note:2N only SRG potentials.

Truncate the full model space to a smaller feasible space. Importance Truncation.

Do a scattering calculation of a neutron on He8 – look for bound states in He9. NCSM/RGM calculation

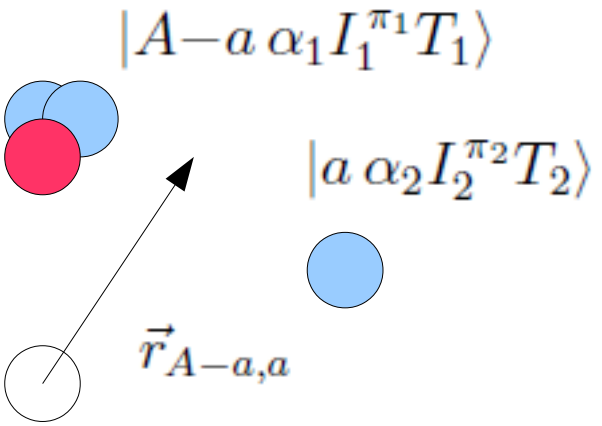
PRL 101, 092501 (2008)
PHYSICAL REVIEW C 79, 044606 (2009)

Sofia Quaglioni and Petr Navrátil

NCSM/RGM key ideas

- **Expand wavefunction on a basis of binary clusters.**
- **Clusters themselves are anti-symmetric, but not anti-symmetric with each respect to each other.**

$$|\Phi^{J^\pi T}_{\nu r}\rangle = \Big[\big(\,|A{-}a\,\alpha_1 I_1^{\pi_1} T_1\rangle\,|a\,\alpha_2 I_2^{\pi_2} T_2\rangle\,\big)^{(sT)} \times Y_\ell(\hat{r}_{A-a,a})\Big]^{(J^\pi T)} \frac{\delta(r - r_{A-a,a})}{r r_{A-a,a}}\,.$$



$$|\Psi^{J^\pi T}\rangle = \sum_\nu \int dr\, r^2 \frac{g_\nu^{J^\pi T}(r)}{r}\,\hat{\mathcal{A}}_\nu\,|\Phi^{J^\pi T}_{\nu r}\rangle$$

- **Clusters determined from NCSM calculation**

$$\sum_\nu \int dr\, r^2 \left[\mathcal{H}^{J^\pi T}_{\nu'\nu}(r', r) - E\,\mathcal{N}^{J^\pi T}_{\nu'\nu}(r', r)\right] \frac{g_\nu^{J^\pi T}(r)}{r} = 0$$

- **Calculate matrix elements for kernels.**

Norm and Hamiltonian kernel

- **The Hamiltonian used**

$$H = T_{\rm rel}(r) + \mathcal{V}_{\rm rel} + \bar{V}_C(r) + H_{(A-a)} + H_{(a)}$$

Relative kinetic between clusters

Relative NN int between clusters (no coulomb)

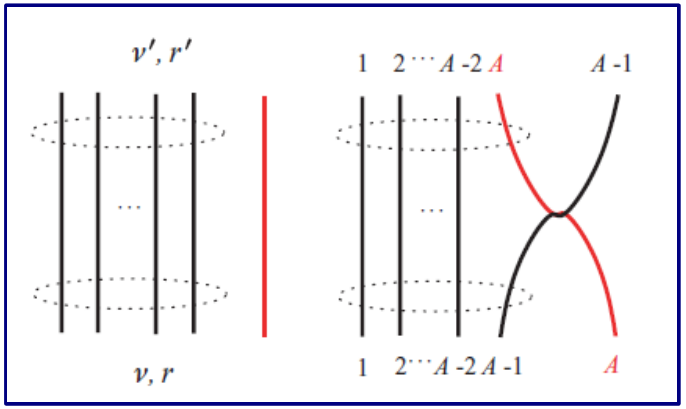
Avg. Coulomb

Determined in same (Nmax,hΩ) space

$$\sum_\nu \int dr\, r^2 \left[\mathcal{H}^{J^\pi T}_{\nu'\nu}(r',r) - E\,\mathcal{N}^{J^\pi T}_{\nu'\nu}(r',r)\right] \frac{g^{J^\pi T}_\nu(r)}{r} = 0$$

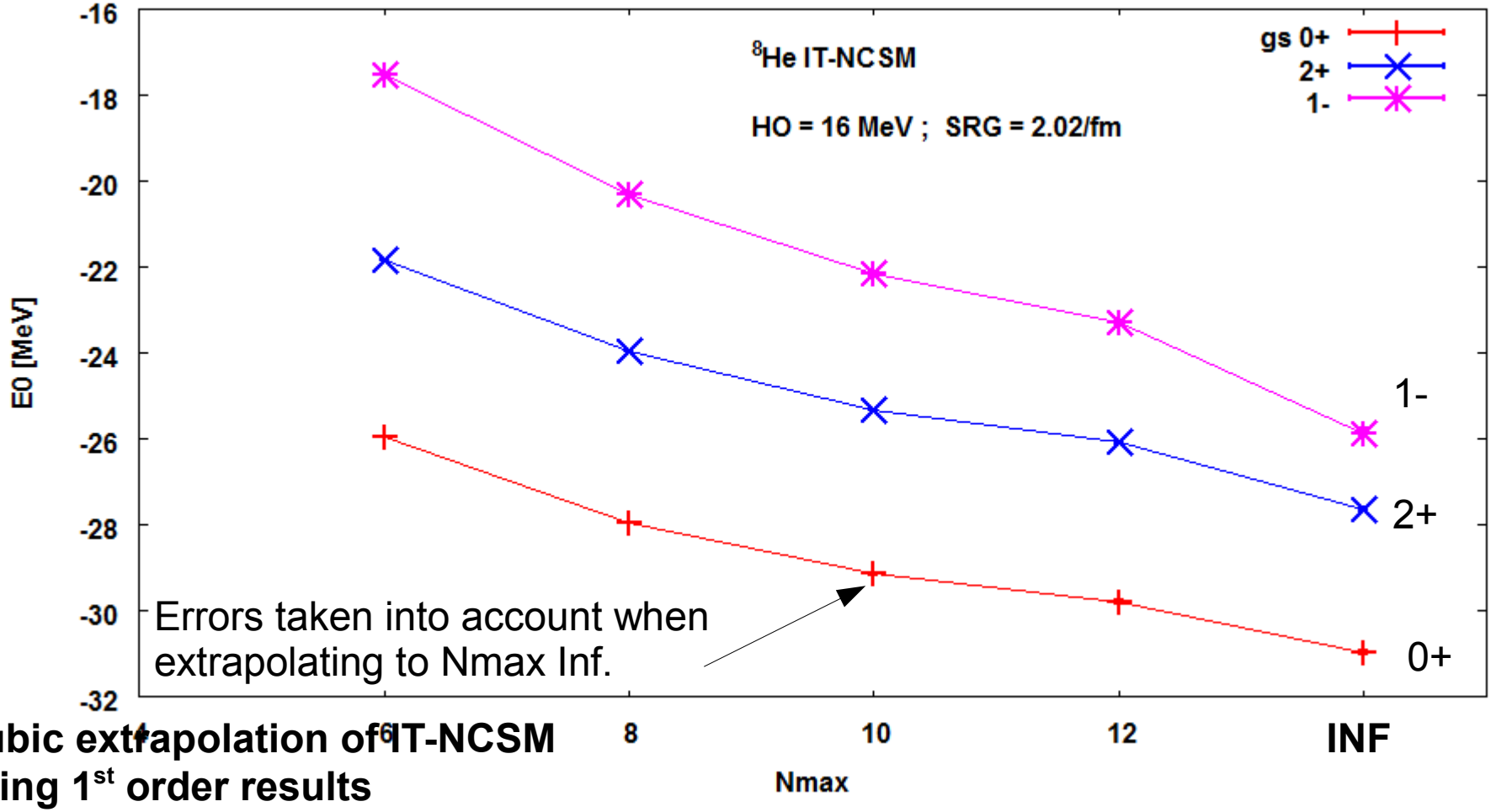
$$\mathcal{H}^{J^\pi T}_{\nu'\nu}(r',r) = \left\langle \Phi^{J^\pi T}_{\nu' r'} \middle| \hat{\mathcal{A}}_{\nu'} H \hat{\mathcal{A}}_\nu \middle| \Phi^{J^\pi T}_{\nu r} \right\rangle$$

$$\mathcal{N}^{J^\pi T}_{\nu'\nu}(r',r) = \left\langle \Phi^{J^\pi T}_{\nu' r'} \middle| \hat{\mathcal{A}}_{\nu'} \hat{\mathcal{A}}_\nu \middle| \Phi^{J^\pi T}_{\nu r} \right\rangle$$



Direct **Exchange**

He 8: Nmax=12 (Importance Truncated)



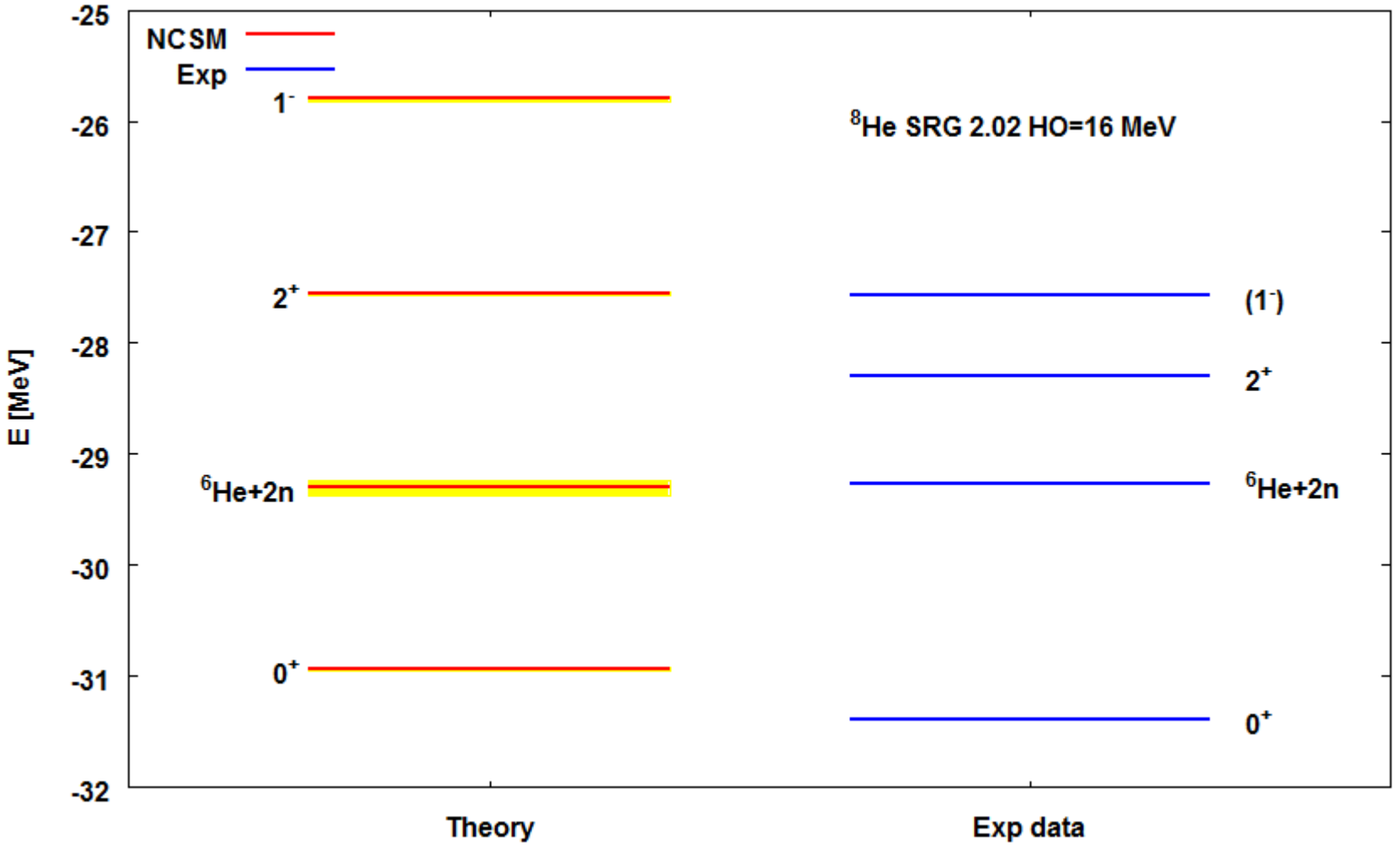
Cubic extrapolation of ^{6}IT-NCSM using 1st order results

Nmax	0+	2+	1-	Gs error (keV)
Infinity	-30.994	-27.653	-25.870	37

Nmax	Full space (+) Parity	IT space (k=1E-5)
12	~ 428 million	~ 13.65 million

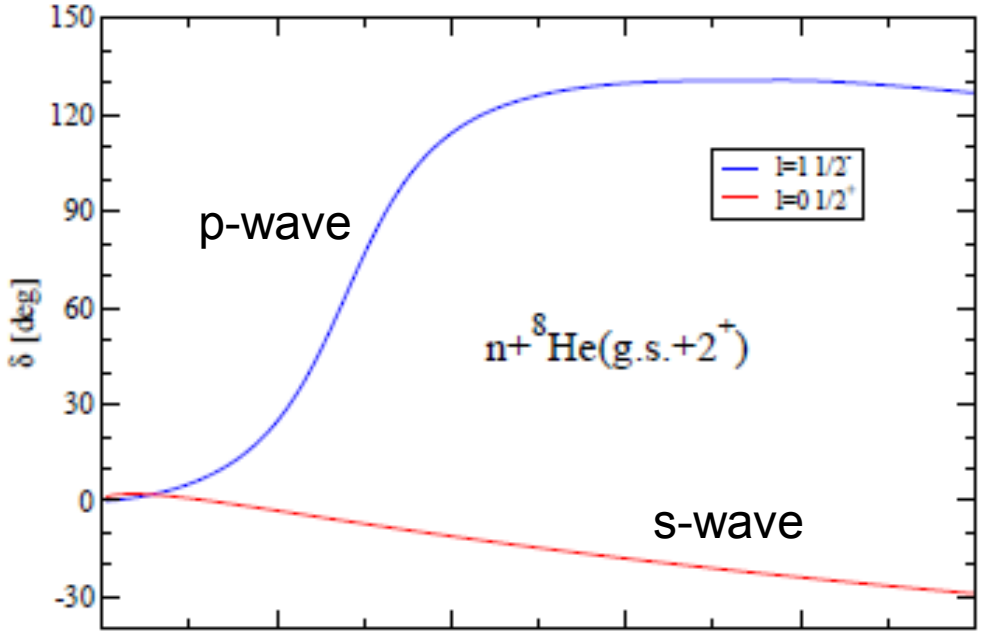
He-8 description

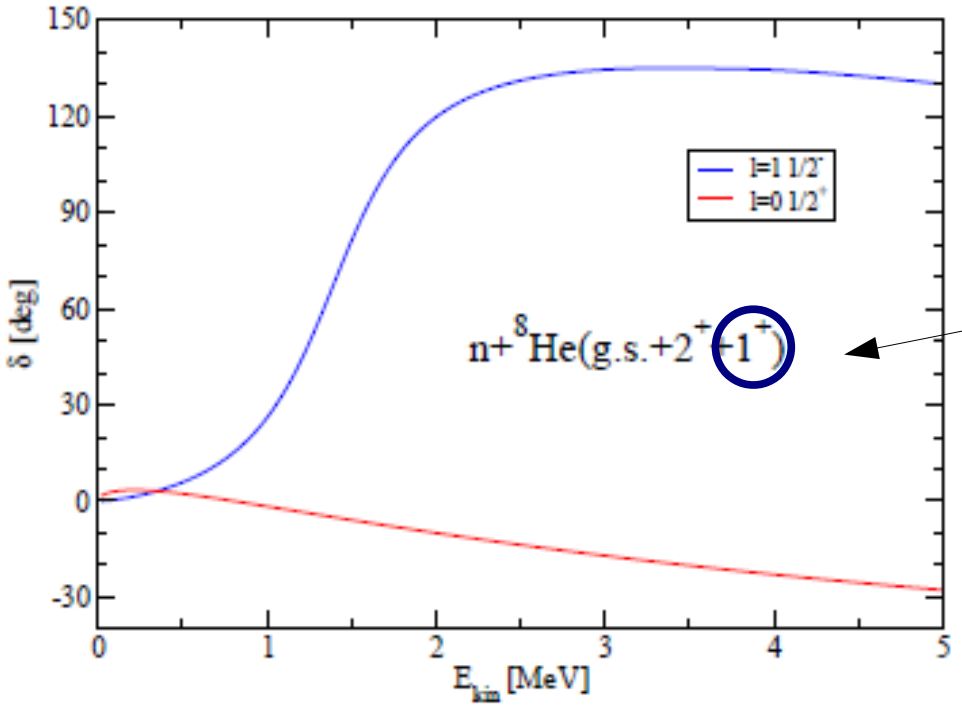
Do we describe the many-body physics of He-8 correctly?
Theory = Nmax extrapolated (IT-NCSM) results.
2+ and 1- state in correct order but splitting to big.

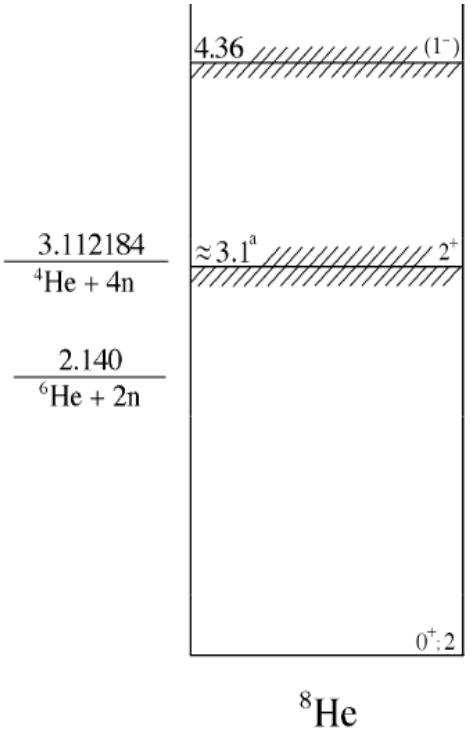


NCSM/RGM: Inclusion of various states

Nuclear Physics A 745 (2004) 155–362







(1-) state: 4.36 MeV

2+ state: 3.1 MeV

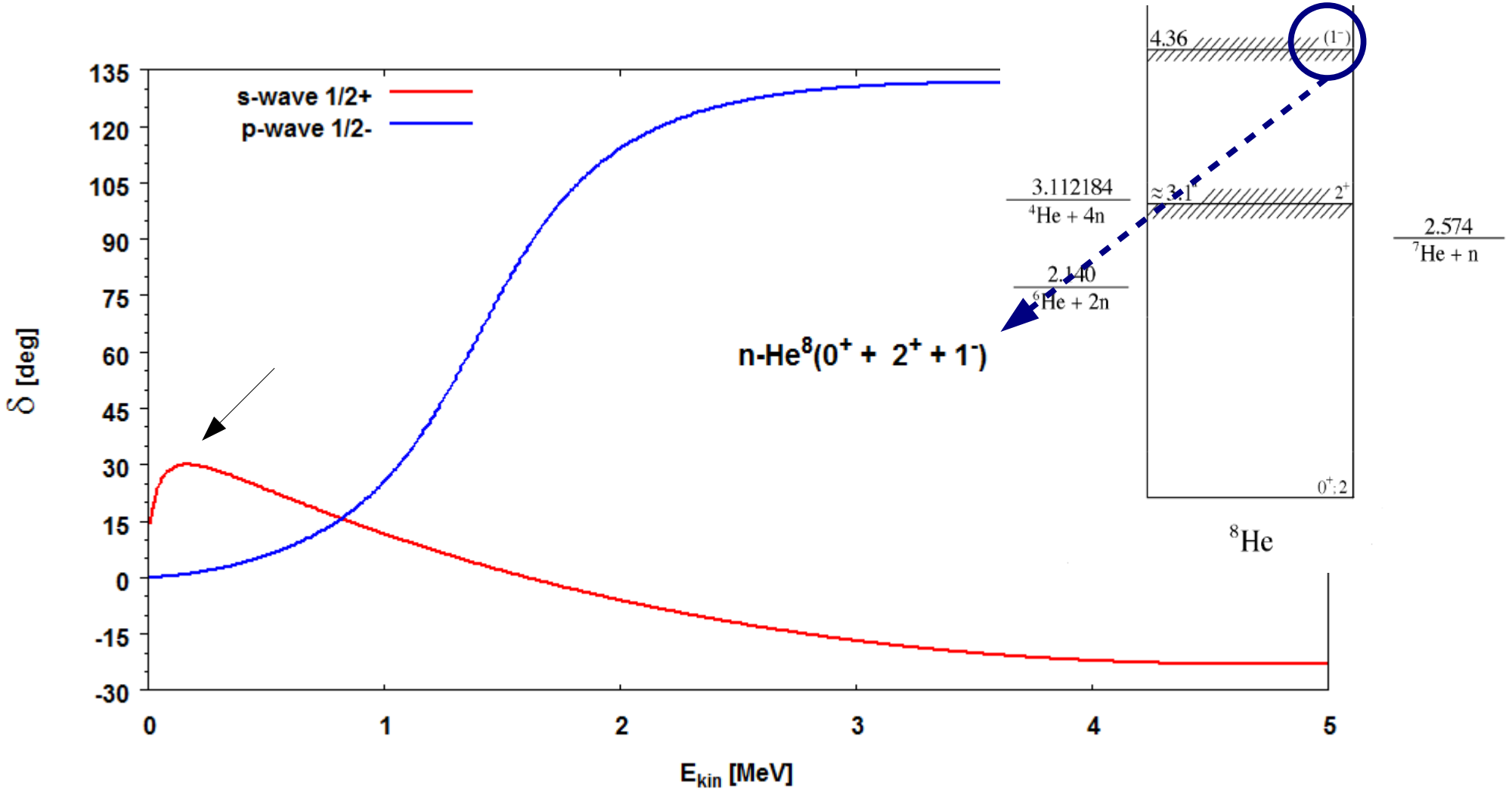
0+ state

There is almost no difference in the calculated phase shifts for including the 1+ state as well. Scattering length a_0 = -1 fm.

No bound state.

Feb 2011

NCSM/RGM: Inclusion of negative parity state



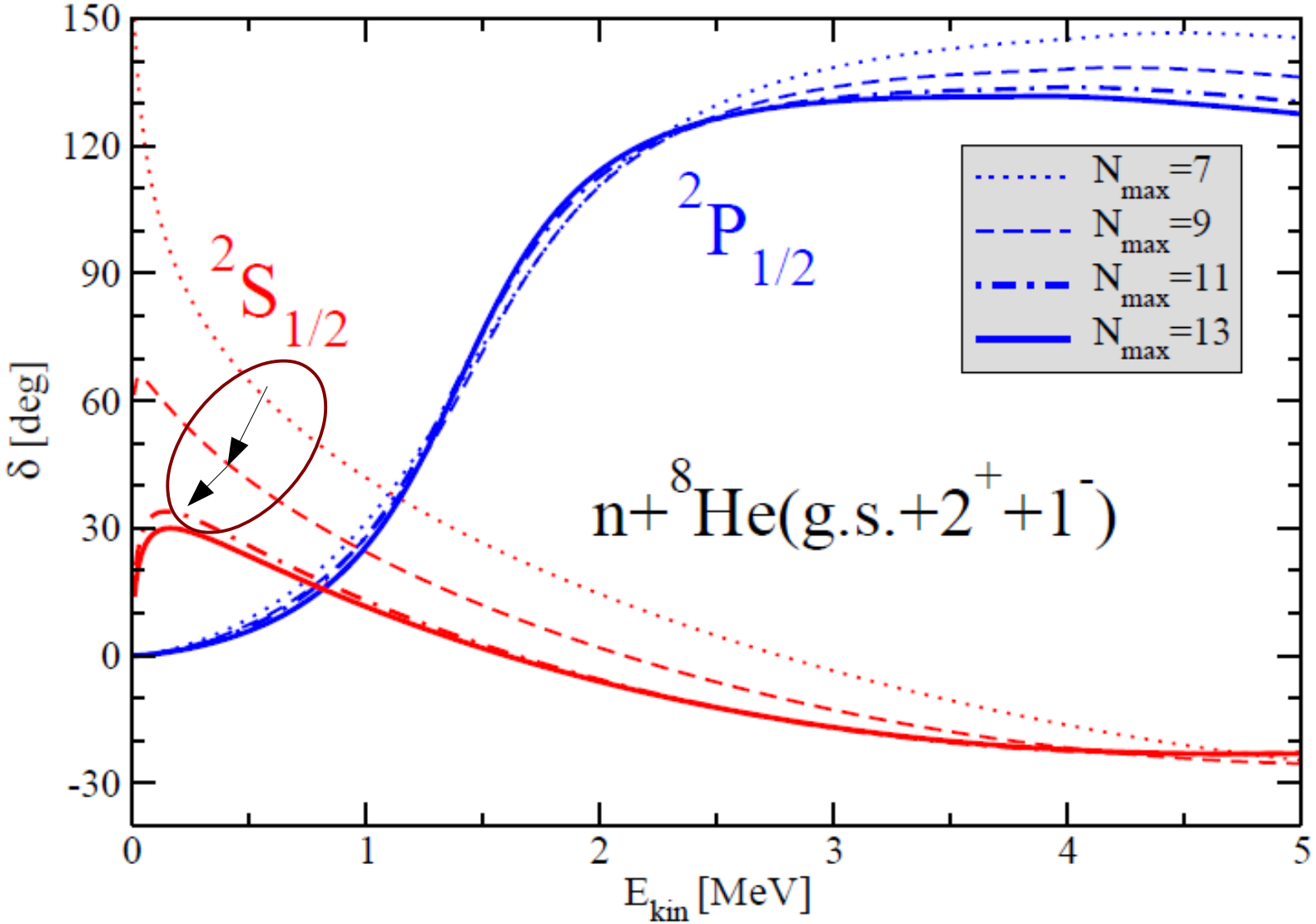
The inclusion of the negative parity state has a large effect! Now the predicted scattering length a_0 = -12.59 fm.

Experiments:
a_0 **< -10 fm** (Chen et al.) [PLB 505, 2001]
a_0**~ -3 fm** (Al Falou, et al.) [arxiv:nucl-ex: 1008:0543]

Note: How hard are expts?

Jan16-2012

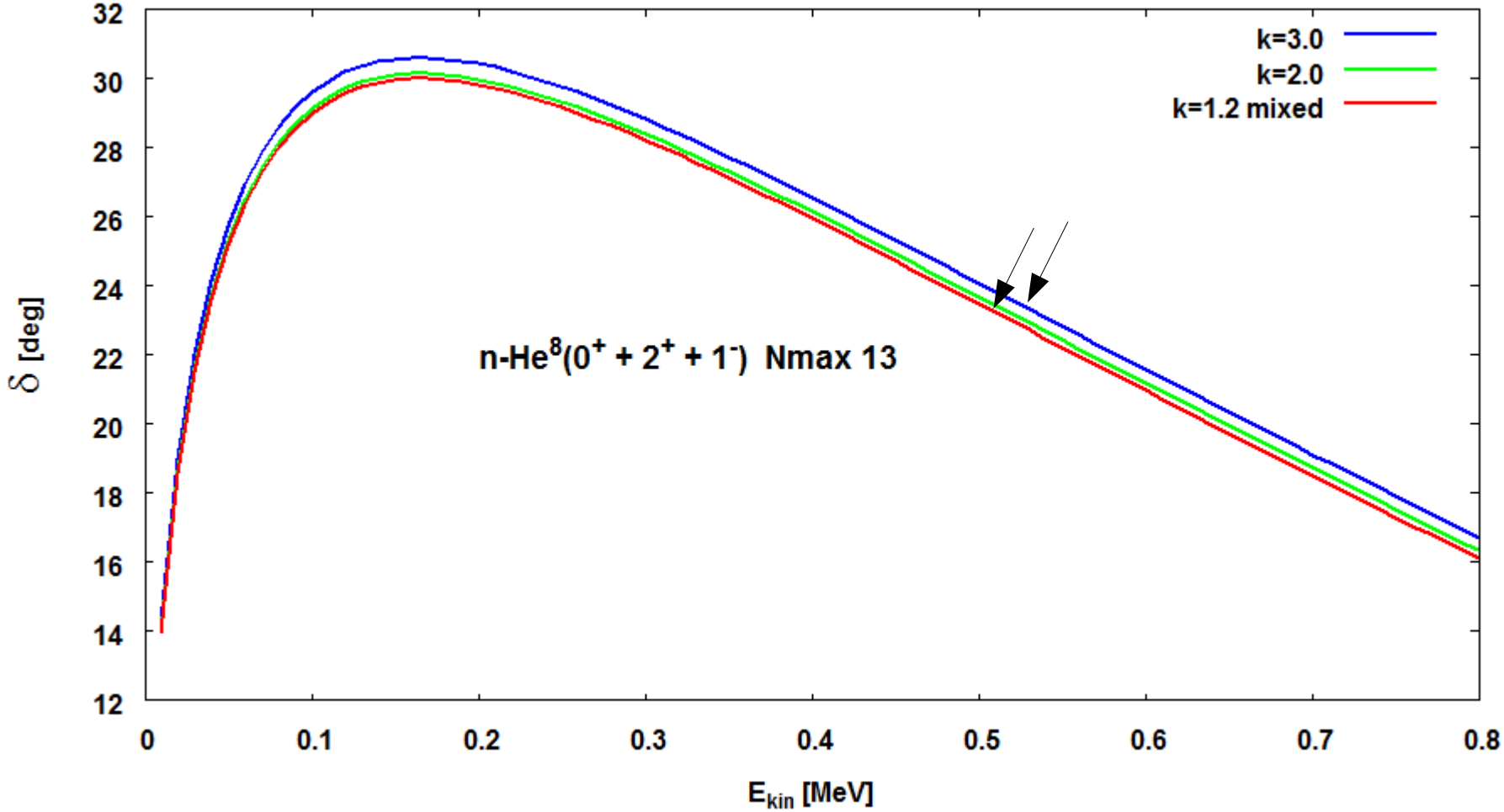
Phase-shifts converge with Nmax



Nmax 7-11: FULL space

Nmax13: IT

S-wave converges with kappa



As kappa decreases, exponentially more states are added to the wavefunction. Note that phase-shift decreases but is nearly converged. P-wave phase-shift is entirely converged.

Status of He-9 calculation

- **Predict a resonance in the ½+ channel.**
- **½- resonance agrees with experiments.**
- **Agrees with other theory calculations, but only agrees with the 2001 MSU experiment.**
- **Need to do a NCSM He-9 calculation and use the NCSM/(NCSM/RGM) technique to study the missing many-body correlations in the n-He8 calculation.**
- **Bench-mark calculation of He-8 with Roth in Nmax=14. Agreement seems pretty good, even though he uses a different IT-NCSM scheme.**
- **What about the 2- state? Close in energy to the 1-.**

UV and IR investigations of the No-Core Shell Model

- **Project started with Sid Coon, James Vary, Pieter Maris, with some contributions from Bira van Kolck.**
- **Supervise an undergrad, Matthew Avetian.**
- **Matthew and me did all the calculations for Sid in the following work (Matthew presented this work at the undergrad symposium at Arizona).**
- **Note: Preliminary (so wait for the arXiv paper).**
-
- **[Slides removed since results deemed "preliminary"]**

People I am in debt to

- **Bruce Barrett**
- **Petr Navratil**
- **Eric Jurgenson**
- **Sid Coon**
- **Alexander Lisetsky**
- **Erich Ormand**
- **Sofia Quaglioni**
- **Bira van Kolck**
- **Hank Miller**







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