CLUSTERING IN LIGHT NUCLEI

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Perspectives of the Ab Initio No Core Shell Model, TRIUMF, Vancouver, 2012

Outline

- Three Steps Between QCD and Nuclear Structure
- From few to many to few
  - Clustering in light nuclei
  - Borromean nucleus 6-He
  - Core+N+N structure in the A=6 isobar
- Outlook

Three Steps Between QCD And Nuclear Structure

From "QCD" To Nuclei

Nuclear Structure

Many-body Methods

ab initio no-core shell model

- A-body HO model space (m scheme)
- Full-space N_{max} energy cutoff

Renormalization Scheme

Similarity Renormalization Group

- SRG flow in NN momentum space
- Study of cutoff dependence

Chiral Effective Field Theory

chiral EFT NN interaction

- Entem and Machleidt (2003)
- N^3LO, $\Lambda = 500$ MeV

Low-energy QCD

From "QCD" To Nuclei

Nuclear Structure

Many-body Methods

Renormalization Scheme

Chiral Effective Field Theory

Low-energy QCD

Non-observables

- For many light nuclei most properties are determined by long-range cluster dynamics.
- **How does clustering emerge from a microscopic theory?**
- We will mostly be dealing with non-observables; but will provide good visualization!

Clustering In Light Nuclei

^{6}He: Facts And Fiction

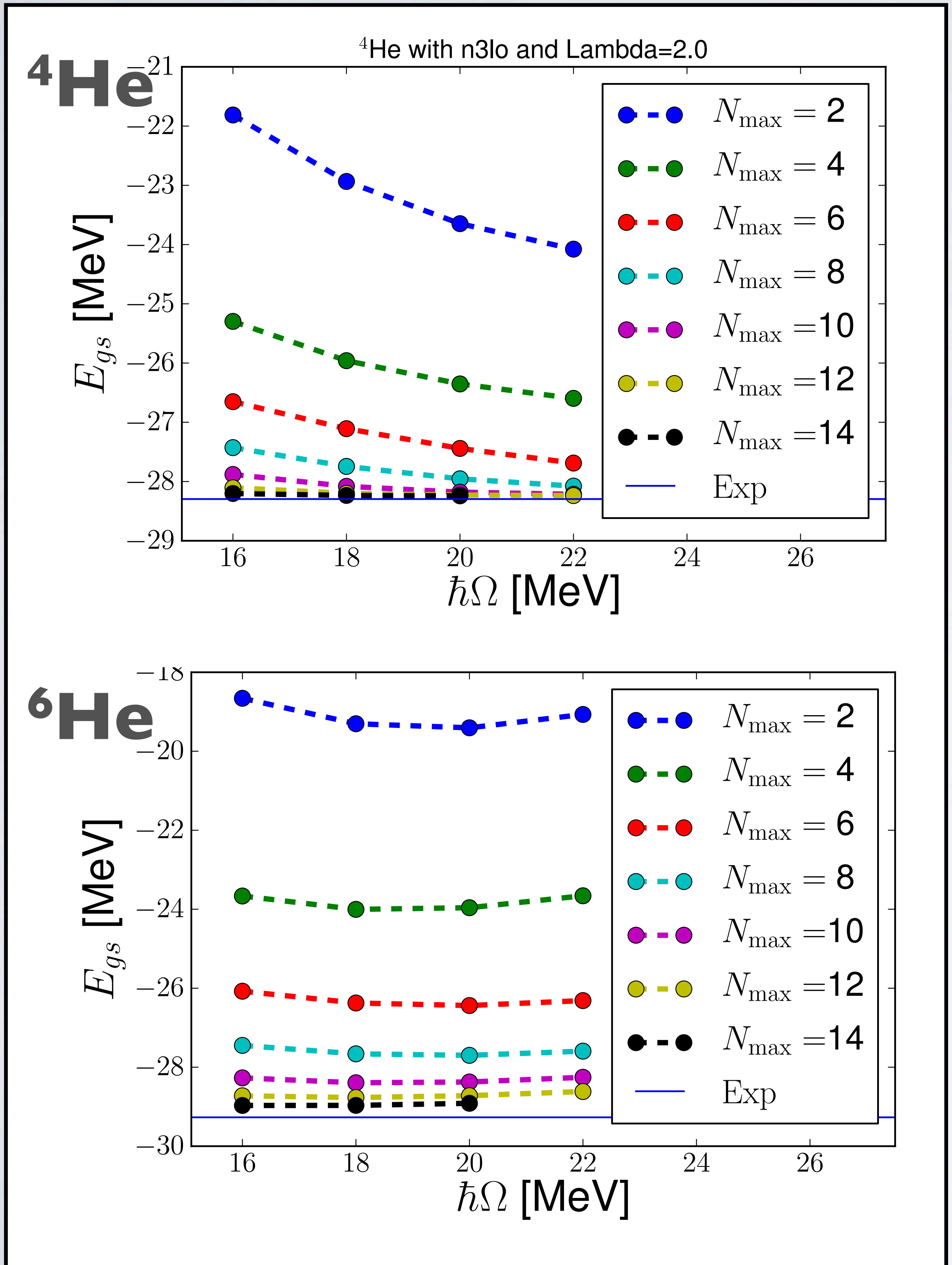
- **Properties**
  Small 2n separation energy, Borromean nucleus, halo ground state

- **Three-body model**
  See, e.g., M.V. Zhukov et al, Phys. Rep. **231**(1993)151

- **Ground-state properties**
  Recent precision measurements of charge radius, mass
  M. Brodeur et al., 2011. Phys. Rev. Lett. 108(2011)052504. P. Mueller et al., Phys. Rev. Lett. 99(2007) 252501.

- **Ab initio approaches**
  GFMC, FMD, EIHH, NCSM, CC

Energy Convergence



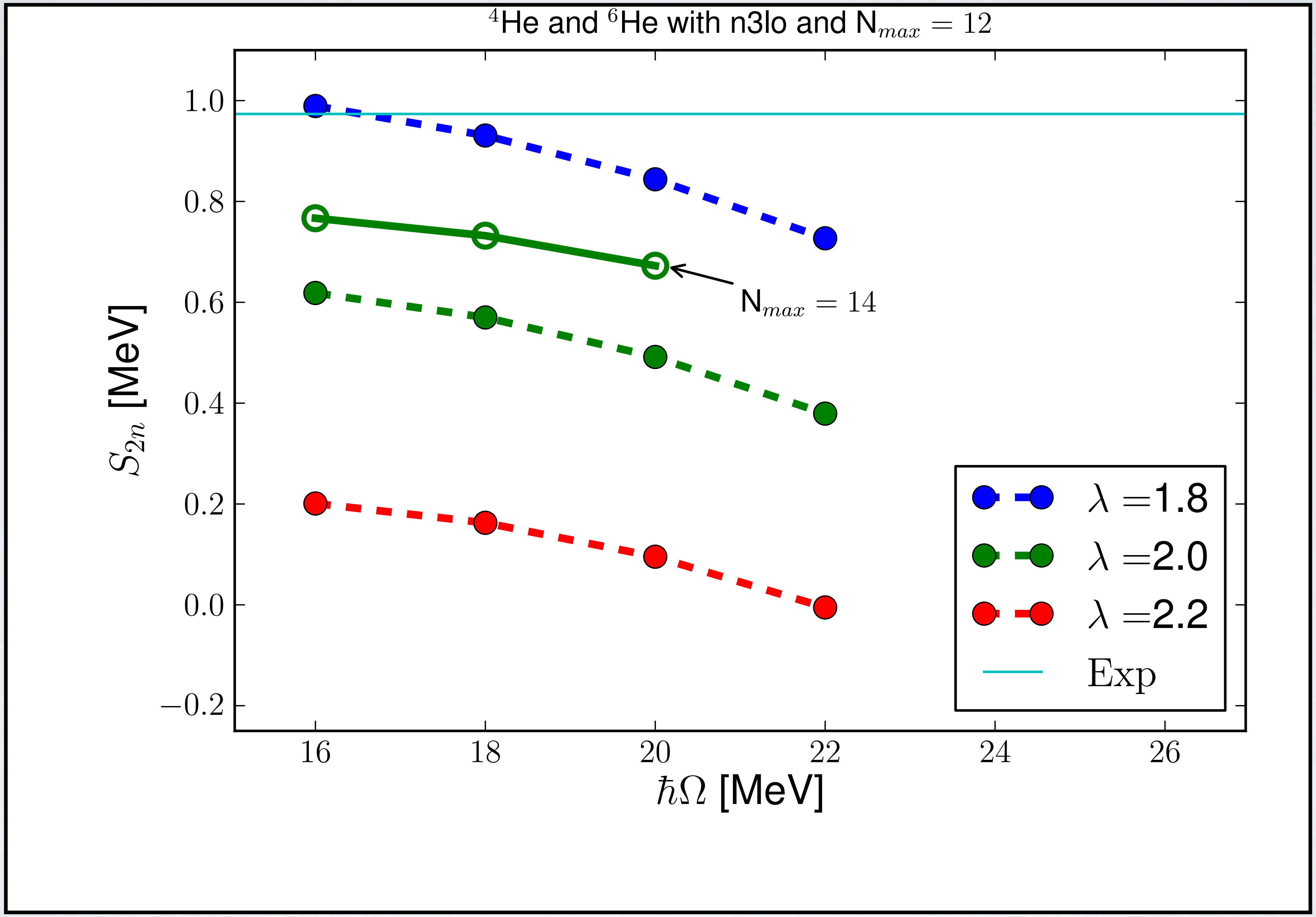
Binding energies

N^3LO, SRG (NN only, Λ = 2.0 fm^{-1})

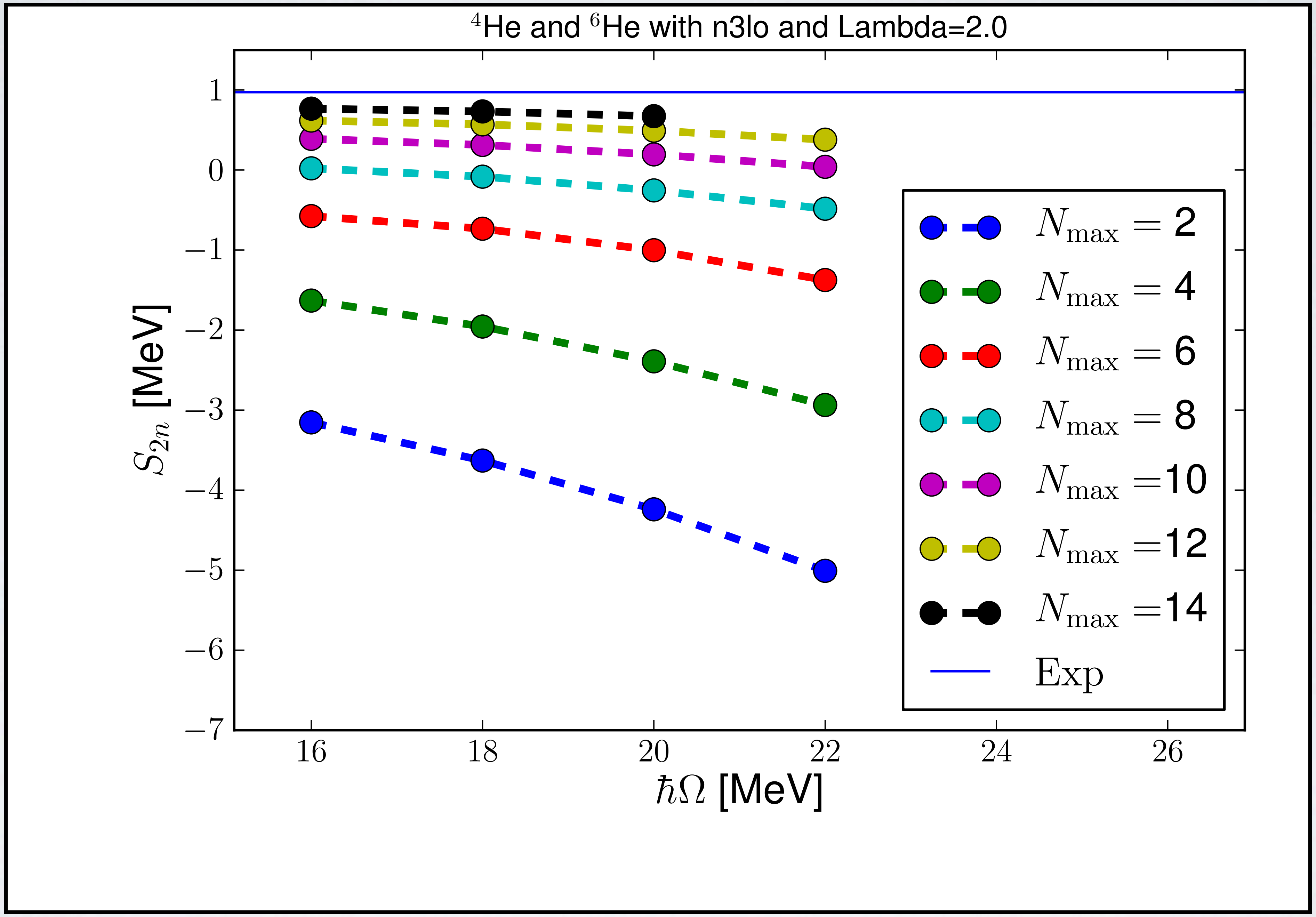
Λ	E(^{4}He)	E(^{6}He)
1.8	-28.45	-29.29
2.0	-28.23	-28.72
2.2	-27.87	-27.96

N_{max}=12, HO=20 MeV

^{6}He: Two-Neutron Separation Energy



^{6}He: Two-Neutron Separation Energy



^{6}He: Point-Proton Radius

- Very accurate charge-radius measurements using laser spectroscopy

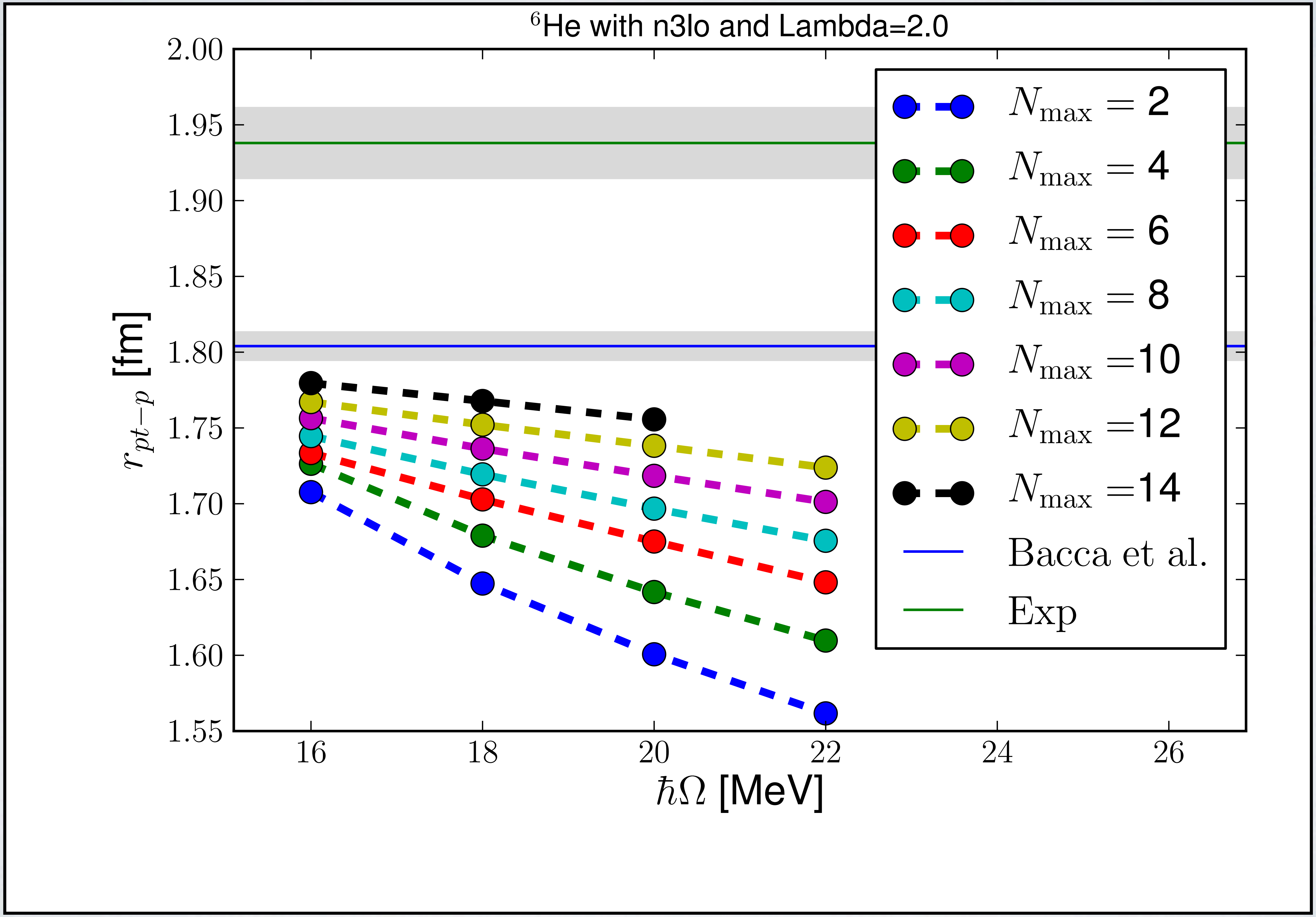
- Relation between charge and point-proton radius:

$$r_{pp}^2 = r_{\mathrm{ch}}^2 - R_p^2 - \frac{N}{Z} R_n^2 - r_{\Delta,\mathrm{rel}}^2 - r_{\mathrm{so}}^2$$

- Several ab initio calculations

- Most recently by Bacca et al
  - using EIHH and V_{lowk} NN potential based on (EM) N^{3}LO.
  - Study of Λ-dependence and observable correlations.

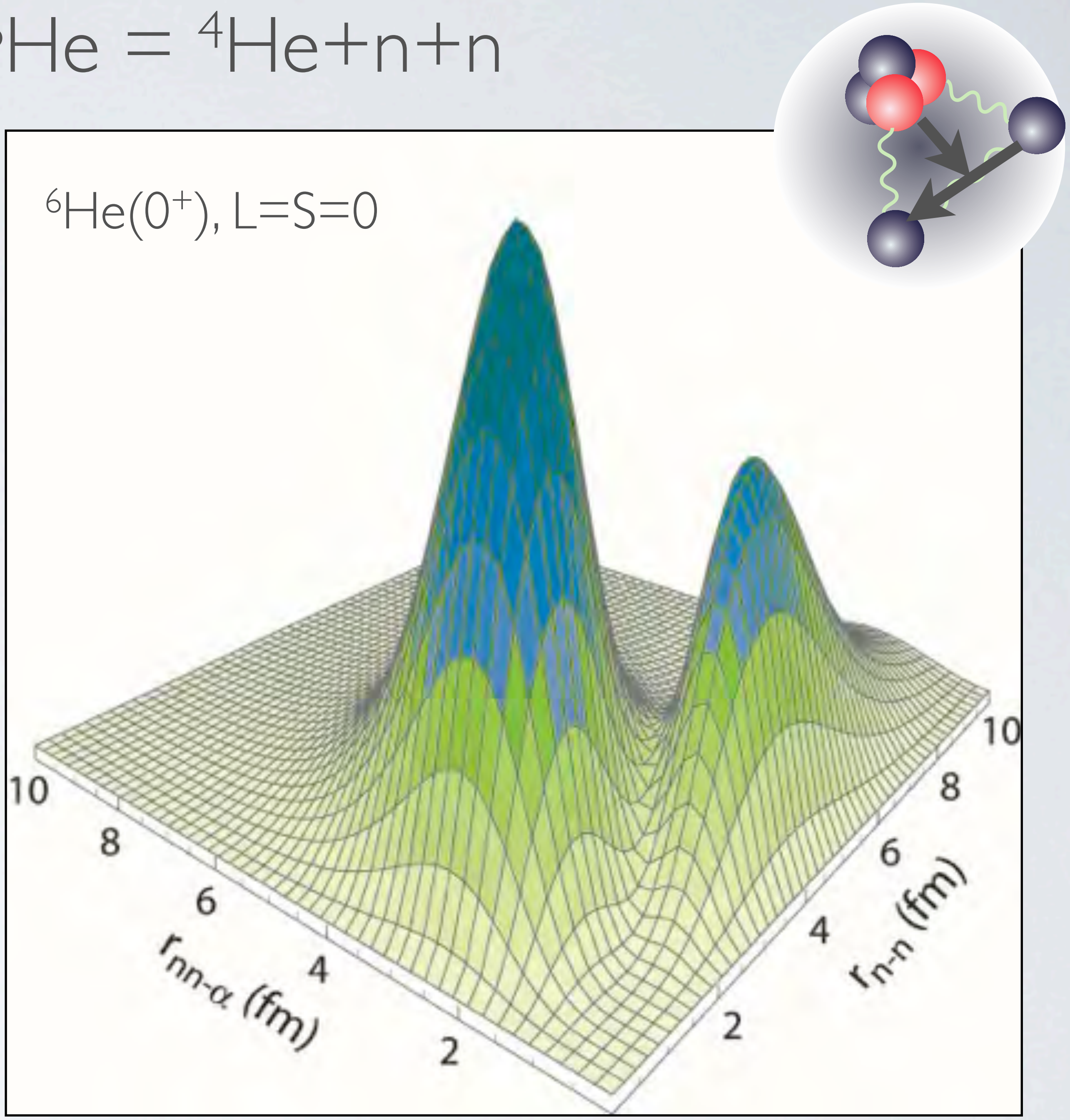
^{6}He: Point-Proton Radius



^{6}He As A Three-Body System

- Borromean nucleus
- HH and CSF three-body models with inert cluster. V_{nn} and $V_{n\alpha}$
  - Core polarization needed $r_{n\alpha} = 1.03 r_{n\alpha(free)}$ (cf. three-body force)
  - Repulsive s-wave potential ("Pauli core")
  - HH expansion K=2 (90%) with L=S=0 (80%) and L=S=1 (10%)
- Pauli focusing.

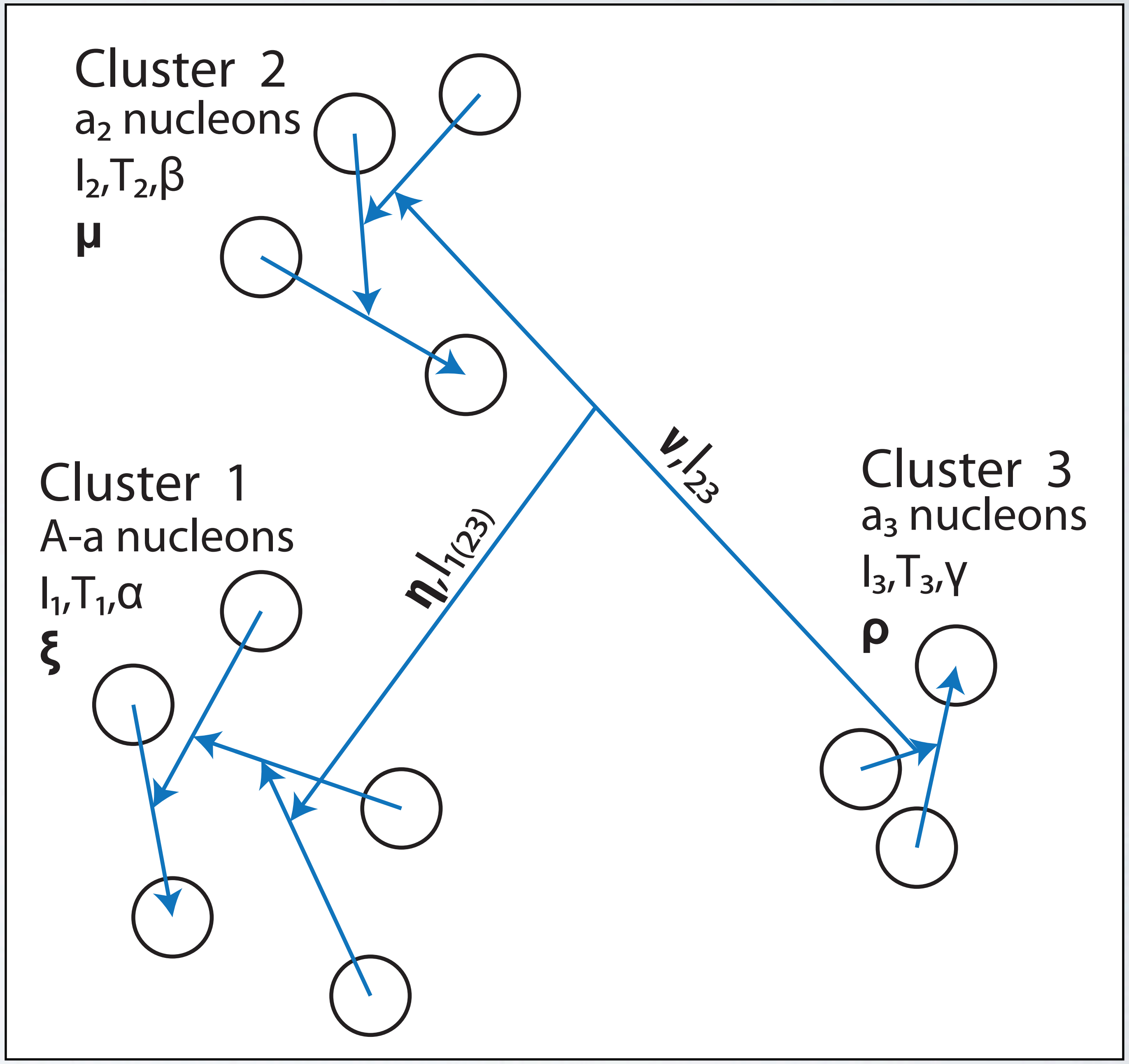
^{6}He = ^{4}He+n+n



M.V. Zhukov et al.— Phys. Rep. **231**, 151 (1993),

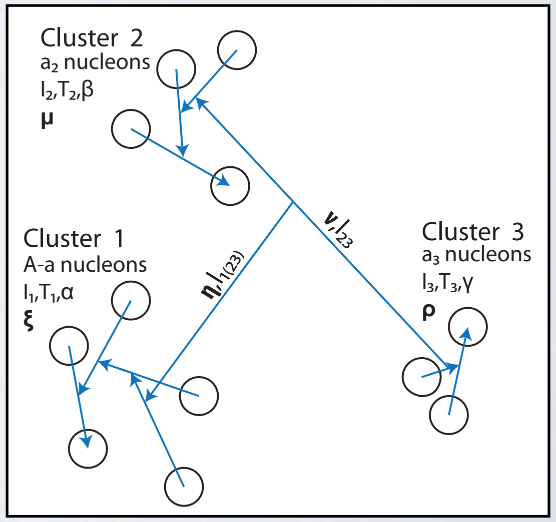
Three-body Cluster Overlap Functions

- Investigate clustering in NCSM wave functions
- Preserve translational invariance
- Harmonic oscillator SD many-body basis
- Transformation between single-particle and Jacobi coordinates



Three-body Cluster Overlap Functions

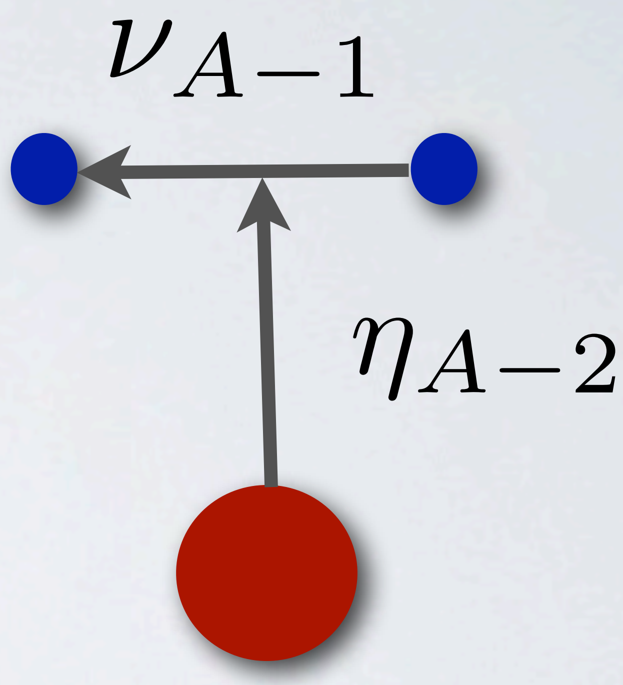
$$
\begin{aligned}
& u^{A\lambda JT}_{A-a\alpha l_1 T_1, a_2 \beta l_2 T_2, a_3 \gamma l_3 T_3; LS}(\eta_{A-a}, \nu_{A-a+1}) \\
&= \left\langle A\lambda JT \middle| \mathcal{A}_{A-a,a_2,a_3} \Phi^{(A-a,a_2,a_3)JT}_{\alpha l_1 T_1, \beta l_2 T_2, \gamma l_3 T_3; LS} : \delta_{\eta_{A-a}}, \delta_{\nu_{A-a+1}} \right\rangle \\
&= \sum_{n_{1(23)}, n_{23}} R_{n_{1(23)} l_{1(23}}(\eta_{A-a}) R_{n_{23} l_{23}}(\nu_{A-a+1}) \\
&\times \sqrt{\frac{A!}{(A-a)!a_2!a_3!}} \left\langle A\lambda JT \middle| \Phi^{(A-a,a_2,a_3)JT}_{\alpha l_1 T_1, \beta l_2 T_2, \gamma l_3 T_3; LS} : n_{1(23)} l_{1(23)}, n_{23} l_{23} \right\rangle
\end{aligned}
$$



Overlap Function For Core+N+N

Start with the **core+N+N** case:

- Do a couple of coordinate transformations (between relative and s.p.)
- Do a number of spin re-couplings
- Integrate over coordinates



Overlap Function For Core+N+N

$$
u^{A\lambda JT}_{A-2\alpha I_1 T_1, \frac{1}{2}\frac{1}{2}, \frac{1}{2}\frac{1}{2}; LS}(\eta_{A-2}, \nu_{A-1})
= \sum \frac{R_{n_{1(23)} l_{1(23)}}(\eta_{A-2}) R_{n_{23} l_{23}}(\nu_{A-1})}{\left\langle n_{1(23)} l_{1(23)} 0 0 l_{1(23)} \right| \left. 0 0 n_{1(23)} l_{1(23)} l_{1(23)} \right\rangle_{\frac{2}{A-2}}}
$$

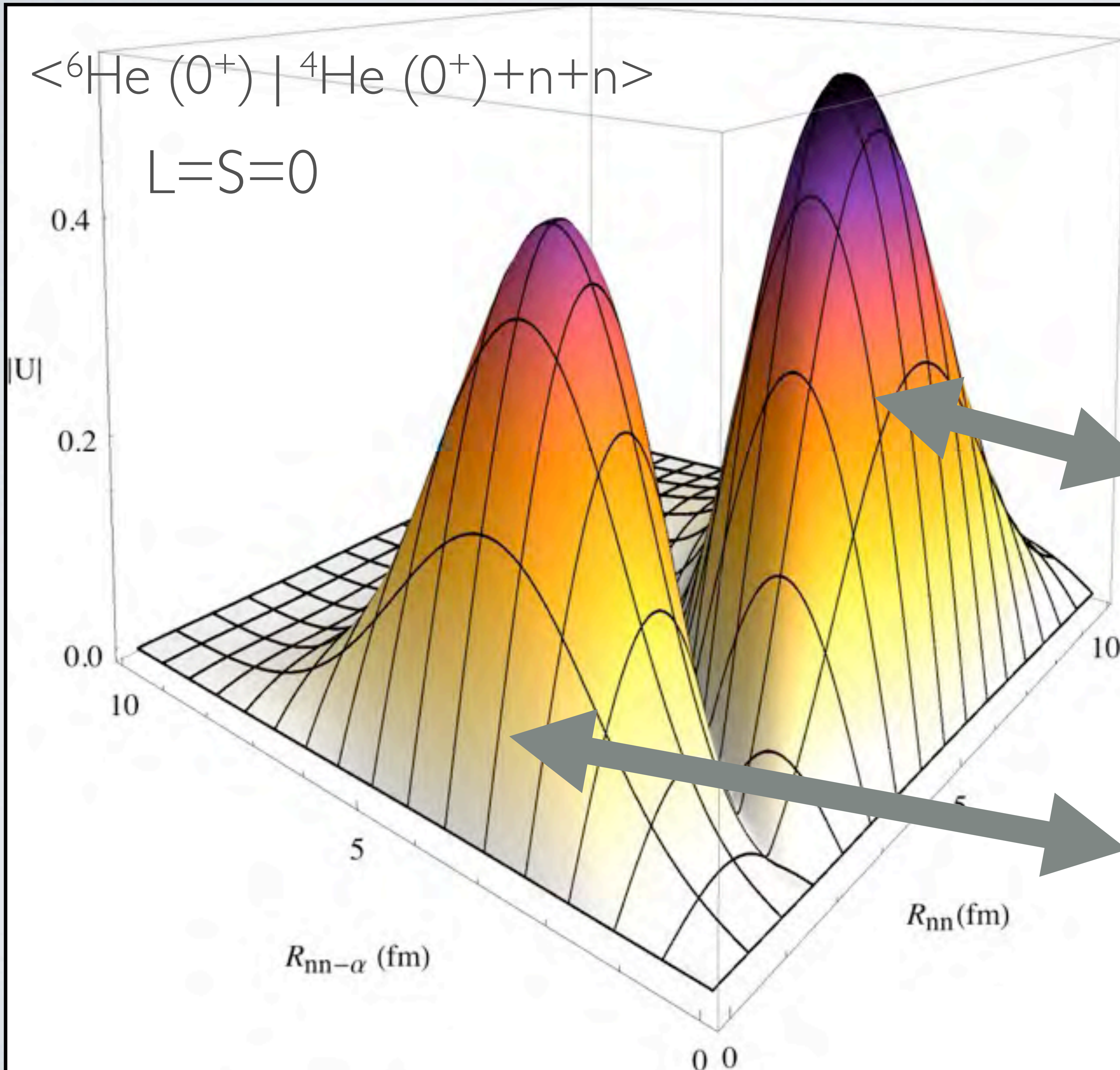
$$
\times \left\langle n_a l_a n_b l_b L \right| \left. n_{1(23)} l_{1(23)} n_{23} l_{23} L \right\rangle_1 (-1)^{3I_1 + l_{23} + l_{ab} - T_{23} - S + L}
$$

$$
\times \frac{\hat{L}\hat{S}\hat{I}^2_{ab}\hat{j}_a\hat{j}_b}{\hat{J}\hat{T}} \begin{Bmatrix} L & l_{23} & I_{ab} \\ I_1 & J & S \end{Bmatrix} \begin{Bmatrix} l_a & l_b & L \\ \frac{1}{2} & \frac{1}{2} & l_{23} \\ j_a & j_b & I_{ab} \end{Bmatrix}
$$

$$
\times {}_{SD} < A\lambda JT ||| \left[a^\dagger_{n_a l_a j_a t_a} a^\dagger_{n_b l_b j_b t_b} \right]^{I_{ab} T_2} ||| (A-2)\alpha I_1 T_1 >_{SD} .
$$

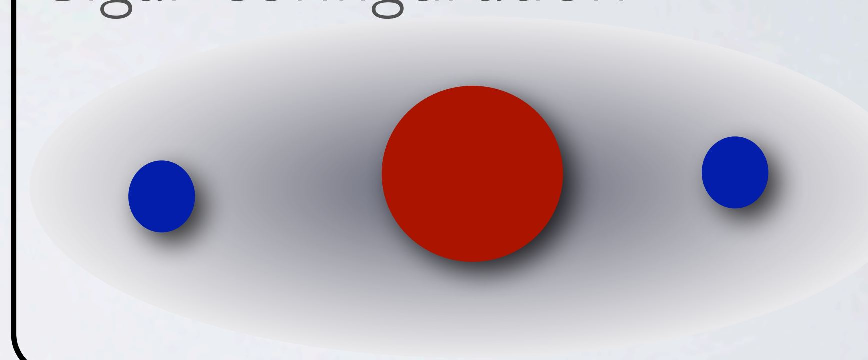
D. Sääf and CF, - in preparation

Ab Initio < ^{6}He | ^{4}He+n+n > Overlap

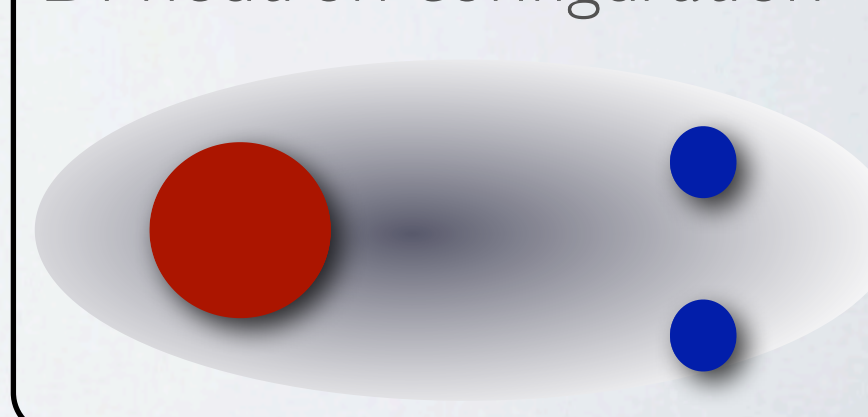


N^{3}LO, SRG
(NN only, Λ = 2.0 fm^{-1})

Cigar configuration



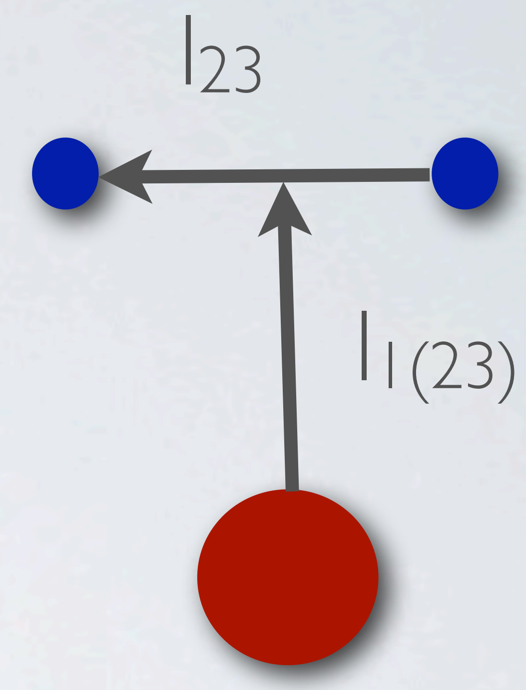
Di-neutron configuration



Pauli Focusing

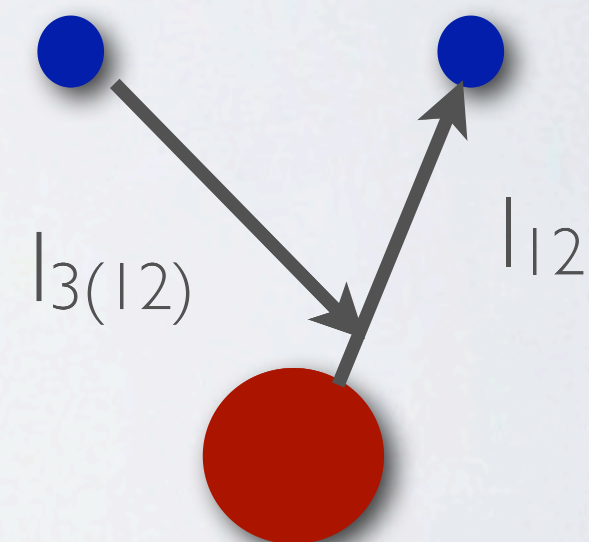
$\langle {}^6He\ (0^+)\ |\ {}^4He\ (0^+) + n + n \rangle$

- Dominance of the $l_{1(23)} = l_{23} = 0$ component.



- RR coefficients determine HH under coordinate-system transformation.

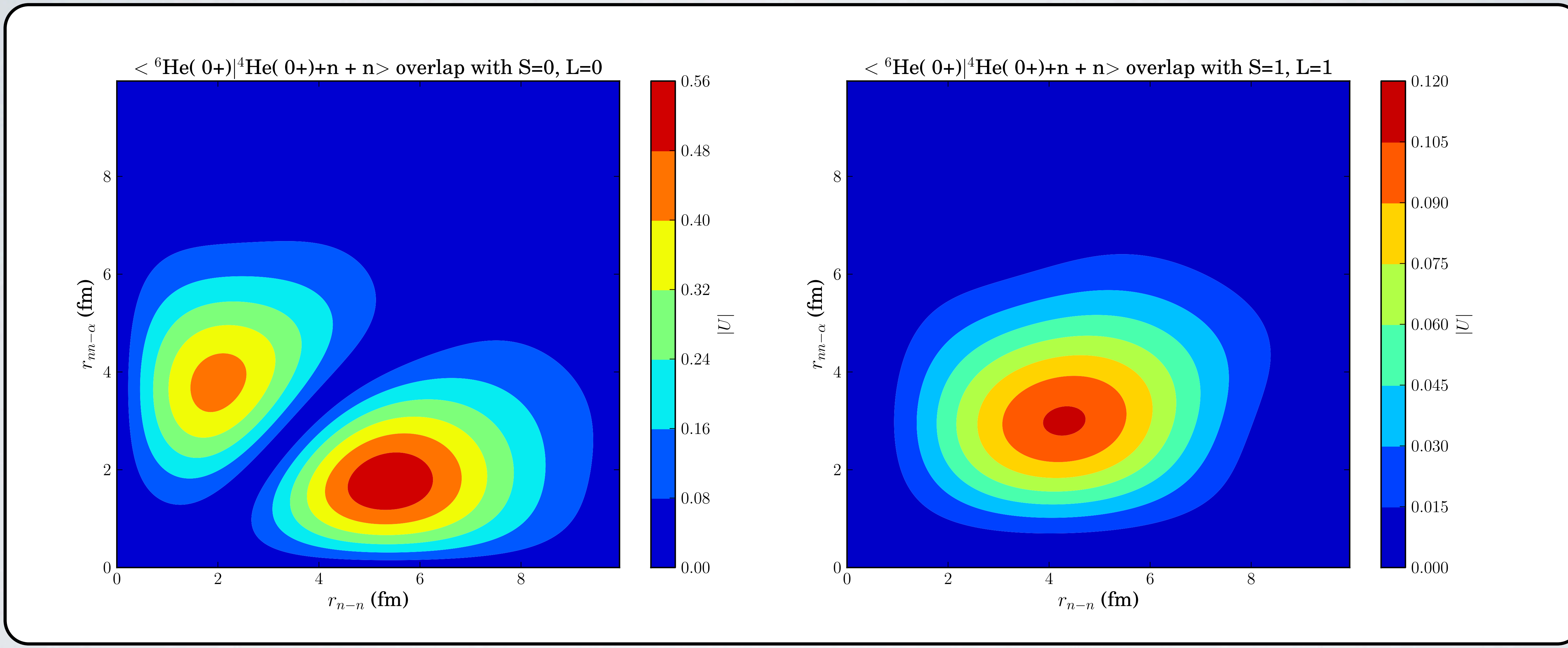
- E.g. with $l_{1(23)} = l_{23} = 0$ we get:
  - $l_{3(12)} = l_{12} = 0$ for K=0 (almost Pauli forbidden)
  - Dominating $l_{3(12)} = l_{12} = 1$ for K=2



Ab Initio $\langle {}^6\mathrm{He} \mid {}^4\mathrm{He}+n+n \rangle$ Overlap

N^{3}LO, SRG
NN only, Λ = 2.0 fm^{-1},
N_{max}=14, HO=20 MeV

$\langle {}^6\mathrm{He}\,(0^+) \mid {}^4\mathrm{He}\,(0^+)+n+n \rangle$



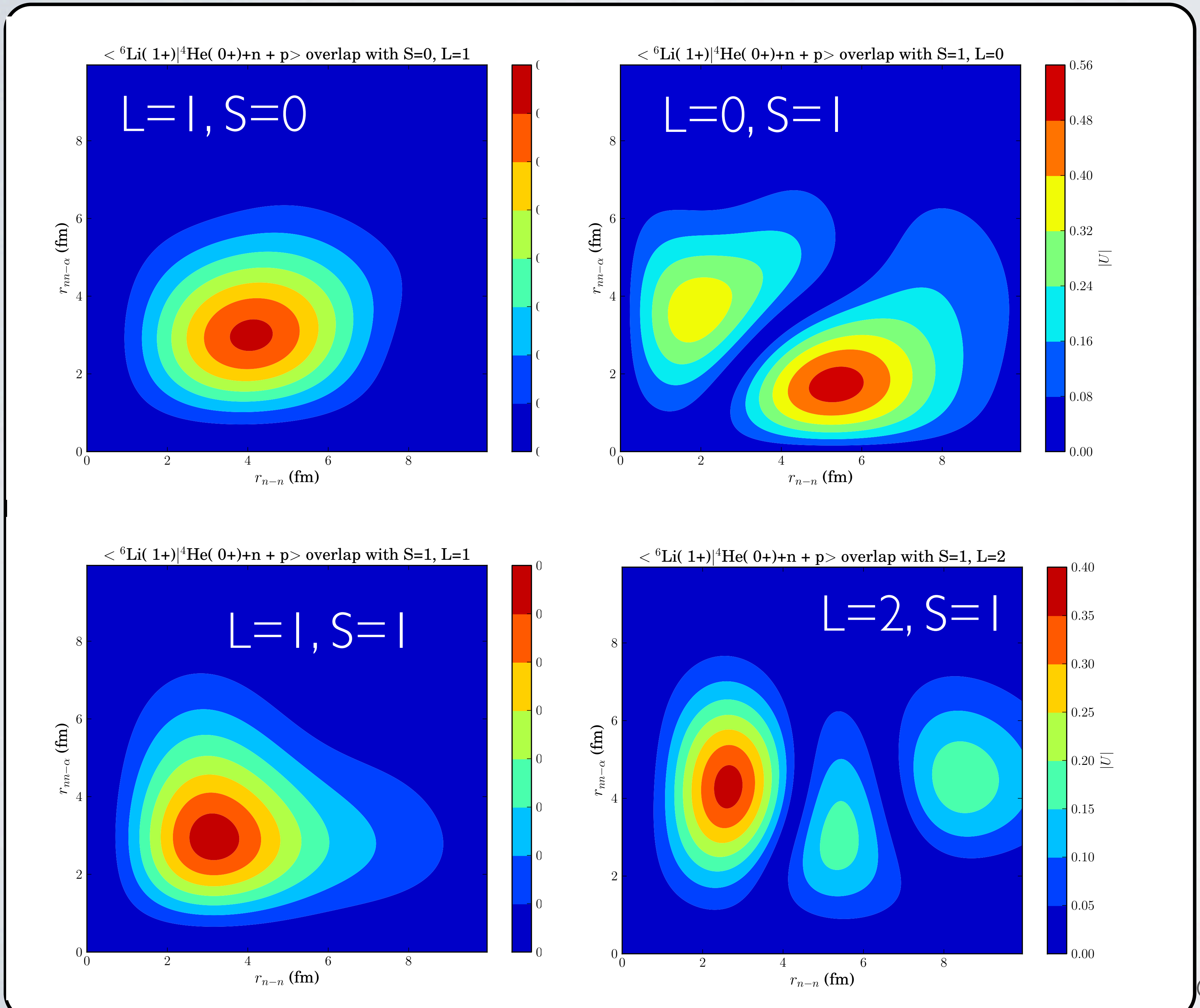
L=S=0

L=S=1

Ab Initio < ^{6}Li | ^{4}He+n+p > Overlap

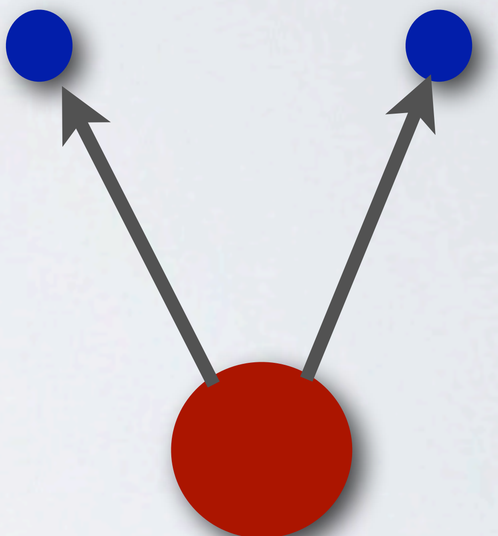
<^{6}Li (1$^+$) | ^{4}He (0$^+$)+n+n>

N^{3}LO, SRG
NN only, Λ = 2.0 fm^{-1},
N_{max}=14, HO=20 MeV



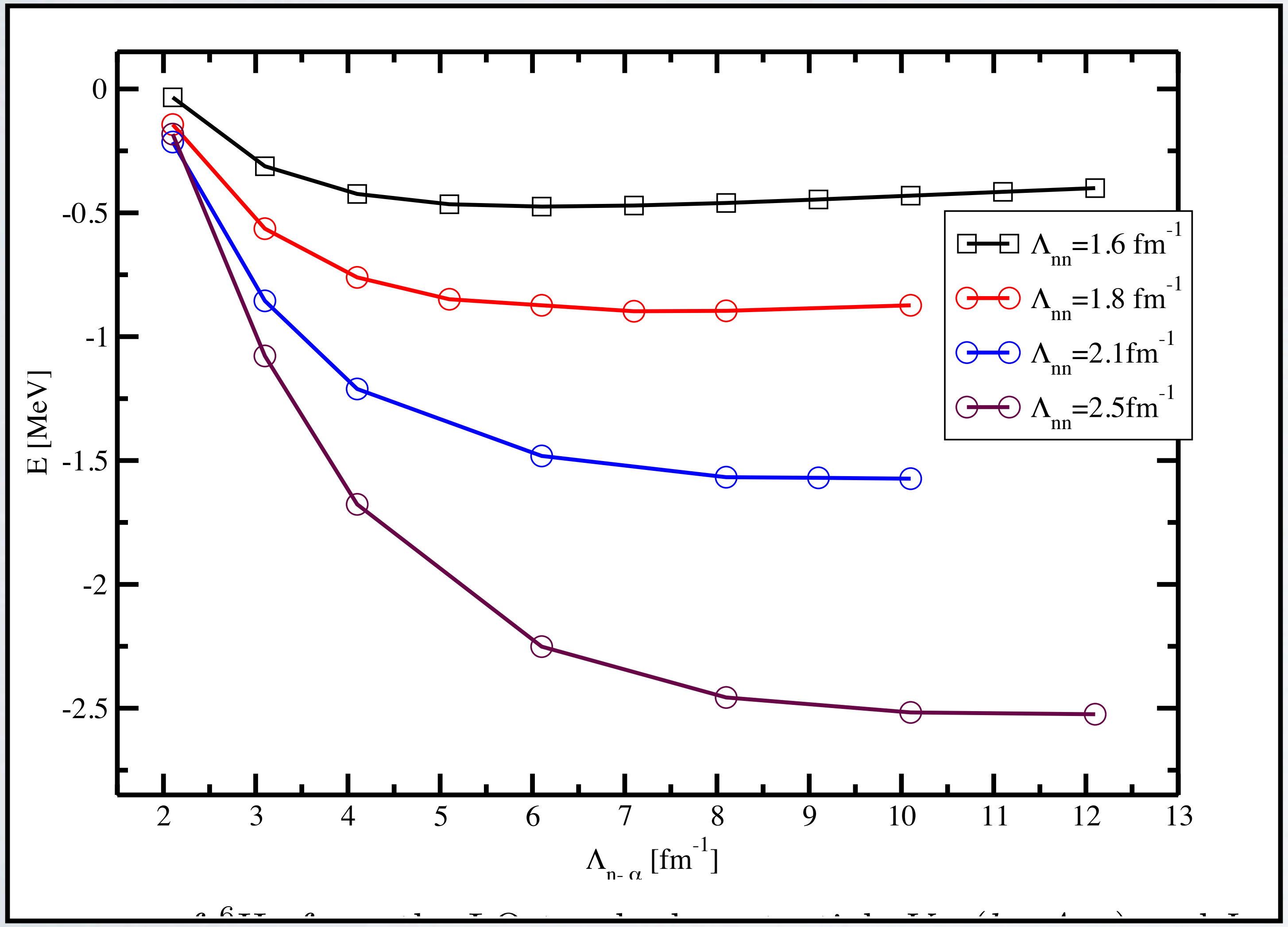
Halo Effective Field Theory

- **J. Rotureau and U. Van Kolck**; arXiv:1201.3351
- Separation of scales
  - $E(^5He)-E(^4He) \sim -0.9$ MeV
  - $E(^6He)-E(^4He) \sim 1$ MeV
  - $E_{exc}(^4He) \sim 20$ MeV
- Two-body potentials at leading order
  - n-^4He in $p_{3/2}$ (Bertulani et al 2002)
  - n-n in $^1S_0$
  - Reproduce Effective Range Expansion



Halo EFT And Gamow Shell Model

- **J. Rotureau and U. Van Kolck**; arXiv:1201.3351
- Solution of three-body problem using Gamow Shell Model (including continuum states)



Cutoff dependence of ground-state energy

Collapse of g.s. under short-range V_{2b}

Halo EFT And Gamow Shell Model

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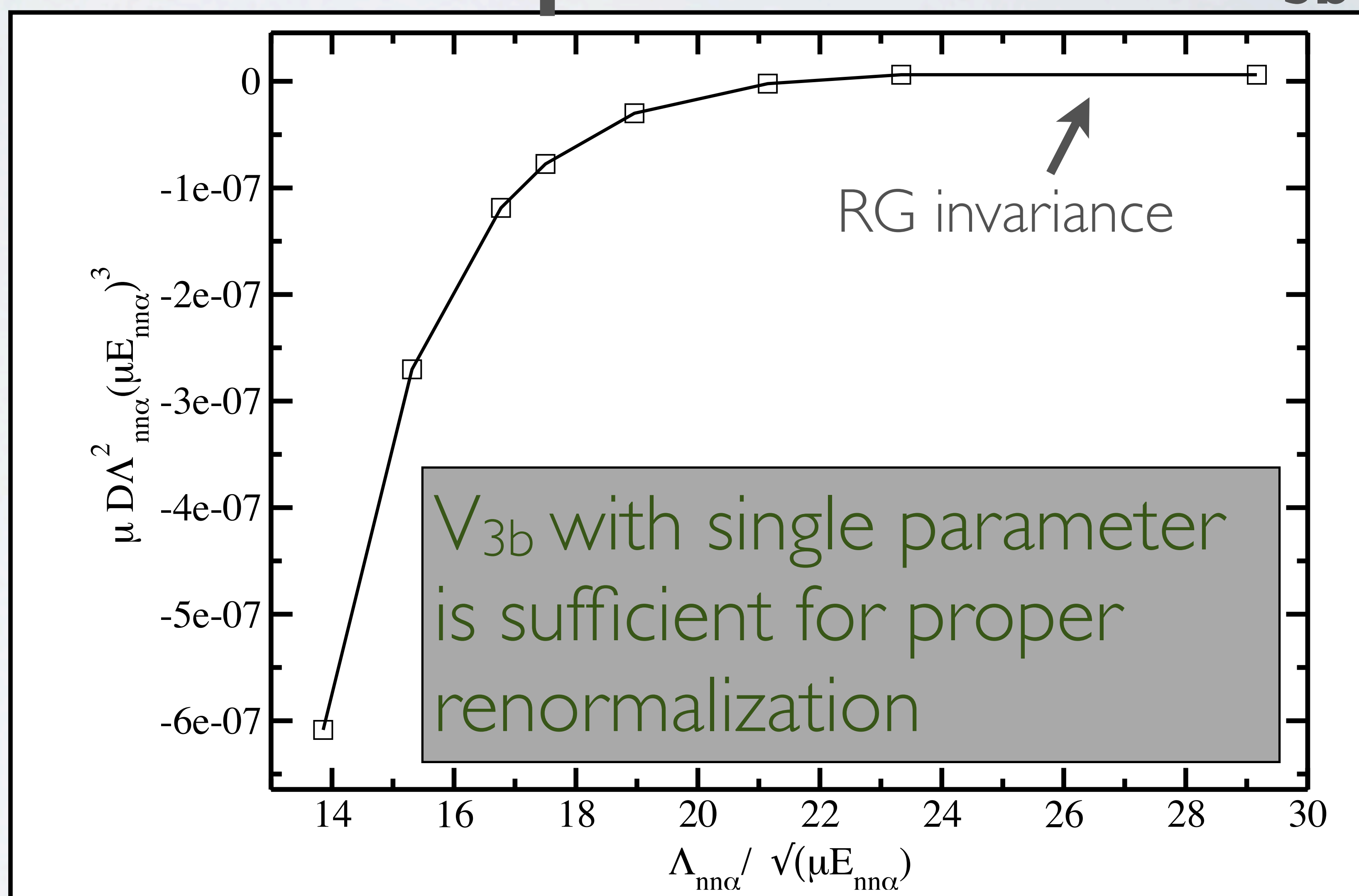
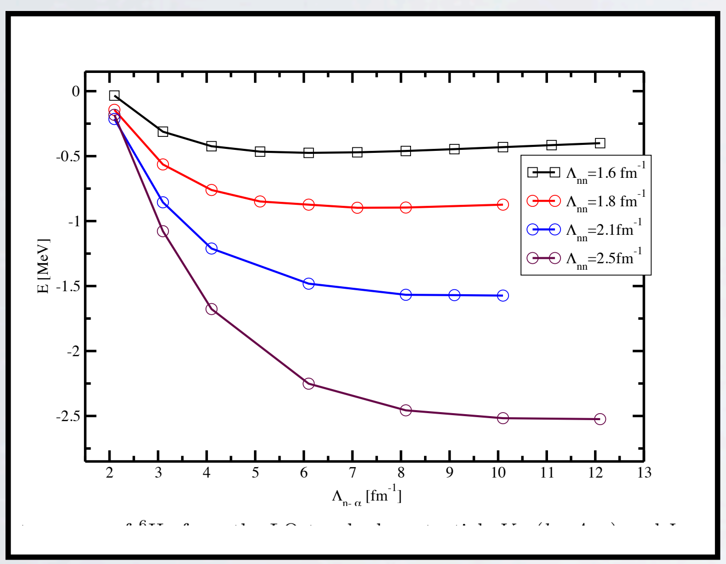
- Solution of three-body problem using Gamow Shell Model (including continuum states)

p-wave contact V_{3b}:



V_{2b}



Conclusion And Outlook

Summary And Outlook

- ab initio nuclear structure calculations with realistic NN(+3N) interactions and unitary transformations for use in finite model space.
- Emerging cluster structures in light nuclei.
- NCSM/RGM calculations with three-cluster states.
- Additional scale separation for clusterized systems: halo EFT
- Constrain/guide cluster calculations.

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