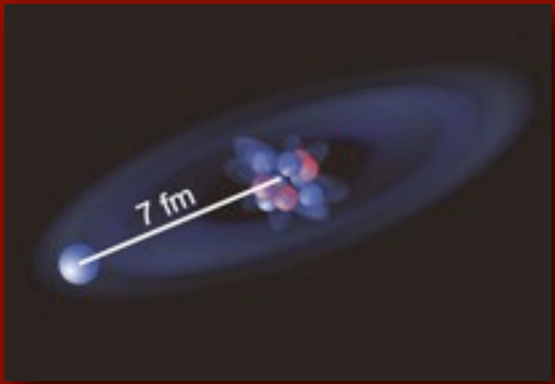
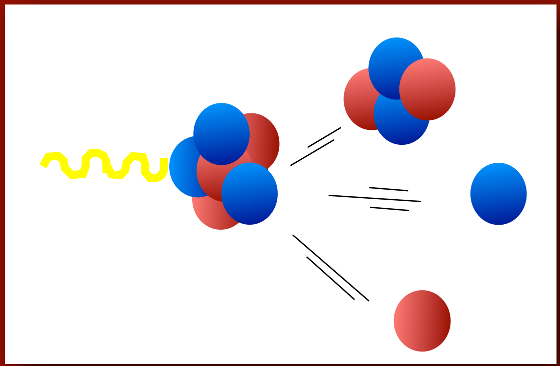
3NF effects in few-body electromagnetic observables

Sonia Bacca | Theory Group | TRIUMF

In collaboration with: N. Barnea, W. Leidemann, G.Orlandini and A. Schwenk



Nuclear Halo



Nuclear Reactions

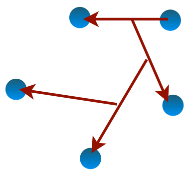
Outline

- The LIT/HH approach
- Electron scattering off ^{4}He
- Halo nuclei: the case of ^{6}He
- Outlook

Hyper-spherical Harmonics

- Few-body method - uses relative coordinates

$$|\psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A)\rangle = |\varphi(\vec{R}_{CM})\Psi(\vec{\eta}_1, \vec{\eta}_2, \ldots, \vec{\eta}_{A-1})\rangle$$



$\vec{\eta}_0 = \sqrt{A}\vec{R}_{CM} \quad \vec{\eta}_1, \ldots, \vec{\eta}_{A-1}$

Recursive definition of hyper-spherical coordinates

$$\rho, \Omega \qquad \rho^2 = \sum_{i=1}^{A} r_i^2 = \sum_{i=1}^{A-1} \eta_i^2$$

Hyper-spherical Harmonics

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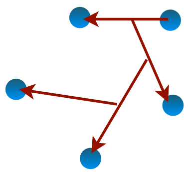

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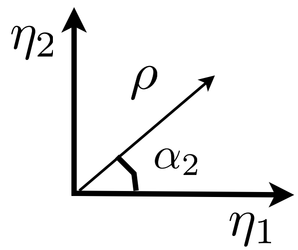
A=3

$$\begin{cases} \vec{\eta}_1 = \{\eta_1, \theta_1, \phi_1\} \\ \vec{\eta}_2 = \{\eta_2, \theta_2, \phi_2\} \end{cases} \qquad \begin{cases} \rho = \sqrt{\eta_1^2 + \eta_2^2} \\ \sin\alpha_2 = \frac{\eta_2}{\rho} \end{cases}$$



Hyper-spherical Harmonics

- Few-body method - uses relative coordinates

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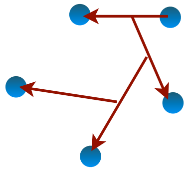

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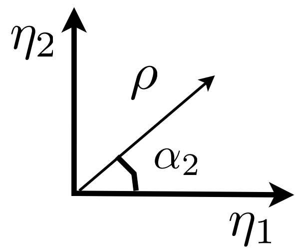
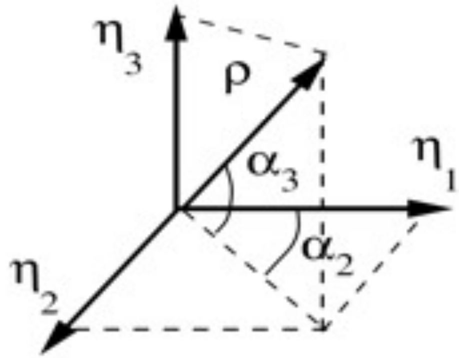


A=4

$$\begin{cases} \vec{\eta}_1 = \{\eta_1, \theta_1, \phi_1\} \\ \vec{\eta}_2 = \{\eta_2, \theta_2, \phi_2\} \\ \vec{\eta}_3 = \{\eta_3, \theta_3, \phi_3\} \end{cases} \qquad \begin{cases} \rho = \sqrt{\eta_1^2 + \eta_2^2 + \eta_3^2} \\ \sin\alpha_2 = \frac{\eta_2}{\rho} \\ \sin\alpha_3 = \frac{\eta_3}{\rho} \end{cases}$$



Hyper-spherical Harmonics

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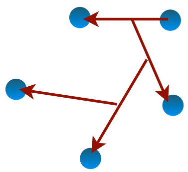

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Recursive definition of hyper-spherical coordinates

$$\rho, \Omega \qquad \rho^2 = \sum_{i=1}^{A} r_i^2 = \sum_{i=1}^{A-1} \eta_i^2$$

$$H(\rho, \Omega) = T_\rho - \frac{K^2(\Omega)}{\rho^2}$$

$$\Psi = \sum_{[K],\nu}^{K_{max},\nu_{max}} c_\nu^{[K]} e^{-\rho/2b} \rho^{n/2} L_\nu^n\left(\frac{\rho}{b}\right) [\mathcal{Y}_{[K]}^{\mu}(\Omega) \chi_{ST}^{\bar\mu}]_{JT}^{a}$$

Asymptotic $\quad e^{-a\rho} \quad \rho \rightarrow \infty$

Model space truncation $K \leq K_{max}$, Matrix Diagonalization $\quad \langle\psi|H_{(2)}|\psi\rangle = \frac{A(A-1)}{2}\langle\psi|H_{(A,A-1)}|\psi\rangle$

Can use non-local interactions

Most applications in few-body; challenge in A>4 $\quad$ Barnea and Novoselsky, Ann. Phys. 256 (1997) 192

The Lorentz Integral Transform Metohd

Efros et al, PLB 338 (1994) 130

Response in the continuum

$$R(\omega) = \sum_f \left| \left\langle \psi_f \left| \hat{O} \right| \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$$

The Lorentz Integral Transform Metohd

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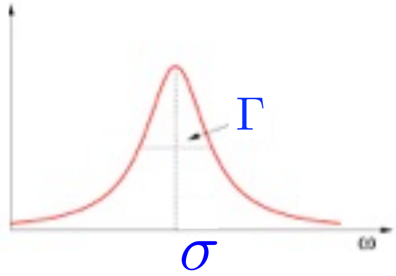

$$L(\sigma, \Gamma) = \int d\omega \frac{R(\omega)}{(\omega - \sigma)^2 + \Gamma^2} = \langle \tilde{\psi} | \tilde{\psi} \rangle$$

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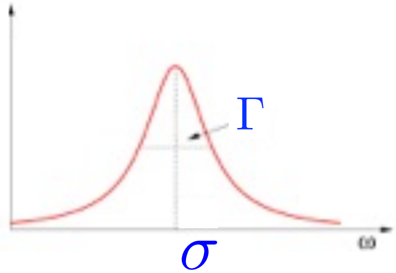

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$$(H - E_0 - \sigma + i\Gamma) \mid \tilde{\psi} \rangle = \hat{O} \mid \psi_0 \rangle$$

- Due to imaginary part Γ the solution $|\tilde{\psi}\rangle$ is unique
- Since the r.h.s. is finite, then $|\tilde{\psi}\rangle$ has bound state asymptotic behaviour

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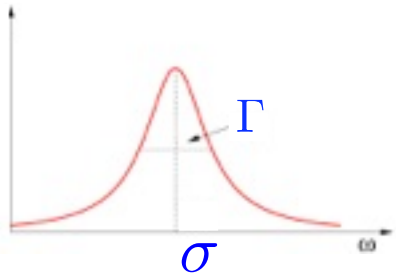

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$L(\sigma, \Gamma)$ ←inversion→ $R(\omega)$ with the exact final state interaction

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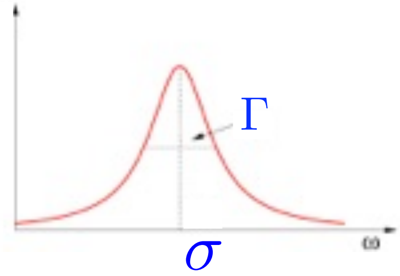

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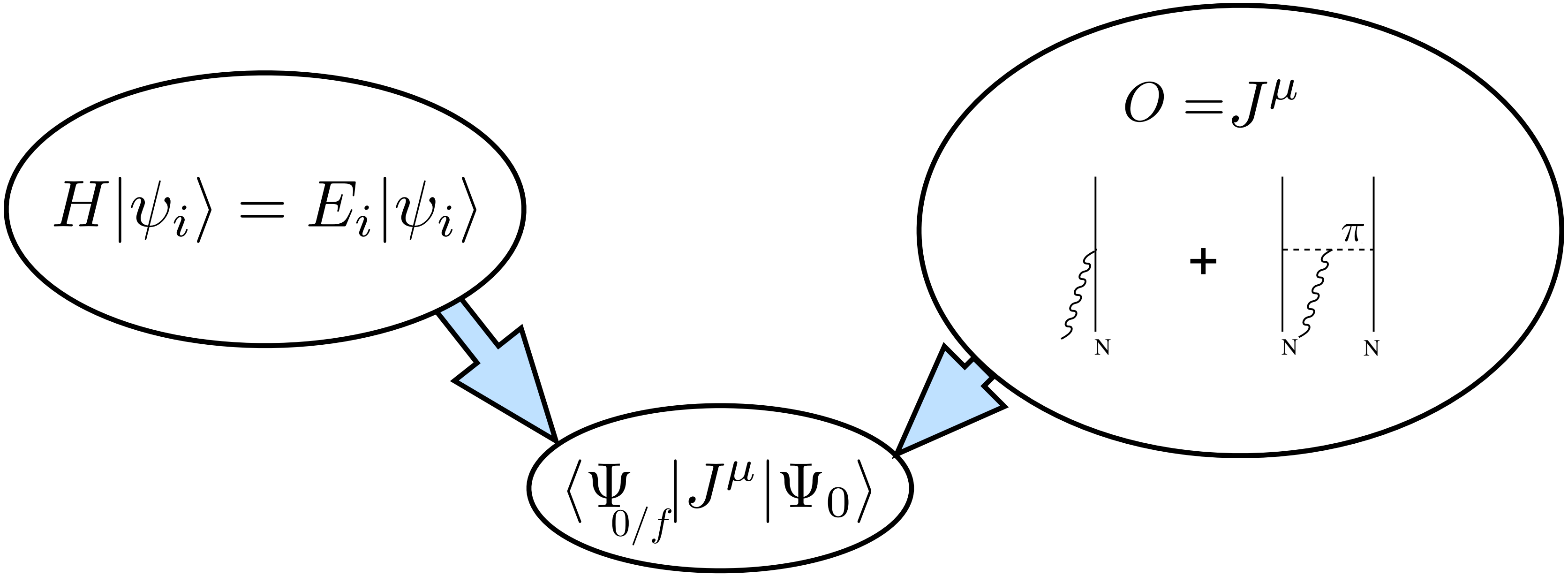
$L(\sigma, \Gamma)$ $\xleftrightarrow{\text{inversion}}$ $R(\omega)$ with the exact final state interaction

You can use any good bound state method

Electromagnetic Observables



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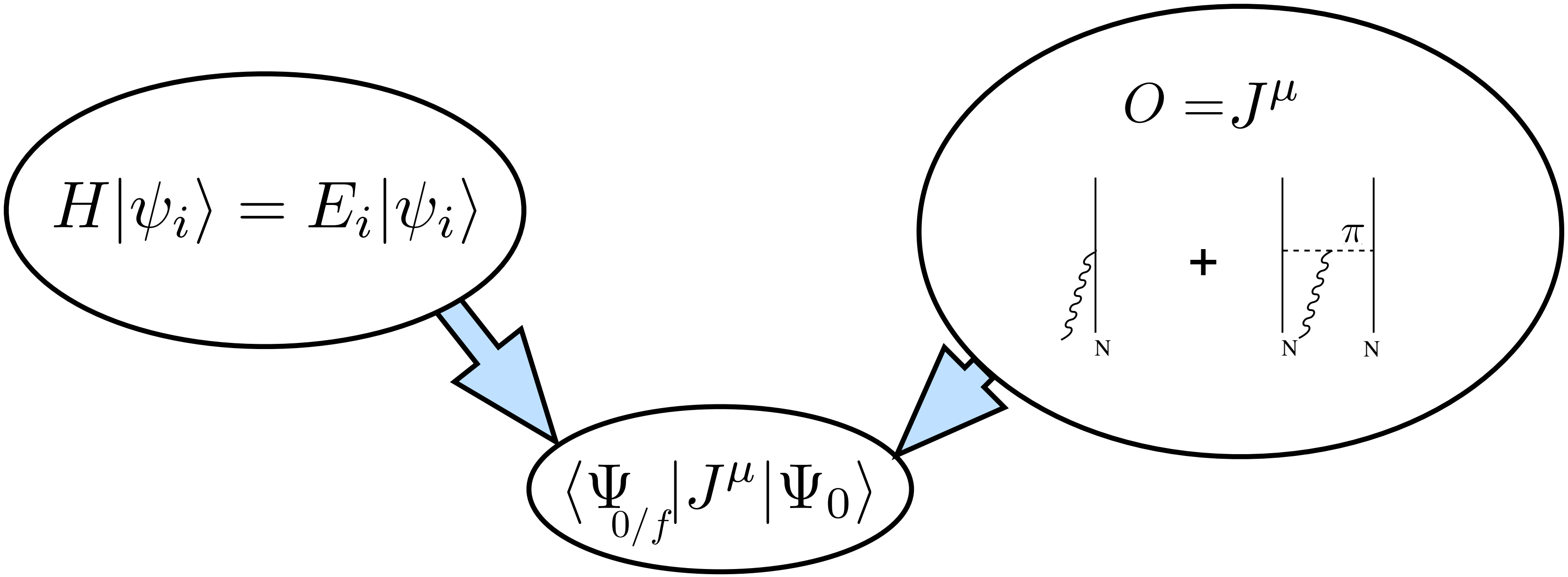
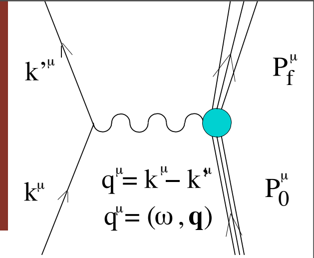
Electromagnetic Observables



$$\boldsymbol{\nabla}\cdot\mathbf{J} = -i[H,\rho]$$

The current should be consistent with the Hamiltonian

Electron scattering reaction



Virtual photon exchange: one can vary the energy and momentum transfer independently

Inclusive cross section A(e,e')X

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[\frac{Q^4}{\mathbf{q}^4} R_L(\omega, \mathbf{q}) + \left(\frac{Q^2}{2\mathbf{q}^2} + \tan^2\frac{\theta}{2} \right) R_T(\omega, \mathbf{q}) \right]$$

Mott cross section

Response Functions, can be studied with the LIT method

$$R_L(\omega, \mathbf{q}) = \sum\!\!\!\!\!\!\int_f |\langle \Psi_f | \rho(\mathbf{q}) | \Psi_0 \rangle|^2 \, \delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M} \right)$$

$$R_T(\omega, \mathbf{q}) = \sum\!\!\!\!\!\!\int_f |\langle \Psi_f | J_T(\mathbf{q}) | \Psi_0 \rangle|^2 \, \delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M} \right)$$

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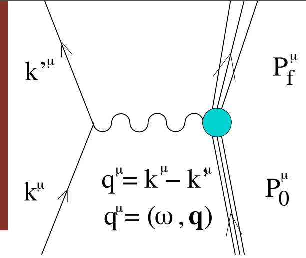

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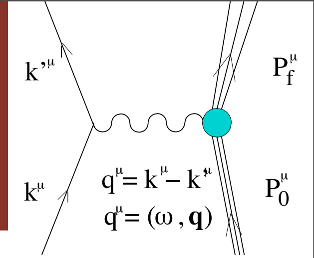
Study R_L for ^{4}He (no exchange currents up to N^3LO)
to investigate the effect of 3NF and help understand the predictive power of the Hamiltonian

$$\rho(\mathbf{q}) = \sum_k \left(\frac{1+\tau_k^3}{2} \right) \exp[i\mathbf{q} \cdot \mathbf{r}_k]$$

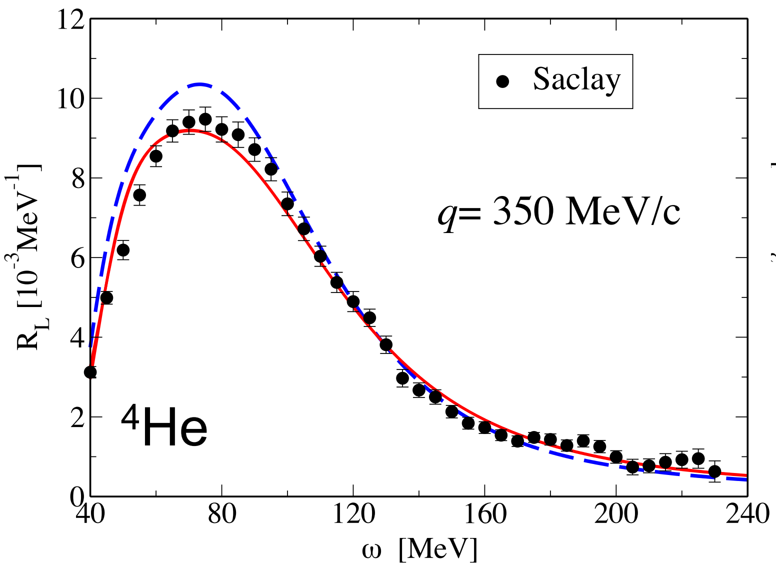
Expand the charge operator into multipoles and use the LIT/HH method for each multipole

Electron scattering reaction

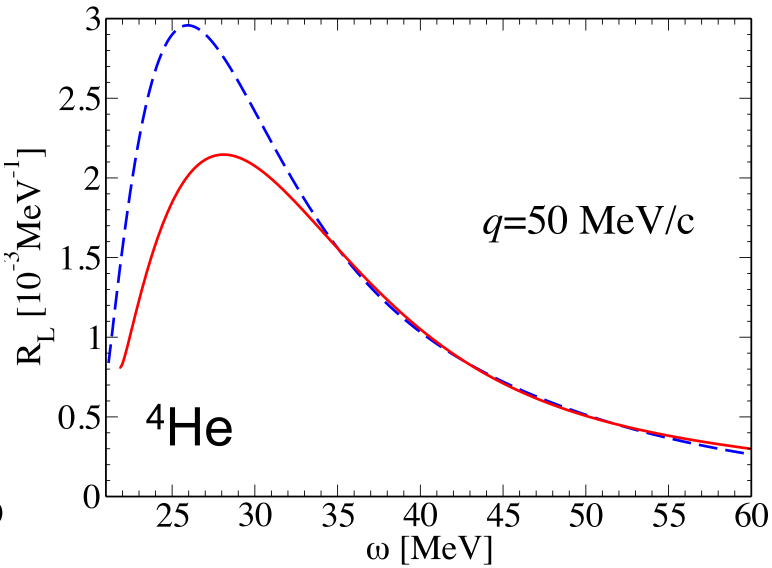


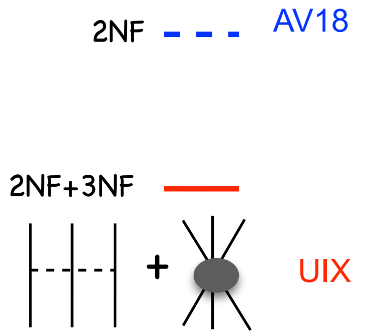
With the LIT/HH method

PRL 102, 162501 (2009)



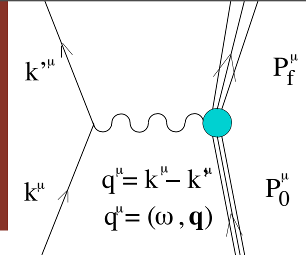
PRC80, 064001 (2009)





Stimulating new experiments

Electron scattering reaction



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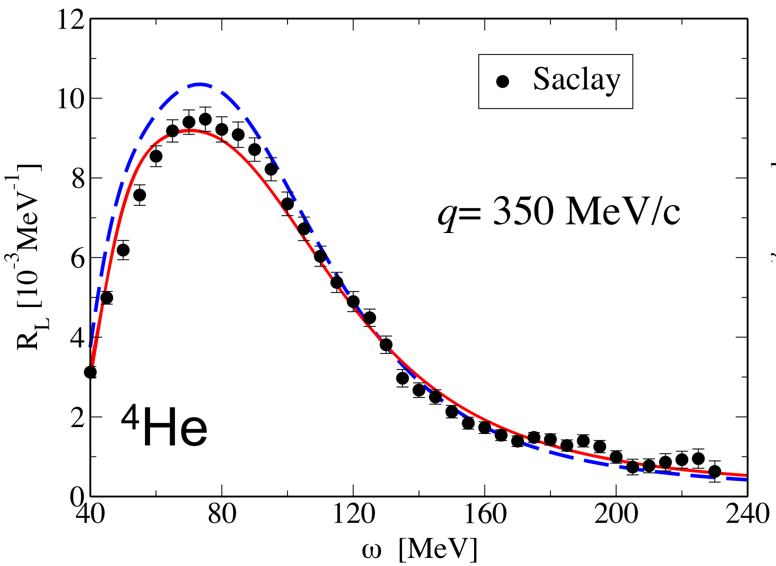

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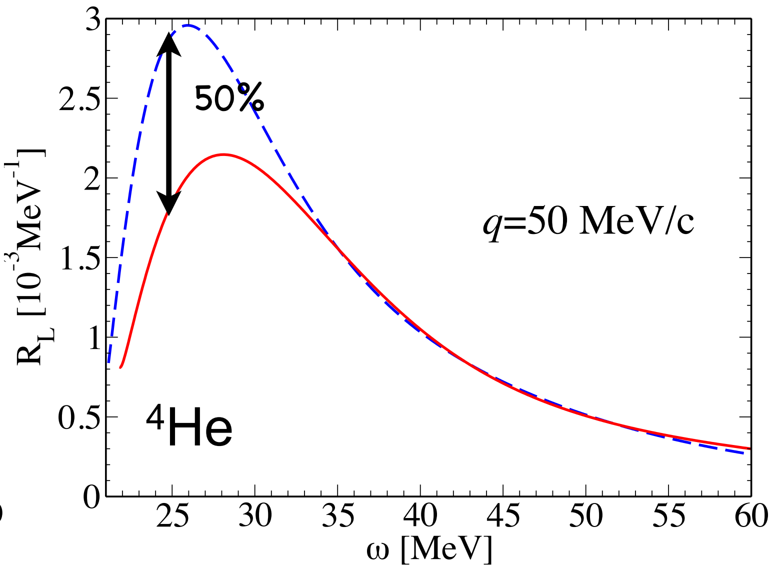

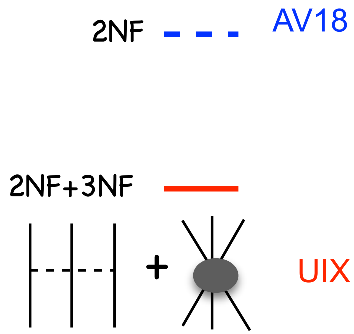

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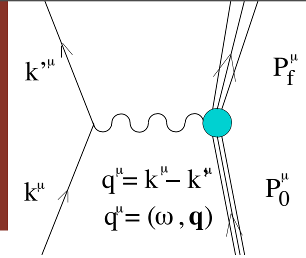

With the LIT/HH method

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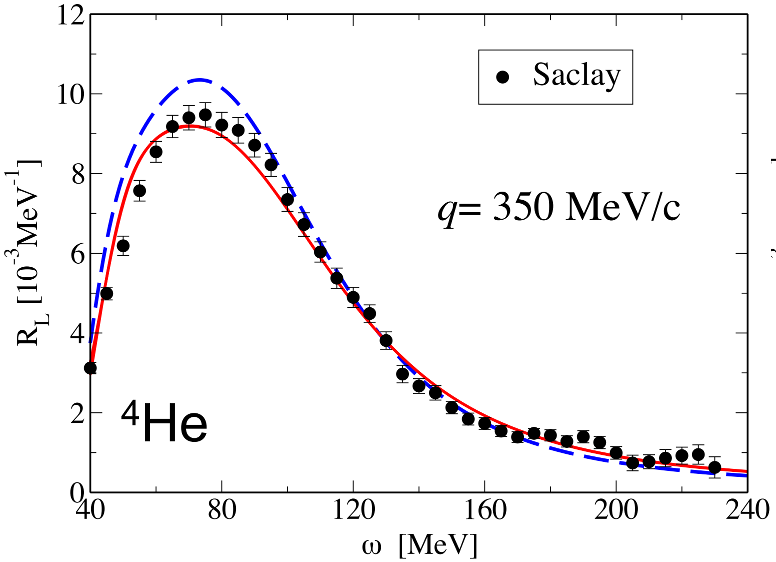

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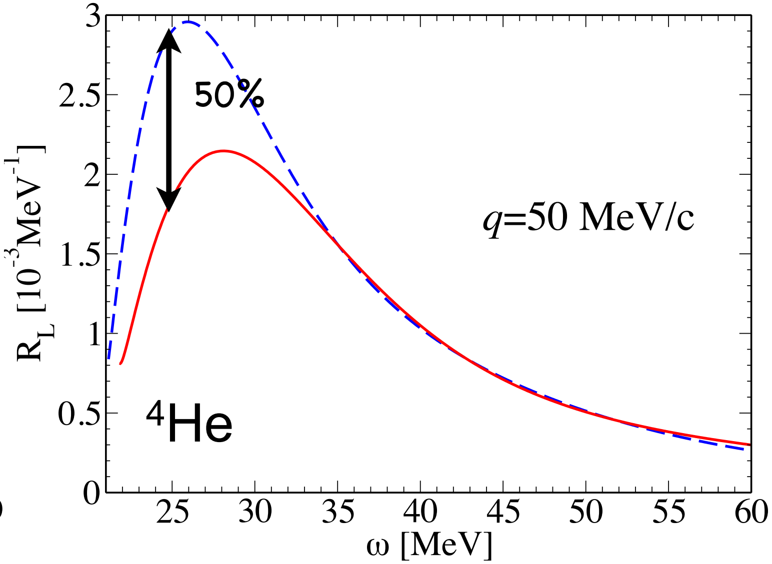
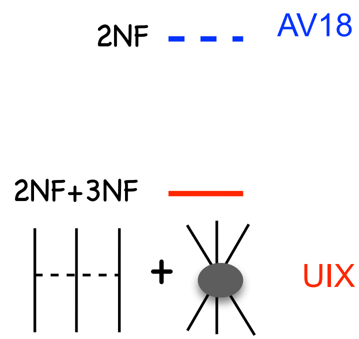
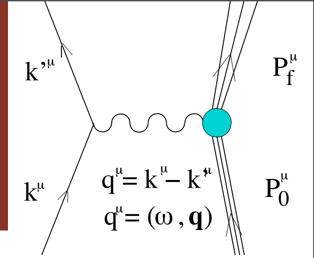




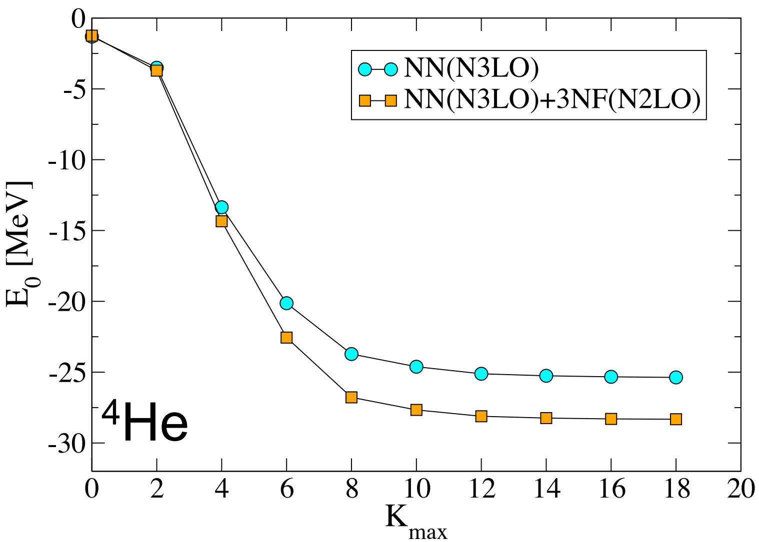
- Stimulating new experiments
- Provide predictions from EFT

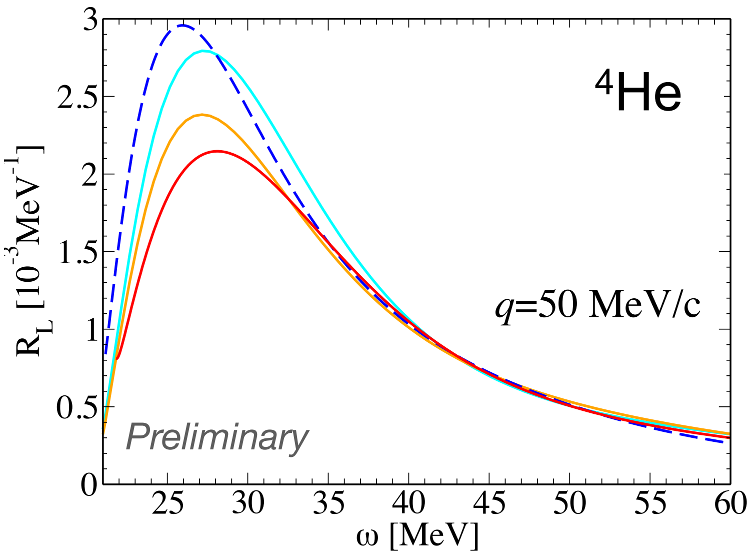
Electron scattering reaction

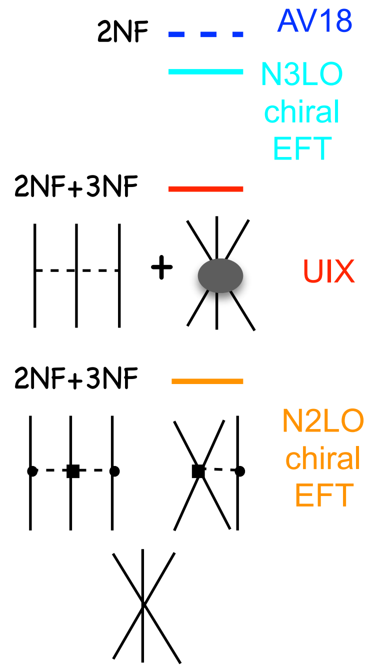


With the LIT/HH method

Work in progress







NN: N^3LO(500 MeV) from Entem-Machleidt PRC68, 041001(R) (2003)

+

3NF: N^2LO from Navratil, FBS 41 (2007) 117-140 c_D= 1

c_E= -0.029

NCSM E_0 = -28.34(2) MeV

HH E_0 = -28.33(3) MeV

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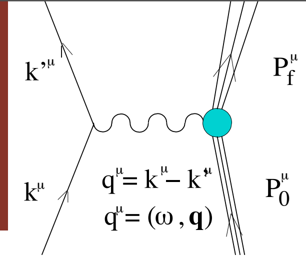
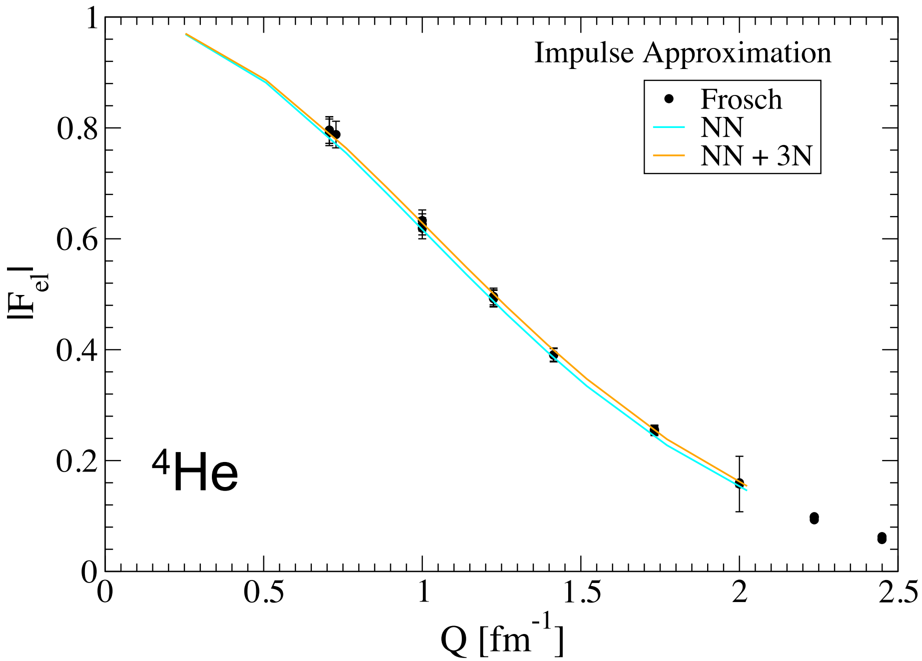
Electron scattering from EFT potentials



Work in progress

Elastic Form Factor

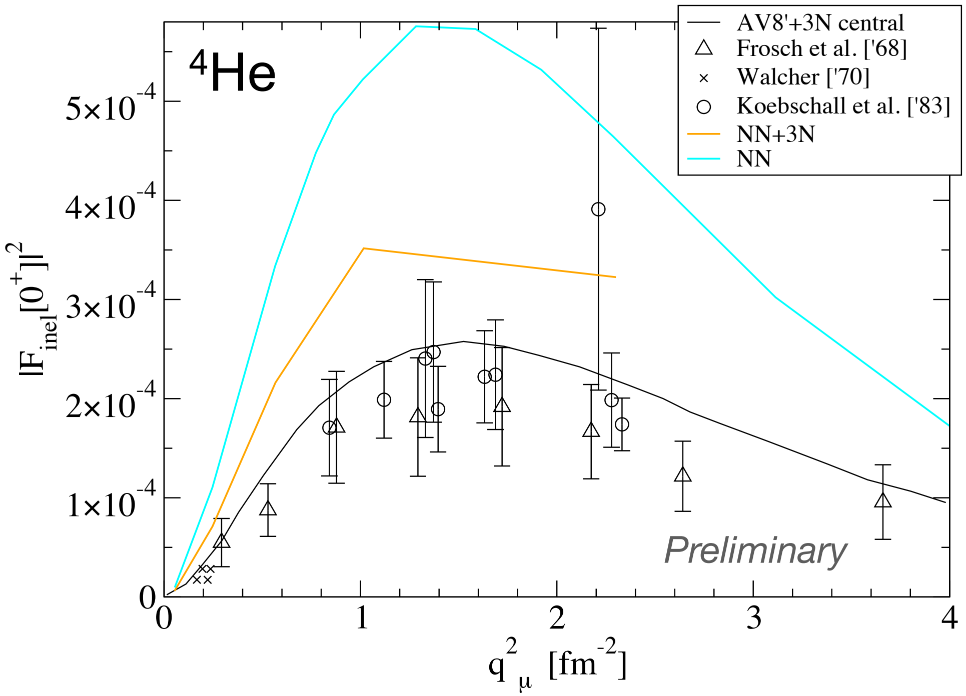


Impulse approximation valid below 2 fm^{-1}
Schiavilla *et al.*, PRC **41** 309 (1990)

NN $\langle r^2_{pp} \rangle = 1.517(2)$

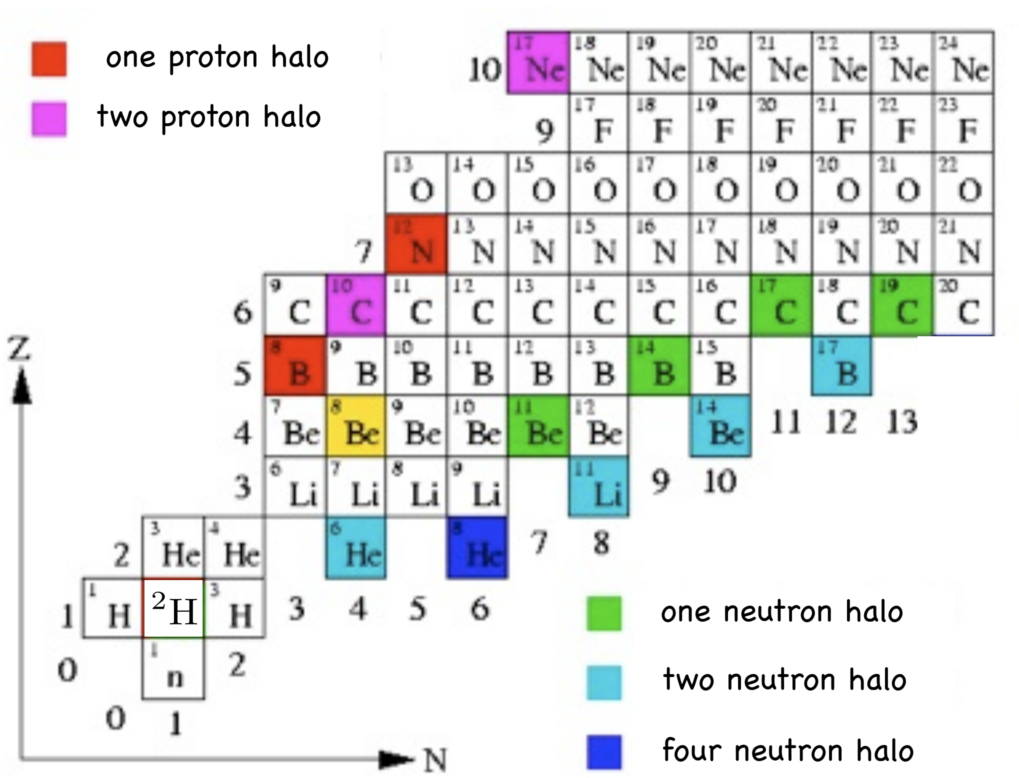
NN+3N $\langle r^2_{pp} \rangle = 1.474(4)$

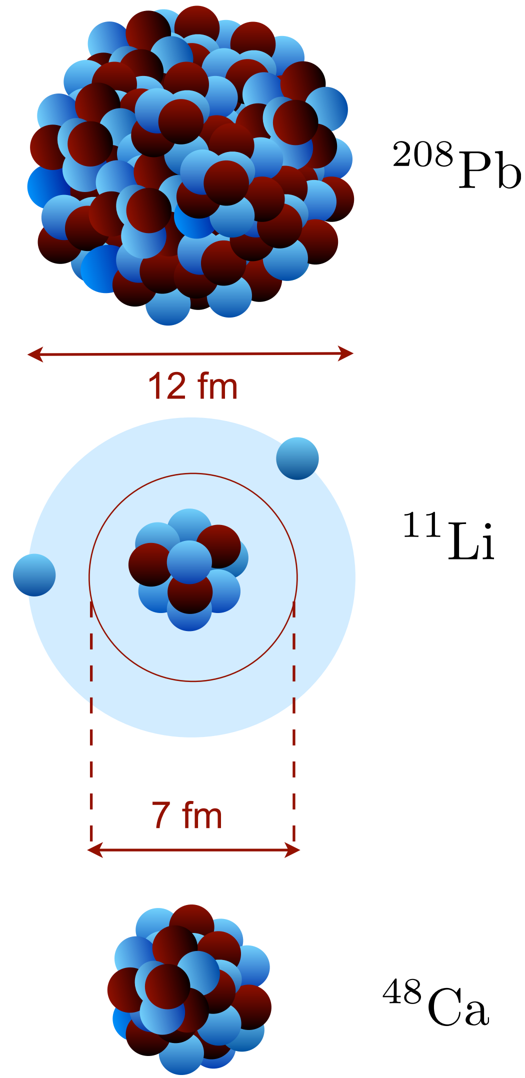
Inelastic Form Factor



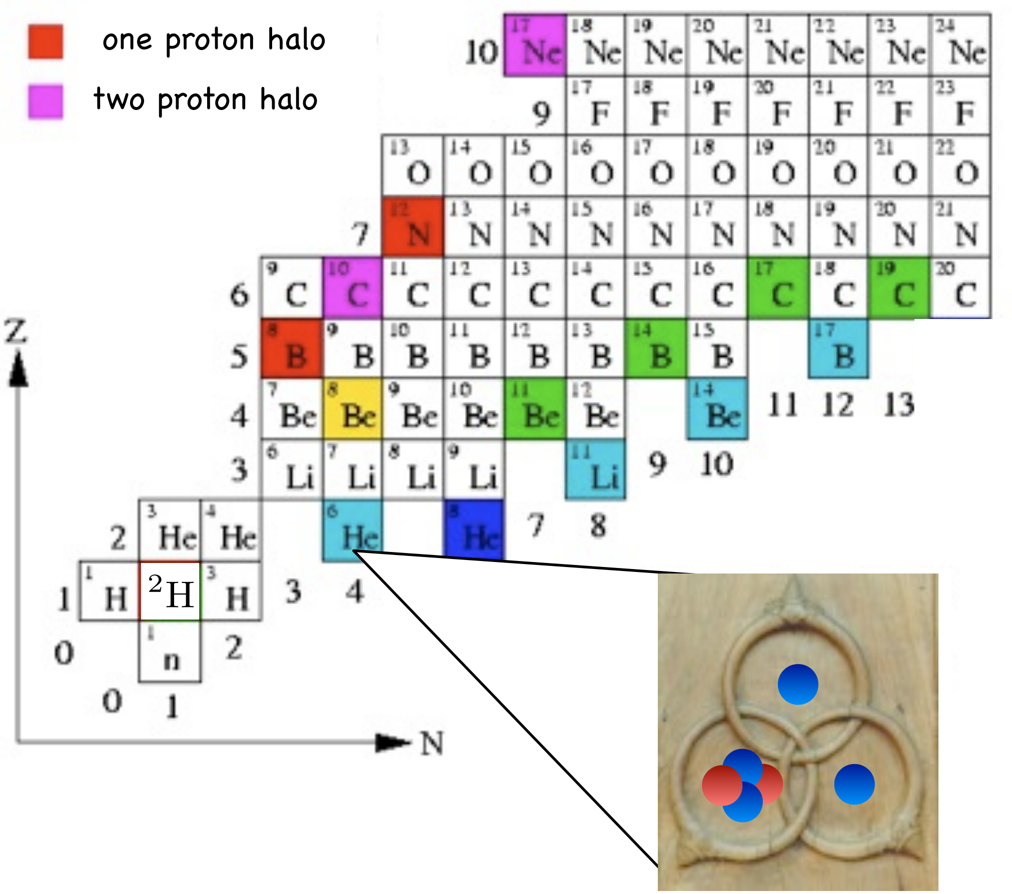
Previous calculation:
Hiyama *et al.*, PRC **70** 031001 (2004)

Halo Nuclei

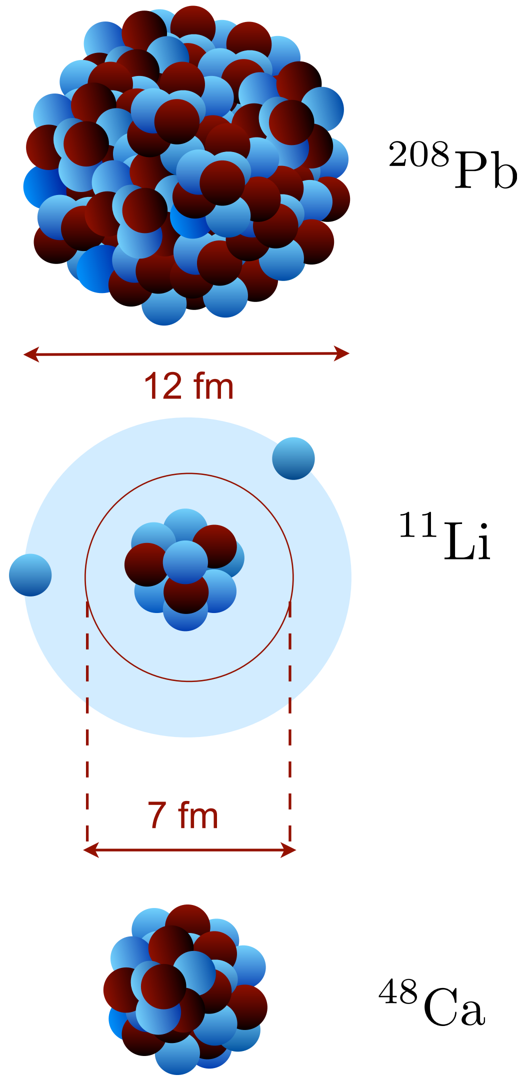

one proton halo
two proton halo
one neutron halo
two neutron halo
four neutron halo
Z
N


^{208}Pb
12 fm
^{11}Li
7 fm
^{48}Ca

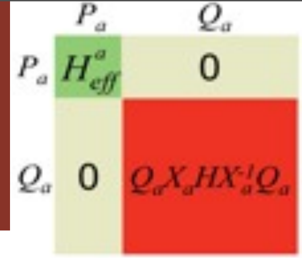
Halo Nuclei



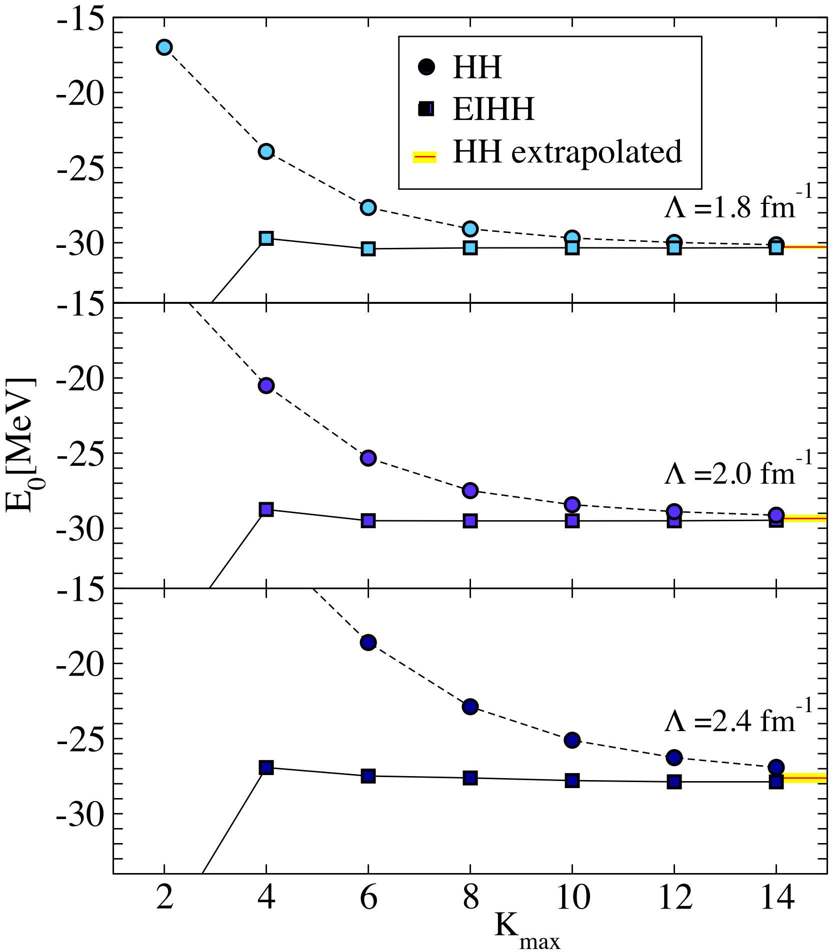
Lightest halo nucleus
and Borromean system

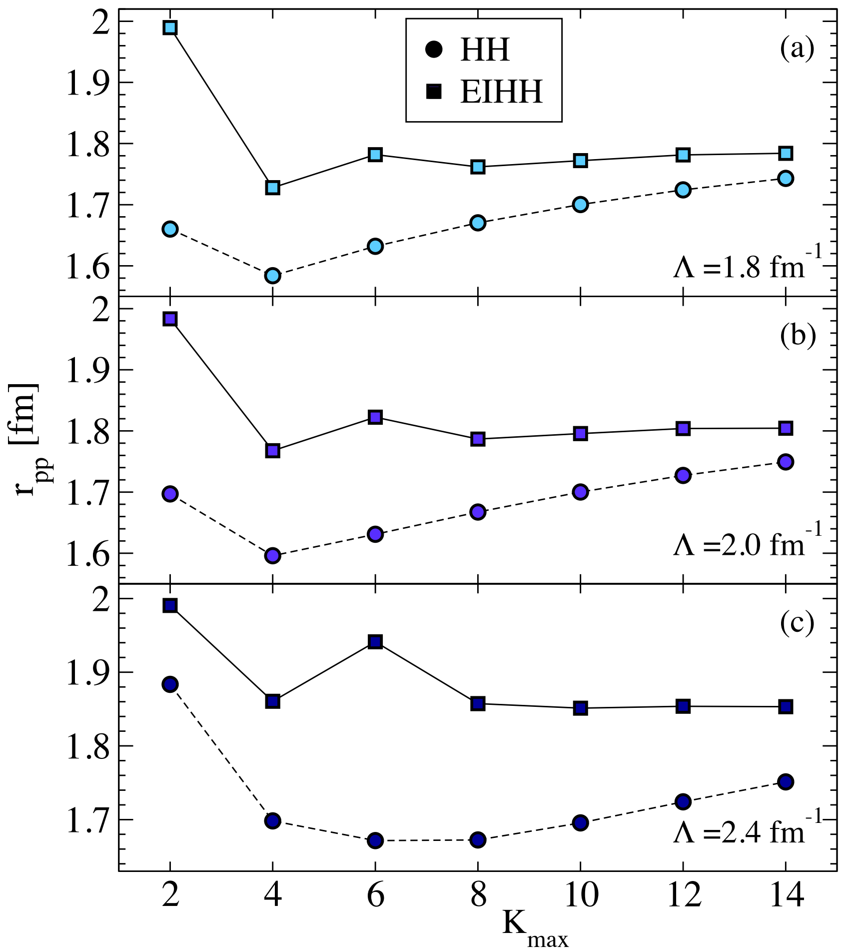


^{6}He from hyper-spherical harmonics



Interaction: $V_{\rm low\ k}$ from N^{3}LO (500 MeV)

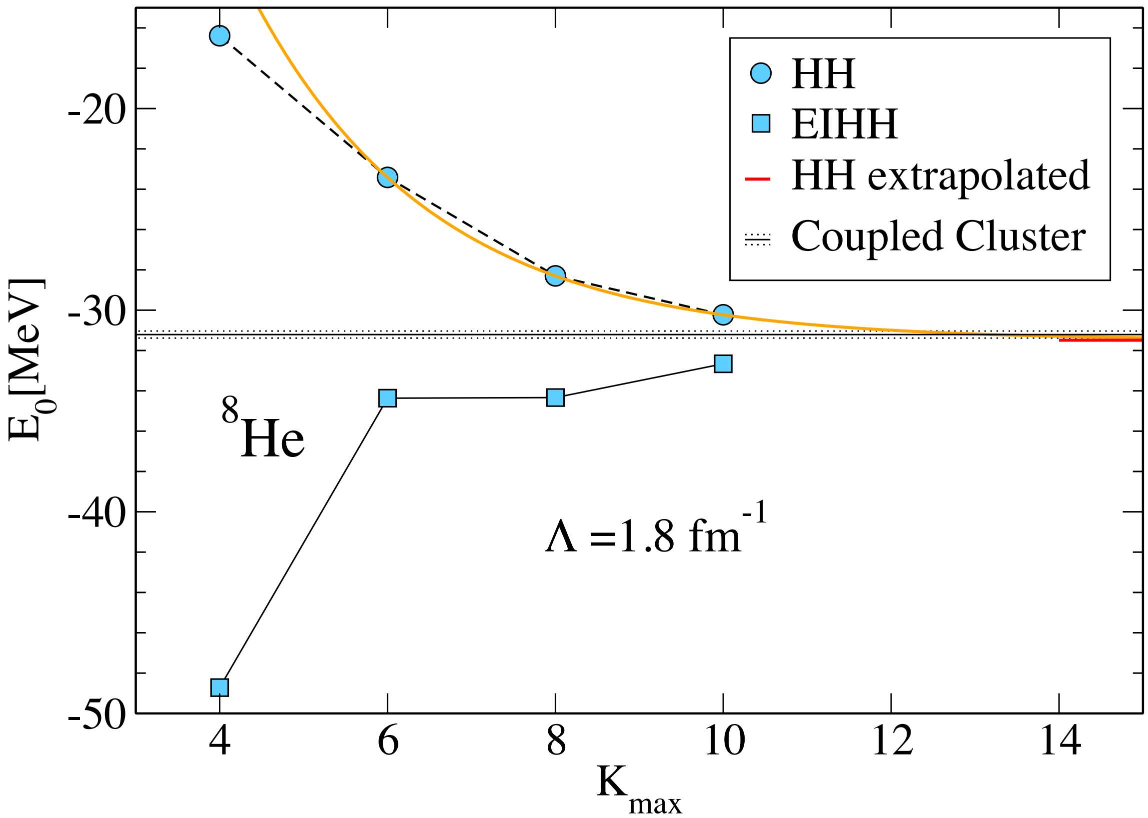




- EIHH agrees with extrapolated HH results from EPJ A 42, 553 (2009)
- EI is key to reach a reliable convergence of radii

^{8}He from hyper-spherical harmonics

$V_{\text{low k}}$ from N^{3}LO (500 MeV)



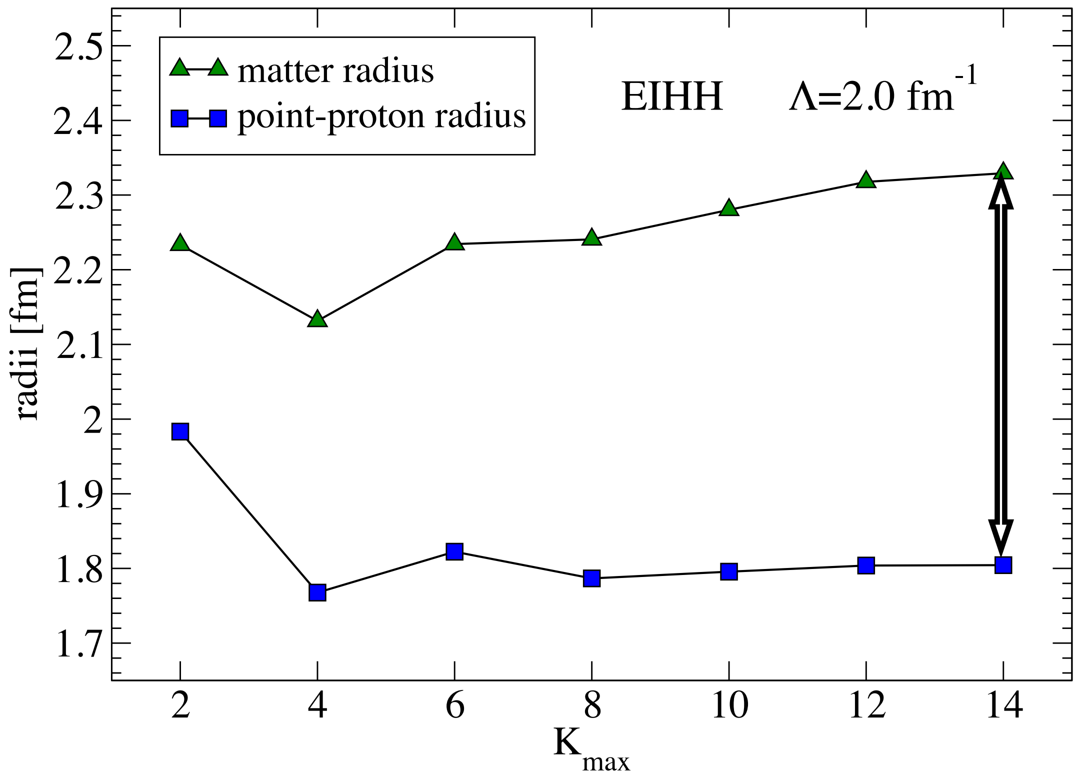
- Difference between Bare and EIHH is about 2.4 MeV
- EIHH seems less effective than for ^{6}He
- Extrapolating Bare results get

$$E_\infty = -31.49\text{MeV}$$

$$E_{\Lambda-\text{CCSD(T)}} = -31.21\text{MeV}$$

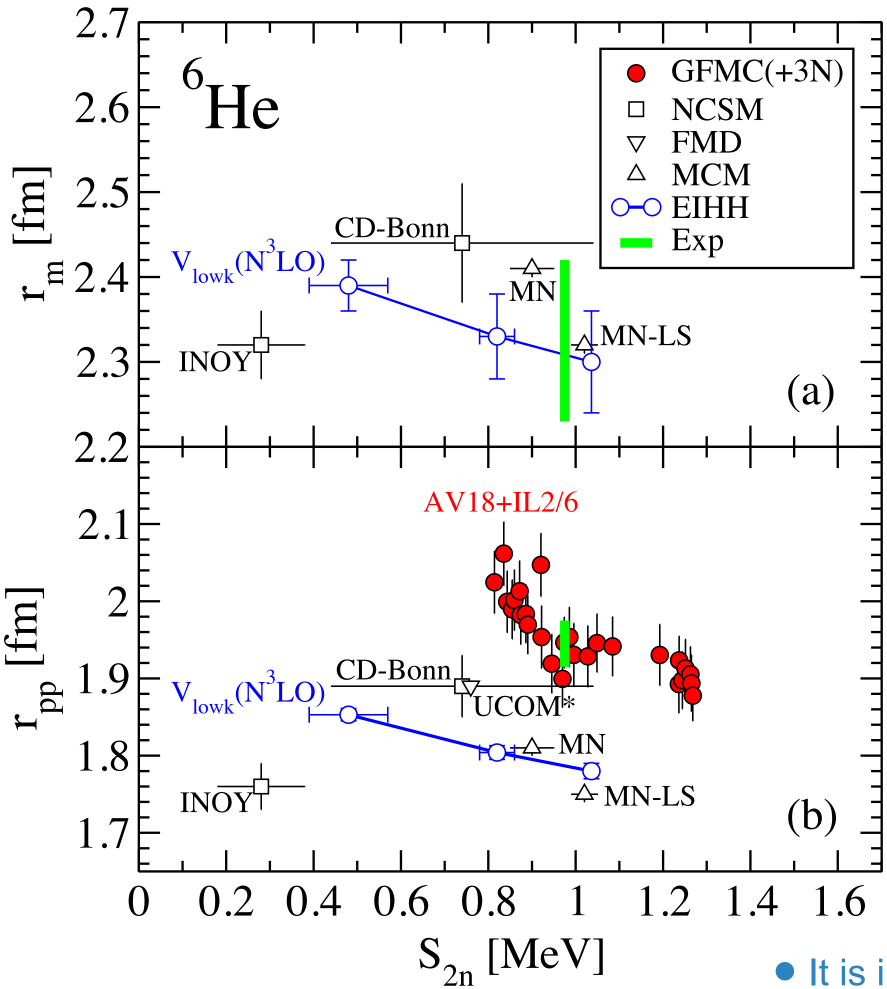
^{6}He from hyper-spherical harmonics

Signatures of the halo



- Point-Proton radii converge better and are smaller than matter radii ⇨ halo structure

Comparison with experiment



Phys. Rev. Lett. 108, 052504 (2012) & arXiv:1202.0516

(a) Experimental matter radius relatively uncertain

(b) Experimental charge radius well constrained

Relativistic corrections

$$r_{pp}^2 = r_c^2 - R_p^2 - \frac{N}{Z} R_n^2 - \frac{3}{4M_p^2} - r_{so}^2$$

0.877(7) fm from electron scattering and H spectroscopy

0.84184(67) from spectroscopy of muonic hydrogen

Calculated ab-initio ~-0.082 fm^2

- It is important to compare more than one observable together
- We observe a correlation between radii and separation energy
- Theory needs (improved) 3NFs

Outlook

- EM observables show sensitivity to the 3NF
- Hyper-spherical harmonics (together with the LIT) provide a tool to perform accurate studies of bound (and continuum) observables for light nuclei

Future:

- Room to study further 3NF effects and to add exchange currents for consistent EFT calculations