Momentum space evolution of chiral three-nucleon forces

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Perspectives of the Ab Initio No-Core Shell Model

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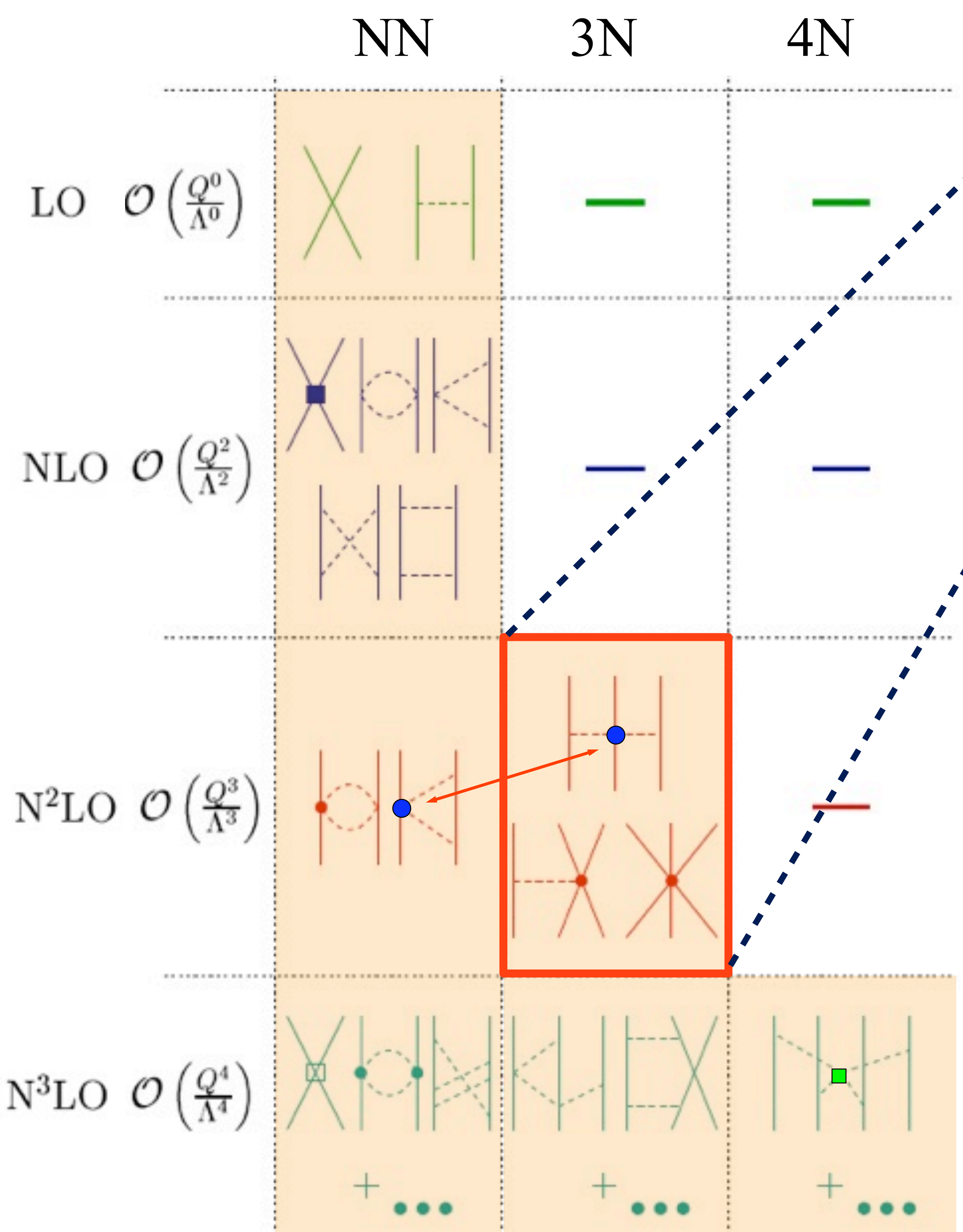
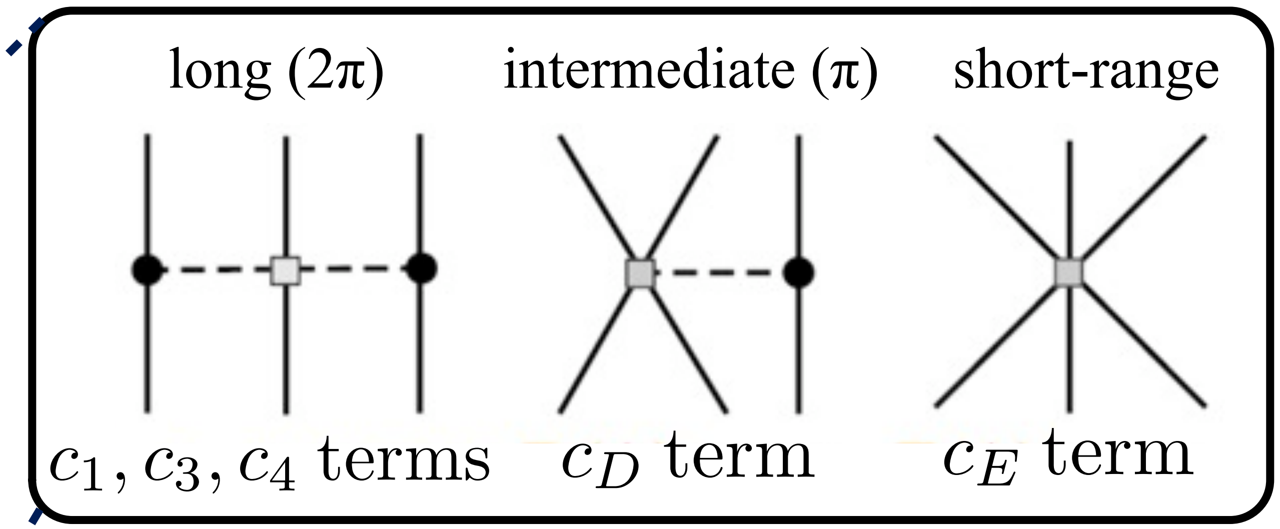
THE OHIO STATE UNIVERSITY

Chiral EFT for nuclear forces, leading order 3N forces





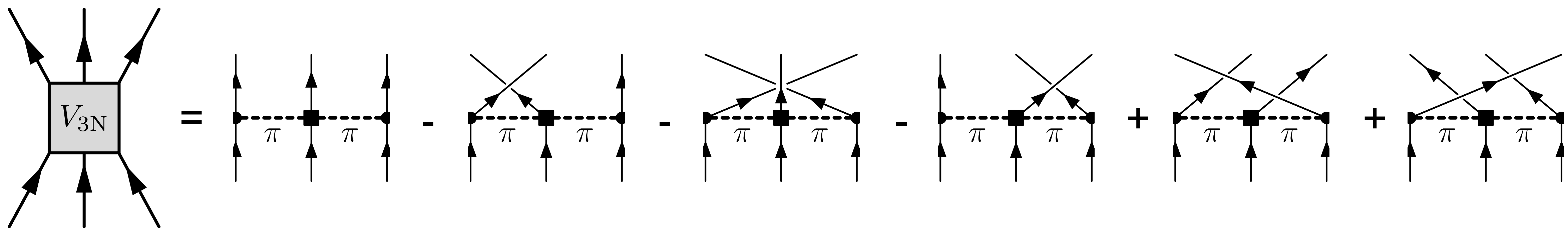
large uncertainties in coupling constants at present:

$$c_1 = -0.9^{+0.2}_{-0.5}\,,\quad c_3 = -4.7^{+1.5}_{-1.0}\,,\quad c_4 = 3.5^{+0.5}_{-0.2}$$

lead to theoretical uncertainties in many-body observables

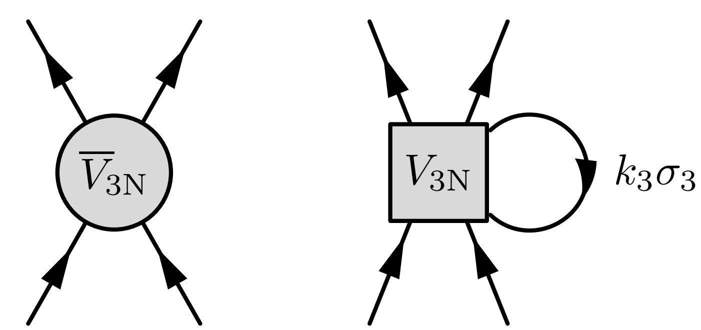
Chiral 3N interaction as density-dependent two-body interaction

(1) calculate antisymmetrized 3N interaction



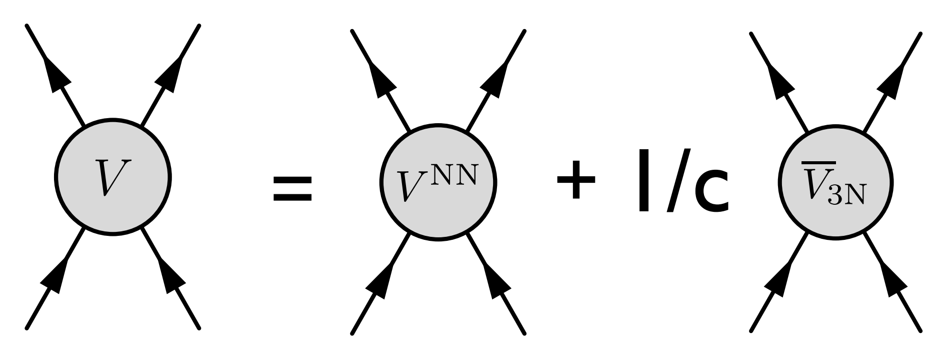
(2) construct effective density-dependent NN interaction

Basic idea:
Sum one particle over occupied states in the Fermi sea, normal ordering

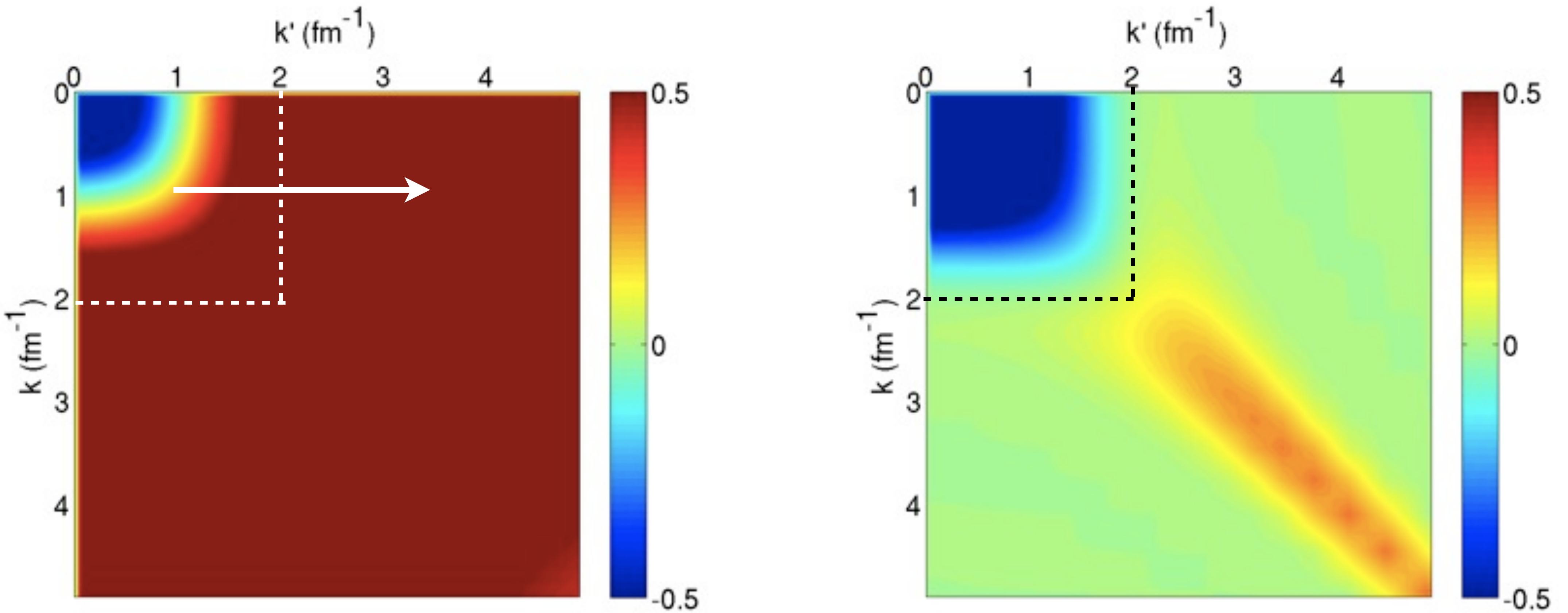


(3) combine with free-space NN interaction

combinatorial factor c depends on type of diagram



Changing the resolution:
The (Similarity) Renormalization Group

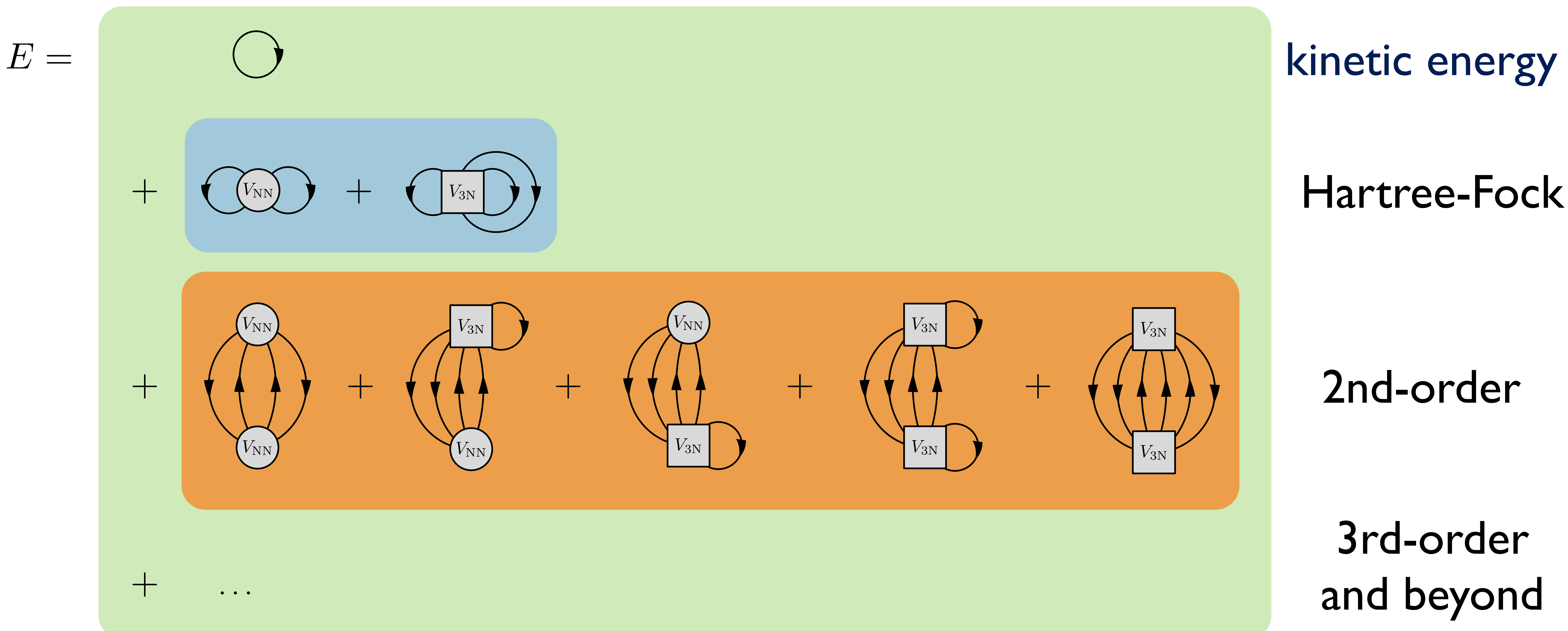


- elimination of coupling between low- and high momentum components
  $\longrightarrow$ simplified calculations
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations
- RG transformation also changes many-body interactions

Equation of state: Many-body perturbation theory

central quantity of interest: energy per particle E/N

$$H(\lambda) = T + V_{\mathrm{NN}}(\lambda) + V_{\mathrm{3N}}(\lambda) + \ldots$$

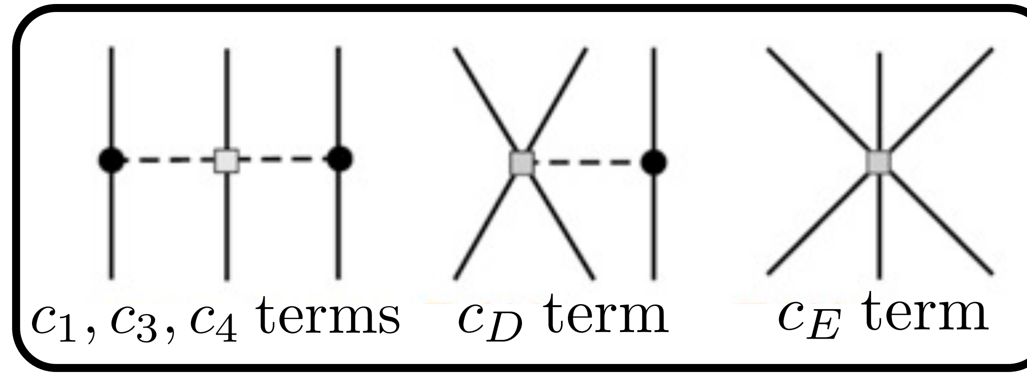


- "hard" interactions require non-perturbative summation of diagrams
- with low-resolution interactions much more perturbative
- inclusion of 3N interaction contributions crucial
- use chiral interactions as initial input for RG evolution

RG evolution of 3N interactions

- So far:

intermediate (c_D) and short-range (c_E) 3NF couplings fitted to few-body systems at different resolution scales:



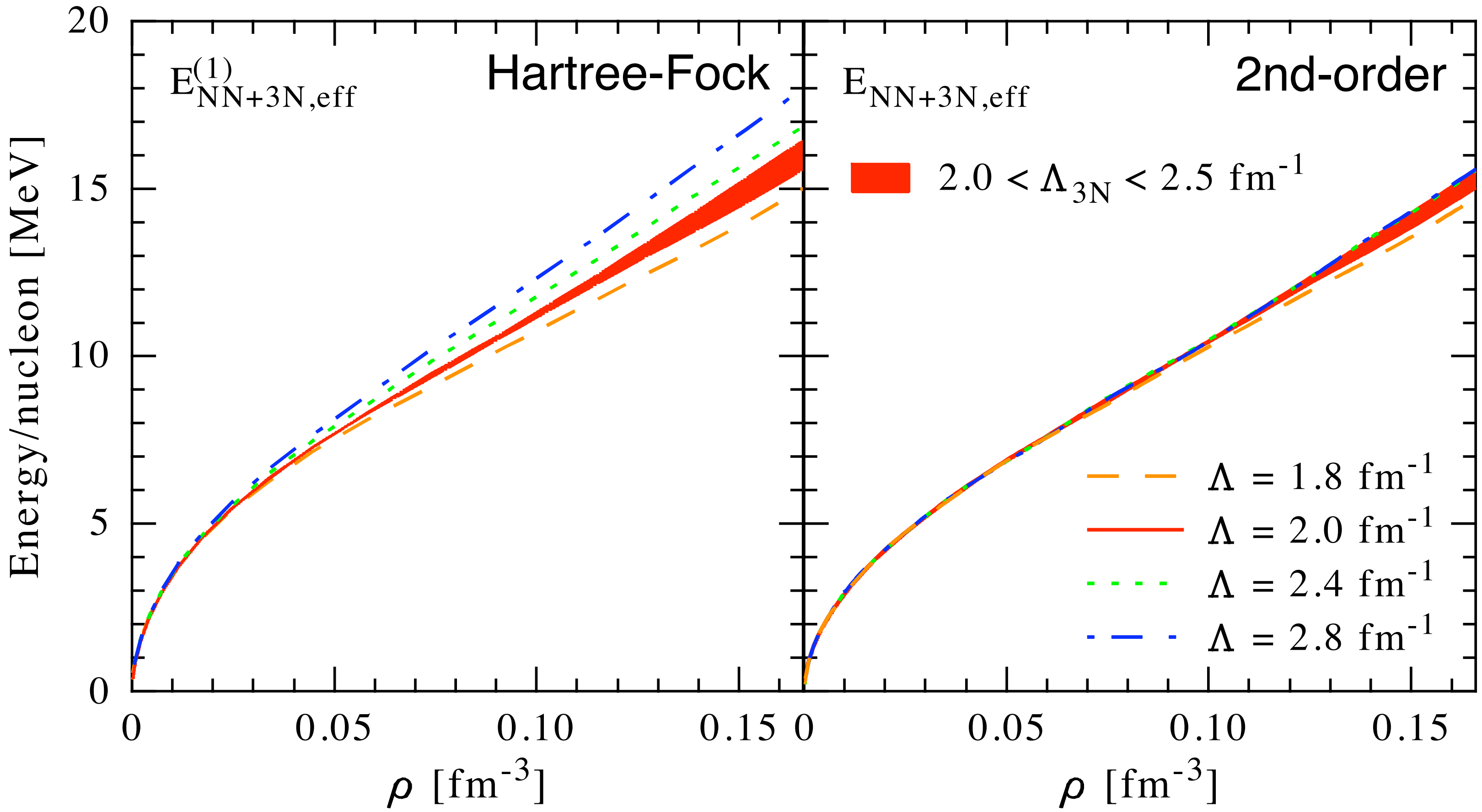
$$E_{^3\mathrm{H}} = -8.482\,\mathrm{MeV} \quad \text{and} \quad r_{^4\mathrm{He}} = 1.95 - 1.96\,\mathrm{fm}$$

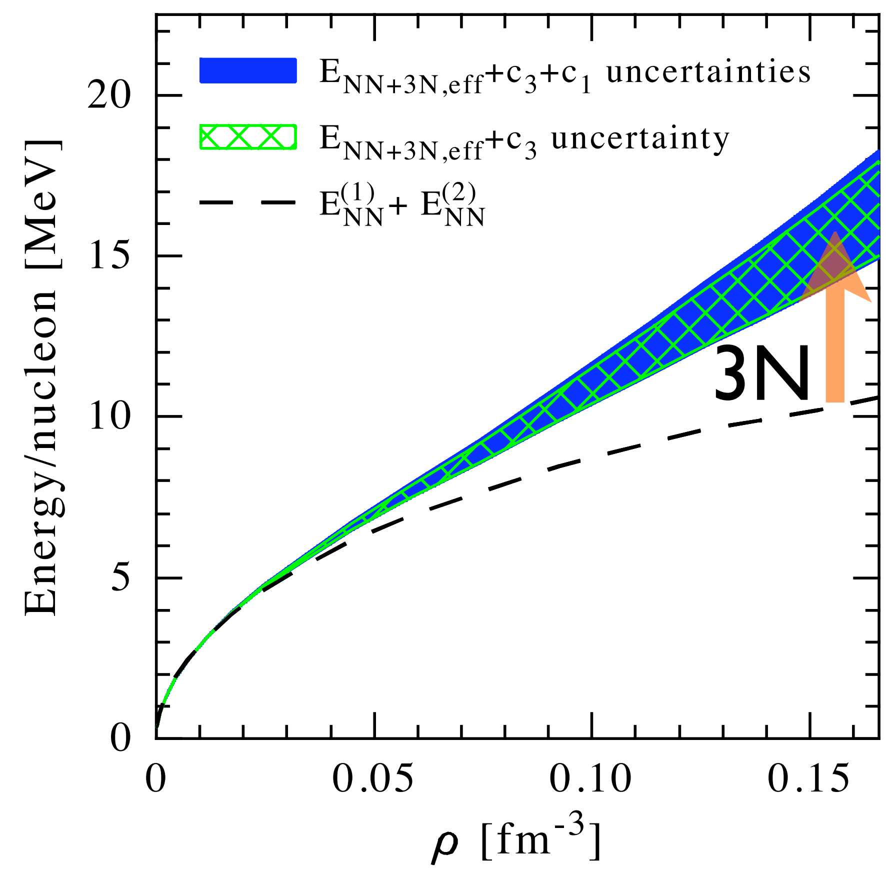
$\longrightarrow$ coupling constants of natural size

in neutron matter contributions from c_D , c_E and c_4 terms vanish

- Ideal case: evolve 3NF consistently with NN to lower resolution using the RG
  - has been achieved in oscillator basis (Jurgenson, Roth)
  - promising results in very light nuclei
  - problems in heavier nuclei
  - not suitable for infinite systems

Equation of state of pure neutron matter





KH and Schwenk PRC 82, 014314 (2010)

- significantly reduced cutoff dependence at 2nd order perturbation theory
- small resolution dependence indicates converged calculation
- variation due to 3N input uncertainty much larger than resolution dependence

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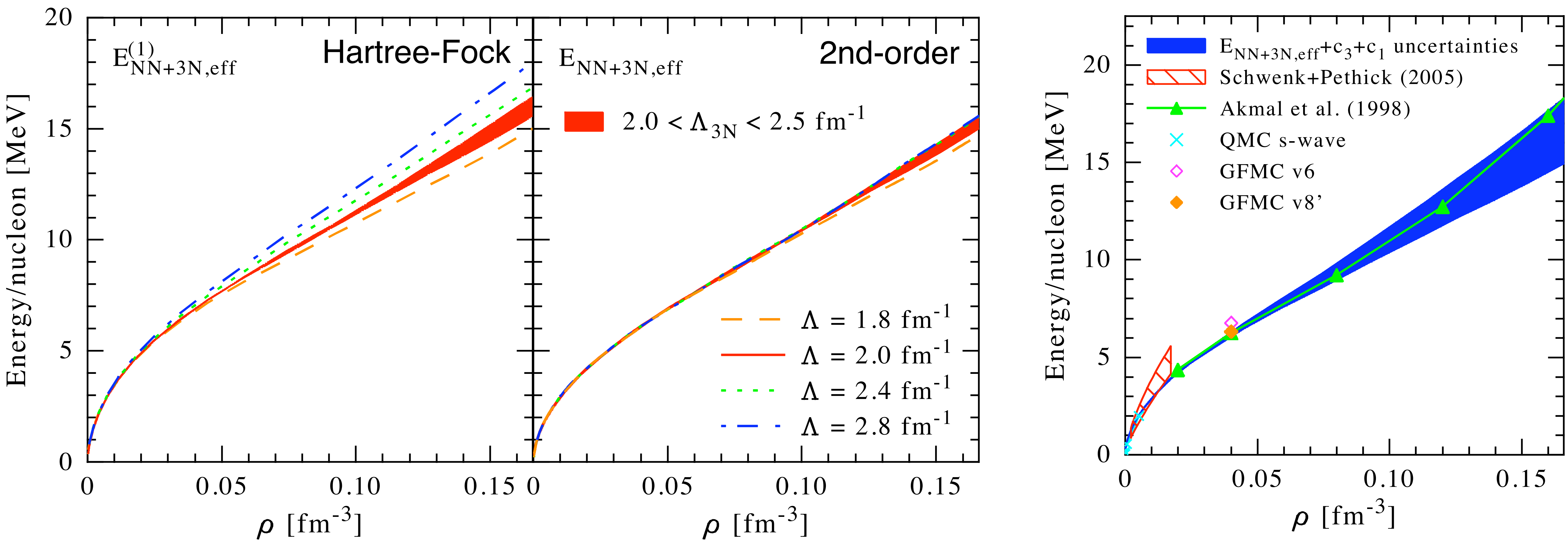

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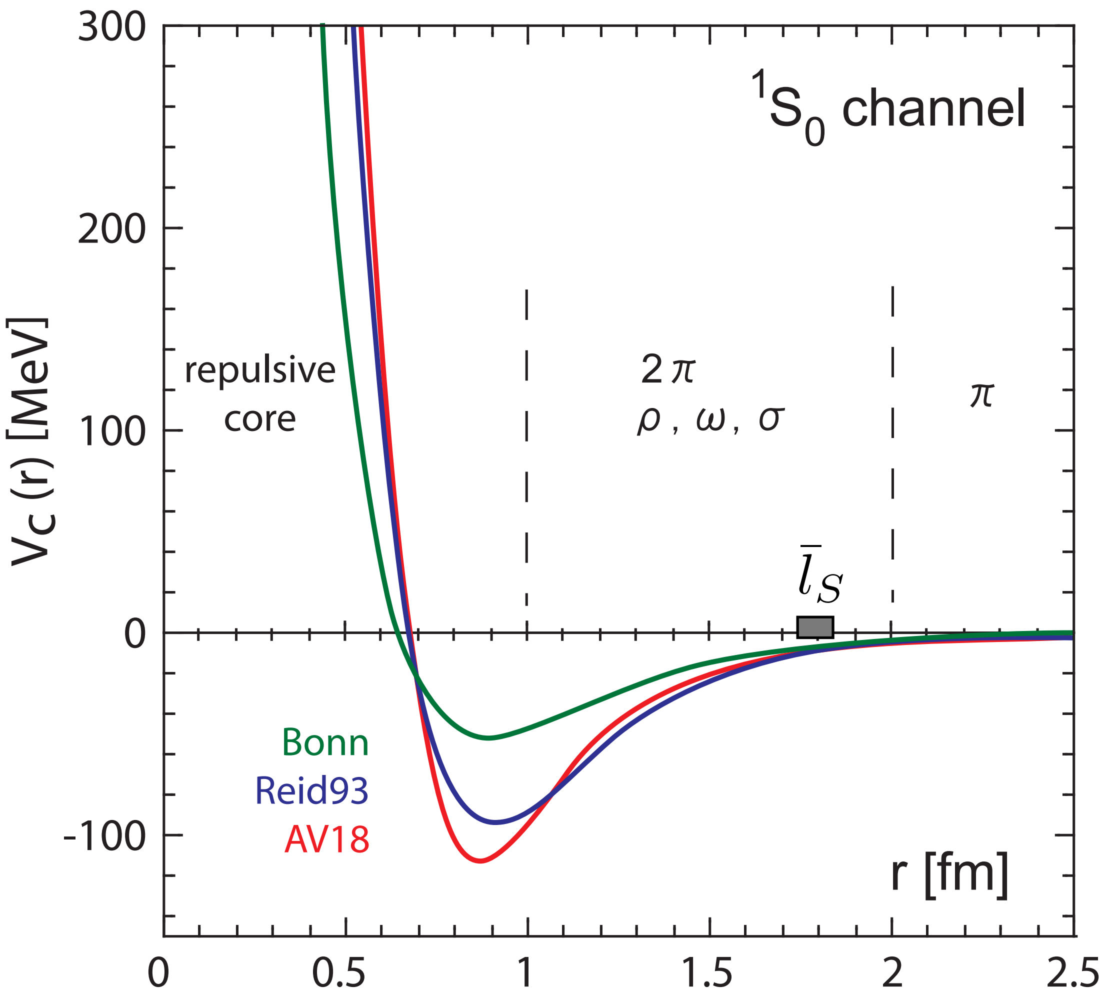
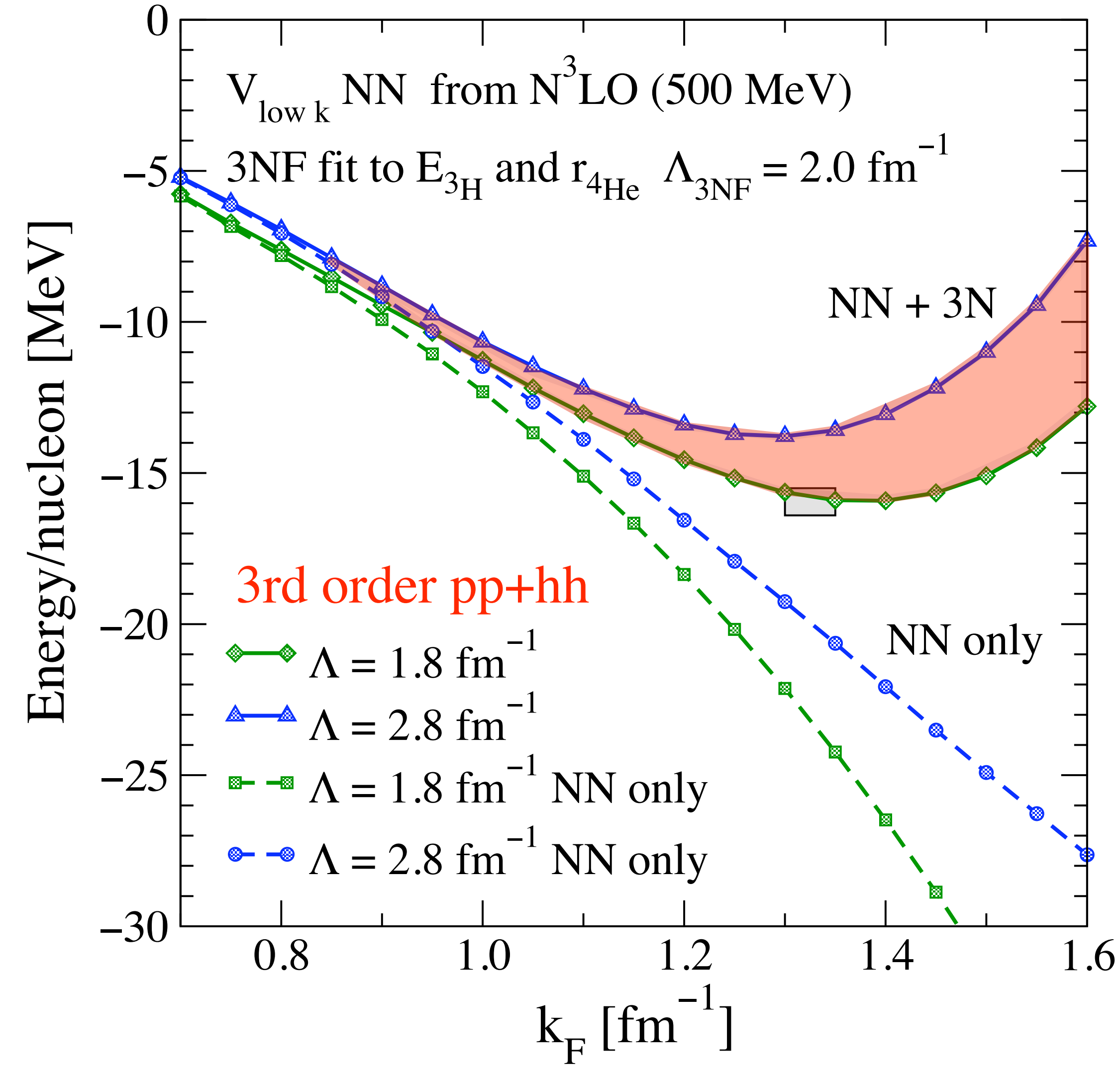
- significantly reduced cutoff dependence at 2nd order perturbation theory
- small resolution dependence indicates converged calculation
- variation due to 3N input uncertainty much larger than resolution dependence
- good agreement with other approaches (different NN interactions)

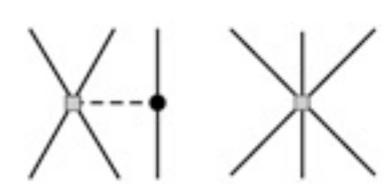
Equation of state of symmetric nuclear matter, Nuclear saturation



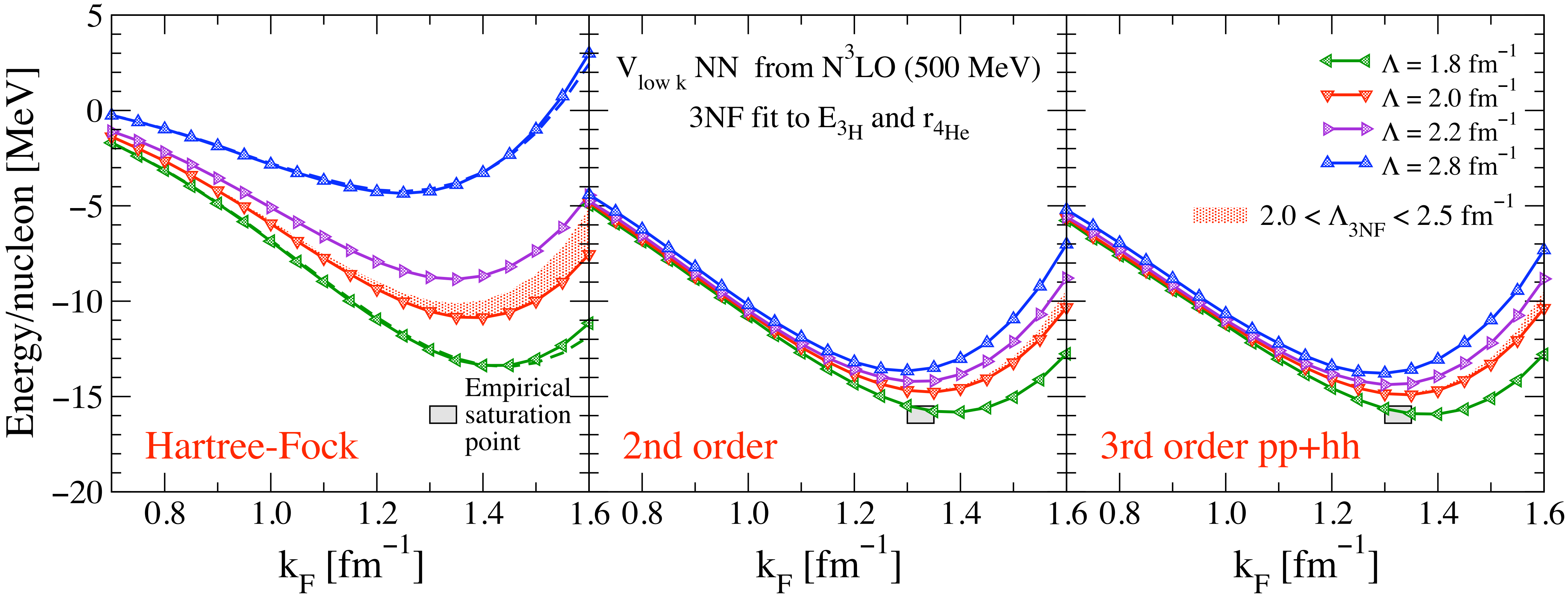


KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)

- nuclear saturation delicate due to cancellations of large kinetic and potential energy contributions
- 3N forces are essential! 3N interactions fitted to ^{3}H and ^{4}He properties



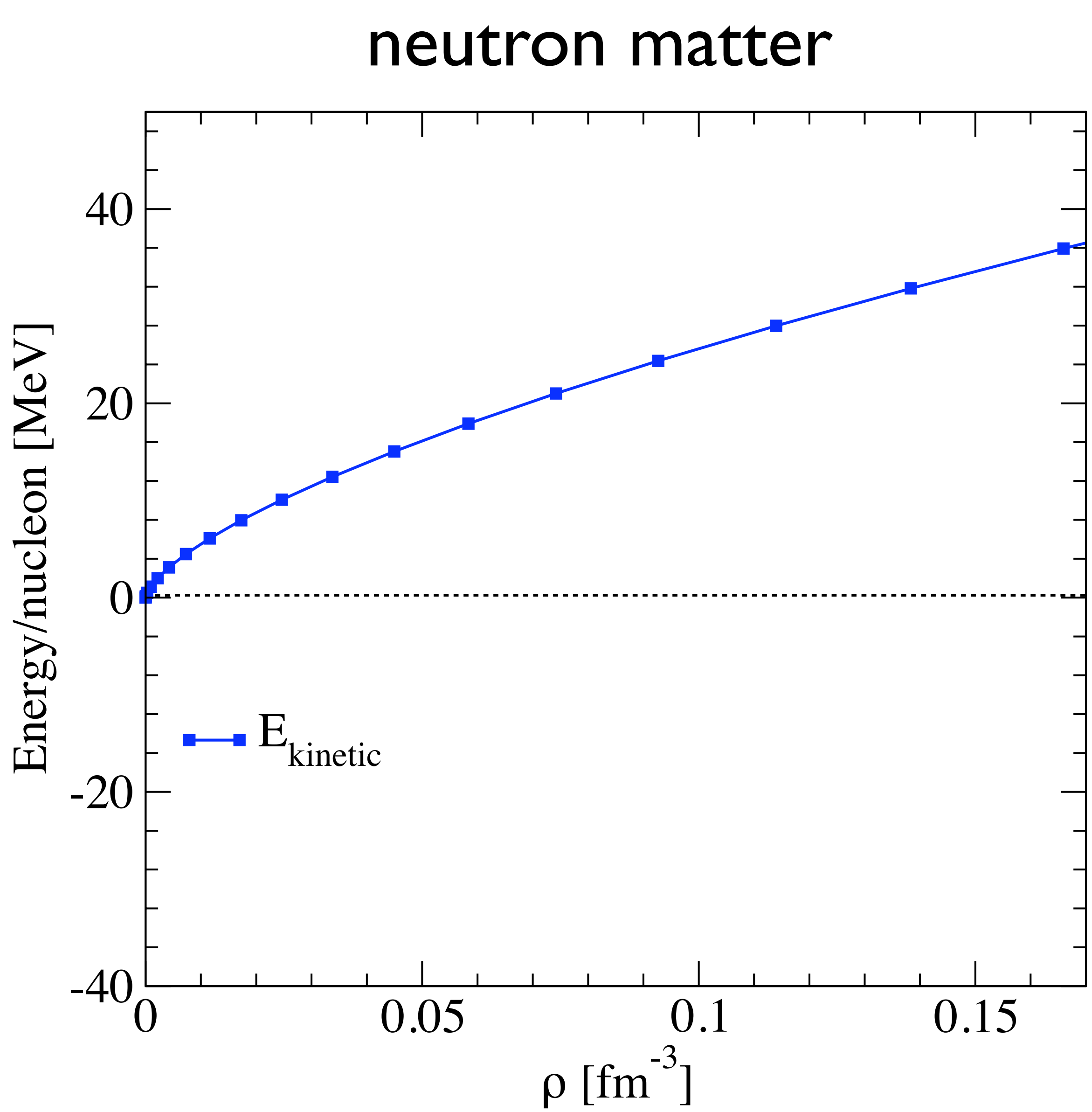
Equation of state of symmetric nuclear matter, Nuclear saturation

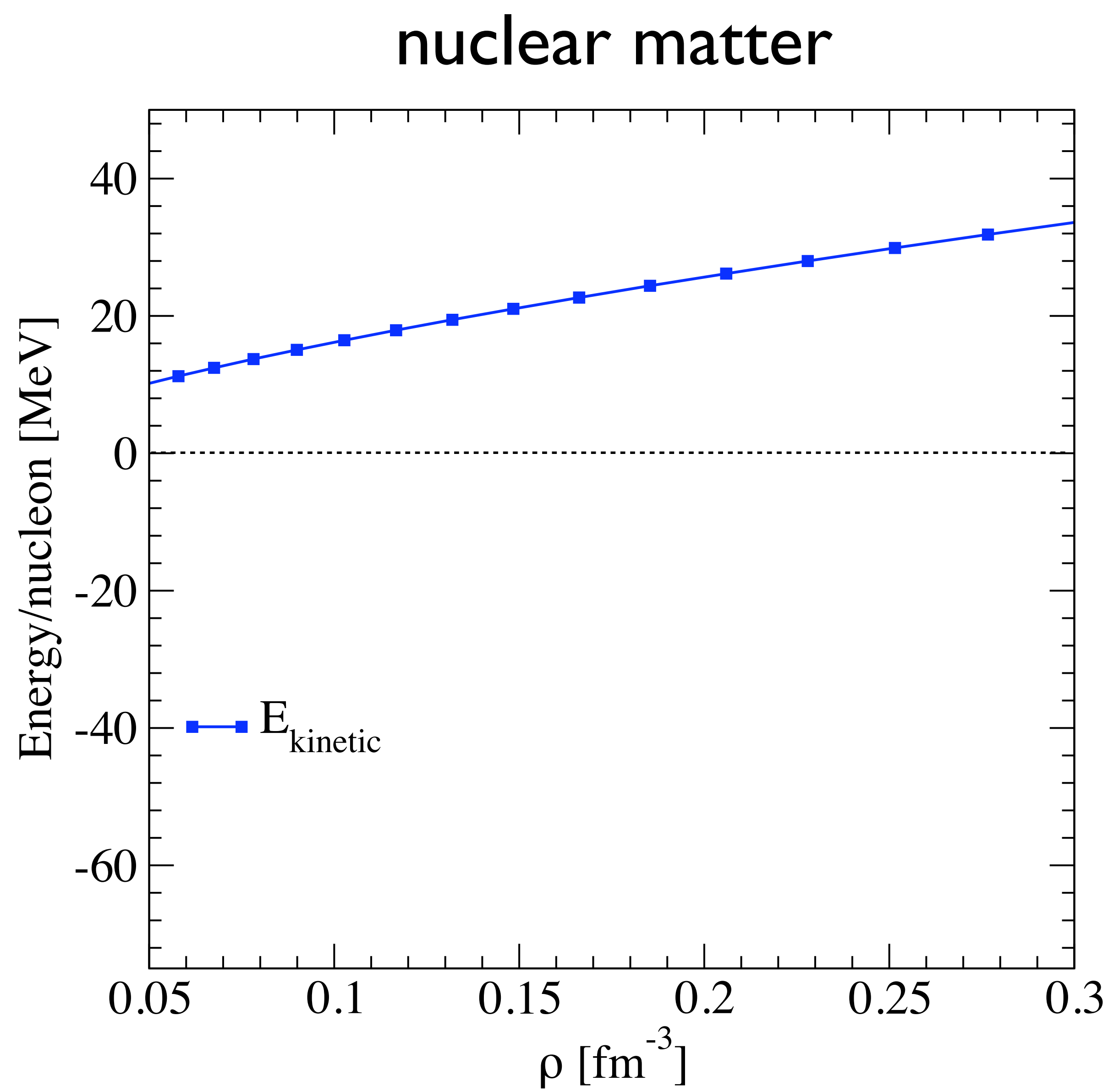


KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)

- saturation point consistent with experiment, without free parameters
- cutoff dependence at 2nd order significantly reduced
- 3rd order contributions small
- cutoff dependence consistent with expected size of 4N force contributions

Hierarchy of many-body contributions





- binding energy results from cancellations of much larger kinetic and potential energy contributions
- chiral hierarchy of many-body terms preserved for considered density range
- resol. dependence of natural size, consistent with chiral exp. parameter $\sim 1/3$

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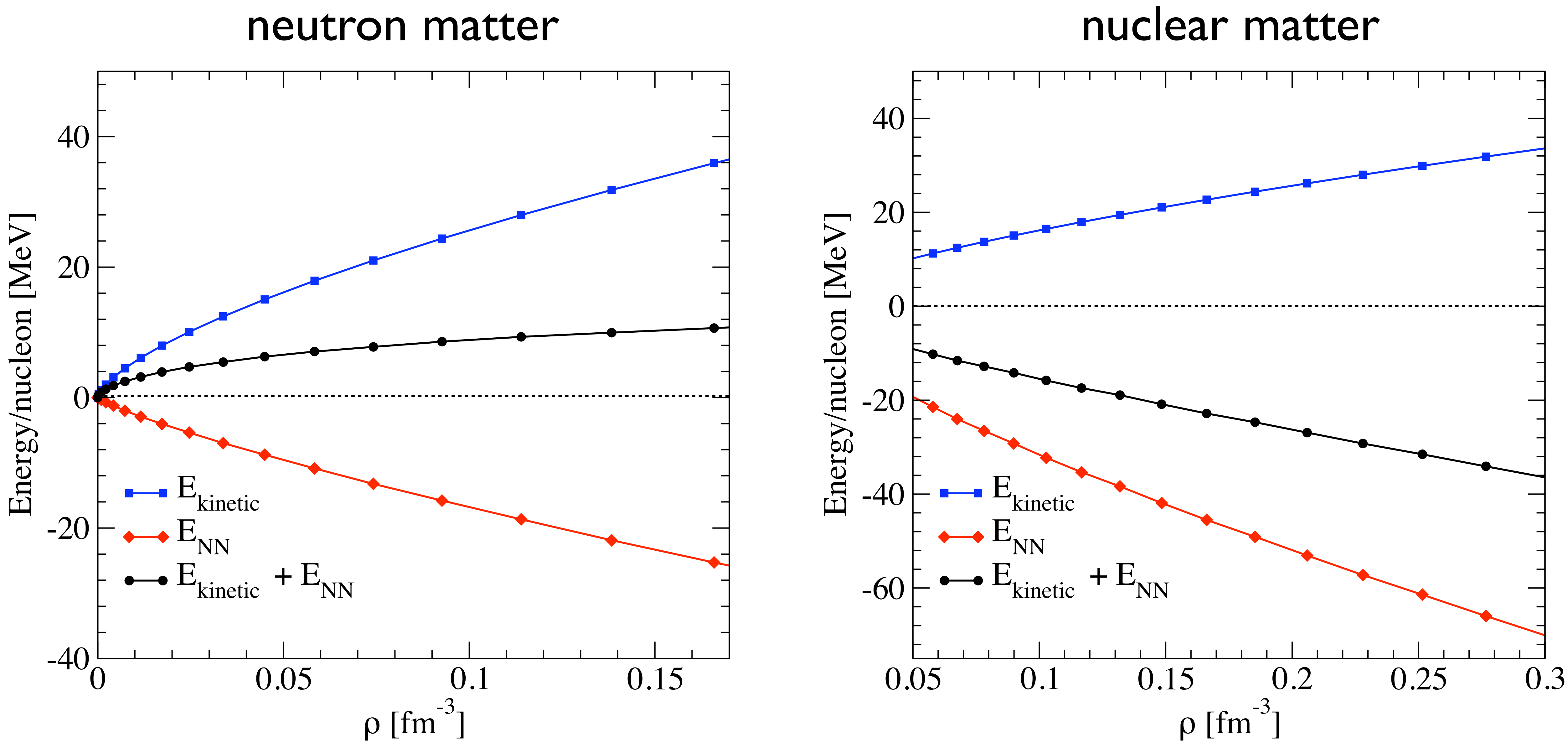

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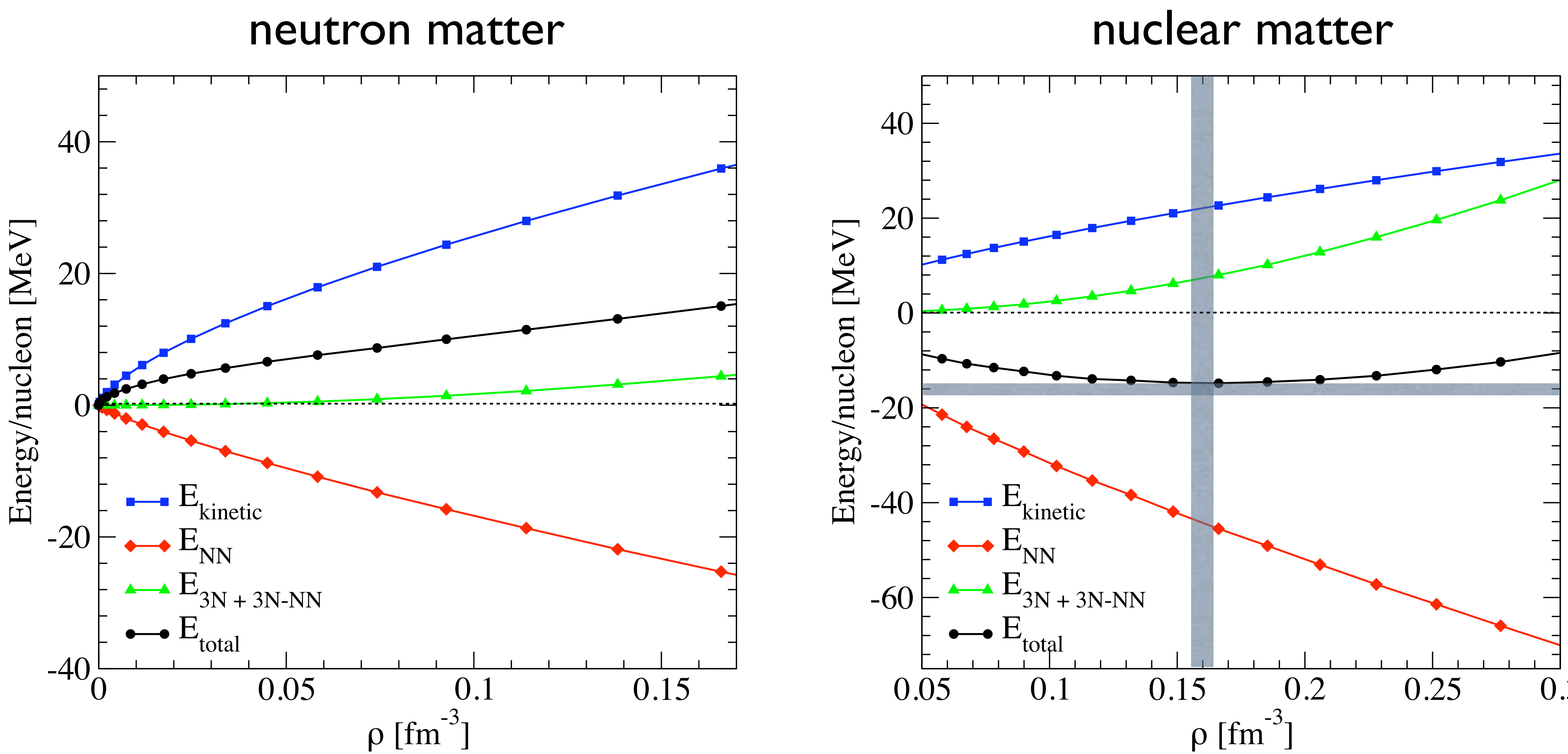

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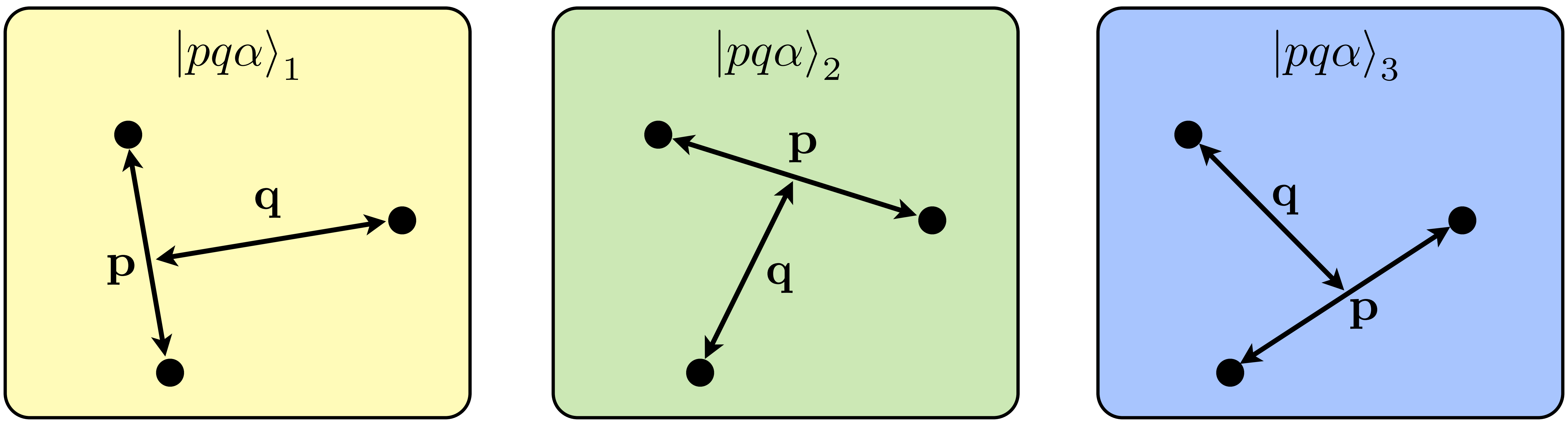
RG evolution of 3N interactions in momentum space

Three-body Faddeev basis:

$$|pq\alpha\rangle_i \equiv |p_i q_i; [(LS)J(ls_i)j]\,\mathcal{J}\mathcal{J}_z(Tt_i)\mathcal{T}\mathcal{T}_z\rangle$$



Faddeev bound state equations:

$$|\psi_i\rangle = G_0\left[2t_iP + (1 + t_iG_0)\,V^i_{3N}(1+2P)\right]|\psi_i\rangle$$

$${}_i\langle pq\alpha|P|p'q'\alpha'\rangle_i = {}_i\langle pq\alpha|p'q'\alpha'\rangle_j$$

SRG flow equations of NN and 3N forces in Faddeev basis

$$\frac{dH_s}{ds} = [\eta_s, H_s] \qquad \eta_s = [T_{\rm rel}, H_s]$$

$$H = T_{\rm rel} + V_{12} + V_{13} + V_{23} + V_{123}$$

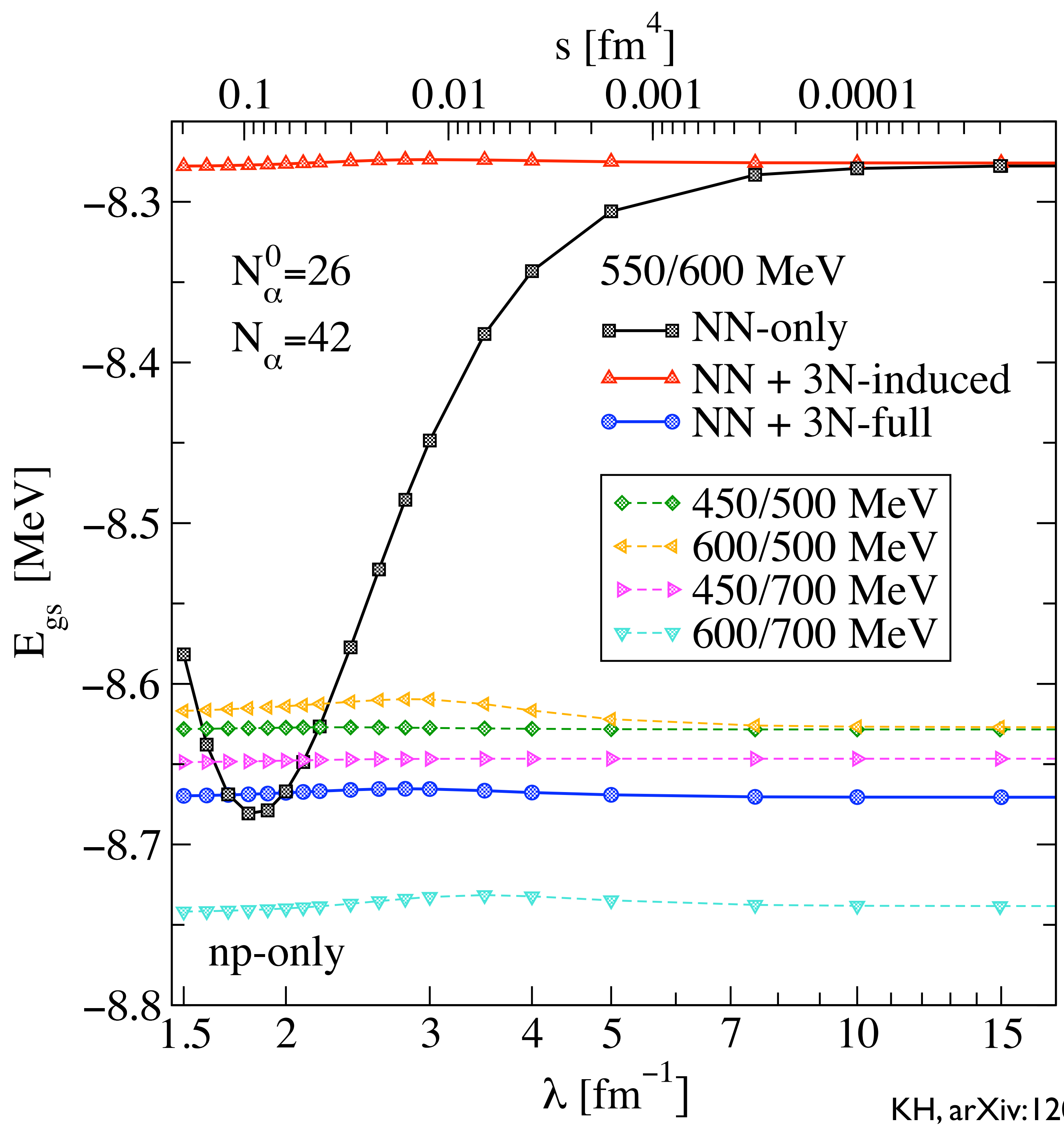
- spectators correspond to delta functions, matrix representation of H_s ill-defined
- solution: explicit separation of NN and 3N flow equations

$$\begin{aligned}
\frac{dV_{ij}}{ds} &= \left[\left[T_{ij}, V_{ij}\right], T_{ij} + V_{ij}\right], \\
\frac{dV_{123}}{ds} &= \left[\left[T_{12}, V_{12}\right], V_{13} + V_{23} + V_{123}\right] \\
&+ \left[\left[T_{13}, V_{13}\right], V_{12} + V_{23} + V_{123}\right] \\
&+ \left[\left[T_{23}, V_{23}\right], V_{12} + V_{13} + V_{123}\right] \\
&+ \left[\left[T_{\rm rel}, V_{123}\right], H_s\right]
\end{aligned}$$

- only connected terms remain in $\frac{dV_{123}}{ds}$,'dangerous' delta functions cancel

see Bogner, Furnstahl, Perry PRC 75, 061001(R) (2007)

RG evolution of 3N interactions in momentum space



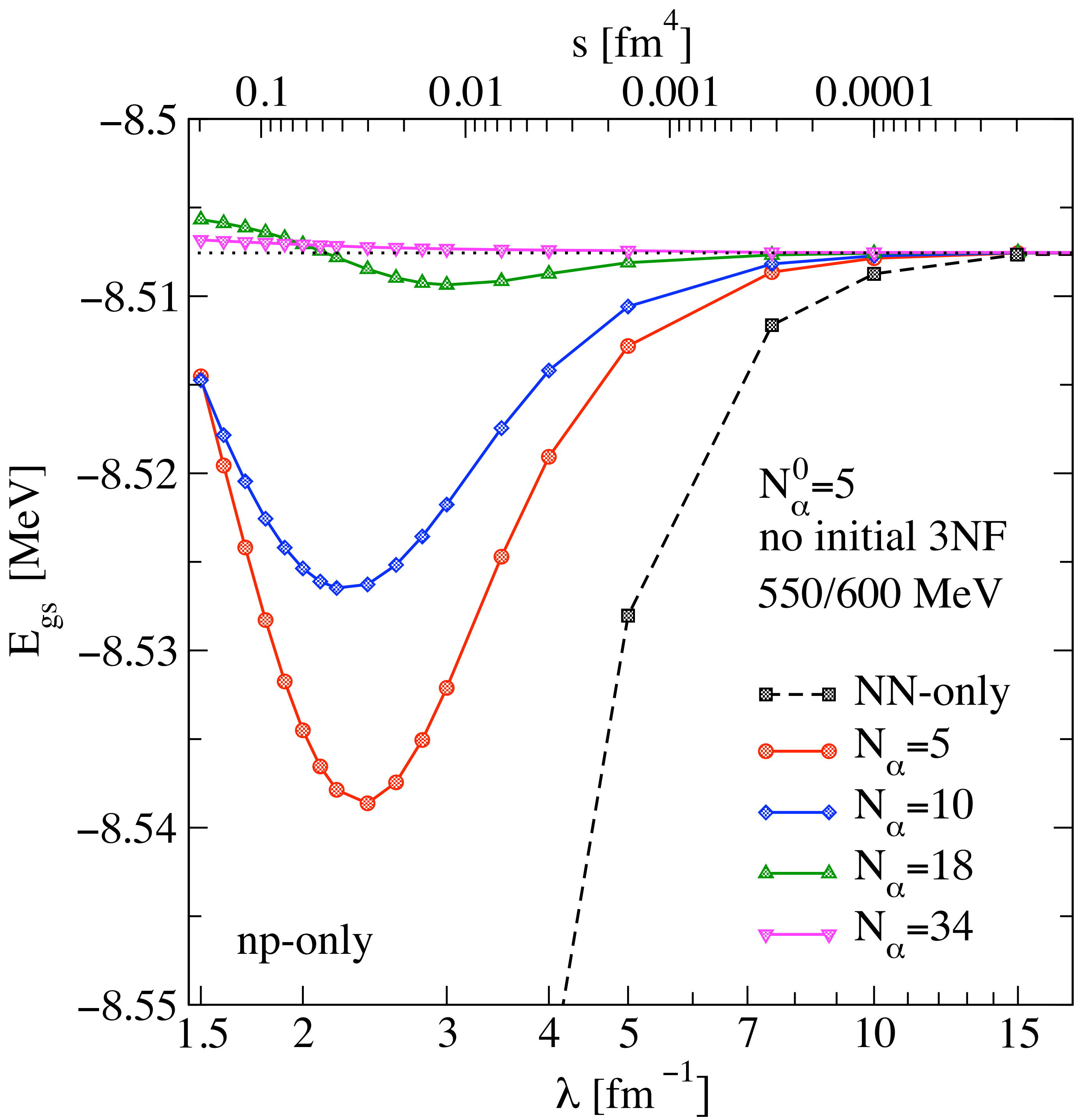
KH, arXiv:1201.0169 (2012)

First implementation:

Invariance of E_{gs}^{3H} within $16\,\mathrm{keV}$ for consistent chiral interactions at $\mathrm{N^2LO}$

Unitarity of SRG evolution

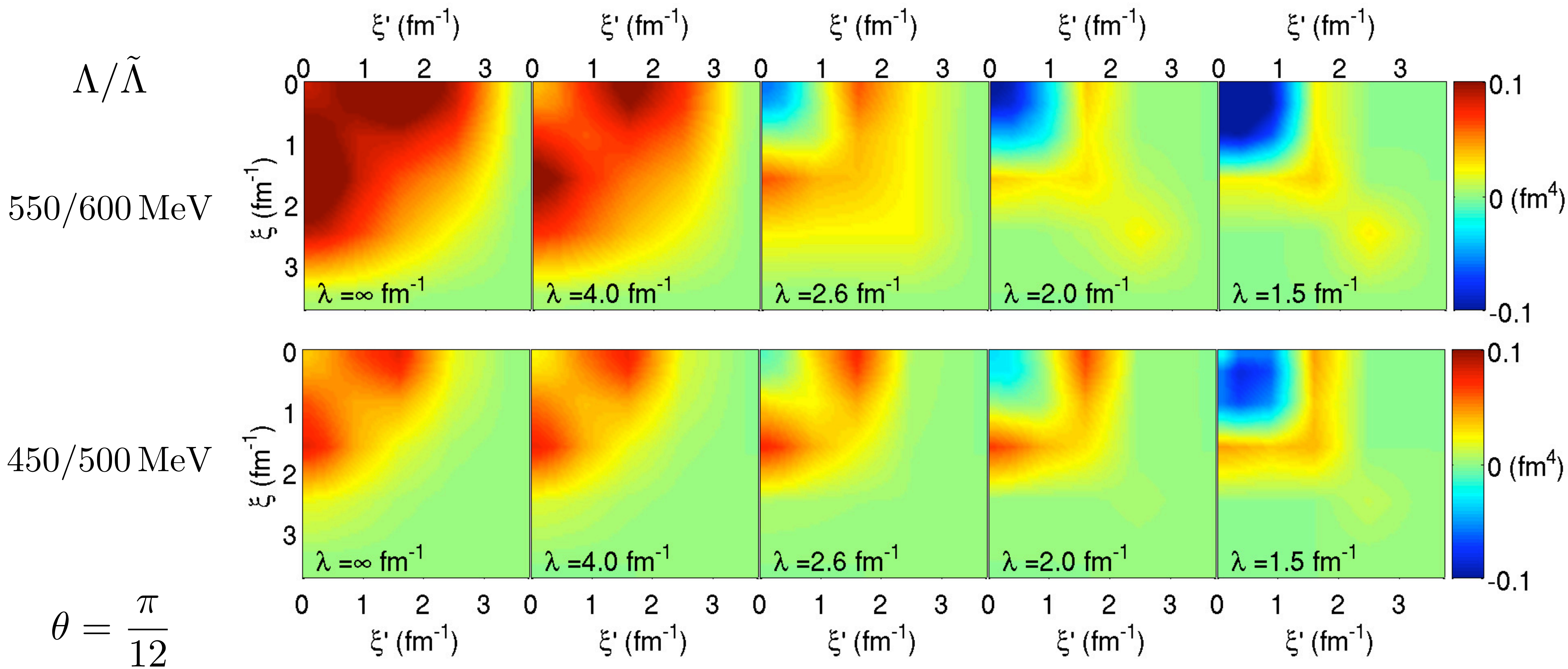
- Faddeev basis not complete under permutation of particles
- embedding of NN forces in 3N basis not exact for bases $V_{12} = P V_{23} P^{-1}, \ldots$



KH, arXiv:1201.0169 (2012)

violation of unitarity can be systematically reduced by increasing the model space

Decoupling of matrix elements



KH, arXiv:1201.0169 (2012)

hyperradius: $\xi^2 = p^2 + \frac{3}{4}q^2$

hyperangle: $\tan\theta = \frac{2\,p}{\sqrt{3}\,q}$

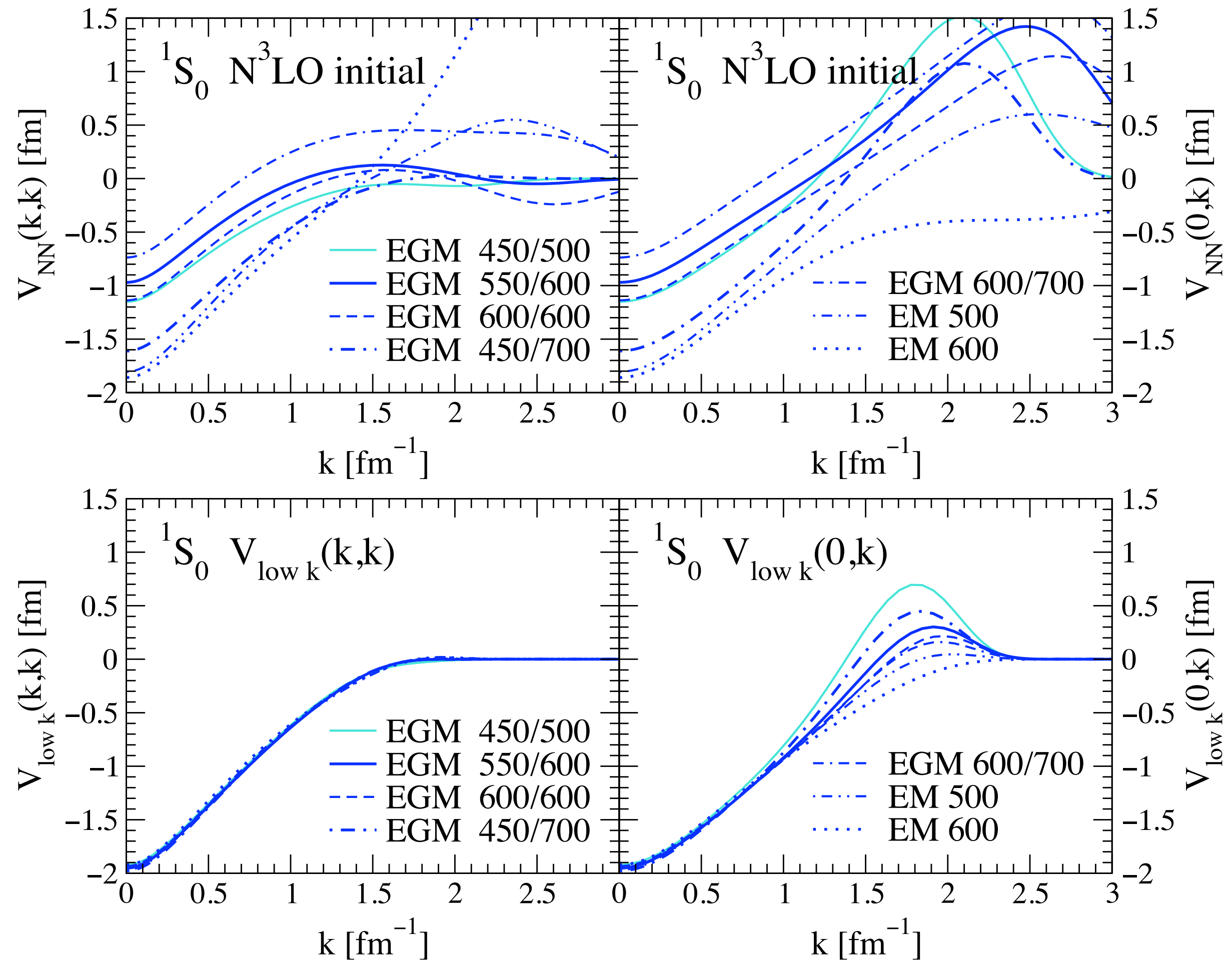
same decoupling patterns like in NN interactions

Universality in 3N interactions at low resolution

phase-shift equivalence

common long-range physics

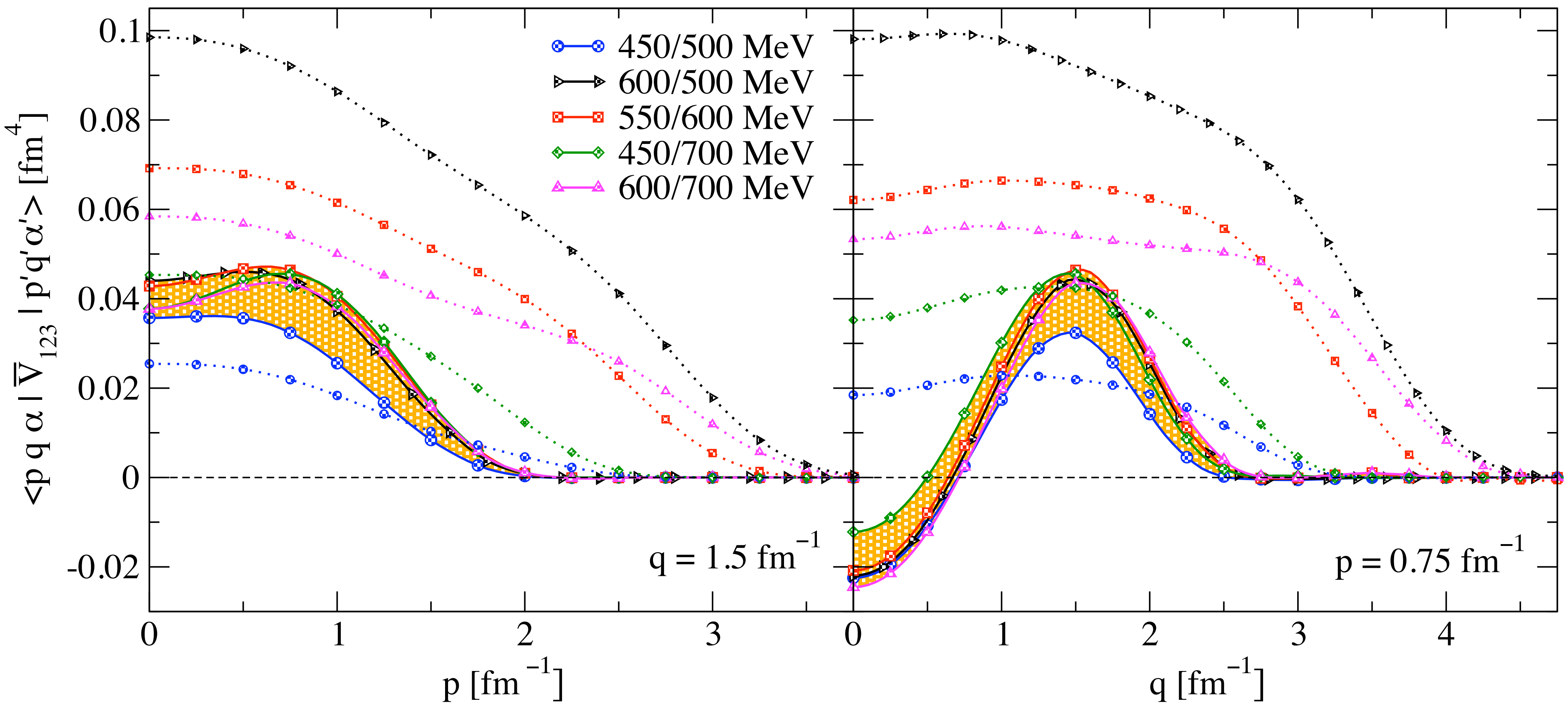
(approximate) universality of low-resolution NN interactions



To what extent are 3N interactions constrained at low resolution?

- only two low-energy constants c_D and c_E
- 3N interactions give only subleading contributions to observables

Universality in 3N interactions at low resolution



KH, arXiv:1201.0169 (2012)

- remarkably reduced model dependence for typical momenta $\sim 1\,\mathrm{fm}^{-1}$,

matrix elements with significant phase space well constrained at low resolution

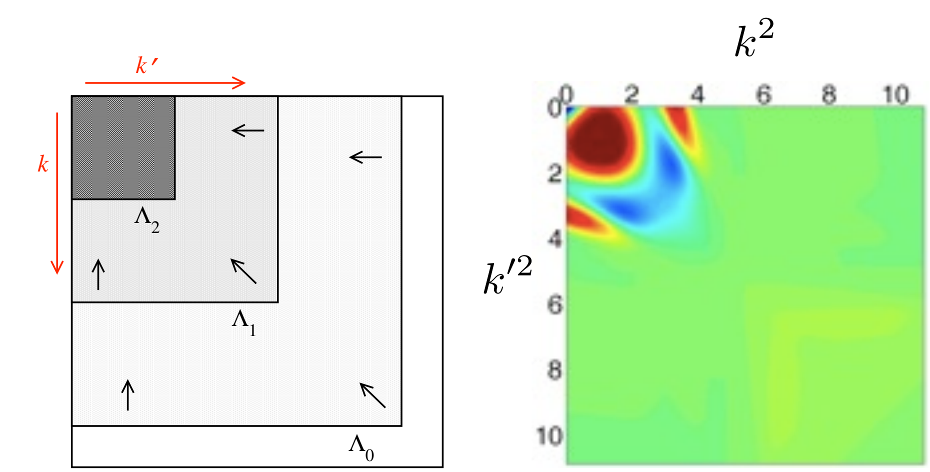
- new momentum structures induced at low resolution
- study based on $\mathrm{N^2LO}$ chiral interactions, improved universality at $\mathrm{N^3LO}$?

Future applications

- application to infinite systems
  - equation of state
  - systematic study of induced many-body contributions

- transformation of evolved interactions to oscillator basis
  - application to finite nuclei, complimentary to HO evolution (no core shell model, coupled cluster)

- study of alternative generators
  - different decoupling patterns (e.g. $V_{\text{low k}}$)
  - improved efficiency of evolution
  - suppression of many-body forces



Anderson et al. , PRC 77, 037001 (2008)

- explicit calculation of unitary 3N transformation
  - RG evolution of operators
  - study of correlations in nuclear systems $\longrightarrow$ factorization