

## Outline

- Nuclear forces from chiral EFT:Overview & achievements
- Beyond the current status
- 3NFs at N3LO
- 3NFs at N4LO
- When does it stop?
- Outlook: How not to get crushed by the Dinosaur?

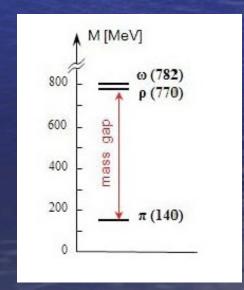
# From QCD to nuclear physics via chiral EFT (in a nutshell)

- QCD at low energy is strong.
- Quarks and gluons are confined into colorless hadrons.

Nuclear forces are residual forces (similar to

van der Waals forces)

Separation of scales



- Calls for an EFT
   soft scale: Q ≈ m<sub>π</sub>, hard scale: Λ<sub>χ</sub> ≈ m<sub>ρ</sub>;
   pions and nucleon relevant d.o.f.
- Low-energy expansion:  $(Q/\Lambda_x)^v$  with v bounded from below.
- Most general Lagrangian consistent with all symmetries of low-energy QCD.
- п-п and п-N perturbatively
- NN has bound states:
  - (i) NN potential perturbatively
  - (ii) apply nonpert. in LS equation.
    (Weinberg)

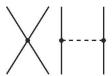
#### 2N forces

**3N forces** 

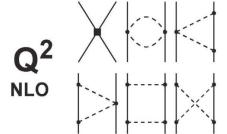
**4N forces** 

Leading Order

Q<sup>0</sup>



Next-to Leading Order

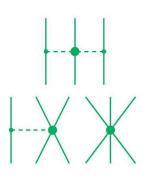


The Hierarchy of Nuclear Forces

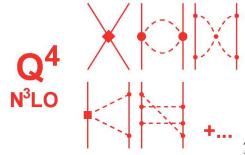
Next-to-Next-to Leading Order

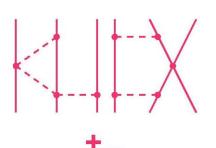
 $Q^3$   $N^2LO$ 

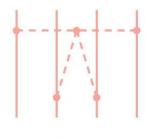




Next-to-Next-to-Next-to Leading Order







5

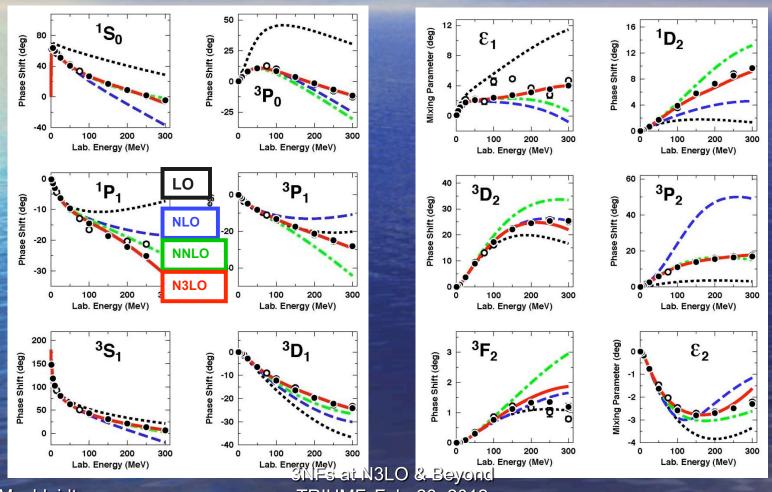
## NN phase shifts up to 300 MeV

Red Line: N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).

Green dash-dotted line: NNLO Potential, and

blue dashed line: NLO Potential

by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).



# $\chi^2/{ m datum}$ for the reproduction of the 1999 np database

Bin (MeV)	# of data	${f N}^3{f L}{f O}$	NNLO	NLO	AV18
0-100	1058	1.05	1.7	4.5	0.95
100 – 190	501	1.08	22	100	1.10
190-290	843	1.15	47	180	1.11
0-290	<b>2402</b>	1.10	20	86	1.04

N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003). NNLO and NLO Potentials by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).



#### **Medium-Mass Nuclei from Chiral Nucleon-Nucleon Interactions**

G. Hagen, <sup>1</sup> T. Papenbrock, <sup>2,1</sup> D. J. Dean, <sup>1</sup> and M. Hjorth-Jensen <sup>3</sup>

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<sup>3</sup>Department of Physics and Center of Mathematics for Applications, University of Oslo, N-0316 Oslo, Norway (Received 20 June 2008; published 29 August 2008)

We compute the binding energies, radii, and densities for selected medium-mass nuclei within coupled-cluster theory and employ a bare chiral nucleon-nucleon interaction at next-to-next-to-next-to-leading order. We find rather well-converged results in model spaces consisting of 15 oscillator shells, and the doubly magic nuclei <sup>40</sup>Ca, <sup>48</sup>Ca, and the exotic <sup>48</sup>Ni are underbound by about 1 MeV per nucleon within the coupled-cluster singles-doubles approximation. The binding-energy difference between the mirror nuclei <sup>48</sup>Ca and <sup>48</sup>Ni is close to theoretical mass table evaluations. Our computation of the one-body density matrices and the corresponding natural orbitals and occupation numbers provides a first step to a microscopic foundation of the nuclear shell model.

Chiral NN potential at N<sup>3</sup>LO underbinds by ~1MeV/nucleon. (Size extensivity at its best.)

Nucleus	ΔE / A [MeV]
<sup>4</sup> He	1.08 (0.73 <sup>FY</sup> )
<sup>16</sup> O	1.25
<sup>40</sup> Ca	0.84
<sup>48</sup> Ca	1.27
<sup>48</sup> Ni	1.21

#### PHYSICAL REVIEW C 82, 034330 (2010)

#### Ab initio coupled-cluster approach to nuclear structure with modern nucleon-nucleon interactions

G. Hagen, <sup>1</sup> T. Papenbrock, <sup>1,2</sup> D. J. Dean, <sup>1</sup> and M. Hjorth-Jensen <sup>3</sup>

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<sup>2</sup>Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA

<sup>3</sup>Department of Physics and Center of Mathematics for Applications, University of Oslo, N-0316 Oslo, Norway (Received 17 May 2010; revised manuscript received 20 August 2010; published 30 September 2010)

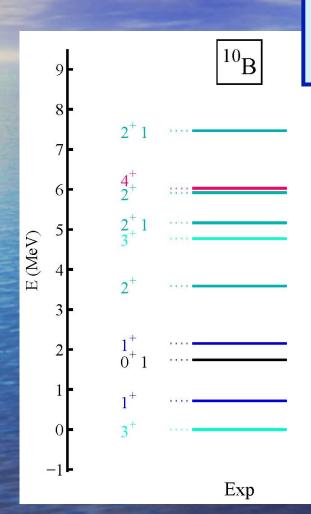
We perform coupled-cluster calculations for the doubly magic nuclei <sup>4</sup>He, <sup>16</sup>O, <sup>40,48</sup>Ca, for neutron-rich isotopes of oxygen and fluorine, and employ "bare" and secondary renormanzed nucleon-nucleon interactions. For the nucleon-nucleon interaction from chiral effective field theory at order next-to-next-to-next-to leading order, we find that the coupled-cluster approximation including triples corrections binds nuclei within 0.4 MeV per nucleon compared to data. We employ interactions from a resolution-scale dependent similarity renormalization group transformations and assess the validity of power counting estimates in medium-mass nuclei. We find that the missing contributions from three-nucleon forces are consistent with these estimates. For the unitary correlator model potential, we find a slow convergence with respect to increasing the size of the model space. For the *G*-matrix approach, we find a weak dependence of ground-state energies on the starting energy combined with a rather slow convergence with respect to increasing model spaces. We also analyze the center-of-mass problem and present a practical and efficient solution.



For medium-mass nuclei, see talks by Papenbrock; Roth et al.; and others.

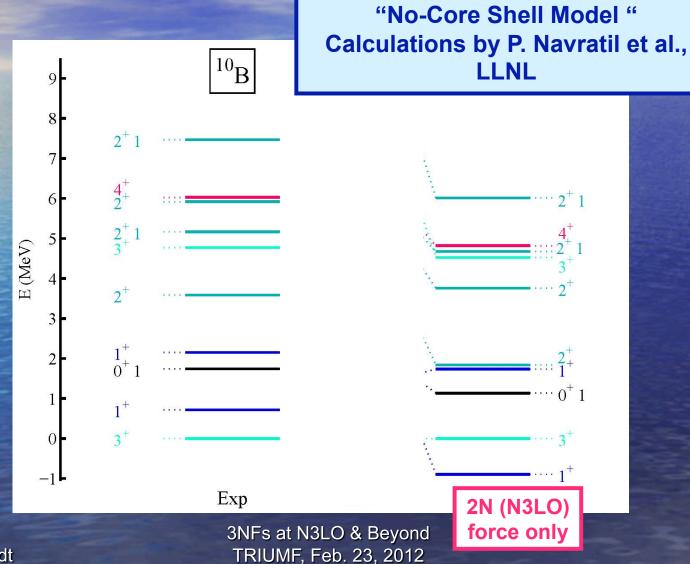
3NFs at N3LO & Beyond TRIUMF, Feb. 23, 2012

## Calculating the properties of light nuclei using chiral 2N and 3N forces

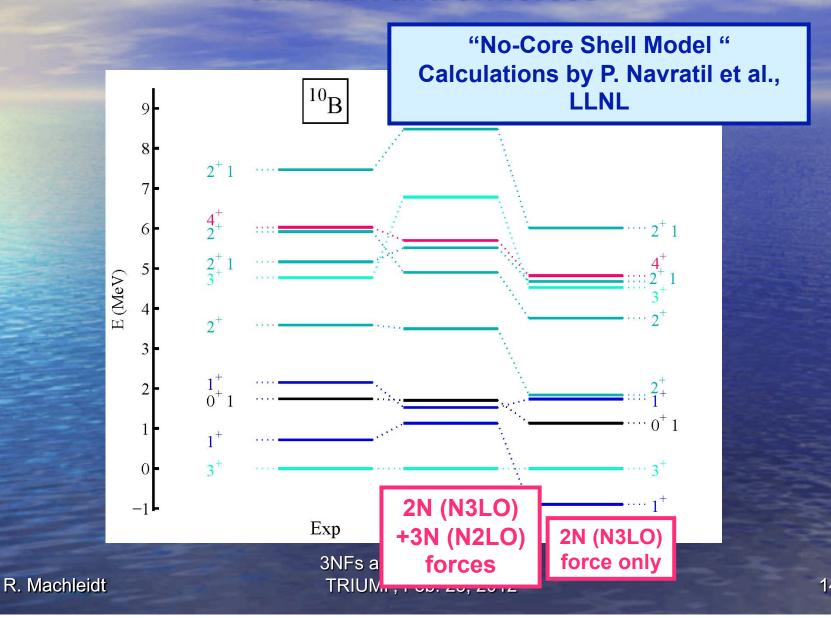


"No-Core Shell Model "
Calculations by P. Navratil et al.,
LLNL

## Calculating the properties of light nuclei using chiral 2N and 3N forces



## Calculating the properties of light nuclei using chiral 2N and 3N forces



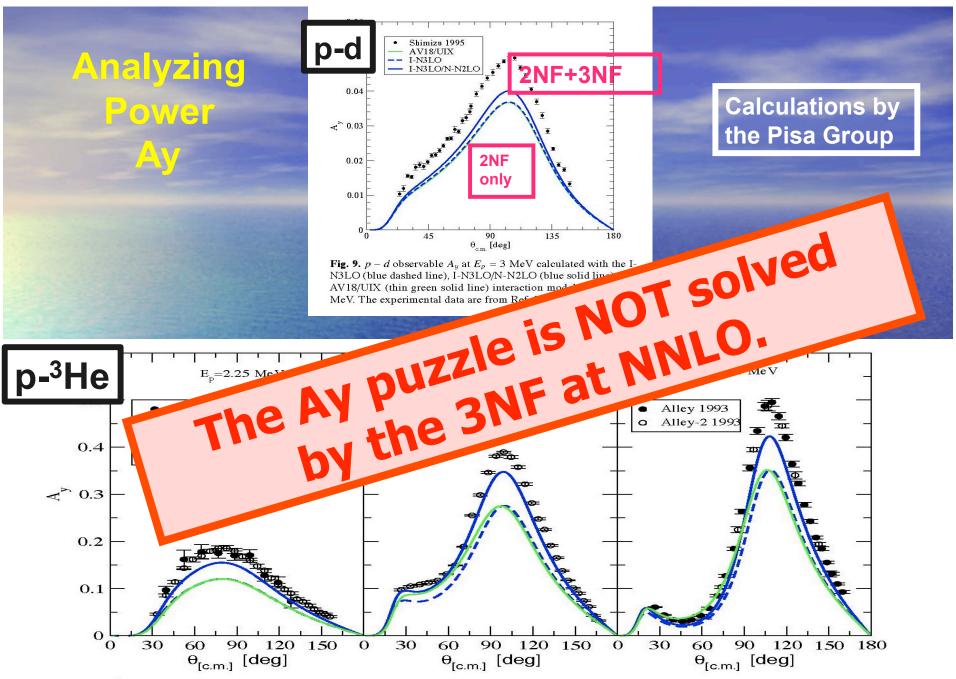
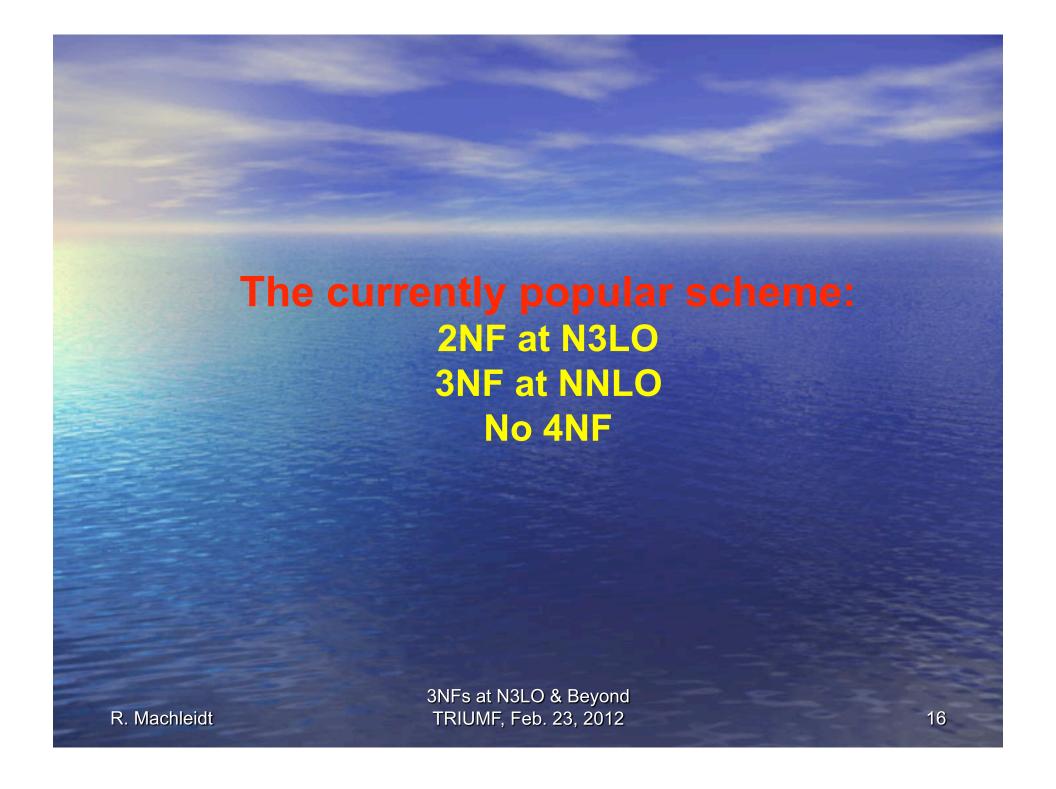


Fig. 6.  $p - {}^{3}$ He  $A_{y}$  observable calculated with the I-N3LO (blue dashed line), the I-N3LO/N-N2LO (blue solid line), and the AV18/UIX (thin green solid line) interaction models for three different incident proton energies. The experimental data are from Refs. [37,22,36].



## Going beyond the current scheme

- The 2NF is N3LO; consistency requires that all contributions are at the same order, so 3NF has to be N3LO, too.
- There are unresolved problems in 3N, 4N scattering and nuclear structure, where additional 3NFs (and 4NFs) may help.



## **Chiral 3N Force** $\Delta$ -less LO $(Q/\Lambda_\chi)^0$ NLO $(Q/\Lambda_\chi)^2$ The 3NF NNLO at NNLO; $(Q/\Lambda_\chi)^3$ used so far. $N^3LO$ $(Q/\Lambda_\chi)^4$ ${f N}^4{f L}{f O}$ $(Q/\Lambda_\chi)^5$ R. Machleidt



## The 3NF at N3LO explicitly

One-loop, leading vertices

#### $2\pi$ -exchange

$$| \phi - \phi - \phi | = | \frac{1}{1 + 1} + | \frac{1}{1 + 1$$

Ishikawa & Robilotta, PRC 76, 014006 (2007)

#### $2\pi$ - $1\pi$ -exchange

$$0 = \frac{1}{2} + \frac{1}{2} +$$

#### ring diagrams

#### contact- $1\pi$ -exchange

#### contact- $2\pi$ -exchange

Bernard, Epelbaum, Krebs, Meissner, PRC 77, 064004 (2008); PRC 84, 054001 (2011). virtual institute "Spin and strong QCD" (grant VH-VI-231). This work was further supported by the DFG (SFB/TR 16 "Submedear Structure of Matter") and by the EU Integrated Infrastructure Initiative Hadron Physics Project under contract number RIB-CT-2004-506078.

#### APPENDIX A: EXPRESSIONS FOR RING DIAGRAMS IN MOMENTUM-SPACE

In this appendix we give lengthy expressions for ring diagrams in Fig. 4 in momentum space. The contributions from

$$\begin{split} V_{\text{sing}} &= \vec{\sigma}_1 \cdot \vec{\sigma}_2 \cdot \tau_2 \cdot \tau_3 \cdot R_1 + \vec{\sigma}_1 \cdot \vec{\phi}_1 \vec{\sigma}_2 \cdot \vec{\phi}_1 \cdot \tau_2 \cdot \tau_3 \cdot R_2 + \vec{\sigma}_1 \cdot \vec{\phi}_1 \vec{\sigma}_2 \cdot \vec{\phi}_1 \cdot \tau_2 \cdot \tau_3 \cdot R_3 + \vec{\sigma}_1 \cdot \vec{\phi}_2 \vec{\sigma}_2 \cdot \vec{\phi}_1 \cdot \tau_2 \cdot \tau_3 \cdot R_4 \\ &+ \vec{\sigma}_1 \cdot \vec{\phi}_2 \vec{\sigma}_2 \cdot \vec{\phi}_1 \cdot \tau_2 \cdot \tau_3 \cdot R_3 + \tau_1 \cdot \tau_3 \vec{\sigma}_3 \cdot \vec{\phi}_1 \cdot \vec{\phi}_1 \cdot \vec{\phi}_1 \cdot \vec{\phi}_1 \cdot \vec{\phi}_2 \cdot \vec{\phi}_2 \cdot \vec{\phi}_1 \cdot \vec{\phi}_2 \cdot \vec{\phi}_1 \cdot \vec{\phi}_2 \cdot \vec{\phi}_1 \cdot \vec{\phi}_2 \cdot \vec{\phi}_2$$

where the functions  $R_i \equiv R_i(q_1,\,q_3,\,z)$  with  $z=\hat{q}_1\cdot\hat{q}_3$  are defined as follows:

 $R_{1} \, = \, \frac{\left(-1+z^{2}\right)g_{A}^{5}M_{\pi}\left(2M_{\pi}^{2}+q_{3}^{2}\right)\left\langle q_{3}^{2}q_{3}+4M_{\pi}^{2}\left(zq_{1}+q_{3}\right)\right.}{128F^{6}\pi\left(4\left(-1+z^{2}\right)M_{\pi}^{2}-q_{2}^{2}\right)\left(4M_{\pi}^{2}q_{3}+q_{3}^{2}\right)} - \frac{A\left(q_{2}\right)g_{A}^{6}q_{2}^{2}\left(2M_{\pi}^{2}\left(q_{1}+zq_{3}\right)+zq_{3}^{2}\right)}{128F^{6}\pi\left(-1+z^{2}\right)q_{3}q_{3}^{2}} - \frac{128F^{6}\pi\left(-1+z^{2}\right)q_{3}q_{3}^{2}}{128F^{6}\pi\left(-1+z^{2}\right)q_{3}q_{3}^{2}} - \frac{128F^{6}\pi\left(-1+z^{2}\right)q_{$ 

 $\begin{array}{ll} A\left(q_{0}\right)g_{A}^{S}\left(2g_{2}^{S}\left(2q_{1}-q_{2}\right)g_{2}-2G_{2}^{S}\left(z\left(2+2^{2}\right)q_{1}^{2}-\left(1+z^{2}\right)q_{1}q_{2}-zq_{2}^{2}\right)\right)\\ A\left(q_{0}\right)g_{A}^{S}\left(2g_{2}^{S}\left(2q_{1}-q_{2}\right)g_{2}^{S}-2G_{2}^{S}\right)\\ 128F^{S}\left(-1+z^{2}\right)q_{1}q_{3}\\ A\left(q_{0}\right)g_{A}^{S}\left(2M_{x}^{2}q_{2}^{2}+q_{3}-zq_{1}^{2}+\left(2-3z^{2}\right)q_{1}^{2}q_{3}-z\left(-2+z^{2}\right)q_{1}q_{2}^{2}+q_{3}^{2}\right)\\ 128F^{S}\left(-1+z^{2}\right)q_{3}^{S}\end{array}\right).$ 

$$\begin{split} &\frac{I(4:0,-q_1,q_2;(0)g_1^4q_2^2}{32F^3(-1+z^2)\left(4(-1+z^2)M_\pi^2-q_2^2\right)q_3}\left(8\left(-1+z^2\right)M_\pi^4\left(2zq_1+\left(1+z^2\right)q_1\right)+q_2^2q_3\left(z^2q_1^2+z^2\right)+q_1^2q_2^2+q_2^2+q_3^2+q$$

 $R_2 \ = \ \frac{A \left(q_2\right) g_A^6 q_2^2 \left(-2 M_\pi^2 \left(\left(1+z^2\right) q_1+2 z q_3\right)+z q_3 \left(\left(1+z^2\right) q_1^2-2 q_3^2\right)\right)}{2 \left(1+z^2\right) q_1^2 + 2 \left(1+z^2\right) q_2^2 + 2 \left(1+z^2\right) q_1^2 + 2 \left(1+z^2\right) q_2^2 + 2 \left(1+z^2\right) q_1^2 + 2 \left(1+z^2\right) q_1^$ 

 $\frac{A\left(q_{3}\right)g_{A}^{6}\left(M_{\pi}^{2}\left(2zq_{1}^{2}+\left(1+3z^{2}\right)q_{1}q_{3}+zq_{3}^{2}\right)+zq_{3}\left(-zq_{1}^{2}-z^{2}q_{1}^{2}q_{3}+zq_{1}q_{3}^{2}+q_{3}^{2}\right)\right)}{64F^{5}\pi\left(-1+z^{2}\right)^{2}q_{1}^{2}q_{3}}+$ 

$$\begin{split} \frac{A(\eta)\,g_A^6}{128\,F^6\pi\,(-1+z^2)^2\,q_1^2q_2^2}\,(2M_{\gamma}^2\,((1+z^2)\,q_1^2+z\,(3+z^2)\,q_1q_2+(1+z^2)\,q_2^2) +\\ q_3\,(-(z+z^2)\,q_1^2+(2-5z^2+z^4)\,q_1^2q_2+(1+z^2)\,q_2q^2+(1+z^2)\,q_2^2) -\\ \frac{I(4:0,-q_1,q_2;0)g_A^6}{32F^6\,(-4(-1+z^2)\,q_1^2+4(1+z^2)\,q_2^2)}\,(q_2^4q_2\,(-2z^2q_1^2+(1+z^2)\,q_2^2) -\\ (g_2^4g_3\,(-2z^2q_1^2+(1+z^2)\,q_2^2+(1+z^2)\,q_2^2+(1+z^2)\,q_2^2) -\\ g_3^4g_3^2\,(-2z^2q_1^2+(1+z^2)\,q_2^2+(1+z^2)\,q_2^2+(1+z^2)\,q_2^2) -\\ g_3^2g_3^2\,(-2z^2q_1^2+(1+z^2)\,q_2^2+(1+z^2)\,q_2^2+(1+z^2)\,q_2^2) -\\ g_3^2g_3^2\,(-2z^2q_1^2+(1+z^2)\,q_2^2$$

$$\begin{split} &8(-1+z)(1+z)M_{\pi}^{\frac{1}{2}}\left(z\left(2+z^{2}\right)q_{1}^{2}+\left(1+2z^{2}\right)^{2}q_{1}^{2}q_{2}+z\left(2+7z^{2}\right)q_{2}q_{2}^{2}+\left(1+2z^{2}\right)q_{3}^{2}\right)+\\ &2M_{\pi}^{2}q_{2}^{2}\left(2zq_{1}^{2}+\left(1-z^{2}+2z^{2}\right)q_{1}^{2}q_{2}^{2}+3y^{2}q_{1}^{2}q_{2}^{2}+z^{4}\right)q_{1}q_{3}^{2}+\left(3+3z^{2}-4z^{4}\right)q_{3}^{2}\right))\\ &-\frac{g_{2}^{2}M_{\pi}\left(2M_{\pi}^{2}+q_{2}^{2}\right)\left(q_{2}^{2}q_{2}+4M_{\pi}^{2}\left(2q_{2}+q_{3}\right)\right)}{122S^{2}w_{2}q_{3}^{2}\left(4(-1+z^{2})M_{\pi}^{2}-q_{3}^{2}\right)\left(4M_{2}q_{3}+q_{3}^{2}\right)}, \end{split}$$

 $R_{5} \; = \; - \frac{z A \left(q_{2}\right) g_{A}^{6} q_{2}^{2} \left(-4 M_{\pi}^{2} \left(q_{1} + z q_{5}\right) + q_{5} \left(2z q_{1}^{2} + \left(-1 + z^{2}\right) q_{1} q_{3} - 2z q_{5}^{2}\right)\right)}{128 F^{6} \pi \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{3}^{3}} \; - \; 0$ 

 $\frac{zA\left(q_{3}\right)g_{3}^{6}}{128F^{6}\pi\left(-1+z^{2}\right)^{2}q_{1}^{2}q_{3}^{2}}\left(M_{\pi}^{2}\left(-2z\left(-3+z^{2}\right)q_{1}^{2}+4\left(1+z^{2}\right)q_{1}q_{3}+4zq_{3}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{2}-2z^{3}q_{1}^{2}q_{3}+\left(1+z^{2}\right)q_{1}q_{3}^{2}+2zq_{3}^{2}\right)\right)-$ 

 $\begin{array}{l} 2z^{2}q_{1}^{2}q_{3}+\left(1+z^{2}\right)q_{1}q_{3}+2zq_{3}^{2}\right)-\\ zA\left(q_{1}\right)g_{A}^{6}\left(2M_{\pi}^{2}\left(2q_{1}^{2}+4zq_{1}q_{3}+\left(1+z^{2}\right)q_{3}^{2}\right)+q_{3}\left(-2zq_{1}^{2}+\left(1-3z^{2}\right)q_{1}^{2}q_{3}+2zq_{1}q_{3}^{2}+\left(1+z^{2}\right)q_{3}^{2}\right)\right)-\\ 12SP^{6}\pi\left[-1+z^{2}\right)^{2}q_{1}q_{3}^{2} \end{array}$ 

 $\frac{I(4:0,-q_1,q_2;0)zg_A^6}{32F^6\left(-1+z^2\right)^2q_1\left(-4\left(-1+z^2\right)M_{\pi}^2+q_2^2\right)q_3^2}\left(q_2^4q_3\left(\left(1+z^2\right)q_1^2+z\left(-1+z^2\right)q_1q_3-\left(1+z^2\right)q_3^2\right)+\frac{1}{2}q_3^2q_3^2+\frac{1}{2}q_3^2+\frac{1}{2}q_$ 

#### The ring diagrams

From: Bernard, Epelbaum, Krebs, Meissner, PRC 77, 064004 (2008)

 $\begin{array}{l} 10z^{2}/q_{1}^{2}q_{3}+3z\left(1+2z^{2}\right)q_{2}q_{3}^{2}+\left(1+2z^{2}\right)q_{3}^{2}\right)+\\ r_{1}^{2}q_{3}-z\left(5+z^{2}\right)q_{1}q_{2}^{2}+\left(-3-3z^{2}+4z^{4}\right)q_{3}^{2}\right))+\\ \frac{\left[sq_{1}+q_{3}\right)}{\frac{1}{3}\left(4M_{1}^{2}+q_{3}^{2}\right)}, \end{array}$ 

 $+2M_{\pi}^{2}\left(2zq_{1}+\left(1+z^{2}\right)q_{3}\right)\right)$ 

 $\frac{v^{4})^{2}q_{1}^{2}q_{3}^{2}}{q_{1}q_{3}+2zq_{3}^{2})+q_{3}\left(2z^{2}q_{1}^{3}+2z^{3}q_{1}^{2}q_{5}+\left(1-4z^{2}+z^{4}\right)q_{1}q_{5}^{2}-2zq_{3}^{2}\right)\right)}{128F^{6}\pi\left(-1+z^{2}\right)^{2}q_{1}q_{3}^{3}}-$ 

 $\left(-3+z^{2}\right)q_{1}^{2}+z\left(3+z^{2}\right)q_{1}q_{3}+\left(1+z^{2}\right)q_{3}^{2}\right)+$ 

 $_{1}^{2}q_{3}+z\left( 1+z^{2}\right) q_{3}q_{3}^{2}+\left( 1+z^{2}\right) q_{3}^{3}\right) \big) - \\$ 

 $\begin{array}{l} \frac{7}{2^{2}+q_{2}^{2})q_{3}^{2}}\left(q_{2}^{4}q_{3}\left(\left(z+z^{3}\right)q_{1}^{2}+\left(-1+z^{2}\right)^{2}q_{1}q_{3}-2zq_{3}^{2}\right)+\right.\\ \left.q_{3}^{2}\left(-2+9z^{2}+2z^{4}\right)q_{1}q_{3}^{2}+z\left(2+z^{2}\right)q_{3}^{2}\right)+\right.\\ \left.q_{1}^{2}q_{3}+\left(4+5z^{2}\left(-3+z^{2}\right)q_{1}q_{3}^{2}+2z\left(-3+z^{2}+z^{4}\right)q_{3}^{2}\right)\right)+\right. \end{array}$ 

 $q_1^2q_3 + (4 + 5z^2(-3 + z^2))q_1q_3^2 + 2z(-3 + z^2 + q_3) \over 4M^2 + a^2)$ ,

 $\frac{2zq_1^2 + \left(-1 + z^2\right)q_1q_3 - 2zq_3^2))}{z^2)^2q_1q_3^4} -$ 

 $3+z^{2}\right) q_{1}^{2}-2 \left(1+z^{2}\right) q_{1} q_{3}-2 z q_{3}^{2})+q_{3} \left(\left(1+z^{2}\right) q_{1}^{3}+2 z^{3} q_{1}^{2} q_{3}-\right.$ 

 $\frac{-z^{2})\,q_{3}^{2})+q_{3}\left(-2zq_{1}^{3}+\left(1-3z^{2}\right)q_{1}^{2}q_{3}+2zq_{1}q_{3}^{2}+\left(1+z^{2}\right)q_{3}^{2}\right))}{128F^{6}\pi\left(-1+z^{2}\right)^{2}q_{3}^{4}}+\\$ 

$$\begin{split} & + \frac{1}{(q_2^2)\,q_3^2}\left(g_2^4g_3\left(\left(1+z^2\right)g_1^2 + z\left(-1+z^2\right)g_1g_3 - \left(1+z^2\right)g_2^2\right) + \\ & + \frac{1}{(q_2^2)}g_1^2g_3 + 3z\left(1+2z^2\right)g_1g_3^2 + \left(1+2z^2\right)g_1^2\right) + \\ & + \frac{2}{(q_3^2)}g_3^2 + \left(3+2z^2\right)g_2^2 + \left(-3-3z^2+4z^4\right)g_3^2\right) - \end{split}$$

 $\frac{q_1 + q_3)}{4M_\pi^2 + q_3^2}$ ,  $\frac{q_1 + q_3}{4M_\pi^2 + q_3^2}$ ,  $\frac{q_1 + q_3}{4M_\pi^2 + q_3^2}$ ,  $\frac{q_1 + q_2}{4M_\pi^2 + q_3^2 + q_3^2}$ ,

 $\frac{!z\left(M_{\pi}^{2}+q_{1}^{2}\right)q_{3}+q_{1}\left(8M_{\pi}^{2}+3q_{1}^{2}+q_{3}^{2}\right))}{128F^{6}\pi q_{1}}+\\ \frac{z}{z}+q_{1}^{2}+3q_{3}^{2}\right))_{-}$ 

$$\begin{split} & \overline{q_{\pi}^2 - q_{2}^2}) q_{3} \left(4M_{\pi}^2 + q_{3}^2\right) \left(\left(5 + z^2\right) q_{1}^2 q_{2}^2 q_{3}^2 + 8M_{\pi}^6 \left(z \left(-3 + 4z^2\right) q_{1}^2 + 4z^2\right) q_{2}^2 + \\ & [] + 2M_{\pi}^4 \left(4z \left(-1 + z^2\right) q_{1}^4 + \left(77 - 36z^2\right) q_{3}^2 q_{3} + 2z \left(33 + 8z^2\right) q_{3}^2 q_{3}^2 + \\ & + 2M_{\pi}^2 q_{3} q_{3} \left(\left(10 + z^2\right) q_{1}^4 + 2z \left(9 + 2z^2\right) q_{1}^2 q_{3} + \left(29 - 7z^2\right) q_{1}^2 q_{3}^2 + \end{split}$$

 $\frac{q_1 \sqrt{-s} \left(-s + s + j \cdot m_3 + q_2 / q_3}{q_1 q_2^2 q_3^2 + 6q_1 q_2 + q_1^2\right) + 4 M_s^4 \left(sq_1^2 - 2 \left(-2 + s^2\right) q_1 q_1 + sq_1^2\right) + 2 M_s^2 \left(4 q_1 q_1 \left(q_1^2 + q_1^2\right) + z \left(q_1^2 + q_1^2 q_2^2 + q_2^2\right)\right)\right)},$ 

 $\frac{3A\left(q_{2}\right)g_{A}^{2}\left(2M_{\pi}^{2}+q_{2}^{2}\right)\left(\left(1+z^{2}\right)q_{1}+2zq_{3}\right)}{256F^{6}\pi\left(-1+z^{2}\right)^{2}q_{1}^{2}} \\ +z^{2}\left(q_{3}^{2}\right) \\ +3A\left(q_{3}\right)g_{A}^{2}\left(2M_{\pi}^{2}+q_{2}^{2}\right)\left(2zq_{1}^{2}+\left(1+3z^{2}\right)q_{1}q_{3}+2zq_{3}^{2}\right)}$ 

 $\begin{array}{l} +\frac{256F^6\pi\left(-1+z^2\right)^2q_1^3q_3}{2[e^2-q_2^2]\left(-q_2^2\left(\left(1+z^2\right)q_1^2+z\left(3+z^2\right)q_1q_3+\left(1+z^2\right)q_3^2\right)+\\ :\left(2+z^2\right)q_1q_3+\left(1+z^2\right)q_3^2\right)\right),} \end{array}$ 

 $\frac{3A\langle q_1\rangle g_A^2 \left(2M_q^2+q_1^2\right) \left((1+z^2)q_1+2zq_1\right)}{226E^{q_1}(-1+z^2)^2q_1^2q_2} + \\ \frac{1+z^2}{2}q_1\right) - \frac{3zA\langle q_2\rangle g_A^2 \left(2M_q^2+q_1^2\right) \left(2zq_1^2+\left(1+3z^2\right)q_1q_2+2zq_2^2\right)}{226E^{q_2}(-1+z^2)^2q_1^2q_2^2} - \\ \frac{226E^{q_1}}{2}\left(2zq_1^2+q_1^2+q_2^2+q_2^2+q_1^2+q_2^2+$ 

 $\begin{array}{l} \frac{+q_{2}^{2}}{\left(\frac{q}{s}-q_{2}^{2}\right)q_{3}}\left(-q_{2}^{2}\left(\left(1+z^{2}\right)q_{1}^{2}+z\left(3+z^{2}\right)q_{1}q_{3}+\left(1+z^{2}\right)q_{3}^{2}\right)+\right.\\ \left.\left.\left(2+z^{2}\right)q_{1}q_{3}+\left(1+2z^{2}\right)q_{3}^{2}\right)\right),\end{array}$ 

 $\frac{q_1^2 + z(3 + z^2)q_1q_3 + (1 + z^2)q_3^2)}{q_1^2 + z(3 + z^2)q_1q_3 + (1 + z^2)q_3^2)} +$ 

 $\frac{q_1}{(1+z^2)^2}\frac{q_1^2q_3^2}{q_1^2q_3^2} + \frac{1}{(1+z^2)^2}\frac{q_1^2q_3^2}{q_1^2q_3^2} + \frac{2^2}{(2+z^2)^2}\frac{q_1q_3^2}{q_1q_3^2} + 2M_{\pi}^2\left(\left(1+z^2\right)q_1 + 2zq_3\right)\right)}{256F^6\pi\left(-1+z^2\right)^2q_1q_3^2} +$ 

 $\begin{array}{l} \omega o u \tau n \left(-1+z^2\right) \cdot \eta_1 q_3^2 \\ \vdots g_3 + 2z \left(2+z^2\right) \cdot \eta_1 q_3^2 + \left(1+z^2\right) \cdot q_3^3 + 2M_{\pi}^2 \left(2zq_1 + \left(1+z^2\right)q_3\right)\right) \\ -256 F_{01}^2 \left(-1+z^2\right)^2 q_1^2 q_3^2 \\ \vdots + q_2^2\right) & \qquad (24 \cdot q_1 \cdot 2z + c \cdot (z+z^2) - z - q_2 \cdot 2z\right) \\ \end{array}$ 

$$\begin{split} & \frac{(+ \cdot q_2^2)}{M_\pi^2 + q_2^2) \, q_3} \left( g_2^2 \left( - 2 g_1^2 + z \left( -5 + z^2 \right) g_1 q_3 - 2 g_3^2 \right) + \right. \\ & \left. \left. \left. \left( g_2^2 + g_3^2 + g_3^2 \right) - \frac{3 z g_A^2 M_\pi \left( 2 M_\pi^2 + g_2^2 \right)}{256 F^6 \pi q_1 \left( -4 \left( -1 + z^2 \right) M_\pi^2 + g_2^2 \right) q_3} \right. \right) \end{split}$$

 $-\frac{3A\left(q_{2}\right)g_{A}^{6}\left(2M_{\pi}^{2}+g_{2}^{2}\right)\left(zq_{1}+q_{3}\right)\left(q_{1}+zq_{3}\right)}{256P^{6}\pi\left(-1+z^{2}\right)q_{1}q_{3}}-\\ +z\left(-4+z^{2}\right)q_{1}q_{3}^{2}+g_{3}^{3}+2M_{\pi}^{2}\left(zq_{1}+q_{3}\right)\right)$ 

 $\frac{\pi \left(-1+x^2\right) q_3}{+\left(1+2z^2\right) q_1 q_3^2+z q_3^3+2 M_{\pi}^2 \left(q_1+z q_3\right)\right)} + \\ \frac{\pi \left(-1+z^2\right) q_1}{z_1^2} + \\$ 

 $\begin{array}{l} \frac{2}{1} \\ -\frac{2}{q_{2}^{2}} \left( -q_{2}^{2} \left( q_{1}^{2}-z \left( -3+z^{2} \right) q_{1}q_{3}+q_{3}^{2} \right) + \right. \\ \left. q_{1}q_{3}+\left( 1+z^{2} \right) q_{3}^{2} \right) \right) \, , \end{array}$ 

 $\frac{iq_1q_3 + 2q_5^2) + q_3\left(zq_1^3 + \left(-1 + 2z^2\right)q_1^2q_3 + zq_1q_2^2 + q_3^3\right)\right)}{256F^6\pi\left(-1 + z^2\right)q_1^2q_3^2} +$ 

 $\frac{zA\left(q_{2}\right)g_{A}^{4}\left(\left(1+z^{2}\right)g_{1}^{2}+z\left(3+z^{2}\right)q_{1}q_{3}+\left(1+z^{2}\right)g_{3}^{2}\right)}{128F^{6}\pi\left(-1+z^{2}\right)^{2}q_{1}q_{3}}$ 

 $\frac{A\left(q_{1}\right)g_{A}^{5}\left(2M_{\pi}^{2}\left(2q_{1}^{2}+2zq_{1}q_{5}+\left(-1+z^{2}\right)q_{3}^{2}\right)+q_{1}\left(q_{1}^{2}+zq_{1}^{2}q_{5}+\left(-1+2z^{2}\right)q_{1}q_{3}^{2}+zq_{3}^{2}\right)\right)}{256F^{2}\pi\left(-1+z^{2}\right)q_{2}^{2}q_{5}^{2}}$ 

 $\frac{I(4:0,-q_1,q_2;0)g_{s,q}^2}{(64F^6\,(-1+z^2)\,q_1^2\,(-4\,(-1+z^2)\,M_\pi^2+q_2^2)\,q_3^2)} - (-(2M_\pi^2+q_1^2)\,(2M_\pi^2+q_3^2)\,(4M_\pi^2+q_1^2+q_2^2) + \\ 2z^3q_1q_2\,(-4M_\pi^4+q_1^2q_2^2) + z^2\,(4M_\pi^2+q_1^2+q_2^2)\,(4M_\pi^4+3q_1^2q_2^2+2M_\pi^2\,(q_1^2+q_2^2)) + \\$ 

$$\begin{split} & 13 \\ \hline & : + q_{1}^{2} \left( -2z^{2} \left( 4M_{\pi}^{2}q_{1}^{2} \left( 4M_{\pi}^{2} + q_{1}^{2} \right) + \right. \\ & + q_{1}^{2} + q_{1}^{2} \right) \left( q_{1}^{2}q_{1}^{2} + 2M_{\pi}^{2} \left( q_{1}^{2} + q_{1}^{2} \right) \right) - \\ & \left. \left( q_{1}^{2} + 4q_{1}^{2}q_{2}^{2} + q_{1}^{2} \right) \right) \right). \end{split} \tag{A.2}$$
 at the magnitudes of the corresponding three-momenta by Further, the function  $I(t^{2}, p_{1}, p_{2}, p_{2}, p_{3})$  references

; |. Further, the function  $I(d:p_1,p_2,p_3;p_4)$  refers to  $\frac{1}{-M_\pi^2+i\epsilon} \frac{1}{(l+p_3)^2-M_\pi^2+i\epsilon} \frac{1}{v \cdot (l+p_4)+i\epsilon} \cdot (A.3)$ 

 $y_i$ . For the case  $p_i^0 = 0$  which we are interested in, it is an space  $J(d : \vec{p}_1, \vec{p}_2, \vec{p}_3)$  $\frac{1}{(\vec{l} + \vec{p}_3)^2 + M_Z^2} \frac{1}{(\vec{l} + \vec{p}_3)^2 + M_Z^2}.$  (A.4)

xpressions for  $R_1$  can be written as  $3: \vec{0}, -\vec{q_1}, \vec{q_3} \Big) \, . \tag{A}.$ 

 $_{i}\vec{\sigma}_{3} \cdot \vec{q}_{1} \cdot \tau_{1} \cdot \tau_{2} \cdot S_{3} + \vec{\sigma}_{1} \cdot \vec{q}_{1}\vec{\sigma}_{3} \cdot \vec{q}_{3} \cdot \tau_{1} \cdot \tau_{2} \cdot S_{4}$  $_{i}\vec{q}_{1} \cdot \vec{q}_{3} \times \vec{\sigma}_{1} \cdot \tau_{1} \cdot \tau_{2} \times \tau_{3} \cdot S_{7}$ , (2)

 $\frac{g_3^2)}{128 F^6 \pi} - \frac{A \left( q_3 \right) g_A^4 \left( 2 M_\pi^2 + g_3^2 \right)}{128 F^6 \pi} +$ 

 $\frac{+z^2) q_3)}{r^2 q_1^2} + \frac{3z^2) q_1 q_3 + 2z q_3^2}{r^2}$ 

 $\begin{aligned} &\frac{^{2})\left( q_{3}\right) }{i_{3}}+\frac{zA\left( q_{2}\right) g_{A}^{A}\left( -2q_{1}^{2}+z\left( -5+z^{2}\right) q_{1}q_{3}-2q_{3}^{2}\right) }{128P^{6}\pi\left( -1+z^{2}\right) ^{2}q_{1}q_{3}} \\ &\cdot \left( 3+z^{2}\right) q_{1}q_{3}+\left( 1+z^{2}\right) q_{3}^{2} \end{aligned}.$ 

 $\frac{z^2(q_3)}{q_1} + \frac{z^2(q_3)}{q_1q_3 - 2zq_3^2}$ 

 $\frac{1}{(\vec{l} + \vec{p_1})^d} \frac{1}{(\vec{l} + \vec{p_1})^2 + M_{\pi}^2} \frac{1}{(\vec{l} + \vec{p_2})^2 + M_{\pi}^2}$ 

 $\frac{q_A^4 \left(2zq_1 + (1+z^2) q_3\right)}{4}$ 

 $+ z^{2}) q_{5}^{2})$ 

 $+2zq_1^2 + (1 + 3z^2) q_1q_3 + 2zq_3^2$ 

 $\frac{(-3+z^2)q_1q_3+q_3^2}{(-1+z^2)} - \frac{A(q_1)g_A^4q_1(q_1+zq_3)}{128F^6\pi(-1+z^2)} +$ 

 $\frac{+\,q_{3})+2M_{\pi}^{2}\,(q_{1}+zq_{3}))}{-1+z^{2})\,q_{1}q_{3}^{2}}+\frac{zA\,(q_{3})\,g_{A}^{4}\,\left(2M_{\pi}^{2}+q_{3}^{2}\right)}{256F^{6}\pi\,\left(-1+z^{2}\right)\,q_{1}q_{3}}.$ 

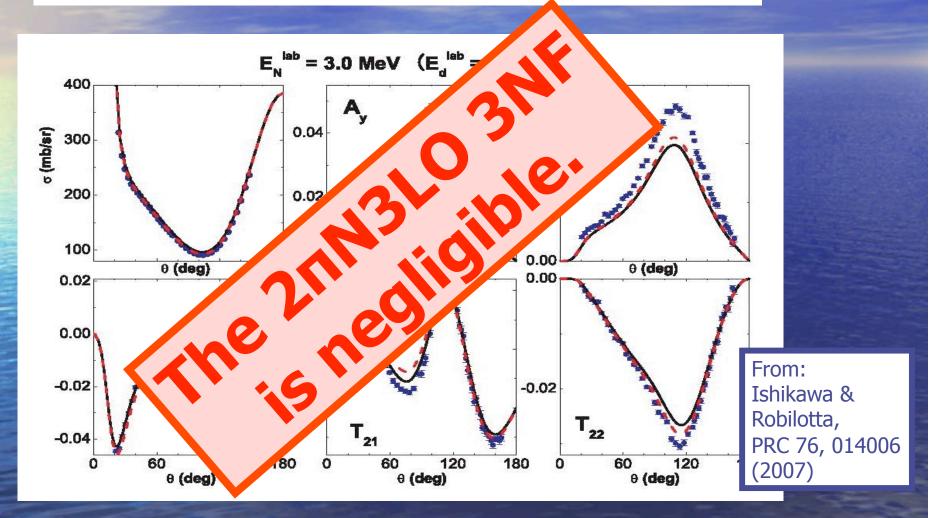
N. Kalantar-Nayestanaki and E. Epelbaum, Nucl. Phys. News 17, 22 (2007) [arXiv:nucl-th/0703089]
 D. R. Entem and R. Machleidt, Phys. Rev. C 68, 041001 (2003) [arXiv:nucl-th/9204018].

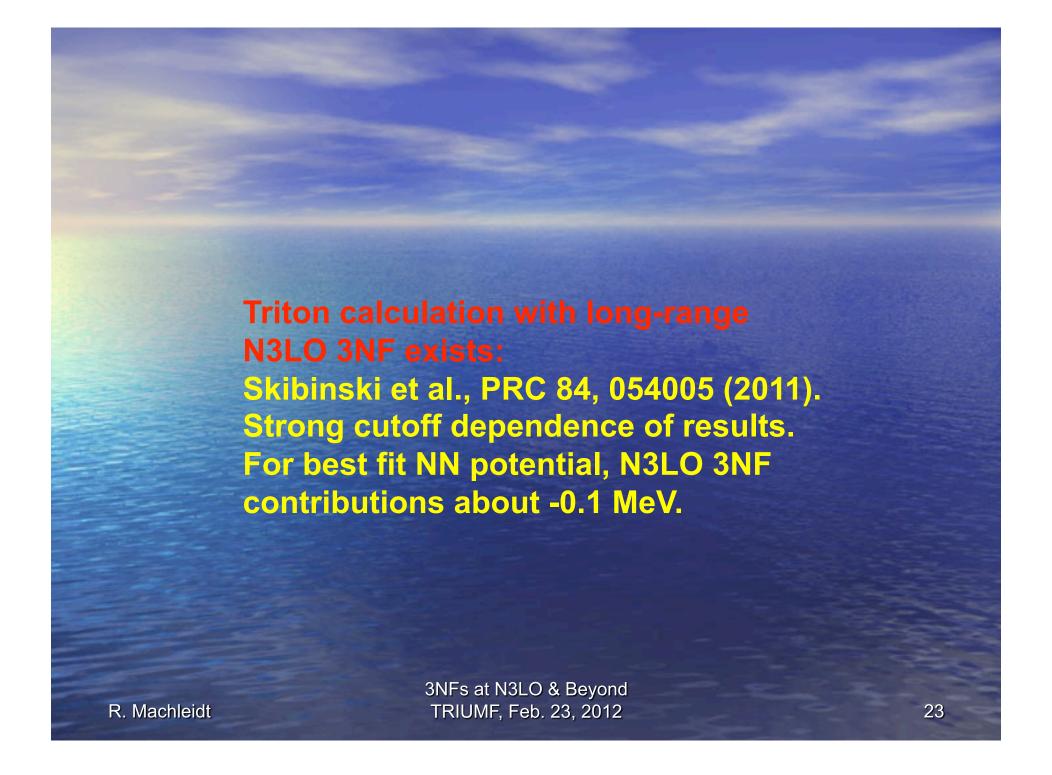
3NFs at N3LO & Beyond TRIUMF, Feb. 23, 2012

### **Proton-deuteron elastic scattering**

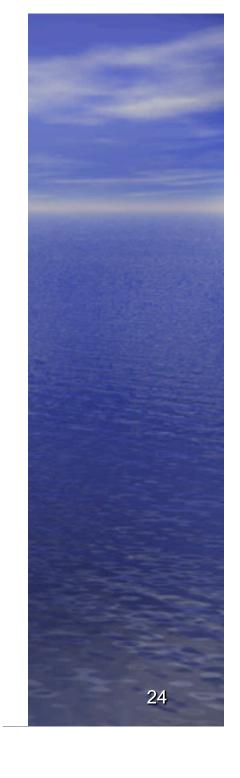
Black line: 2NF only

Red dashed line:  $2NF + 3NF(N2LO + 2\pi N3LO) \approx 2NF + 3NF(N2LO)$ 





## **Chiral 3N Force** $\Delta$ -less LO $(Q/\Lambda_\chi)^0$ NLO $(Q/\Lambda_\chi)^2$ The 3NF NNLO at NNLO; $(Q/\Lambda_\chi)^3$ used so far. Small? $N^3LO$ $(Q/\Lambda_\chi)^4$ $N^4LO$ $(Q/\Lambda_\chi)^5$ R. Machleidt



## **Chiral 3N Force** $\Delta$ -less LO $(Q/\Lambda_\chi)^0$ NLO $(Q/\Lambda_\chi)^2$ The 3NF NNLO at NNLO; $(Q/\Lambda_\chi)^3$ used so far. Small? $N^3LO$ $(Q/\Lambda_\chi)^4$ $N^4LO$ $(Q/\Lambda_\chi)^5$ R. Machleidt



# What to expect from those N4LO 1-loop diagrams?

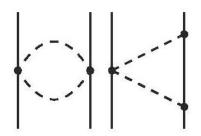
Compare to "similar" 2NF diagrams.

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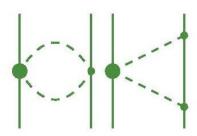
### **Corresponding 2NF contributions**

2п

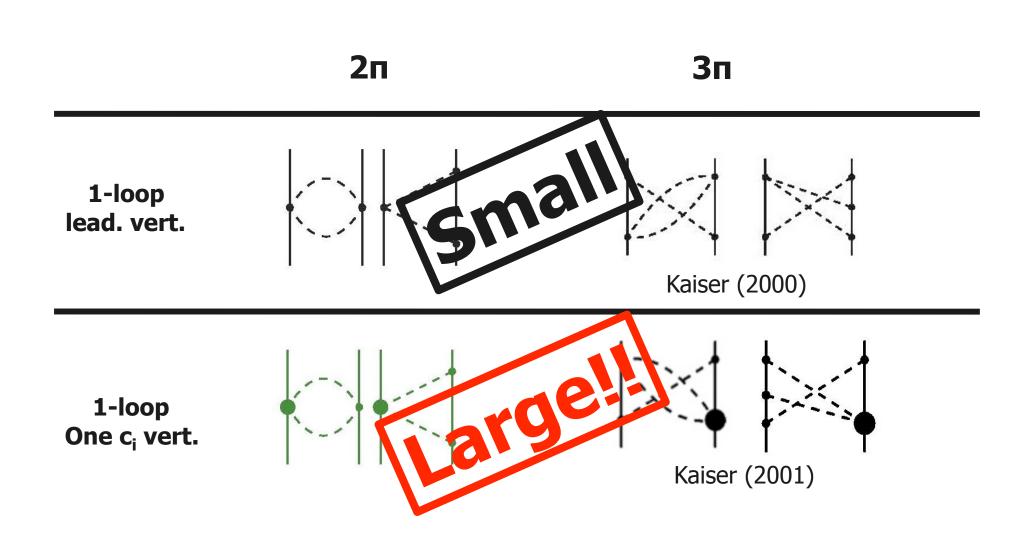
1-loop lead. vert.



1-loop One c<sub>i</sub> vert.

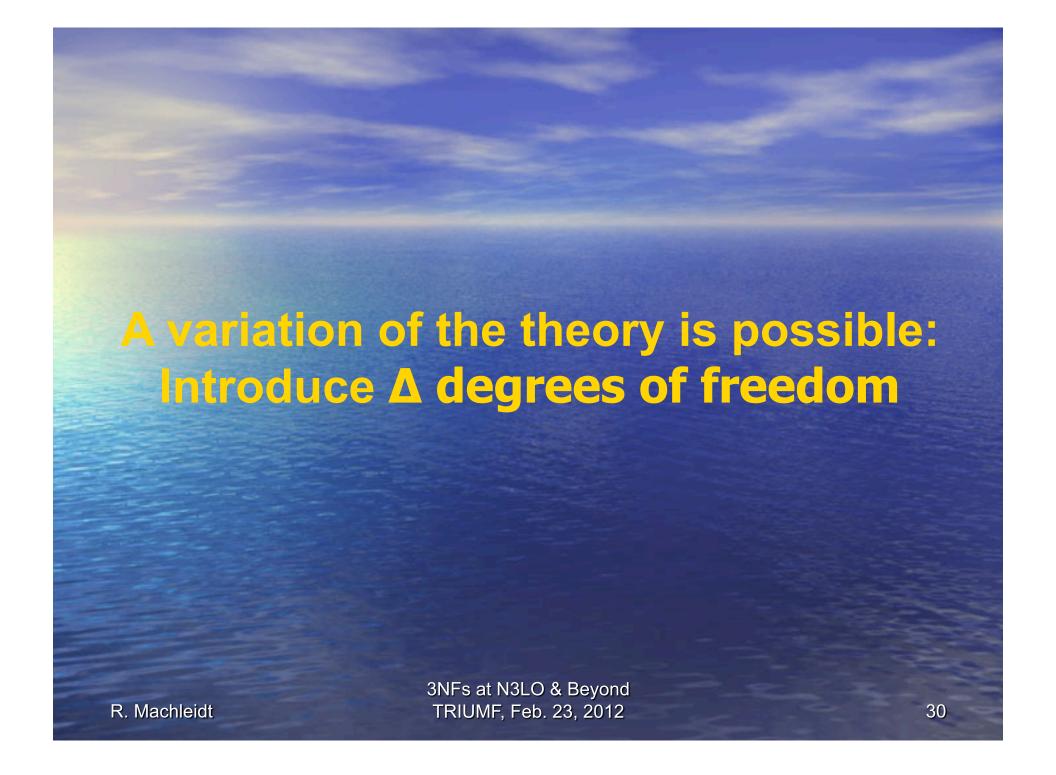


### **Corresponding 2NF contributions**



## **Chiral 3N Force** $\Delta$ -less LO $(Q/\Lambda_\chi)^0$ NLO $(Q/\Lambda_\chi)^2$ The 3NF NNLO at NNLO; $(Q/\Lambda_\chi)^3$ used so far. Small? $N^3LO$ $(Q/\Lambda_\chi)^4$ $N^4LO$ Large!! $(Q/\Lambda_\chi)^5$ R. Machleidt



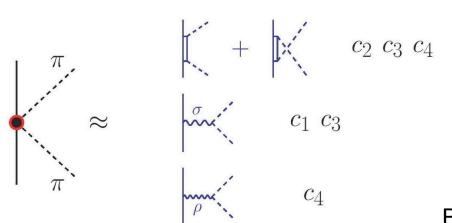


## **Chiral 3N Force** $\Delta$ -less LO $(Q/\Lambda_\chi)^0$ NLO $(Q/\Lambda_\chi)^2$ The 3NF at NNLO; used so far. NNLO $(Q/\Lambda_\chi)^3$ Small? $N^3LO$ $(Q/\Lambda_\chi)^4$ $N^4LO$ $(Q/\Lambda_\chi)^5$ R. Machleidt



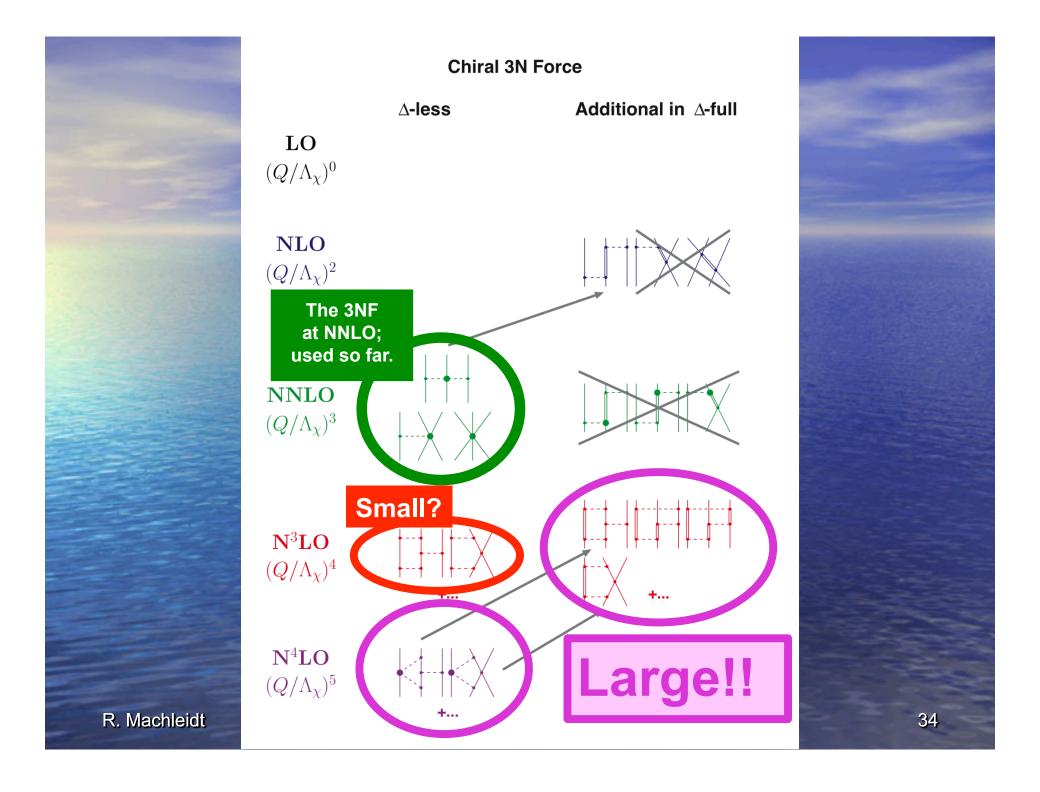
## pi-N Lagrangian with two derivatives ("next-to-leading" order)

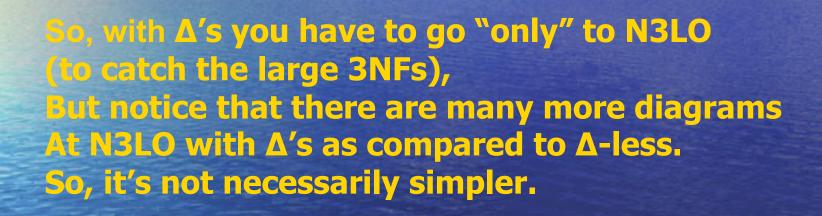
$$\mathcal{L}_{\pi N, c_{i}}^{(2)} = \bar{N} \left[ 2 c_{1} m_{\pi}^{2} (U + U^{\dagger}) + \left( c_{2} - \frac{g_{A}^{2}}{8M_{N}} \right) u_{0}^{2} + c_{3} u_{\mu} u^{\mu} + \frac{i}{2} \left( c_{4} + \frac{1}{4M_{N}} \right) \vec{\sigma} \cdot (\vec{u} \times \vec{u}) \right] N$$



Bernard et al. '97

### **Chiral 3N Force** $\Delta$ -less Additional in ∆-full LO $(Q/\Lambda_\chi)^0$ NLO $(Q/\Lambda_\chi)^2$ The 3NF at NNLO; used so far. **NNLO** $(Q/\Lambda_\chi)^3$ Small? $N^3LO$ $(Q/\Lambda_\chi)^4$ $N^4LO$ $(Q/\Lambda_{\chi})^5$ R. Machleidt 33





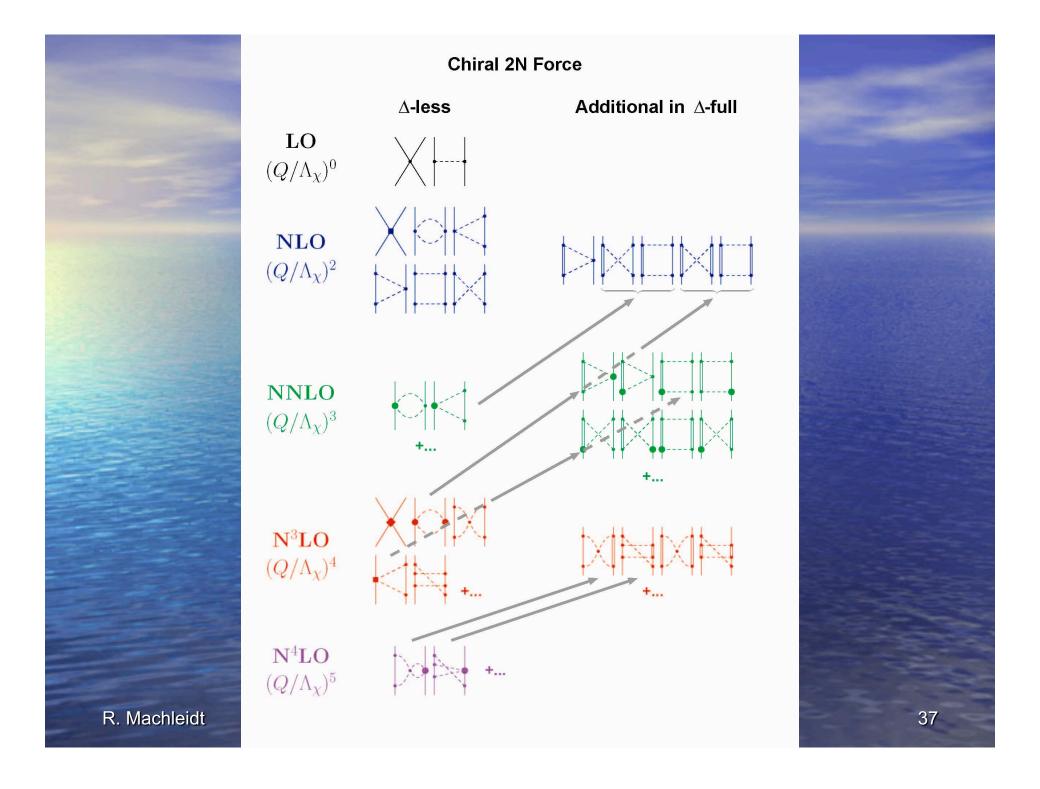
Notice further that in the  $\Delta$ -full theory the 2NF

Gets also more complicated.

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## **Chiral 2N Force** $\Delta$ -less LO $(Q/\Lambda_\chi)^0$ NLO $(Q/\Lambda_\chi)^2$ NNLO $(Q/\Lambda_\chi)^3$ $N^3LO$ $(Q/\Lambda_\chi)^4$ ${f N}^4{f L}{f O}$ $(Q/\Lambda_\chi)^5$ R. Machleidt







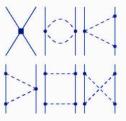
#### **Chiral 3N Force**

#### ∆-less









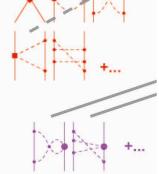




 ${f N}^3{f L}{f O} \ (Q/\Lambda_\chi)^4$ 

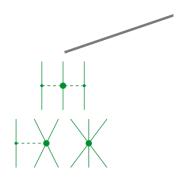
 $N^4LO$ 

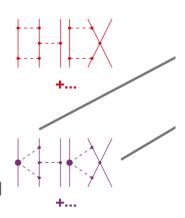
 $(Q/\Lambda_\chi)^5$ 



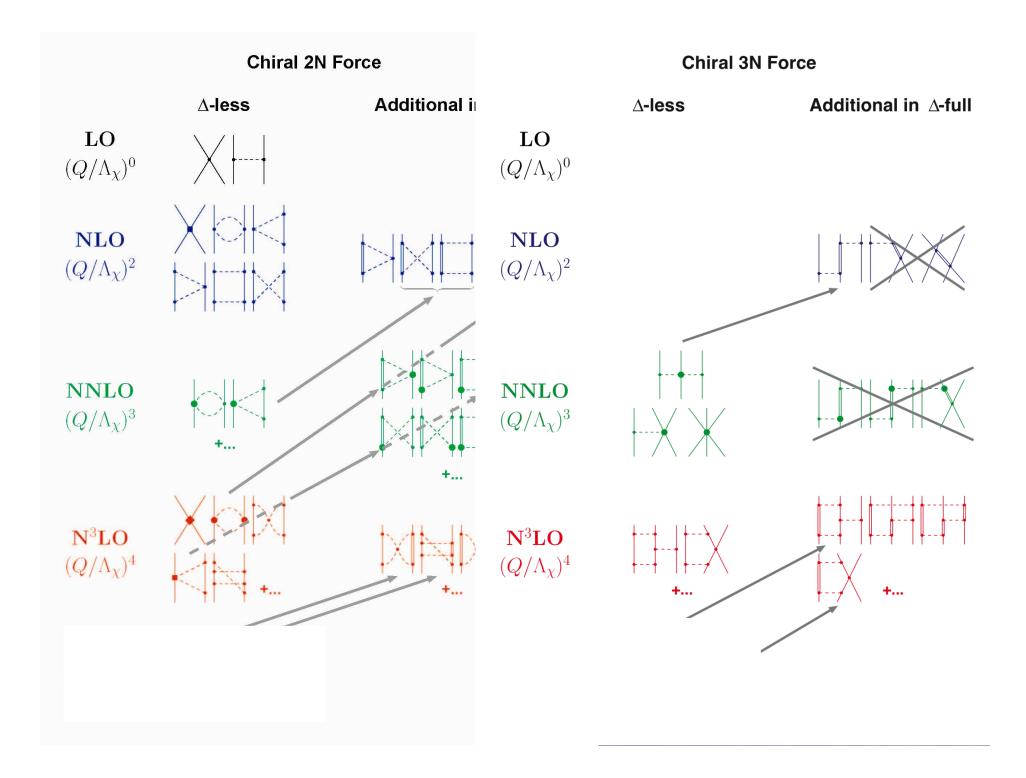
R. Machleidt

 $\Delta$ -less





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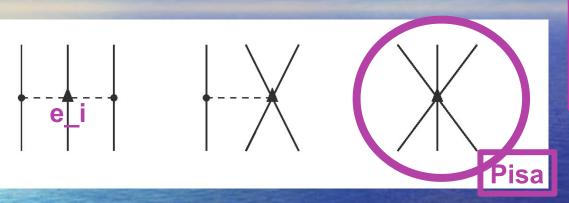
# What's the better approach is a matter of taste

 Bochum (E. Epelbaum & H. Krebs) pursues Δ-full

Idaho (R. M. & D. R. Entem) Δ-less

# **Chiral 3N Force** $\Delta$ -less LO $(Q/\Lambda_\chi)^0$ NLO $(Q/\Lambda_\chi)^2$ The 3NF NNLO at NNLO; $(Q/\Lambda_\chi)^3$ used so far. Small? $N^3LO$ $(Q/\Lambda_\chi)^4$ $N^4LO$ Large!! $(Q/\Lambda_\chi)^5$ R. Machleidt

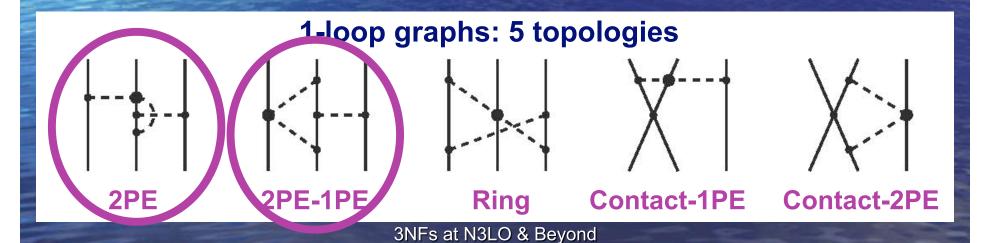
# The A-less N4LO (v=5) 3NF in more detail: Tree diagrams and 1-loop graphs



R. Machleidt

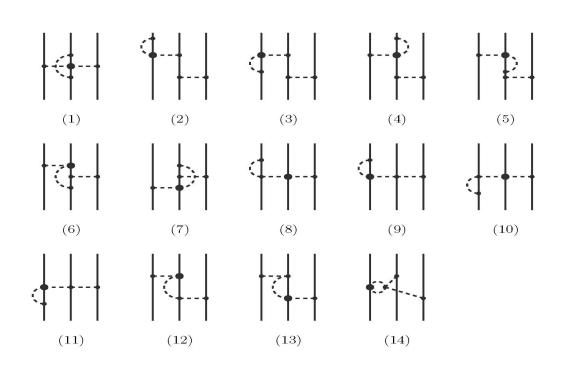
Power = 2+2L + 
$$\sum_{\text{all vertices}} \Delta_{j}$$
with  $\Delta_{j} = d_{j} + \frac{n_{j}}{2} - 2$ 

43



TRIUMF, Feb. 23, 2012

## The N4LO 3NF 2PE diagrams



$$V_{2PE}^{(N4LO)} = \frac{g_A^2}{8f_\pi^4} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \ \vec{\sigma}_2 \cdot \vec{q}_2}{(q_1^2 + m_\pi^2)(q_2^2 + m_\pi^2)} \left\{ \vec{\tau}_1 \cdot \vec{\tau}_2 (-4 \ \tilde{\textbf{c}_1} \ m_\pi^2 + 2 \ \tilde{\textbf{c}_3} \ \vec{q}_1 \cdot \vec{q}_2 + \tilde{\textbf{c}_4} \ \vec{\tau}_1 \cdot (\vec{\tau}_2 \times \vec{\tau}_3) \ \vec{\sigma}_3 \cdot (\vec{q}_1 \times \vec{q}_2) \right\}$$

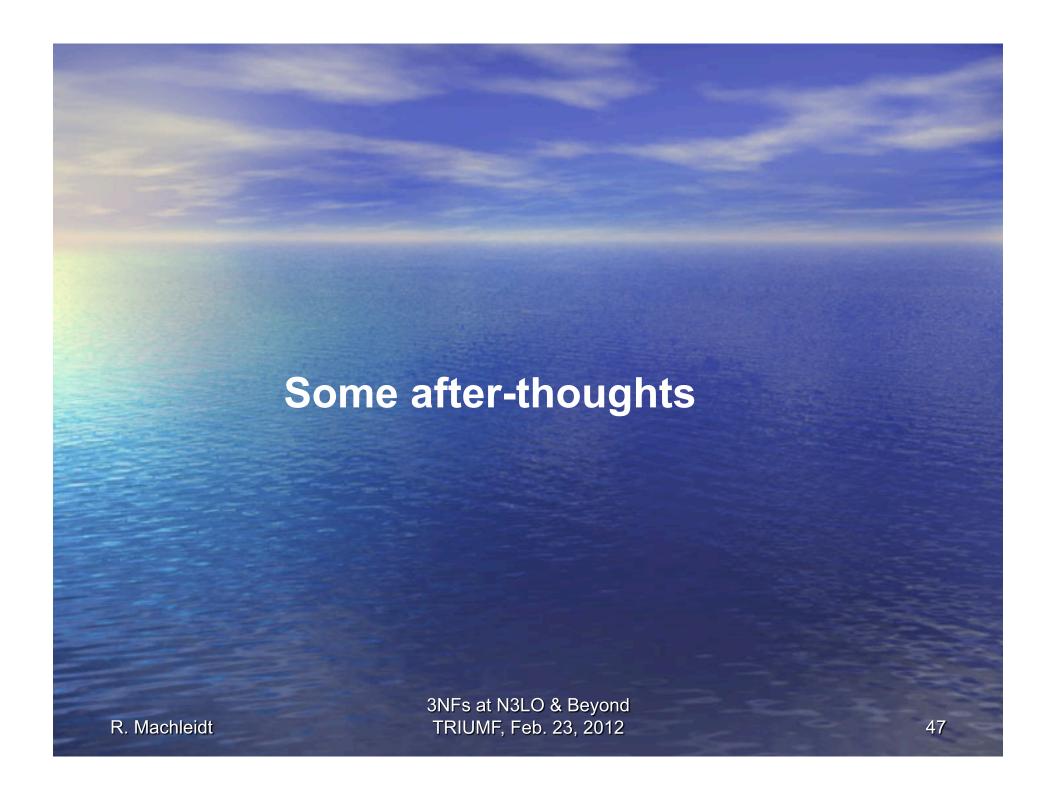
same structure as NNLO 2PE 3NF, but  $\tilde{c}_1$ ,  $\tilde{c}_3$ , and  $\tilde{c}_4$  get renormalized.

## The N4LO 3NF 2PE-1PE diagrams



# **General Summary**

- Substantial advances in chiral nuclear forces during the past decade. The major milestone of the decade: "high precision" NN pots. at N3LO; good for nuclear structure.
- But there are still important open issues concerning three-nucleon forces (3NF), which is one reason why we are here.
- Large 3NFs with many new structures to be expected at N3LO of Δ-full or N4LO at Δ-less. Construction is under way.
- There will be many new 3NFs to check out in the near future. Stay tuned.



"There will be many new 3NFs ..."

How many?

Too many?!

How not to get crushed by the Dinosaur?

3NFs at N3LO & Beyond TRIUMF, Feb. 23, 2012

# **Chiral 3N Force** Additional in $\Delta$ -full $\Delta$ -less LO $(Q/\Lambda_\chi)^0$ NLO $(Q/\Lambda_\chi)^2$ The 3NF at NNLO; used so far. NNLO $(Q/\Lambda_\chi)^3$ Small? $N^3LO$ $(Q/\Lambda_\chi)^4$ $N^4LO$ $(Q/\Lambda_\chi)^5$ R. Machleidt 49

∆-less

Additional in △-full

 $\mathbf{LO} \\ (Q/\Lambda_\chi)^0$ 

NLO  $(Q/\Lambda_{\gamma})^2$ 

#### $2\pi$ -exchange

#### $2\pi$ - $1\pi$ -exchange

$$0 = \frac{1}{2} + \frac{1}{2} +$$

#### ring diagrams

#### contact- $1\pi$ -exchange

#### contact- $2\pi$ -exchange

#### ∧-less

LO  $(Q/\Lambda_{\chi})^0$ 

virtual institute "Spin and strong QCD" (grant VH-VI-231). This work was further supported by the DFG (SFB/TR 16 "Subundear Structure of Matter") and by the EU Integrated Infrastructure Initiative Hadron Physics Project under contract number RIB-CT-2004-050078.

#### APPENDIX A: EXPRESSIONS FOR RING DIAGRAMS IN MOMENTUM-SPACE

In this appendix we give lengthy expressions for ring diagrams in Fig. 4 in momentum space. The contributions from diagrams (1) and (2) can be expressed as:

 $V_{\rm rinr} \; = \; \vec{\sigma}_1 \cdot \vec{\sigma}_2 \; \tau_2 \cdot \tau_3 \; R_1 + \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_2 \cdot \vec{q}_1 \; \tau_2 \cdot \tau_3 \; R_2 + \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_2 \cdot \vec{q}_3 \; \tau_2 \cdot \tau_3 \; R_3 + \vec{\sigma}_1 \cdot \vec{q}_3 \vec{\sigma}_2 \cdot \vec{q}_1 \; \tau_2 \cdot \tau_5 \; R_4$  $\vec{\sigma}_1 \cdot \vec{q}_3 \vec{\sigma}_2 \cdot \vec{q}_3 \tau_2 \cdot \tau_3 R_8 + \vec{\tau}_1 \cdot \tau_3 R_6 + \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_1 R_7 + \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3 R_8 + \vec{\sigma}_1 \cdot \vec{q}_3 \vec{\sigma}_3 \cdot \vec{q}_1 R_9 \qquad (A.1)$ +  $\vec{\sigma}_1 \cdot \vec{\sigma}_3 R_{10} + \vec{q}_1 \cdot \vec{q}_3 \times \vec{\sigma}_2 \tau_1 \cdot \tau_2 \times \tau_3 R_{11}$ .

where the functions  $R_i \equiv R_i(q_1, q_3, z)$  with  $z = \hat{q}_1 \cdot \hat{q}_3$  are defined as follows:

where the functions  $R_1 = \mathcal{N}(q_1, q_2, z)$  with  $z = q_1$ ,  $q_2$  and orbinod as follows:  $R_1 = \frac{(-1+x^2)^2 g_1^2 M_1^2 (2l_2^2 + q_1^2)}{(2S^2 F^2 (4(z+z^2)M_1^2 - q_1^2) (2l_2^2 (q_1 + 2q_1) + 2q_1 (-q_1^2 + q_1^2))}{(2S^2 F^2 (4(z+z^2)M_1^2 - q_1^2) (4l_2^2 q_2 + q_1^2)} - \frac{A(\eta) g_2^2 (2l_2^2 (q_1 + 2q_1) + 2q_1 (-q_1^2 + q_1^2) - (1+x^2) q_1 q_2}{(2S^2 F^2 (4(z+z^2)M_1^2 - q_1^2) (4l_1^2 + q_1^2) (4l_1^2 + q_1^2 - q_1^2 - q_1^2 + q_1^2)} + \frac{A(\eta) g_2^2 (2l_1^2 q_1^2 q_1 - q_1^2 + (2-3x^2) q_1^2 q_1 - (z-2z^2) q_1^2 q_1^2 + q_1^2)}{(2S^2 F^2 (-1+x^2) q_1^2 - q_1^2 + q_1^2 - q_1^2 + q_1^2)} - \frac{1}{2S^2 F^2 (-1+x^2) q_1^2}$ 

 $\frac{I(4:0,-q_{1},q_{5};0)g_{A}^{2}q_{2}^{2}}{32F^{6}\left(-1+z^{2}\right)\left(4\left(-1+z^{2}\right)M_{x}^{2}-q_{2}^{2}\right)q_{3}}\left(8\left(-1+z^{2}\right)M_{A}^{4}\left(2zq_{1}+\left(1+z^{2}\right)q_{3}\right)+q_{2}^{2}q_{3}\left(z^{2}q_{1}^{2}+q_{3}^{2}\right)q_{3}^{2}+q_{3}^{2}+q_{3}^{2}q_{3}^{2}+q_{3}^{2}q_{3}^{2}+q_{3}^{2}q_{3}^{2}+q_{3}^{2}q_{3}^{2}+q_{3}^{2}q_{3}^{2}+q_{3}^{2}q_{3}^{2}+q_{3}^{2}q_{3}^{2}+q_{3}^{2}+q_{3}^{2}q_{3}^{2}+q_{3}^{2}q_{3}^{2}+q_{3}^{2}$  $z \left( -1 + z^2 \right) q_1 q_3 - q_3^2 \right) + 2 M_\pi^2 \left( z \left( -2 + z^2 \right) q_1^3 - \left( 1 + 2 z^2 \right) q_1^2 q_3 + 3 z \left( -2 + z^2 \right) q_1 q_3^2 + \left( -3 + 2 z^4 \right) q_3^3 \right) \, ,$ 

$$\begin{split} R_2 &= \frac{A\left(q_2\right)g_A^2q_2^2\left(-2M_\pi^2\left(\left(1+z^2\right)q_1+2v_{\theta 3}\right)+zq_3\left(\left(1+z^2\right)q_1^2-2q_2^2\right)\right)}{128F^6\pi\left(-1+z^2\right)^2q_1^2q_3^2} + \\ &\frac{A\left(q_3\right)g_A^2\left(M_\pi^2\left(2zq_1^2+\left(1+3z^2\right)q_1q_2+zq_3^2\right)+zq_3\left(-zq_1^2-z^2q_1^2q_2+zq_1q_2^2+q_3^2\right)\right)}{4} + \\ &\frac{A\left(q_3\right)g_A^2\left(M_\pi^2\left(2zq_1^2+\left(1+3z^2\right)q_1q_3+zq_3^2\right)+zq_3\left(-zq_1^2-z^2q_1^2q_2+zq_1q_2^2+q_3^2\right)\right)}{4} + \\ &\frac{A\left(q_3\right)g_A^2\left(M_\pi^2\left(2zq_1^2+\left(1+3z^2\right)q_1q_3+zq_3^2\right)+zq_3\left(-zq_1^2-z^2q_1^2q_2+zq_1q_2^2+zq_3^2\right)\right)}{4} + \\ &\frac{A\left(q_3\right)g_A^2\left(M_\pi^2\left(2zq_1^2+\left(1+3z^2\right)q_1q_2+zq_3^2\right)+zq_3\left(-zq_1^2-zq_1^2+zq_3^2\right)+zq_3^2\left(-zq_1^2-zq_1^2+zq_1^2+zq_2^2\right)\right)}{4} + \\ &\frac{A\left(q_3\right)g_A^2\left(M_\pi^2\left(2zq_1^2+\left(1+3z^2\right)q_1q_3+zq_3^2\right)+zq_3^2\left(-zq_1^2-zq_2^2\right)\right)}{4} + \\ &\frac{A\left(q_3\right)g_A^2\left(M_\pi^2\left(2zq_1^2+q_1^2+zq_1^2+zq_2^2\right)+zq_3^2\left(-zq_1^2-zq_2^2\right)}{4} + \\ &\frac{A\left(q_3\right)g_A^2\left(M_\pi^2\left(2zq_1^2+q_1^2+zq_2^2\right)+zq_3^2\left(-zq_1^2-zq_2^2\right)\right)}{4} + \\ &\frac{A\left(q_3\right)g_A^2\left(M_\pi^2\left(2zq_1^2+zq_2^2\right)+zq_3^2\left(-zq_1^2-zq_2^2\right)}{4} + \\ &\frac{A\left(q_3\right)g_A^2\left(q_3^2+zq_3^2\right)+zq_3^2\left(-zq_3^2+zq_3^2\right)}{4} + \\ &\frac{A\left(q_3\right)g_A^2\left(q_3^2+zq_3^2\right)+zq_3^2\left(-zq_3^2+zq_3^2\right)}{4} + \\ &\frac{A\left(q_3\right)g_A^2\left(q_3^2+zq_3^2\right)+zq_3^2\left(-zq_3^2+zq_3^2\right)}{4} + \\ &\frac{A\left(q_3\right)g_A^2\left(q_3^2+zq_3^2\right)+zq_3^2\left(-zq_3^2\right)}{4} + \\ &\frac{A\left(q_3\right)g_A^2\left(q_3^2+zq_3^2\right)+zq_3^2\left(-zq_3^2\right)}{4} + \\ &\frac{A\left(q_3\right)g_A^2\left(q_3^2+zq_3^2\right)+zq_3^2\left(-zq_3^2+zq_3^2\right)}{4} + \\ &\frac{A\left(q_3\right)g_A^2\left(q_3^2+zq_3^2\right)+zq_3^2\left(-zq_3^2+zq_3^2\right)}{4} + \\ &\frac{A\left(q_3\right)g_A^2\left(q_3^2+zq_3^2\right)+zq_3^2\left(-zq_3^2+zq_3^2\right)}{4} + \\ &\frac{A\left(q_3\right)g_A^$$

 $64F^6\pi (-1+z^2)^2 q_1^3q_3$ 

 $\frac{A\left(q_{1}\right)g_{A}^{6}}{128F^{6}\pi\left(-1+z^{2}\right)^{2}q_{1}^{2}q_{3}^{2}}\left(2M_{\pi}^{2}\left(\left(1+z^{2}\right)q_{1}^{2}+z\left(3+z^{2}\right)q_{1}q_{3}+\left(1+z^{2}\right)q_{3}^{2}\right)+\right.$  $q_3\left(-\left(z+z^3\right)q_1^3+\left(2-5z^2+z^4\right)q_1^2q_3+z\left(1+z^2\right)q_1q_3^2+\left(1+z^2\right)q_3^3\right)\right)-$ 

 $\frac{I(4:0,-q_{1},q_{3};0)g_{3}^{6}}{32F^{5}\left(-1+z^{2}\right)^{2}q_{1}^{2}\left(-4\left(-1+z^{2}\right)M_{\pi}^{2}+q_{2}^{2}\right)q_{3}}\left(q_{2}^{4}q_{3}\left(-2z^{2}q_{1}^{2}+\left(1+z^{2}\right)q_{3}^{2}\right)-\right.$ 

 $8(-1+z)(1+z)M_{\pi}^{4}\left(z\left(2+z^{2}\right)q_{1}^{3}+\left(1+2z^{2}\right)^{2}q_{1}^{2}q_{5}+z\left(2+7z^{2}\right)q_{1}q_{5}^{2}+\left(1+2z^{2}\right)q_{5}^{3}\right)+$  $2M_{\pi}^{2}q_{2}^{2}\left(2zq_{1}^{3}+\left(1-z^{2}+6z^{4}\right)q_{1}^{2}q_{3}-2z\left(-1-3z^{2}+z^{4}\right)q_{1}q_{3}^{2}+\left(3+3z^{2}-4z^{4}\right)q_{3}^{3}\right)\right)$ 

 $+\frac{g_A^6 M_\pi \left(2 M_\pi^2+q_5^2\right) \left(q_2^2 q_3+4 M_\pi^2 \left(z q_1+q_5\right)\right)}{128 F^6 \pi q_1^2 \left(4 \left(-1+z^2\right) M_\pi^2-q_2^2\right) \left(4 M_\pi^2 q_3+q_3^2\right)},$ 

 $R_{8} \; = \; - \frac{zA\left(q_{2}\right)g_{A}^{6}g_{2}^{2}\left(-4M_{\pi}^{2}\left(q_{1}+zq_{3}\right)+q_{3}\left(2zq_{1}^{2}+\left(-1+z^{2}\right)q_{1}q_{3}-2zq_{8}^{2}\right)\right)}{2}$  $128F^6\pi (-1+z^2)^2 q_1^2 q_3^3$ 

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 $\frac{zA\left(q_{3}\right)g_{3}^{6}}{128F^{6}\pi\left(-1+z^{2}\right)^{2}q_{1}^{2}q_{3}^{2}}\left(M_{x}^{2}\left(-2z\left\langle-3+z^{2}\right)q_{1}^{2}+4\left(1+z^{2}\right)q_{1}q_{3}+4zq_{3}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{3}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{3}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{3}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{3}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{3}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{3}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{3}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{3}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{3}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{2}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{2}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{2}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{2}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{2}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{2}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{2}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{2}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{2}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{1}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{1}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{1}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{1}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{1}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{1}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{1}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{1}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{1}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{1}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{1}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{1}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{1}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{3}+4z^{2}q_{1}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{2}+4z^{2}q_{1}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{2}+4z^{2}q_{1}^{2}\right)+q_{3}\left(-\left(1+z^{2}\right)q_{1}^{2}+4z^{2}q_{1}^{2}+4z^{2}q_{1}^{2}+4z^{2}q_{1}^{2}+4z^{2}q_{1}^{2}+4z^{2}q_{1}^{2}+4z^{2}q_{1}^{2}+4z^{2}q_{1}^{2}+4z^{2}q_{1}^{2}+4z^{2}q_{1}^{2}+4z^{2}q_{1}^{2}+4z^{2}q_{1}^{2}+4z^{2}q_{1}^{2}+4z^{2}q_{1}^{2}+4z^{2}q_{1}^{2}+4z^{2}q_{1}^{2}+4z^{2}q_{1}^{2}+4z^{2}q_{1}^{2}+4z$  $2z^3q_1^2q_3 + (1+z^2)q_1q_3^2 + 2zq_3^3)) -$ 

 $zA\left(q_{1}\right)g_{A}^{6}\left(2M_{z}^{2}\left(2q_{1}^{2}+4zq_{1}q_{3}+\left(1+z^{2}\right)g_{3}^{2}\right)+q_{3}\left(-2zq_{1}^{3}+\left(1-3z^{2}\right)q_{1}^{2}q_{3}+2zq_{1}q_{3}^{2}+\left(1+z^{2}\right)q_{3}^{3}\right)\right)$ 

 $\frac{I(4:0,-g_1,g_2;0)zg_{\rm A}^2}{32F^3\left(-1+z^2\right)^2g_1\left(-4\left(-1+z^2\right)M_{\pi}^2+g_2^2\right)g_2^2}\left(g_2^4g_1\left(\left(1+z^2\right)q_1^2+z\left(-1+z^2\right)q_1q_3-\left(1+z^2\right)q_2^2\right)+\frac{1}{2}g_2^2g_3^2\right)g_3^2}$ 

 $q_1^2q_3 + 3z(1 + 2z^2)q_1q_3^2 + (1 + 2z^2)q_3^3$  +  $z \left(5 + z^2\right) q_1 q_1^2 + \left(-3 - 3z^2 + 4z^4\right) q_1^3\right)\right) +$ 

 $M_{\pi}^{2} \left(2zq_{1} + \left(1 + z^{2}\right)q_{5}\right)\right)_{\perp}$ 

 $\left. +2zq_3^2\right) + q_3\left(2z^2q_1^3 + 2z^3q_1^2q_3 + \left(1 - 4z^2 + z^4\right)q_1q_3^2 - 2zq_3^3\right)\right)$ 

 $+z^{2}$ )  $q_{1}^{2}+z$   $(3+z^{2})$   $q_{1}q_{3}+(1+z^{2})$   $q_{2}^{2}$ ) +

 $z(1 + z^2)q_1q_3^2 + (1 + z^2)q_3^3)$  -

 $\frac{1}{s^2 \ln a^2} \left( q_2^4 q_3 \left( (z + z^3) q_1^2 + (-1 + z^2)^2 q_1 q_3 - 2z q_3^2 \right) + \right)$ 

 $-2 + 9z^2 + 2z^4$ )  $q_1q_3^2 + z(2 + z^2)q_3^3$ ) +  $+(4 + 5z^2(-3 + z^2))q_1q_1^2 + 2z(-3 + z^2 + z^4)q_1^3)) +$  $\frac{+q_{3})}{f_{\pi}^{2}+q_{3}^{2}}$ ,

 $+(-1+z^2)q_1q_3-2zq_3^2)$ 

 $a_1^2 - 2(1 + z^2)q_1q_3 - 2zq_2^2 + q_3((1 + z^2)q_1^3 + 2z^3q_1^2q_3$ 

 $q_3^2$ ) +  $q_3$   $\left(-2zq_1^3 + (1 - 3z^2)q_1^2q_3 + 2zq_1q_2^2 + (1 + z^2)q_3^2\right)$ +  $28F^6\pi (-1 + z^2)^2q_1^4$ 

 $\frac{1}{3}$   $\left(q_2^4q_3\left(\left(1+z^2\right)q_1^2+z\left(-1+z^2\right)q_1q_3-\left(1+z^2\right)q_3^2\right)+$ 

 $\left| \begin{array}{l} q_{1}^{2}q_{3}+3z\left(1+2z^{2}\right)q_{1}q_{3}^{2}+\left(1+2z^{2}\right)q_{3}^{3}\right) + \\ -z\left(5+z^{2}\right)q_{1}q_{3}^{2}+\left(-3-3z^{2}+4z^{4}\right)q_{3}^{3}\right) - \end{array} \right|$ 

 $\frac{I_{\pi}^2 + q_1^2) q_3 + q_1 \left(8M_{\pi}^2 + 3q_1^2 + q_3^2\right)\right)}{128F^6 \pi q_1} +$ 

 $\frac{1}{q_1^2 q_3 \left(4M_{\pi}^2 + q_3^2\right)} \left(\left(5 + z^2\right) q_1^3 q_2^2 q_3^3 + 8M_{\pi}^6 \left(z \left(-3 + 4z^2\right) q_1^2 + 4z^2\right) q_2^2 + 4z^2\right) q_1^2 + 4z^2 q_2^2 + 4z^2 q_3^2 + 4z^2 q_3^2$  $q_2/q_3 \left( e m_\pi + q_3 \right)$   $2M_\pi^4 \left( 4z \left( -1 + z^2 \right) q_1^4 + \left( 77 - 36z^2 \right) q_1^3 q_3 + 2z \left( 33 + 8z^2 \right) q_1^2 q_3^2 +$  $M_{\pi}^{2}q_{1}q_{3}\left(\left(10+z^{2}\right)q_{1}^{4}+2z\left(9+2z^{2}\right)q_{1}^{3}q_{3}+\left(29-7z^{2}\right)q_{1}^{2}q_{3}^{2}+\right.$ 

 $(q_1^2 + zq_1q_3 + q_3^2) + 4M_\pi^4(zq_1^2 - 2(-2 + z^2)q_1q_3 + zq_3^2) +$ 

 $3A\left(q_{3}\right)g_{A}^{6}\left(2M_{\pi}^{2}+q_{2}^{2}\right)\left(\left(1+z^{2}\right)q_{1}+2zq_{3}\right)$  $\frac{^{2}) q_{3})}{} + \frac{3A(q_{2}) g_{A}^{2} \left(2M_{\pi}^{2} + q_{2}^{2}\right) \left(2zq_{1}^{2} + \left(1 + 3z^{2}\right) q_{1}q_{3} + 2zq_{3}^{2}\right)}{256F^{6}\pi \left(-1 + z^{2}\right)^{2} q_{1}^{2}q_{3}} +$  $q_1^2$   $\left(-q_2^2\left(\left(1+z^2\right)q_1^2+z\left(3+z^2\right)q_1q_3+\left(1+z^2\right)q_3^2\right)+\right.$  $z^{2}$ )  $q_{1}q_{3} + (1 + 2z^{2}) q_{5}^{2}$ ),

 $\begin{array}{l} + & 3zA\left(g_3\right)g_A^2\left(2M_\pi^2+g_2^2\right)\left(\left(1+z^2\right)g_1+2zg_3\right) \\ + & 25EF^2\left(-1+z^2\right)^2g_{13}^2 \\ + & 25EF^2\left(-1+z^2\right)^2g_{13}^2 \\ - & 3zA\left(g_2\right)g_A^2\left(2M_\pi^2+g_2^2\right)\left(2zg_1^2+\left(1+3z^2\right)g_1g_3+2zg_3^2\right) \\ - & 25EF^2\left(-1+z^2\right)^2g_1^2g_3^2 \end{array}$ 

 $\frac{|q_2|}{|q_2^2|} \frac{|q_3|}{|q_3|} \left( -q_2^2 \left( (1+z^2) q_1^2 + z (3+z^2) q_1 q_3 + (1+z^2) q_3^2 \right) + \right)$  $-z^2$ )  $q_1q_3 + (1 + 2z^2) q_3^2$ ),

 $\frac{+z(3+z^2)q_1q_3+(1+z^2)q_3^2)}{z^2)^2q_1^2q_3^2}$  +  $\frac{q_1^2q_3 - z^2(-7 + z^2)q_1q_3^2}{256F^6\pi(-1 + z^2)^2q_1q_3^2}$  $-\frac{z^{2}\left(-7+z^{2}\right)}{2}q_{1}q_{3}^{2}+2zq_{3}^{3}+2M_{\pi}^{2}\left(\left(1+z^{2}\right)q_{1}+2zq_{3}\right)\right)}{+}$ 

 $+2z\left(2+z^{2}\right)q_{1}q_{3}^{2}+\left(1+z^{2}\right)q_{3}^{3}+2M_{\pi}^{2}\left(2zq_{1}+\left(1+z^{2}\right)q_{3}\right)\right)}{256F^{6}\pi\left(-1+z^{2}\right)^{2}q_{1}^{2}q_{3}}+$ 

 $\frac{q_2^2}{+q_2^2}$   $\frac{q_2^2}{q_1} \left(q_2^2 \left(-2q_1^2 + z \left(-5 + z^2\right) q_1 q_3 - 2q_3^2\right) + \right)$ 

 $+(2+z^2)q_s^2)) - \frac{3zg_A^6M_\pi(2M_\pi^2+q_2^2)}{256F^6\pi q_1(-4(-1+z^2)M_\pi^2+q_2^2)q_8}$  $\frac{1 \left(q_2\right) g_A^6 \left(2 M_\pi^2 + q_2^2\right) \left(z q_1 + q_3\right) \left(q_1 + z q_3\right)}{256 F^6 \pi \left(-1 + z^2\right) q_1 q_3}.$ 

 $-4 + z^2$ )  $q_1q_3^2 + q_3^3 + 2M_{\pi}^2(zq_1 + q_3)$ )  $-1 + z^2$ )  $q_3$ 

 $+2z^{2}$ ) $q_{1}q_{3}^{2}+zq_{3}^{3}+2M_{\pi}^{2}(q_{1}+zq_{3}))$  $+z^{2}$  $)q_{1}$ + $\left(-q_{1}^{2}\left(q_{1}^{2}-z\left(-3+z^{2}\right)q_{1}q_{3}+q_{2}^{2}\right)+\right.$ 

 $+(1+z^2)q_3^2)$ ,

 $\frac{1}{3} + 2q_3^2\right) + q_3\left(zq_1^3 + \left(-1 + 2z^2\right)q_1^2q_3 + zq_1q_3^2 + q_3^3\right)\right) + q_3\left(zq_1^3 + \left(-1 + z^2\right)q_1^2q_3^2\right) + q_3\left(zq_1^3 + \left(-1 + z^2\right)q_1^2q_3^2\right) + q_3\left(zq_1^3 + \left(-1 + zz^2\right)q_1^2q_3^2\right) + q_3\left(zq_1^3 + \left(-1 + zz^2\right)q_1^2q_3^2\right)\right)$ 

 $\frac{A\left(q_{1}\right)g_{A}^{6}\left(2M_{\pi}^{2}\left(2q_{1}^{2}+2zq_{1}q_{5}+\left(-1+z^{2}\right)q_{3}^{2}\right)+q_{1}\left(q_{1}^{2}+zq_{1}^{2}q_{5}+\left(-1+2z^{2}\right)q_{1}q_{3}^{2}+zq_{3}^{2}\right)\right)}{256F^{2}\left(-1+z^{2}\right)q_{2}^{2}q_{5}^{2}}$ 

 $\frac{I(4:0_1-q_1,q_3;0)g_{\Lambda}^6q_2^2}{(64F^6\left(-1+z^2\right)q_1^2\left(-4\left(-1+z^2\right)M_{\pi}^2+q_2^2\right)q_3^2)}\left(-\left(2M_{\pi}^2+q_1^2\right)\left(2M_{\pi}^2+q_3^2\right)\left(4M_{\pi}^2+q_1^2+q_3^2\right)+\frac{1}{2}\left(4M_{\pi}^2+q_1^2+q_2^2\right)q_3^2\right)}{(64F^6\left(-1+z^2\right)q_1^2\left(-4\left(-1+z^2\right)M_{\pi}^2+q_2^2\right)q_3^2)}\right)$  $2z^3q_1q_3\left(-4M_\pi^4+q_1^2q_3^2\right)+z^2\left(4M_\pi^2+q_1^2+q_3^2\right)\left(4M_\pi^4+3q_1^2q_3^2+2M_\pi^2\left(q_1^2+q_3^2\right)\right)+$  $zq_1q_3\left(8M_\pi^4+q_1^4+q_3^4+4M_\pi^2\left(q_1^2+q_3^2\right)\right)\right)-$ 

 $128F^{6}\pi (-1+z^{2})^{2}q_{1}q_{3}$ 

(A.5)

 $2zq_1 + (1 + z^2) q_3$ 

 $q_1^2 + (1 + 3z^2) q_1q_5 + 2zq_3^2$ 

 $\frac{1 + 2M_{\pi}^{2}(q_{1} + zq_{3})}{256F^{6}\pi(-1 + z^{2})q_{4}^{2}} + \frac{zA(q_{5})g_{4}^{4}(2M_{\pi}^{2} + q_{3}^{2})}{256F^{6}\pi(-1 + z^{2})q_{1}q_{3}}$ 

 $\frac{(d : \vec{0}, \vec{q}_1)}{\hat{s} + q_1q_3} + \frac{J(d : \vec{0}, \vec{q}_3)}{q_1^2 + q_1q_3} - \frac{J(d : \vec{0}, \vec{q}_1 + \vec{q}_3)}{q_1q_3}$ 

 $^{2}\Big[\frac{J(d:\vec{0},\vec{q_{1}})}{q_{3}^{2}-q_{1}q_{3}}+\frac{J(d:\vec{0},\vec{q_{3}})}{q_{1}^{2}-q_{1}q_{3}}+\frac{J(d:\vec{0},\vec{q_{1}}+\vec{q_{3}})}{q_{1}q_{3}}$ 

 $J(d: \vec{0}, \vec{q_1} + \vec{q_3})$  },

 $J_3)(2q_1 - q_3) J(d : \vec{0}, \vec{q_1})$ 

 $(\vec{l} + \vec{p_1})^2 + M_\pi^2 (\vec{l} + \vec{p_2})^2 + M_\pi^2$ 

 $J(d : \vec{0}, \vec{q}_1 + \vec{q}_3)$  $2q_1 + q_3$  $J(d: \vec{0}, \vec{q_1})$ 

adividual terms in the expressions for  $R_i$  and  $S_i$  are singular es, however, cancel in such a way that the resulting terms in ble to obtain a representation for functions  $R_i$  and  $S_i$  which is the singularities at  $z=\pm 1$  can be avoided if one expresses the

 $J(d : \vec{0}, \vec{q}_{5}) + \frac{2(4M^{2} + (d - 2)(q_{1} - q_{3})^{2})}{(d - 1)q_{1}q_{3}(q_{1} - q_{3})^{2}}J(d : \vec{0}, \vec{q}_{1} + \vec{q}_{5})$ 

 $\int J(d : \vec{0}, \vec{q}_{3}) - \frac{2(4M^{2} + (d - 2)(q_{1} + q_{3})^{2})}{(d - 1)q_{1}q_{3}(q_{1} + q_{3})^{2}} J(d : \vec{0}, \vec{q}_{1} + \vec{q}_{3})],$ 

(A.8)

(A.9)

 $-2z^{2}\left(4M_{\pi}^{4}q_{1}^{2}\left(4M_{\pi}^{2}+q_{1}^{2}\right)+\right.$  $-q_3^2$ )  $(q_1^2q_3^2 + 2M_\pi^2(q_1^2 + q_3^2)) -4q_1^2q_3^2+q_3^4)))$ . (A.2)

 $f_{\pi}^2 + i\epsilon \frac{i}{(l+p_3)^2 - M_{\pi}^2 + i\epsilon} \frac{1}{v \cdot (l+p_4) + i\epsilon}$ . (A.3) for the case  $p_1^0 = 0$  which we are interested in, it space  $J(d : \vec{p_1}, \vec{p_2}, \vec{p_3})$ 

 $\vec{p}_2$ )<sup>2</sup> +  $M_{\pi}^2$   $(\vec{l} + \vec{p}_3)^2 + M_{\pi}^2$ 

ons for  $R_i$  can be written as  $-\vec{q}_1, \vec{q}_3$ .

 $\vec{q}_1 \cdot \tau_1 \cdot \tau_2 \cdot S_3 + \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3 \cdot \tau_1 \cdot \tau_2 \cdot S_4$   $\cdot \vec{q}_3 \times \vec{\sigma}_1 \cdot \tau_1 \cdot \tau_2 \times \tau_3 \cdot S_7$ ,

 $\frac{A (q_3) g_A^4 (2M_{\pi}^2 + q_3^2)}{128P^6\pi} +$ 

 $q_1q_3 + 2zq_3^2$ 

 $\frac{1}{128P^{6}\pi\left(-1+z^{2}\right)^{2}q_{1}q_{3}-2q_{1}^{2}}$  +  $\frac{zA\left(q_{2}\right)g_{4}^{4}\left(-2q_{1}^{2}+z\left(-5+z^{2}\right)q_{1}q_{3}-2q_{1}^{2}\right)}{128P^{6}\pi\left(-1+z^{2}\right)^{2}q_{1}q_{3}}$  -

 $-z^2$ )  $q_1q_3 + (1 + z^2) q_3^2$ )

contact- $2\pi$ -exchange

N. Kalantar-Nayestanaki and E. Epelbaum, Nucl. Phys. News 17, 22 (2007) [arXiv:nucl-th/0703089]
 D. R. Entem and R. Machleidt, Phys. Rev. C 68, 041001 (2003) [arXiv:nucl-th/6304018].

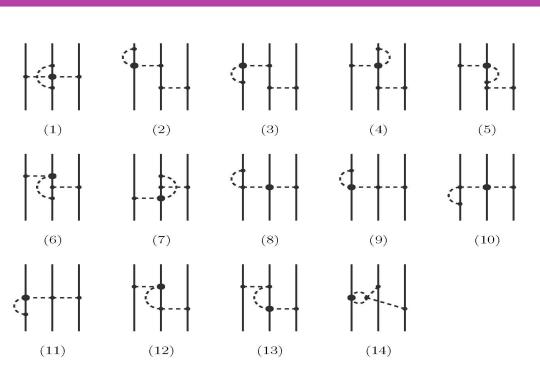
# **Chiral 3N Force** Additional in $\Delta$ -full $\Delta$ -less LO $(Q/\Lambda_\chi)^0$ NLO $(Q/\Lambda_\chi)^2$ The 3NF at NNLO; used so far. NNLO $(Q/\Lambda_\chi)^3$ Small? $N^3LO$ $(Q/\Lambda_\chi)^4$ $N^4LO$ $(Q/\Lambda_\chi)^5$ R. Machleidt 52

∆-less

Additional in △-full

 $\mathbf{LO} \\ (Q/\Lambda_\chi)^0$ 

NIT (



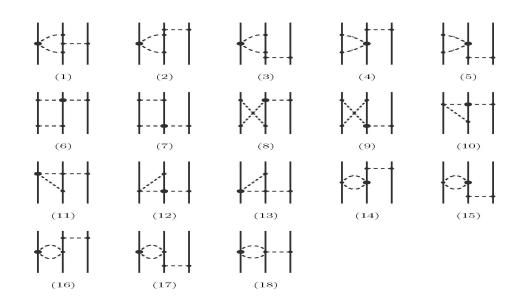
$$V_{2PE}^{(N4LO)} = \frac{g_A^2}{8f_{\pi}^4} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \ \vec{\sigma}_2 \cdot \vec{q}_2}{(q_1^2 + m_{\pi}^2)(q_2^2 + m_{\pi}^2)} \left\{ \vec{\tau}_1 \cdot \vec{\tau}_2 (-4 \ \tilde{\boldsymbol{c}_1} \ m_{\pi}^2 + 2 \ \tilde{\boldsymbol{c}_3} \ \vec{q}_1 \cdot \vec{q}_2 + \tilde{\boldsymbol{c}_4} \ \vec{\tau}_1 \cdot (\vec{\tau}_2 \times \vec{\tau}_3) \ \vec{\sigma}_3 \cdot (\vec{q}_1 \times \vec{q}_2) \right\}$$

same structure as NNLO 2PE 3NF, but  $\tilde{c}_1$ ,  $\tilde{c}_3$ , and  $\tilde{c}_4$  get renormalized.

#### ∆-less

#### Additional in △-full

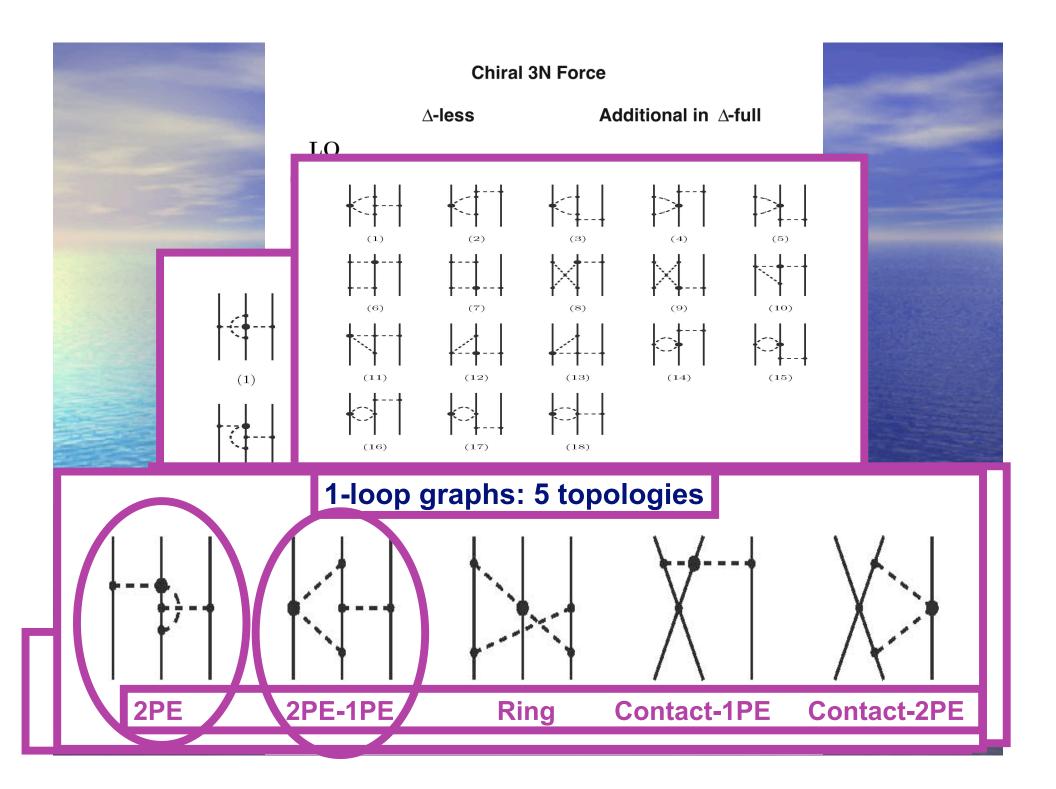
 $\mathbf{LO}$ 



$$V_{2PE-1PE}^{N4LO} = \frac{\vec{\sigma}_2 \cdot \vec{q}_2}{{q_2}^2 + {m_\pi}^2} \times \{\vec{\tau}_1 \cdot \vec{\tau}_2 [\vec{\sigma}_3 \cdot \vec{q}_4]\}$$

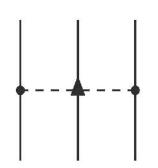
 $V_{_{2PE}}^{\scriptscriptstyle (N4LO)}$ 

same struc





∆-less







**3NF contacts** at N4LO (Pisa)

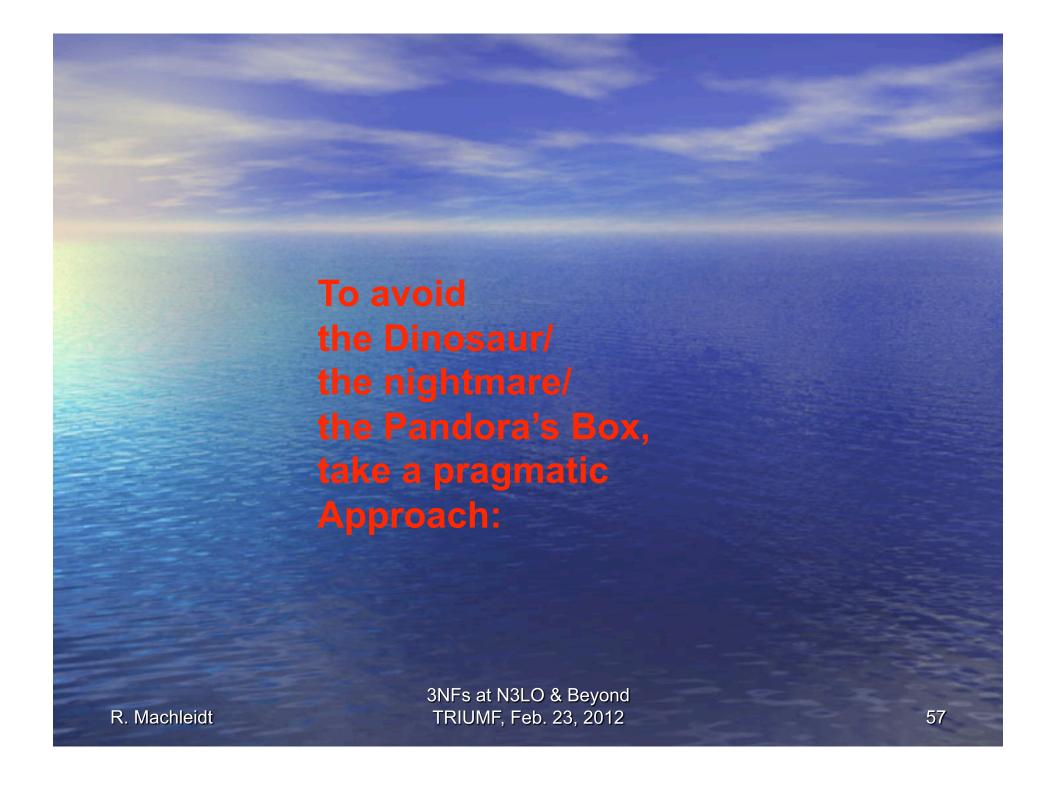
## **3NF** contacts at N4LO

Girlanda, Kievsky, Viviani, PRC 84, 014001 (2011)

 $\mathbf{k}_i = \mathbf{p}_i - \mathbf{p}_i'$  and  $\mathbf{Q}_i = \mathbf{p}_i + \mathbf{p}_i'$ ,  $\mathbf{p}_i$  and  $\mathbf{p}_i'$  being the initial and final momenta of nucleon i, the potential in momentum space is found to be

$$V = \sum_{i \neq j \neq k} \left[ -E_1 \mathbf{k}_i^2 - E_2 \mathbf{k}_i^2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j - E_3 \mathbf{k}_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j - E_4 \mathbf{k}_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \right]$$

Spin-Orbit
$$\begin{bmatrix}
-E_{\tau}\left(2\mathbf{k}_{i} - i\mathbf{k}_{i} - \sigma_{j} - \mathbf{k}_{i}^{2}\right) - E_{6}\left(3\mathbf{k}_{i} \cdot \boldsymbol{\sigma}_{i}\mathbf{k}_{i} \cdot \boldsymbol{\sigma}_{j} - \mathbf{k}_{i}^{2}\right))\boldsymbol{\tau}_{i} \cdot \boldsymbol{\tau}_{j} \\
+ \frac{i}{2}E_{7}\mathbf{k}_{i} \times \left(\mathbf{Q}_{i} - \mathbf{Q}_{j}\right) \cdot \left(\boldsymbol{\sigma}_{i} + \boldsymbol{\sigma}_{j}\right) + \frac{i}{2}E_{8}\mathbf{k}_{i} \times \left(\mathbf{Q}_{i} - \mathbf{Q}_{j}\right) \cdot \left(\boldsymbol{\sigma}_{i} + \boldsymbol{\sigma}_{j}\right)\boldsymbol{\tau}_{j} \cdot \boldsymbol{\tau}_{k} \\
-E_{9}\mathbf{k}_{i} \cdot \boldsymbol{\sigma}_{i}\mathbf{k}_{j} \cdot \boldsymbol{\sigma}_{j} - E_{10}\mathbf{k}_{i} \cdot \boldsymbol{\sigma}_{i}\mathbf{k}_{j} \cdot \boldsymbol{\sigma}_{j}\boldsymbol{\tau}_{i} \cdot \boldsymbol{\tau}_{j},
\end{bmatrix}, \tag{15}$$



## A working model:

- use ∆-less
- include NNLO 3NF.
- skip N3LO 3NF (small in Δ-less!)
- at N4LO start with contact 3NF, use one term at a time, e.g. spin-orbit
- that may already solve your problems.

#### **Chiral 3N Force**

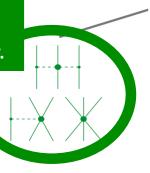
∆-less

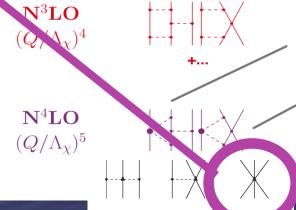
 ${\bf LO} \\ (Q/\Lambda_\chi)^0$ 

 $\mathbf{NLO} \\ (Q/\Lambda_\chi)^2$ 

The 3NF at NNLO;

NNLo  $(Q/\Lambda_{\chi})^3$ 





3NFs at N3LO & Beyond TRIUMF, Feb. 23, 2012

# A working 3NF model (to be used on an "investigational" basis)

### NNLO 3NF

$$V_{\text{TPE}}^{3\text{NF}} = \left(\frac{g_A}{2f_\pi}\right)^2 \frac{1}{2} \sum_{i \neq j \neq k} \frac{(\vec{\sigma}_i \cdot \vec{q}_i)(\vec{\sigma}_j \cdot \vec{q}_j)}{(q_i^2 + m_\pi^2)(q_j^2 + m_\pi^2)} F_{ijk}^{\alpha\beta} \tau_i^{\alpha} \tau_j^{\beta}$$
(2)

with  $\vec{q}_i \equiv \vec{p}_i' - \vec{p}_i$ , where  $\vec{p}_i$  and  $\vec{p}_i'$  are the initial and final momenta of nucleon i, respectively, and

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta} \left[ -\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2} \vec{q}_i \cdot \vec{q}_j \right] + \frac{c_4}{f_\pi^2} \sum_{\gamma} \epsilon^{\alpha\beta\gamma} \tau_k^{\gamma} \vec{\sigma}_k \cdot [\vec{q}_i \times \vec{q}_j] . (3)$$

$$V_{\mathrm{OPE}}^{\mathrm{3NF}} = D \; \frac{g_A}{8 f_\pi^2} \sum_{i \neq j \neq k} \frac{\vec{\sigma}_j \cdot \vec{q}_j}{q_j^2 + m_\pi^2} (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) (\vec{\sigma}_i \cdot \vec{q}_j)$$

$$V_{
m ct}^{
m 3NF} = E \; rac{1}{2} \sum_{j 
eq k} m{ au}_j \cdot m{ au}_k \; .$$

### + N4LO 3NF Contacts

 $\mathbf{k}_i = \mathbf{p}_i - \mathbf{p}_i'$  and  $\mathbf{Q}_i = \mathbf{p}_i + \mathbf{p}_i'$ ,  $\mathbf{p}_i$  and  $\mathbf{p}_i'$  being the initial and final momenta of nucleon i, the potential in momentum space is found to be

$$V = \sum_{i \neq j \neq k} \left[ -E_1 \mathbf{k}_i^2 - E_2 \mathbf{k}_i^2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j - E_3 \mathbf{k}_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j - E_4 \mathbf{k}_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \right]$$

# Spin-Orbit Force!

$$E_{6}\left(3\mathbf{k}_{i}\cdot\boldsymbol{\sigma}_{i}\mathbf{k}_{i}\cdot\boldsymbol{\sigma}_{j}-\mathbf{k}_{i}^{2}\right) - E_{6}\left(3\mathbf{k}_{i}\cdot\boldsymbol{\sigma}_{i}\mathbf{k}_{i}\cdot\boldsymbol{\sigma}_{j}-\mathbf{k}_{i}^{2}\right))\boldsymbol{\tau}_{i}\cdot\boldsymbol{\tau}_{j}$$

$$+\frac{i}{2}E_{7}\mathbf{k}_{i}\times\left(\mathbf{Q}_{i}-\mathbf{Q}_{j}\right)\cdot\left(\boldsymbol{\sigma}_{i}+\boldsymbol{\sigma}_{j}\right)+\left[E_{8}\mathbf{k}_{i}\times\left(\mathbf{Q}_{i}-\mathbf{Q}_{j}\right)\cdot\left(\boldsymbol{\sigma}_{i}+\boldsymbol{\sigma}_{j}\right)\boldsymbol{\tau}_{j}\cdot\boldsymbol{\tau}_{k}\right]$$

$$-E_{9}\mathbf{k}_{i} \quad \mathcal{F}_{j} \quad E_{10}\mathbf{k}_{i}\cdot\boldsymbol{\sigma}_{i}\mathbf{k}_{j}\cdot\boldsymbol{\sigma}_{j}\boldsymbol{\tau}_{i}\cdot\boldsymbol{\tau}_{j}\right],$$
(15)