
Three-Nucleon Forces
at N3LO and Beyond
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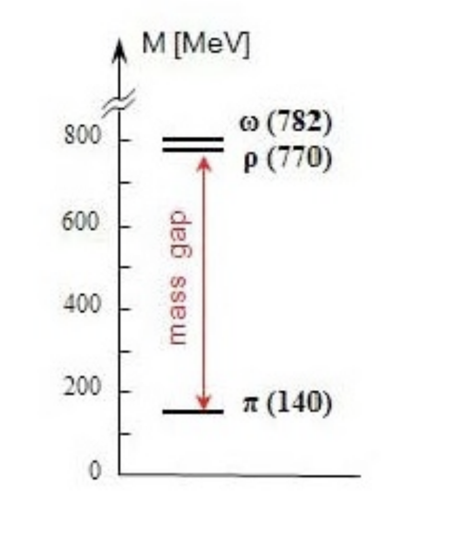
Outline

- Nuclear forces from chiral EFT: Overview & achievements
- Beyond the current status
- 3NFs at N3LO
- 3NFs at N4LO
- When does it stop?
- Outlook: How not to get crushed by the Dinosaur?

From QCD to nuclear physics via chiral EFT (in a nutshell)

- **QCD at low energy is strong.**
- **Quarks and gluons are confined into colorless hadrons.**
- **Nuclear forces are residual forces (similar to van der Waals forces)**
- **Separation of scales**



- **Calls for an EFT**

  soft scale: $Q \approx m_\pi$, hard scale: $\Lambda_\chi \approx m_\rho$;
  pions and nucleon relevant d.o.f.
- **Low-energy expansion: $(Q/\Lambda_\chi)^\nu$**

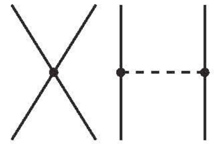
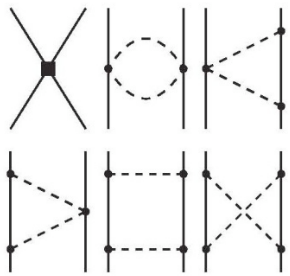
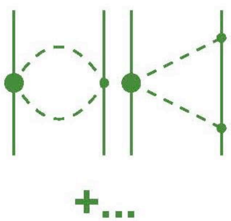
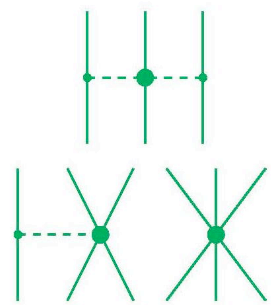
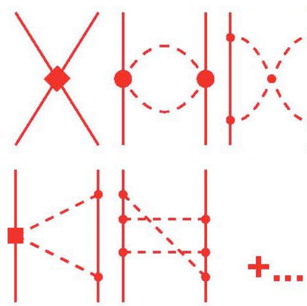
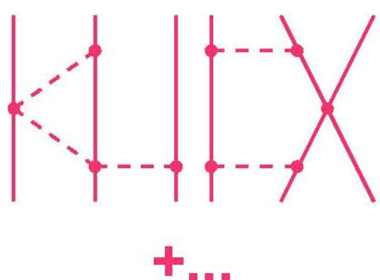
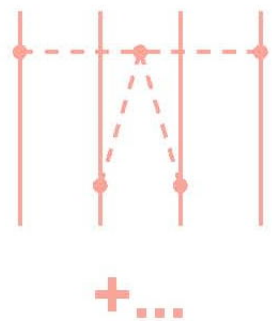
  with ν bounded from below.
- **Most general Lagrangian consistent with all symmetries of low-energy QCD.**
- **π-π and π-N perturbatively**
- **NN has bound states:**

  (i) NN potential perturbatively

  (ii) apply nonpert. in LS equation.

  (Weinberg)

The Hierarchy of Nuclear Forces

		2N forces	3N forces	4N forces
Leading Order	Q^0 LO			
Next-to Leading Order	Q^2 NLO			
Next-to-Next-to Leading Order	Q^3 N^2LO			
Next-to-Next-to-Next-to Leading Order	Q^4 N^3LO			

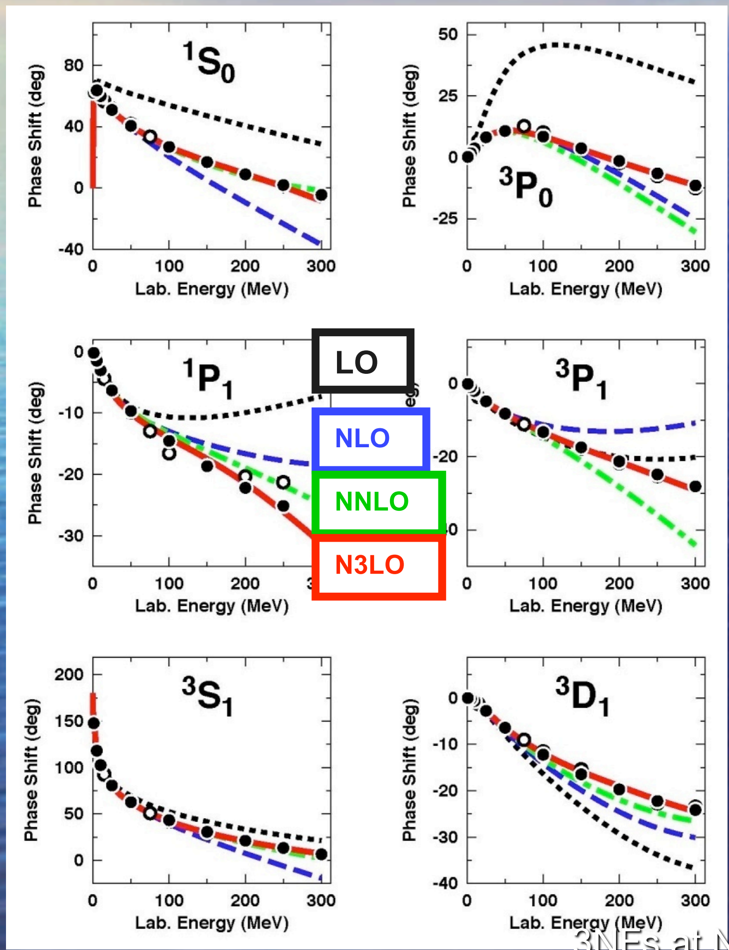
NN phase shifts up to 300 MeV

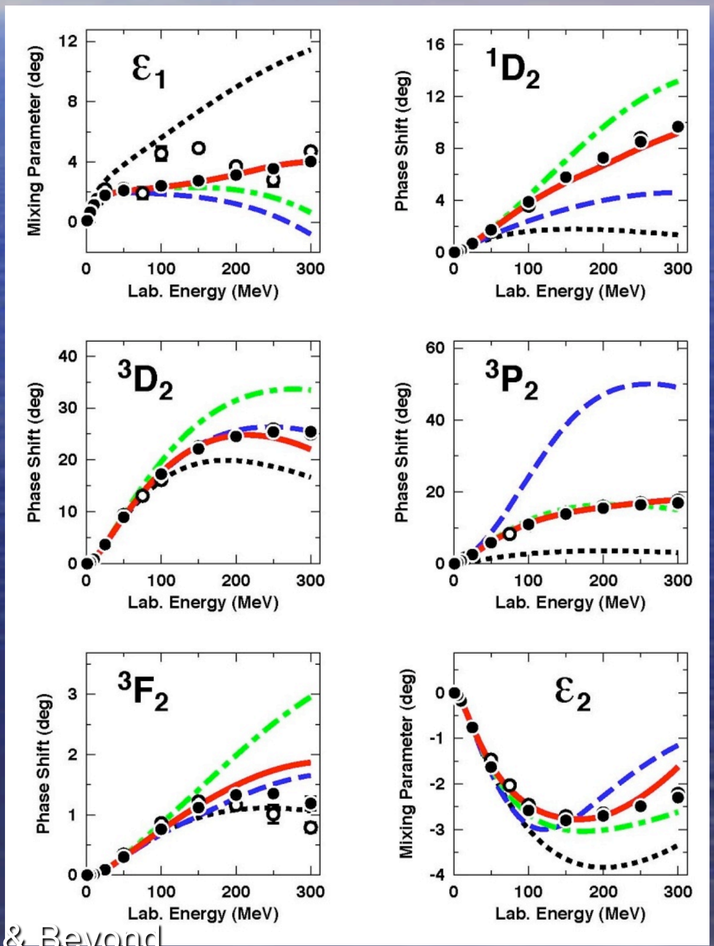
Red Line: N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).

Green dash-dotted line: NNLO Potential, and

blue dashed line: NLO Potential

by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).





χ^2/datum for the reproduction of the 1999 *np* database

Bin (MeV)	# of data	N^3LO	NNLO	NLO	AV18
0–100	1058	1.05	1.7	4.5	0.95
100–190	501	1.08	22	100	1.10
190–290	843	1.15	47	180	1.11
0–290	2402	1.10	20	86	1.04

N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).
NNLO and NLO Potentials by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).

Applications of the chiral NN potential at N3LO

PRL **101,** 092502 (2008) PHYSICAL REVIEW LETTERS week ending 29 AUGUST 2008

Medium-Mass Nuclei from Chiral Nucleon-Nucleon Interactions

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(Received 20 June 2008; published 29 August 2008)

We compute the binding energies, radii, and densities for selected medium-mass nuclei within coupled-cluster theory and employ a bare chiral nucleon-nucleon interaction at next-to-next-to-next-to-leading order. We find rather well-converged results in model spaces consisting of 15 oscillator shells, and the doubly magic nuclei ^{40}Ca, ^{48}Ca, and the exotic ^{48}Ni are underbound by about 1 MeV per nucleon within the coupled-cluster singles-doubles approximation. The binding-energy difference between the mirror nuclei ^{48}Ca and ^{48}Ni is close to theoretical mass table evaluations. Our computation of the one-body density matrices and the corresponding natural orbitals and occupation numbers provides a first step to a microscopic foundation of the nuclear shell model.

Chiral NN potential at N^{3}LO underbinds by ~1MeV/nucleon. (Size extensivity at its best.)

Nucleus	**ΔE / A [MeV]**
^{4}He	**1.08 (0.73^{FY})**
^{16}O	**1.25**
^{40}Ca	**0.84**
^{48}Ca	**1.27**
^{48}Ni	**1.21**

PHYSICAL REVIEW C **82**, 034330 (2010)

Ab initio coupled-cluster approach to nuclear structure with modern nucleon-nucleon interactions

G. Hagen,[1] T. Papenbrock,[1,2] D. J. Dean,[1] and M. Hjorth-Jensen[3]

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(Received 17 May 2010; revised manuscript received 20 August 2010; published 30 September 2010)

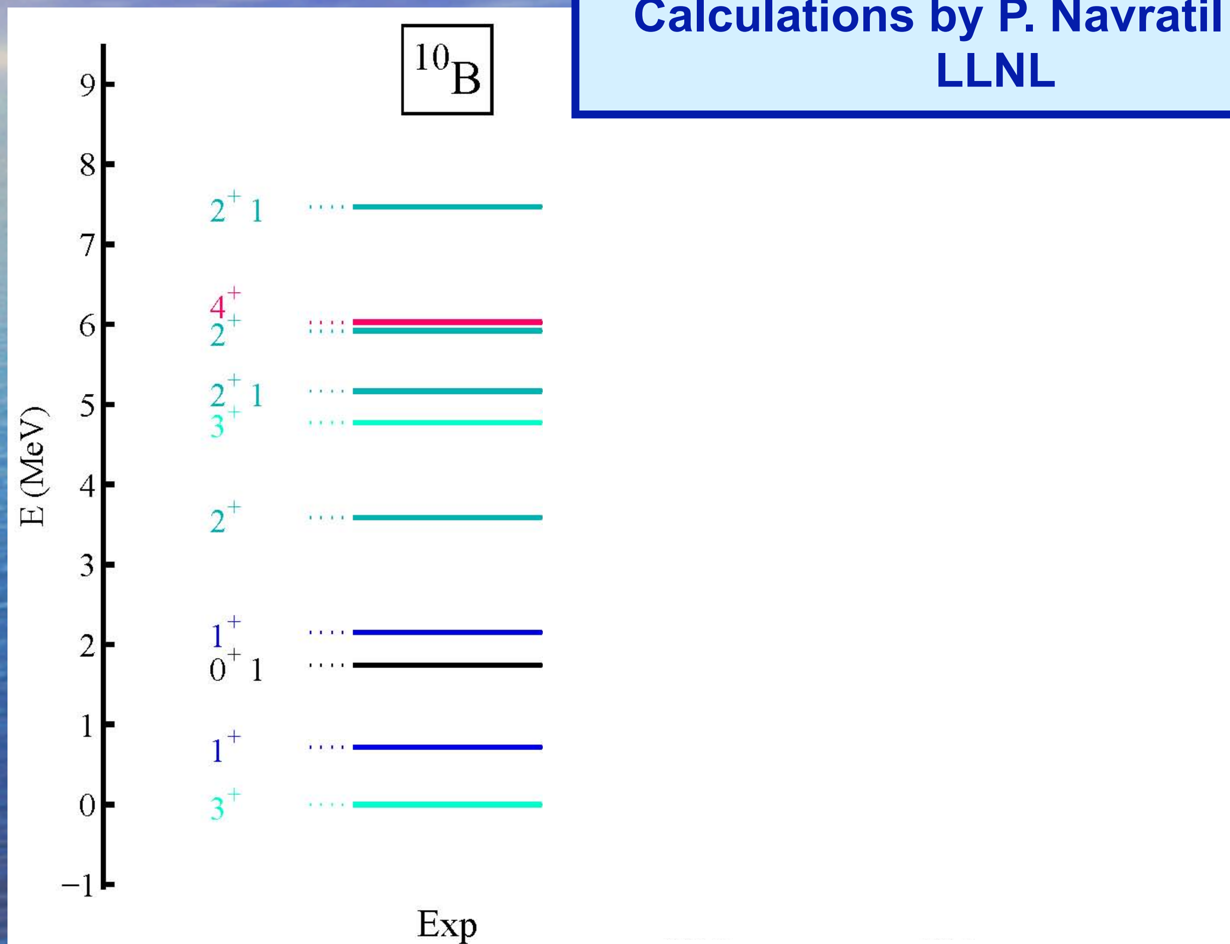
We perform coupled-cluster calculations for the doubly magic nuclei ^{4}He, ^{16}O, ^{40,48}Ca, for neutron-rich isotopes of oxygen and fluorine, and employ "bare" and secondary renormalized nucleon-nucleon interactions. For the nucleon-nucleon interaction from chiral effective field theory at order next-to-next-to-next-to leading order, we find that the coupled-cluster approximation including triples corrections binds nuclei within 0.4 MeV per nucleon compared to data. We employ interactions from a resolution-scale dependent similarity renormalization group transformations and assess the validity of power counting estimates in medium-mass nuclei. We find that the missing contributions from three-nucleon forces are consistent with these estimates. For the unitary correlator model potential, we find a slow convergence with respect to increasing the size of the model space. For the G-matrix approach, we find a weak dependence of ground-state energies on the starting energy combined with a rather slow convergence with respect to increasing model spaces. We also analyze the center-of-mass problem and present a practical and efficient solution.

… including the chiral 3NF at N2LO

For medium-mass nuclei, see talks by Papenbrock; Roth et al.; and others.

Calculating the properties of light nuclei using chiral 2N and 3N forces

"No-Core Shell Model " Calculations by P. Navratil et al., LLNL



Calculating the properties of light nuclei using chiral 2N and 3N forces

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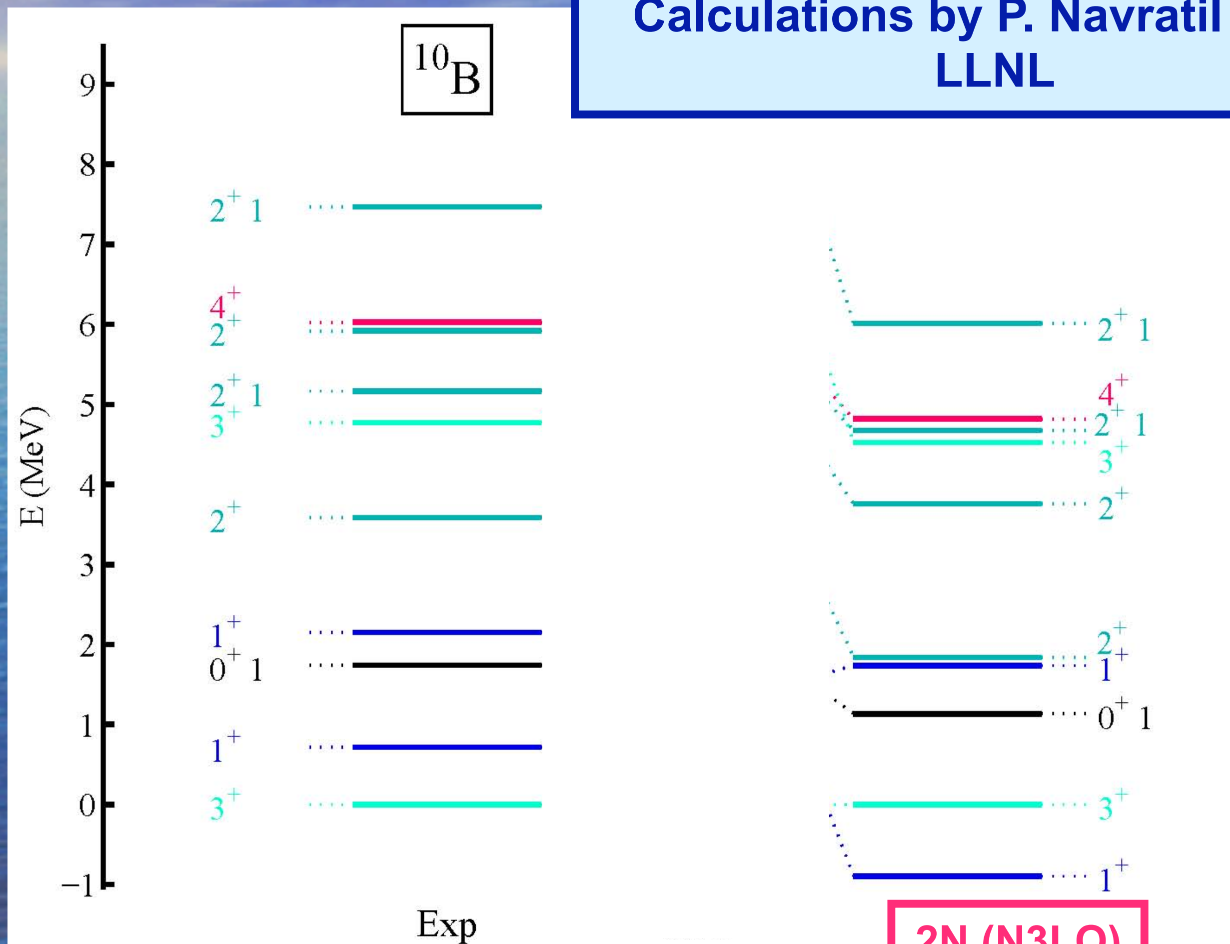
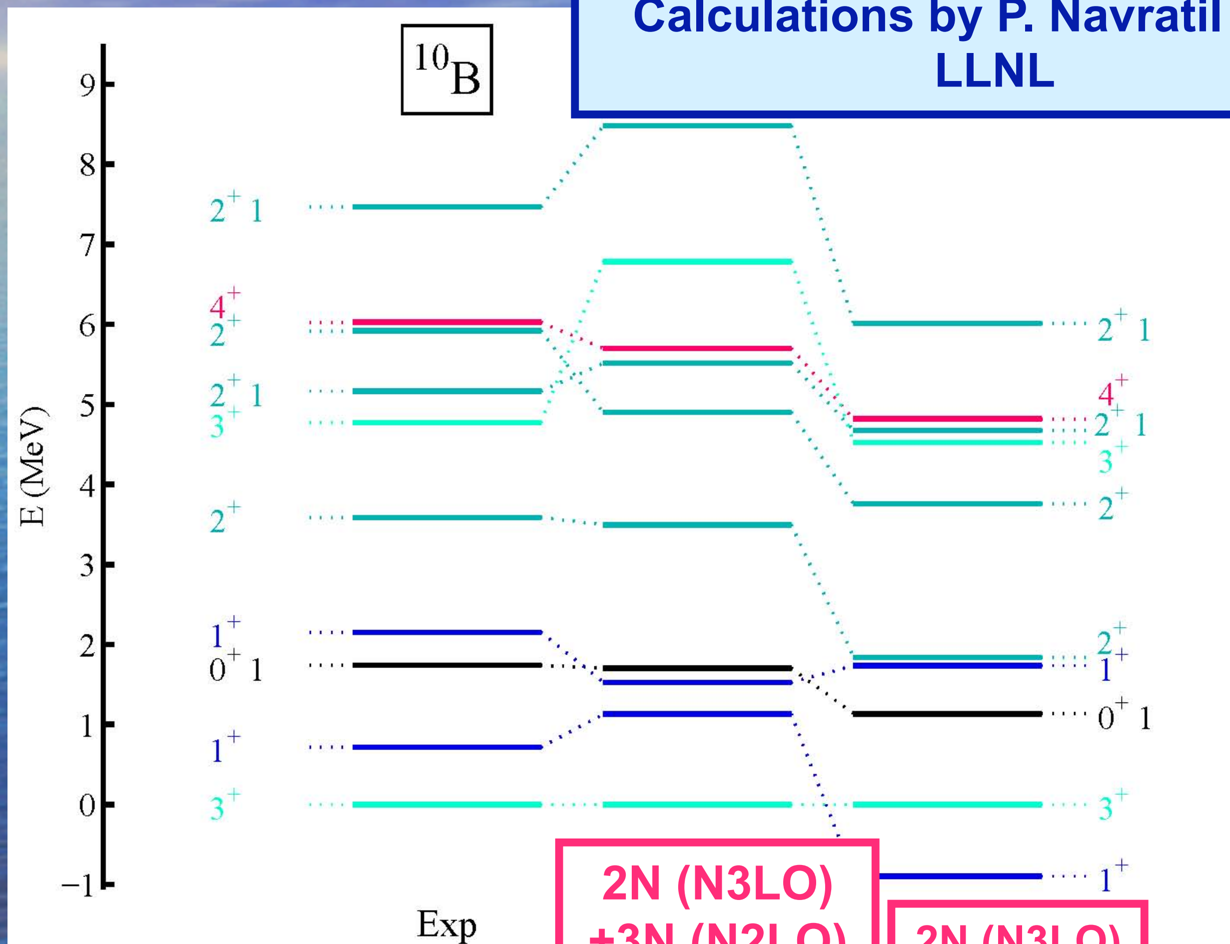


2N (N3LO) force only

Calculating the properties of light nuclei using chiral 2N and 3N forces

"No-Core Shell Model " Calculations by P. Navratil et al., LLNL



2N (N3LO) +3N (N2LO) forces

2N (N3LO) force only

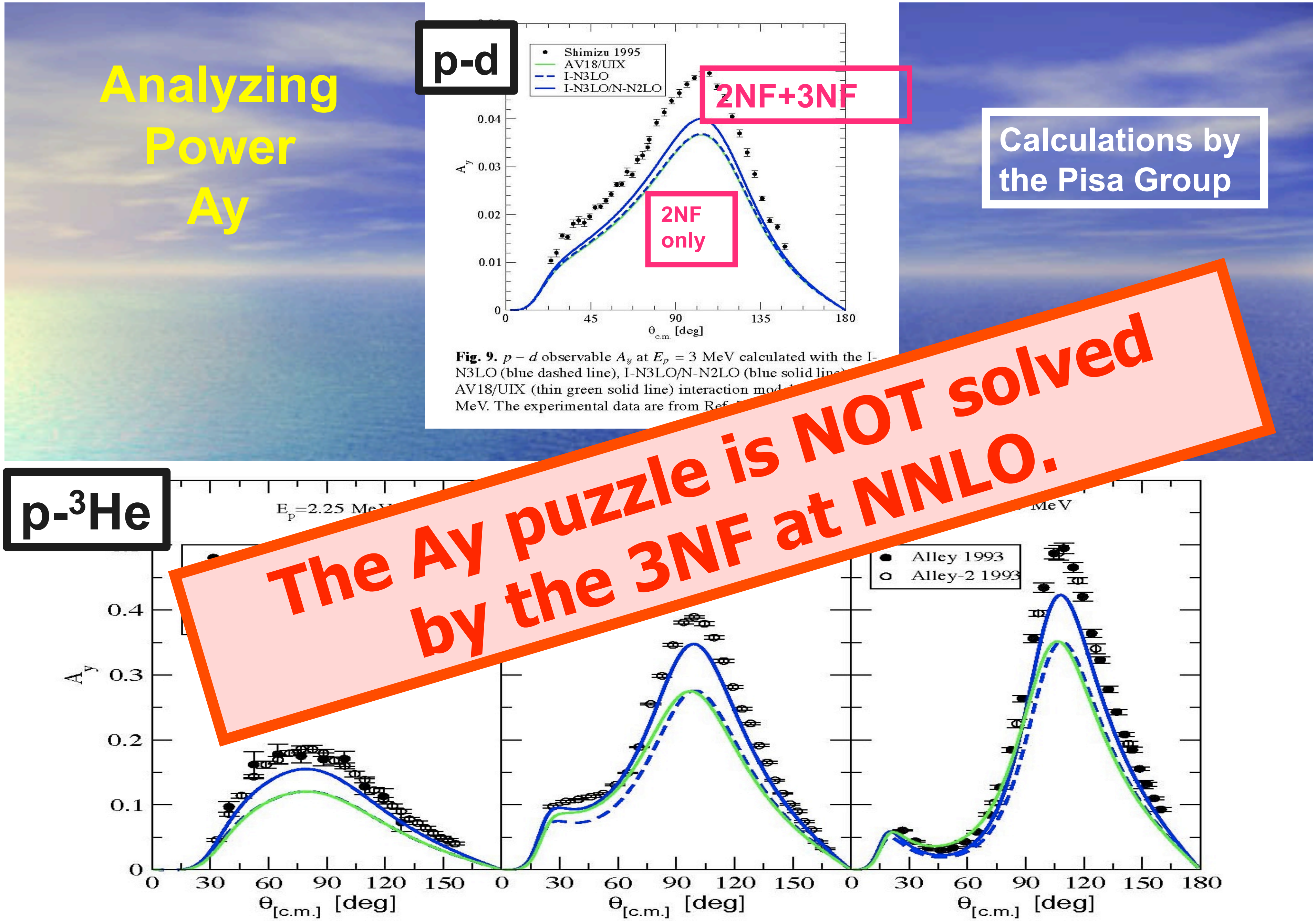


Fig. 6. $p - ^3$He A_y observable calculated with the I-N3LO (blue dashed line), the I-N3LO/N-N2LO (blue solid line), and the AV18/UIX (thin green solid line) interaction models for three different incident proton energies. The experimental data are from Refs. [37,22,36].

The currently popular scheme:

2NF at N3LO

3NF at NNLO

No 4NF

Going beyond the current scheme

- The 2NF is N3LO;
  consistency requires that all contributions are at the same order, so **3NF has to be N3LO, too.**

- There are unresolved problems in 3N, 4N scattering and nuclear structure, where additional 3NFs (and 4NFs) may help.

So, let's take a close look at the development of chiral 3NFs.

Chiral 3N Force

Δ-less

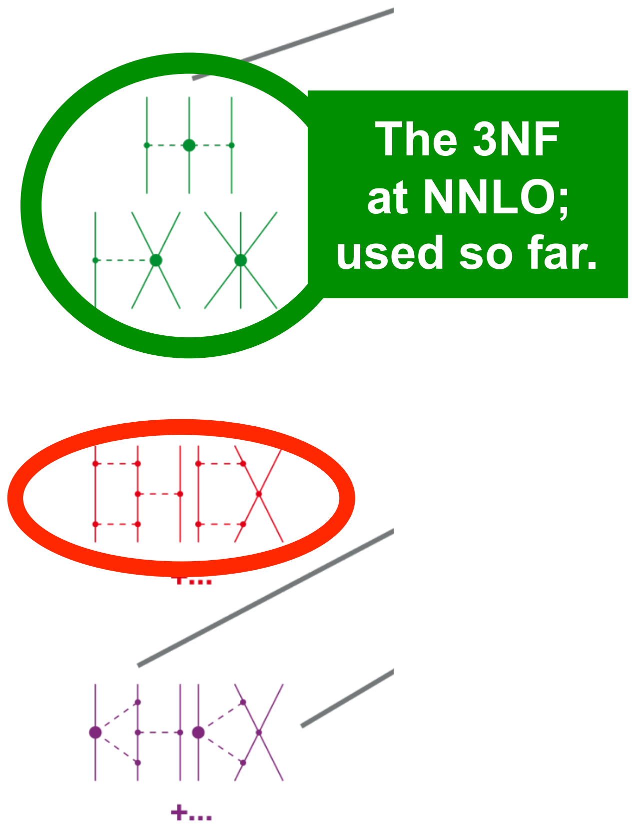
LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

NNLO
$(Q/\Lambda_\chi)^3$

N^{3}LO
$(Q/\Lambda_\chi)^4$

N^{4}LO
$(Q/\Lambda_\chi)^5$

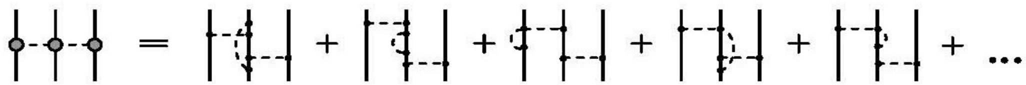


The 3NF at N3LO explicitly

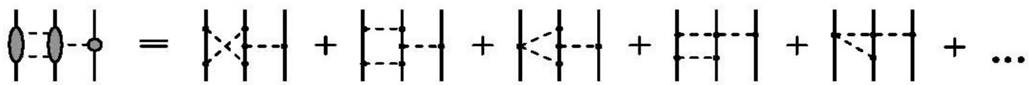
One-loop, leading vertices

Ishikawa & Robilotta, PRC 76, 014006 (2007)

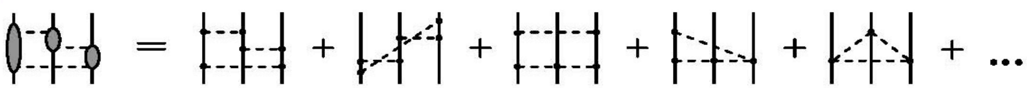
2π−exchange



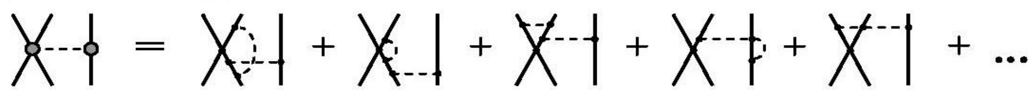
2π-1π−exchange



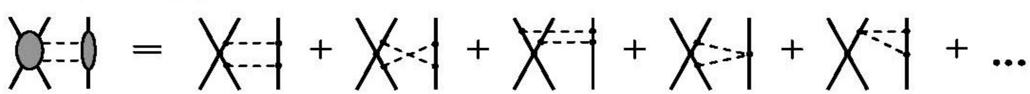
ring diagrams



contact-1π−exchange



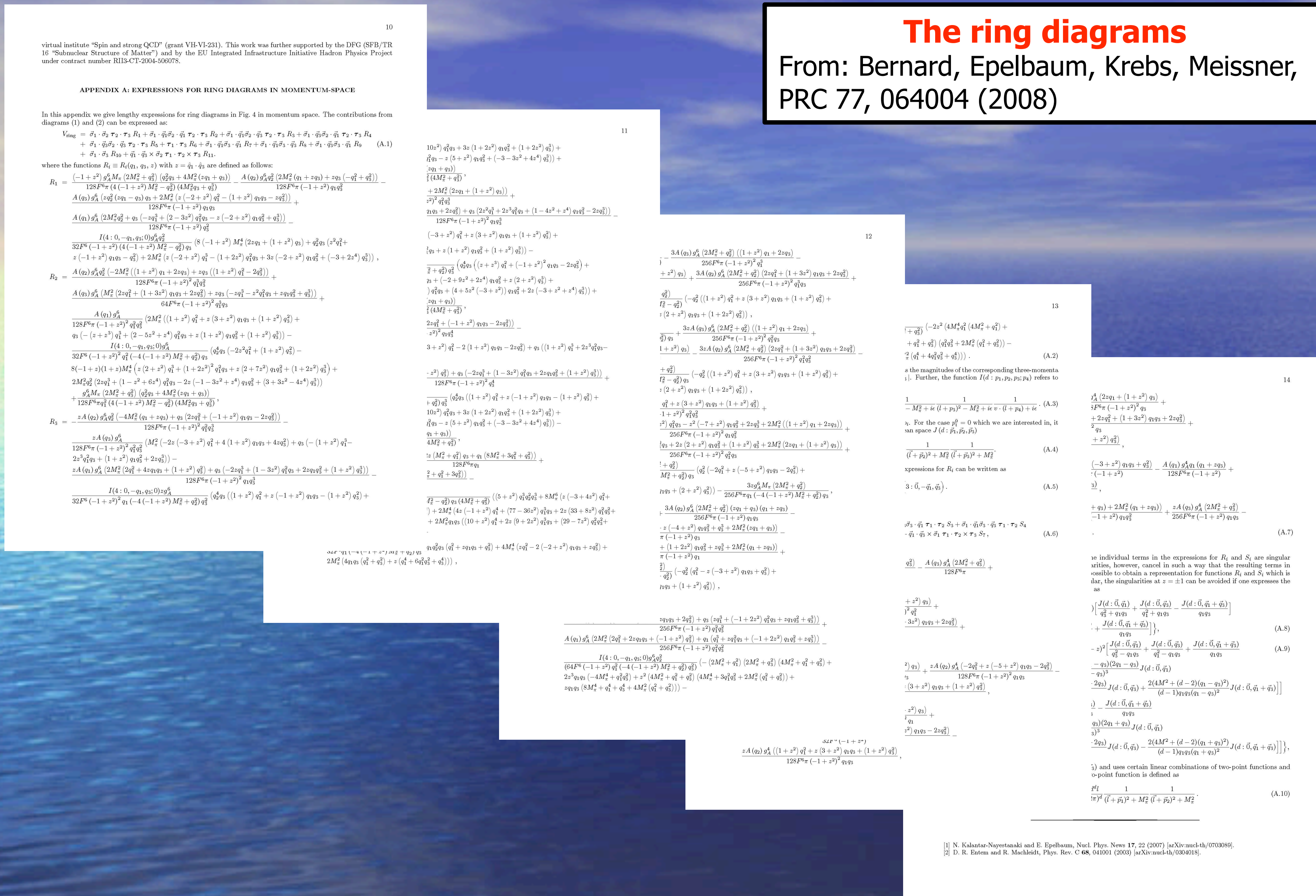
contact-2π−exchange



Bernard, Epelbaum, Krebs, Meissner, PRC 77, 064004 (2008); PRC 84, 054001 (2011).

The ring diagrams

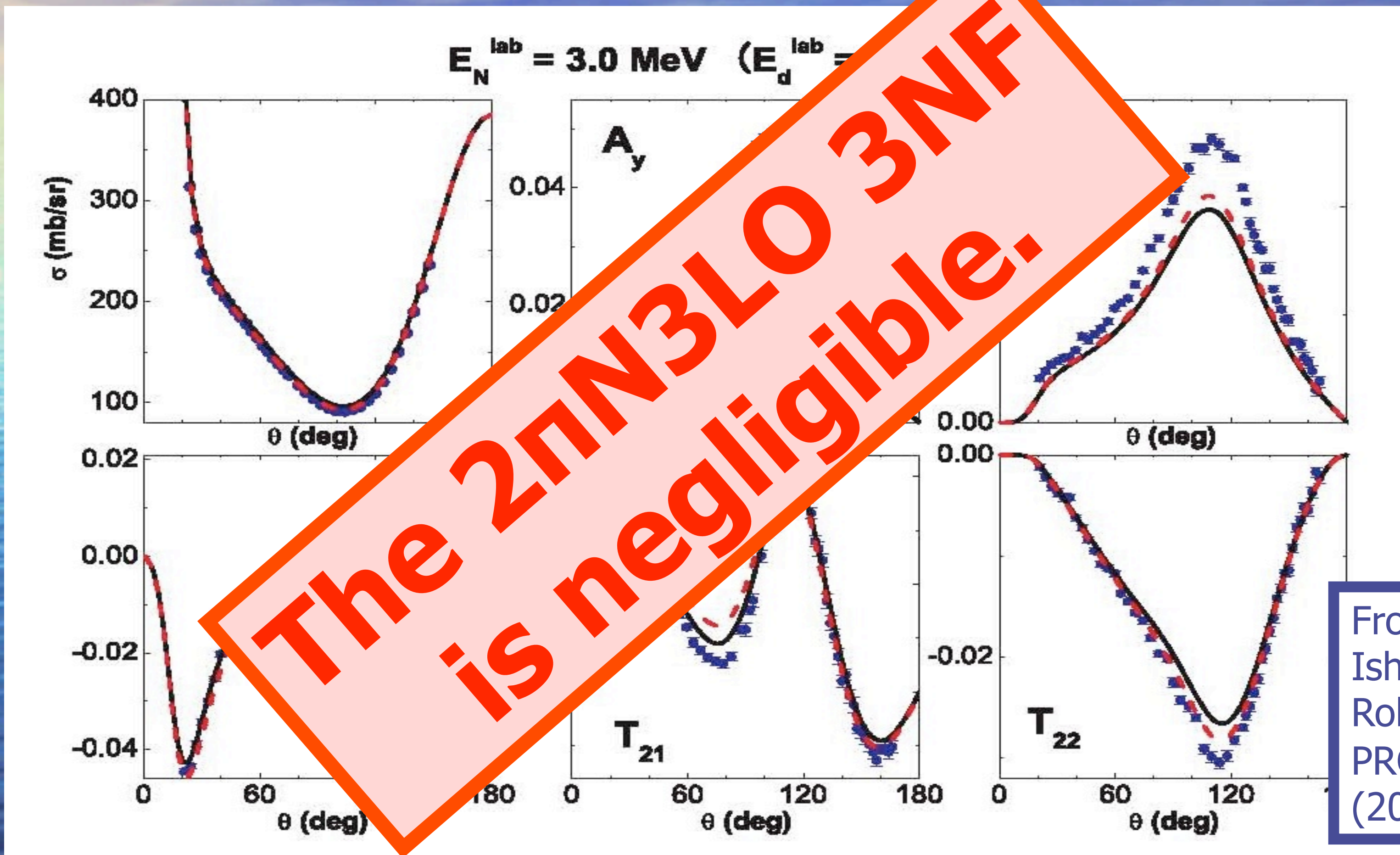
From: Bernard, Epelbaum, Krebs, Meissner, PRC 77, 064004 (2008)



Proton-deuteron elastic scattering

Black line: 2NF only

Red dashed line: 2NF + 3NF(N2LO+2πN3LO) ≈ 2NF + 3NF(N2LO)



From:
Ishikawa & Robilotta,
PRC 76, 014006 (2007)

Triton calculation with long-range N3LO 3NF exists:

Skibinski et al., PRC 84, 054005 (2011).
Strong cutoff dependence of results.
For best fit NN potential, N3LO 3NF contributions about -0.1 MeV.

Chiral 3N Force

Δ-less

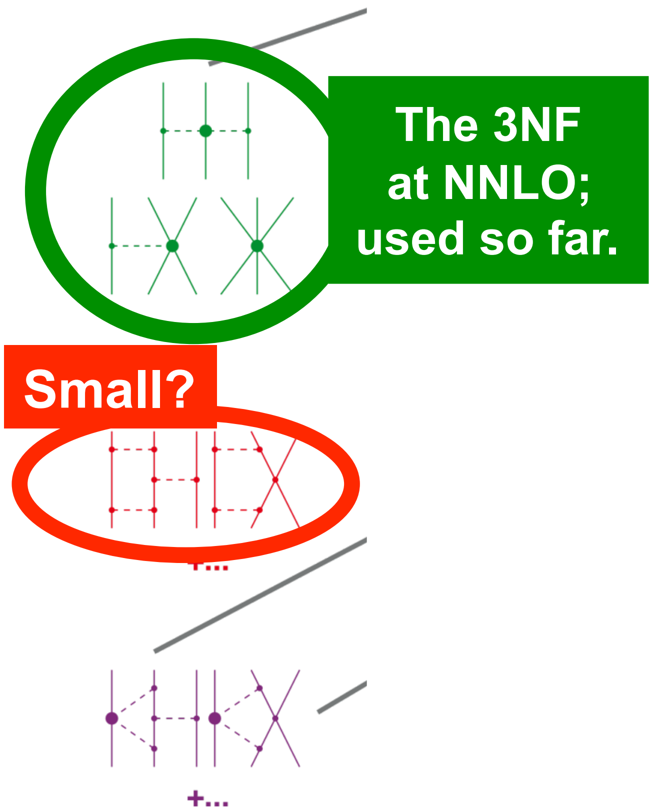
LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

NNLO
$(Q/\Lambda_\chi)^3$

N^{3}LO
$(Q/\Lambda_\chi)^4$

N^{4}LO
$(Q/\Lambda_\chi)^5$



Chiral 3N Force

Δ-less

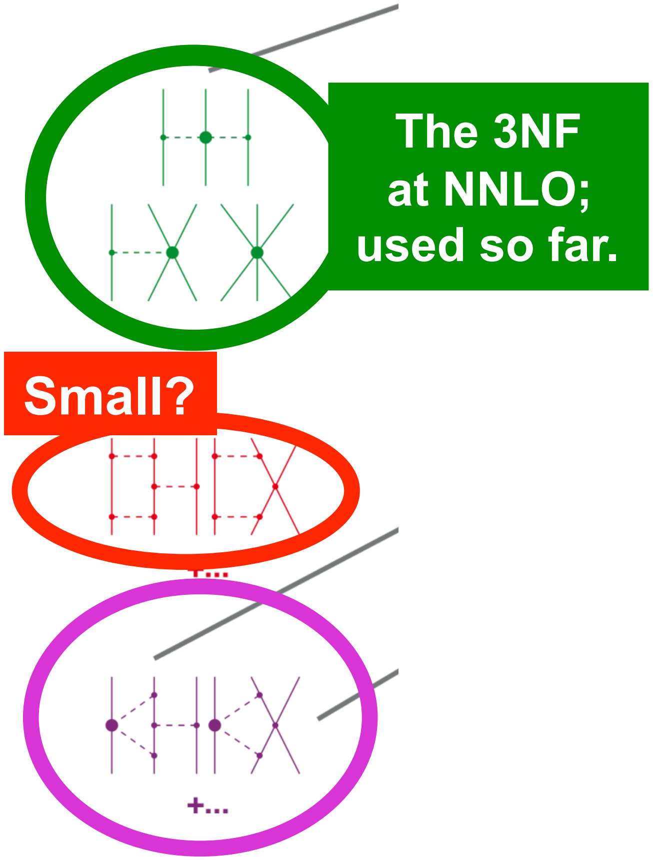
LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

NNLO
$(Q/\Lambda_\chi)^3$

N^{3}LO
$(Q/\Lambda_\chi)^4$

N^{4}LO
$(Q/\Lambda_\chi)^5$



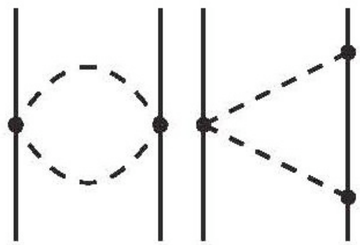
What to expect from those N4LO 1-loop diagrams?

Compare to "similar" 2NF diagrams.

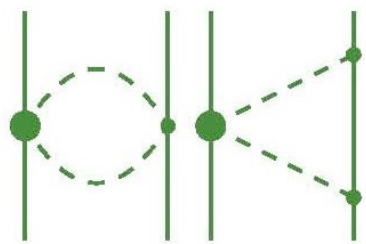
Corresponding 2NF contributions

2π

1-loop lead. vert.



1-loop One c_i vert.



Corresponding 2NF contributions

	2π	3π

1-loop lead. vert.

Kaiser (2000)

1-loop One c_i vert.

Kaiser (2001)

Chiral 3N Force

Δ-less

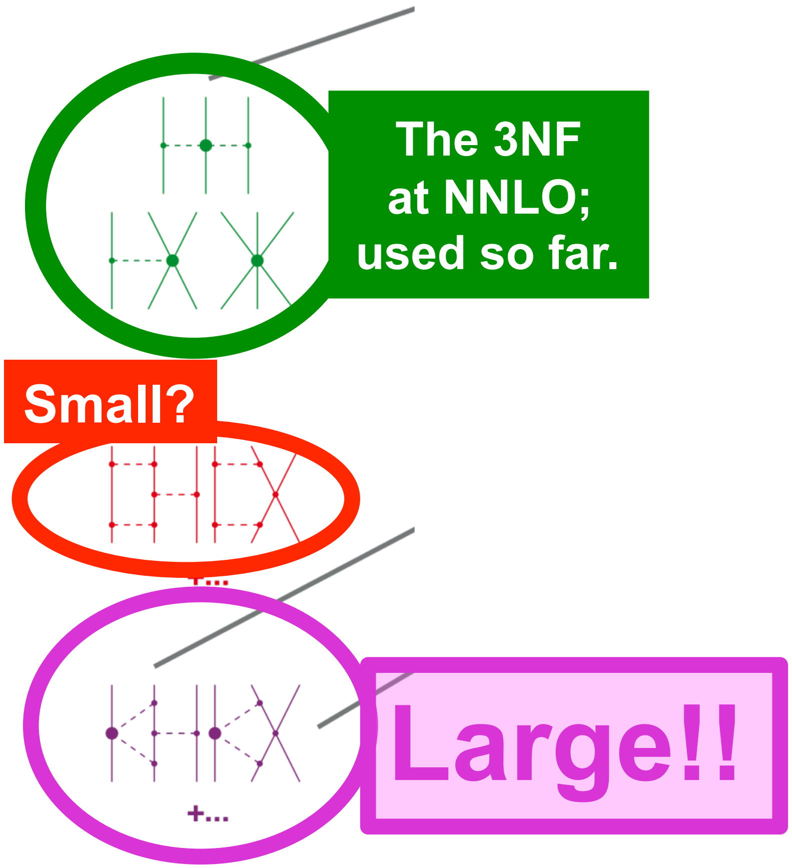
LO $(Q/\Lambda_\chi)^0$

NLO $(Q/\Lambda_\chi)^2$

NNLO $(Q/\Lambda_\chi)^3$

N^{3}LO $(Q/\Lambda_\chi)^4$

N^{4}LO $(Q/\Lambda_\chi)^5$



A variation of the theory is possible: Introduce Δ degrees of freedom

Chiral 3N Force

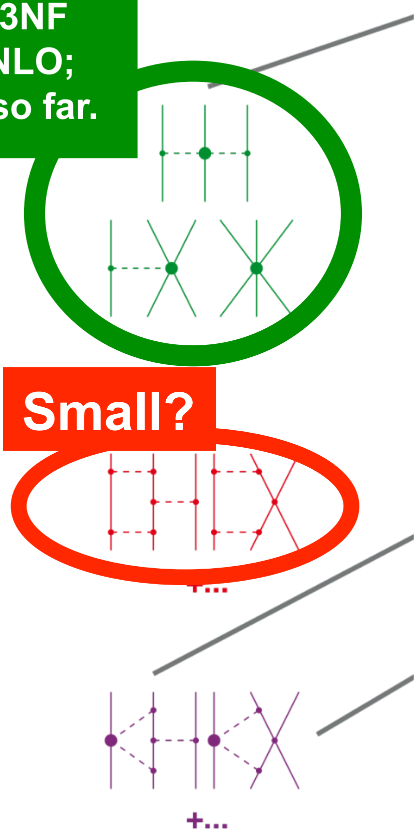
Δ-less

LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

The 3NF at NNLO; used so far.

NNLO
$(Q/\Lambda_\chi)^3$



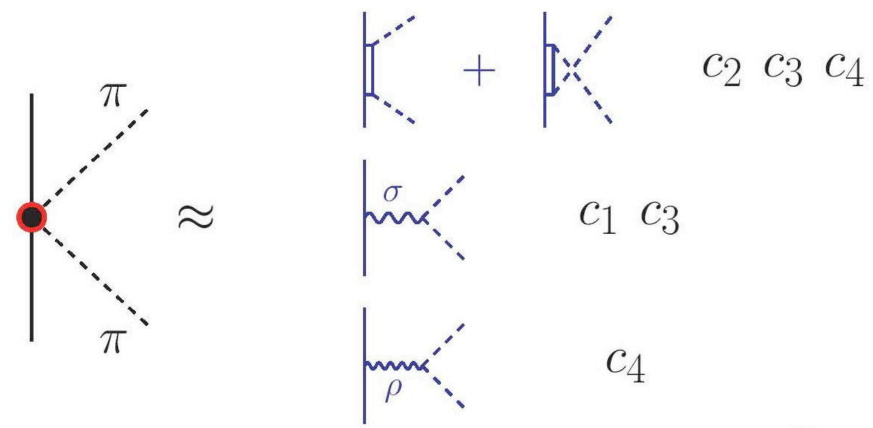
Small?

N^{3}LO
$(Q/\Lambda_\chi)^4$

N^{4}LO
$(Q/\Lambda_\chi)^5$

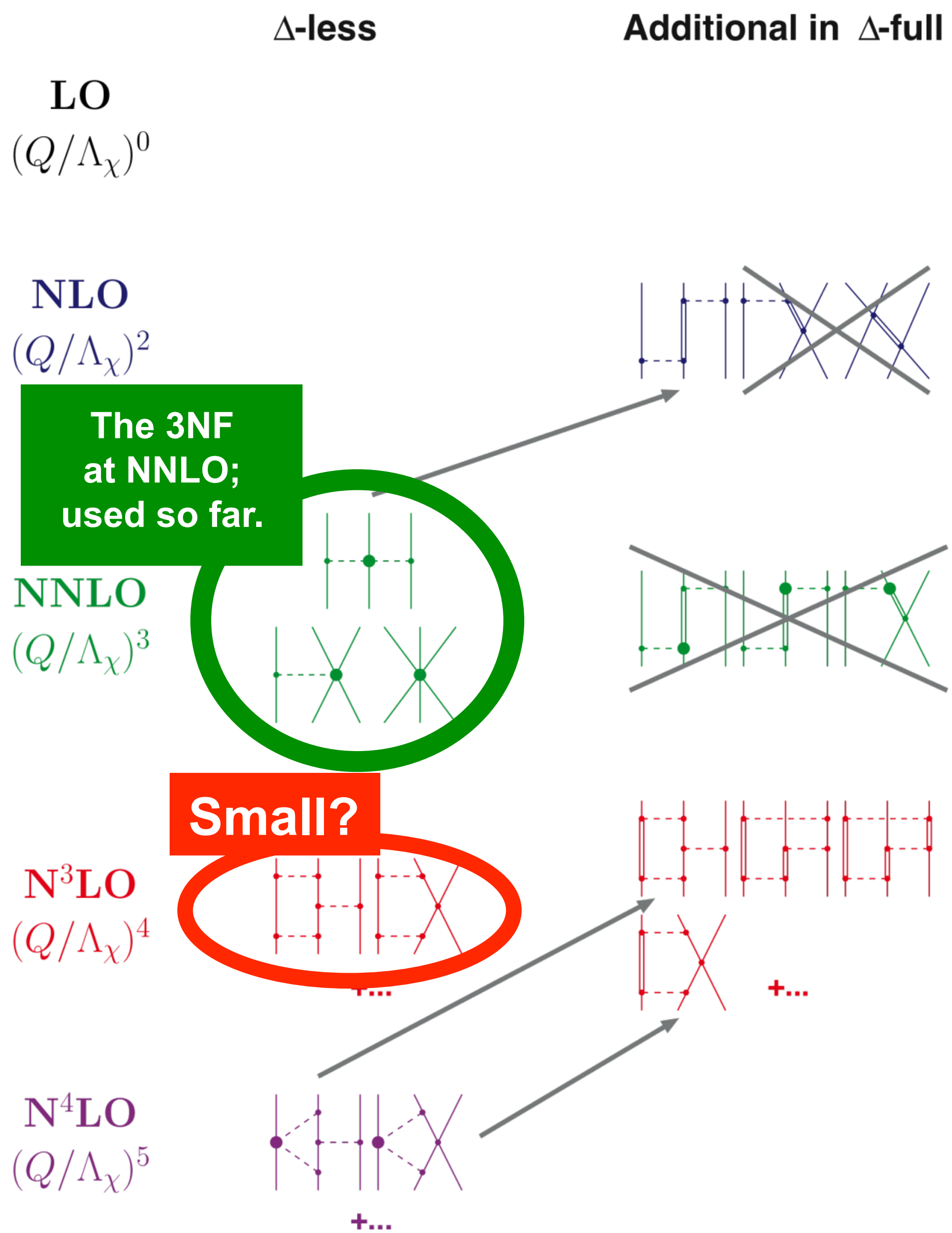
pi-N Lagrangian with two derivatives ("next-to-leading" order)

$$\mathcal{L}^{(2)}_{\pi N, c_i} = \bar{N}\Big[2\, c_1\, m_\pi^2\,(U+U^\dagger) + \left(c_2 - \frac{g_A^2}{8M_N}\right) u_0^2 + c_3\, u_\mu\, u^\mu + \frac{i}{2}\left(c_4 + \frac{1}{4M_N}\right)\vec{\sigma}\cdot(\vec{u}\times\vec{u})\Big] N$$

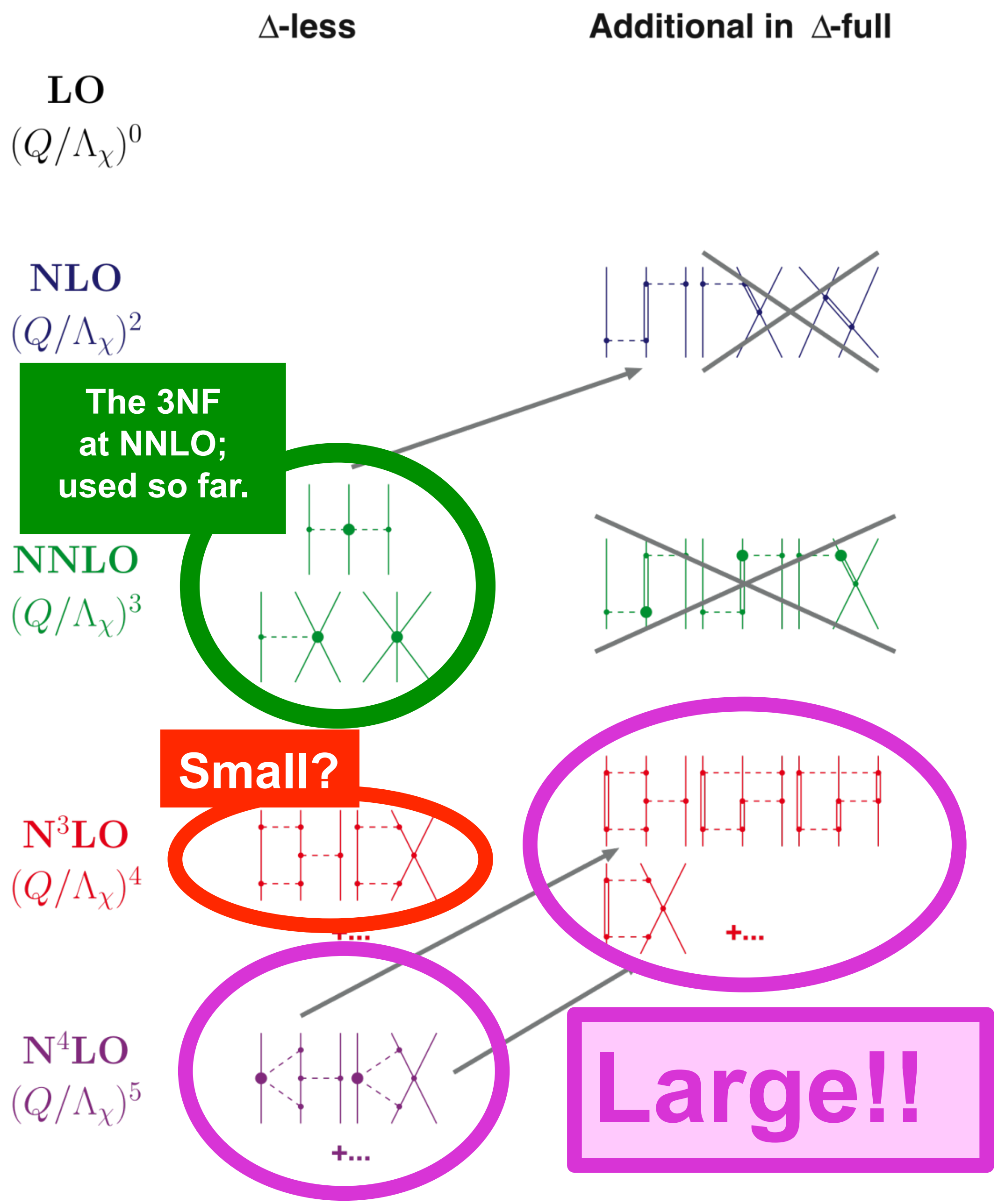


Bernard et al. '97

Chiral 3N Force



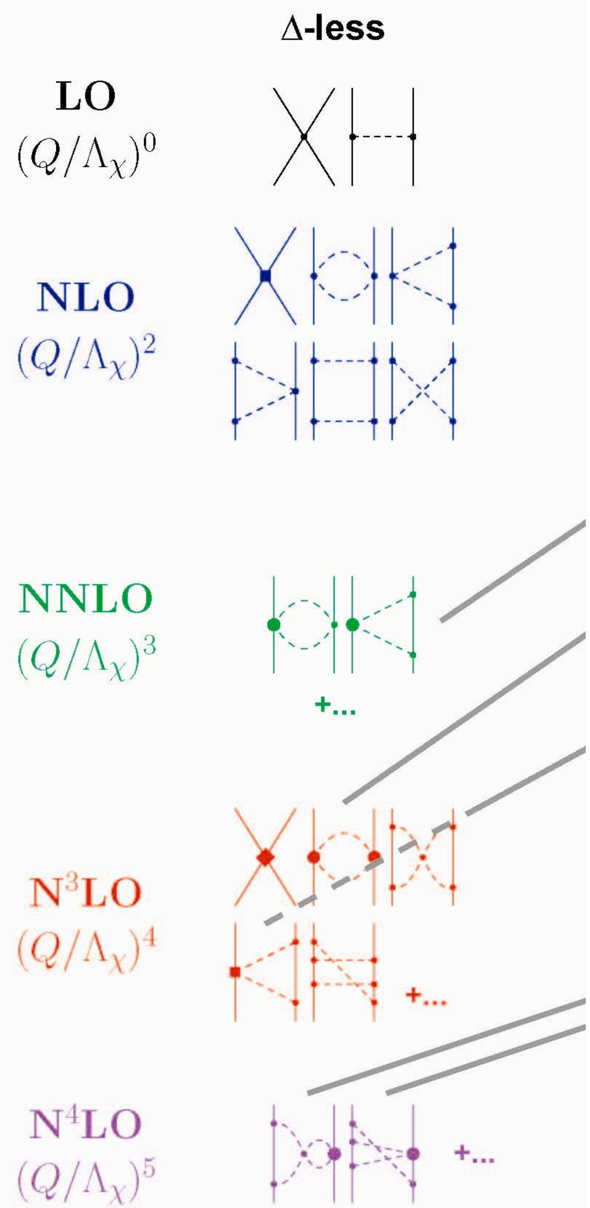
Chiral 3N Force



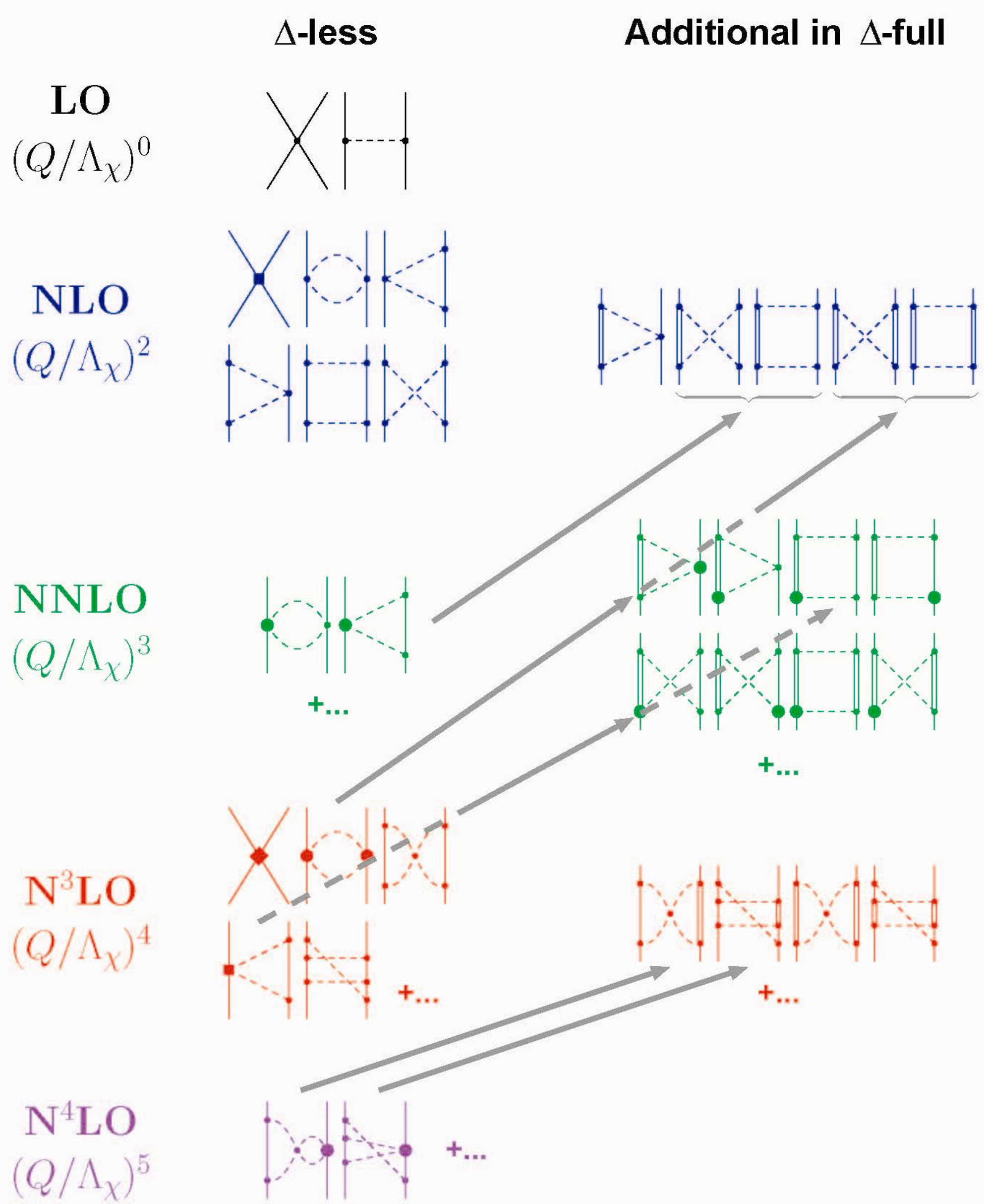
So, with Δ's you have to go "only" to N3LO (to catch the large 3NFs),
But notice that there are many more diagrams At N3LO with Δ's as compared to Δ-less.
So, it's not necessarily simpler.

Notice further that in the Δ-full theory the 2NF Gets also more complicated.

Chiral 2N Force



Chiral 2N Force



Summarize: Δ-full vs. Δ-less (2NF and 3NF).

Chiral 2N Force

Δ-less

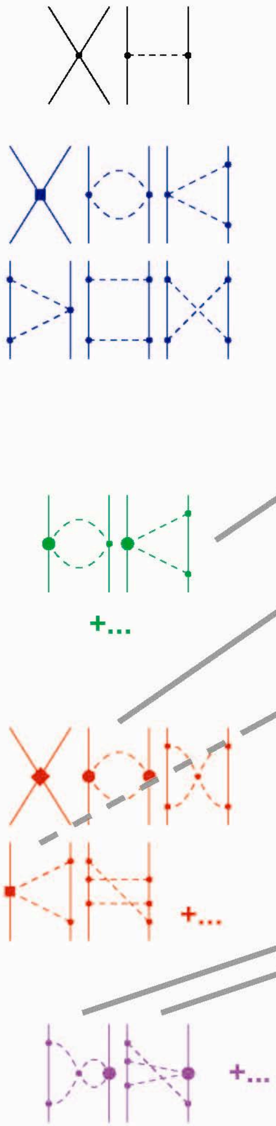
LO $(Q/\Lambda_\chi)^0$

NLO $(Q/\Lambda_\chi)^2$

NNLO $(Q/\Lambda_\chi)^3$

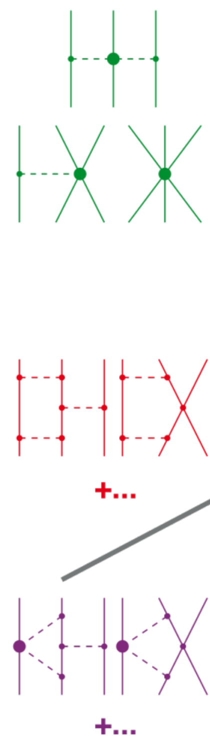
N^{3}LO $(Q/\Lambda_\chi)^4$

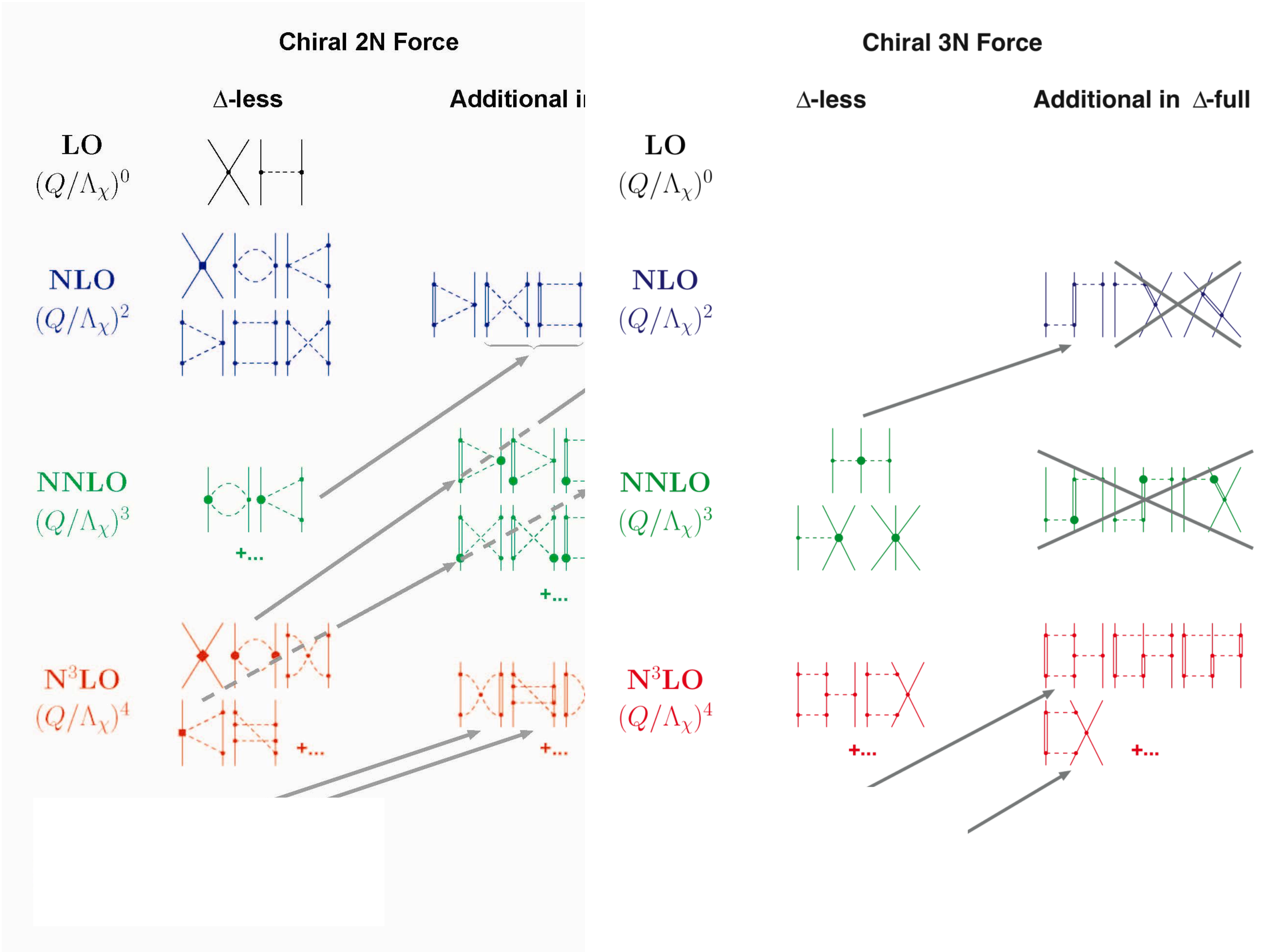
N^{4}LO $(Q/\Lambda_\chi)^5$



Chiral 3N Force

Δ-less





Chiral 2N Force
Δ-less
Additional i
LO
$(Q/\Lambda_\chi)^0$
NLO
$(Q/\Lambda_\chi)^2$
NNLO
$(Q/\Lambda_\chi)^3$
+...
+...
N^{3}LO
$(Q/\Lambda_\chi)^4$
+...
+...
Chiral 3N Force
Δ-less
Additional in Δ-full
LO
$(Q/\Lambda_\chi)^0$
NLO
$(Q/\Lambda_\chi)^2$
NNLO
$(Q/\Lambda_\chi)^3$
N^{3}LO
$(Q/\Lambda_\chi)^4$
+...
+...

What's the better approach is a matter of taste

- Bochum (E. Epelbaum & H. Krebs) pursues Δ-full
- Idaho (R. M. & D. R. Entem) Δ-less

Chiral 3N Force

Δ-less

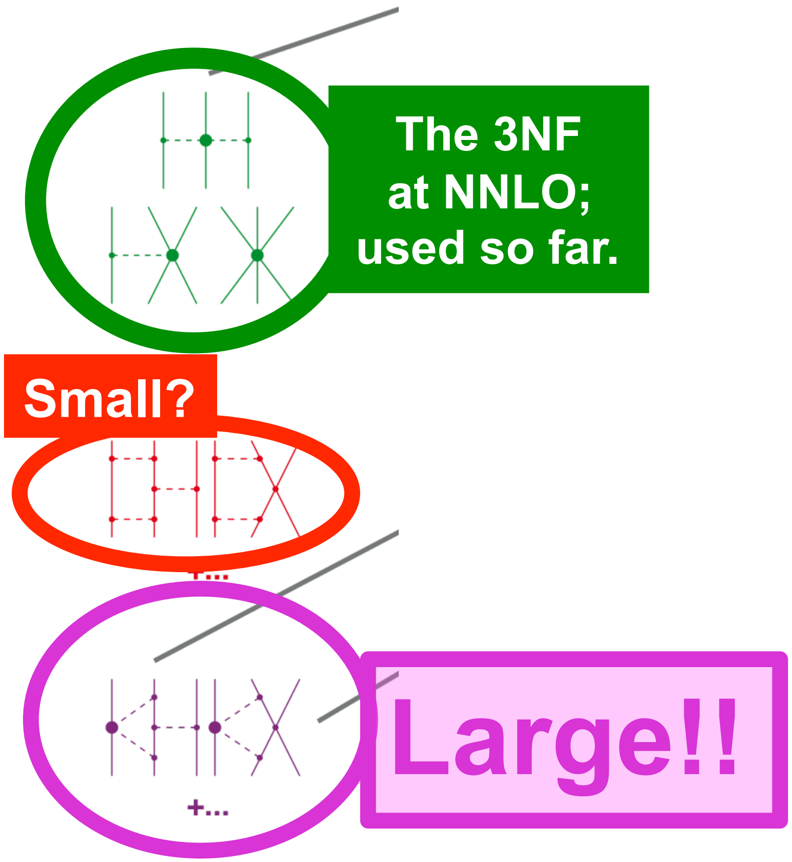
LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

NNLO
$(Q/\Lambda_\chi)^3$

N^{3}LO
$(Q/\Lambda_\chi)^4$

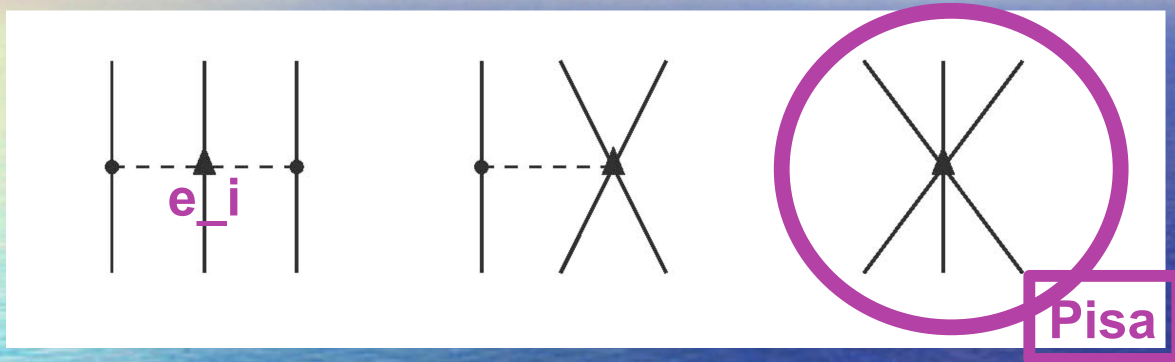
N^{4}LO
$(Q/\Lambda_\chi)^5$



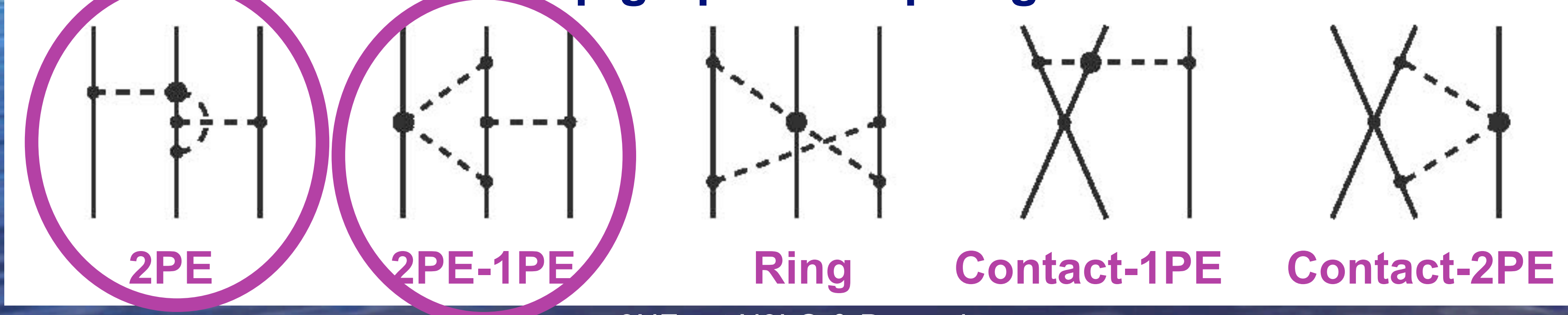
The Δ-less N4LO (ν=5) 3NF in more detail: Tree diagrams and 1-loop graphs

$$\text{Power} = 2 + 2L + \sum_{\text{all vertices}} \Delta_i$$

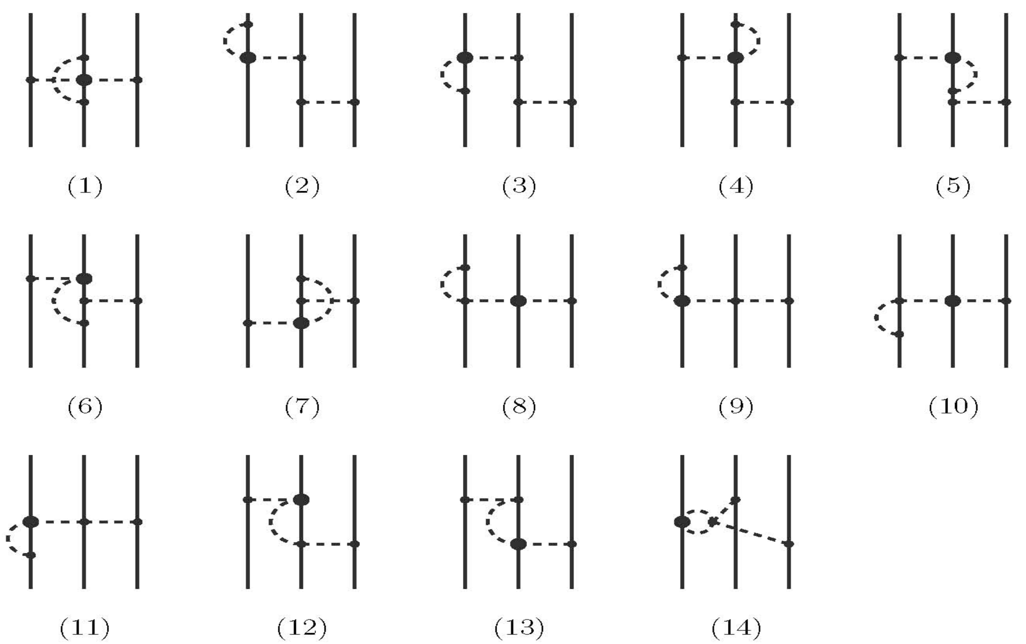
$$\text{with } \Delta_i = d_i + \frac{n_i}{2} - 2$$



1-loop graphs: 5 topologies



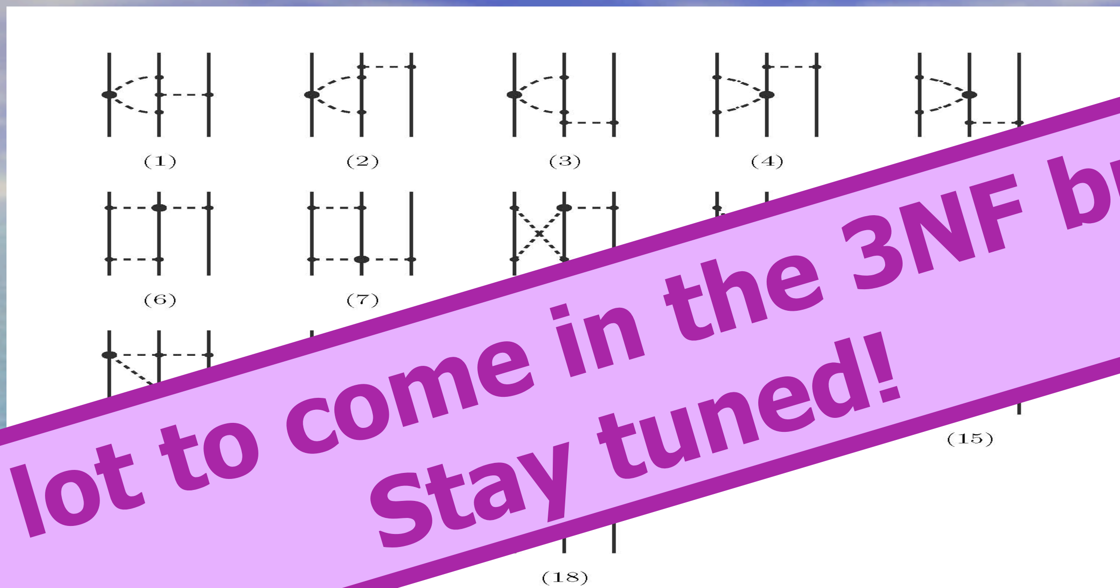
The N4LO 3NF 2PE diagrams



$$V_{2PE}^{(N4LO)} = \frac{g_A{}^2}{8 f_\pi{}^4} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \; \vec{\sigma}_2 \cdot \vec{q}_2}{(q_1{}^2 + m_\pi{}^2)(q_2{}^2 + m_\pi{}^2)} \left\{ \vec{\tau}_1 \cdot \vec{\tau}_2 (-4 \; \tilde{c}_1 \; m_\pi{}^2 + 2 \; \tilde{c}_3 \; \vec{q}_1 \cdot \vec{q}_2 + \tilde{c}_4 \; \vec{\tau}_1 \cdot (\vec{\tau}_2 \times \vec{\tau}_3) \; \vec{\sigma}_3 \cdot (\vec{q}_1 \times \vec{q}_2) \right\}$$

same structure as NNLO 2PE 3NF, but $\tilde{c}_1$, $\tilde{c}_3$, and $\tilde{c}_4$ get renormalized.

The N4LO 3NF 2PE-1PE diagrams



Still a lot to come in the 3NF business. Stay tuned!

Many new isospin/spin/momentum structures, some similar to N3LO 2PE-1PE 3NF.

$$
_{2PE-1PE} = \frac{\vec{\sigma}_2 \cdot \vec{q}_2}{{q_2}^2 + {m_\pi}^2} \times
\begin{aligned}[t]
&\{\vec{\tau}_1 \cdot \vec{\tau}_2 [\vec{\sigma}_3 \cdot \vec{q}_1 \;\; \vec{q}_1 \cdot \vec{q}_2 \;\; F_1(q_1) + \vec{\sigma}_1 \cdot \vec{q}_1 \;\; \vec{q}_1 \cdot \vec{q}_2 \;\; F_2(q_1) \\
&\qquad + \vec{\sigma}_3 \cdot \vec{q}_1 \;\; F_3(q_1) + \vec{\sigma}_3 \cdot \vec{q}_2 \;\; F_4(q_1) + \vec{\sigma}_1 \cdot \vec{q}_2 \;\; F_5(q_1)] \\
&+ \vec{\tau}_2 \cdot \vec{\tau}_3 [\vec{\sigma}_1 \cdot \vec{q}_1 \;\; \vec{q}_1 \cdot \vec{q}_2 \;\; F_6(q_1) + \vec{\sigma}_3 \cdot \vec{q}_1 \;\; \vec{q}_1 \cdot \vec{q}_2 \;\; F_7(q_1) \\
&\qquad + \vec{\sigma}_1 \cdot \vec{q}_2 \;\; F_8(q_1) + \vec{\sigma}_3 \cdot \vec{q}_1 \;\; F_9(q_1) + \vec{\sigma}_3 \cdot \vec{q}_2 \;\; F_{10}(q_1)] \\
&+ \vec{\tau}_1 \cdot (\vec{\tau}_2 \times \vec{\tau}_3) \;\; \vec{q}_1 \cdot (\vec{\sigma}_1 \times \vec{\sigma}_3) \;\; F_{11}(q_1)\}
\end{aligned}
$$

General Summary

- Substantial advances in chiral nuclear forces during the past decade. The major milestone of the decade: "high precision" NN pots. at N3LO; good for nuclear structure.
- But there are still important open issues concerning three-nucleon forces (3NF), which is one reason why we are here.
- Large 3NFs with many new structures to be expected at N3LO of Δ-full or N4LO at Δ-less. Construction is under way.
- There will be many new 3NFs to check out in the near future. Stay tuned.

Some after-thoughts

“There will be many new 3NFs …”

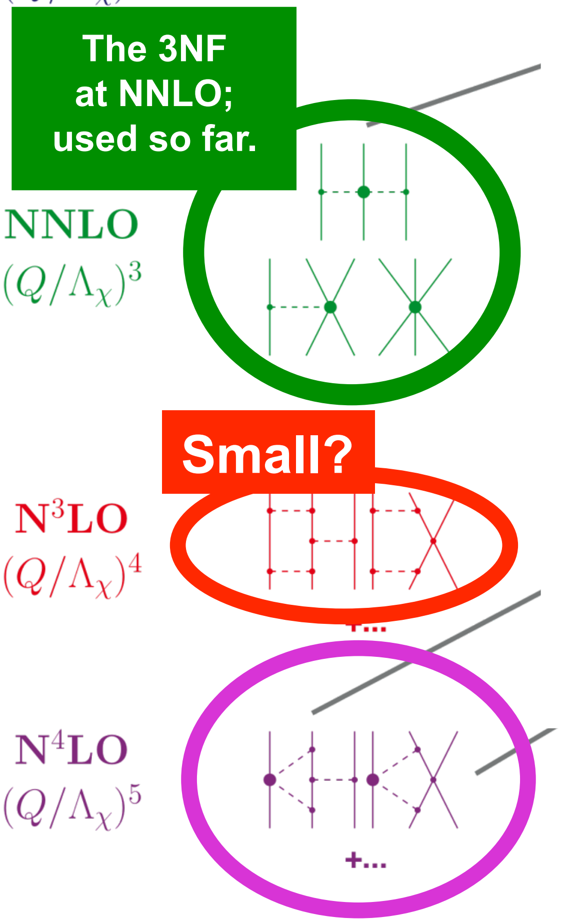
How many?

Too many?!

How not to get crushed
by the Dinosaur?

Chiral 3N Force

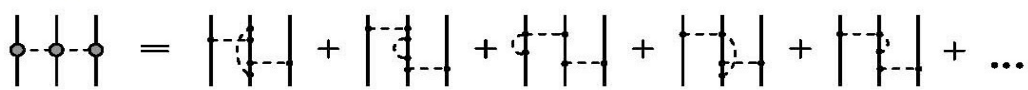
	Δ-less	Additional in Δ-full
LO $(Q/\Lambda_\chi)^0$		
NLO $(Q/\Lambda_\chi)^2$		
NNLO $(Q/\Lambda_\chi)^3$		
N^{3}LO $(Q/\Lambda_\chi)^4$		
N^{4}LO $(Q/\Lambda_\chi)^5$		



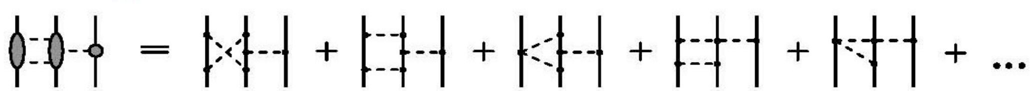
Chiral 3N Force

	Δ-less	Additional in Δ-full
LO $(Q/\Lambda_\chi)^0$		
NLO $(Q/\Lambda_\chi)^2$		

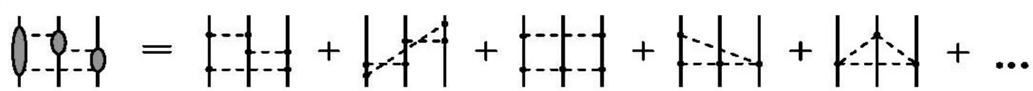
2π–exchange



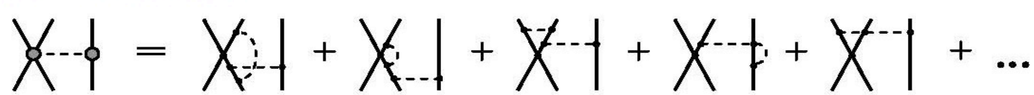
2π-1π–exchange



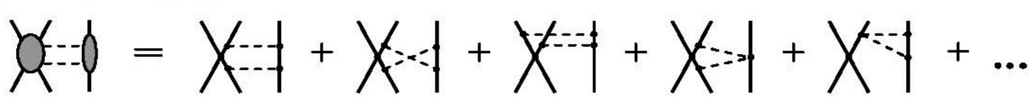
ring diagrams



contact-1π–exchange



contact-2π–exchange

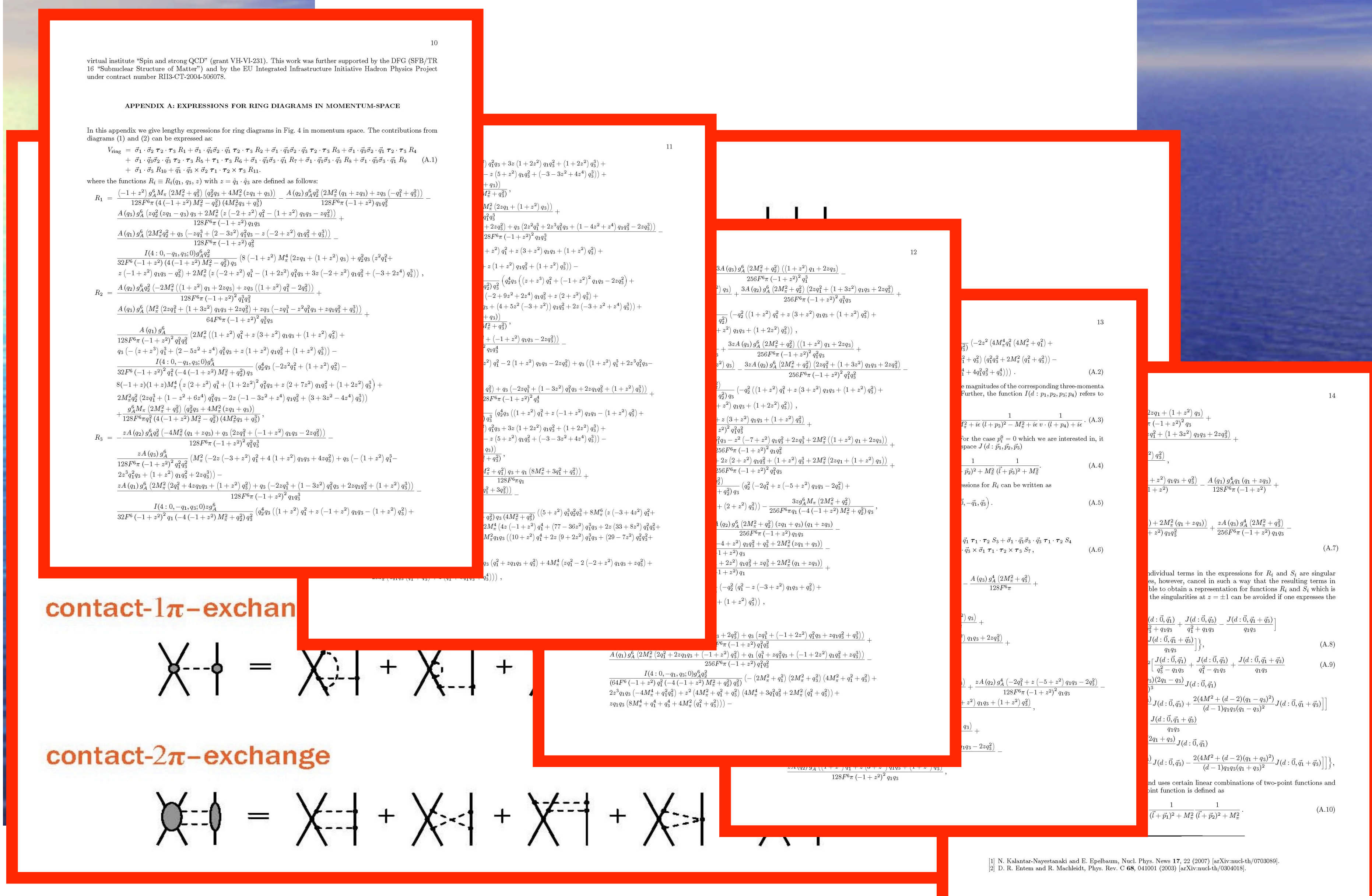


Chiral 3N Force

Δ-less

LO

$(Q/\Lambda_\chi)^0$

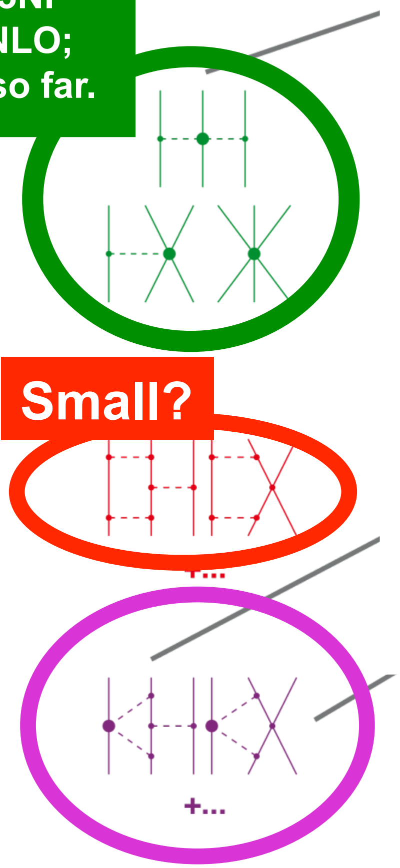


Chiral 3N Force

	Δ-less	Additional in Δ-full
LO $(Q/\Lambda_\chi)^0$		
NLO $(Q/\Lambda_\chi)^2$		
NNLO $(Q/\Lambda_\chi)^3$		
N^{3}LO $(Q/\Lambda_\chi)^4$		
N^{4}LO $(Q/\Lambda_\chi)^5$		

The 3NF at NNLO; used so far.

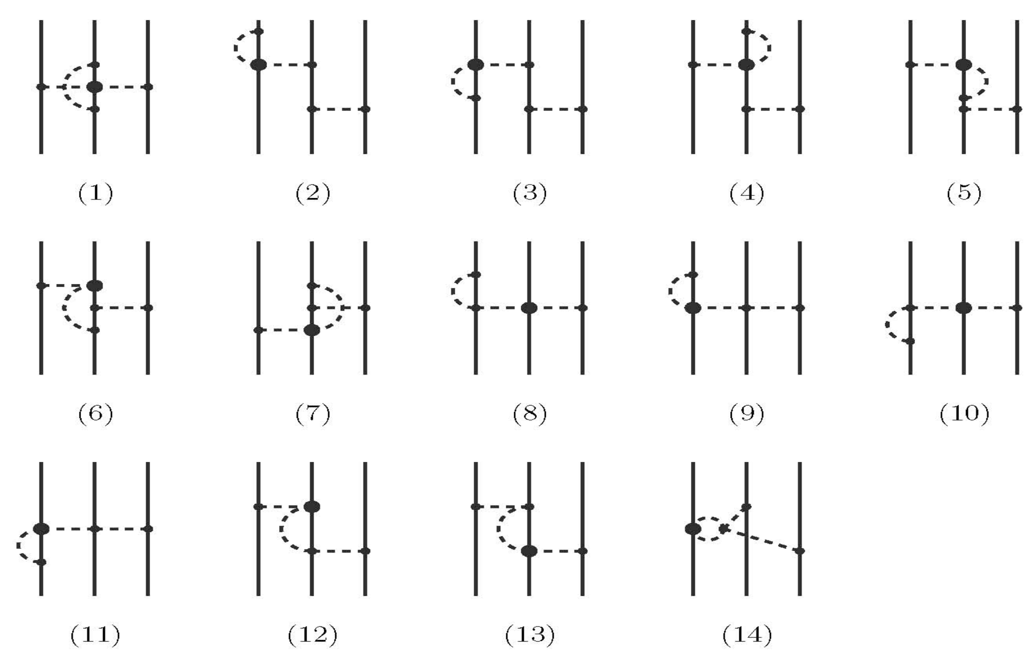
Small?



Chiral 3N Force

	Δ-less	Additional in Δ-full
LO $(Q/\Lambda_\chi)^0$		

NLO



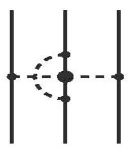
$$V^{(N4LO)}_{2PE} = \frac{g_A{}^2}{8 f_\pi{}^4} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \;\; \vec{\sigma}_2 \cdot \vec{q}_2}{(q_1{}^2 + m_\pi{}^2)(q_2{}^2 + m_\pi{}^2)} \{\vec{\tau}_1 \cdot \vec{\tau}_2 (-4 \; \tilde{c}_1 \; m_\pi{}^2 + 2 \; \tilde{c}_3 \; \vec{q}_1 \cdot \vec{q}_2 + \tilde{c}_4 \;\; \vec{\tau}_1 \cdot (\vec{\tau}_2 \times \vec{\tau}_3) \;\; \vec{\sigma}_3 \cdot (\vec{q}_1 \times \vec{q}_2)\}$$

same structure as NNLO 2PE 3NF, but $\tilde{c}_1$, $\tilde{c}_3$, and $\tilde{c}_4$ get renormalized.

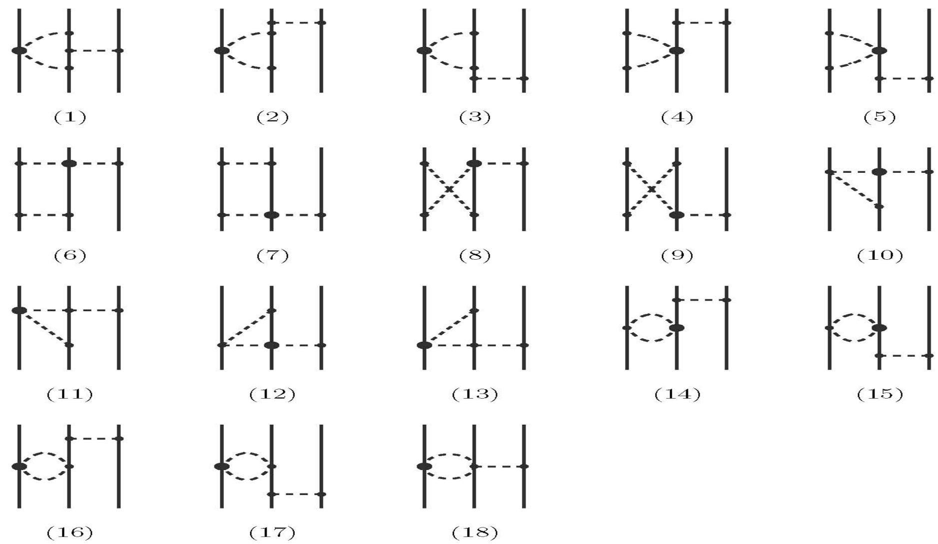
Chiral 3N Force

Δ-less | **Additional in Δ-full**

LO



(1)



$$V^{N4LO}_{2PE-1PE} = \frac{\vec{\sigma}_2 \cdot \vec{q}_2}{q_2{}^2 + m_\pi{}^2} \times$$

$$\{\vec{\tau}_1 \cdot \vec{\tau}_2[\vec{\sigma}_3 \cdot \vec{q}_1 \;\; \vec{q}_1 \cdot \vec{q}_2 \;\; F_1(q_1) + \vec{\sigma}_1 \cdot \vec{q}_1 \;\; \vec{q}_1 \cdot \vec{q}_2 \;\; F_2(q_1)$$

$$+\vec{\sigma}_3 \cdot \vec{q}_1 \;\; F_3(q_1) + \vec{\sigma}_3 \cdot \vec{q}_2 \;\; F_4(q_1) + \vec{\sigma}_1 \cdot \vec{q}_2 \;\; F_5(q_1)]$$

$$+\vec{\tau}_2 \cdot \vec{\tau}_3[\vec{\sigma}_1 \cdot \vec{q}_1 \;\; \vec{q}_1 \cdot \vec{q}_2 \;\; F_6(q_1) + \vec{\sigma}_3 \cdot \vec{q}_1 \;\; \vec{q}_1 \cdot \vec{q}_2 \;\; F_7(q_1)$$

$$+\vec{\sigma}_1 \cdot \vec{q}_2 \;\; F_8(q_1) + \vec{\sigma}_3 \cdot \vec{q}_1 \;\; F_9(q_1) + \vec{\sigma}_3 \cdot \vec{q}_2 \;\; F_{10}(q_1)]$$

$$+\vec{\tau}_1 \cdot (\vec{\tau}_2 \times \vec{\tau}_3) \;\; \vec{q}_1 \cdot (\vec{\sigma}_1 \times \vec{\sigma}_3) \;\; F_{11}(q_1)\}$$

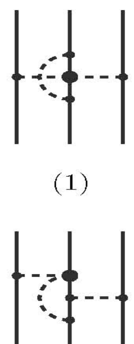
$V^{(N4LO)}_{2PE}$

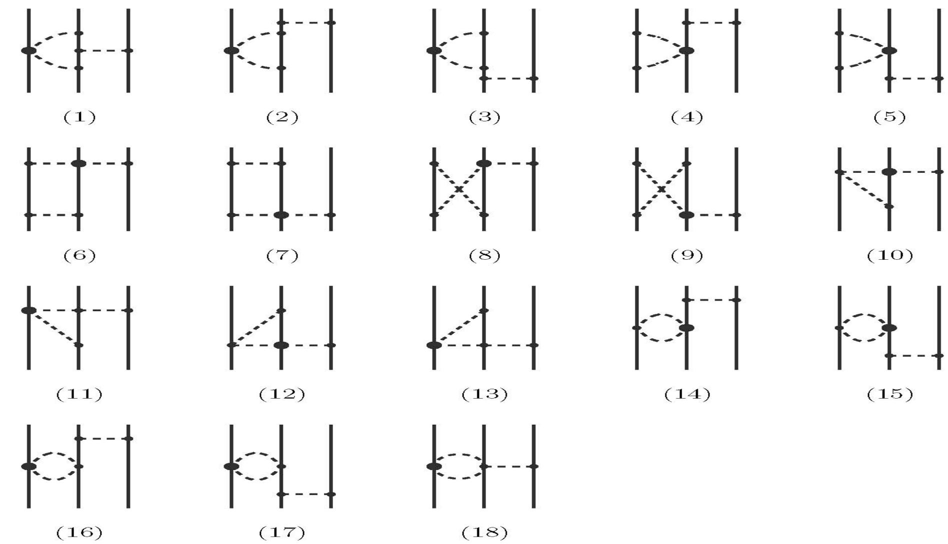
same struc

Chiral 3N Force

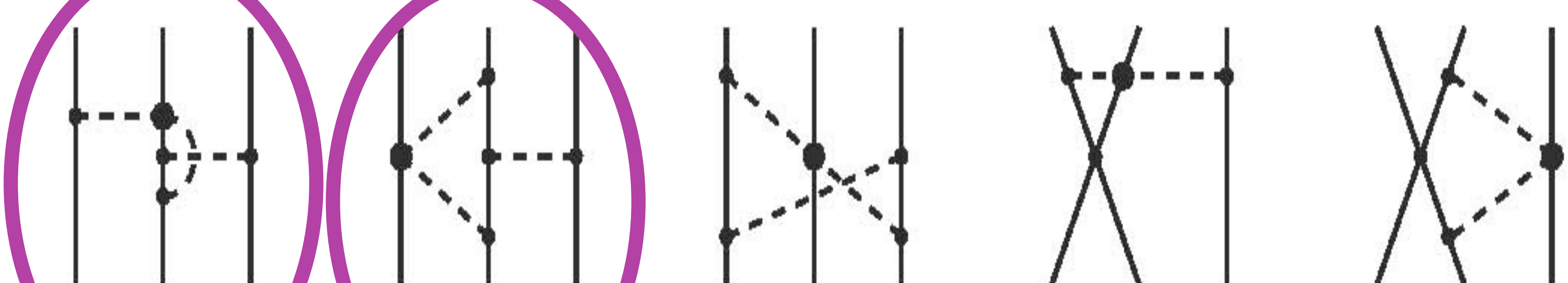
Δ-less | Additional in Δ-full

LO





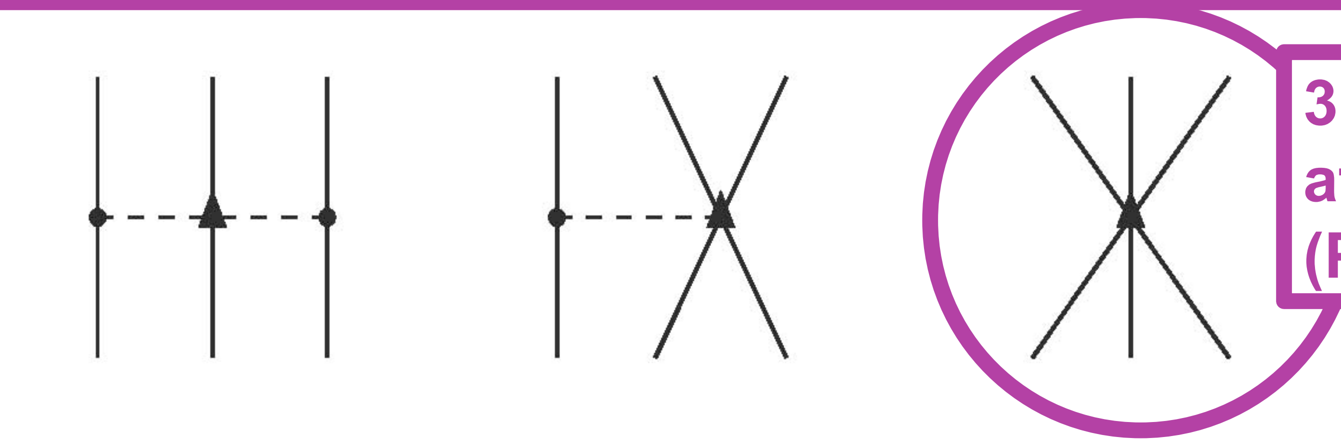
1-loop graphs: 5 topologies



2PE | 2PE-1PE | Ring | Contact-1PE | Contact-2PE

Chiral 3N Force

Δ-less



3NF contacts at N4LO (Pisa)

3NF contacts at N4LO

Girlanda, Kievsky, Viviani, PRC 84, 014001 (2011)

$\mathbf{k}_i = \mathbf{p}_i - \mathbf{p}'_i$ and $\mathbf{Q}_i = \mathbf{p}_i + \mathbf{p}'_i$, $\mathbf{p}_i$ and $\mathbf{p}'_i$ being the initial and final momenta of nucleon i, the potential in momentum space is found to be

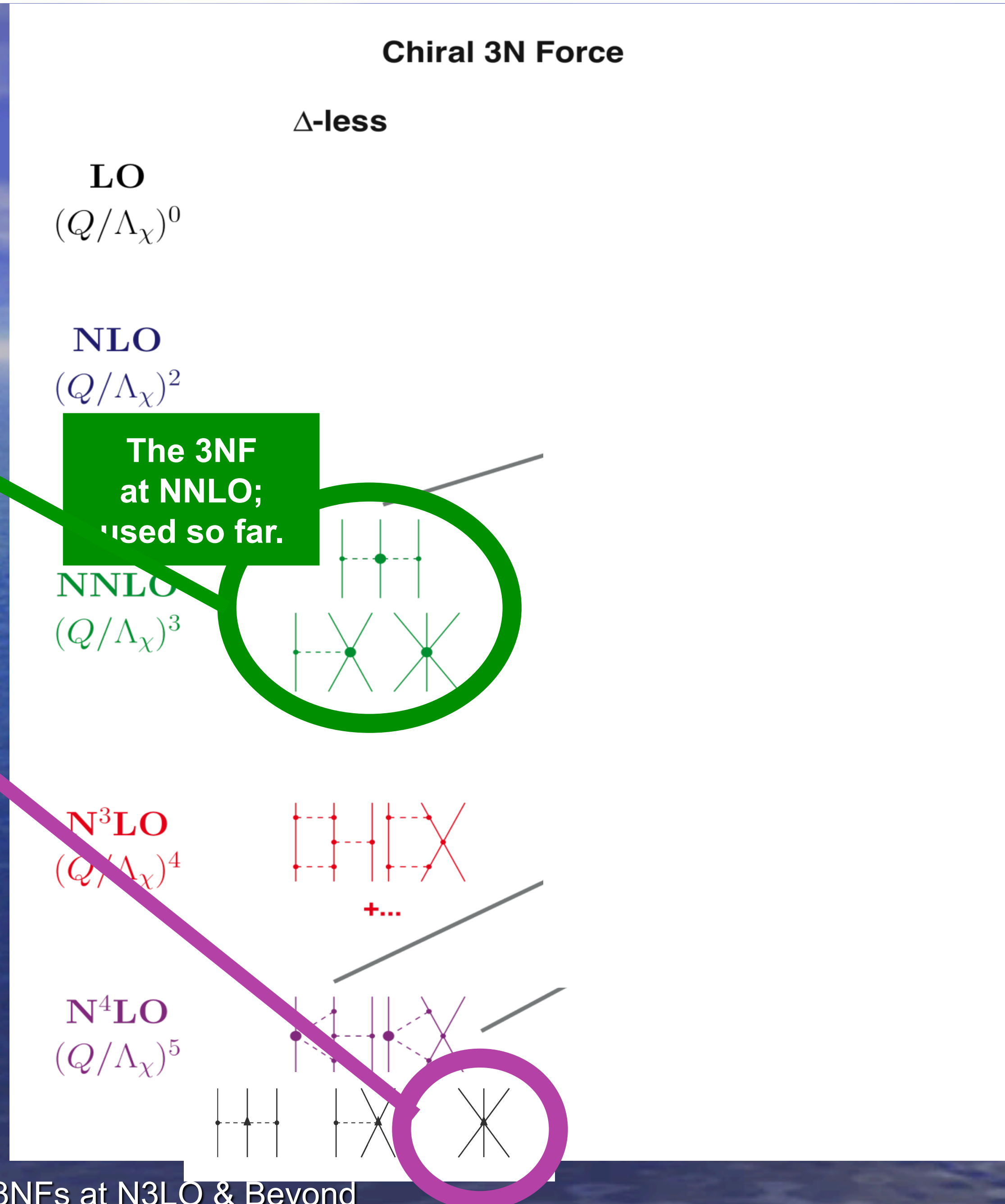
$$V = \sum_{i\neq j\neq k} \Big[-E_1\mathbf{k}_i^2 - E_2\mathbf{k}_i^2\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j - E_3\mathbf{k}_i^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j - E_4\mathbf{k}_i^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$$
$$-E_5\,(3\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_i\cdot\boldsymbol{\sigma}_j - \mathbf{k}_i^2) - E_6\,(3\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_i\cdot\boldsymbol{\sigma}_j - \mathbf{k}_i^2))\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$$
$$+\frac{i}{2}E_7\mathbf{k}_i\times(\mathbf{Q}_i-\mathbf{Q}_j)\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j) + \frac{i}{2}E_8\mathbf{k}_i\times(\mathbf{Q}_i-\mathbf{Q}_j)\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k$$
$$-E_9\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_j\cdot\boldsymbol{\sigma}_j - E_{10}\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_j\cdot\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\Big], \qquad (15)$$

Spin-Orbit Force!

To avoid
the Dinosaur/
the nightmare/
the Pandora's Box,
take a pragmatic
Approach:

A working model:

- use Δ-less
- include NNLO 3NF
- skip N3LO 3NF
  (small in Δ-less!)
- at N4LO start with contact 3NF, use one term at a time, e.g. spin-orbit
- that may already solve your problems.



A working 3NF model
(to be used on an “investigational” basis)

NNLO 3NF

$$V^{3\mathrm{NF}}_{\mathrm{TPE}} = \left(\frac{g_A}{2f_\pi}\right)^2 \frac{1}{2} \sum_{i\neq j\neq k} \frac{(\vec{\sigma}_i\cdot\vec{q}_i)(\vec{\sigma}_j\cdot\vec{q}_j)}{(q_i^2+m_\pi^2)(q_j^2+m_\pi^2)} F^{\alpha\beta}_{ijk}\,\tau_i^\alpha\tau_j^\beta \quad (2)$$

with $\vec{q}_i \equiv \vec{p_i}' - \vec{p}_i$, where $\vec{p}_i$ and $\vec{p_i}'$ are the initial and final momenta of nucleon i, respectively, and

$$F^{\alpha\beta}_{ijk} = \delta^{\alpha\beta}\left[-\frac{4c_1m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2}\,\vec{q}_i\cdot\vec{q}_j\right] + \frac{c_4}{f_\pi^2}\sum_\gamma \epsilon^{\alpha\beta\gamma}\,\tau_k^\gamma\,\vec{\sigma}_k\cdot[\vec{q}_i\times\vec{q}_j]\,. (3)$$

$$V^{3\mathrm{NF}}_{\mathrm{OPE}} = D\,\frac{g_A}{8f_\pi^2}\sum_{i\neq j\neq k}\frac{\vec{\sigma}_j\cdot\vec{q}_j}{q_j^2+m_\pi^2}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)(\vec{\sigma}_i\cdot\vec{q}_j)$$

$$V^{3\mathrm{NF}}_{\mathrm{ct}} = E\,\frac{1}{2}\sum_{j\neq k}\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k\,.$$

+ N4LO 3NF Contacts

$\mathbf{k}_i = \mathbf{p}_i - \mathbf{p}_i'$ and $\mathbf{Q}_i = \mathbf{p}_i + \mathbf{p}_i'$, $\mathbf{p}_i$ and $\mathbf{p}_i'$ being the initial and final momenta of nucleon i, the potential in momentum space is found to be

$$\begin{aligned} V = \sum_{i\neq j\neq k} \Big[& -E_1\mathbf{k}_i^2 - E_2\mathbf{k}_i^2\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j - E_3\mathbf{k}_i^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j - E_4\mathbf{k}_i^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j \\ & - E_5(3\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_i\cdot\boldsymbol{\sigma}_j - \mathbf{k}_i^2) - E_6(3\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_i\cdot\boldsymbol{\sigma}_j - \mathbf{k}_i^2))\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j \\ & + \frac{i}{2}E_7\mathbf{k}_i\times(\mathbf{Q}_i-\mathbf{Q}_j)\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j) + \frac{i}{2}E_8\mathbf{k}_i\times(\mathbf{Q}_i-\mathbf{Q}_j)\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k \\ & - E_9\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_j\cdot\boldsymbol{\sigma}_j - E_{10}\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_j\cdot\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j \Big], \end{aligned} \quad (15)$$

Spin-Orbit Force!