Ab initio description of reactions in light nuclei

Perspectives of the *Ab Initio* No-Core Shell Model
Vancouver, TRIUMF
February 23-25, 2012

Wataru Horiuchi (RIKEN)

Collaborators:
P. Navratil (TRIUMF), S. Quaglioni (LLNL)
Y. Suzuki (Niigata, RIKEN), K. Arai (Nagaoka)

Introduction

- To understand dynamics of light nuclei
  - Important for astrophysical reactions
  - *Ab initio* calculation: Predictability
    - Structures and reactions
    - Interactions
- Nuclear reactions with realistic interactions
  - Realistic nuclear interaction
    - Hard (short-range, tensor force or high momentum)
  - Bound state problem can be solved up to A～10.
  - Continuum description

    → much more difficult (boundary conditions etc.)
    - Use of a square integrable (L^2) basis
      - Easy to handle

Unifying bound and scattering states

Contents

- Correlated Gaussian approach: application to photoabsorption reaction of ^{4}He
- Recent progress on NCSM/RGM calculation for three-particle projectile

Explicitly correlated basis function

Correlated Gaussian with global vectors: explicitly correlated

$$\phi^{(N)\pi}_{L_1L_2(L_{12})L_3LM_L}(A,u_1,u_2,u_3)$$
$$=\exp(-\tilde{x}Ax)[[\mathcal{Y}_{L_1}(\tilde{u}_1x)\mathcal{Y}_{L_2}(\tilde{u}_2x)]_{L_{12}}\mathcal{Y}_{L_3}(\tilde{u}_3x)]_{LM_L},$$

x: any set of relative coordinates

$$\tilde{x}Ax=\sum_{i,j=1}^{N-1}A_{ij}x_i\cdot x_j$$
$$\tilde{u}_ix=\sum_{k=1}^{N-1}(u_i)_kx_k$$

Flexible basis

Formulation for *N*-particle system

Functional form unchanged under any coordinate transformations

LS coupling scheme: $\Phi^{(N)\pi}_{(LS)JMTM_T}=\mathcal{A}\left[\phi^{(N)\pi}_L\chi^{(N)}_S\right]_{JM}\eta^{(N)}_{TM_T},$

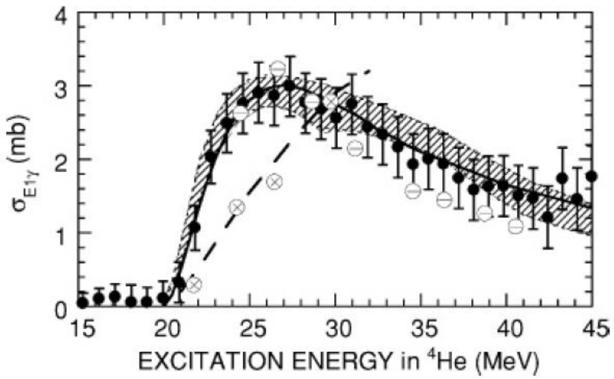
$$\chi^{(N)}_{S_{12}S_{123}\ldots SM_S}=[\ldots[[\chi_{\frac{1}{2}}(1)\chi_{\frac{1}{2}}(2)]_{S_{12}}\chi_{\frac{1}{2}}(3)]_{S_{123}}\ldots]_{SM_S}$$

Ground state of ^{4}He agrees with the benchmark cal.

H. Kamada et al., PRC64, 044001 (2001)

Application of correlated Gaussian basis to reactions in light nuclei

- n+^{4}He scattering: With Green's function method
  Y. Suzuki, WH, K. Arai, NPA823, 1 (2009).
- d+d, n+^{3}He reactions: With microscopic R-matrix method
  K. Arai, S. Aoyama, Y. Suzuki, P. Descouvemont, D. Baye, PRL107, 132502(2011)
  S. Aoyama, K. Arai, Y. Suzuki, P. Descouvemont, D. Baye, FBS52, 97 (2012)
- Inclusive reaction ^{4}He(γ,X)
  - Recent measurements
    - Peak ～27MeV S. Nakayama et al., PRC 76, 021305 (2007).
    - Peak ～30 MeV T. Shima et al., PRC 72, 044004 (2005).
  - Theoretical studies
    D. Gazit et al. PRL 96, 112302 (2006)
    S. Bacca, PRC75, 044001(2007)
    S. Quaglioni and P. Navratil, PLB652, 370 (2007)
    Basis function: HH, NCSM
    Interactions: Effective interaction based on realistic force
    Continuum: Lorentz Integral Transform Method



Taken from S. Nakayama et al. PRC 76, 021305 (2007).

In order to understand this problem

- To use a realistic interaction as it is
- To include couplings with final decay channels explicitly
- Perform different method
  - Complex scaling method (CSM)
  - Microscopic R-matrix method (MRM)

Complex scaling method (CSM)

Photoabsorption cross section $$\sigma_\gamma(E_\gamma) = \frac{4\pi^2}{\hbar c} E_\gamma \frac{1}{3} S(E_\gamma)$$

Strength function $$S(E) = \mathcal{S}_{\mu f} \left| \langle \Psi_f | \mathcal{M}_{1\mu} | \Psi_0 \rangle \right|^2 \delta(E_f - E_0 - E)$$
$$= -\frac{1}{\pi} \mathrm{Im} \sum_\mu \langle \Psi_0 | \mathcal{M}_{1\mu}^\dagger \frac{1}{E - H + i\epsilon} \mathcal{M}_{1\mu} | \Psi_0 \rangle$$

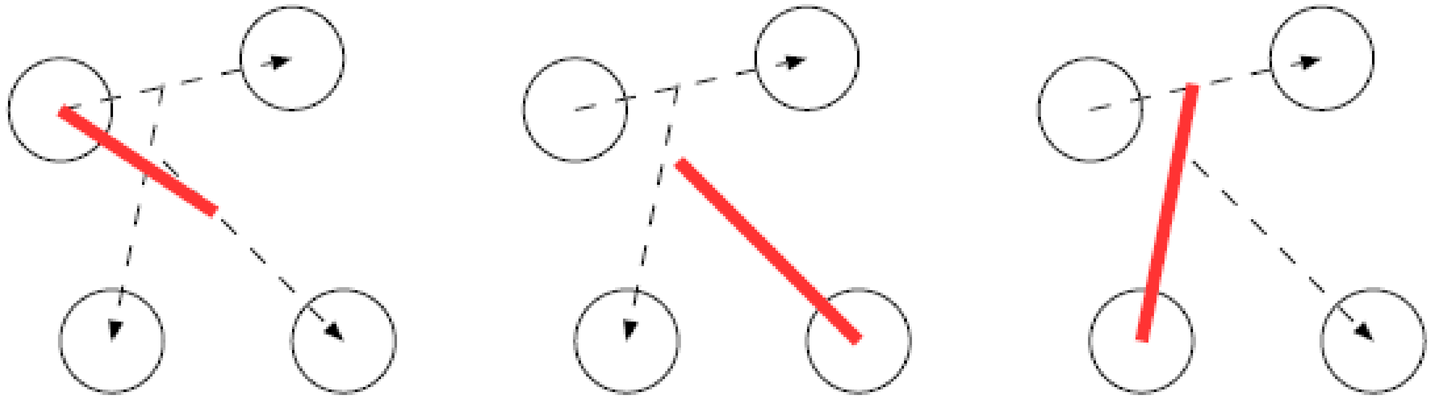
Electric dipole operator $$\mathcal{M}_{1\mu} = \sum_{i=1}^{4} \frac{e}{2}(1 - \tau_{3i})(\boldsymbol{r}_i - \boldsymbol{x}_4)_\mu,$$
$$= -\frac{e}{2}\sqrt{\frac{4\pi}{3}} \sum_{i=1}^{4} \tau_{3i} \mathcal{Y}_{1\mu}(\boldsymbol{r}_i - \boldsymbol{x}_4),$$ Isovector type

Complex rotation $$U(\theta): \quad \boldsymbol{r}_j \to \boldsymbol{r}_j e^{i\theta}, \quad \boldsymbol{p}_j \to \boldsymbol{p}_j e^{-i\theta}$$ Outgoing-wave B.C.

Expanded in L^2 basis $$\Psi_\lambda^{JM\pi}(\theta) = \sum_i C_i^\lambda(\theta) \Phi_i(\boldsymbol{x}) \qquad H(\theta)\Psi_\lambda^{JM\pi}(\theta) = E_\lambda(\theta)\Psi_\lambda^{JM\pi}(\theta)$$

Strength function $$S(E) = -\frac{1}{\pi} \sum_{\mu\lambda} \mathrm{Im} \frac{\widetilde{\mathcal{D}}_\mu^\lambda(\theta) \mathcal{D}_\mu^\lambda(\theta)}{E - E_\lambda(\theta) + i\epsilon}$$
$$\mathcal{D}_\mu^\lambda(\theta) = \left\langle (\Psi_\lambda^{JM\pi}(\theta))^* \right| \mathcal{M}_{1\mu}(\theta) \left| U(\theta)\Psi_0 \right\rangle$$
$$\widetilde{\mathcal{D}}_\mu^\lambda(\theta) = \left\langle (U(\theta)\Psi_0)^* \right| \widetilde{\mathcal{M}}_{1\mu}(\theta) \left| \Psi_\lambda^{JM\pi}(\theta) \right\rangle$$

Configuration for final state



(i) Single-particle excitation

(ii) 3N+N two-body disintegration

(iii) d+p+n three-body disintegration

(i) $\Psi_f^{\rm sp} = \mathcal{A}\left[\Phi_0^{(4)}(i)\mathcal{Y}_1(r_1 - x_4)\right]_{1M}\eta^{(4)}_{T_{12}T_{123}10}$

- The ground state combined with $Y_1(r_1\text{-}x_4)$
- Complete set of isospin wave function(T=1)
- Basis and possible angular momentum couplings are included independently.

(ii)

3N: three-body cal.

3N-N: p-wave

(iii)

2N: two-body cal.

2N-N: p-wave

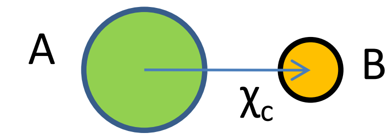
3N*-N: s-wave

Note: The coherent E1 state $\sum_\mu \mathcal{M}_{1\mu}|0\rangle$ exhausts 100% of the non-energy-weighted sum rule.

Explicit correlation of 3+1 and 2+1+1

Microscopic R-matrix Method (MRM)

Wave function $$\Psi_{\rm AB}^{JM\pi} = \sum_{i=1}^{N_A}\sum_{j=1}^{N_B}\sum_{I,\ell} \mathcal{A}[[\Phi_{J_A\pi_A}^{\rm A,\it i}\,\Phi_{J_B\pi_B}^{\rm B,\it j}]_I\,\chi_c]_{JM}$$



- Internal region: Relative motion between two clusters χ_c is expanded in several Gaussians $r^\ell \exp(-\rho r^2) Y_\ell(\hat{r})$
- External region: asymptotic wave function (e.g. Coulomb w.f.)
- Matching internal and external regions at a channel radius

Schroedinger eq. $$[H - E + \widetilde{L}]\Psi_{\rm int}^{JM\pi} = \widetilde{L}\Psi_{\rm ext}^{JM\pi}$$ *L*: Bloch operator

The continuity condition: $$\Psi_{\rm int}^{JM\pi} = \Psi_{\rm ext}^{JM\pi}$$

- Distortion of the cluster is taken into account by adding pseudostates $3N(1/2^+)+N$, $d(1^+)+d(1^+)$, $pn(0^+)+pn(0^+)$, $pp(0^+)+nn(0^+)$

Photoabsorption cross section ⇔ radiative capture cross section

^{4}He(γ, N)3N ⇔ N(3N,γ)^{4}He

$$\sigma_\gamma^{\rm AB}(E_\gamma) = \frac{k^2(2J_A+1)(2J_B+1)}{2k_\gamma^2(2J_0+1)}\sigma_{\rm cap}^{\rm AB}(E_{\rm in})$$

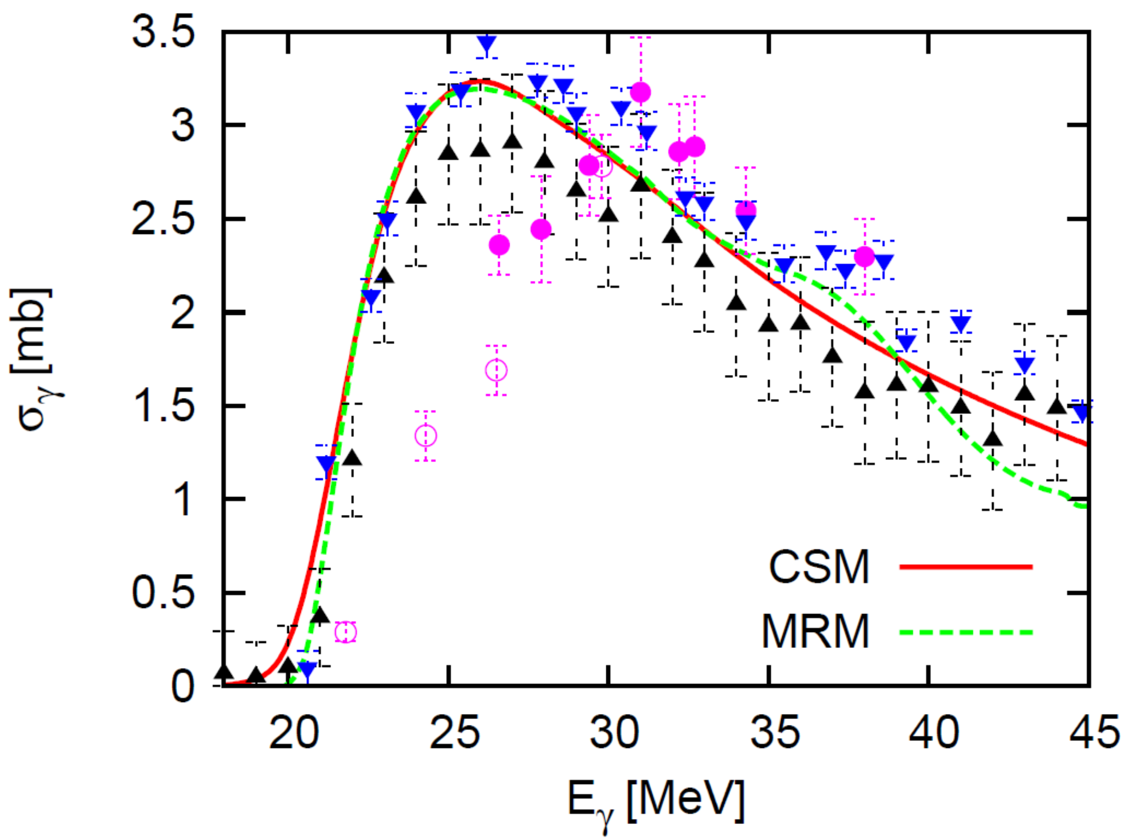
Total photoabsorption cross sections

Interaction: AV8'+3NF

3NF: E. Hiyama et al., PRC70, 031001(2004).

Comparison of two methods

→agree in the low-energy region



Data taken from

- ■ S. Nakayama et al., (2007)
- ▼ Y. M. Arkatov et al.,(1974).
- ○ T. Shima et al., (2005).
- ● T. Shima et al., new measurement

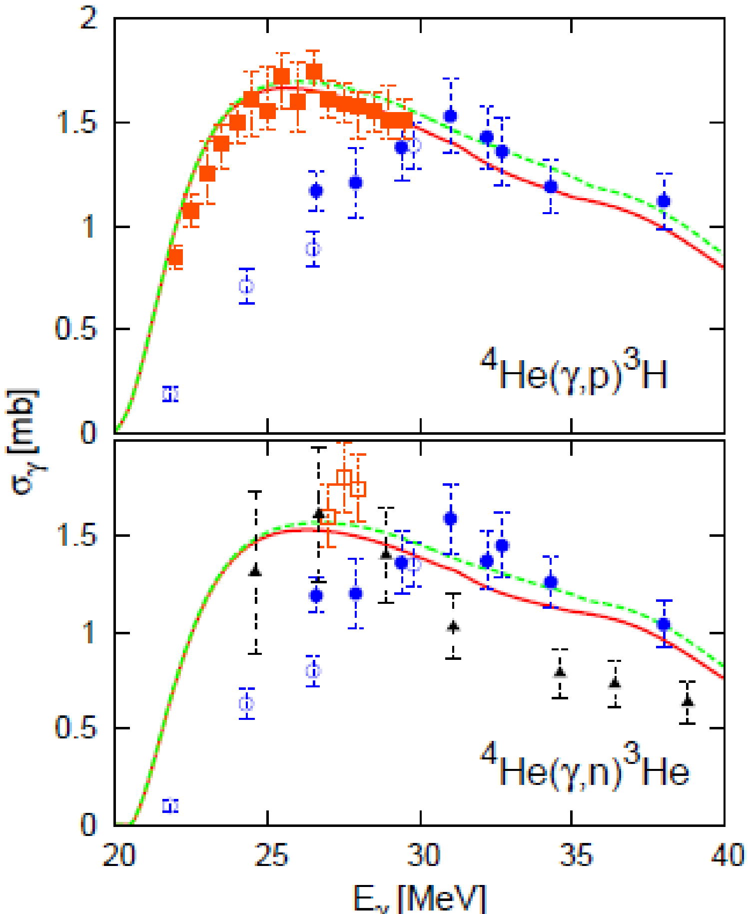
Good reproduction of the experiments

D. Gazit et al. PRL 96, 112302 (2006)

S. Bacca, PRC75, 044001(2007)

S. Quaglioni and P. Navratil, PLB652, 370 (2007)

Photoabsorption cross sections



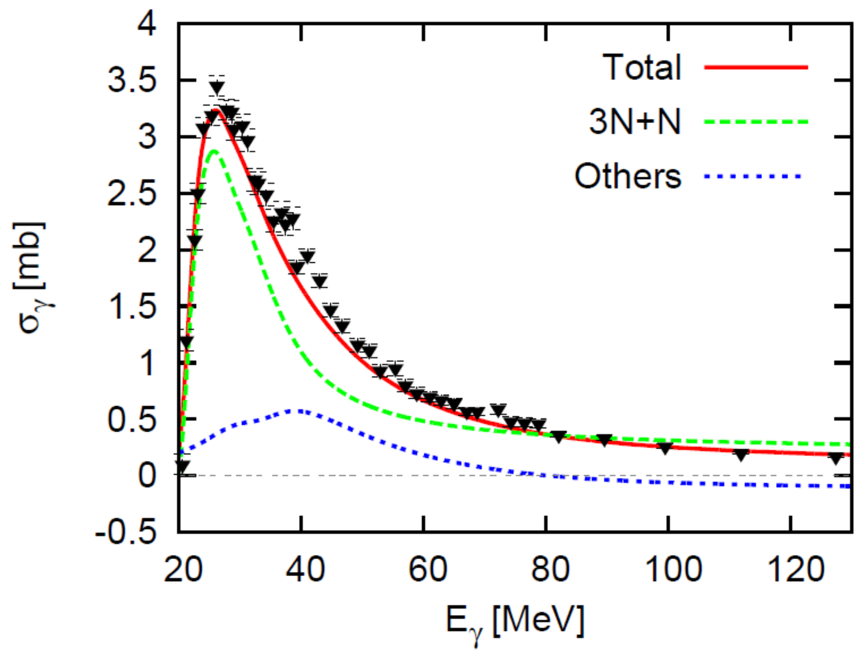
■ R. Raut et al. (2012)

□ W. Tornow et al. (2011), preliminary

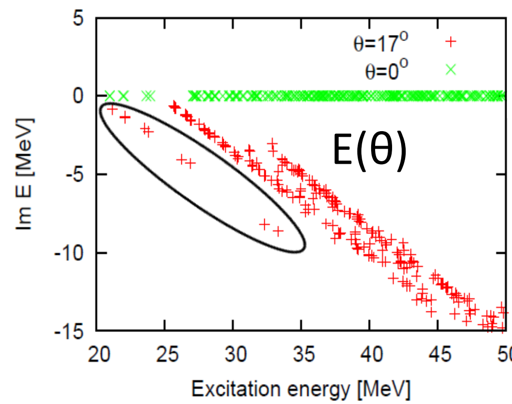
○ T. Shima et al. (2005)

● T. Shima et al., new measurements

▲ B.Nilsson et al. (2007)



▼ Y. M. Arkatov et al., Yad. Fiz. 19, 1172 (1974).



$$S(E) = \sum_{\lambda} S_\lambda = \sum_{\lambda \in 3NN} S_\lambda + \sum_{\lambda \notin 3NN} S_\lambda$$

3N+N configurations are dominant in the low-energy cross section

Summary and outlook

- Photoabsorption cross sections of ^{4}He based on four-body calculation
  - Correlated Gaussian with global vectors, flexible basis!
    - "Bare" interaction can be used.
  - Explicit cluster correlations in the final state
  - Continuum states are properly treated **using the square integrable (L^2) basis** by the Complex Scaling Method and Microscopic R-matrix Method

- The calculations with two different methods
  - Virtually the same results in the low-energy
  - Good agreement with the recent experiments
  - 3N+N cluster structure is important

WH, Y. Suzuki, K. Arai, arXiv: 1202.0268

- Future works
  - More particle systems (A>5)
  - Electro-weak responses (Gamow-Teller, spin-dipole etc.)

NCSM/RGM calculation

- Consistent description of bound and scattering states
  - No core shell model (NCSM)
    - Effective interaction starting from realistic force
    - Applicable A<16
  - Resonating Group Method (RGM)
    - Fully microscopic
    - Proper treatment of continuum states with the microscopic R-matrix

- Single-particle projectile (N-^{3}H, N-^{4}He, N-^{10}Be)

  S. Quaglioni, P. Navratil, PRC79, 044606 (2009).
- Combined with IT-NCSM (N-^{7}Li, N-^{7}Be, N-^{12}C, N-^{16}O)

  P. Navratil, R. Roth, S. Quaglioni, PRC82, 034609 (2010).
- Two-particle projectile (d-^{4}He, d-^{3}H->n-^{4}He)

  P. Navratil, S. Quaglioni, PRC83, 044609 (2011).

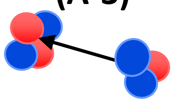
  P. Navratil, S. Quaglioni, PRL108, 042503 (2012).
- Three-particle projectile (^{3}H-^{3}H, ^{3}He-^{4}He, ...)

  This work

Formalism

Scattering wave function

$$\left|\Psi^{J^\pi T}\right\rangle = \sum_\nu \int dr\, r^2 \frac{g_\nu^{J^\pi T}(r)}{r} \hat{\mathcal{A}}_\nu \left|\Phi_{\nu r}^{J^\pi T}\right\rangle$$



Basis function (NCSM basis)

$$\left|\Phi_{\nu r}^{J^\pi T}\right\rangle = \left[\left(\left|A-3\,\alpha_1 I_1^{\pi_1} T_1\right\rangle \left|a=3\,\alpha_2 I_2^{\pi_2} T_2\right\rangle\right)^{(sT)} Y_\ell(\hat{r}_{A-3,3})\right]^{(J^\pi T)} \frac{\delta(r - r_{A-3,3})}{r r_{A-3,3}}$$

Schroedinger equation (RGM equation)

$$\sum_\nu \int dr\, r^2 \left[\mathcal{H}_{\nu'\nu}^{J^\pi T}(r', r) - E \mathcal{N}_{\nu'\nu}^{J^\pi T}(r', r)\right] \frac{g_\nu^{J^\pi T}(r)}{r} = 0$$

Microscopic R-matrix method is used to solve the RGM eq.

Non-local matrix elements

Norm kernel

$$\mathcal{N}_{\nu'\nu}^{J^\pi T}(r', r) = \left\langle \Phi_{\nu' r'}^{J^\pi T} \right| \hat{\mathcal{A}}_{\nu'} \hat{\mathcal{A}}_\nu \left| \Phi_{\nu r}^{J^\pi T} \right\rangle$$

Hamiltonian kernel

$$\mathcal{H}_{\nu'\nu}^{J^\pi T}(r', r) = \left\langle \Phi_{\nu' r'}^{J^\pi T} \right| \hat{\mathcal{A}}_{\nu'} H \hat{\mathcal{A}}_\nu \left| \Phi_{\nu r}^{J^\pi T} \right\rangle$$

$$H = T_{\text{rel}} + \mathcal{V}_{\text{rel}} + V_C(r) + H_{A-3} + H_{a=3}$$

Calculation of RGM matrix elements

1. Basis: Combination of Slater determinant (SD) and Jacobi basis obtained by NCSM

$$\left|\Phi^{J^\pi T}_{\nu n}\right\rangle_{\rm SD} = \Big[\Big(\underbrace{|A-3\,\alpha_1 I_1^{\pi_1} T_1\rangle_{\rm SD}}_{\text{SD basis}}\,\underbrace{|a=3\,\alpha_2 I_2^{\pi_2} T_2\rangle}_{\text{Jacobi basis}}\Big)^{(sT)} Y_\ell(\hat{R}^{(a=3)}_{\rm cm})\Big]^{(J^\pi T)} R_{n\ell}(R^{(a=3)}_{\rm cm})$$

SD basis is computationally advantageous for A>5.

Spurious c.m. motion can be subtracted.

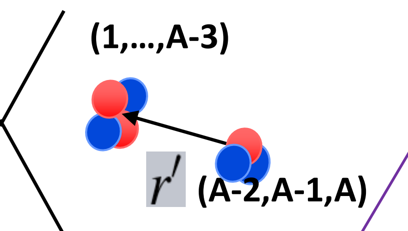
2. Write it down with single particle HO states
3. Calculate the Norm and Hamiltonian kernels
   – Expression: Sum of A-body (up to 4) densities

Norm kernel

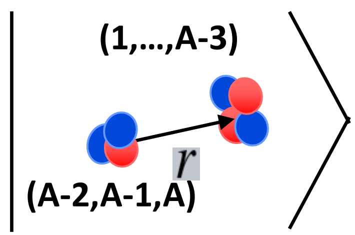
- Antisymmetrizer between two clusters

$$\mathcal{A}^{(A-3,3)} = \sqrt{\frac{6}{A(A-1)(A-2)}}\Bigg[1 - \sum_{i=1}^{A-3}\hat{P}_{i,A} - \sum_{i=1}^{A-3}\hat{P}_{i,A-1} - \sum_{i=1}^{A-3}\hat{P}_{i,A-2} + \frac{1}{2}\sum_{i\neq j}\left(\hat{P}_{i,A}\hat{P}_{j,A-1} + \hat{P}_{i,A-2}\hat{P}_{j,A} + \hat{P}_{i,A-1}\hat{P}_{j,A-2}\right) - \frac{1}{6}\sum_{i\neq j\neq k}\hat{P}_{i,A}\hat{P}_{j,A-1}\hat{P}_{k,A-2}\Bigg]$$

- Norm kernel $\mathcal{N}^{J^\pi T}_{\nu'\nu}(r',r) = \left\langle \Phi^{J^\pi T}_{\nu' r'}\right| \mathcal{A}^2 \left|\Phi^{J^\pi T}_{\nu r}\right\rangle$

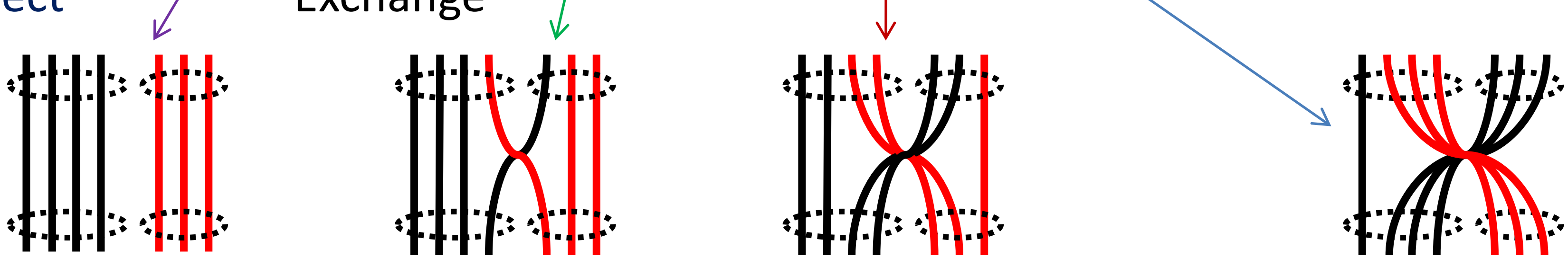


$$1 - 3(A-3)\hat{P}_{A-4,A-3} + \frac{3}{2}(A-3)(A-4)\hat{P}_{A-4,A}\hat{P}_{A-5,A-1} - \frac{1}{6}(A-3)(A-4)(A-5)\hat{P}_{A-4,A}\hat{P}_{A-5,A-1}\hat{P}_{A-6,A-2}$$



"Direct"

"Exchange"



$${}_{SD}\langle \psi^{A-3}_{\mu'}|a^\dagger a|\psi^{A-3}_{\mu}\rangle_{SD} \qquad {}_{SD}\langle \psi^{A-3}_{\mu'}|a^\dagger a^\dagger a a|\psi^{A-3}_{\mu}\rangle_{SD} \qquad {}_{SD}\langle \psi^{A-3}_{\mu'}|a^\dagger a^\dagger a^\dagger a a a|\psi^{A-3}_{\mu}\rangle_{SD}$$

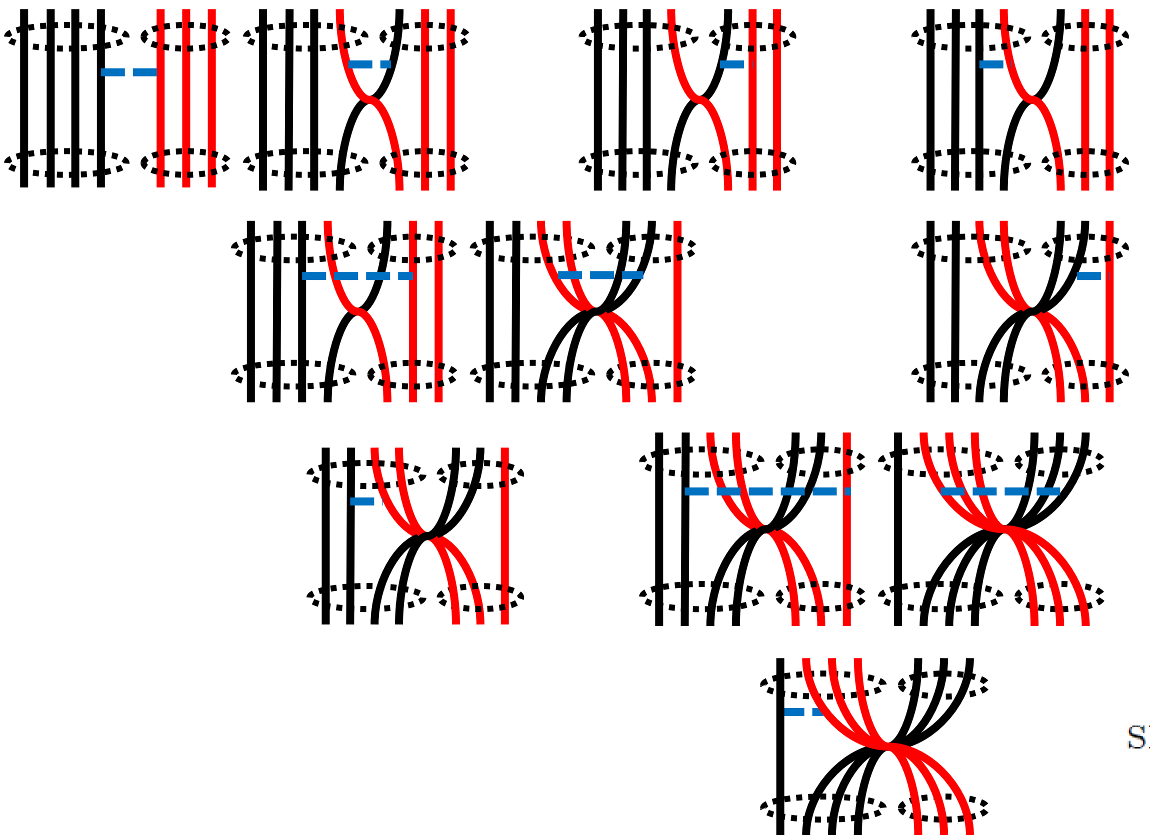
Hamiltonian kernel

$$\mathcal{H}^{J^\pi T}_{\nu'\nu}(r',r) = \left\langle \Phi^{J^\pi T}_{\nu' r'} \right| \mathcal{A} H \mathcal{A} \left| \Phi^{J^\pi T}_{\nu r} \right\rangle = \left\langle \Phi^{J^\pi T}_{\nu' r'} \right| H \mathcal{A}^2 \left| \Phi^{J^\pi T}_{\nu r} \right\rangle$$

$$\mathcal{V}_{\nu',\nu}(r',r) = \left\langle \Phi^{J^\pi T}_{\nu' r'} \right| \mathcal{V}_{\rm rel} \mathcal{A}^2 \left| \Phi^{J^\pi T}_{\nu r} \right\rangle$$

8 terms

$$\begin{aligned}
&3(A-3)V_{A-3,A-2}(1-\hat{P}_{A-3,A-2}) - 6(A-3)V_{A-3,A-1}\hat{P}_{A-3,A-2} \\
&- 3(A-3)(A-4)V_{A-4,A-2}\hat{P}_{A-3,A-2} \\
&- 6(A-3)(A-4)V_{A-3,A}(1-\hat{P}_{A-3,A})\hat{P}_{A-4,A-1} \\
&+ 3(A-3)(A-4)V_{A-3,A-2}\hat{P}_{A-3,A}\hat{P}_{A-4,A-1} \\
&+ 3(A-3)(A-4)(A-5)V_{A-5,A}\hat{P}_{A-3,A}\hat{P}_{A-4,A-1} \\
&+ \tfrac{3}{2}(A-3)(A-4)(A-5)V_{A-5,A-2}\hat{P}_{A-3,A}\hat{P}_{A-4,A-1}(1-\hat{P}_{A-5,A-2}) \\
&- \tfrac{1}{2}(A-3)(A-4)(A-5)(A-6)V_{A-6,A}\hat{P}_{A-3,A}\hat{P}_{A-4,A-1}\hat{P}_{A-5,A-2}
\end{aligned}$$



$${}_{\rm SD}\langle \psi^{A-3}_{\mu'} | a^\dagger a | \psi^{A-3}_{\mu} \rangle_{\rm SD}$$

$${}_{\rm SD}\langle \psi^{A-3}_{\mu'} | a^\dagger a^\dagger a a | \psi^{A-3}_{\mu} \rangle_{\rm SD}$$

$${}_{\rm SD}\langle \psi^{A-3}_{\mu'} | a^\dagger a^\dagger a^\dagger a a a | \psi^{A-3}_{\mu} \rangle_{\rm SD}$$

$${}_{\rm SD}\langle \psi^{A-3}_{\mu'} | a^\dagger a^\dagger a^\dagger a^\dagger a a a a | \psi^{A-3}_{\mu} \rangle_{\rm SD}$$

Explicit formula of the RGM kernels

$$\left\langle \Phi^{(A-3,3)J^\pi T}_{\alpha_1' I_1' T_1', \alpha_2' I_2' T_2'; s'\ell'}; n'\ell' \middle| \mathcal{O} \middle| \Phi^{(A-3,3)J^\pi T}_{\alpha_1 I_1 T_1, \alpha_2 I_2 T_2; s\ell}; n\ell \right\rangle$$
$$= \sum \langle (n_2\ell_2 s_2 j_2 t_2; \mathcal{N}_2\mathcal{L}_2\mathcal{J}_2\tfrac{1}{2})\, I_2T_2 | a=3\,\alpha_2 I_2 T_2\rangle$$
$$\times \langle a=3\,\alpha_2' I_2' T_2' | \,(n_2'\ell_2' s_2' j_2' t_2'; \mathcal{N}_2'\mathcal{L}_2'\mathcal{J}_2'\tfrac{1}{2})\, I_2'T_2'\rangle$$
$$\times\, \hat{s}\hat{I}\hat{s}_2\hat{j}_2\hat{I}_2\hat{\mathcal{J}}_2\hat{j}_a\hat{j}_b\hat{j}_c\hat{I}_{ab}\hat{\lambda}^2\hat{L}^2_{ab}\hat{s}'\hat{I}'\hat{s}_2'\hat{j}_2'\hat{I}_2'\hat{\mathcal{J}}_2'\hat{j}_a'\hat{j}_b'\hat{j}_c'\hat{I}_{ab}'\hat{\lambda}'^2\hat{L}'^2_{ab}$$
$$\times\, (-1)^{I_1-I+\ell_c+\ell+\mathcal{J}_2+\ell_2+t_2+I_{ab}+I_1'-I'+\ell_c'+\ell'+\mathcal{J}_2'+\ell_2'+t_2'+I_{ab}'+2J+1}$$
$$\times\, \langle n_a\ell_a n_b\ell_b L_{ab}|N_2L_2n_2\ell_2L_{ab}\rangle_{d=1}\, \langle n_c\ell_c N_2L_2\lambda|n\ell\mathcal{N}_2\mathcal{L}_2\lambda\rangle_{d=1/2}$$
$$\times\, \langle n_a'\ell_a' n_b'\ell_b' L_{ab}'|N_2'L_2'n_2'\ell_2'L_{ab}'\rangle_{d=1}\, \langle n_c'\ell_c' N_2'L_2'\lambda'|n'\ell'\mathcal{N}_2'\mathcal{L}_2'\lambda'\rangle_{d=1/2}$$
$$\times \begin{Bmatrix} I_1 & I_2 & s \\ \ell & J & I \end{Bmatrix} \begin{Bmatrix} I_1' & I_2' & s' \\ \ell' & J & I' \end{Bmatrix} \begin{Bmatrix} L_2 & L_{ab} & \ell_2 \\ s_2 & j_2 & I_{ab} \end{Bmatrix} \begin{Bmatrix} L_2' & L_{ab}' & \ell_2' \\ s_2' & j_2' & I_{ab}' \end{Bmatrix}$$
$$\times \begin{Bmatrix} \ell_a & \ell_b & L_{ab} \\ \frac{1}{2} & \frac{1}{2} & s_2 \\ j_a & j_b & I_{ab} \end{Bmatrix} \begin{Bmatrix} \ell_a' & \ell_b' & L_{ab}' \\ \frac{1}{2} & \frac{1}{2} & s_2' \\ j_a' & j_b' & I_{ab}' \end{Bmatrix} \begin{Bmatrix} \ell & \lambda & L_2 & j_2 \\ \mathcal{L}_2 & \ell_c & I_{ab} & I_2 \\ \mathcal{J}_2 & \frac{1}{2} & j_c & I \end{Bmatrix} \begin{Bmatrix} \ell' & \lambda' & L_2' & j_2' \\ \mathcal{L}_2' & \ell_c' & I_{ab}' & I_2' \\ \mathcal{J}_2' & \frac{1}{2} & j_c' & I' \end{Bmatrix} \mathcal{M}(\mathcal{O})$$

Coefficients come from the transformation from Jacobi to single particle coordinates.

$$\mathcal{M}(\mathcal{O}) = \int d\zeta_1 \cdots d\zeta_A$$
$$\times \left\{ \left[\langle \zeta_1 \cdots \zeta_{A-3} | A-3\,\alpha_1' I_1' T_1'\rangle_{\mathrm{SD}} \left[\left[\varphi_{n_a'\ell_a'j_a'\frac{1}{2}}(\zeta_A)\varphi_{n_b'\ell_b'j_b'\frac{1}{2}}(\zeta_{A-1})\right]^{(I_{ab}'t_2')} \varphi_{n_c'\ell_c'j_c'\frac{1}{2}}(\zeta_{A-2})\right]^{(I'T_2')}\right]^{(JT)}\right\}^*$$
$$\times\, \mathcal{O} \left\{ \left[\langle \zeta_1 \cdots \zeta_{A-3} | A-3\,\alpha_1 I_1 T_1\rangle_{\mathrm{SD}} \left[\left[\varphi_{n_a\ell_aj_a\frac{1}{2}}(\zeta_A)\varphi_{n_b\ell_bj_b\frac{1}{2}}(\zeta_{A-1})\right]^{(I_{ab}t_2)} \varphi_{n_c\ell_cj_c\frac{1}{2}}(\zeta_{A-2})\right]^{(IT_2)}\right]^{(JT)}\right\}$$

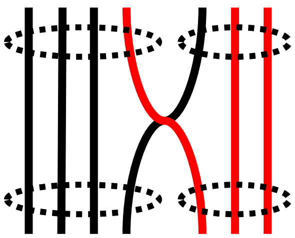
Matrix element for operator *O*

Norm kernel (3 terms)

One-particle exchange

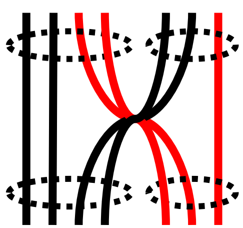
$$\mathcal{M}(\hat{P}_{A-3,A-2}) = \delta_{n'_a,n_a}\delta_{\ell'_a,\ell_a}\delta_{j'_a,j_a}\delta_{n'_b,n_b}\delta_{\ell'_b,\ell_b}\delta_{j'_b,j_b}\delta_{I'_{ab},I_{ab}}\delta_{t'_2,t_2}\frac{1}{A-3}$$
$$\times \sum_{K\tau}(-1)^{I_{ab}+I_1+K+J+j'_c+t_2+T_1+\tau+T+\frac{1}{2}}\hat{I}\hat{I}'\hat{K}\hat{T}_2\hat{T}'_2\hat{\tau}$$
$$\times \begin{Bmatrix} I_1 & K & I'_1 \\ I' & J & I \end{Bmatrix}\begin{Bmatrix} I_{ab} & j'_c & I' \\ K & I & j_c \end{Bmatrix}\begin{Bmatrix} T_1 & \tau & T'_1 \\ T'_2 & T & T_2 \end{Bmatrix}\begin{Bmatrix} t_2 & \frac{1}{2} & T'_2 \\ \tau & T_2 & \frac{1}{2} \end{Bmatrix}$$
$$\times \langle A-3\,\alpha'_1 I'_1 T'_1 ||| \left(a^{\dagger}_{n_c\ell_c j_c\frac{1}{2}}\tilde{a}_{n'_c\ell'_c j'_c\frac{1}{2}}\right)^{(K\tau)} ||| A-3\,\alpha_1 I_1 T_1\rangle$$

One-body density



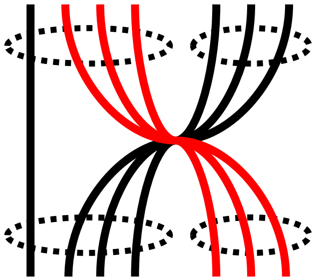
Two-particle exchange

$$\mathcal{M}(\hat{P}_{A-3,A}\hat{P}_{A-4,A-1}) = \delta_{n'_c,n_c}\delta_{\ell'_c,\ell_c}\delta_{j'_c,j_c}\frac{1}{(A-3)(A-4)}$$
$$\times \sum_{K\tau}(-1)^{j_c+I_1+I+K+J+I'+I_{ab}+I'_{ab}+j'_a+j'_b+\frac{1}{2}+T_1+T_2+\tau+T+T'_2+t_2+t'_2+1}\hat{I}\hat{I}'\hat{K}\hat{T}_2\hat{T}'_2\hat{\tau}$$
$$\times \begin{Bmatrix} I_1 & K & I'_1 \\ I' & J & I \end{Bmatrix}\begin{Bmatrix} j_c & I'_{ab} & I' \\ K & I & I_{ab} \end{Bmatrix}\begin{Bmatrix} T_1 & \tau & T'_1 \\ T'_2 & T & T_2 \end{Bmatrix}\begin{Bmatrix} \frac{1}{2} & t'_2 & T'_2 \\ \tau & T_2 & t_2 \end{Bmatrix}$$
$$\times \langle A-3\,\alpha'_1 I'_1 T'_1 ||| \left[\left(a^{\dagger}_{n_a\ell_a j_a\frac{1}{2}}a^{\dagger}_{n_b\ell_b j_b\frac{1}{2}}\right)^{(I_{ab}t_2)}\left(\tilde{a}_{n'_b\ell'_b j'_b\frac{1}{2}}\tilde{a}_{n'_a\ell'_a j'_a\frac{1}{2}}\right)^{(I'_{ab}t'_2)}\right]^{(K\tau)} ||| A-3\,\alpha_1 I_1 T_1\rangle$$



Two-body density

Norm kernel



Three-particle exchange

$$\mathcal{M}(\hat{P}_{A-3,A}\hat{P}_{A-4,A-1}\hat{P}_{A-5,A-2})$$
$$= \frac{1}{(A-3)(A-4)(A-5)}(-1)^{I_1+I+J+I'+j'_a+j'_b+j'_c+T_1+T_2+T+T'_2+\frac{3}{2}}$$
$$\times \sum_{K\tau}\hat{K}\hat{\tau}\begin{Bmatrix} I_1 & K & I'_1\\ I' & J & I\end{Bmatrix}\begin{Bmatrix}T_1 & \tau & T'_1\\ T'_2 & T & T_2\end{Bmatrix}\langle A-3\,\alpha'_1 I'_1 T'_1 ||| \{[(a^\dagger_{n_a\ell_a j_a\frac{1}{2}} a^\dagger_{n_b\ell_b j_b \frac{1}{2}})^{(I_{ab}t_2)} a^\dagger_{n_c\ell_c j_c\frac{1}{2}}]^{(IT_2)}$$
$$\times [\tilde{a}_{n'_c\ell'_c j'_c\frac{1}{2}}(\tilde{a}_{n'_b\ell'_b j'_b\frac{1}{2}}\tilde{a}_{n'_a\ell'_a j'_a\frac{1}{2}})^{(I'_{ab}t'_2)}]^{(I'T'_2)}\}^{(K\tau)} ||| A-3\,\alpha_1 I_1 T_1\rangle$$

Three-body density

$$= \frac{1}{(A-3)(A-4)(A-5)}\sum_{I_\beta T_\beta}(-1)^{j'_a+j'_b+j'_c+I-I'+I'_1+I_\beta+\frac{3}{2}+T_2-T'_2+T'_1+T_\beta}\begin{Bmatrix} I & I'_1 & I_\beta\\ I' & I_1 & J\end{Bmatrix}\begin{Bmatrix}T_2 & T'_1 & T_\beta\\ T'_2 & T_1 & T\end{Bmatrix}$$
$$\times \langle A-3\,\alpha'_1 I'_1 T'_1 ||| [(a^\dagger_{n_a\ell_a j_a\frac{1}{2}} a^\dagger_{n_b\ell_b j_b \frac{1}{2}})^{(I_{ab}t_2)} a^\dagger_{n_c\ell_c j_c\frac{1}{2}}]^{(IT_2)} ||| A-6\,\beta I_\beta T_\beta\rangle$$
$$\times \langle A-6\,\beta I_\beta T_\beta ||| [\tilde{a}_{n'_c\ell'_c j'_c\frac{1}{2}}(\tilde{a}_{n'_b\ell'_b j'_b\frac{1}{2}}\tilde{a}_{n'_a\ell'_a j'_a\frac{1}{2}})^{(I'_{ab}t'_2)}]^{(I'T'_2)} ||| A-3\,\alpha_1 I_1 T_1\rangle$$

Completeness (closure) relation

A=7 case $|A\text{-}6\ \beta I_\beta T_\beta\rangle \rightarrow |n_\beta l_\beta j_\beta 1/2\rangle$:single particle HO basis

Hamiltonian Kernel (8 terms)

Term #1

$$\mathcal{M}(V_{A-3,A-2}(1-\hat{P}_{A-3,A-2})) = \delta_{(n'_a\ell'_a j'_a),(n_a\ell_a j_a)}\delta_{(n'_b\ell'_b j'_b),(n_b\ell_b j_b)}\delta_{I'_{ab},I_{ab}}\delta_{t'_2,t_2}\frac{1}{A-3}$$

$$\times \sum_{n_d\ell_d j_d}\sum_{n'_d\ell'_d j'_d}\sum_{K\tau J_0 T_0}(-1)^{I_{ab}+I_1+J+K-J_0+j'_d+t_2+T_1+T+\tau-T_0+1/2}\hat{I}\hat{I}'\hat{K}\hat{J}_0^2\hat{T}_2\hat{T}'_2\hat{\tau}\hat{T}_0^2$$

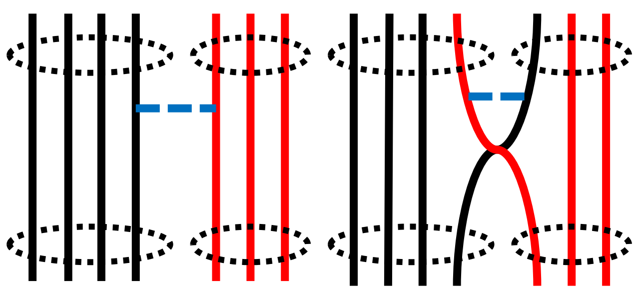
$$\times \begin{Bmatrix} I_1 & K & I'_1 \\ I' & J & I \end{Bmatrix}\begin{Bmatrix} I_{ab} & j'_c & I' \\ K & I & j_c \end{Bmatrix}\begin{Bmatrix} j'_c & K & j_c \\ j_d & J_0 & j'_d \end{Bmatrix}\begin{Bmatrix} T_1 & \tau & T'_1 \\ T'_2 & T & T_2 \end{Bmatrix}\begin{Bmatrix} t_2 & \frac{1}{2} & T'_2 \\ \tau & T_2 & \frac{1}{2} \end{Bmatrix}\begin{Bmatrix} \frac{1}{2} & \tau & \frac{1}{2} \\ \frac{1}{2} & T_0 & \frac{1}{2} \end{Bmatrix}$$

$$\times \sqrt{1+\delta_{(n'_c\ell'_c j'_c),(n'_d\ell'_d j'_d)}}\sqrt{1+\delta_{(n_d\ell_d j_d),(n_c\ell_c j_c)}}\,\langle (n'_c\ell'_c j'_c\tfrac{1}{2})(n'_d\ell'_d j'_d\tfrac{1}{2})J_0T_0|\,V\,|(n_c\ell_c j_c\tfrac{1}{2})(n_d\ell_d j_d\tfrac{1}{2})J_0T_0\rangle$$

Two-body interaction

$$\times \langle A-3\,\alpha'_1 I'_1 T'_1 |||\left(a^\dagger_{n'_d\ell'_d j'_d\frac{1}{2}}\tilde{a}_{n_d\ell_d j_d\frac{1}{2}}\right)^{(K\tau)}|||A-3\,\alpha_1 I_1 T_1\rangle$$

One-body density



Term #8

$$\mathcal{M}(V_{A-6,A-2}\hat{P}_{A-3,A}\hat{P}_{A-4,A-1}\hat{P}_{A-5,A-2})$$

$$= \frac{1}{2(A-3)(A-4)(A-5)(A-6)} \sum_{n_f\ell_f j_f}\sum_{n_g\ell_g j_g}\sum_{n'_g\ell'_g j'_g}\sum_{J_0 K K_1 K_2 k_2}\sum_{T_0\tau\tau_1\tau_2 q_2}$$

$$\times (-1)^{I_1+J+K+j'_a+j'_b+j'_c+T_1+T+\tau+3/2}\hat{I}'\hat{K}\hat{K}_1\hat{K}_2\hat{k}_2\hat{J}_0^2\hat{T}'_2\hat{\tau}\hat{\tau}_1\hat{\tau}_2\hat{q}_2\hat{T}_0^2$$

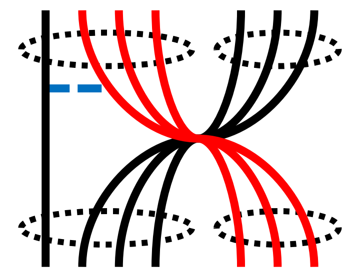
$$\times \begin{Bmatrix} I_1 & K & I'_1 \\ I' & J & I \end{Bmatrix} \begin{Bmatrix} K & I' & I \\ j'_g & K_1 & K_2 \end{Bmatrix} \begin{Bmatrix} I'_{ab} & j_f & k_2 \\ j_g & K_2 & J_0 \end{Bmatrix} \begin{Bmatrix} I'_{ab} & j'_c & I' \\ j'_g & K_2 & J_0 \end{Bmatrix}$$

$$\times \begin{Bmatrix} T_1 & \tau & T'_1 \\ T'_2 & T & T_2 \end{Bmatrix} \begin{Bmatrix} \tau & T'_2 & T_2 \\ \frac{1}{2} & \tau_1 & \tau_2 \end{Bmatrix} \begin{Bmatrix} t'_2 & \frac{1}{2} & q_2 \\ \frac{1}{2} & \tau_2 & T_0 \end{Bmatrix} \begin{Bmatrix} t'_2 & \frac{1}{2} & T'_2 \\ \frac{1}{2} & \tau_2 & T_0 \end{Bmatrix}$$

$$\times \sqrt{1+\delta_{(n'_c\ell'_c j'_c),(n'_g\ell'_g j'_g)}}\sqrt{1+\delta_{(n_f\ell_f j_f),(n_g\ell_g j_g)}}\,\underline{\langle (n'_c\ell'_c j'_c\tfrac{1}{2})(n'_g\ell'_g j'_g\tfrac{1}{2})J_0T_0|\,V\,|(n_f\ell_f j_f\tfrac{1}{2})(n_g\ell_g j_g\tfrac{1}{2})J_0T_0\rangle}$$

$$\times \underline{\langle A-3\,\alpha'_1 I'_1 T'_1||| \Bigg(\Bigg\{ \Big[\Big(a^\dagger_{n_a\ell_a j_a\frac{1}{2}} a^\dagger_{n_b\ell_b j_b\frac{1}{2}} \Big)^{(I_{ab}t_2)} a^\dagger_{n_c\ell_c j_c\frac{1}{2}} \Big]^{(IT_2)} a^\dagger_{n'_g\ell'_g j'_g\frac{1}{2}} \Bigg\}^{(K_1\tau_1)}}$$

Four-body density



$$\underline{\times \Bigg\{ \tilde{a}_{n_g\ell_g j_g\frac{1}{2}} \Big[\tilde{a}_{n_f\ell_f j_f\frac{1}{2}} \Big(\tilde{a}_{n'_b\ell'_b j'_b\frac{1}{2}} \tilde{a}_{n'_a\ell'_a j'_a\frac{1}{2}} \Big)^{(I'_{ab}t'_2)} \Big]^{(k_2q_2)} \Bigg\}^{(K_2\tau_2)} \Bigg)^{(K\tau)}}\; |||A-3\,\alpha_1 I_1 T_1\rangle$$

$$= \frac{1}{2(A-3)(A-4)(A-5)(A-6)} \sum_{n_f\ell_f j_f}\sum_{n_g\ell_g j_g}\sum_{n'_g\ell'_g j'_g}\sum_{J_0 K_1 K_2 k_2 I_\beta}\sum_{T_0\tau_1\tau_2 q_2 T_\beta}$$

$$\times (-1)^{2I_1+I'_1+J+j'_a+j'_b+j'_c+2T_1+T'_1+T+3/2}\hat{I}'\hat{K}_1\hat{K}_2\hat{k}_2\hat{J}_0^2\hat{T}'_2\hat{\tau}_1\hat{\tau}_2\hat{q}_2\hat{T}_0^2$$

$$\times \begin{Bmatrix} I'_{ab} & j_f & k_2 \\ j_g & K_2 & J_0 \end{Bmatrix} \begin{Bmatrix} I'_{ab} & j'_c & I' \\ j'_g & K_2 & J_0 \end{Bmatrix} \begin{Bmatrix} I_1 & K_2 & I_\beta \\ I & j'_g & K_1 \\ J & I' & I'_1 \end{Bmatrix} \begin{Bmatrix} t'_2 & \frac{1}{2} & q_2 \\ \frac{1}{2} & \tau_2 & T_0 \end{Bmatrix} \begin{Bmatrix} t'_2 & \frac{1}{2} & T'_2 \\ \frac{1}{2} & \tau_2 & T_0 \end{Bmatrix} \begin{Bmatrix} T_1 & \tau_2 & T_\beta \\ T_2 & \frac{1}{2} & \tau_1 \\ T & T'_2 & T'_1 \end{Bmatrix}$$

$$\times \sqrt{1+\delta_{(n'_c\ell'_c j'_c),(n'_g\ell'_g j'_g)}}\sqrt{1+\delta_{(n_f\ell_f j_f),(n_g\ell_g j_g)}}\,\underline{\langle (n'_c\ell'_c j'_c\tfrac{1}{2})(n'_g\ell'_g j'_g\tfrac{1}{2})J_0T_0|\,V\,|(n_f\ell_f j_f\tfrac{1}{2})(n_g\ell_g j_g\tfrac{1}{2})J_0T_0\rangle}$$

$$\times \underline{\langle A-3\,\alpha'_1 I'_1 T'_1||| \Bigg\{ \Big[\Big(a^\dagger_{n_a\ell_a j_a\frac{1}{2}} a^\dagger_{n_b\ell_b j_b\frac{1}{2}} \Big)^{(I_{ab}t_2)} a^\dagger_{n_c\ell_c j_c\frac{1}{2}} \Big]^{(IT_2)} a^\dagger_{n'_g\ell'_g j'_g\frac{1}{2}} \Bigg\}^{(K_1\tau_1)} |||A-7\,\beta I_\beta T_\beta\rangle}$$

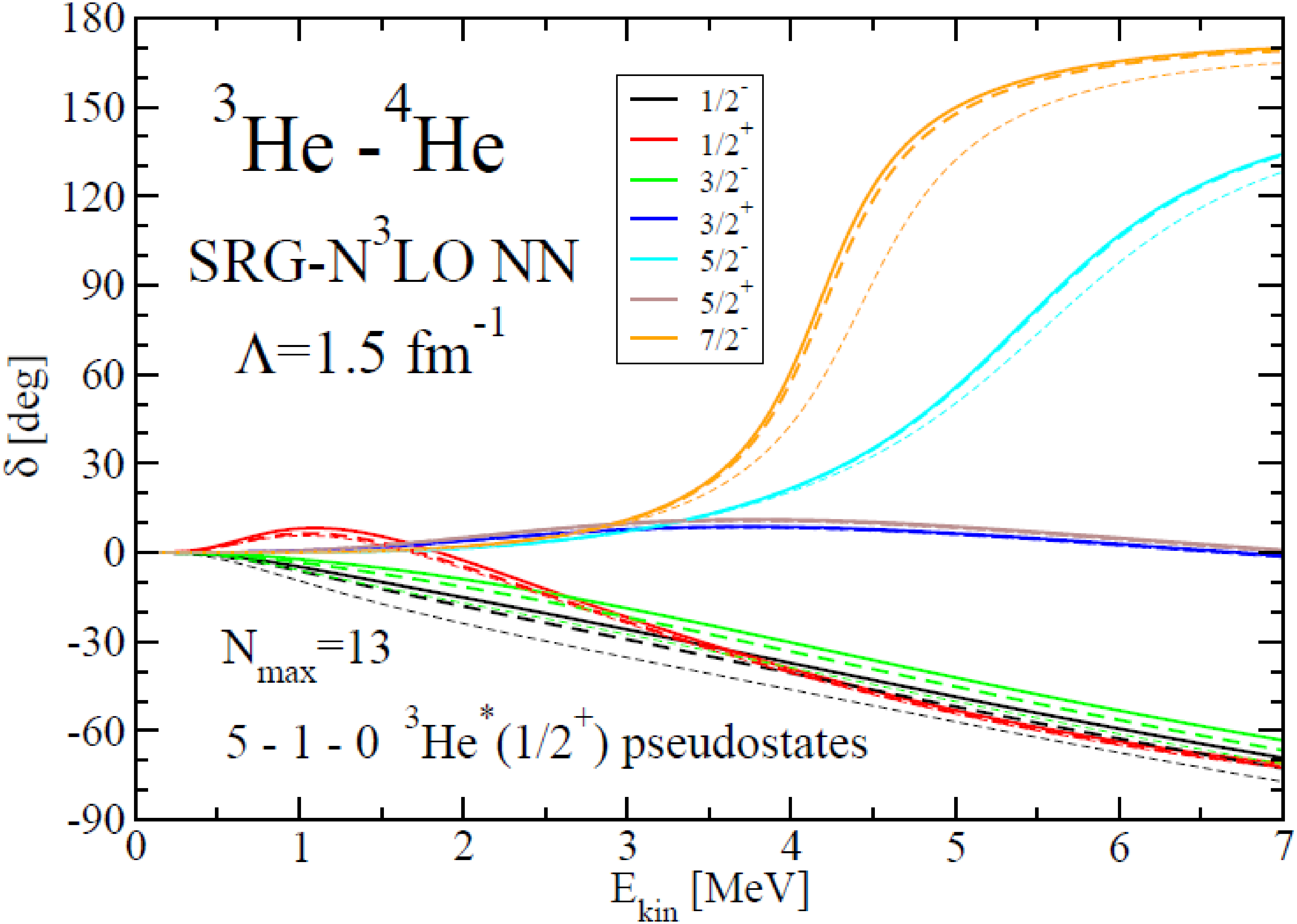
Closure relation

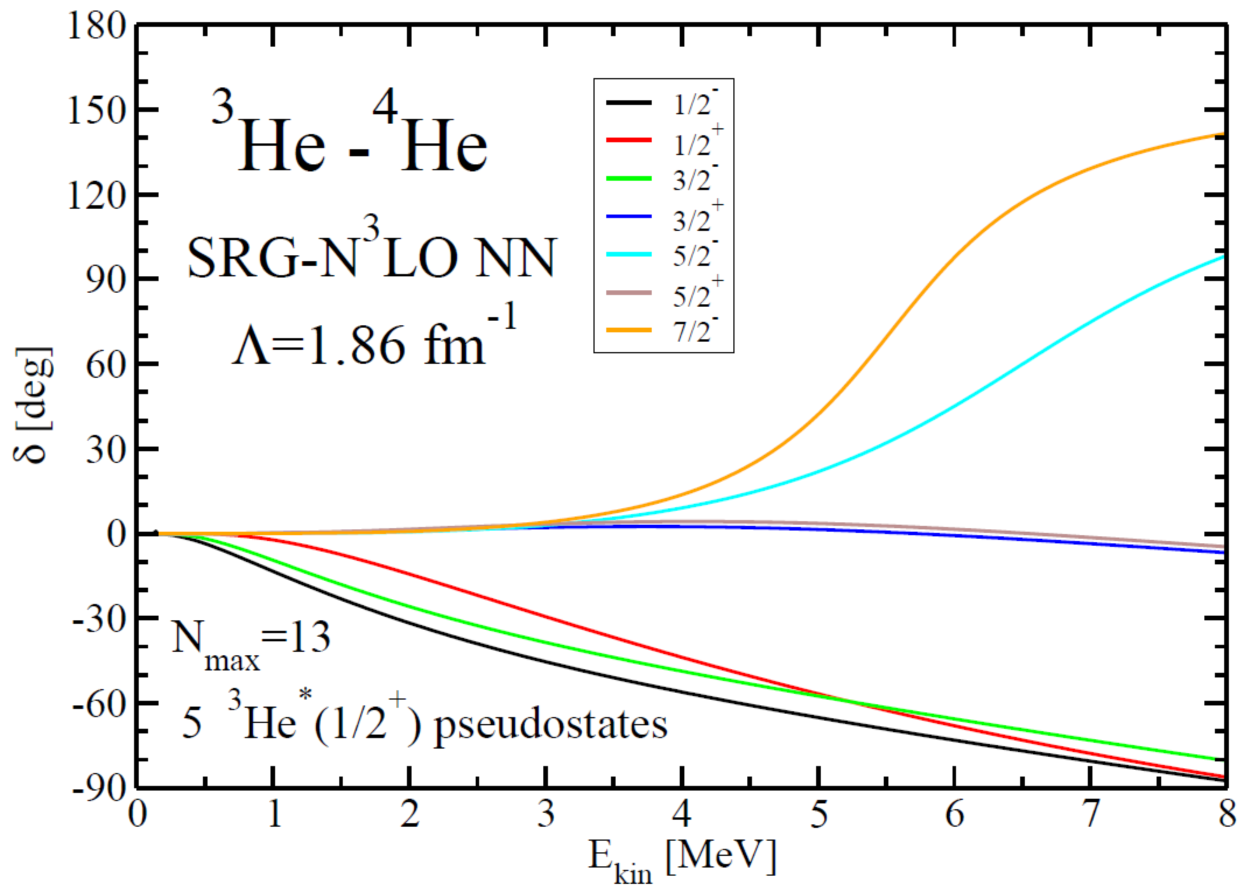
$$\times \underline{\langle A-7\,\beta I_\beta T_\beta||| \Bigg\{ \tilde{a}_{n_g\ell_g j_g\frac{1}{2}} \Big[\tilde{a}_{n_f\ell_f j_f\frac{1}{2}} \Big(\tilde{a}_{n'_b\ell'_b j'_b\frac{1}{2}} \tilde{a}_{n'_a\ell'_a j'_a\frac{1}{2}} \Big)^{(I'_{ab}t'_2)} \Big]^{(k_2q_2)} \Bigg\}^{(K_2\tau_2)} |||A-3\,\alpha_1 I_1 T_1\rangle}$$

A=7 case

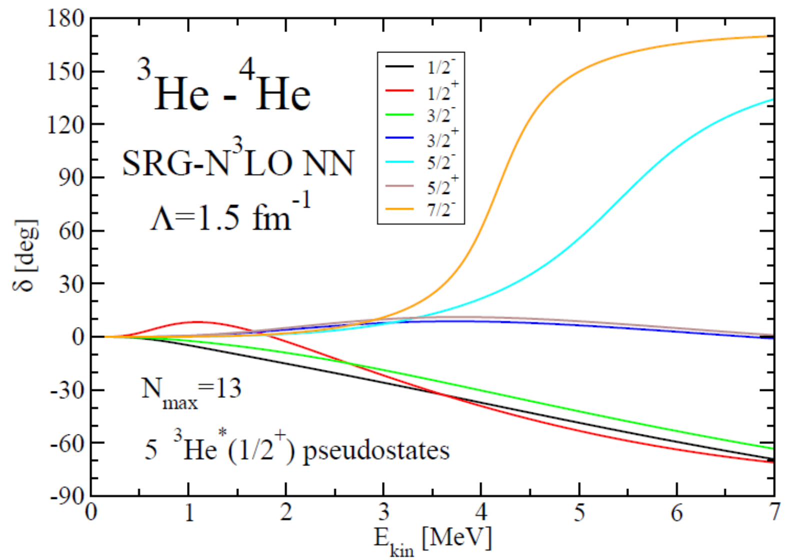
$|A-7\, \beta I_\beta T_\beta\rangle \rightarrow$ vacuum

Phase shifts (preliminary)

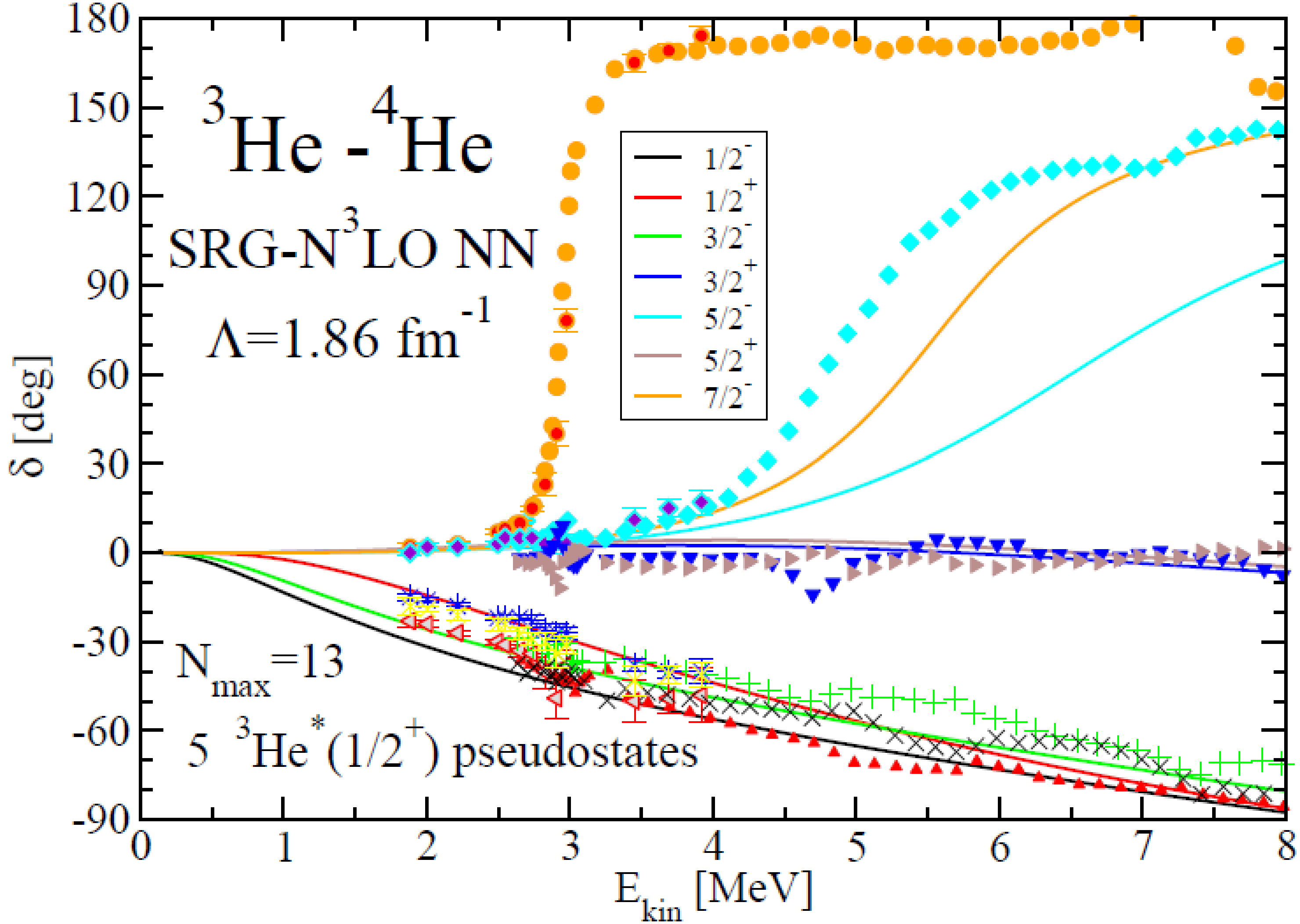




Phase shifts (preliminary)



Phase shifts (preliminary) with Exp.



Other pseudo states (e. g. $3/2^+$) should be added.

Summary and future

- NCSM/RGM for a three-body projectile
  - Algebraic derivation and coding completed
  - Preliminary $^{3}He+^{4}He$ Phase shift
- To be done
  - Check the convergence
    - Pseudo states ($3/2^{+}$, negative parity)
- Future developments
  - Radiative capture reaction of $^{3}He(^{4}He, \gamma)^{7}Be$
  - Heavier target (many-body density)
  - Derive the RGM kernels for a four-particle projectile