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Canada's national laboratory for particle and nuclear physics
Laboratoire national canadien pour la recherche en physique nucléaire et en physique des particules

Recent developments of the NCSM/ RGM approach to nuclear reactions

2nd Workshop on
Perspectives of the *Ab Initio* No-Core Shell Model
February 23-25, 2012
TRIUMF, Vancouver, BC, Canada

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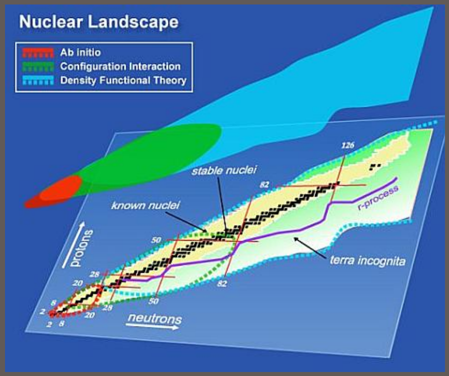

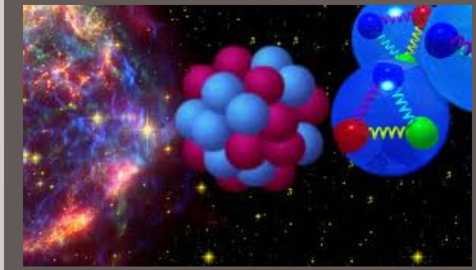

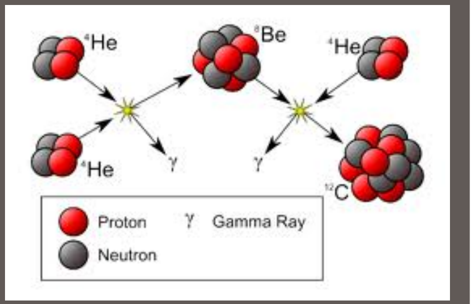

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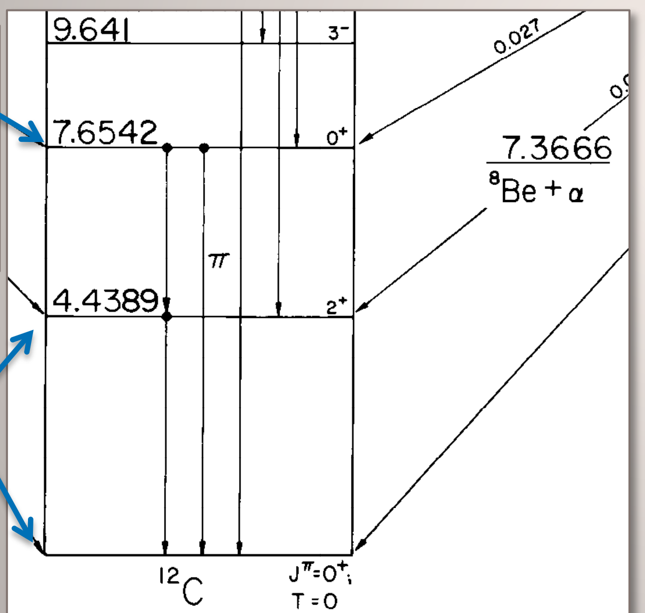
Light nuclei from first principles

- Nuclear **structure** and **reaction** theory for light nuclei cannot be uncoupled
  - Well-bound nuclei, e.g. ^{12}C, have low-lying **cluster-dominated resonances**
  - Bound states of exotic nuclei, e.g. ^{11}Be, manifest **many-nucleon correlations**

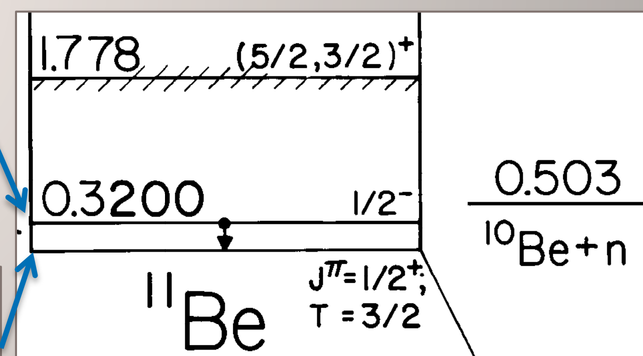
Hoyle state: α-cluster resonance with *p*-shell components

p-shell states



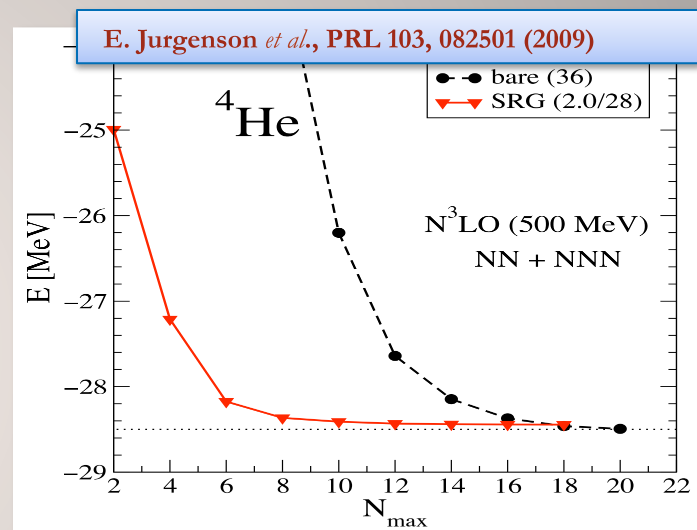
p-shell state with extended tail

$^{10}Be_{gs}$ + (*l*=0) *n* with significant $^{10}Be^*$ components

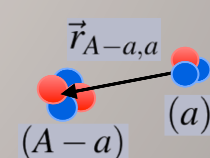


No-core shell model combined with the resonating group method (NCSM/RGM)

- **The NCSM:** An approach to the solution of the *A*-nucleon bound-state problem
  - Accurate nuclear Hamiltonian
  - Finite harmonic oscillator (HO) basis
    - Complete $N_{max}\hbar\Omega$ model space
  - Effective interaction due to the model space truncation
    - Similarity-Renormalization-Group evolved NN(+NNN) potential
  - Short & medium range correlations
  - No continuum



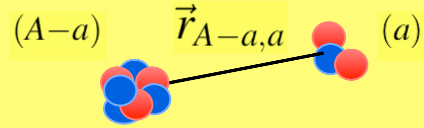
- **The RGM:** A microscopic approach to the A-nucleon scattering of clusters
  - Nuclear Hamiltonian may be simplistic
  - Cluster wave functions may be simplified and inconsistent with the nuclear Hamiltonian
  - Long range correlations, relative motion of clusters



***Ab initio* NCSM/RGM**: Combines the best of both approaches
Accurate nuclear Hamiltonian, consistent cluster wave functions
Correct asymptotic expansion, Pauli principle and translational invariance

The *ab initio* NCSM/RGM in a snapshot

- Ansatz: $\Psi^{(A)} = \sum_{\nu} \int d\vec{r}\, \varphi_\nu(\vec{r})\, \hat{\mathcal{A}}\, \Phi^{(A-a,a)}_{\nu\vec{r}}$



$\psi^{(A-a)}_{1\nu}\, \psi^{(a)}_{2\nu}\, \delta(\vec{r} - \vec{r}_{A-a,a})$

eigenstates of $H_{(A\text{-}a)}$ and $H_{(a)}$ in the *ab initio* NCSM basis

- Many-body Schrödinger equation:

$$H\Psi^{(A)} = E\Psi^{(A)}$$

$$\sum_{\nu} \int d\vec{r} \left[\mathcal{H}^{(A-a,a)}_{\mu\nu}(\vec{r}\,', \vec{r}) - E\mathcal{N}^{(A-a,a)}_{\mu\nu}(\vec{r}\,', \vec{r}) \right] \varphi_\nu(\vec{r}) = 0$$

$T_{\rm rel}(r) + \mathcal{V}_{\rm rel} + \bar{V}_{\rm Coul}(r) + H_{(A-a)} + H_{(a)}$

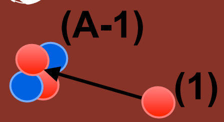
realistic nuclear Hamiltonian

$\langle \Phi^{(A-a,a)}_{\mu\vec{r}\,'} | \hat{\mathcal{A}} H \hat{\mathcal{A}} | \Phi^{(A-a,a)}_{\nu\vec{r}} \rangle$

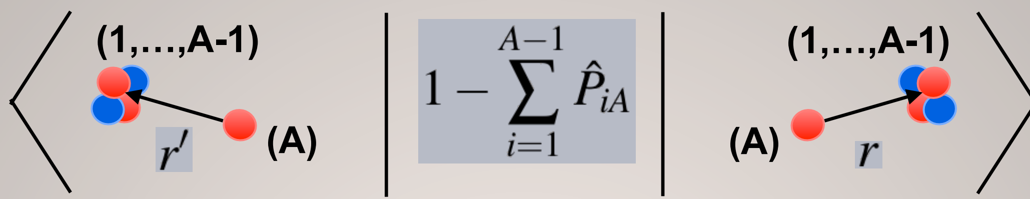
Hamiltonian kernel

$\langle \Phi^{(A-a,a)}_{\mu\vec{r}\,'} | \hat{\mathcal{A}}^2 | \Phi^{(A-a,a)}_{\nu\vec{r}} \rangle$

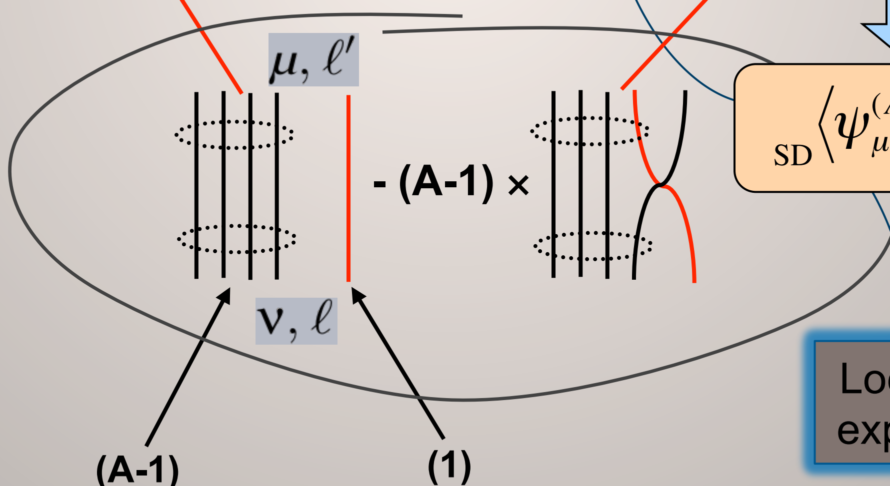
Norm kernel



Single-nucleon projectile: the norm kernel



$$\mathcal{N}^{(A-1,1)}_{\mu\ell',\nu\ell}(r',r) = \delta_{\mu\nu}\,\delta_{\ell'\ell}\frac{\delta(r'-r)}{r'r} - (A-1)\sum_{n'n} R_{n'\ell'}(r')\langle\Phi^{(A-1,1)JT}_{\mu n'\ell'}|P_{A,A-1}|\Phi^{(A-1,1)JT}_{\nu n\ell}\rangle R_{n\ell}(r)$$



$$_{\rm SD}\left\langle \psi^{(A-1)}_{\mu_1}\right| a^+ a \left|\psi^{(A-1)}_{\nu_1}\right\rangle_{\rm SD}$$

Localized parts of kernels expanded in the HO basis

Solving the RGM equations

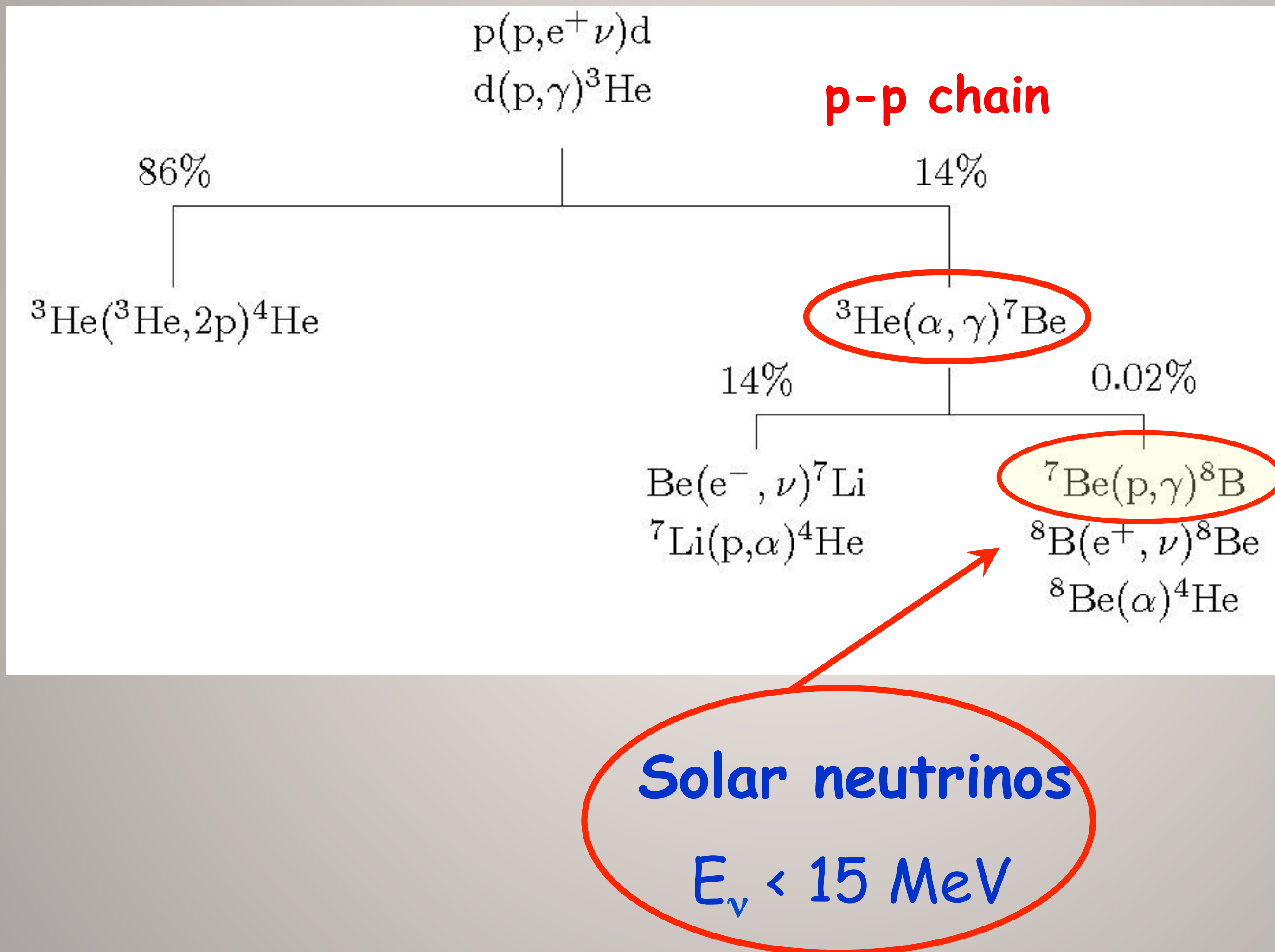
- The many-body problem has been reduced to a two-body problem!
  - Macroscopic degrees of freedom: nucleon clusters
  - Unknowns: relative wave function between the two clusters
- Non-local integral-differential coupled-channel equations:

$$\left[T_{rel}(r) + V_C(r) + E_{\alpha_1}^{(A-a)} + E_{\alpha_2}^{(a)}\right] u_\nu^{(A-a,a)}(r) + \sum_{a'\nu'} \int dr' r'\, W_{a\nu,a'\nu'}(r,r') u_{\nu'}^{(A-a',a')}(r') = 0$$

- Solve with R-matrix theory on Lagrange mesh imposing
  - Bound state boundary conditions ➔ eigenenergy + eigenfunction
  - Scattering state boundary conditions ➔ Scattering matrix
    - Phase shifts
    - Cross sections
    - …

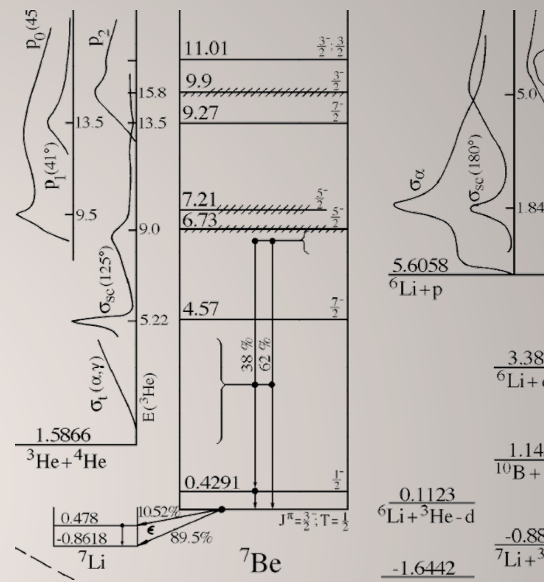
The R-matrix theory on Lagrange mesh is an elegant and powerful technique, particularly for calculations with non-local potentials

Solar *p-p* chain

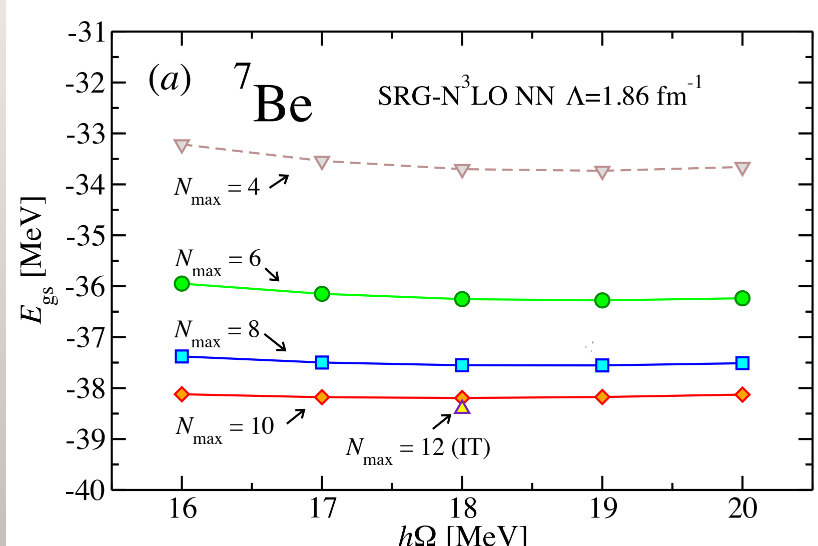


^{7}Be(*p*,γ)^{8}B radiative capture: Input - *NN* interaction, ^{7}Be eigenstates

- Similarity-Renormalization-Group (SRG) evolved chiral N^{3}LO *NN* interaction
    - Accurate
    - Soft: Evolution parameter Λ
        - Study dependence on Λ



- ^{7}Be
    - NCSM up to N_{max}=10, Importance Truncated NCSM up to N_{max}=14
    - Variational calculation
        - optimal HO frequency from the ground-state minimum
        - For the selected NN potential with Λ=1.86 fm^{-1}: hΩ=18 MeV



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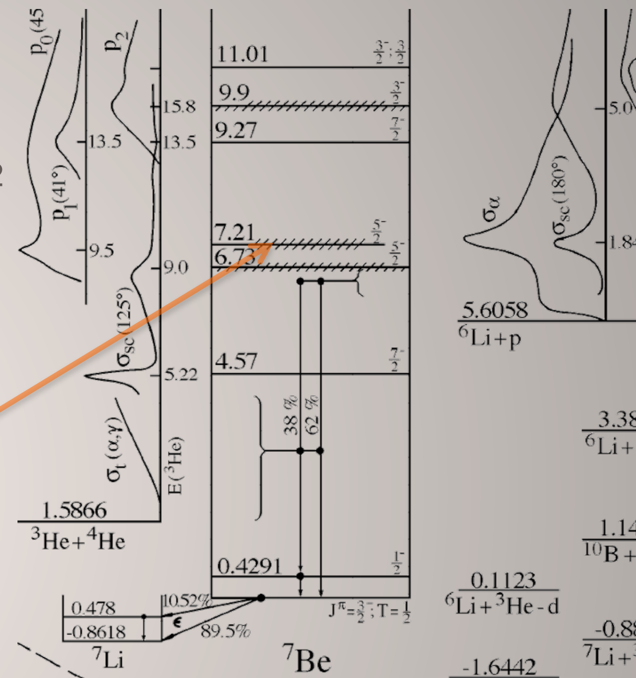
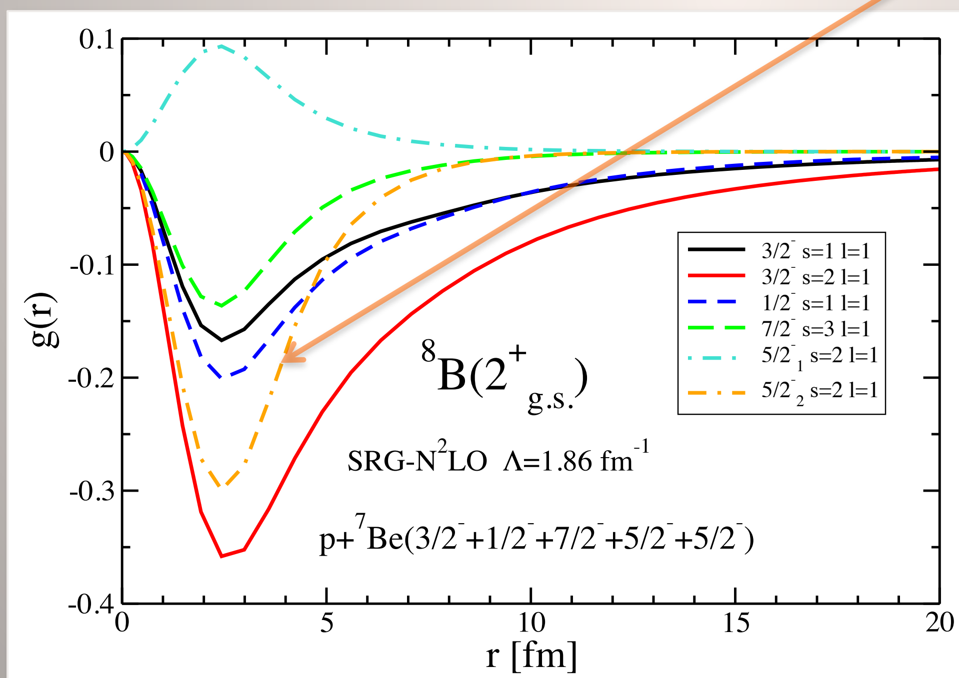
Structure of the ^{8}B ground state

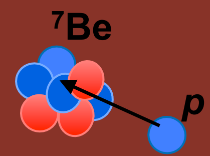
- NCSM/RGM p-^{7}Be calculation
  - five lowest ^{7}Be states: $3/2^-$, $1/2^-$, $7/2^-$, $5/2^-_1$, $5/2^-_2$
  - Soft NN SRG-N^{3}LO with Λ = 1.86 fm^{-1}
- ^{8}B 2^+ g.s. bound by 136 keV (Expt 137 keV)
  - Large P-wave $5/2^-_2$ component





$5/2^-_2$ state of ^{7}Be should be included in ^{7}Be(p, γ)^{8}B calculations

p-^{7}Be scattering

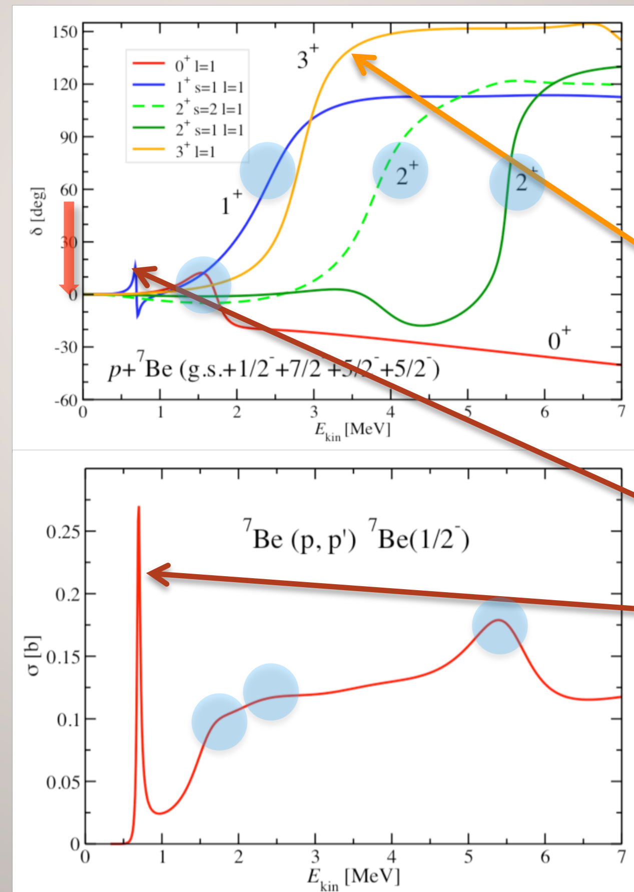


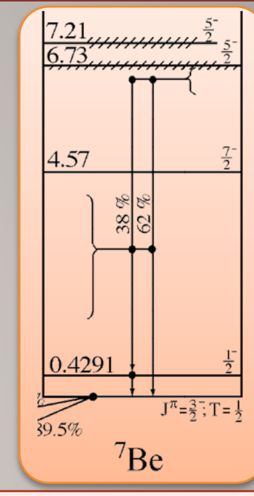
- NCSM/RGM calculation of p-^{7}Be scattering
  - ^{7}Be states $3/2^-$, $1/2^-$, $7/2^-$, $5/2^-_1$, $5/2^-_2$
  - Soft NN potential (SRG-N^{3}LO with Λ = 1.86 fm^{-1})

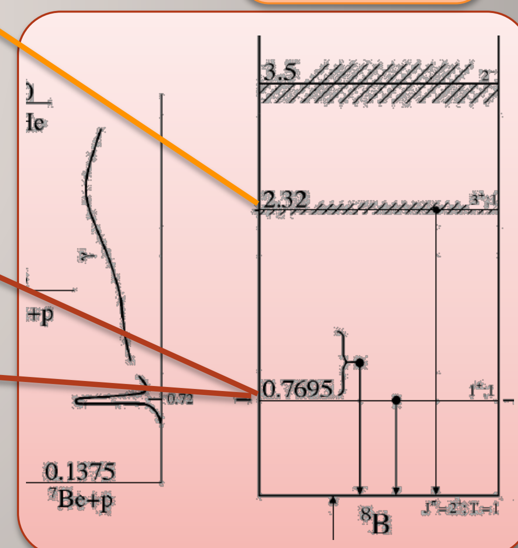
^{8}B 2^+ g.s. **bound** by 136 keV
(expt. bound by 137 keV)

New 0^+, 1^+, and two 2^+ resonances **predicted**

s =1 l =1 2^+ clearly visible in (p,p') cross sections







PRC **82**, 034609 (2010)

$^7Be(p,\gamma)^8B$ radiative capture

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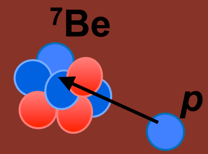
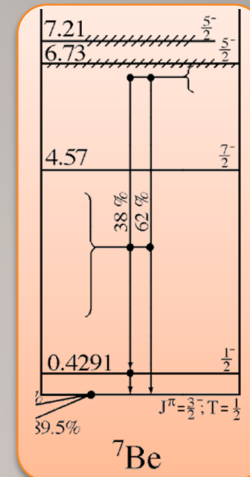


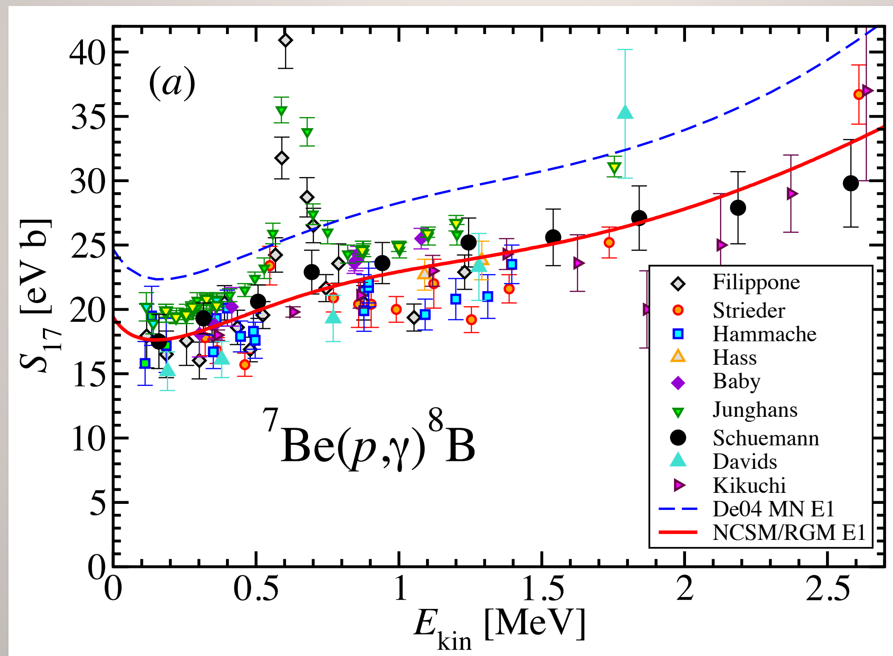
- NCSM/RGM calculation of $^7Be(p,\gamma)^8B$ radiative capture
  - ^7Be states $3/2^-$, $1/2^-$, $7/2^-$, $5/2^-_1$, $5/2^-_2$
  - Soft NN potential (SRG-N^{3}LO with Λ = 1.86 fm^{-1})

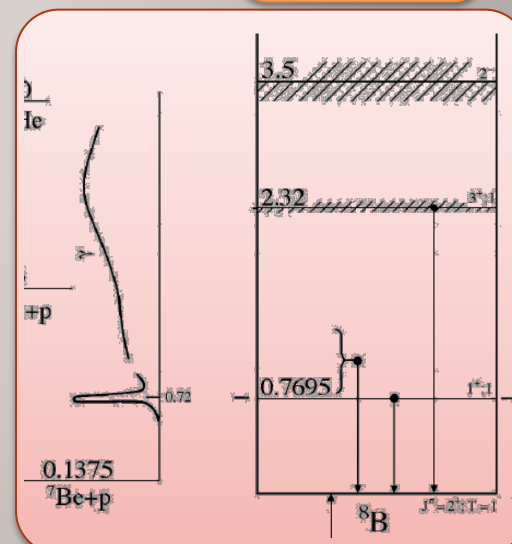


^8B 2^+ g.s. **bound** by 136 keV (expt. 137 keV)

S(0) ~ 19.4(0.7) eV b

Data evaluation: S(0)=20.8(2.1) eV b





The first ever *ab initio* calculations of $^7Be(p,\gamma)^8B$

Physics Letters B 704 (2011) 379

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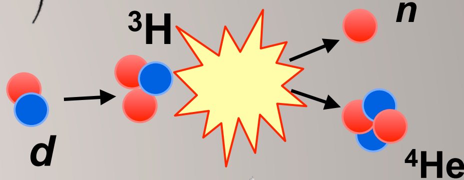
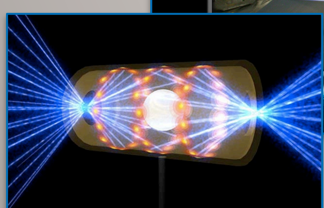
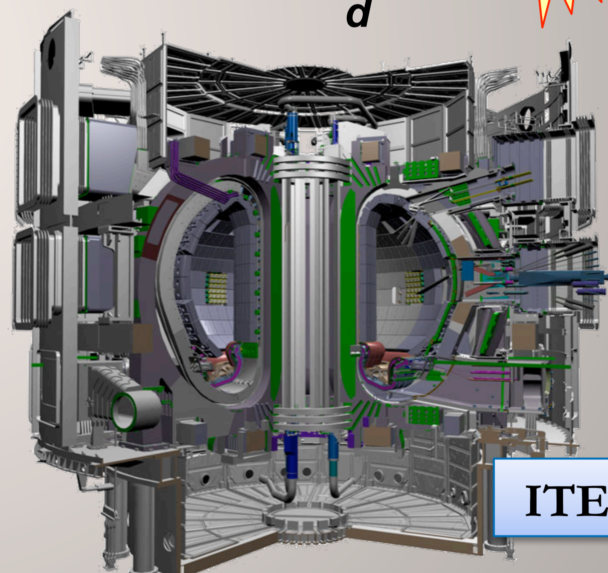
Ab initio calculation of the ^{3}H(d,n)^{4}He fusion

$$\int dr\, r^2 \begin{pmatrix} \left\langle \overset{r'}{n\,\alpha} \right| \hat{A}_1 (H-E)\hat{A}_1 \left| \overset{r}{\alpha\, n} \right\rangle & \left\langle \overset{r'}{n\,\alpha} \right| \hat{A}_1 (H-E)\hat{A}_2 \left| \overset{r}{{}^3\mathrm{H}\, d} \right\rangle \\ \left\langle \overset{r'}{d\,{}^3\mathrm{H}} \right| \hat{A}_2 (H-E)\hat{A}_1 \left| \overset{r}{\alpha\, n} \right\rangle & \left\langle \overset{r'}{d\,{}^3\mathrm{H}} \right| \hat{A}_2 (H-E)\hat{A}_2 \left| \overset{r}{{}^3\mathrm{H}\, d} \right\rangle \end{pmatrix} \begin{pmatrix} \frac{g_1(r)}{r} \\ \frac{g_2(r)}{r} \end{pmatrix} = 0$$





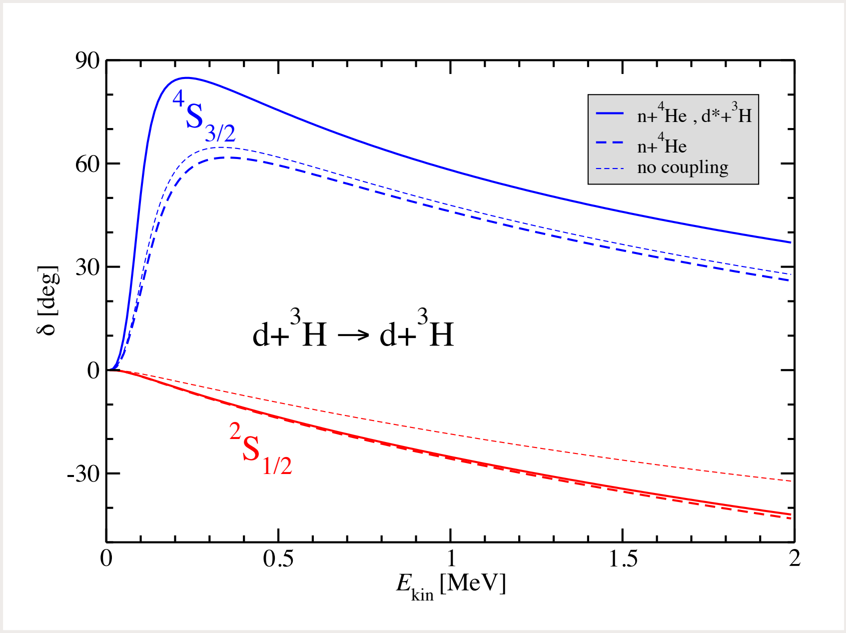
NIF

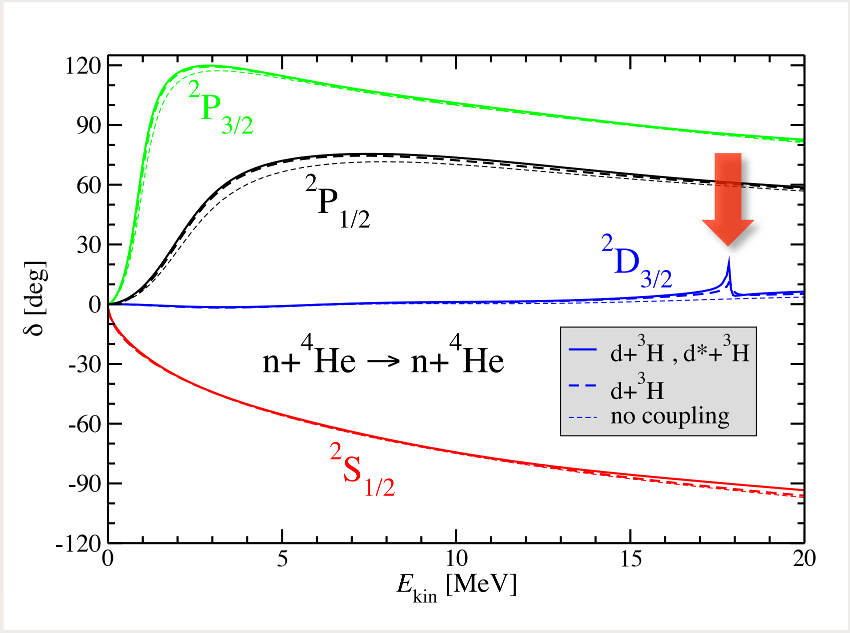


ITER

energy generation

d+^{3}H and n+^{4}He elastic scattering: phase shifts





- d+^{3}H elastic phase shifts:
  - Resonance in the $^4S_{3/2}$ channel
  - Repulsive behavior in the $^2S_{1/2}$ channel ➔ Pauli principle

d^* deuteron pseudo state in $^3S_1$-$^3D_1$ channel: deuteron polarization, virtual breakup

- n+^{4}He elastic phase shifts:
  - d+^{3}H channels produces slight increase of the P phase shifts
  - Appearance of resonance in the 3/2$^+$ D-wave, just above d-^{3}H threshold

The d-^{3}H fusion takes place through a transition of d+^{3}H is S-wave to n+^{4}He in D-wave: Importance of the **tensor force**

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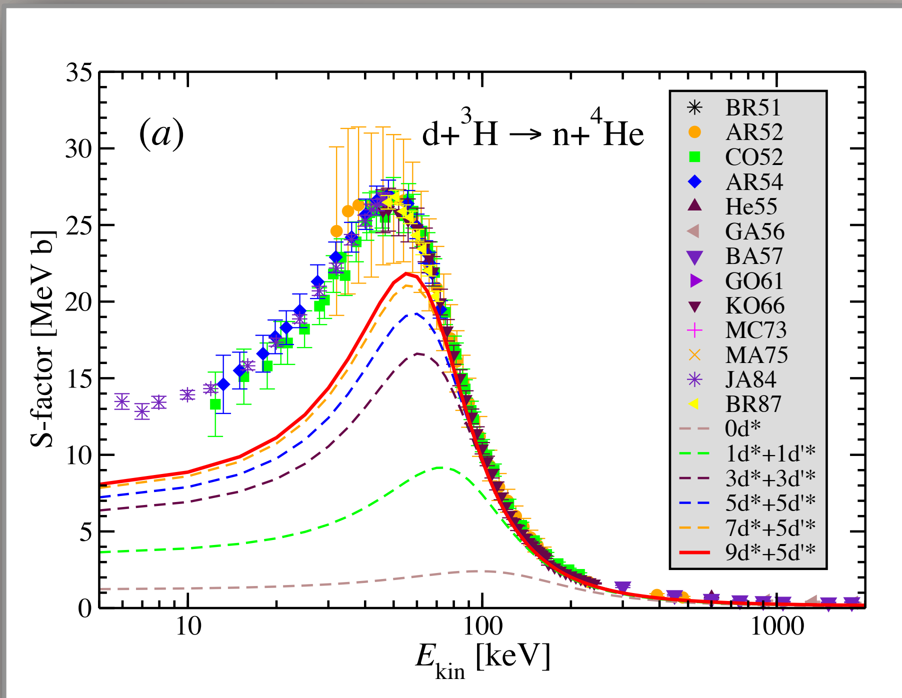
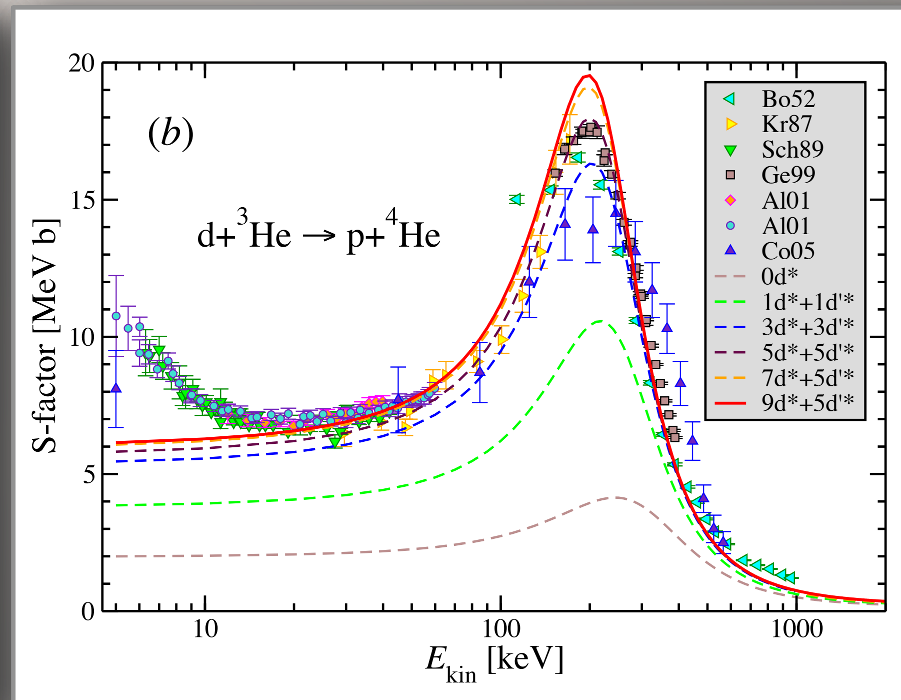
$^3H(d,n)^4He$ and $^3He(d,p)^4He$ cross sections





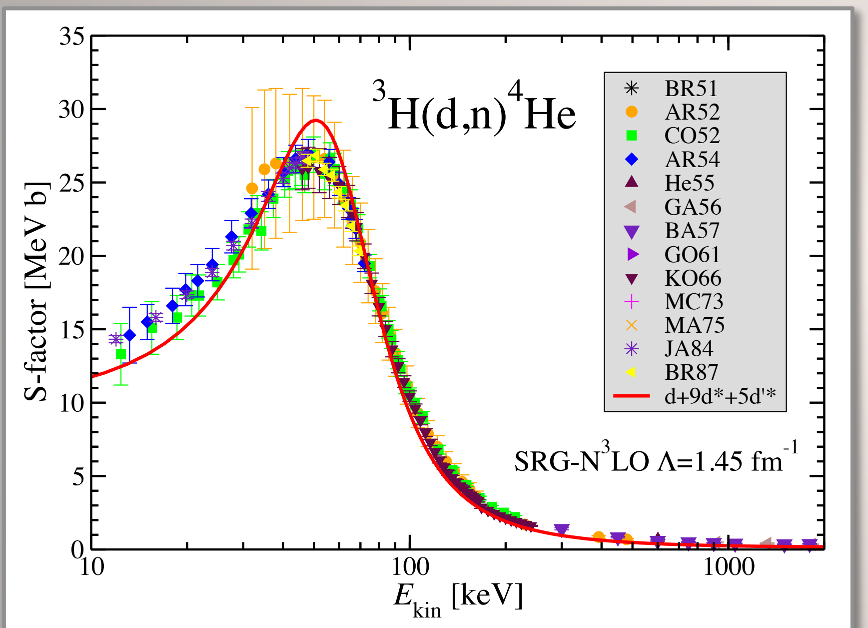
- The cross section improves with the inclusion of virtual breakup of the deuteron
  - Deuteron weakly bound: easily gets polarized and easily breaks
  - These effects included below the breakup threshold with continuum discretized by pseudo-states

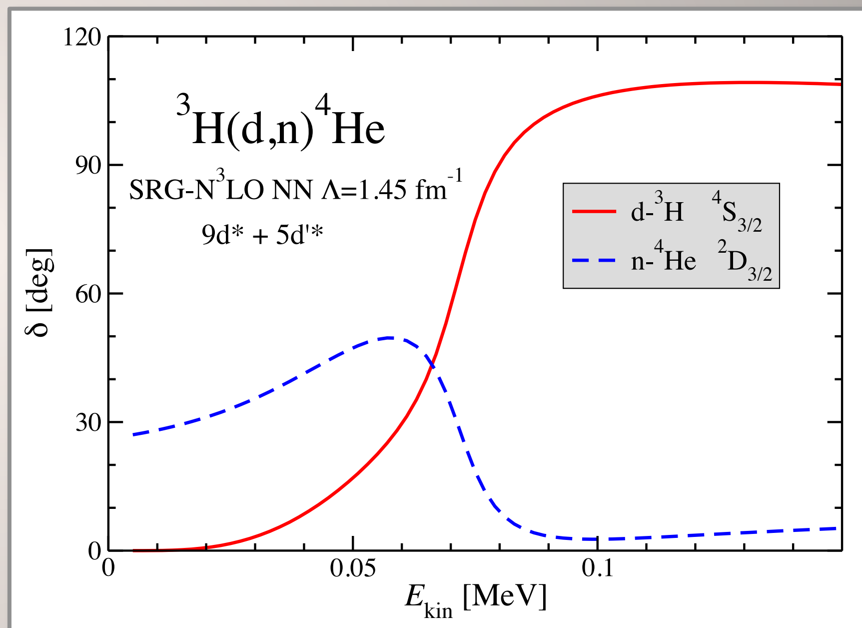
*d** deuteron pseudo state in $^3S_1$-$^3D_1$ channel; *d'** deuteron pseudo state in $^3D_2$ channel

First *ab initio* results for *d*-^{3}H and *d*-^{3}He fusion:
Very promising, correct physics, can become competitive with fitted evaluations …

^{3}H(d,n)^{4}He cross section

- SRG-N^{3}LO (Λ=1.45 fm^{-1}) NN potential
  - Position of the resonance matches experiment





Improvements:
Excitations of ^{3}H, ^{4}He; n-p-^{3}H rather than d^*, d'^*, NNN interaction…
Potential to address unresolved fusion research related questions:
^{3}H(d,n)^{4}He fusion with polarized deuterium and/or tritium,
^{3}H(d,$n\gamma$)^{4}He bremsstrahlung …

PRL **108**, 042503 (2012)

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New results and developments

- Structure of exotic nuclei
  - ^{9}He: Talk by Michael Kruse
- Calculations with the three-nucleon projectile
  - ^{3}He-^{4}He and ^{3}H-^{4}He scattering: Done on *jaguar* with up to 65,536 cores
  - ^{3}H-^{3}H scattering
  - Goal: $^{3}He(\alpha,\gamma)^{7}Be$, $^{3}H(\alpha,\gamma)^{7}Li$
  - Talk by Wataru Horiuchi
- Reactions with three-body final states
  - Goal: $^{3}He(^{3}He,2p)^{4}He$, $^{3}H(^{3}H,2n)^{4}He$,
  - structure of ^{6}He - $^{4}He+n+n$ or ^{11}Li - $^{9}Li+n+n$
  - Talk by Carolina Romero-Redondo
- Ultimate coupling of the bound and continuum states:
  - NCSM with the continuum (NCSMC)
  - the most efficient way to include many-body correlations and speed up convergence of both bound- and scattering state calculations
  - Example: ^{9}Li NCSM eigenstates coupled with the $^{8}Li+n$ NCSM/RGM basis
  - Talk by Simone Baroni

New developments: Including 3N interaction

- Very important in particular since we use soft SRG evolved interactions
  - induced 3N quite significant for A>~10
- Enabled by 3N coupled-*J* scheme
  - Introduced and implemented by Robert Roth *et al.*
  - Now also in my codes
    - Jacobi-Slater Determinant transformation code
    - NCSD code
    - Example: ^{6}He, ^{8}He NCSM calculations up to N_{max}=10 done with moderate resources

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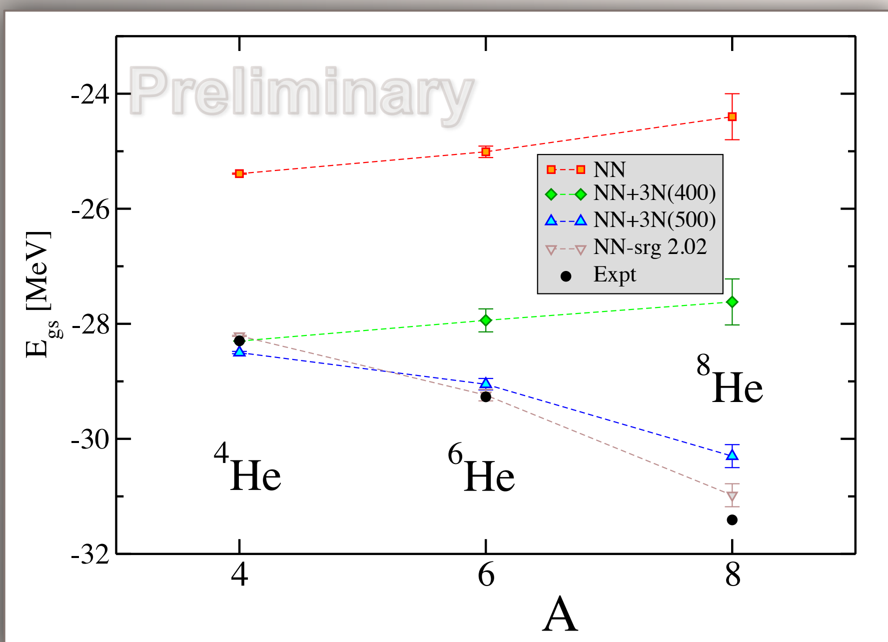
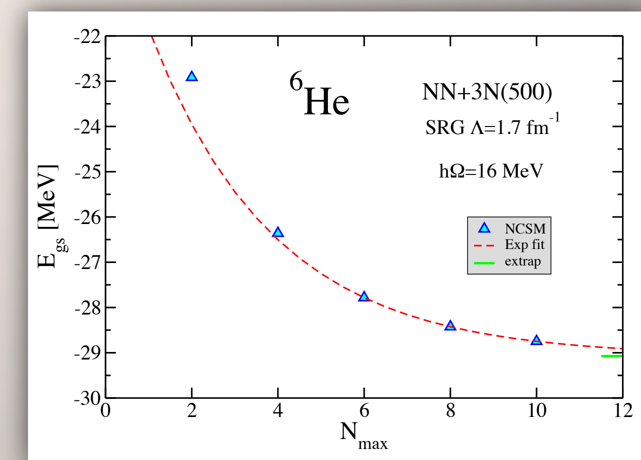
3N interaction effects in neutron rich nuclei: He isotopes

- ^{6}He and ^{8}He with SRG-evolved chiral N^{3}LO NN + N^{2}LO 3N
  - chiral N^{3}LO NN: ^{4}He underbound, ^{6}He and ^{8}He unbound
  - chiral N^{3}LO NN + N^{2}LO 3N(400): ^{4}He fitted, ^{6}He barely unbound, ^{8}He unbound
    - describes quite well binding energies of ^{12}C, ^{16}O, ^{40}Ca, ^{48}Ca
  - chiral N^{3}LO NN + N^{2}LO 3N(500): ^{4}He OK, both ^{6}He and ^{8}He bound
    - does well up to *A*=10, overbinds ^{12}C, ^{16}O, Ca isotopes
  - SRG-N^{3}LO NN Λ=2.02 fm^{-1}: ^{4}He OK, both ^{6}He and ^{8}He bound
    - ^{16}O, Ca strongly overbound





Our knowledge of the 3N interaction is incomplete

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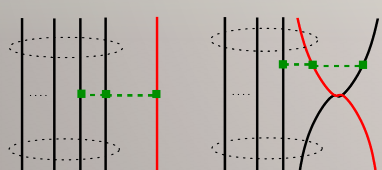
Including 3N interaction in the NCSM/RGM
Single-nucleon projectile:

$$\left\langle \Phi^{J^\pi T}_{\nu' r'} \middle| \hat{A}_{\nu'} V^{NNN} \hat{A}_{\nu} \middle| \Phi^{J^\pi T}_{\nu r} \right\rangle = \left\langle \begin{matrix}(A-1)\\ r' \quad (a'=1)\end{matrix} \middle| V^{NNN}\left(1-\sum_{i=1}^{A-1}\hat{P}_{iA}\right) \middle| \begin{matrix}(A-1)\\ (a=1) \quad r\end{matrix} \right\rangle$$

$$\mathcal{V}^{NNN}_{\nu'\nu}(r,r') = \sum R_{n'l'}(r')R_{nl}(r)\left[\frac{(A-1)(A-2)}{2}\langle\Phi^{J^\pi T}_{\nu' n'}|V_{A-2A-1A}(1-2P_{A-1A})|\Phi^{J^\pi T}_{\nu n}\rangle - \frac{(A-1)(A-2)(A-3)}{2}\langle\Phi^{J^\pi T}_{\nu' n'}|P_{A-1A}V_{A-3A-2A-1}|\Phi^{J^\pi T}_{\nu n}\rangle\right].$$

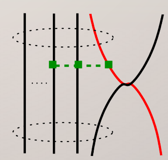
Direct potential: in the model space
(interaction is localized!)



(a) (b)

$$\propto {}_{SD}\left\langle \psi^{(A-1)}_{\alpha_1'} \middle| a^+_i a^+_j a_l a_k \middle| \psi^{(A-1)}_{\alpha_1} \right\rangle_{SD}$$

Exchange potential: in the model space
(interaction is localized!)



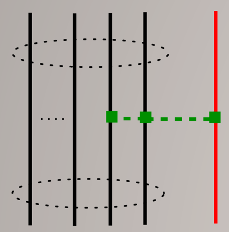
(c)

$$\propto {}_{SD}\left\langle \psi^{(A-1)}_{\alpha_1'} \middle| a^+_h a^+_i a^+_j a_m a_l a_k \middle| \psi^{(A-1)}_{\alpha_1} \right\rangle_{SD}$$

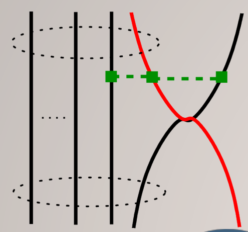
Including 3N interaction challenging: more than *2 body density required*

TRIUMF

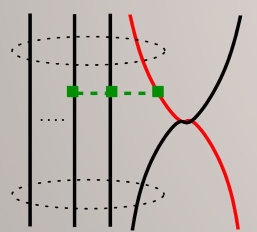
Including 3N interaction in the NCSM/RGM: Direct and exchange terms



(a)



(b)



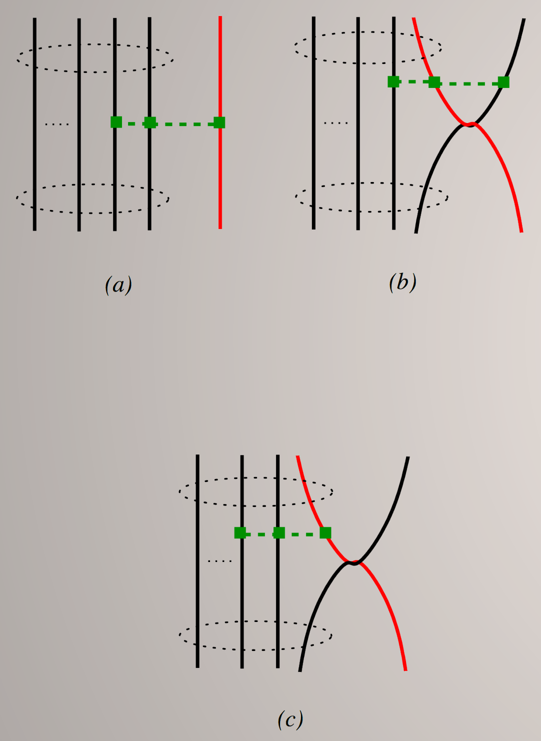
(c)

$$
\begin{aligned}
&\sum \hat{s}\hat{s}'\hat{j}\hat{j}'\hat{\tau}\hat{K}\,(-1)^{J_1'+j'+J}(-1)^{j_0+J_0-j+2j'}(-1)^{T_1+1/2+T}(-1)^{1/2+T_0+t_0+1}\\
&\quad\left\{\begin{array}{ccc} J_1 & 1/2 & s\\ l & J & j\end{array}\right\}\left\{\begin{array}{ccc} J_1' & 1/2 & s'\\ l' & J & j'\end{array}\right\}\\
&\quad\left\{\begin{array}{ccc} J_1 & K & J_1'\\ j' & J & j\end{array}\right\}\left\{\begin{array}{ccc} j_0' & j_0 & K\\ j & j' & J_0\end{array}\right\}\\
&\quad\left\{\begin{array}{ccc} T_1 & \tau & T_1'\\ 1/2 & T & 1/2\end{array}\right\}\left\{\begin{array}{ccc} t_0' & t_0 & \tau\\ 1/2 & 1/2 & T_0\end{array}\right\}\\
&\quad\Big\langle\Big[(n_a'l_a'j_a':n_b'l_b'j_b')\,j_0't_0':\ n'l'j'\Big]\,J_0T_0\Big|V_{A-2A-1A}(1-2P_{A-1A})\Big|\Big[\Big(n_al_aj_a:\ n_bl_bj_b\Big)\,j_0t_0:nlj\Big]\,J_0T_0\Big\rangle\\
&\quad\left\langle\alpha_{A-1}'J_1'T_1'\left|\left|\left|\left[\left[a^\dagger_{n_a'l_a'j_a'}a^\dagger_{n_b'l_b'j_b'}\right]^{j_0't_0'}\left[\tilde{a}_{n_al_aj_a}\tilde{a}_{n_bl_bj_b}\right]^{j_0t_0}\right]^{K\tau}\right|\right|\right|\alpha_{A-1}J_1T_1\right\rangle
\end{aligned}
$$

$$
\begin{aligned}
&\sum \hat{s}\hat{s}'\hat{j}\hat{j}'\hat{\tau}\hat{K}\hat{J}_0\hat{T}_0\hat{g}'\hat{t}_g'\hat{j}_0'\hat{t}_0'\hat{k}'\hat{t}_k'\,(-1)^{J_1'+j'+J}(-1)^{j+j_a'+j_b'+j_0'+J_0+k'}(-1)^{T_1+1/2+T}(-1)^{1-T_0-\tau+t_0'+t_k'}\\
&\quad\left\{\begin{array}{ccc} J_1 & 1/2 & s\\ l & J & j\end{array}\right\}\left\{\begin{array}{ccc} J_1' & 1/2 & s'\\ l' & J & j'\end{array}\right\}\\
&\quad\left\{\begin{array}{ccc} J_1 & K & J_1'\\ j' & J & j\end{array}\right\}\left\{\begin{array}{ccc} g' & J_0 & K\\ j_0' & k' & j_b'\end{array}\right\}\left\{\begin{array}{ccc} k' & j_0' & K\\ j' & j & j_a'\end{array}\right\}\\
&\quad\left\{\begin{array}{ccc} T_1 & \tau & T_1'\\ 1/2 & T & 1/2\end{array}\right\}\left\{\begin{array}{ccc} t_g' & T_0 & \tau\\ t_0' & t_k' & 1/2\end{array}\right\}\left\{\begin{array}{ccc} t_k' & t_0' & \tau\\ 1/2 & 1/2 & 1/2\end{array}\right\}\\
&\quad\Big\langle\Big[\Big(n'l'j':\ n_a'l_a'j_a'\Big)\,j_0't_0':n_b'l_b'j_b'\Big]\,J_0T_0\Big|V_{A-3\,A-2\,A-1}\Big|\Big[(n_\alpha l_\alpha j_\alpha:n_al_aj_a)\,j_0t_0:\ n_bl_bj_b\Big]\,J_0T_0\Big\rangle\\
&\quad\left\langle\alpha_{A-1}'J_1'T_1'\left|\left|\left|\left[\left\{\left[a^\dagger_{nlj}a^\dagger_{n_a'l_a'j_a'}\right]^{k't_k'}a^\dagger_{n_b'l_b'j_b'}\right\}^{g't_g'}\left\{\left[\tilde{a}_{n_\alpha l_\alpha j_\alpha}\tilde{a}_{n_al_aj_a}\right]^{j_0t_0}\tilde{a}_{n_bl_bj_b}\right\}^{J_0T_0}\right]^{K\tau}\right|\right|\right|\alpha_{A-1}J_1T_1\right\rangle
\end{aligned}
$$

G. Hupin: Kernel derivations with many-body densities.
Use of existing codes:
Applicable to A=3,4 targets

Including 3N interaction in the NCSM/RGM: Direct and exchange terms

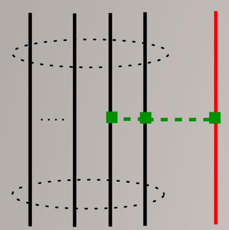


J. Langhammer:

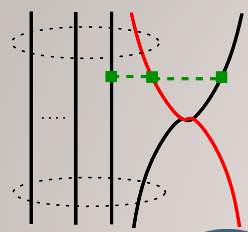
Kernel derivations without the angular momentum re-coupling and the many-body density factorization.

Kernel calculations directly from the target eigenvectors:
Applicable to *p*-shell nuclei targets
The same strategy possible for multi-nucleon projectiles and *A*>4 targets

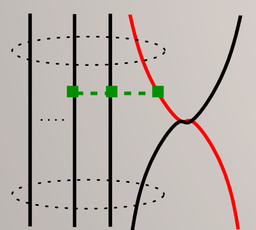
Including 3N interaction in the NCSM/RGM: Direct and exchange terms



(a)



(b)



(c)

$$\langle \epsilon^{J\pi T}_{\nu' n'} | \hat{V}_{A-2\,A-1\,A}(1-\hat{T}_{A-1,A}-\hat{T}_{A-2,A}) | \epsilon^{J\pi T}_{\nu n}\rangle$$
$$= \sum_{M_1 m_j}\sum_{M_{T_1} m_t} \left(\begin{array}{cc|c} I_1 & j & J \\ M_1 & m_j & M_J \end{array}\right)\left(\begin{array}{cc|c} T_1 & \frac{1}{2} & T \\ M_{T_1} & m_t & M_T \end{array}\right)$$
$$\sum_{M'_1 m'_j}\sum_{M'_{T_1} m'_t} \left(\begin{array}{cc|c} I'_1 & j' & J \\ M'_1 & m'_j & M'_J \end{array}\right)\left(\begin{array}{cc|c} T'_1 & \frac{1}{2} & T \\ M'_{T_1} & m'_t & M'_T \end{array}\right)$$
$$\frac{1}{2(A-1)(A-2)}\sum_{\beta_{A-2}}\sum_{\beta_{A-1}}\sum_{\beta'_{A-2}}\sum_{\beta'_{A-1}}$$
$$\langle \Psi' I'_1 M'_1 T'_1 M'_{T_1} | \hat{a}^\dagger_{\beta_{A-1}}\hat{a}^\dagger_{\beta_{A-2}}\hat{a}_{\beta'_{A-2}}\hat{a}_{\beta'_{A-1}} | \Psi I_1 M_1 T_1 M_{T_1}\rangle$$
$${}_a\langle \beta_{A-2}\beta_{A-1} n' l' j' m'_j m'_t | \hat{V} | \beta'_{A-2}\beta'_{A-1} n l j m_j m_t\rangle_a$$

$$\langle \epsilon^{J\pi T}_{\nu' n'} | \hat{V}_{A-3\,A-2\,A}\hat{T}_{A-1,A} | \epsilon^{J\pi T}_{\nu n}\rangle$$
$$= \sum_{M_1 m_j}\sum_{M_{T_1} m_t} \left(\begin{array}{cc|c} I_1 & j & J \\ M_1 & m_j & M_J \end{array}\right)\left(\begin{array}{cc|c} T_1 & \frac{1}{2} & T \\ M_{T_1} & m_t & M_T \end{array}\right)$$
$$\sum_{M'_1 m'_j}\sum_{M'_{T_1} m'_t} \left(\begin{array}{cc|c} I'_1 & j' & J \\ M'_1 & m'_j & M'_J \end{array}\right)\left(\begin{array}{cc|c} T'_1 & \frac{1}{2} & T \\ M'_{T_1} & m'_t & M'_T \end{array}\right)$$
$$\frac{1}{6}\frac{1}{(A-1)(A-2)(A-3)}\sum_{\beta_{A-3}}\sum_{\beta_{A-2}}\sum_{\beta'_{A-3}}\sum_{\beta'_{A-2}}\sum_{\beta'_{A-1}}$$
$$\langle \Psi' I'_1 M'_1 T'_1 M'_{T_1} | \hat{a}^\dagger_{nljm_j\frac{1}{2}m_t}\hat{a}^\dagger_{\beta_{A-2}}\hat{a}^\dagger_{\beta_{A-3}}\hat{a}_{\beta'_{A-3}}\hat{a}_{\beta'_{A-2}}\hat{a}_{\beta'_{A-1}} | \Psi I_1 M_1 T_1 M_{T_1}\rangle$$
$${}_a\langle \beta_{A-3}\beta_{A-2} n' l' j' m'_j \tfrac{1}{2} m'_t | \hat{V}_{A-3\,A-2\,A} | \beta'_{A-3}\beta'_{A-2}\beta'_{A-1}\rangle_a$$

Computational challenge: Large scale parallelization, target eigenvectors for multiple M values

TRIUMF

Conclusions and Outlook

- With the NCSM/RGM approach we are extending the *ab initio* effort to describe low-energy reactions and weakly-bound systems
- The first ^{7}Be(p,γ)^{8}B *ab initio* S-factor calculation

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- Deuteron-projectile results with SRG-N^{3}LO *NN* potentials:
  - *d*-^{4}He scattering
  - First *ab initio* study of ^{3}H(d,n)^{4}He & ^{3}He(d,p)^{4}He fusion

PRC **83**, 044609 (2011)

PRL **108**, 042503 (2012)

- Under way:
  - *n*-^{8}He scattering and ^{9}He structure
  - ^{3}He-^{4}He and ^{3}He-^{3}He scattering calculations
  - *Ab initio* NCSM with continuum (NCSMC)
  - Three-cluster NCSM/RGM and treatment of three-body continuum
  - Inclusion of NNN force
- To do:
  - Alpha clustering: ^{4}He projectile