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Subleading chiral few-nucleon forces

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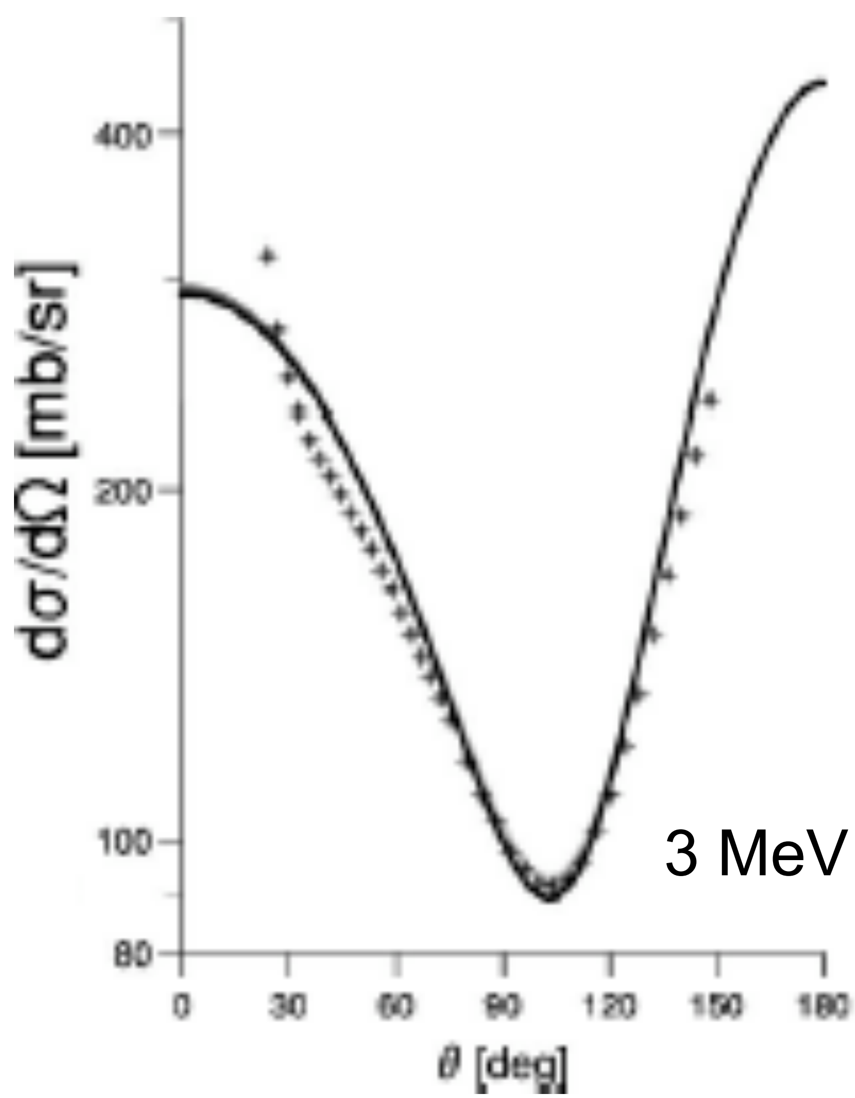
- Motivation
  - need for three-nucleon interactions
  - few-nucleon scattering data
- Leading three-nucleon forces
  - power counting estimates and numerical results
  - discussion of results
- Subleading few-nucleon interactions
  - 4N forces / numerical results
  - 3N forces / status
- Conclusions & Outlook

Phenomenological approach

Several NN force models describe the data (~ 4000 data) up to the pion production threshold perfectly using ~ 40 parameters

Long-range part is driven by one-pion exchange

Predictions based on NN forces are reasonable:
Many low energy few-nucleon observables are well & model independently described !



(see e.g. Witała et al., 2001)

Approximation to the nuclear Hamiltonian does not seem to be too bad, but .....

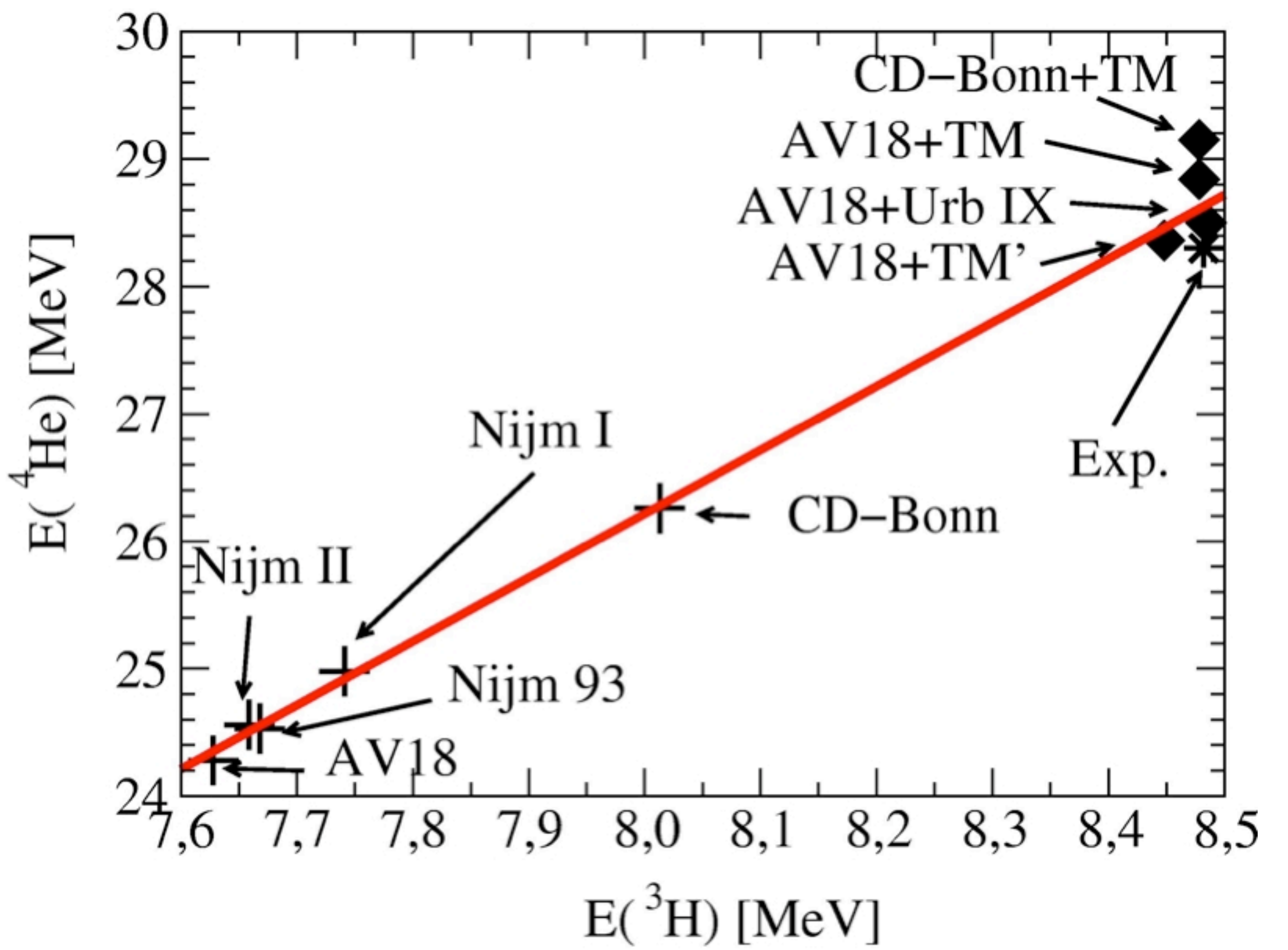
Phenomenological approach

Binding energies are not model-independent

& the results do not agree with experiment

	^3H	^4He
CD-Bonn	-8.013	-26.23
AV18	-7.628	-24.25
Nijm I	-7.741	-24.99
Nijm II	-7.659	-24.55
Expt	-8.482	-28.30

(see e.g. A.N. et al., 2002)

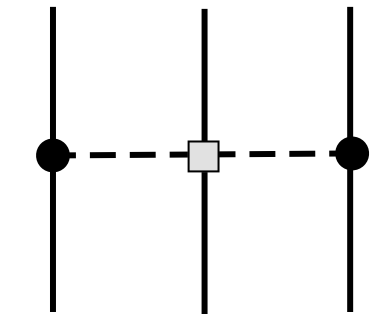


→ **3NF's are quantitatively important for binding energies.**
Modified NN interactions?

Cancelation of kinetic and potential energy!
Small parts of the nuclear Hamiltonian are relevant

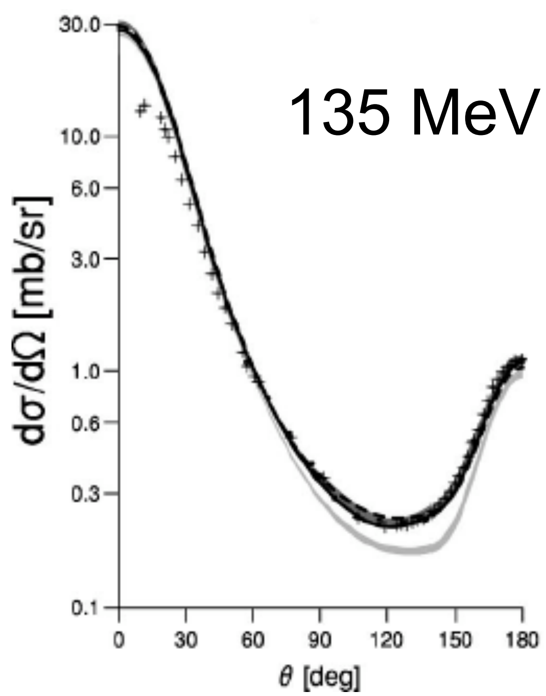
Phenomenological 3NF's

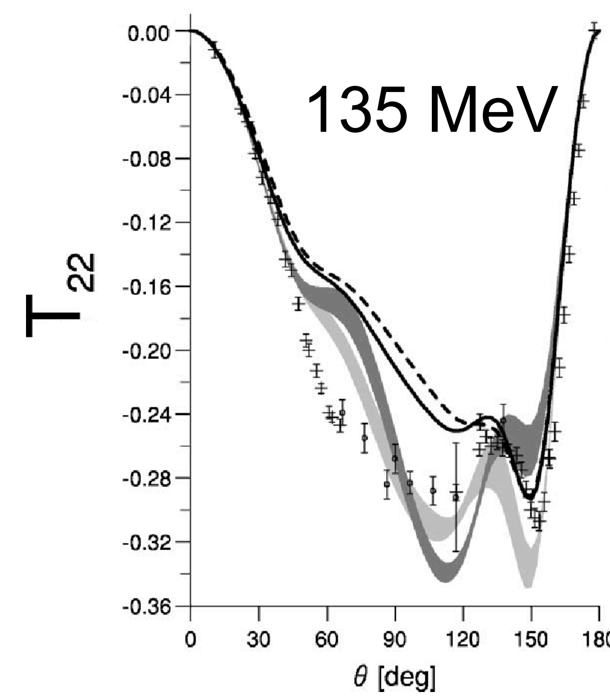
NN interactions can be augmented by phenomenological 3N interactions
(Tuscon-Melbourne, Urbana, Illinois, ...)



usually the 3N force is adjusted so that the
^{3}H binding energy is described correctly (remember Tjon-line correlation)

→ **none of the phenomenological models describes all the data!**





relativistic effects are small at these energies (see e.g. Sekiguchi et al., 2005)

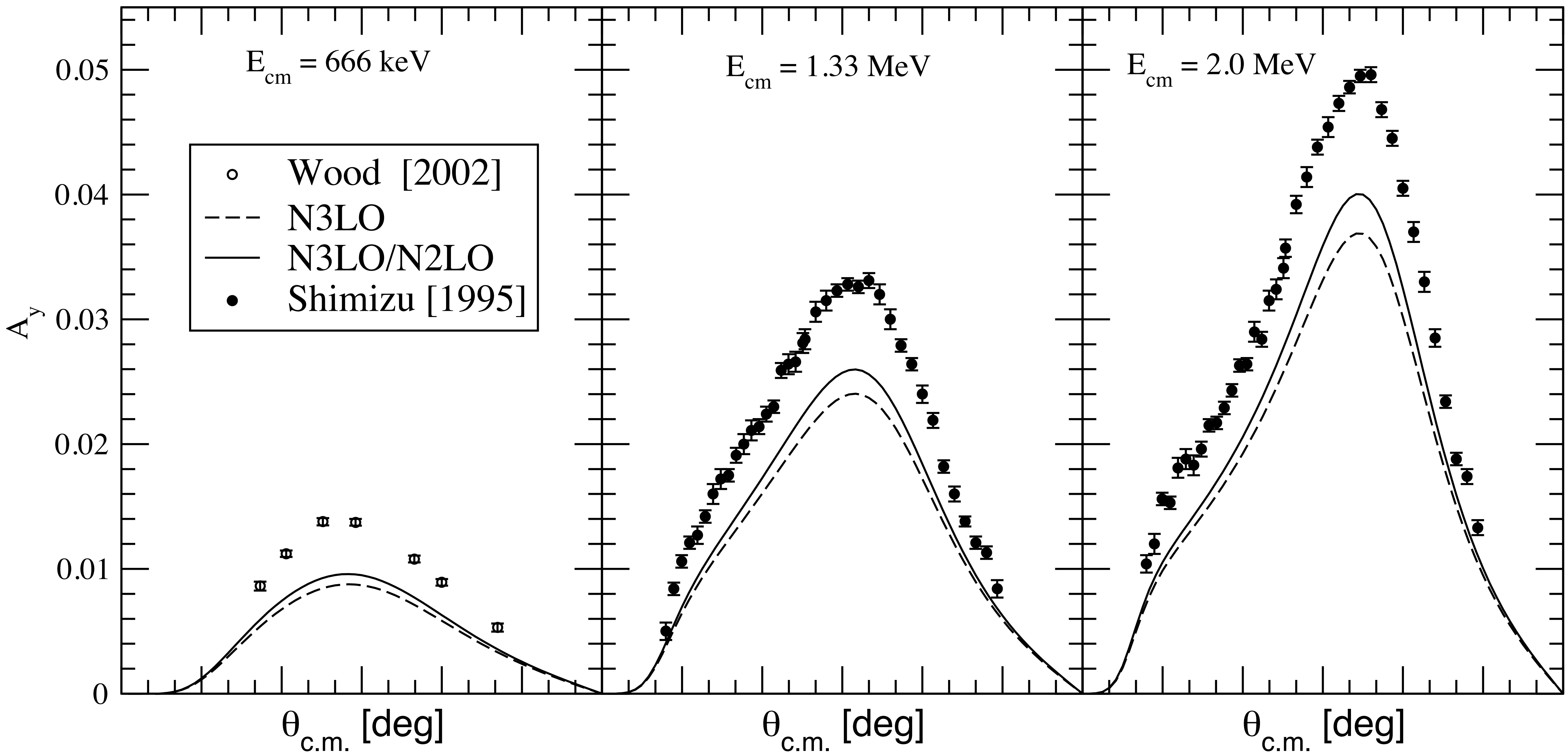
These phenomenological combinations are very useful to identify signatures of 3NF's

→ triggered a lot of experiments for pd scattering (RIKEN, KVI, IUCF, ...)
so that the 3N data base became quite extensive (at intermediate energies)

Low energy puzzles

very few observables at low energy are not well described when 3NF's are included → „puzzles"

e.g. A_y of pd and nd elastic scattering

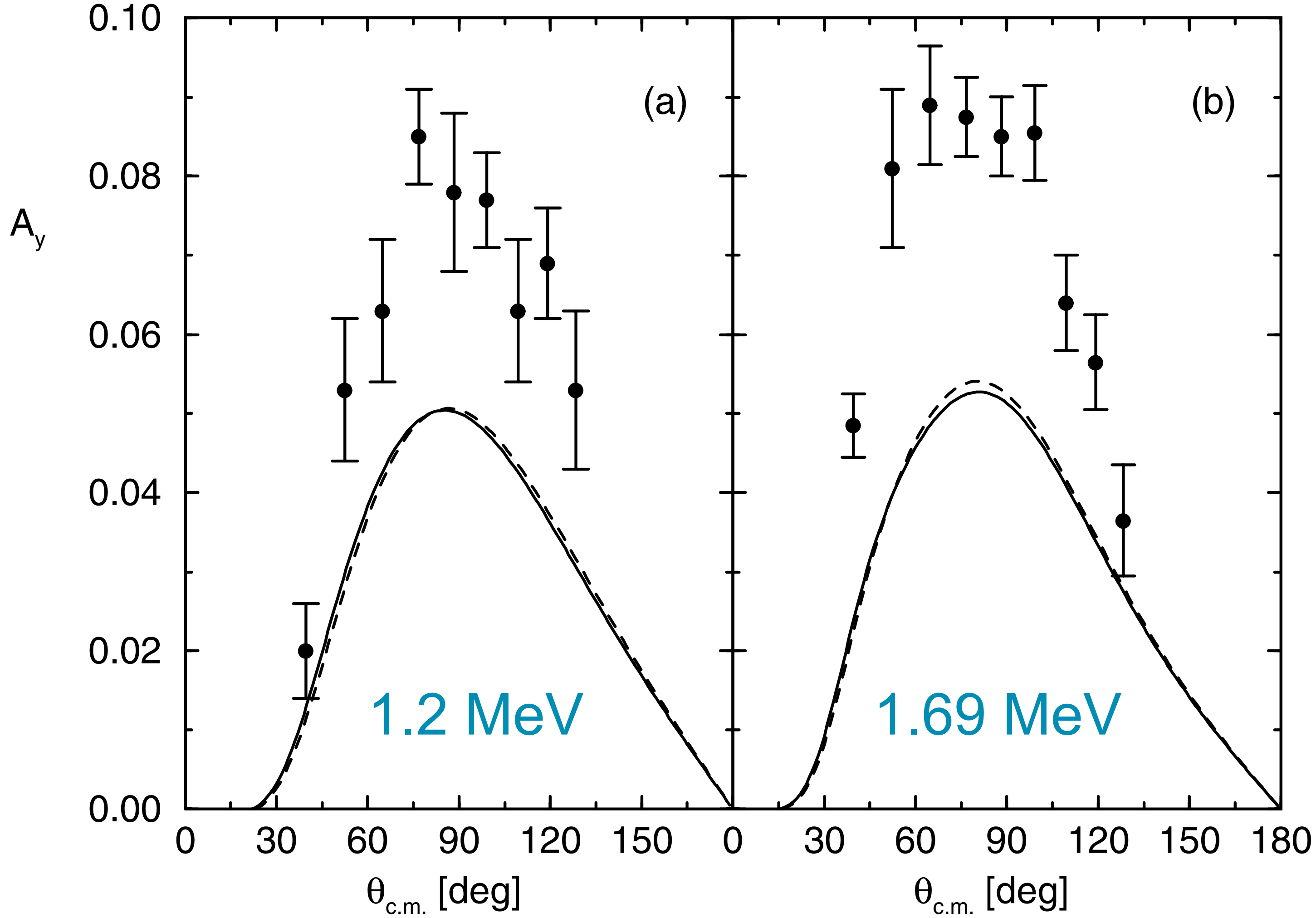


(L.E. Marcucci et al., 2009)

Note that Ay deviation is on the 1 % level !

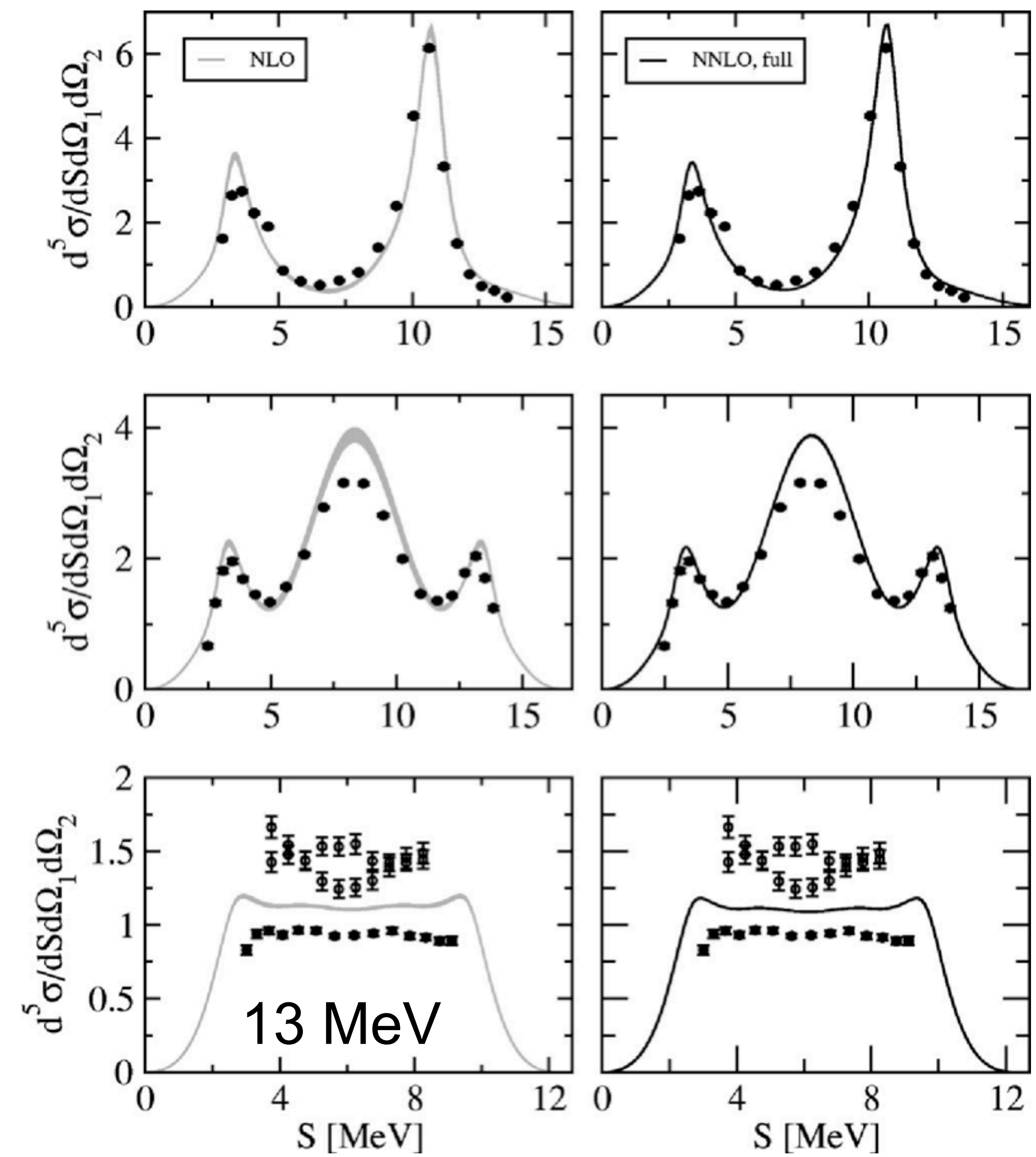
More serious ...

e.g. p - ^{3}He A_y



(Viviani et al., 2001)

e.g. space star configuration in nd breakup

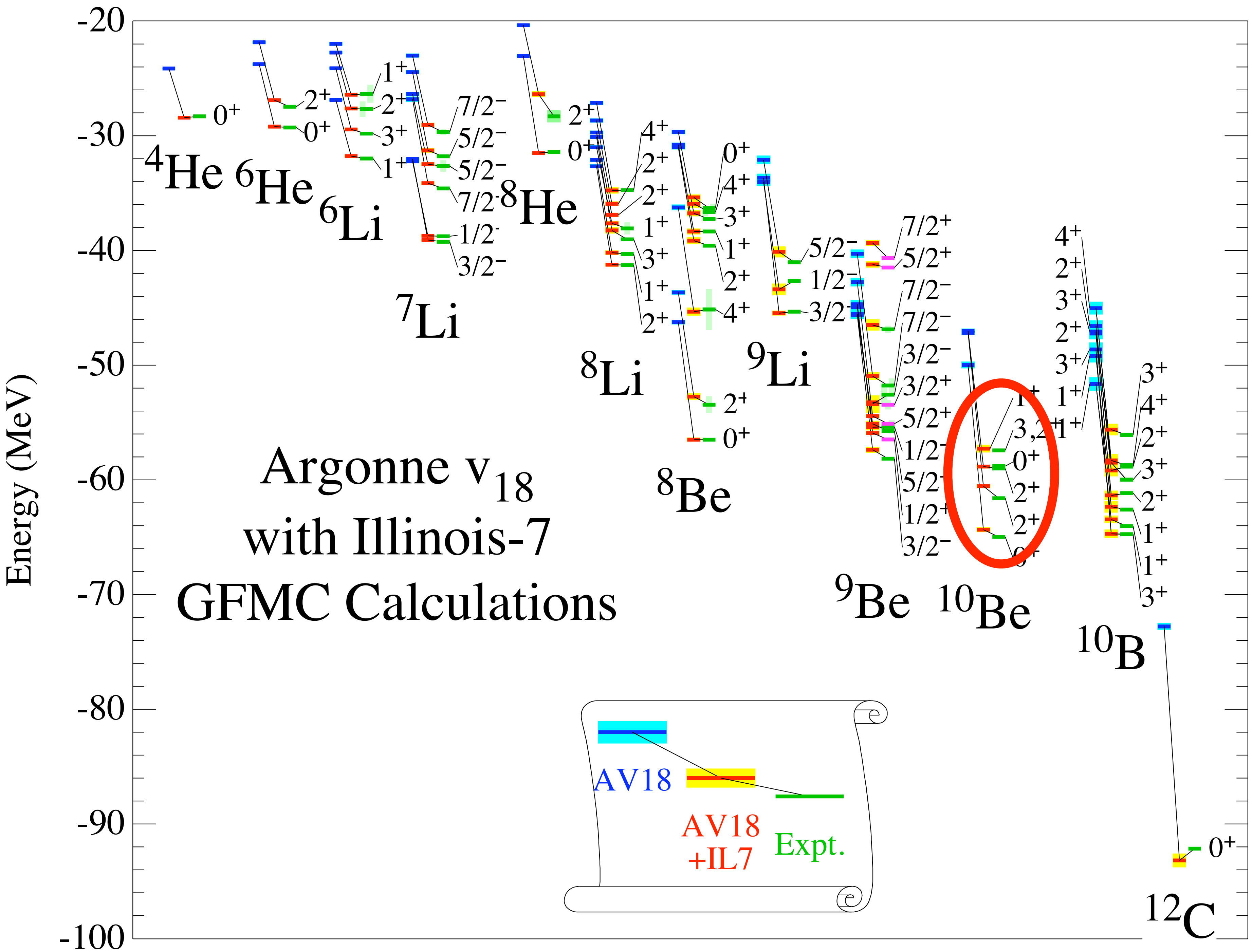


(see e.g. Witała et al., 2001)

Here the deviation for A_y is on the 5 % level and more at other energies !

Binding energies and 3NF's

3NF's improve the description of binding energies, but some discrepancies remain



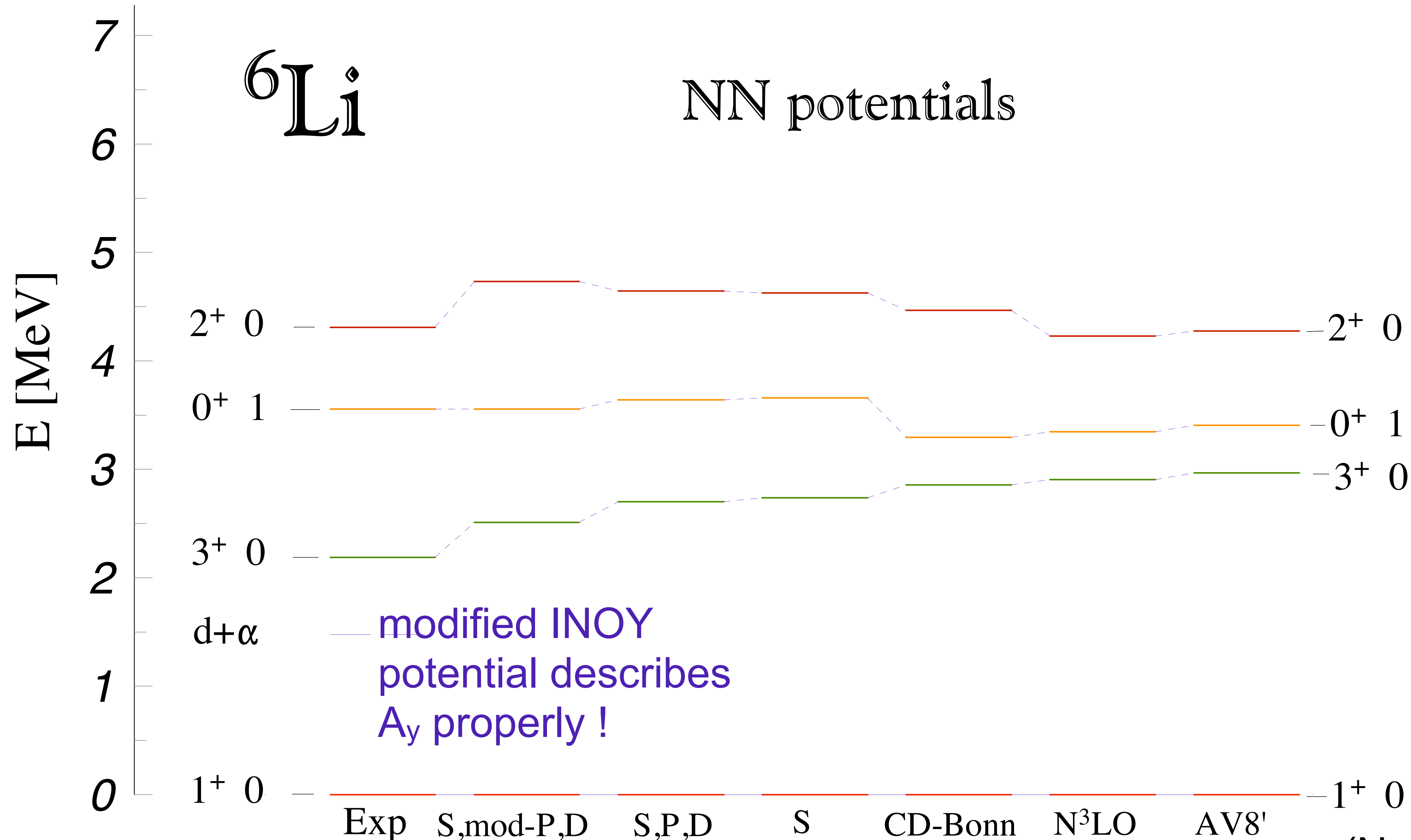
(S. Pieper, 2011)

Discussion: how accurate do we need to describe BE's and excitation energies?

Improvement of 3NF's and/or 4NF's is required

nd scattering and nuclear structure

Some obvious correlations: e.g. LS splittings with the nd A_y



(Navrátil, 2003)

Are there more correlations of 3N scattering and nuclear structure observables?

Up to what energy do we need to describe 3N data?

EFT of QCD: chiral perturbation theory

Aim is the systematically improvement of nuclear forces

EFT enables to related strong interaction to QCD

EFT allows to understand pion mass dependence of nuclear observables
→ connections to lattice QCD results

EFT can be applied to different strong interaction reactions
→ reveals connections of different processes,
connects NN, **3N**, 4N ... interactions

QCD → ChEFT involving π,N,...

→ πN
→ π production
....
→ nuclei

↑ pion mass dependence

↑ symmetries (chiral symmetry)

EFT of QCD: chiral perturbation theory

QCD & approximate chiral symmetry

symmetries →

Effective Field Theory of QCD:
relevant degrees of freedom
nucleons & pions

$$\mathcal{L}_{QCD} = \bar{q}\; i\not{D}\; q - \frac{1}{2}\;\mathrm{Tr}\; G_{\mu\nu}G^{\mu\nu} - \bar{q}\; m\; q$$

expansion in $\frac{Q}{\Lambda_\chi}$

↓ spontaneously & explicitly broken chiral symmetry

$$Q \approx m_\pi, \text{typical momentum}$$

$$\begin{aligned}\Lambda_\chi &\propto m_\Delta - m_N, m_\rho, \sqrt{m_\pi m_N}, 4\pi F_\pi, \ldots \\ &\approx 300\ \mathrm{MeV} \ldots 1200\ \mathrm{MeV}\end{aligned}$$

Goldstone bosons: pions → **Chiral Perturbation Theory** (ChPT)

„power counting“
a systematic scheme to identify
a finite numbers of diagrams
contributing at a given order

Chiral nuclear interactions

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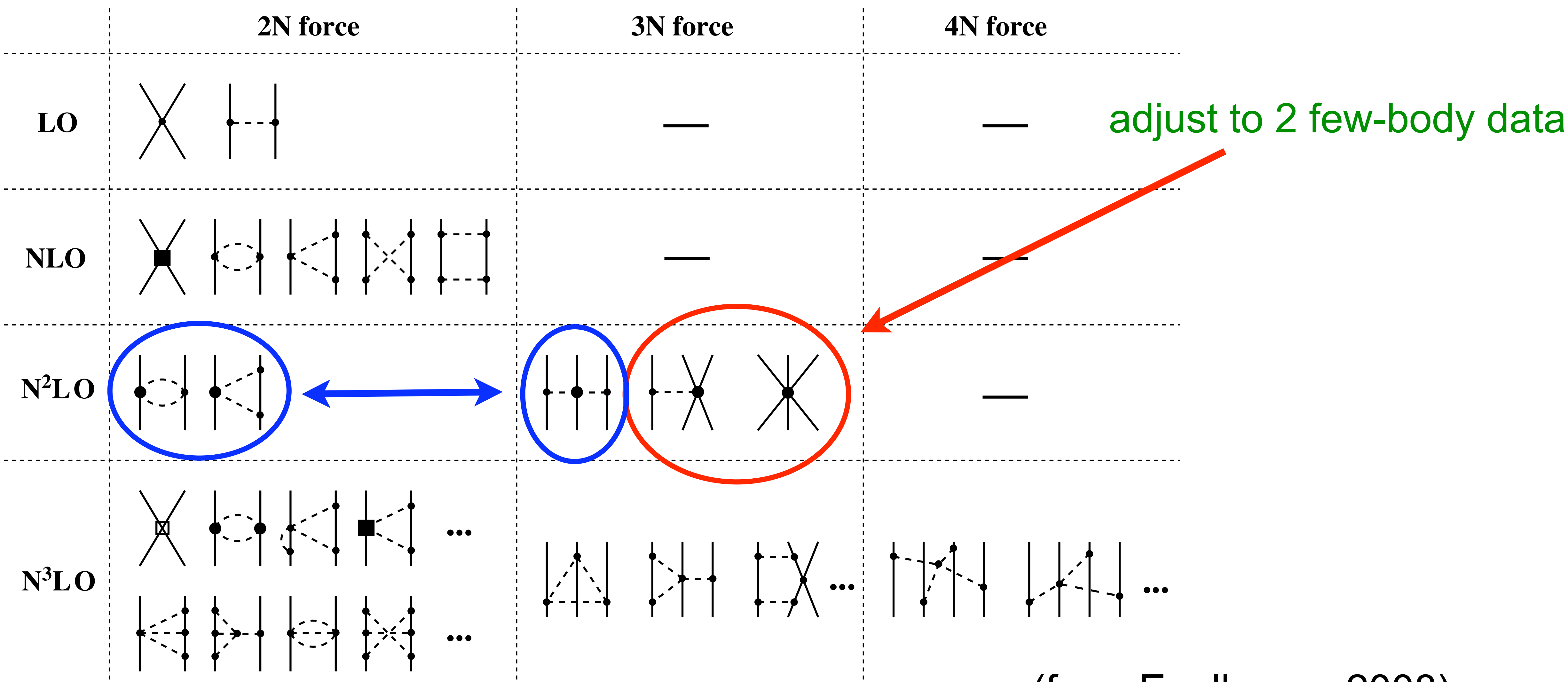
non-perturbativity of A ≥ 2 requires to
perform chiral expansion for a potential which is used to solve a Schrödinger equation



(from Epelbaum, 2008)

Systematically improvable NN, 3N, 4N, ... interactions

Qualitatively: NN >> 3N >> 4N ...

What do we know quantitatively on that hierarchy?

Estimate accuracy using cutoffs of the Lippmann-Schwinger equation

Estimated residual Λ dependence

typical momentum in nuclei is of the order of the pion mass

$$Q \approx \sqrt{2 m_N (E/A)} \approx 130 \text{ MeV/c}$$

typical cutoff value for chiral interactions $\Lambda \approx 500$ MeV

order	NN contact forces omitted	Λ [MeV]	ΔV/V	ΔE/E
LO	$(Q/\Lambda)^2$	500	7%	70%
NLO	$(Q/\Lambda)^4$	500	0.5 %	5%
N^2LO	$(Q/\Lambda)^4$	500	0.5 %	5%
N^3LO	$(Q/\Lambda)^6$	500	0.03 %	0.3 %
N^2LO	$(Q/\Lambda)^4$	700	0.1 %	1%
N^2LO	$(Q/\Lambda)^4$	300	3.5%	35%

Same estimate for NLO and N2LO !

Λ variation gives a **lower bound** of accuracy

(e.g. accuracy of NLO & N^3LO is less than estimated cutoff dependence!)

Binding energies for ^3H (NN forces only)

^3H binding energies are based on NN forces only
(LO from AN et al., 2005
NLO and N^2LO from Epelbaum et al., 2005,
N^3LO from Entem et al., 2003)

	$\Lambda/\tilde{\Lambda}$ [MeV]	E_b [MeV]	V [MeV]	ΔE_b [keV]	$\vert\Delta E_b/V\vert$ [%]
LO	500 / no loops	-7.50	-51.8	1430	3.0 (7.0)
	600 / no loops	-6.07			
NLO	400 / 700	-8.46		650	1.6 (0.5)
	550 / 700	-7.81	-41.1		
N^2LO	450 / 700	-8.42	-38.3	530	1.3 (0.5)
	600 / 700	-7.89			
N^3LO	500 / DR	-7.84	-42.3	40	0.1 (0.03)
	600 / DR	-7.80			

"power counting" estimates in brackets qualitatively agree ✓

To what order to we need to go? I assume N^3LO for this talk.

Explicit form of the leading 3NF's

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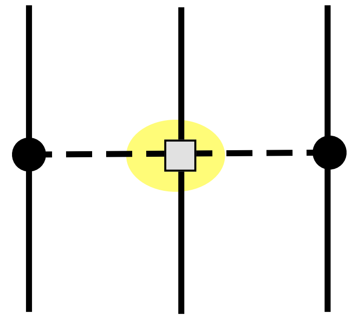
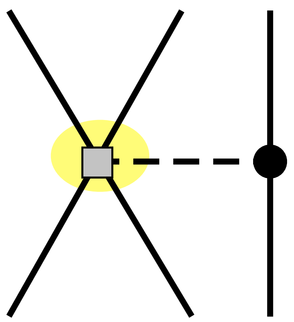
The explicit form of the 3NF at N2LO (van Kolck, 1994)

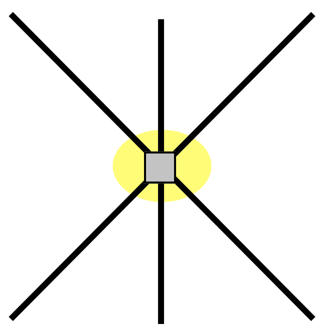


$$V_{3NF}^{2\pi} = \sum_{i<j<k} \left(\frac{g_A}{2F_\pi} \right)^2 \frac{\vec{\sigma}_i \cdot \vec{q}_i \; \vec{\sigma}_j \cdot \vec{q}_j}{(\vec{q}_i^{\,2} + m_\pi^2)(\vec{q}_j^{\,2} + m_\pi^2)} F_{ijk}^{\alpha\beta} \, \tau_i^\alpha \, \tau_j^\beta$$

$$F_{ijk}^{\alpha\beta} = \delta_{\alpha\beta} \left[- \frac{4c_1 m_\pi^2}{F_\pi^2} + \frac{2c_3}{F_\pi^2} \vec{q}_i \cdot \vec{q}_j \right] + \frac{c_4}{F_\pi^2} \epsilon^{\alpha\beta\gamma} \, \tau_k^\gamma \vec{\sigma}_k \cdot [\vec{q}_i \times \vec{q}_j]$$



$$V_{3NF}^{1\pi} = - \sum_{i<j<k} \frac{g_A}{4F_\pi^2} \frac{c_D}{F_\pi^2 \Lambda_x} \frac{\vec{\sigma}_j \cdot \vec{q}_j}{\vec{q}_j^{\,2} + m_\pi^2} \tau_i \cdot \tau_j \; \vec{\sigma}_i \cdot \vec{\sigma}_j$$



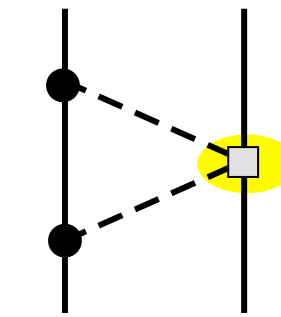
$$V_{3NF}^{c} = \sum_{i<j<k} \frac{c_E}{F_\pi^4 \Lambda_x} \tau_j \cdot \tau_k$$

- c_i are related to πN scattering and also to the N^2LO NN force
- c_i are unnaturally large in EFT without explicit Δ (approximately by factor 3)
- large uncertainties in c_i
- c_D and c_E can be determined using several strategies

c_i constants

How well do we know the strength of the subleading πN vertices ?

c_i constants link 2π-exchange NN-, 3N-force and πN scattering



	c_1	c_3	c_4	
Rentmeester et al. PRC 67, 044001	-0.76	-4.78	3.96	NN
Büttiker et al. NPA 668, 97	-0.81	-4.70	3.40	πN
Fettes et al. NPA 640, 199	-1.23	-5.94	3.47	πN
Meißner, talk at TRIUMF	-0.9	-4.7	3.5	πN
Entem et al. PRC 66,014002	-0.81	-3.40	3.40	NN
Entem et al. PRC 68,041001(R)	-0.81	-3.20	5.40	NN

(red=input to analysis)

There are sizable error bars > 30 % !

Note that the uncertainty is at least comparable to N^3LO contributions !

Discussion can we improve this situation?

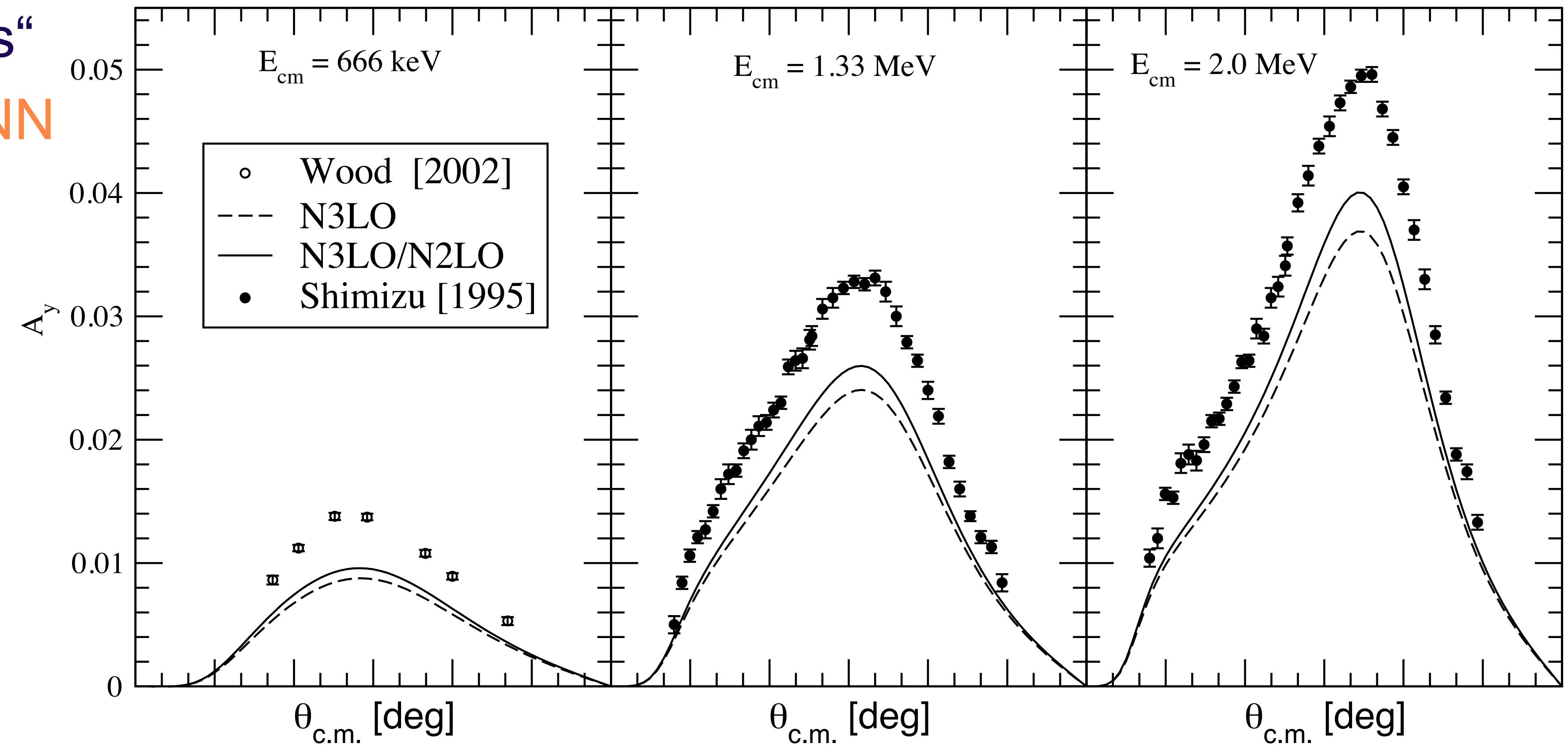
Nevertheless, let's check impact of leading 3NF's on observables

nd scattering and chiral forces

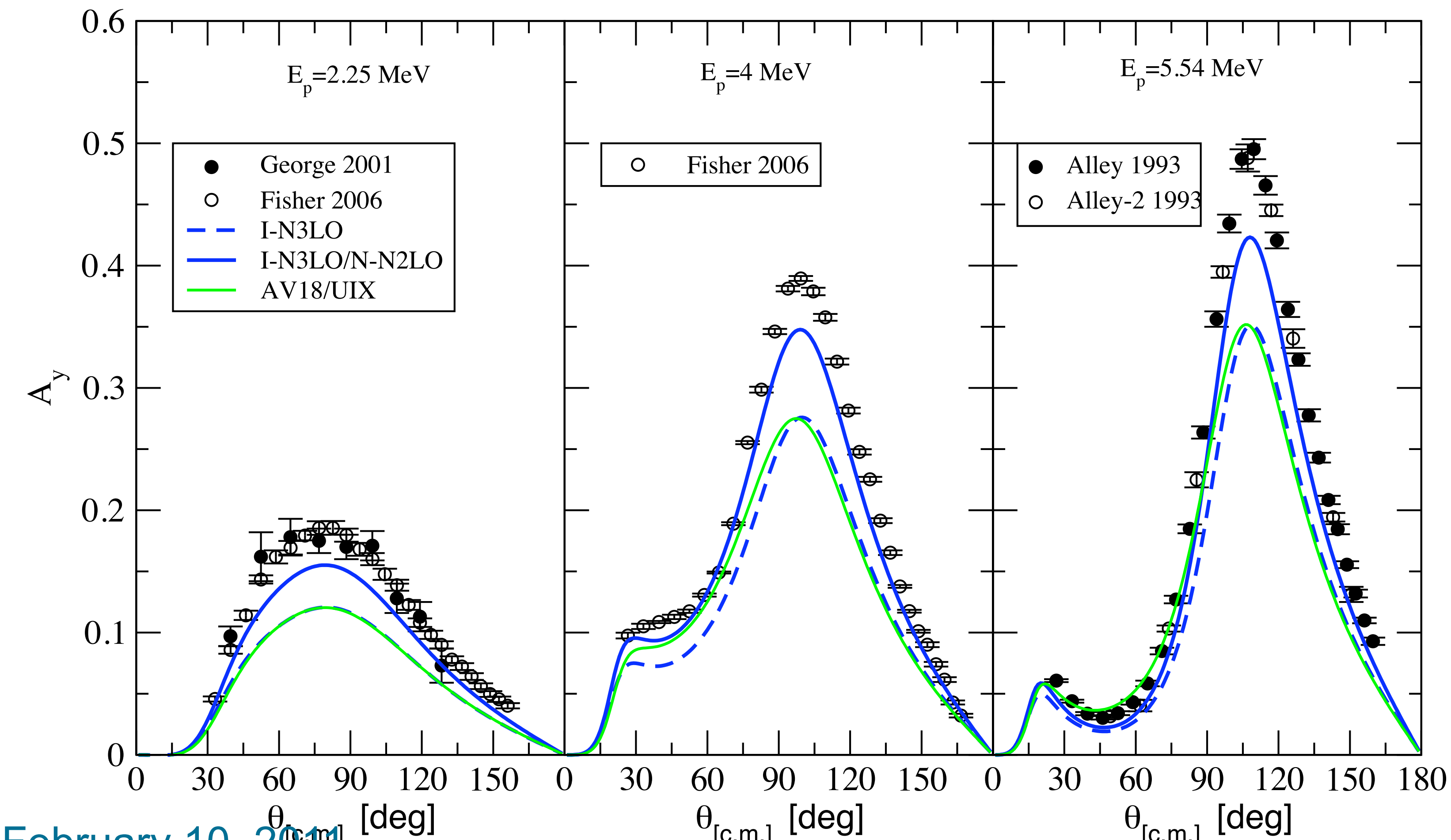
Impact on low energy „puzzles“

c_i are chosen consistently to NN

no change for nd A_y at low energy !



(L.E. Marcucci et al., 2009)

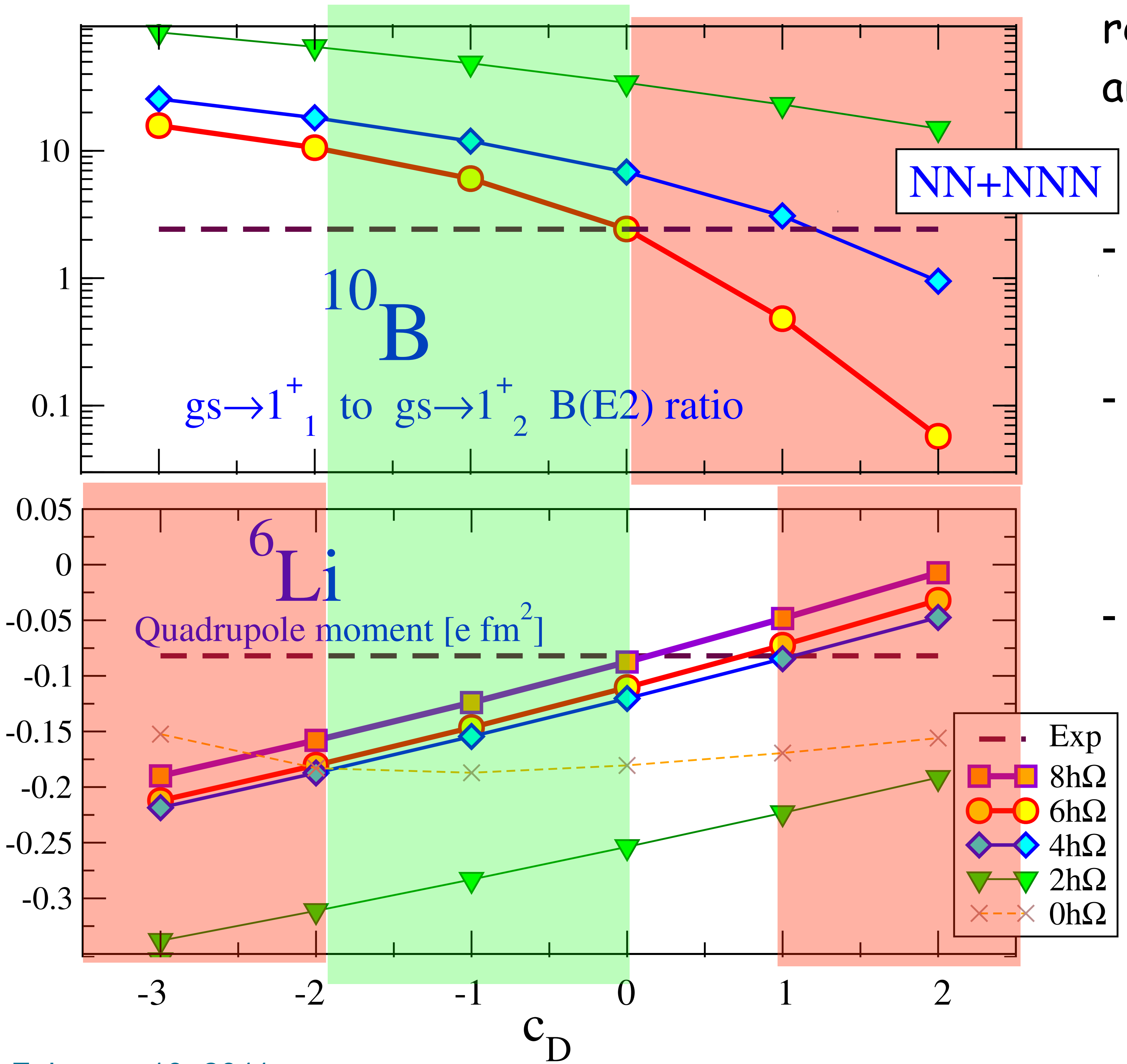


but p-^{3}He A_y is affected !!!

(remember that this is in contrast to Urbana-IX)

(Viviani et al.,2010 preliminary)

Impact of c_D & c_E on p-shell nuclei

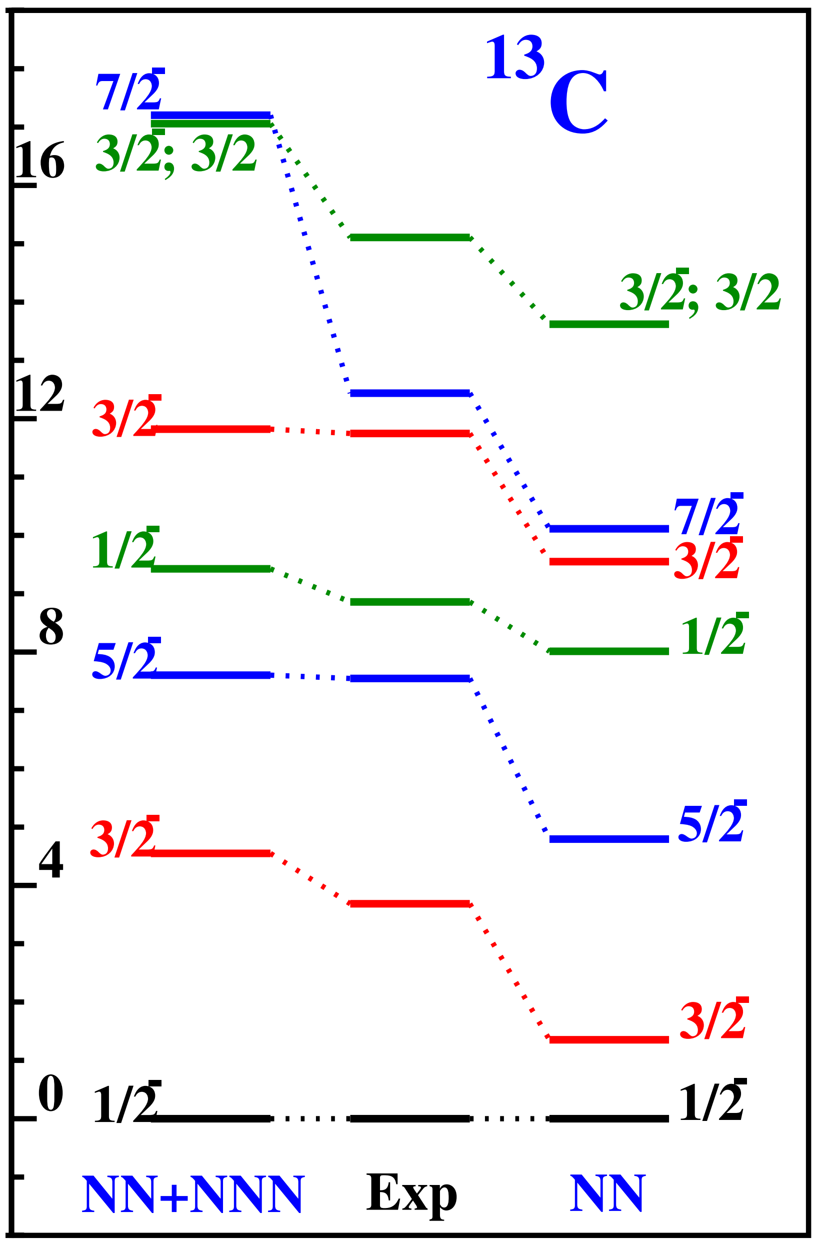


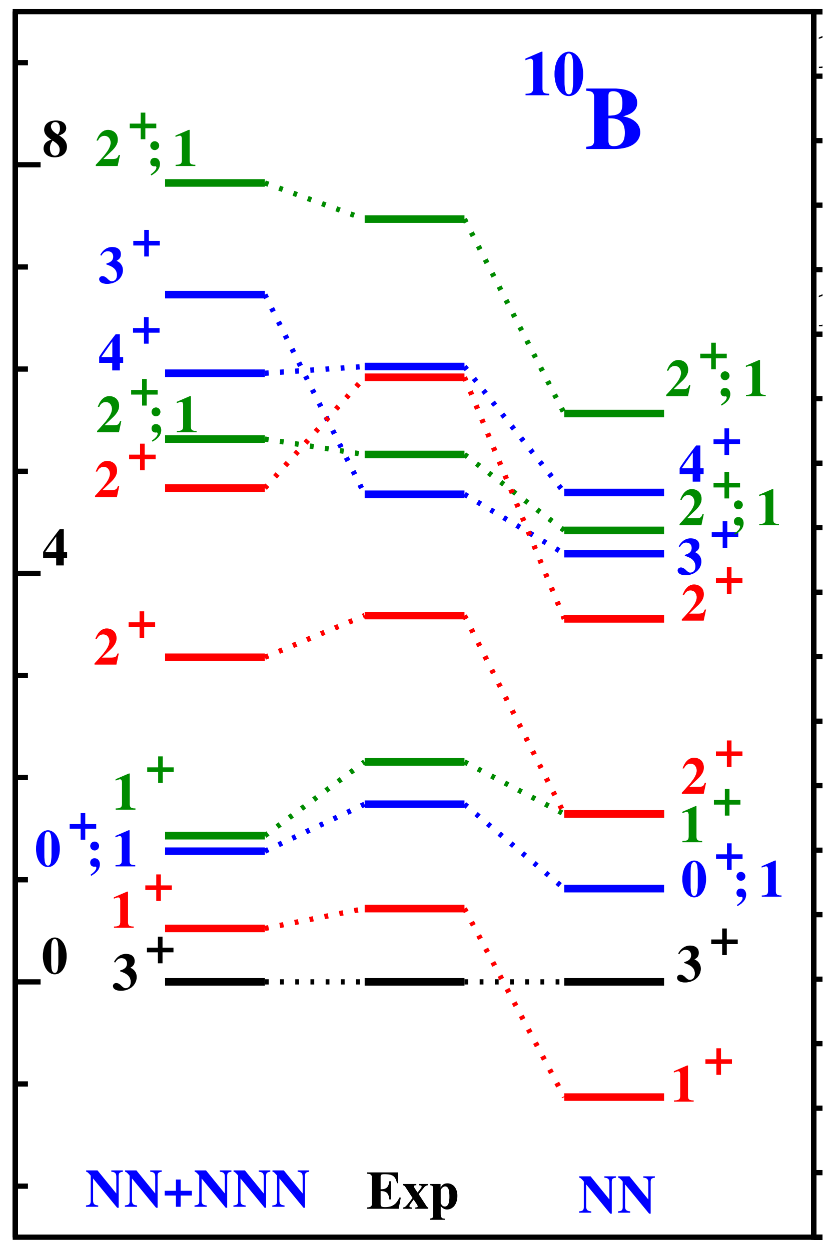
Survey of A=6,10-13 nuclei revealed a **few** observables that are sensitive to c_D/c_E (Navrátil et al., 2007),

- green area accommodates B(E2,^{10}B) and Q(^{6}Li)
- other observables are either insensitive to variation of c_D or are consistently described
- c_D determination difficult because of the numerical accuracy

^{10}B & ^{13}C spectra

Choose c_D =-1 and obtain spectra and their sensitivity on the 3NF ....





(Navrátil et al., 2007)

- Clear improvement of description compared to experiment.
- Some corrections are too strong
- c_i are fixed at EM values, **shall one relax the consistency to the NN force?**

3NF contributions - estimate of N^2LO

- 3NF and NN expectation values for ^4He
- 3NF **power counting estimate: 2 % of V** (based on Λ=500 MeV)

	$\Lambda/\tilde{\Lambda}$ [MeV]	E_b [MeV]	V_{NN} [MeV]	V_{123} [MeV]	$\|V_{123}/V_{NN}\|$ [%]
N^2LO	450 / 700	-27.65	-84.56	-1.11	1.3
	600 / 700	-28.57	-93.73	-6.83	7.3
N^3LO	500 / DR-3NF-A	-28.27	-99.45	-4.06	4.1
	500 / DR-3NF-B	-28.24	-98.92	-7.10	7.2

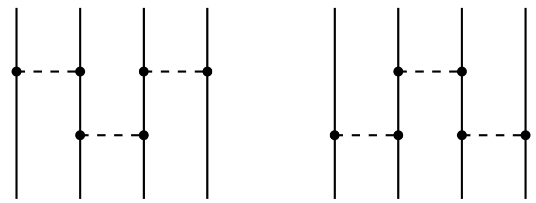
→

- 3NF contributions are somewhat more important (Δ not included in EFT, factor 3)
- Naive estimate for N^3LO contributions (subleading 3NF's and 4NF) based on same expansion parameter is approximately 0.5 % of V (→ 500 keV)
- Estimate N^3LO contributions using **4NF** (because it is actually simpler to obtain)

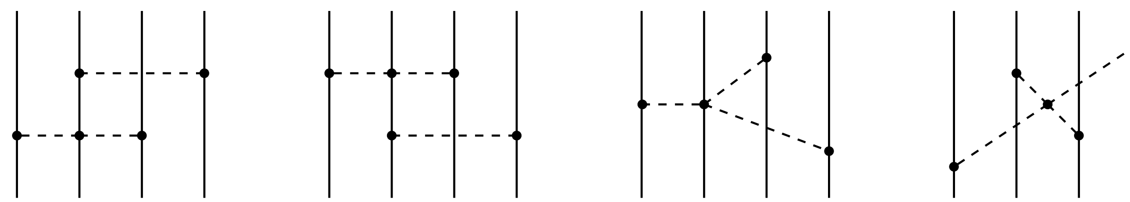
Leading chiral 4NF

Five non-vanishing classes of contributions (see E. Epelbaum, 2006,2007)

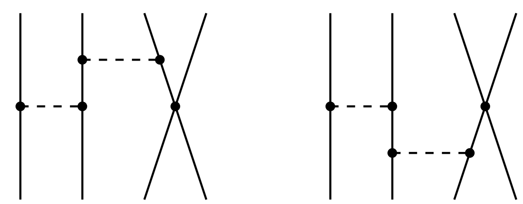
Class I $\propto g_a^6$



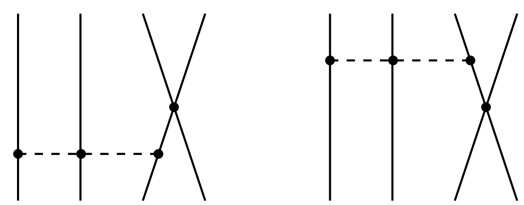
Class II $\propto g_a^4$



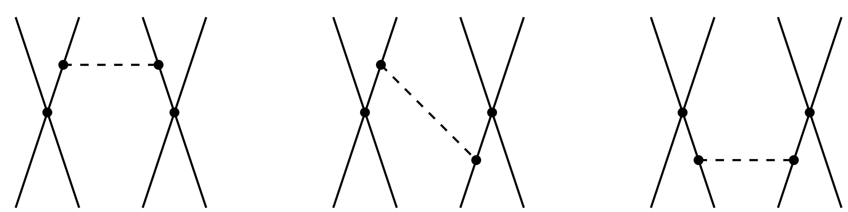
Class IV $\propto g_a^4 C_T$



Class V $\propto g_a^2 C_T$



Class VII $\propto g_a^2 C_T^2$



no Δ involved

Δ induced 4NF's have been estimated in Deltuva et al.,2008 (→ 170 keV in ^{4}He)

- many terms with complicated spin/isospin structure !
- all parameters linked to leading NN interaction

$$V_{LO} = -\left(\frac{g_a}{2F_\pi}\right)^2 \frac{\vec{\sigma}_1 \cdot \vec{q}\,\vec{\sigma}_2 \cdot \vec{q}}{q^2 + m_\pi^2}\,\tau_1 \cdot \tau_2 + C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

- finite range contributions (some discussed in McManus, Riska (1980), Robilotta (1985))
- contact contributions $\propto C_T$ (possibly suppressed due to Wigner symmetry ?)

Ingredients of the calculation

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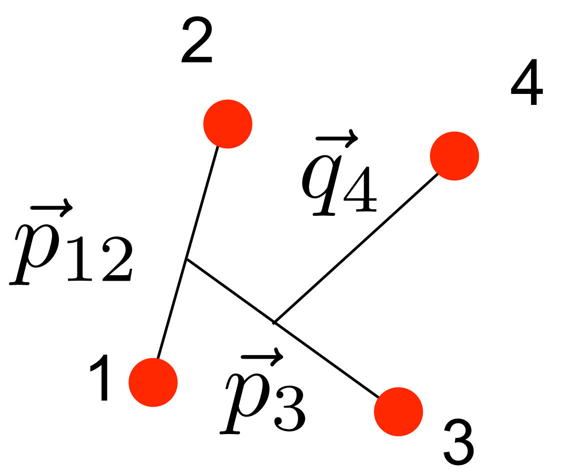
- First attempt:
  - *perturbative estimate of the 4NF contribution (works well for low cutoffs for 3NF)*
- Need to calculate expectation value

$$\langle V_4 \rangle = \sum_{\alpha\alpha'} \int d^3p_{12} d^3p_3 d^3q_4 d^3p'_{12} d^3p'_3 d^3q'_4 \langle \Psi | \vec{p}_{12}\vec{p}_3\vec{q}_4\alpha \rangle \, \langle \ldots | V_4 | \ldots \rangle \, \langle \vec{p}_{12}{}'\vec{p}_3{}'\vec{q}_4{}'\alpha' | \Psi \rangle$$

$$= \sum_{\alpha\alpha'} \int d^3p_{12} d^3p_3 d^3q_4 d^3p'_{12} d^3p'_3 d^3q'_4 w(p_{12}, p_3, q_4; p'_{12}, p'_3, q'_4) \frac{\langle \Psi | \vec{p}_{12}\vec{p}_3\vec{q}_4\alpha \rangle \, \langle \ldots | V_4 | \ldots \rangle \, \langle \vec{p}_{12}{}'\vec{p}_3{}'\vec{q}_4{}'\alpha' | \Psi \rangle}{w(p_{12}, p_3, q_4; p'_{12}, p'_3, q'_4)}$$



- ^{4}He wave function $\Psi\left(\vec{p}_{12}\vec{p}_3\vec{q}_4, \alpha\right)$
- spin-isospin channels $|\alpha\rangle \equiv |m_1 m_2 m_3 m_4 \, m_1^t m_2^t m_3^t m_4^t\rangle$
- 4NF matrix element $\langle \vec{p}_{12}\vec{p}_3\vec{q}_4\alpha | V_4 | \vec{p}_{12}{}'\vec{p}_3{}'\vec{q}_4{}'\alpha' \rangle$ generated using ***Mathematica***

→

- Metropolis walk for evaluation based on weight function

$$w(p_{12}, p_3, q_4; p'_{12}, p'_3, q'_4) \propto \prod_{\substack{i=12,3,4,\\12',3',4'}} \frac{1}{(p_i + C_i)^{n_i}}$$

^{4}He wave functions

all estimates are based on realistic 4He wave functions

results will be shown for

1) AV18 + Urbana IX / CD-Bonn + TM

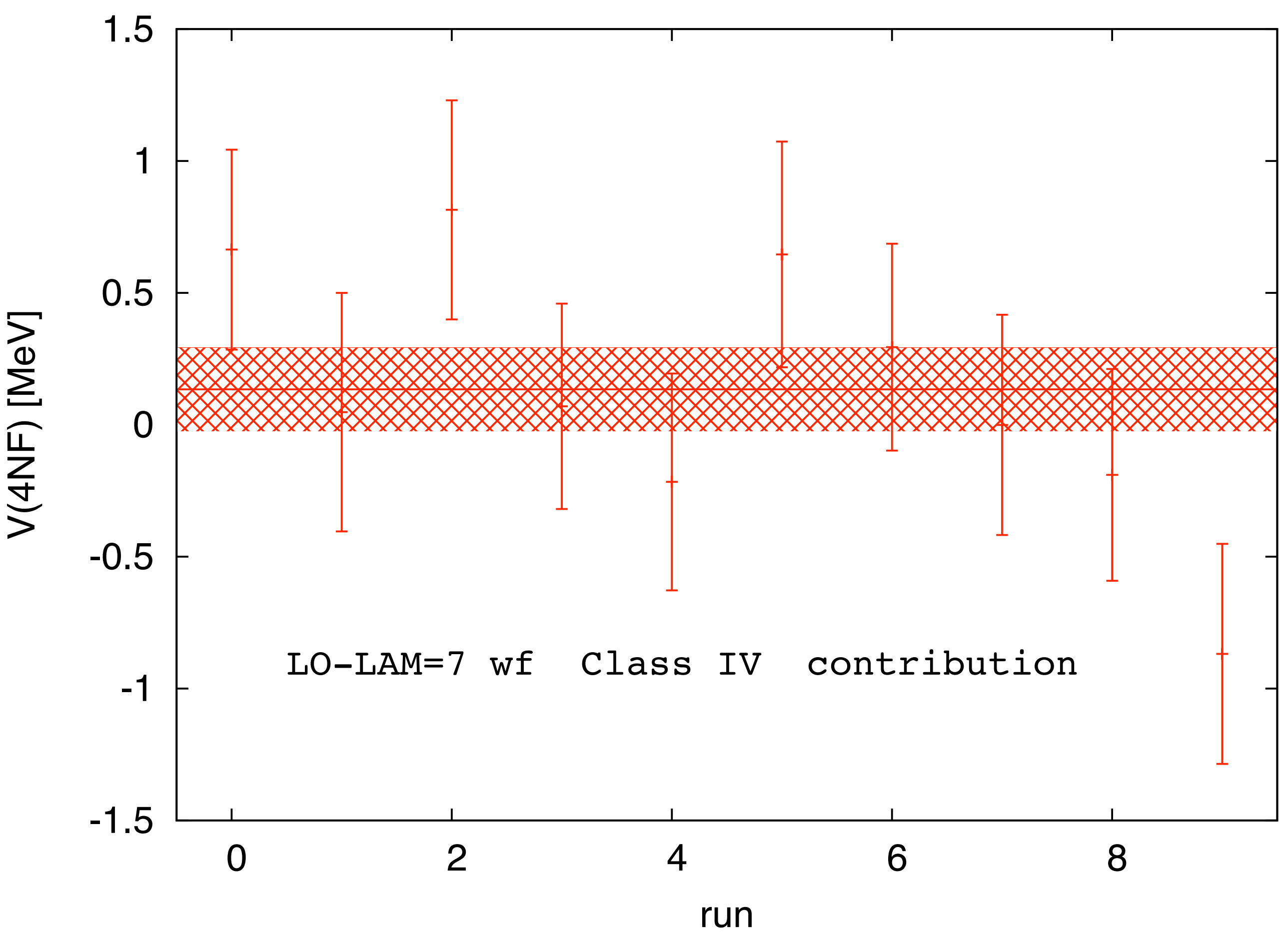
2) LO chiral interactions for cutoffs $\Lambda = 2 \ldots 7\ \mathrm{fm}^{-1}$

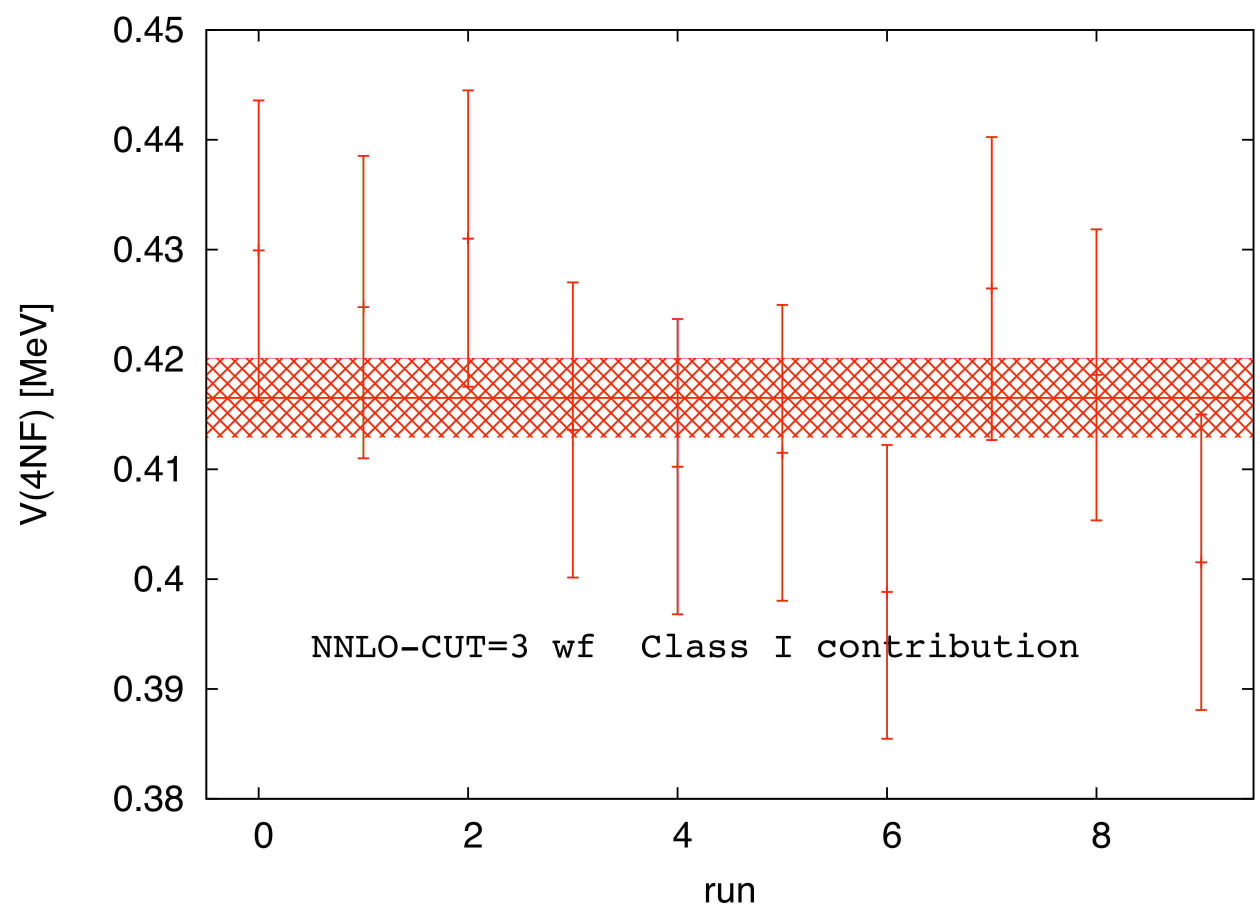
3) NLO & N2LO (including 3NF) wave functions $\Lambda = 2 \ldots 3\ \mathrm{fm}^{-1}$

	^{3}He	^{4}He
AV18+Urbana IX	-7.72	-28.5
CD-Bonn + TM99	-7.74	-28.4
LO	-5.4 ... -11.0	-15.1 ... -39.9
NLO	-6.99 ... -7.70	-24.4 ... -28.8
NNLO	-7.72 ... -7.81	-27.7 ... -28.3
Expt	-7.72	-28.3

Complete calculation

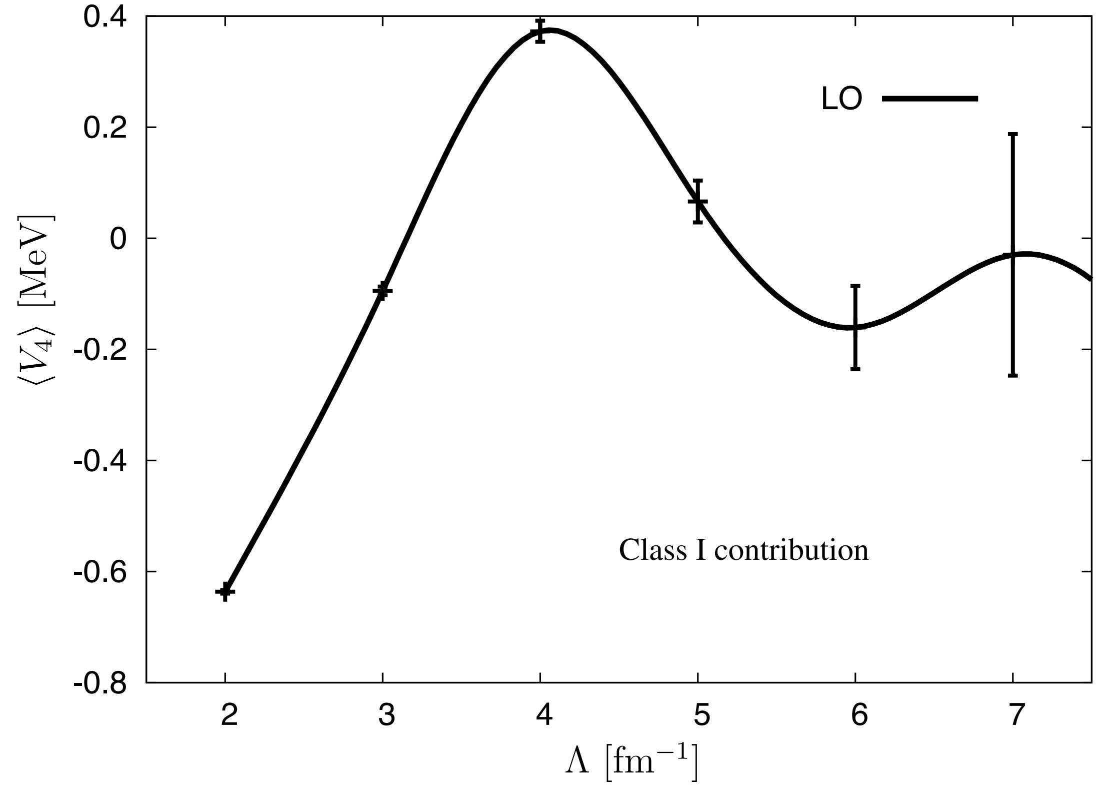
- weight function adjusted for low statistics runs
- each production run requires ≈10^7 sample points
- calculations performed on JUGENE on ≈ 4000 processors
- calculation of wave function most time-consuming
- 10 independent calculations of contributions and standard deviation allow to check consistency of statistics
- Mersenne Twister random number generator (IBM compilers internal one failed !)

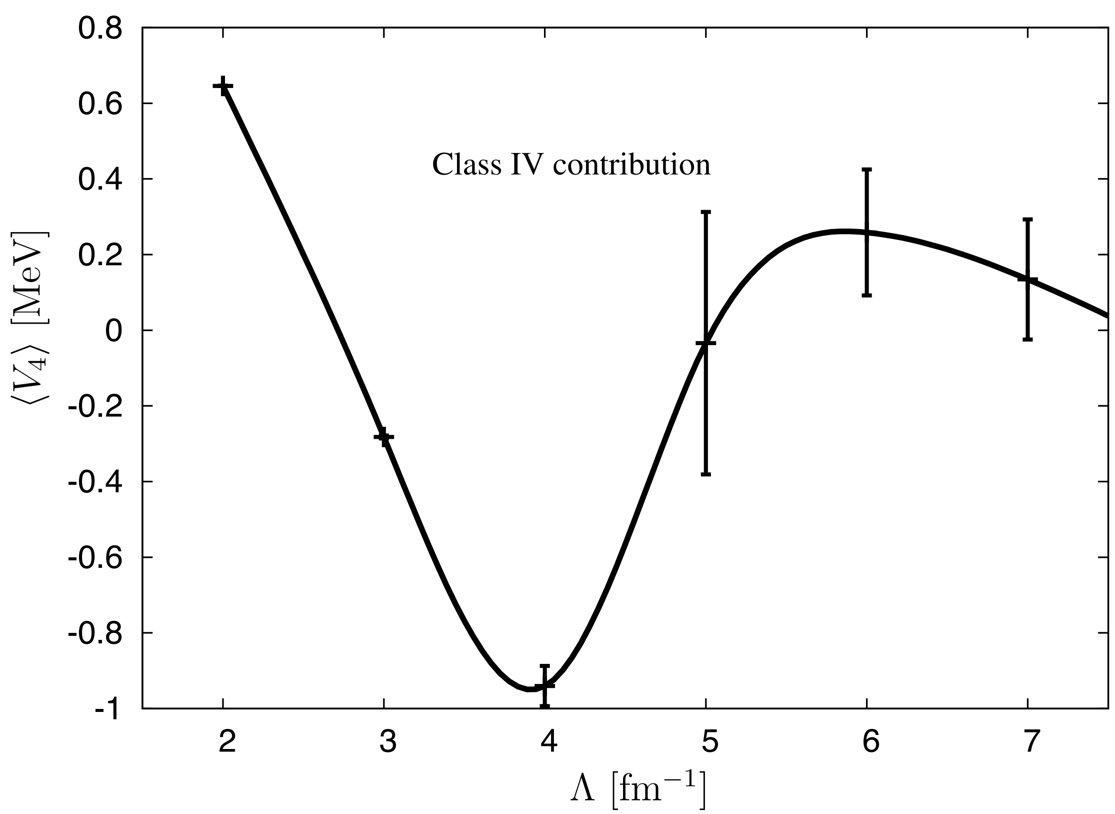


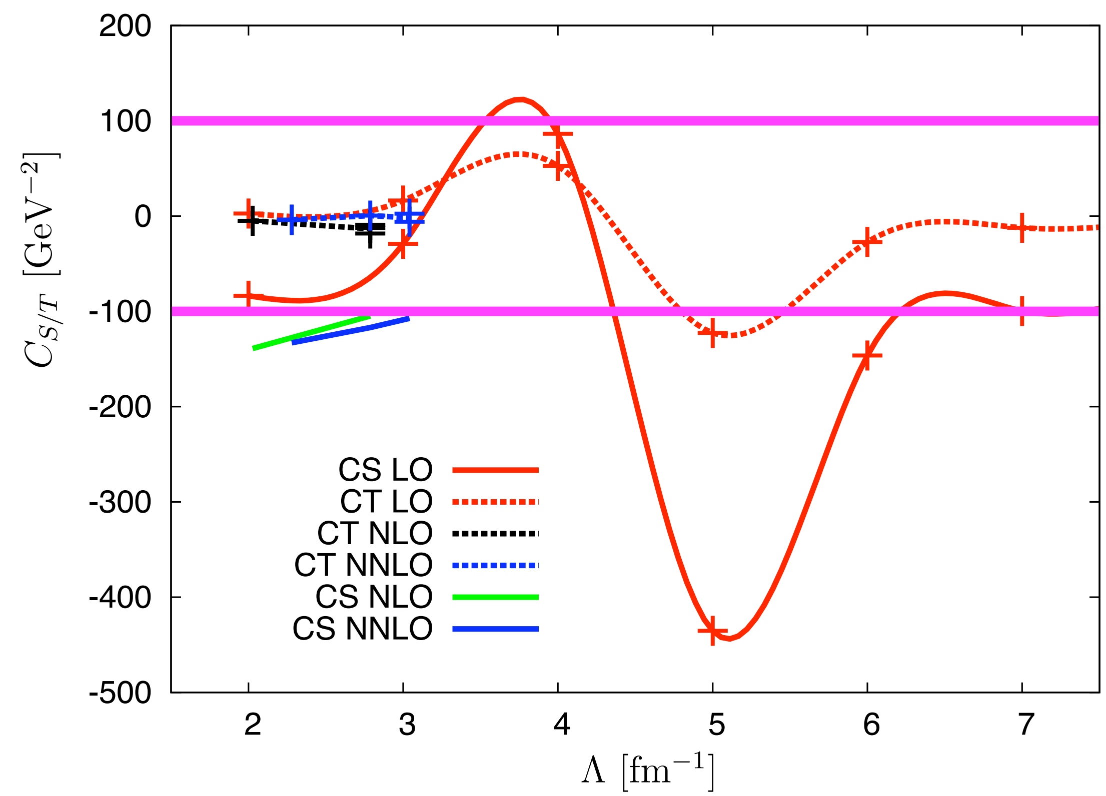


LO wf for large range of cutoffs

- perturbativity of 4NF for large cutoffs ?
- all large cutoff results are within expected bounds
- Wigner symmetry does not suppress 4NF contributions in LO
- estimates for higher order wave functions are more reliable (better description of binding energy)
- typical 4NF contribution is 500 keV

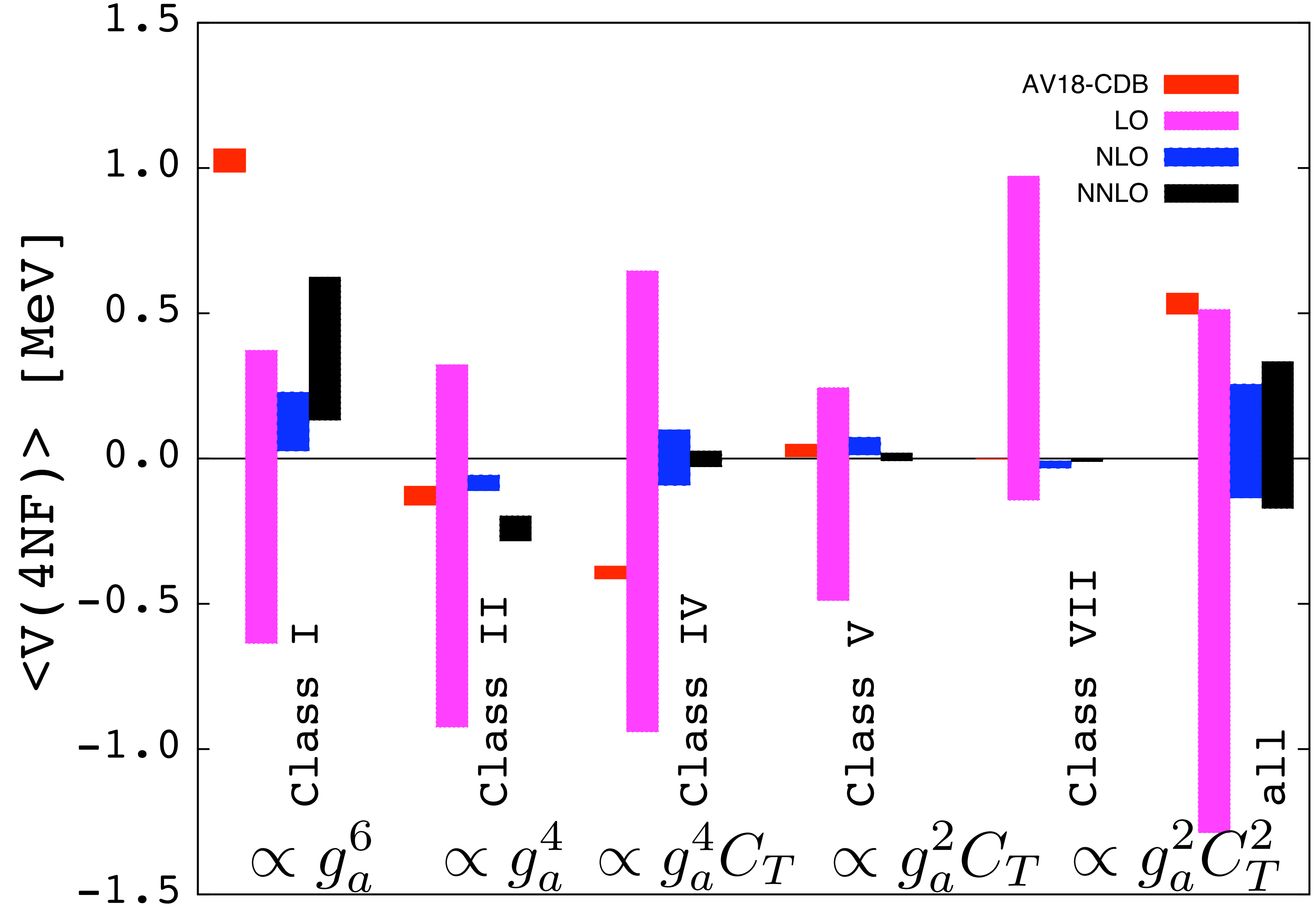






Contribution of the 4NF

- results of chiral wave functions with consistent C_T (C_T=+10 GeV^{-2} for CD-Bonn & AV18)
- 4NF contribution approximately agrees with power counting estimate (≈ 0.5% ≈ 500 keV)
  (some cancelations of individual contributions make it smaller)
- strong model / cutoff dependence (the 4NF contribution is non-observable)



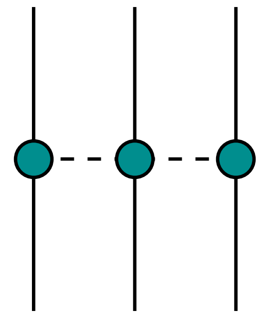
Probably good a estimate of typical N^3LO contribution: → 500 keV

Is this relevant? Implementation of 4NF‘s in NCSM?

subleading 3NF

in part formulated in Bernard et al., 2008 and currently implemented (no results yet)

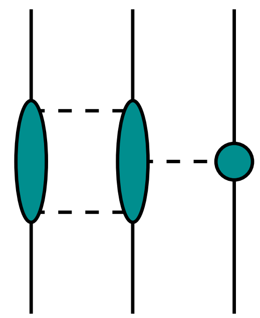
remaining parts are almost finished (Bernard et al., in progress, 2011)



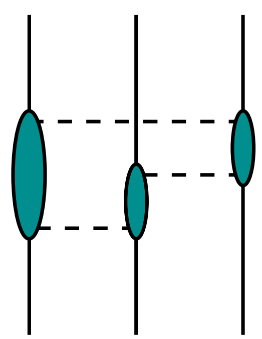
1π-exchange terms
(new spin structures
& shifts!)

$$c_1 \to \bar{c}_1 = c_1 - \frac{g_A^2 M_\pi}{64\pi F_\pi^2}, \qquad c_3 \to \bar{c}_3 = c_3 + \frac{g_A^4 M_\pi}{16\pi F_\pi^2}$$

$$c_4 \to \bar{c}_4 = c_4 - \frac{g_A^4 M_\pi}{16\pi F_\pi^2},$$

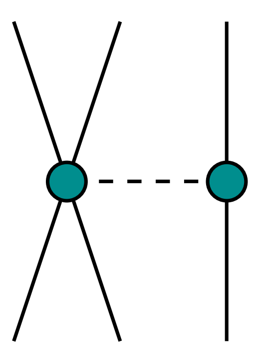


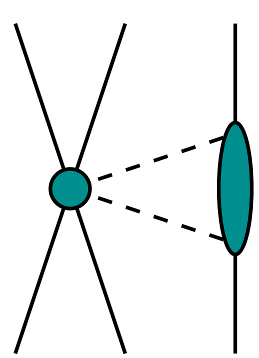
2π-1π exchange terms
(new spin structures!)



ring diagrams not equal to Illinois
(new spin structures!)

these terms do not involve Δ
(→ 500 keV to ^{4}He ?)





shorter-range diagrams and 1/m corrections are not completely formulated yet

subleading 3NF

Issues of the implementation:

- many structures make an analytical partial wave decomposition difficult

  → numerical pwa required (see Golak et al.,2010)

  long range part is local

  1/m corrections will be non-local

  How to get the HO basis version for the NCSM?

- shifts of c_i are sizeable & c_i are not very well known

$$c_1 \to \bar{c}_1 = c_1 - \frac{g_A^2 M_\pi}{64\pi F_\pi^2}, \quad c_3 \to \bar{c}_3 = c_3 + \frac{g_A^4 M_\pi}{16\pi F_\pi^2},$$

$$c_4 \to \bar{c}_4 = c_4 - \frac{g_A^4 M_\pi}{16\pi F_\pi^2},$$

Is an independent fit of the c_i for the 3NF anyway mandatory?

Are the c_i of the NN force after SRG or vlowk evolution still relevant?

Conclusions & Outlook

- **3NF's are necessary**

  INOY has shown deviations previously, JISP?

- **Leading order 3NF improves the description of the data**

  A_y puzzle in 3N and 4N, LS splittings in p-shell nuclei, transition matrix elements, ...

- **Few-nucleon scattering data should constrain 3NF**

  What data is relevant for nuclear structure?

  Energy range? Correlations of few-nucleon data with nuclear structure data?

  Fit 3NF parameters independently of the NN force?

- **N^3LO contributions to ^4He are of the order of 500 keV**

  Naive estimate, **4NF** results and cutoff variation agree
  Is this relevant? Is N^3LO enough?

- **N^3LO 3NF's are partly known and will be completely formulated in short time**

  technical performance of NCSM calculations for these more complicated terms