Significance of SRG-generated many-body interactions

The Interaction

Kristina D. Launey
Tomas Dytrych, Jerry P. Draayer (LSU)
James P. Vary (ISU)

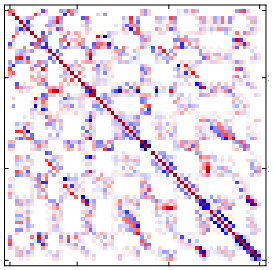
Significance of SRG-generated many-body interactions

Interaction Renormalization

Interactions between nucleons…

What they are…

$H_{s=0}$

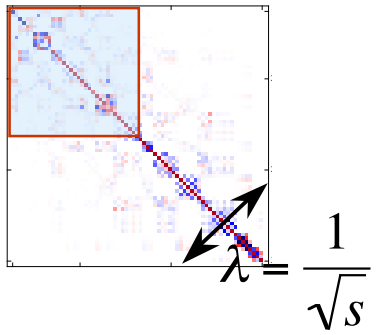


$$H_s = e^{s\eta} H_{s=0} e^{-s\eta}$$

Unitary transformation

Renormalized…

H_s



Large basis space

Ab initio shell model

Small subspace

Simple… bare *NN*, *NNN*

Interaction

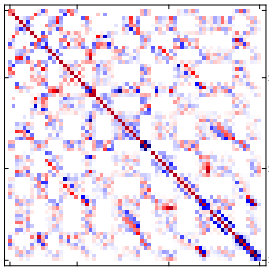
Complex … many-body interactions (3b, 4b, 5b, etc.)

Significance of generated many-body method's applicability

Similarity Renormalization Group

What they are…

$H_{s=0}$



$$H_s = e^{s\eta} H_{s=0} e^{-s\eta}$$

Unitary transformation

Renormalized…

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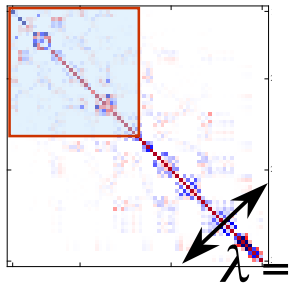


$\lambda = \frac{1}{\sqrt{s}}$

SRG generator: $\eta_s = [C, H_s]$

flow equation: $\frac{dH_s}{ds} = [[[C, H_s], H_s]$

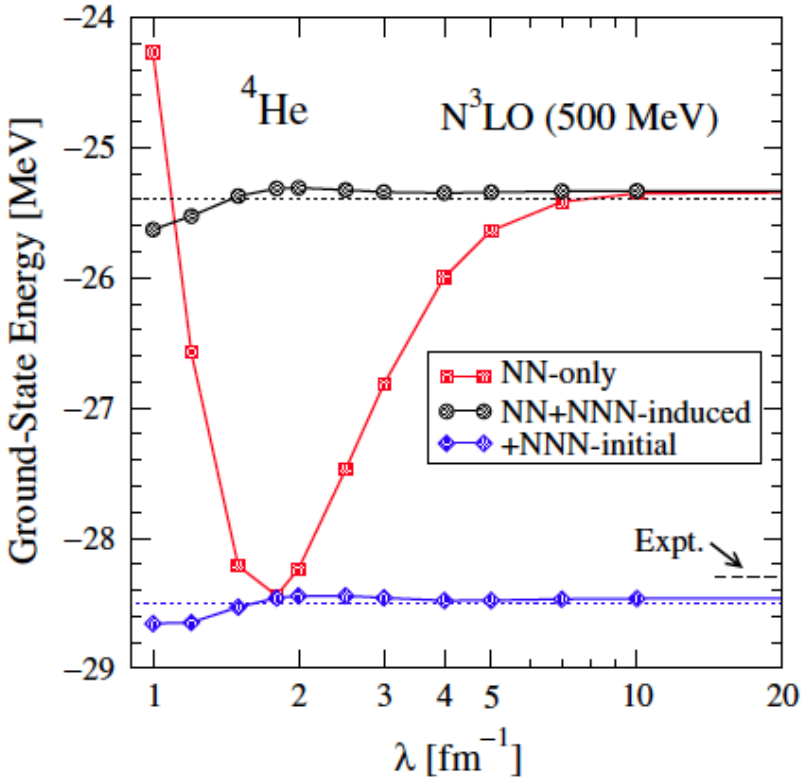
reference operators

With **1b** C: $\xrightarrow[\textbf{2b}]{H_0}$ $\frac{dH_0}{ds} = [[[C, H_0], H_0]$ $\xrightarrow[\textbf{3b}]{H_1}$ $\frac{dH_1}{ds} = [[[C, H_1], H_1]$ … and so on

two-body (3b) three-body up to 5b (5b)

Similarity Renormalization Group for Nuclear Physics

- Recent study: He-4
  - NCSM, SRG-evolved chiral-PT interaction
    - 3-body important
    - 4-body negligible



E.D. Jurgenson, P. Navratil, R.J. Furnstahl, Phys. Rev. Lett. 103 (2009) 082501

- Theoretical underpinning
  - Systematic study of all many-body induced interactions
  - General: for any *C* and initial *H*

Spectral Distribution Theory

- *All* many-body terms included
- Simple quantitative measures

$$\langle \ldots \rangle = \frac{1}{N_d}\mathrm{Tr}(\ldots)$$

  - Strength of interaction (size of operator)

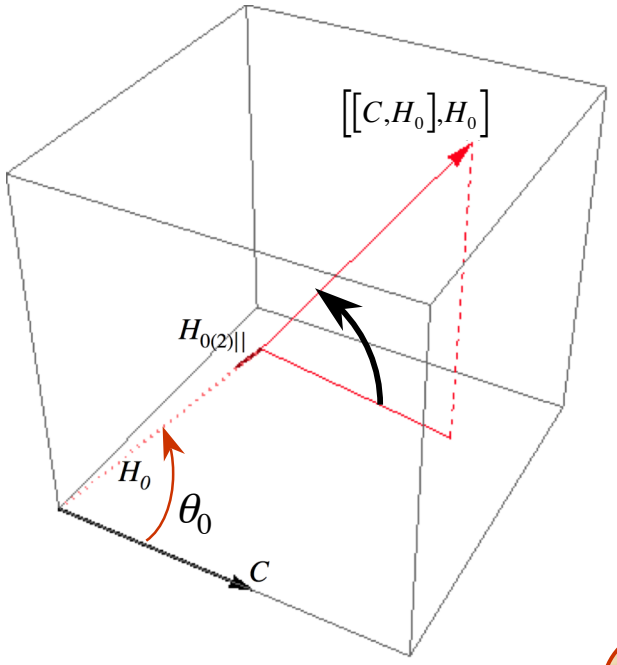
$$\sigma_H^2 = \left\langle (H - \langle H \rangle)^\dagger (H - \langle H \rangle) \right\rangle$$

    - Weaker interaction ⇔ smaller width ⇔ more compressed energy spectrum
  - Correlation between two interactions (similarity)

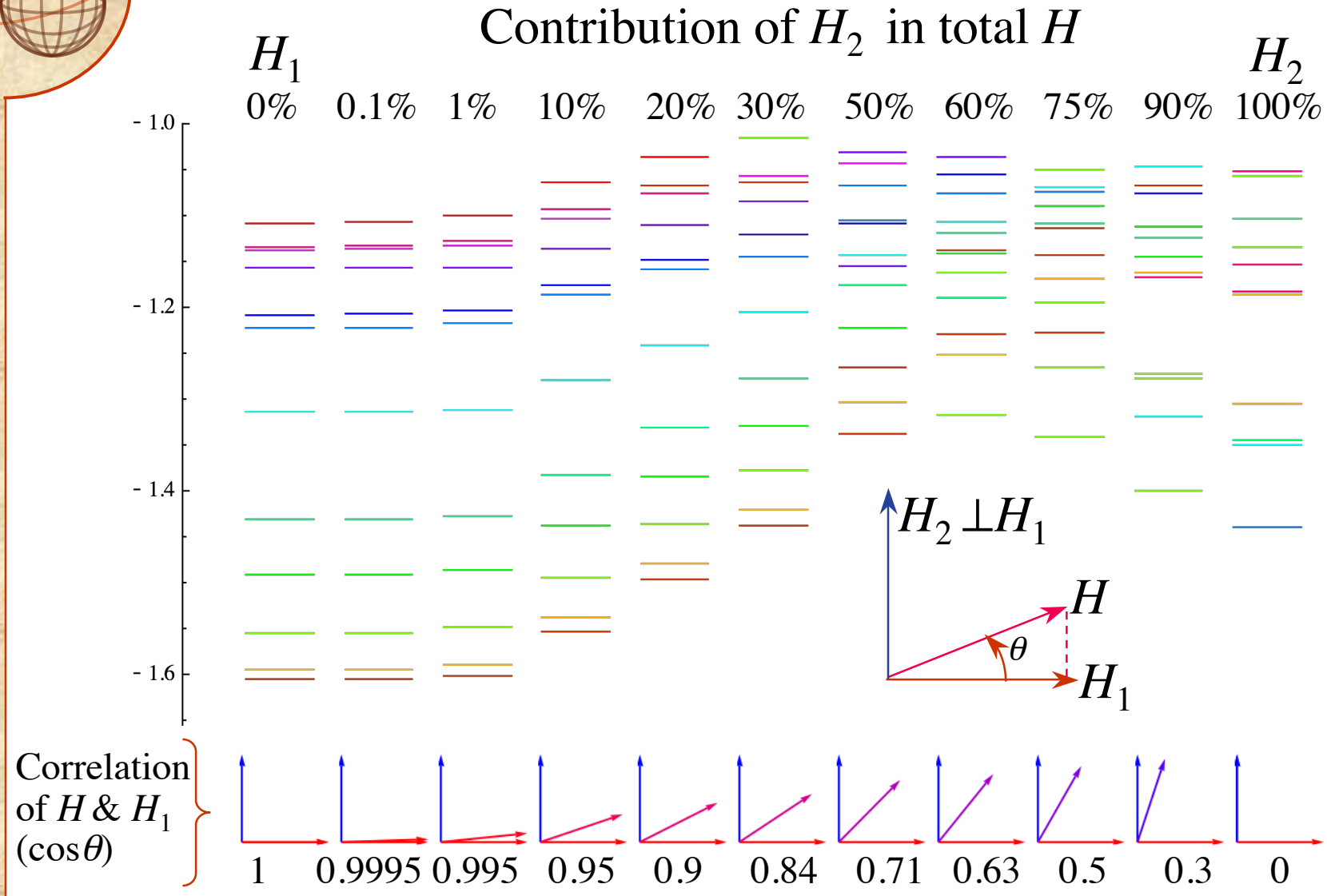
$$\zeta^{\alpha}_{H,H'} = \frac{\langle (H^\dagger - \langle H^\dagger \rangle^\alpha)(H' - \langle H' \rangle^\alpha) \rangle^\alpha}{\sigma_H \sigma_{H'}}$$

$$\cos\theta = \frac{\vec{v}\cdot\vec{v}'}{|\vec{v}||\vec{v}'|}$$

    - Large correlation coefficients yield similar energy spectra

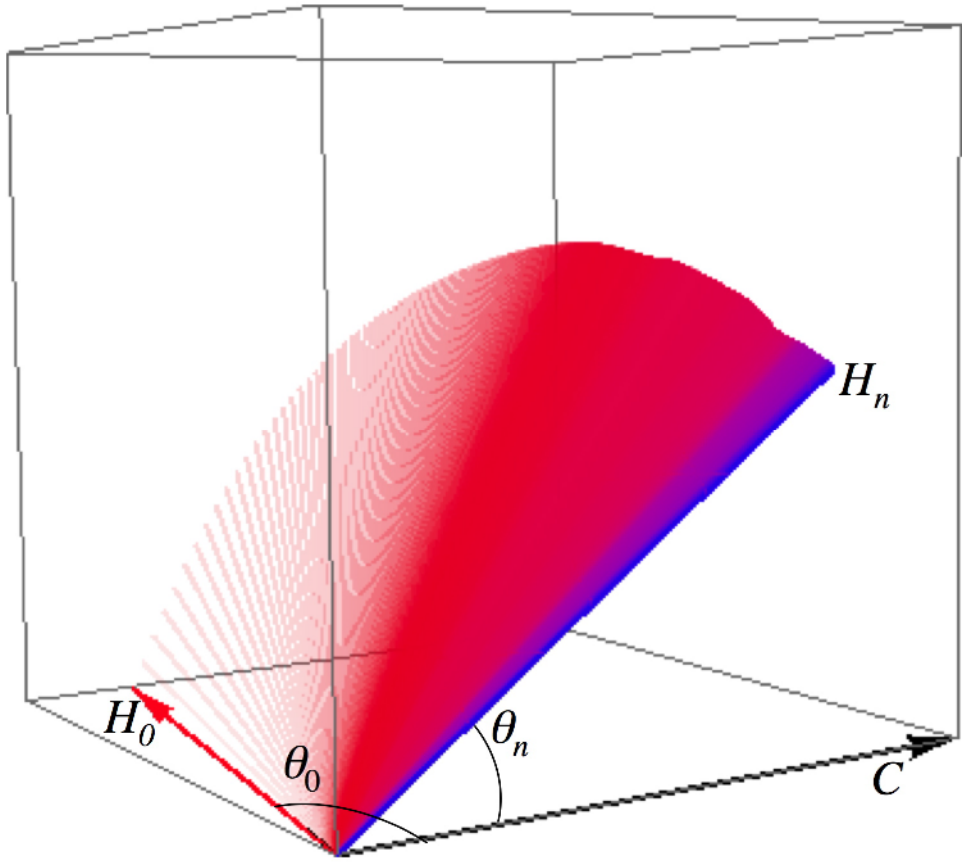


Statistical measures and energy spectra



A snapshot of SRG

- SRG: increases similarity between H_n and C
- In H_n: many-body interactions



H_0: J=0, T= 0 T_{rel}+N^{3}LO (8-shell)

A snapshot of SRG

- SRG: increases similarity between H_n and C
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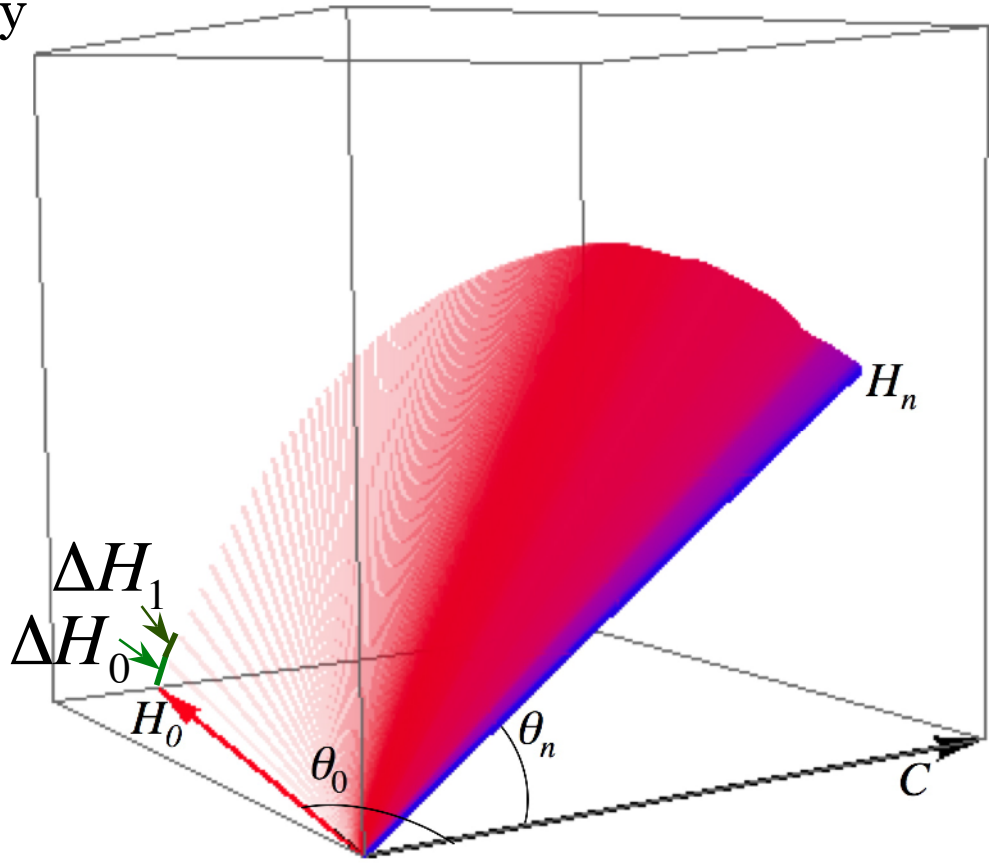
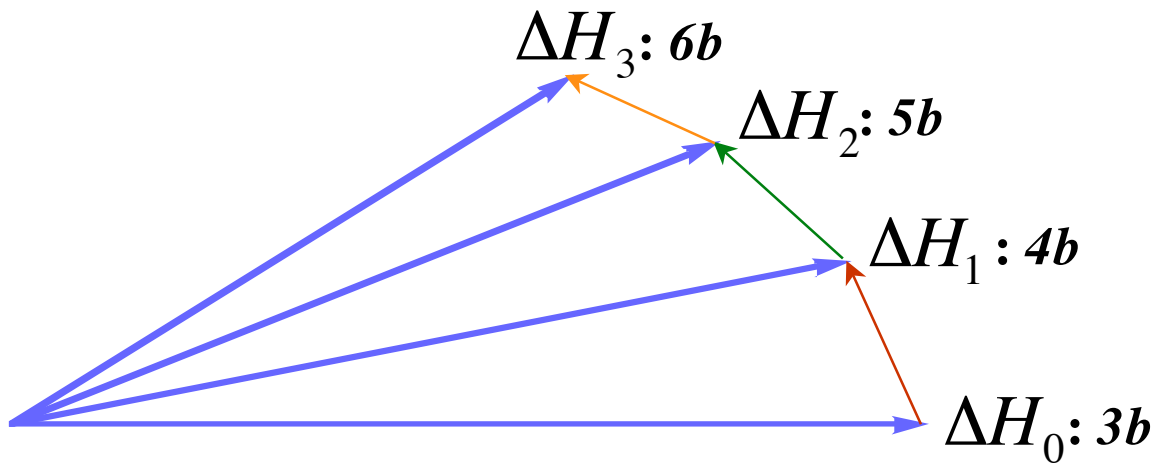


H_0: J=0, T= 0 T_{rel}+N^{3}LO (8-shell)

Hierarchy in particle ranks

General SRG flow (controlled)

- For illustration, take "big" steps, δs
  - Express "new" through "old": power expansion in δs



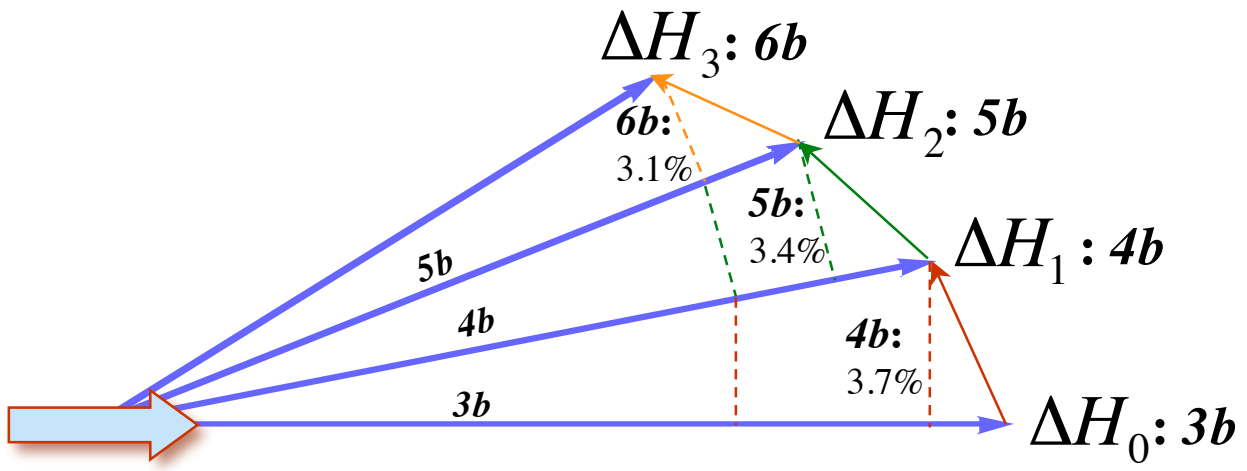
H_0: J=0, T= 0 T_{rel}+N^{3}LO (8-shell)

Hierarchy in particle ranks

General SRG flow (controlled)

- For illustration, take "big" steps, δs
  - Express "new" through "old": power expansion in δs
  - 3-body >> 4-body > 5-body > 6-body ....
  - (ΔH_3 : 90% - 3.5% - 3.3% - 3.1%)

(many-body terms: labeled for 1b C and 2b H_0)



Most important contribution

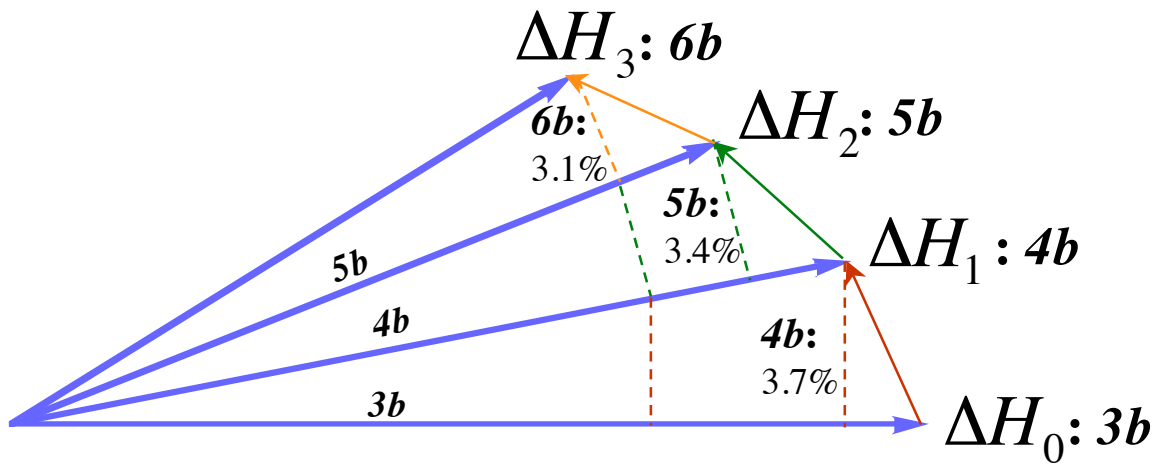
H_0: J=0, T= 0 T_{rel}+N^{3}LO (8-shell)

Accumulated contributions

General SRG flow (controlled)

- At the end of flow,
  - add all 3-body contributions = SRG-3b,
  - add all (perpendicular) 4-body contributions = SRG-4b,
  - ...

(many-body terms: labeled for 1b C and 2b H_0)



H_0: J=0, T= 0 T_{rel}+N^{3}LO (8-shell)

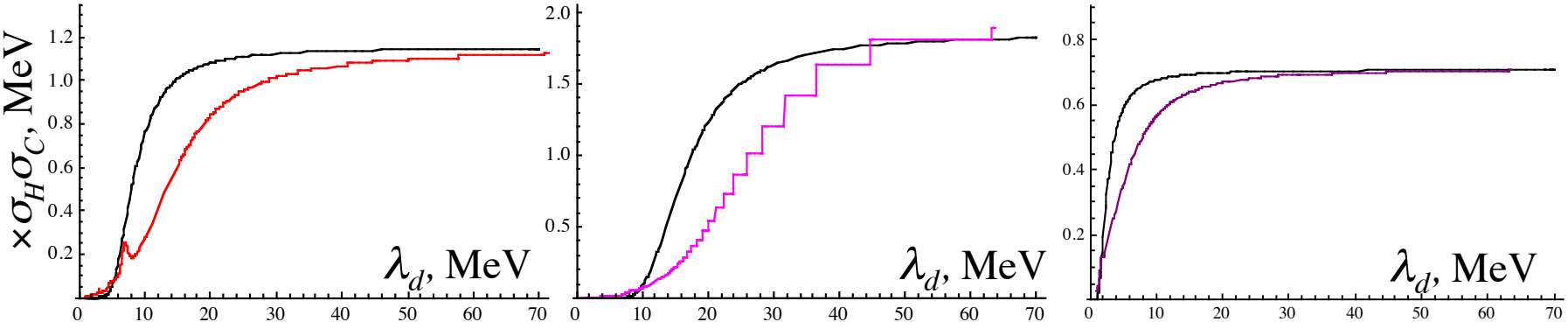
Accumulated contributions

General SRG flow (controlled)

- Accumulated strengths
  - upper limits (slowest flow)
  - near symmetry of C and fully evolved H

(many-body terms: labeled for 1b C and 2b H_0)

Strength of the $[C,H]$ SRG generator



H_0: J=2, T=1 T_{rel}+N^{3}LO
C: C_2^{su3}

H_0: J=2, T=1 T_{rel}+AV18
C: C_2^{su3}

H_0: J=2, T=1 T_{rel}+AV18
C: T_{rel}

…Typically, large overestimate for C_2^{su3}

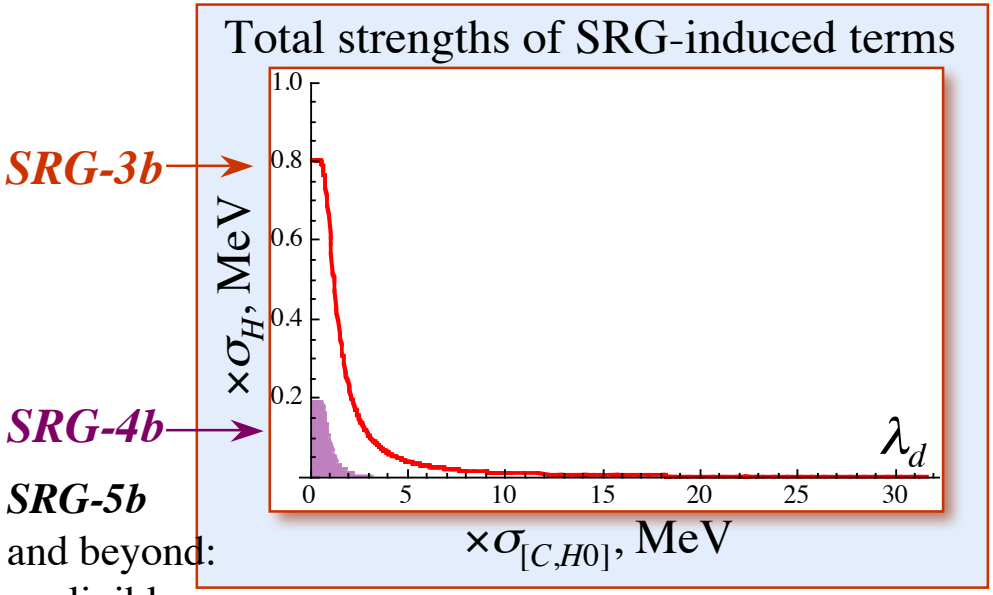
(8-shell)

Accumulated contributions

General SRG flow (controlled)

- Accumulated strengths
  - upper limits (slowest flow)
  - near symmetry of C and fully evolved H
  - SRG-3b: large contribution
  - SRG-4b ...

(many-body terms: labeled for 1b C and 2b H_0)



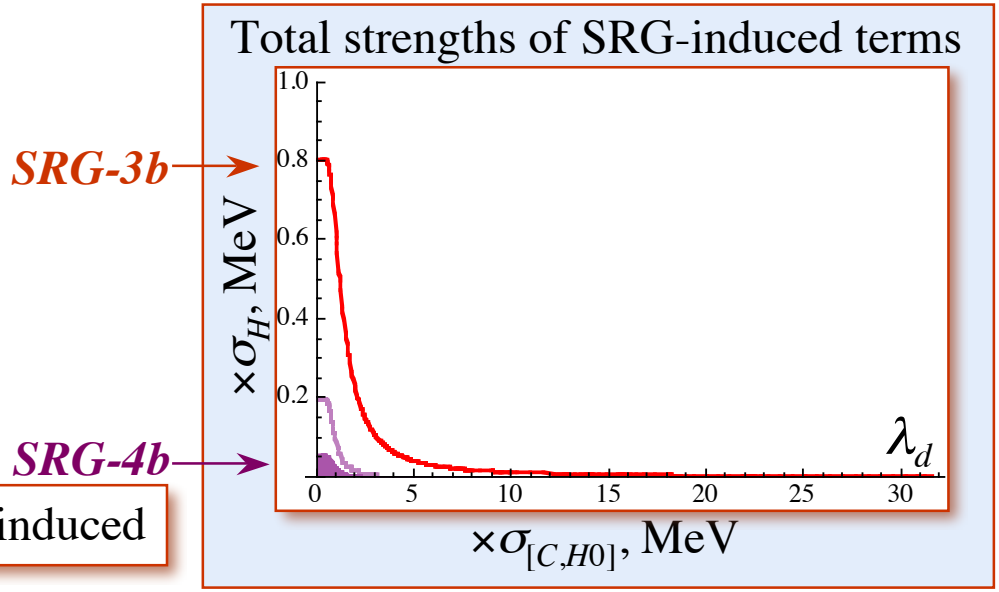
Example shown for: step $\delta = 0.1$, $\xi_{C,H0} = 0.2$, $\sigma^{scaled}_{[[C,H_0],H_0]} = 1$, and $\sigma^{scaled}_{[[C,H_0],[[C,H_0],H_0]]} = 1$

Accumulated contributions

General SRG flow (controlled)

- Accumulated strengths
  - upper limits (slowest flow)
  - near symmetry of C and fully evolved H
  - SRG-3b: large contribution

(many-body terms: labeled for 1b C and 2b H_0)



- SRG-4b decrease for smaller step

~4% of total induced

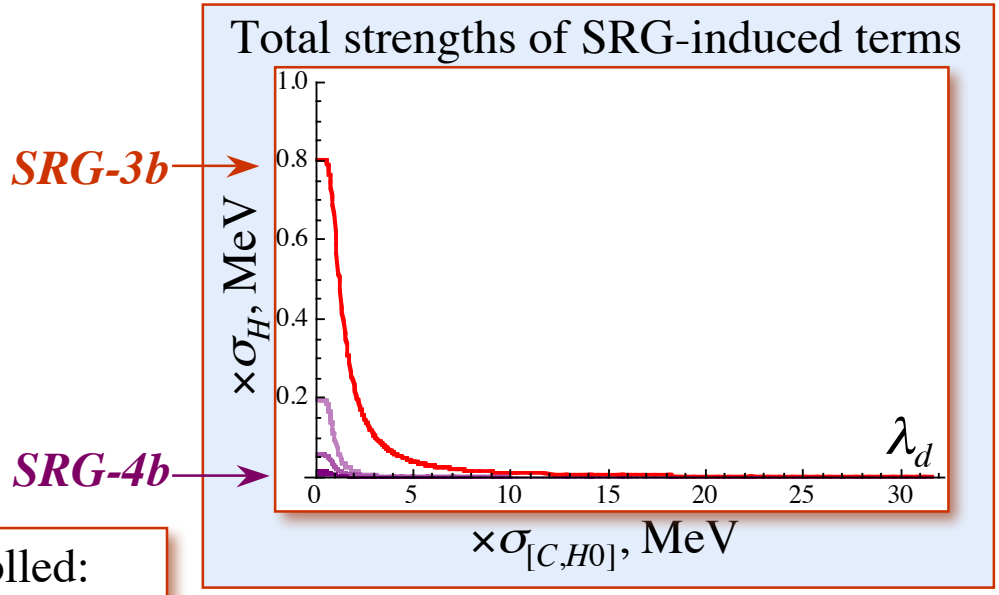
Example shown for: step $\delta = 0.01$, $\xi_{C,H0} = 0.2$, $\sigma^{scaled}_{[[C,H_0],H_0]} = 1$, and $\sigma^{scaled}_{[[C,H_0],[[C,H_0],H_0]]} = 1$

Accumulated contributions

General SRG flow (controlled)

- Accumulated strengths
  - upper limits (slowest flow)
  - near symmetry of C and fully evolved H
  - SRG-3b: large contribution

(many-body terms: labeled for 1b C and 2b H_0)



- SRG-4b decrease for smaller step & increase with $\sigma^{scaled}_{[[C,H_0],[[C,H_0],H_0]]}$

Can be controlled: negligible contribution

Example shown for: step $\delta = 0.001$, $\xi_{C,H0} = 0.2$, $\sigma^{scaled}_{[[C,H_0],H_0]} = 1$, and $\sigma^{scaled}_{[[C,H_0],[[C,H_0],H_0]]} = 1$

Fully evolved Hamiltonian

General SRG flow (controlled)

- Accumulated strengths
  - upper limits (slowest flow)
  - near symmetry of C and fully evolved H
  - Total SRG-3b,

  at saturation:

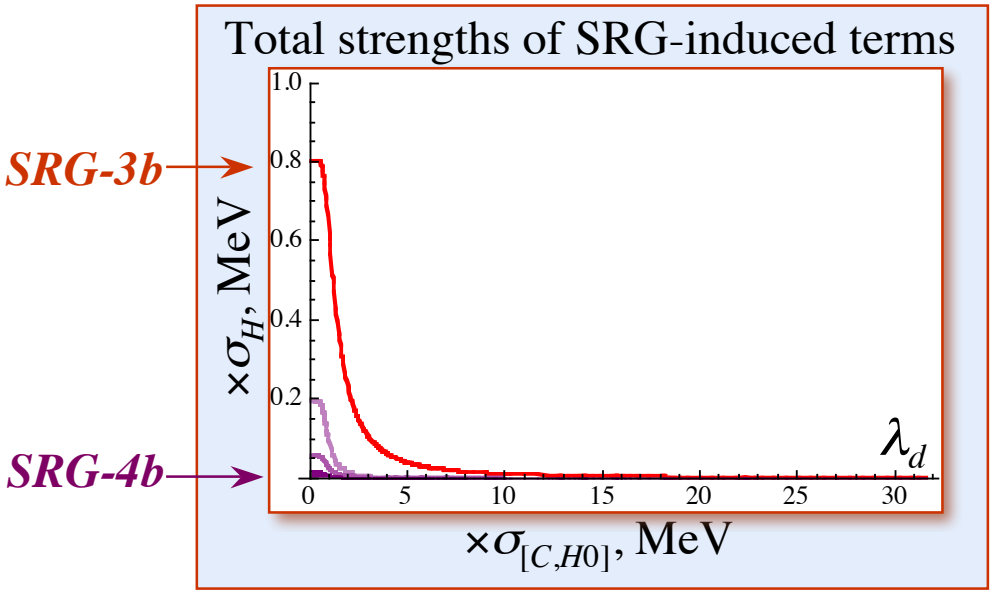
(many-body terms: labeled for 1b C and 2b H_0)

Strength of initial 3b

$$\frac{\sigma_{[[C,H_0],H_0]}\sigma_C}{\sigma^2_{[C,H_0]}}(1-\cos\theta_0)\sigma_H$$

Initial strength of SRG generator

Angle between C and H_0



Example shown for: step $\delta = 0.001$, $\xi_{C,H0} = 0.2$, $\sigma^{scaled}_{[[C,H_0],H_0]} = 1$, and $\sigma^{scaled}_{[[C,H_0],[[C,H_0],H_0]]} = 1$

Fully evolved Hamiltonian

General SRG flow (controlled)

- Accumulated strengths
  - upper limits (slowest flow)
  - near symmetry of C and fully evolved H
  - Total SRG-**L**owest-**P**article-**R**ank,

  at saturation:

LPR=3b, for 1b C & 2b H_0
LPR=4b, for 2b C & 2b H_0
LPR=5b, for 1b C & 3b H_0

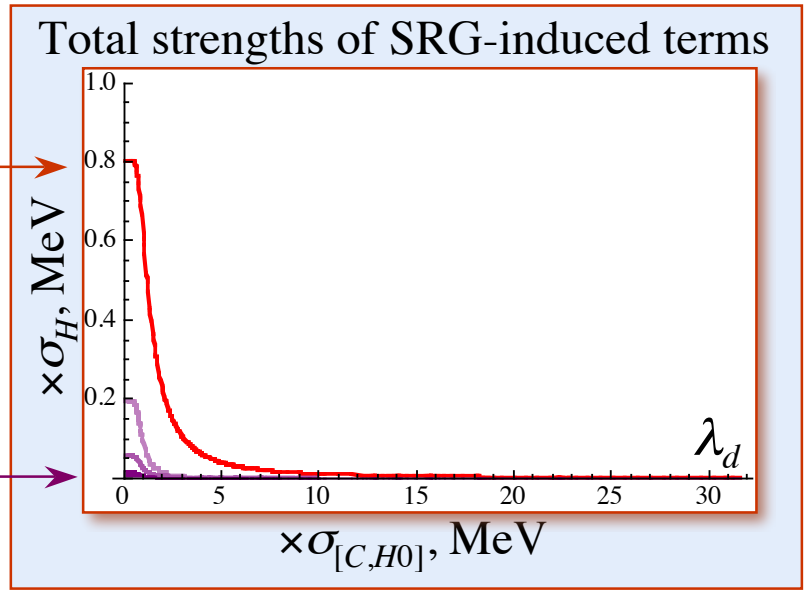
Strength of initial LPR

$$\frac{\sigma_{[[C,H_0],H_0]}\sigma_C}{\sigma^2_{[C,H_0]}}(1-\cos\theta_0)\sigma_H$$

Initial strength of SRG generator

Angle between C and H_0

Total strengths of SRG-induced terms



Example shown for: step $\delta = 0.001$, $\xi_{C,H0} = 0.2$, $\sigma^{scaled}_{[[C,H_0],H_0]} = 1$, and $\sigma^{scaled}_{[[C,H_0],[[C,H_0],H_0]]} = 1$

Fully evolved Hamiltonian

H_0: N^{3}LO (8-shell)

LPR=4b, for 2b C & 2b H_0

- Accumulated strengths
  - upper limits (slowest flow)
  - near symmetry of C and fully evolved H: $C=C_2^{su3}$ or T_{rel}
  - Total SRG-LPR,

  at saturation:

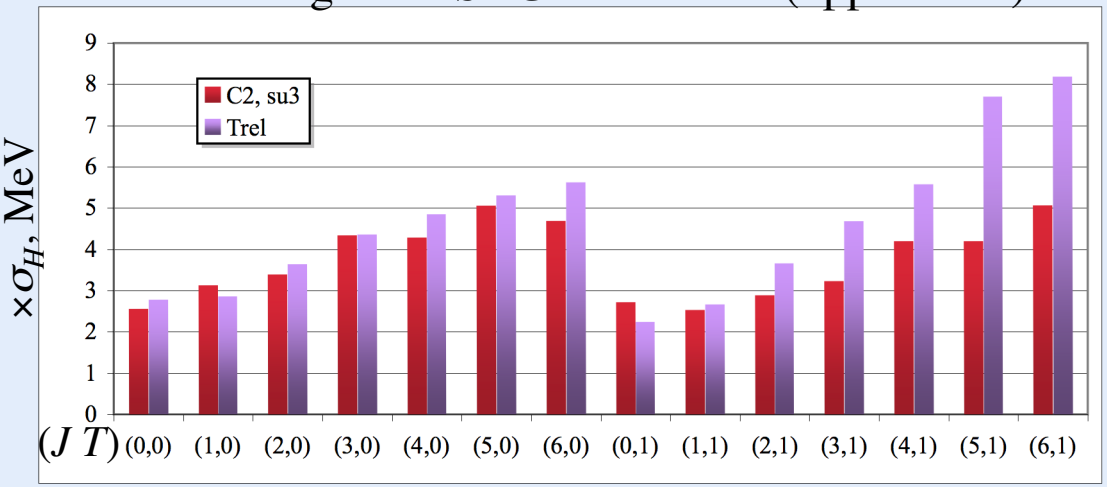
Strength of initial LPR

$$\frac{\sigma_{[[C,H_0],H_0]}\sigma_C}{\sigma^2_{[C,H_0]}}(1-\cos\theta_0)\sigma_H$$

Initial strength of SRG generator

Angle between C and H_0

Total strengths of SRG-LPR terms (upper limit)



•C_2^{su3}: smaller contribution of LPR interactions

Fully evolved Hamiltonian

H_0: T_{rel} +N^{3}LO (8-shell)

- Accumulated strengths
  - upper limits (slowest flow)
  - **near symmetry** of C and fully evolved H: $C=C_2^{su3}$ or T_{rel}
  - Total SRG-LPR,

  at saturation:

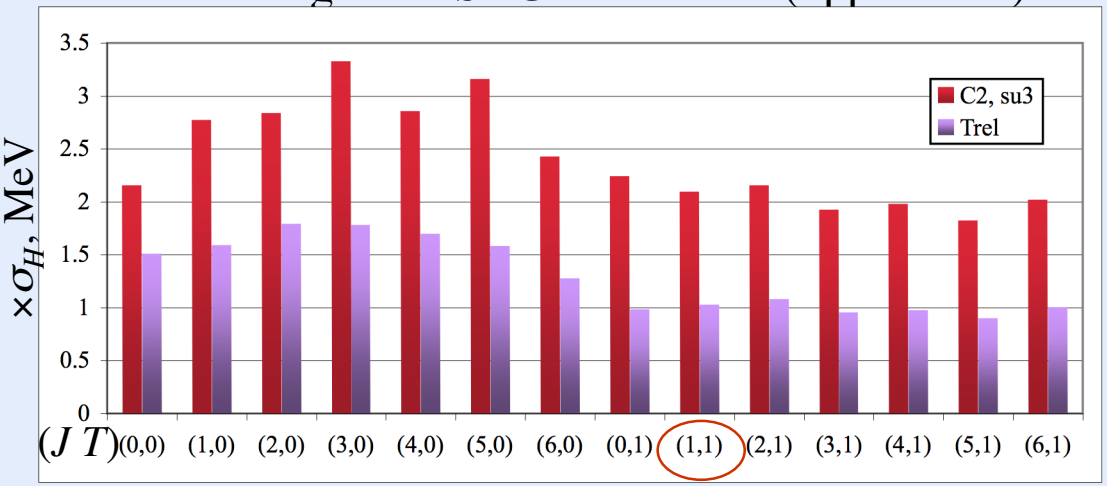
Strength of initial LPR

$$\frac{\sigma_{[[C,H_0],H_0]}\sigma_C}{\sigma^2_{[C,H_0]}}(1-\cos\theta_0)\sigma_H$$

Initial strength of SRG generator

Angle between C and H_0

Total strengths of SRG-LPR terms (upper limit)



•T_{rel}: smaller contribution of LPR interactions

Fully evolved Hamiltonian

H_0: $T_{\rm rel}$ +N^{3}LO (8-shell)

- Accumulated strengths
  - upper limits (slowest flow)
  - **near symmetry** of C and fully evolved H: $C=C_2^{\rm su3}$ or $T_{\rm rel}$
  - Total SRG-LPR,

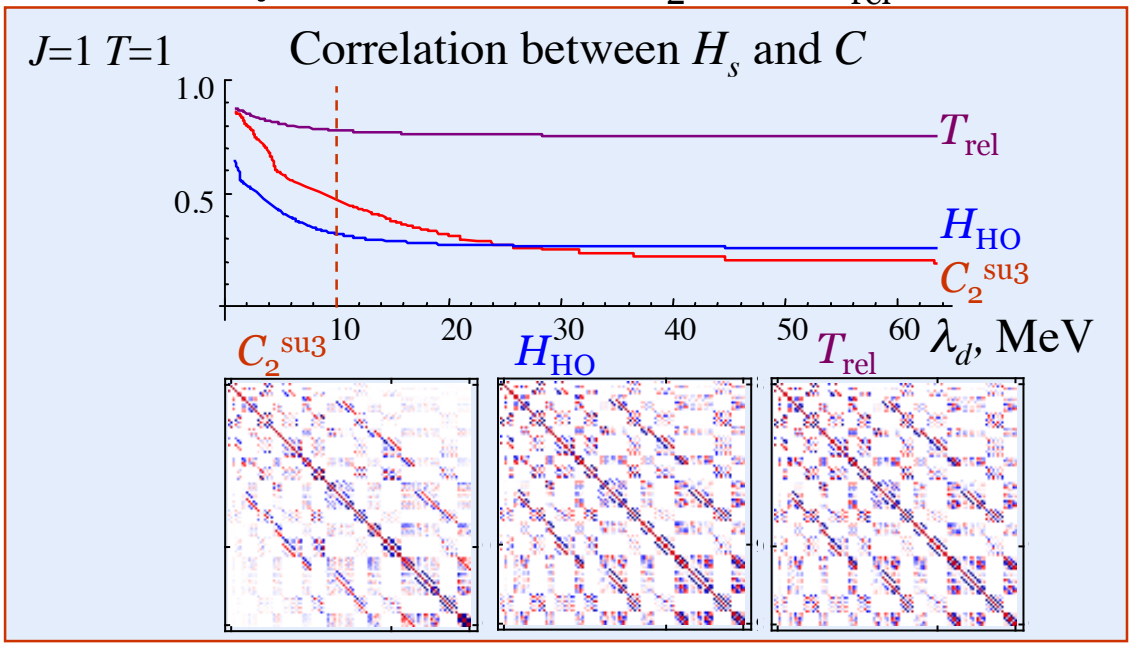
  at saturation:

Strength of initial LPR

$$\frac{\sigma_{[[C,H_0],H_0]}\sigma_C}{\sigma^2_{[C,H_0]}}(1-\cos\theta_0)\sigma_H$$

Initial strength of SRG generator

Angle between C and H_0



- $T_{\rm rel}$: smaller contribution of LPR interactions
- $C_2^{\rm su3}$: faster decoupling

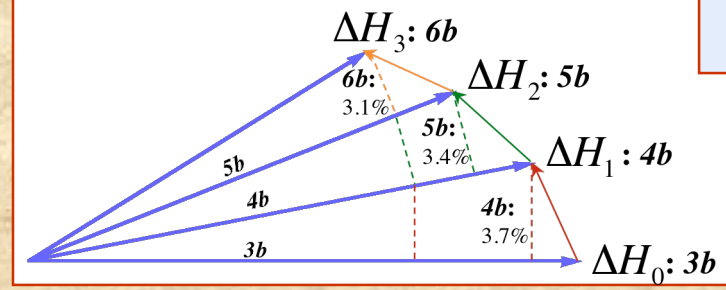
Fully evolved Hamiltonian

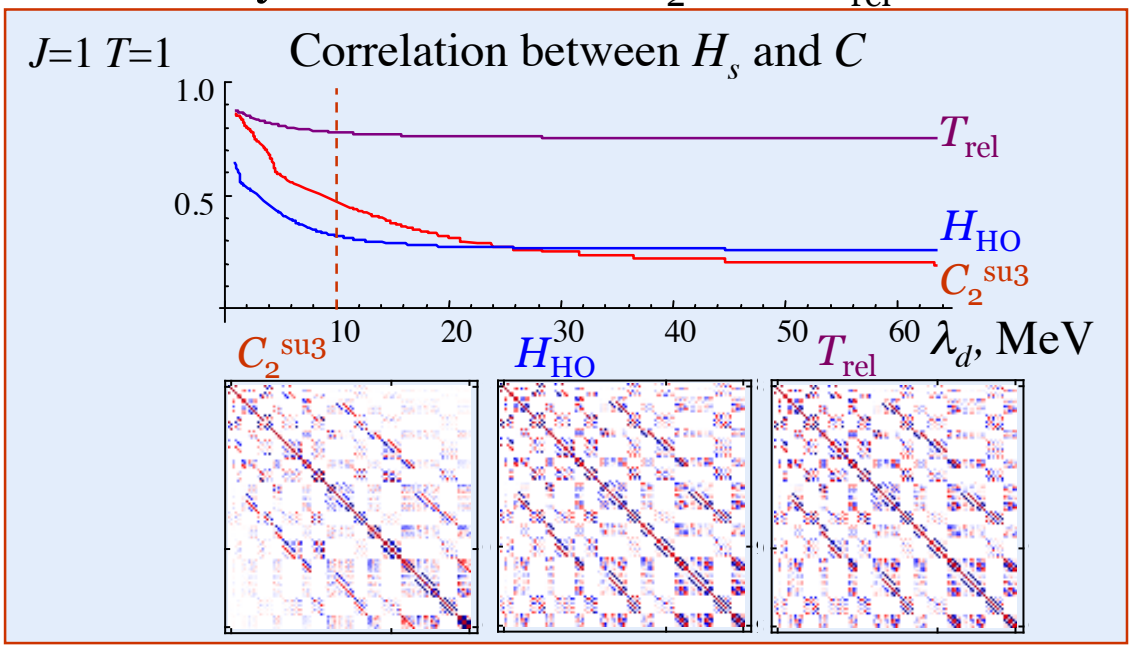
H_0: T_{rel} +N^{3}LO (8-shell)

- Accumulated strengths
  - upper limits (slowest flow)
  - **near symmetry** of C and fully evolved H: C=C$_2^{su3}$ or T_{rel}
  - Total SRG-LPR, at saturation:

Strength of initial LPR

$$\frac{\sigma_{[[C,H_0],H_0]}\sigma_C}{\sigma^2_{[C,H_0]}}(1-\cos\theta_0)\sigma_H$$





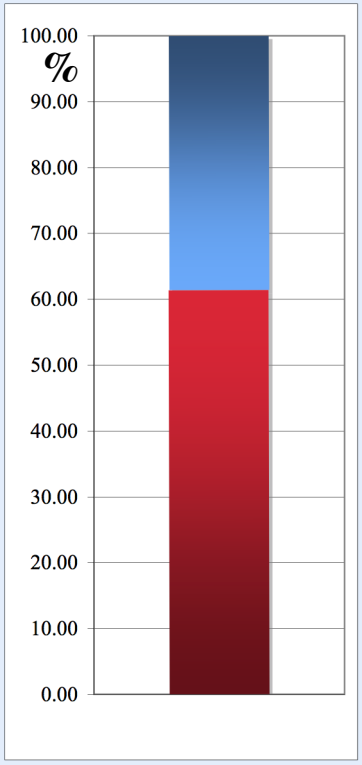
- T_{rel}: smaller contribution of LPR interactions
- C_2^{su3}: faster decoupling

Initial SRG-induced 3b

1b C and 2b H_0: *random* (3-shell)

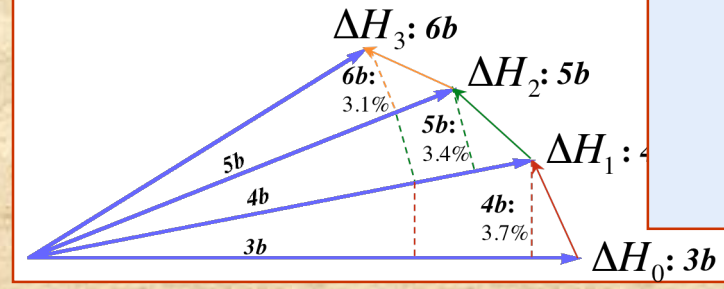
- Initial SRG-induced term: up to 3b
  - dominant role in all SRG-induced interactions
  - 2b: 60%
  - 3b: 40%
- What is the 3b contribution in a 6-particle system?

Percentage of the initial SRG-3b



Pure induced ***3b***

Total induced ***2b***



initial SRG-3b

With **1b** C: H_0 → **2b**

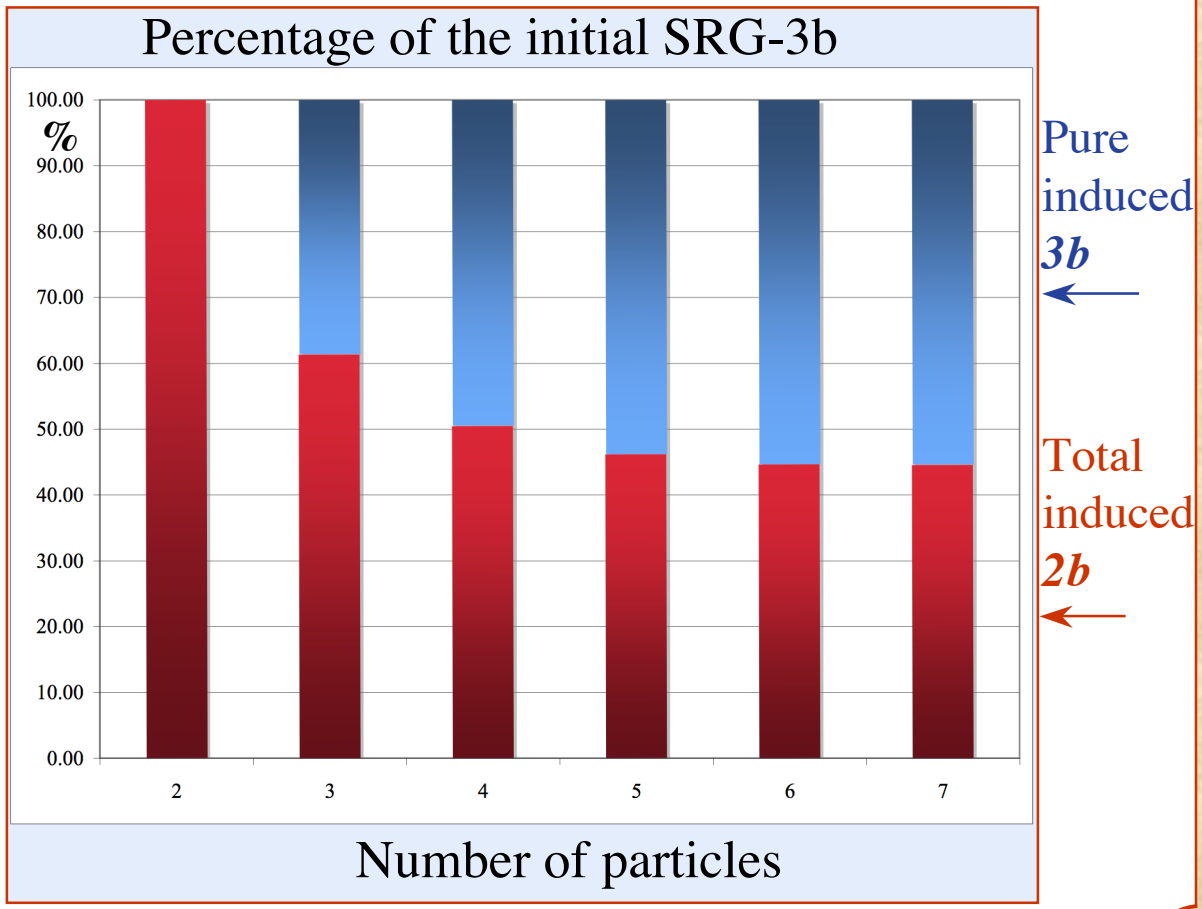
$$\frac{dH_0}{ds} = [[C,H_0],H_0]$$

two-body three-body

Initial SRG-induced 3b

1b C and 2b H_0: *random* (3-shell)

- Initial SRG-induced term: up to 3b
  - dominant role in all SRG-induced interactions
  - 2b: 60%
  - 3b: 40%
- A = 4, 5,…: ~50% contribution due to 3b

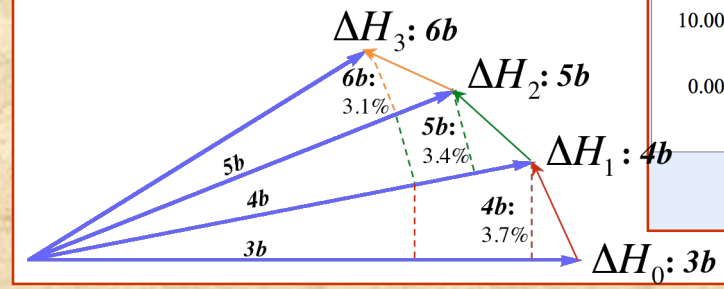


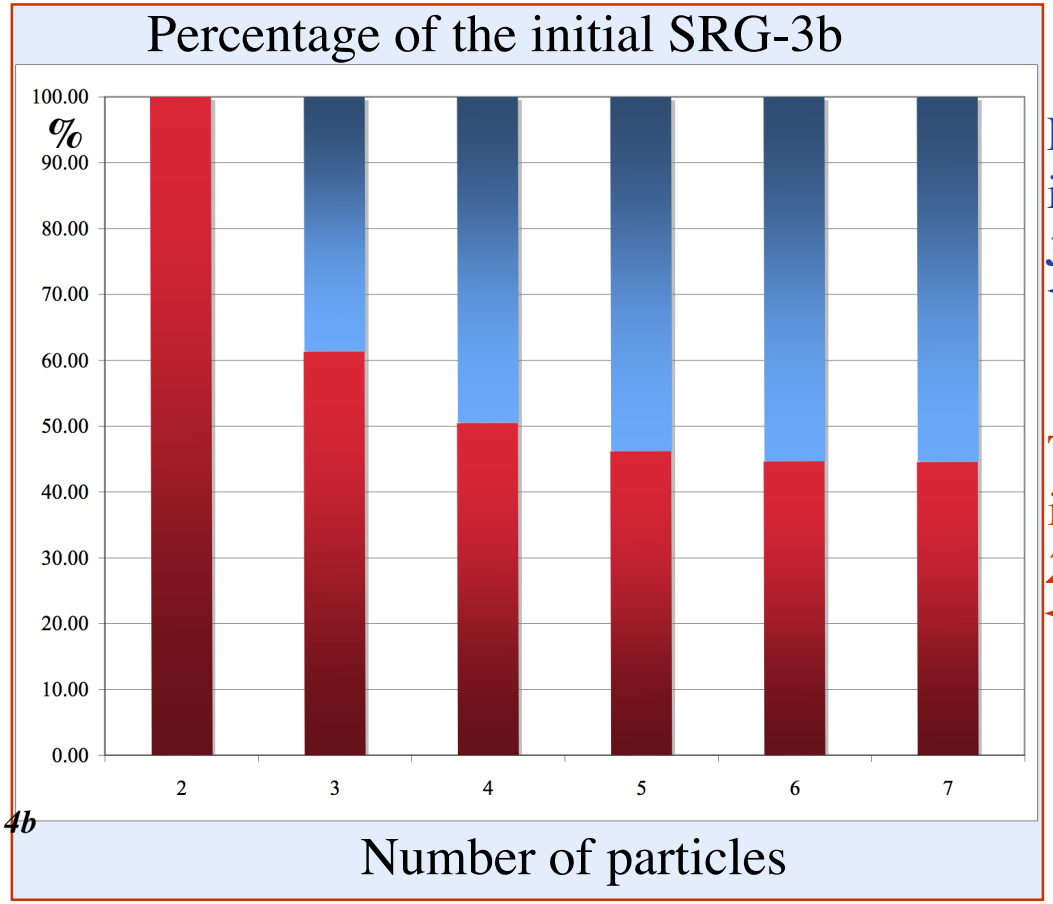
Applicability of SRG

1b C and 2b H_0: *random* (3-shell)

- SRG in A=2: neglects ~50% 3b, <1% up-to-4b, …
- SRG in A=3: neglects <1% up-to-4b, …

(including pure 3b of 4b, 5b, … that enters in SRG-3b)

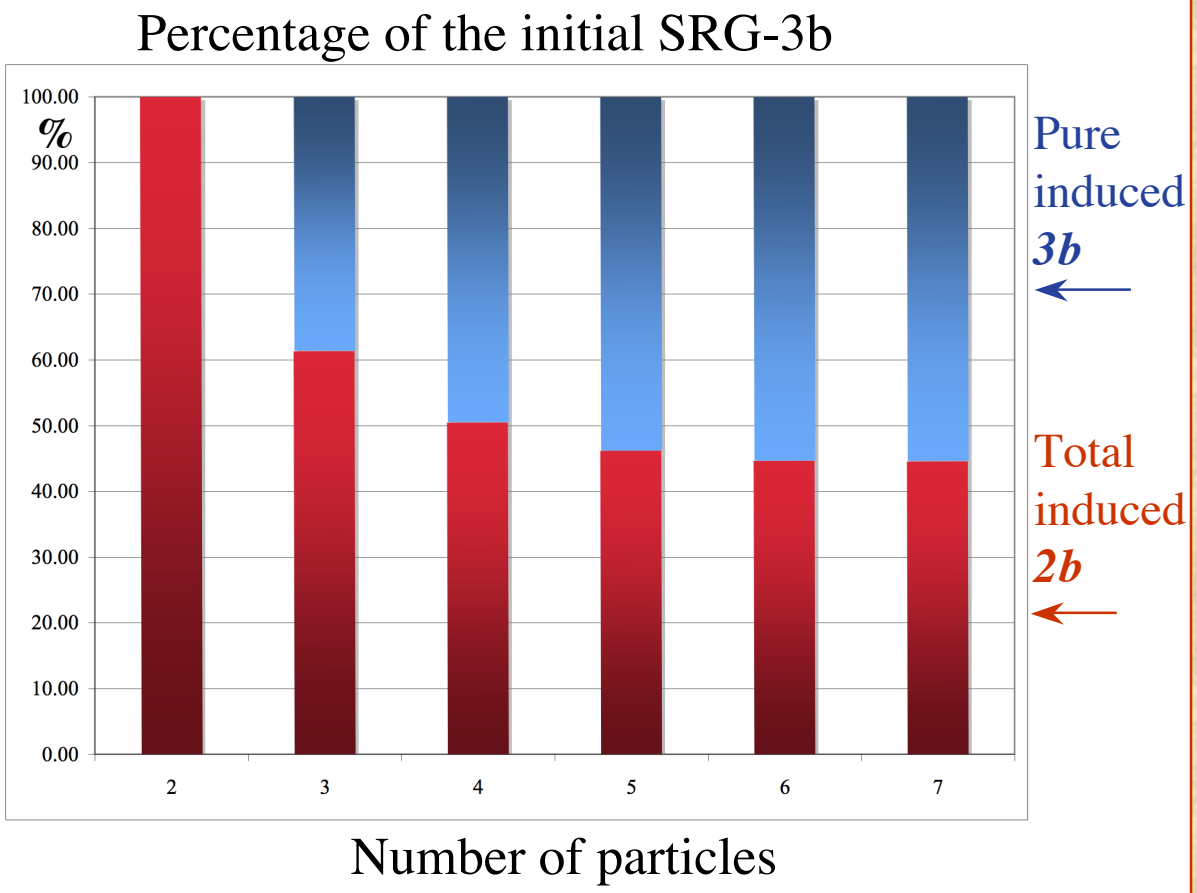




Significance of SRG-induced many-body interactions

1b C and 2b H_0: *random* (3-shell)

- 2b dominant (together with initial 2b H_0)
- 3b important
- 4b and beyond negligible



Significance of SRG-induced many-body interactions

- 2b dominant

(together with initial 2b H_0)

- LPR important
- NLPR and beyond negligible

LPR=3b, for 1b C & 2b H_0 ... $C = H_{HO}$
LPR=4b, for 2b C & 2b H_0 ... $C = C_2^{su3}$ or T_{rel}
LPR=5b, for 1b C & 3b H_0 ... $H_0 = NN+NNN$

Case of C_2^{su3}: SU(3) Tensor Interaction

Example: 4 shells

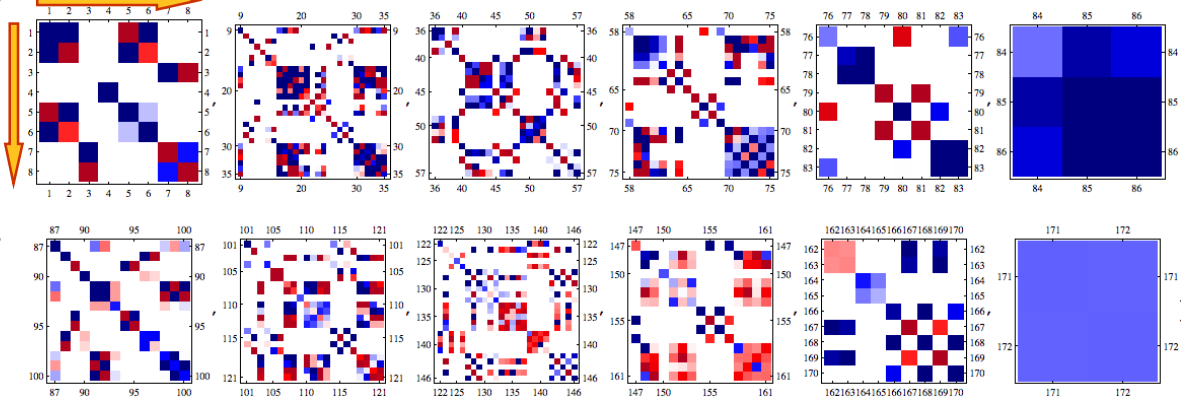
CD-Bonn

$\hbar\Omega$ =15 MeV

Rotational symmetry, SO(3): J_0

initial state (i)

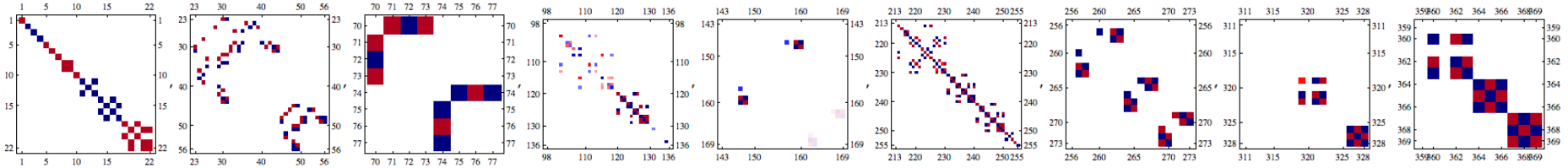
final state (f)



$V_{f,i}^{(J_0=0)T_0}$

SU(3) symmetry, (λ_0 μ_0) …different "shapes"

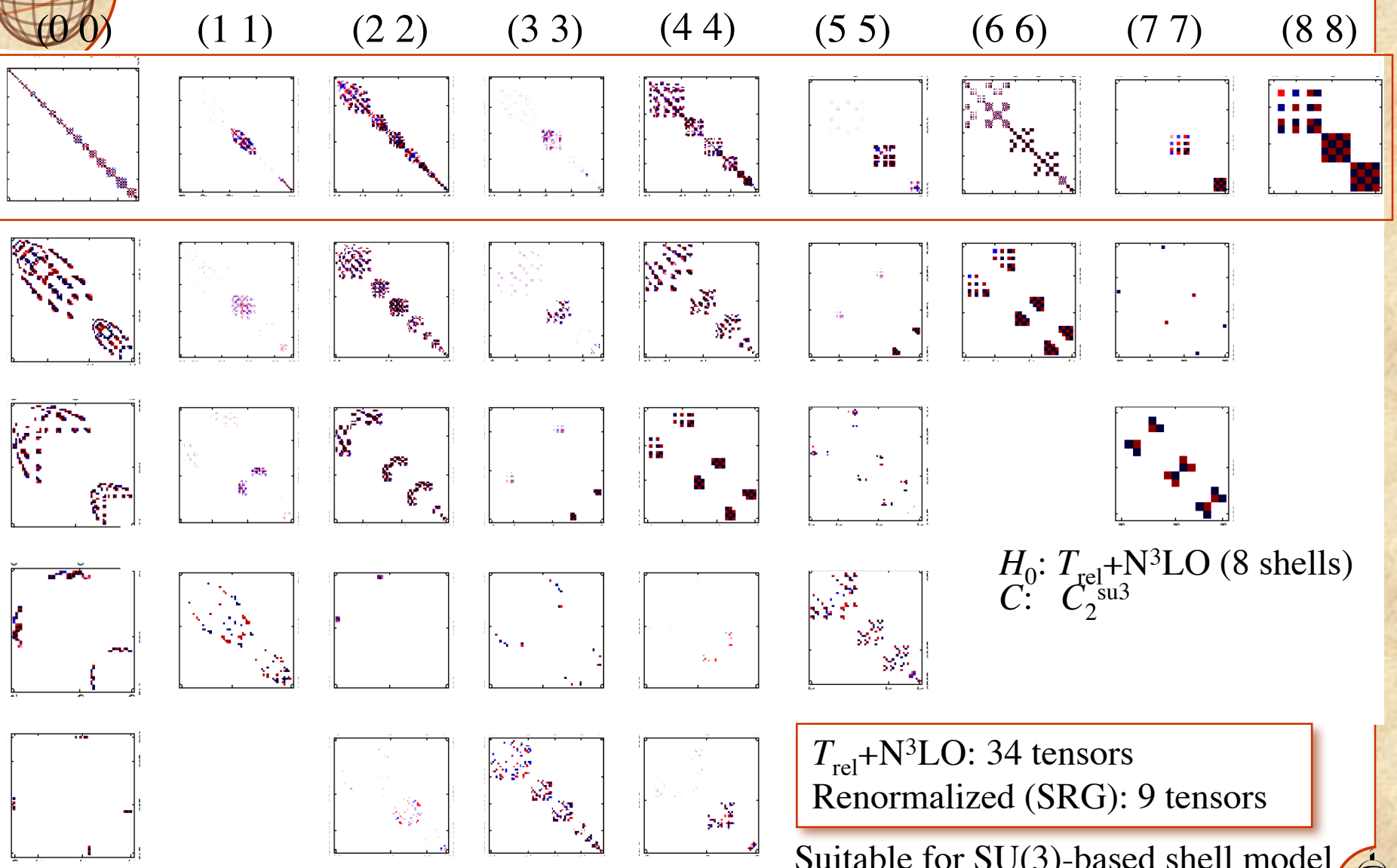
(0 0) (2 0) (4 0) (1 1) (3 1) (2 2) (2 4) (3 3) (4 4) …



$V_{f,i}^{(\lambda_0\ \mu_0)S_0T_0}$

- Pre-diagonalized
- 13 tensors/ smaller dimensions

Case of C_2^{su3}: SU(3) Tensor Interaction



H_0: T_{rel}+N^{3}LO (8 shells)
C: C_2^{su3}

T_{rel}+N^{3}LO: 34 tensors
Renormalized (SRG): 9 tensors

Suitable for SU(3)-based shell model

Summary

- Systematic study of a *general* SRG flow
- Fully evolved interaction:
  - 2-body dominant
  - Lowest-Particle-Rank (3-body) important
  - NLPR (4-body) and beyond: can be controlled toward negligible
- Estimates for many-body interaction contributions – depend only on properties of initial H_0 and C
- Initial SRG-induced interaction – dominant contribution to all SRG-induced terms