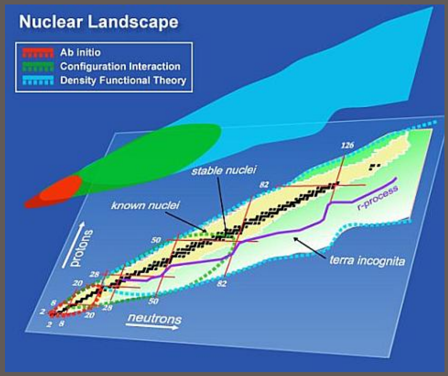
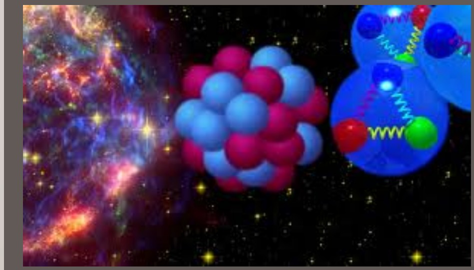
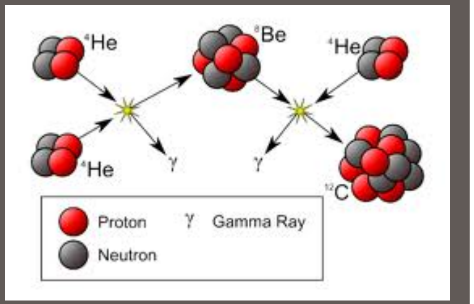
Unifying nuclear structure and reactions within the NCSM/RGM

Perspectives of the *Ab Initio* No-Core Shell Model
TRIUMF, February 10-12, 2011

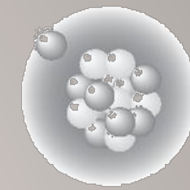
Petr Navratil | TRIUMF



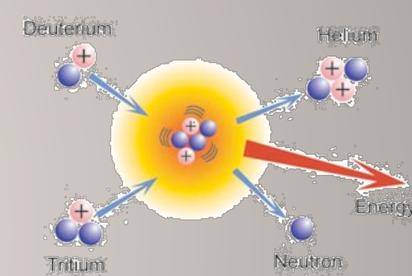




Light nuclei from first principles

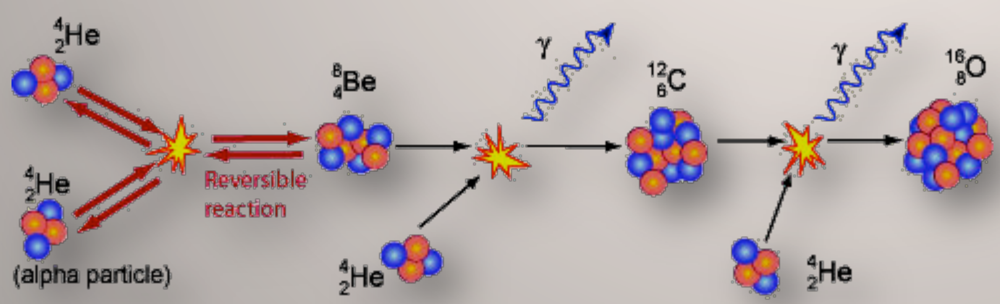


- **<u>Goal</u>**: Predictive theory of structure and reactions of light nuclei
- Needed for
  - Physics of exotic nuclei, tests of fundamental symmetries
  - Understanding of nuclear reactions important for astrophysics
  - Understanding of reactions important for energy generation

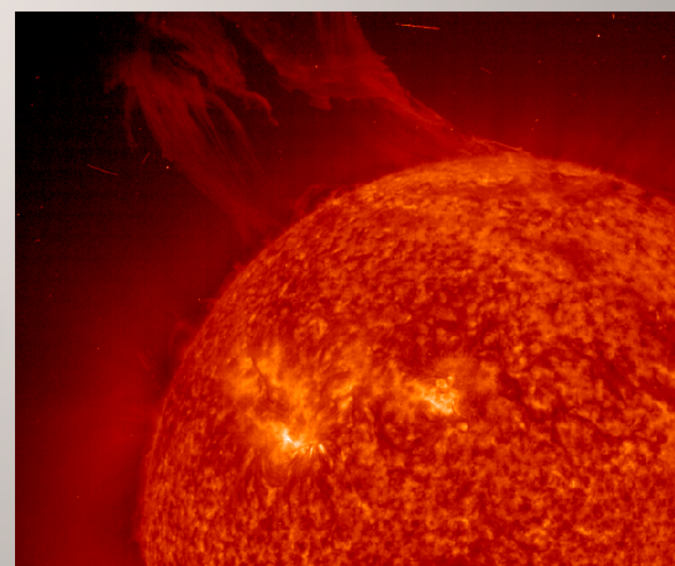


- **From first principles or *ab initio:***

✓ Nuclei as systems of nucleons interacting by nucleon-nucleon (and three-nucleon) forces that describe accurately nucleon-nucleon (and three-nucleon) systems

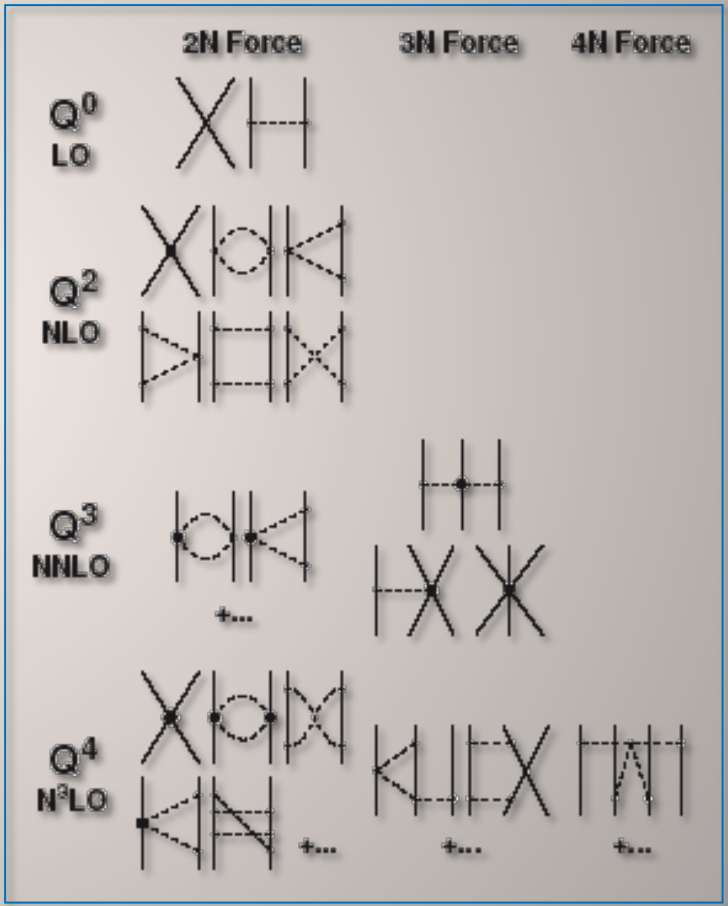


Understanding our Sun



Light nuclei from the first principles

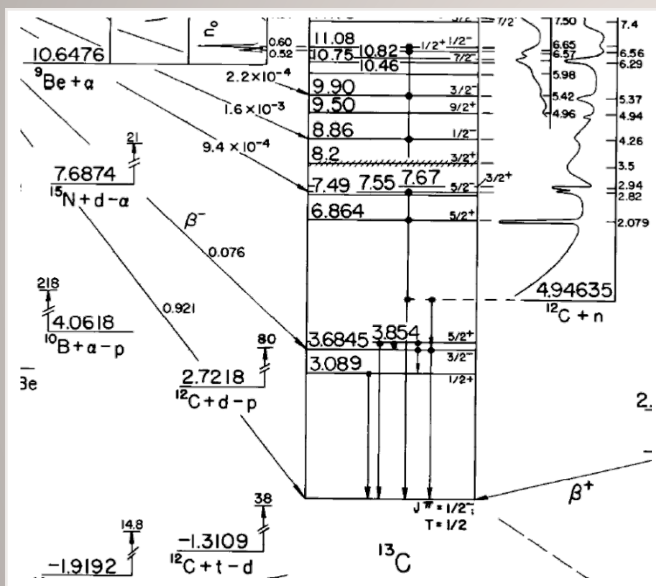
- **First principles for Nuclear Physics:**
  QCD
  - Non-perturbative at low energies
  - Lattice QCD in the future
- ***For now a good place to start*:**
- Inter-nucleon forces from chiral effective field theory
  - Based on the symmetries of QCD
    - Degrees of freedom: nucleons + pions
  - Systematic low-momentum expansion to a given order
  - Hierarchy
  - Consistency
  - Low energy constants (LEC)
    - Fitted to data
    - Can be calculated by lattice QCD



TRIUMF

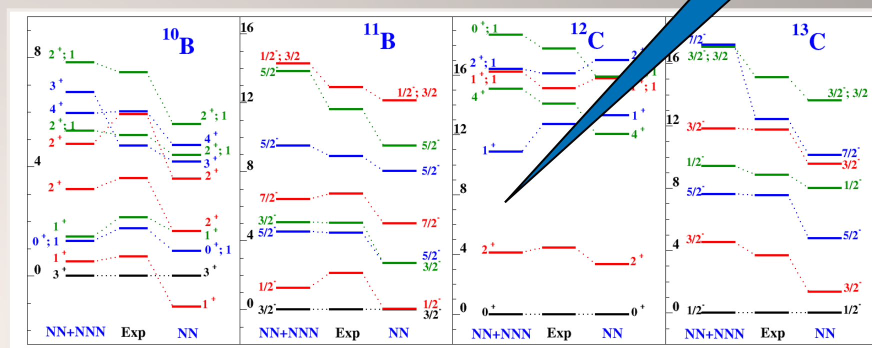
Predictive *ab initio* theory must provide a unified description of structure and reactions of light nuclei

- Nuclei are quantum many-body systems with bound states, resonances, scattering states
  - Bound-state techniques not sufficient

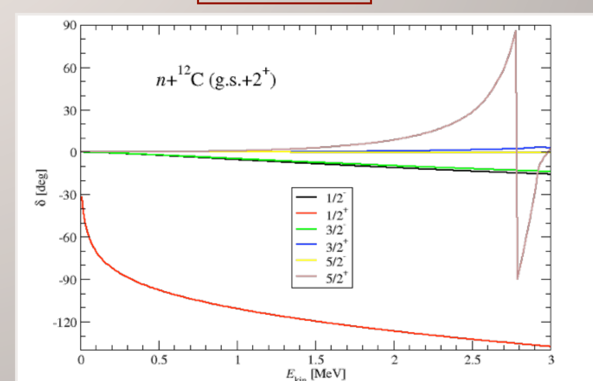


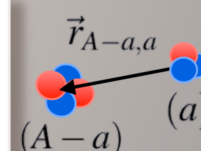
NCSM

The Hoyle state missing



RGM





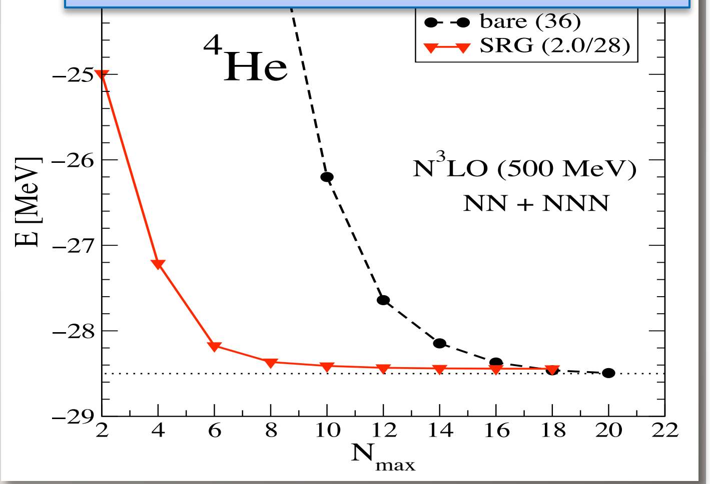
- **Our approach** - combining the *ab initio* no-core shell model (NCSM) with the resonating group method (RGM)
  - $\Rightarrow$ ***ab initio* NCSM/RGM**
  - NCSM - single-particle degrees of freedom
  - RGM - clusters and their relative motion

Our many-body technique:

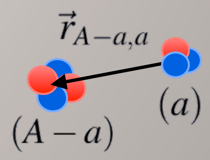
- **Combine** the *ab initio* no-core shell model (NCSM) with the resonating group method (RGM)

- **The NCSM:** An approach to the solution of the *A*-nucleon bound-state problem
  - Accurate nuclear Hamiltonian
  - Finite harmonic oscillator (HO) basis
    - Complete $N_{max}\hbar\Omega$ model space
  - Effective interaction due to the model space truncation
    - Similarity-Renormalization-Group evolved NN(+NNN) potential
  - Short & medium range correlations
  - No continuum

E. Jurgenson *et al.*, PRL 103, 082501 (2009)



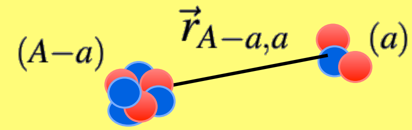
- **The RGM:** A microscopic approach to the *A*-nucleon scattering of clusters
  - Nuclear Hamiltonian may be simplistic
  - Cluster wave functions may be simplified and inconsistent with the nuclear Hamiltonian
  - Long range correlations, relative motion of clusters



***Ab initio* NCSM/RGM**: Combines the best of both approaches
Accurate nuclear Hamiltonian, consistent cluster wave functions
Correct asymptotic expansion, Pauli principle and translational invariance

The *ab initio* NCSM/RGM in a snapshot

- Ansatz: $\Psi^{(A)} = \sum_\nu \int d\vec{r}\, \varphi_\nu(\vec{r})\, \hat{\mathcal{A}}\, \Phi^{(A-a,a)}_{\nu\vec{r}}$



$$\psi^{(A-a)}_{1\nu}\, \psi^{(a)}_{2\nu}\, \delta(\vec{r} - \vec{r}_{A-a,a})$$

eigenstates of $H_{(A\text{-}a)}$ and $H_{(a)}$ in the *ab initio* NCSM basis

- Many-body Schrödinger equation:

$$H\Psi^{(A)} = E\Psi^{(A)}$$

$$T_{\rm rel}(r) + \mathcal{V}_{\rm rel} + \bar{V}_{\rm Coul}(r) + H_{(A-a)} + H_{(a)}$$

realistic nuclear Hamiltonian

$$\sum_\nu \int d\vec{r} \left[\mathcal{H}^{(A-a,a)}_{\mu\nu}(\vec{r}\,', \vec{r}) - E\, \mathcal{N}^{(A-a,a)}_{\mu\nu}(\vec{r}\,', \vec{r}) \right] \varphi_\nu(\vec{r}) = 0$$

$$\langle \Phi^{(A-a,a)}_{\mu\vec{r}\,'} | \hat{\mathcal{A}} H \hat{\mathcal{A}} | \Phi^{(A-a,a)}_{\nu\vec{r}} \rangle$$

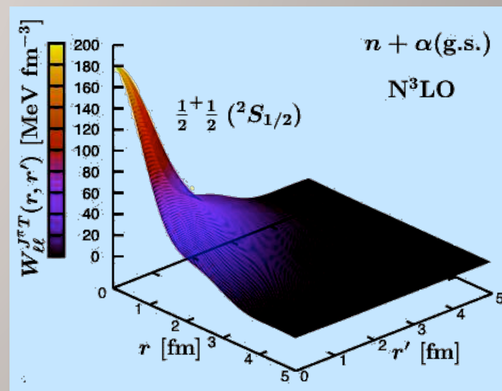
Hamiltonian kernel

$$\langle \Phi^{(A-a,a)}_{\mu\vec{r}\,'} | \hat{\mathcal{A}}^2 | \Phi^{(A-a,a)}_{\nu\vec{r}} \rangle$$

Norm kernel

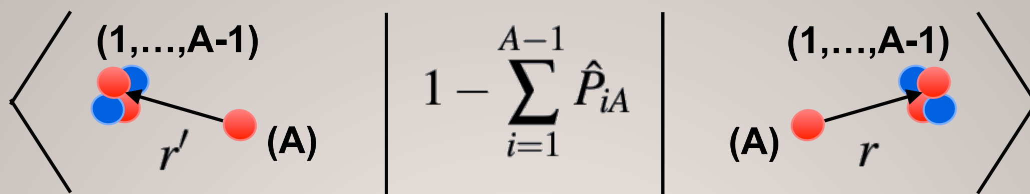
- Non-local integro-differential coupled-channel equations:

$$[\hat{T}_{\rm rel}(r) + \bar{V}_{\rm C}(r) - (E - E_\nu)]\, u_\nu(r) + \sum_\nu \int dr'\, r'\, W_{\nu\nu'}(r, r')\, u_\nu(r') = 0$$

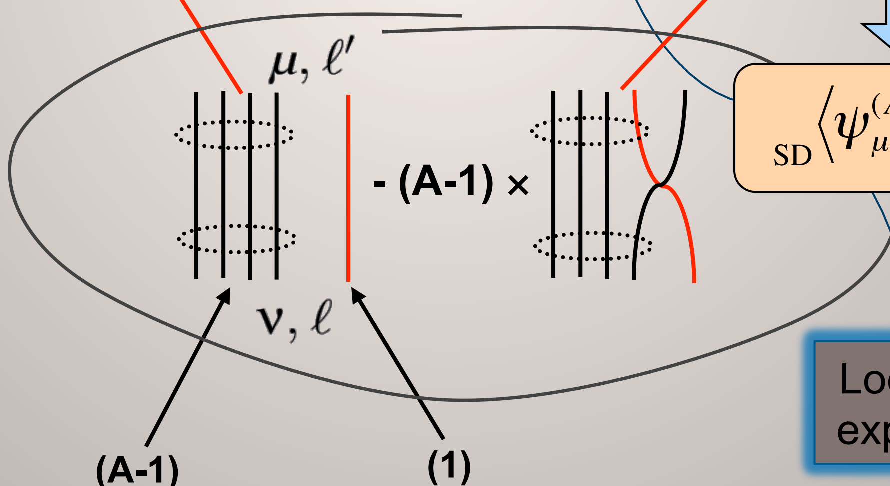


TRIUMF

Single-nucleon projectile: the norm kernel



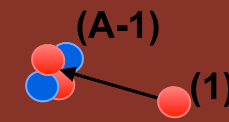
$$\mathcal{N}^{(A-1,1)}_{\mu\ell',\nu\ell}(r',r) = \delta_{\mu\nu}\,\delta_{\ell'\ell}\frac{\delta(r'-r)}{r'r} - (A-1)\sum_{n'n} R_{n'\ell'}(r')\langle\Phi^{(A-1,1)JT}_{\mu n'\ell'}|P_{A,A-1}|\Phi^{(A-1,1)JT}_{\nu n\ell}\rangle R_{n\ell}(r)$$

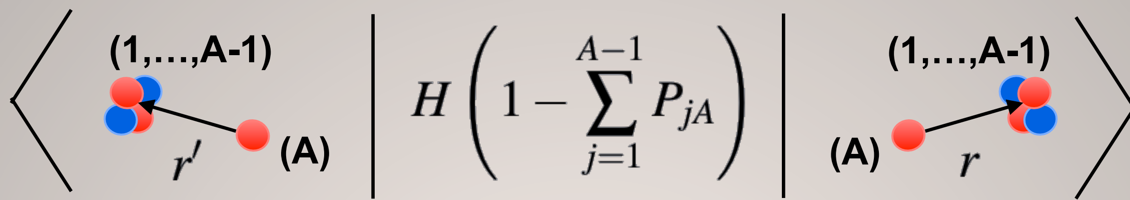


$${}_{\mathrm{SD}}\left\langle \psi^{(A-1)}_{\mu_1} \middle| a^{+}a \middle| \psi^{(A-1)}_{\nu_1} \right\rangle_{\mathrm{SD}}$$

Localized parts of kernels expanded in the HO basis

Single-nucleon projectile basis: the Hamiltonian kernel



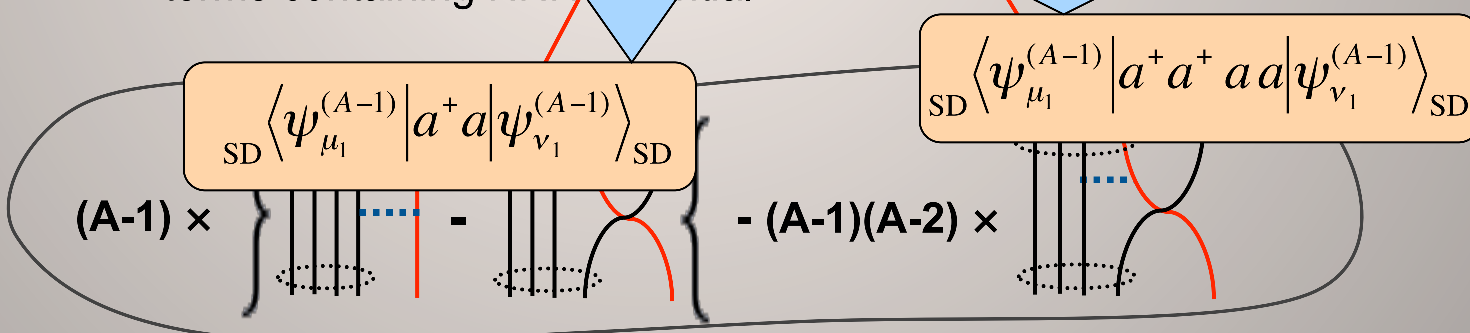


$$\mathcal{H}^{(A-1,1)}_{\mu\ell',\nu\ell}(r',r) = (E_{A-1}+\mathcal{T}_{rel})\,\mathcal{N}^{(A-1,1)}_{\mu\ell',\nu\ell}(r',r)$$

$$+\,(A-1)\boxed{\sum_{n'n} R_{n'\ell'}(r')\langle\Phi^{(A-1,1)JT}_{\mu n'\ell'}|V_{A-1,A}(1-P_{A-1,A})|\Phi^{(A-1,1)JT}_{\nu n\ell}\rangle R_{n\ell}(r)}$$

$$-\,(A-1)(A-2)\boxed{\sum_{n'n} R_{n'\ell'}(r')\langle\Phi^{(A-1,1)JT}_{\mu n'\ell'}|V_{A-2,A}P_{A,A-1}|\Phi^{(A-1,1)JT}_{\nu n\ell}\rangle R_{n\ell}(r)}$$

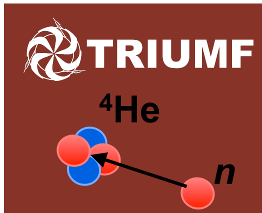
+ terms containing NNN potential



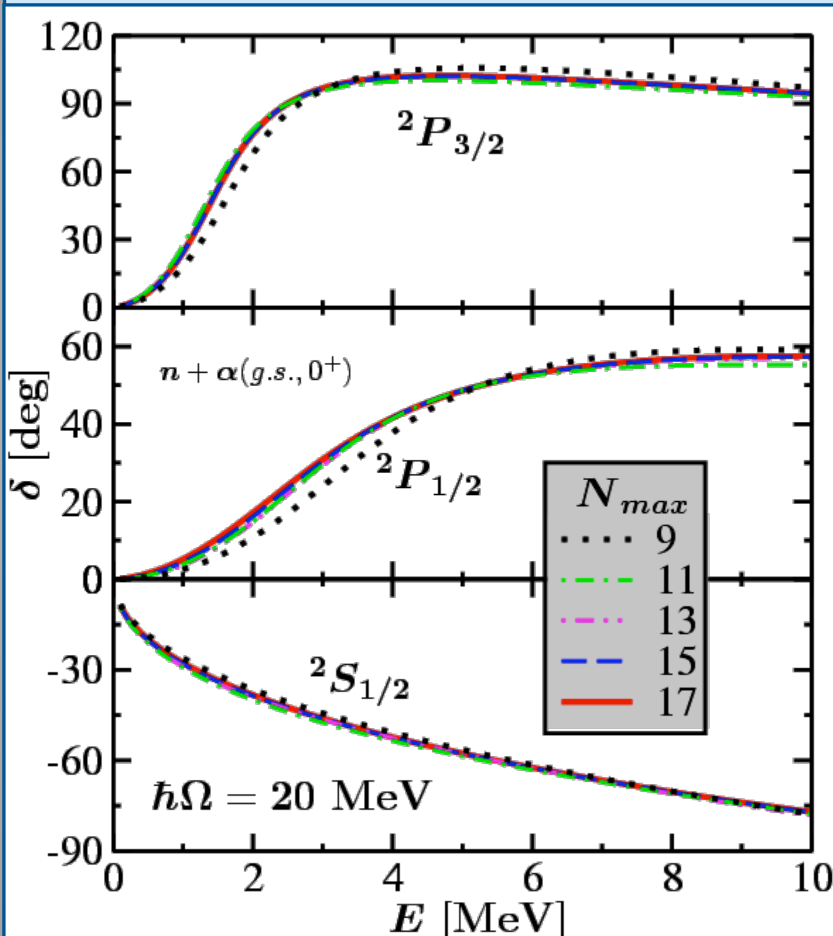
"direct potential"

"exchange potential"

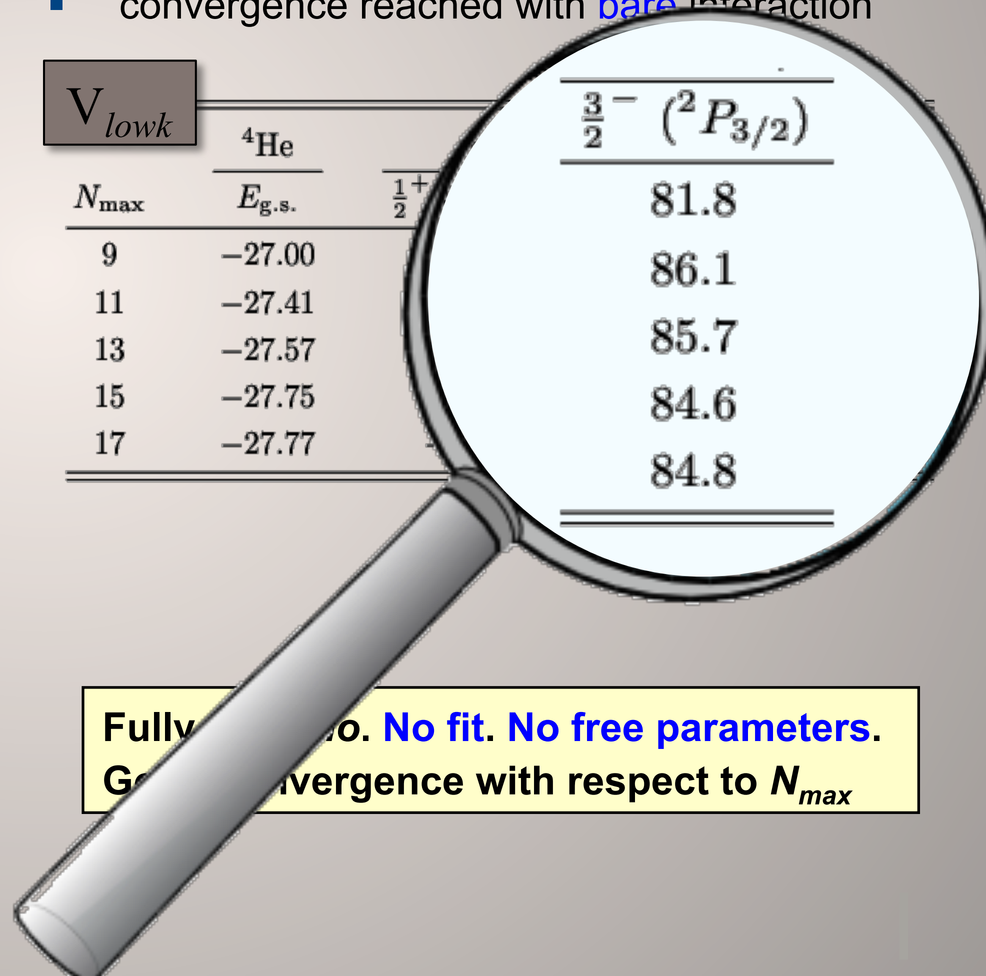
Convergence of the *ab initio* NCSM/RGM: n-^{4}He phase shifts



n-^{4}He phase shifts: SRG-N^{3}LO, λ=2.02 fm^{-1}



- Similarity-renormalization-group (SRG) evolved chiral N^{3}LO NN interaction
- Low-momentum V_{lowk} NN potential
- convergence reached with bare interaction

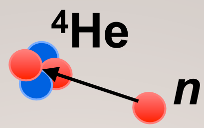


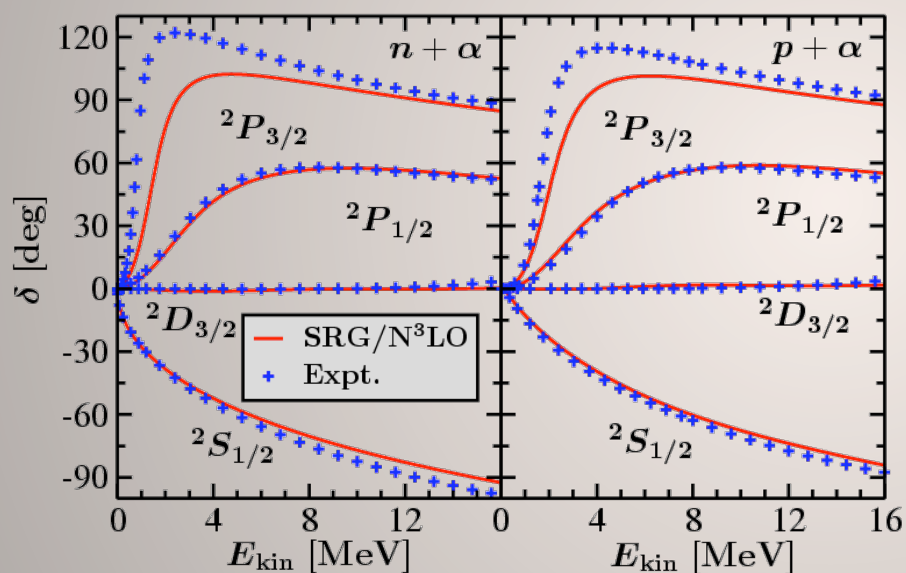
Fully ... io. No fit. No free parameters.
G... vergence with respect to N_{max}

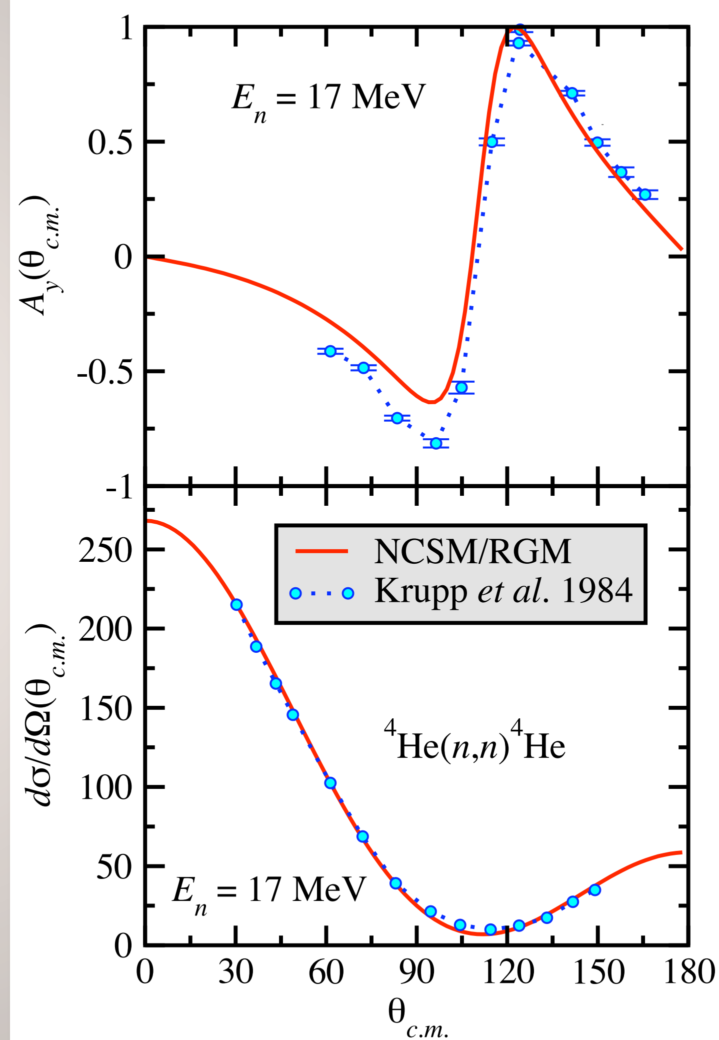
TRIUMF

The best system to start with: $n+{}^4$He, $p+{}^4$He

- NCSM/RGM calculations with
  - $N + {}^4$He(g.s., 0^+0)
  - SRG-N^{3}LO NN potential with Λ=2.02 fm^{-1}



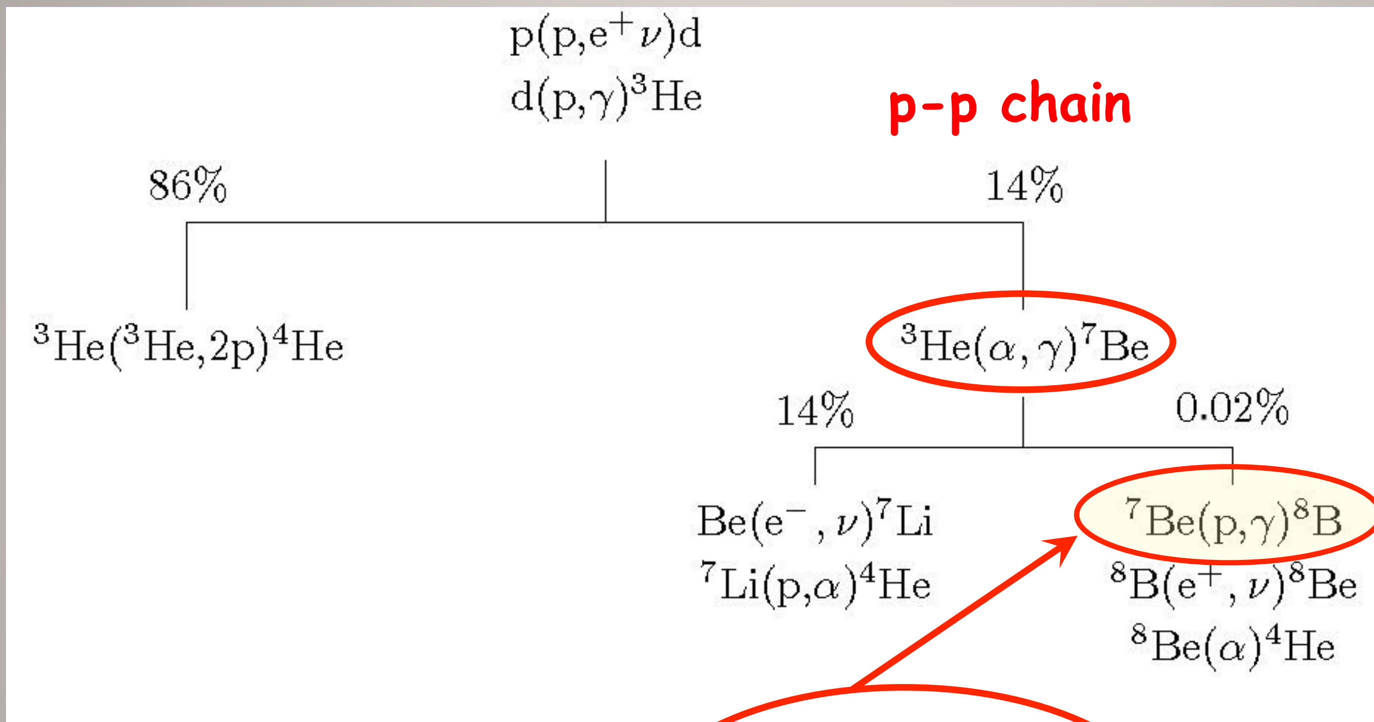




- Differential cross section and analyzing power @17 MeV neutron energy
  - Polarized neutron experiment at Karlsruhe

NNN missing: Good agreement only for energies beyond low-lying 3/2$^-$ resonance

Solar *p-p* chain



Solar neutrinos

E_ν < 15 MeV

Observed at SNO, Super K
- neutrino oscillations

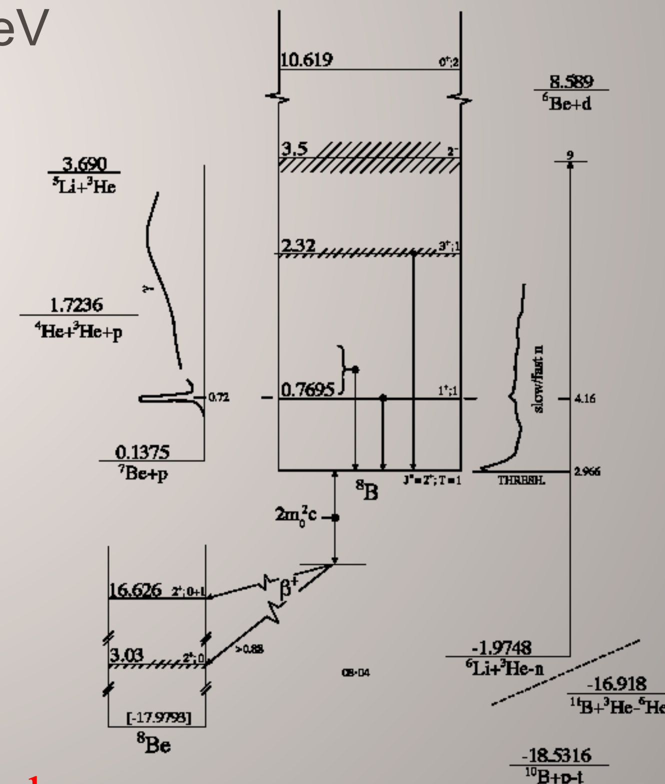
^{7}Be(p,γ)^{8}B S-factor

- S_{17} one of the main inputs in solar neutrino problem
  - Needs to be known with a precision better than 9 %
- Current evaluation has uncertainty >10%
  - Theory needed for extrapolation to ~ 10 keV

$$S(E) = E\sigma(E)\exp[2\pi\eta(E)]$$

$$\eta(E) = Z_{A-a}Z_a e^2 / \hbar v_{A-a,a}$$

$$\boxed{\left\langle {}^{8}\mathrm{B}_{\mathrm{g.s.}} \left| E1 \right| {}^{7}\mathrm{Be}_{\mathrm{g.s.}} + \mathrm{p} \right\rangle}$$

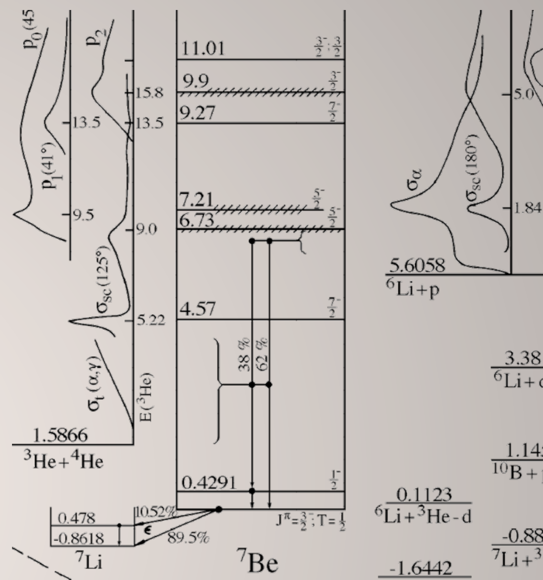


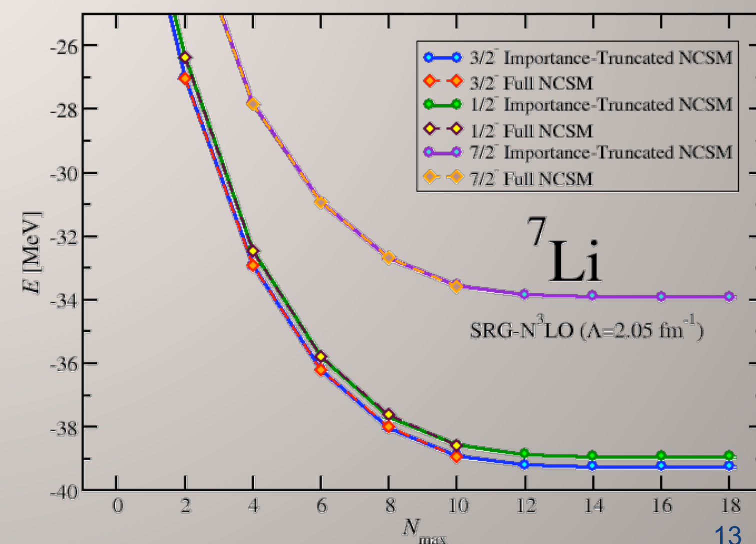
Many theoretical calculations in the past…
…now something new: Starting from first principles

Input: *NN* interaction, ^{7}Be eigenstates

- Similarity-Renormalization-Group (SRG) evolved chiral N^{3}LO *NN* interaction
  - Accurate
  - Soft: Evolution parameter Λ

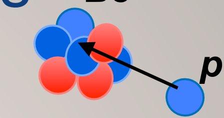
- ^{7}Be (^{7}Li)
  - NCSM up to N_{max}=10 possible
  - Importance Truncated NCSM up to N_{max}=18
    - R. Roth & P. N., PRL **99**, 092501 (2007)
  - large N_{max} needed for convergence of
    - Target eigenstates
    - Localized parts of integration kernels





p-^{7}Be scattering: Impact of 5/2^{-} states

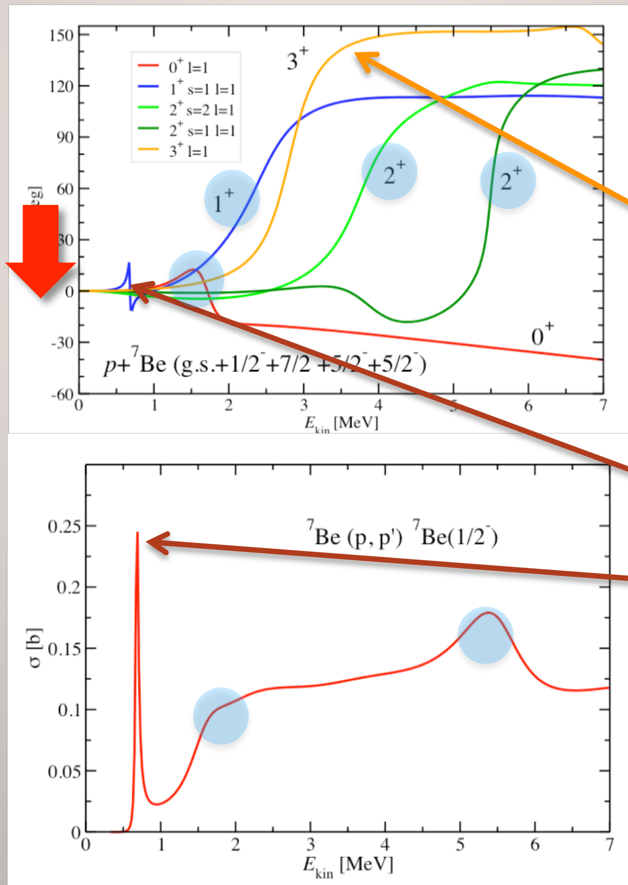
- NCSM/RGM coupled channel calculations
  - ^{7}Be states $3/2^{-}$, $1/2^{-}$, $7/2^{-}$, $5/2^{-}_{1}$, $5/2^{-}_{2}$
  - Soft NN potential (SRG-N^{3}LO with Λ = 1.86 fm^{-1})

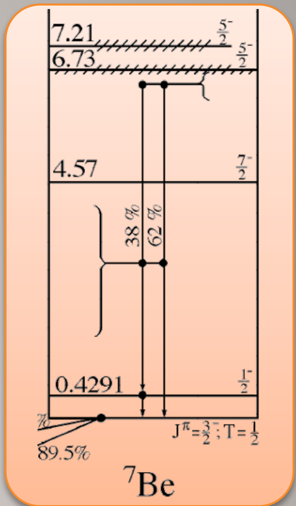


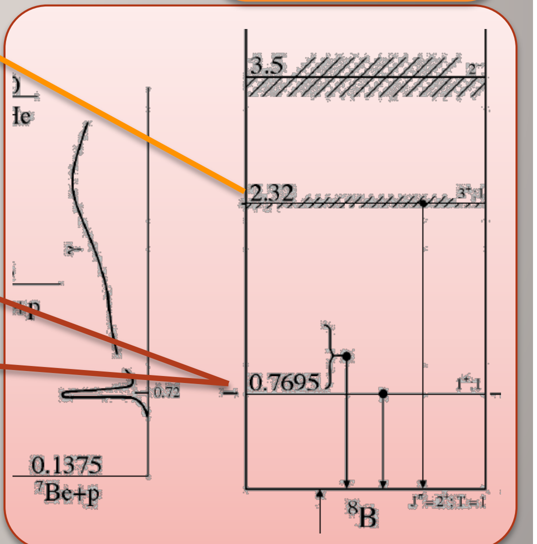
^{8}B 2^{+} g.s. **bound** by 135 keV (expt. bound by 137 keV)

New 0^{+}, 1^{+}, **two 2^{+}** resonances **predicted**

s =1 l =1 2^{+} clearly visible in (p,p') cross sections



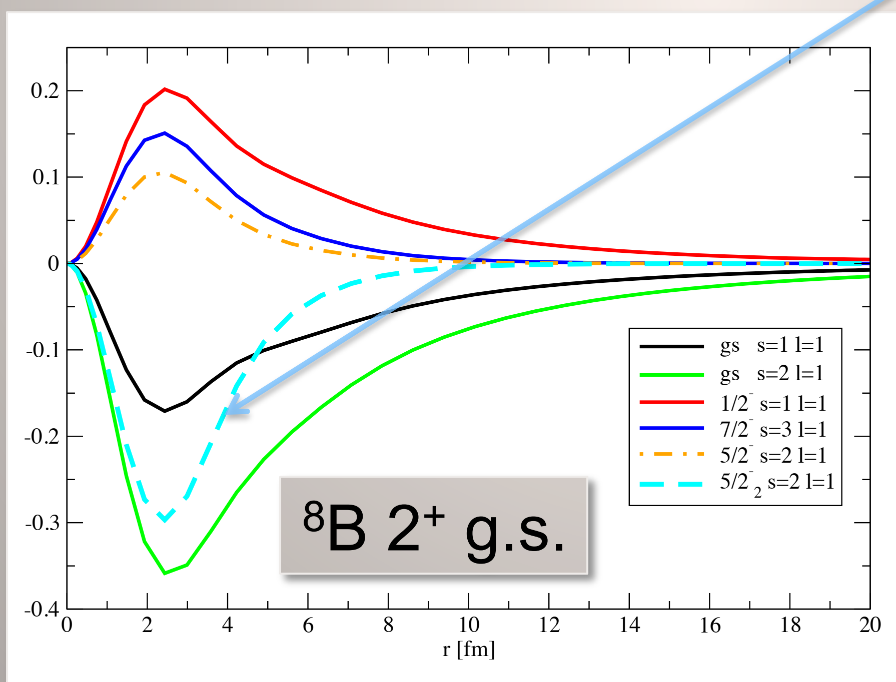


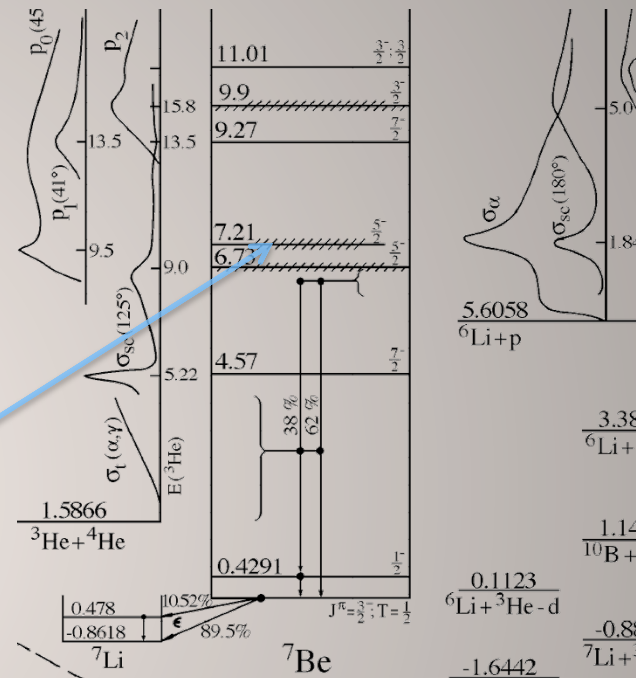


TRIUMF

Impact of higher excited states of ^{7}Be

- NCSM/RGM *p*-^{7}Be calculation with more excited states
  - $1/2^-$, $7/2^-$, $5/2^-_1$, $5/2^-_2$
- ^{8}B 2^+ g.s.
  - Large *P*-wave $5/2^-_2$ component

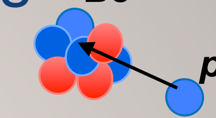


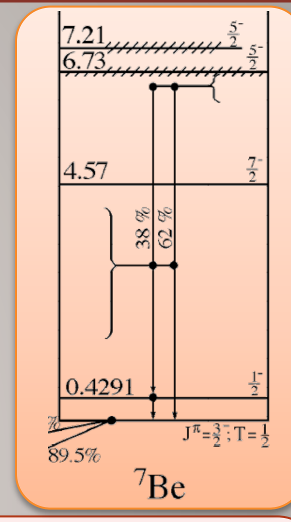


$5/2^-_2$ state of ^{7}Be should be included in ^{7}Be(p, γ)^{8}B calculations

$^7Be(p,\gamma)^8B$: Impact of $5/2^-$ states

- NCSM/RGM coupled channel calculations
  - ^{7}Be states $3/2^-$, $1/2^-$, $7/2^-$, $5/2^-_1$, $5/2^-_2$
  - Soft NN potential (SRG-N^{3}LO with Λ = 1.86 fm^{-1})

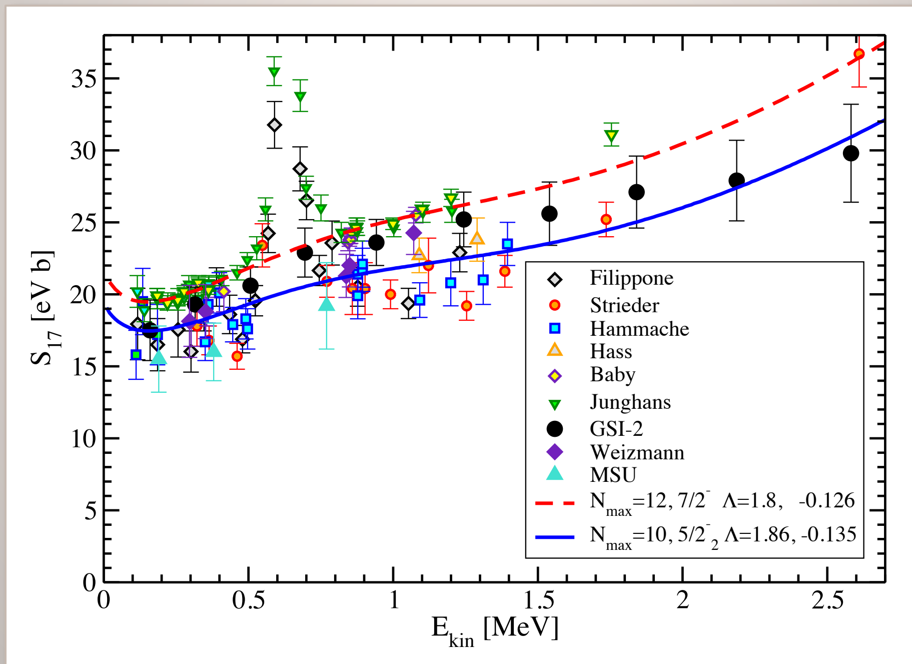


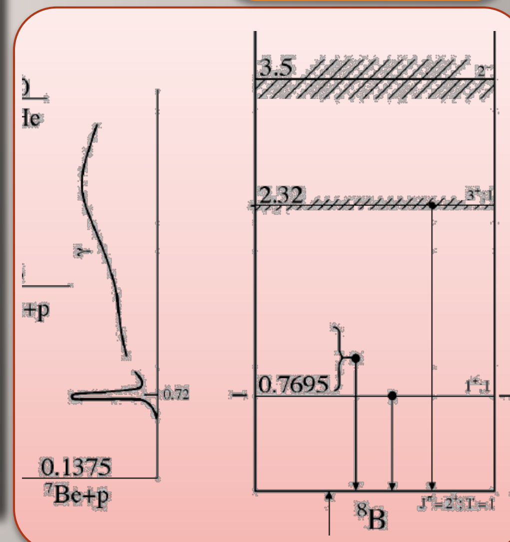


^{8}B 2^+ g.s. **bound** by 135 keV (expt. 137 keV)

S(0) ~ 20 eV b (preliminary)

Data evaluation: S(0)=20.8(2.1) eV b





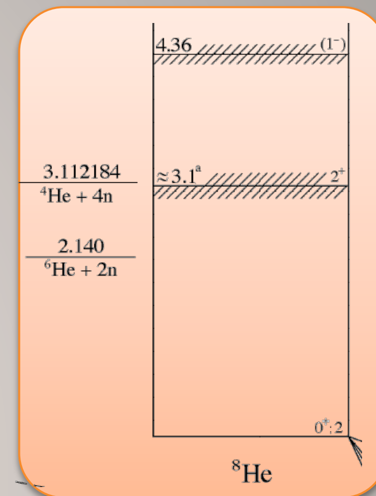
The $5/2^-_2$ state improves $^7Be(p,\gamma)^8B$ S-factor energy dependence for E>0.4 MeV

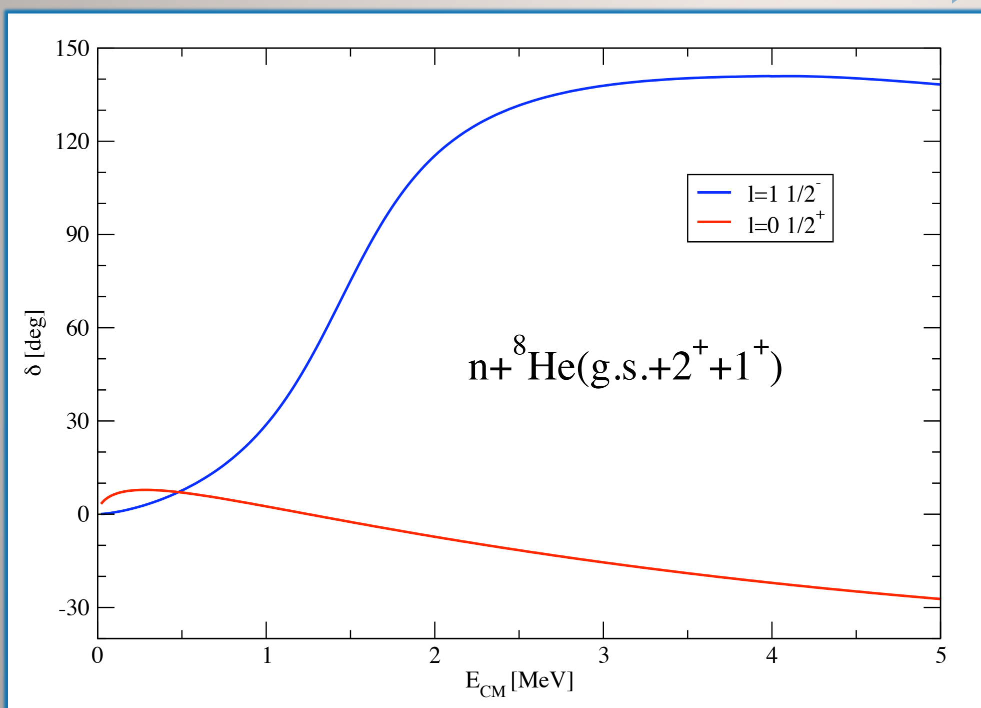
TRIUMF

Is ^{9}He bound? What is its ground state?

- NCSM/RGM calculation of n+^{8}He
  - SRG-N^{3}LO NN potential with Λ = 2.02 fm^{-1}
  - ^{8}He 0$^+$ g.s. and 2$^+$, 1$^+$ excited states included
  - Up to N_{max}= 8

exotic nuclei





No bound state

P-wave resonance at ~1.7 MeV
Expt. ~ 1.27(10) MeV (Bohlen *et al.*)

Weak attraction in the *S*-wave, a_0 ~ -2 fm
Expt. a_0~ -10 fm (Chen *et al.*)
a_0~ -3 fm (Al Falou, *et al.*)

Calculation suggests 1/2$^+$ g.s. ?
More ^{8}He excited states needed

TRIUMF

(A-2) (2)

The deuteron-projectile formalism: norm kernel

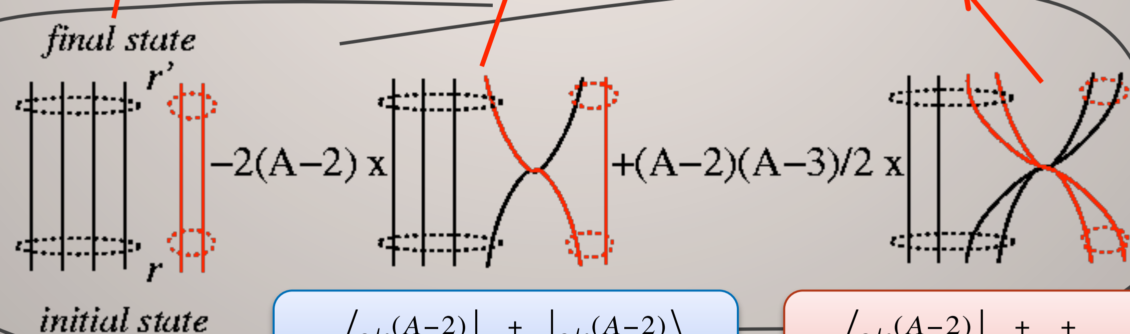
(1,…,A-2) (A-1,A) r' (1,…,A-2) (A-1,A) r

$$\left\langle \;(1,\dots,A\text{-}2)\,(A\text{-}1,A)\,r'\;\right| 1-\sum_{i=1}^{A-2}\sum_{k=A-1}^{A}\hat{P}_{ij}+\sum_{i<j=1}^{A-2}\hat{P}_{i,A}\hat{P}_{j,A-1}\left|\;(1,\dots,A\text{-}2)\,(A\text{-}1,A)\,r\;\right\rangle$$

$$N^{(A-2,2)}_{\mu\ell',\nu\ell}(r',r)=\boxed{\delta_{\mu\nu}\delta_{\ell'\ell}\frac{\delta(r'-r)}{r'r}}$$

$$\boxed{-2(A-2)\sum_{n'n}R_{n'\ell'}(r')\left\langle\Phi^{(A-2,2)JT}_{\mu n'\ell'}\right|P_{A-2,A-1}\left|\Phi^{(A-2,2)JT}_{\nu n\ell}\right\rangle R_{n\ell}(r)}$$

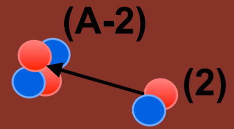
$$\boxed{+\frac{(A-2)(A-3)}{2}\sum_{n'n}R_{n'\ell'}(r')\left\langle\Phi^{(A-2,2)JT}_{\mu n'\ell'}\right|P_{A-2,A-1}P_{A-3,A}\left|\Phi^{(A-2,2)JT}_{\nu n\ell}\right\rangle R_{n\ell}(r)}$$

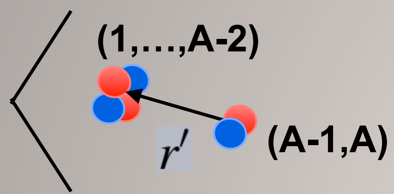


$${}_{\rm SD}\left\langle\psi^{(A-2)}_{\mu_1}\right|a^+a\left|\psi^{(A-2)}_{\nu_1}\right\rangle_{\rm SD}$$

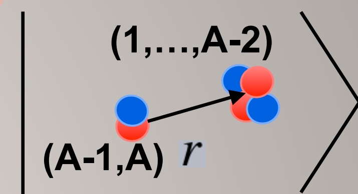
$${}_{\rm SD}\left\langle\psi^{(A-2)}_{\mu_1}\right|a^+a^+\,a\,a\left|\psi^{(A-2)}_{\nu_1}\right\rangle_{\rm SD}$$

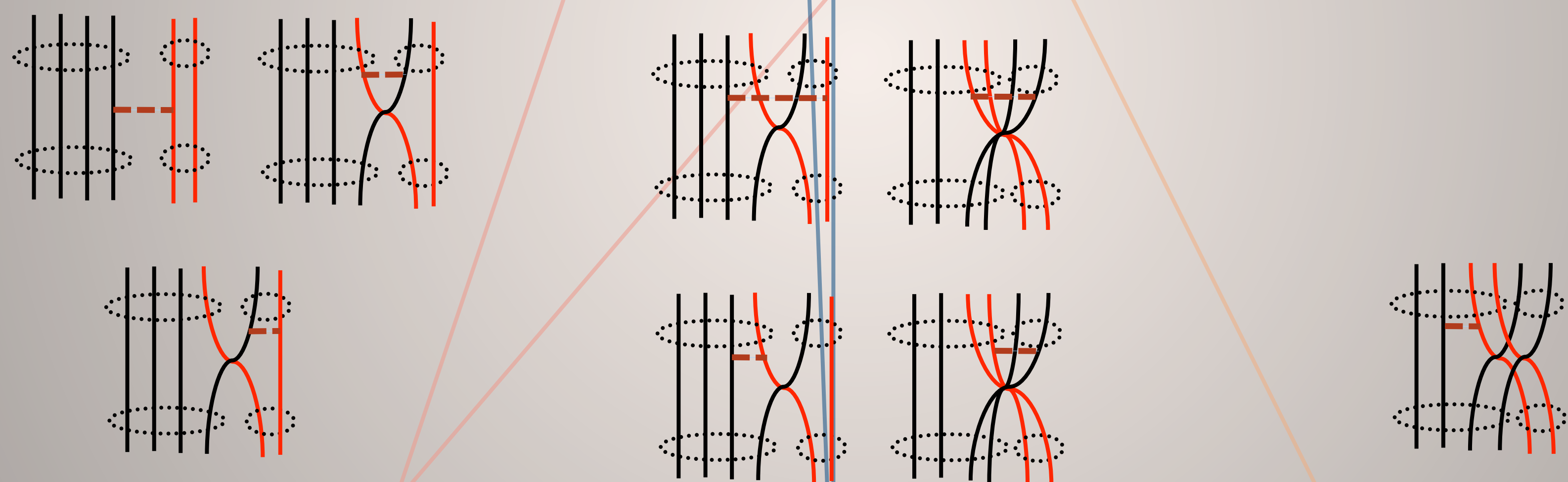
The deuteron projectile: Hamiltonian kernel





$$
\begin{aligned}
&2(A-2)V_{A-1,A-2}(1-P_{A-1,A-2})-2(A-2)V_{A,A-2}\hat{P}_{A-2,A-1}\\
&-2(A-2)(A-3)V_{A,A-3}(1-\hat{P}_{A,A-3})\hat{P}_{A-2,A-1}\\
&-2(A-2)(A-3)V_{A-1,A-3}\hat{P}_{A-2,A-1}\\
&+(A-2)(A-3)(A-4)V_{A-1,A-4}\hat{P}_{A-2,A}\hat{P}_{A-3,A-1}
\end{aligned}
$$





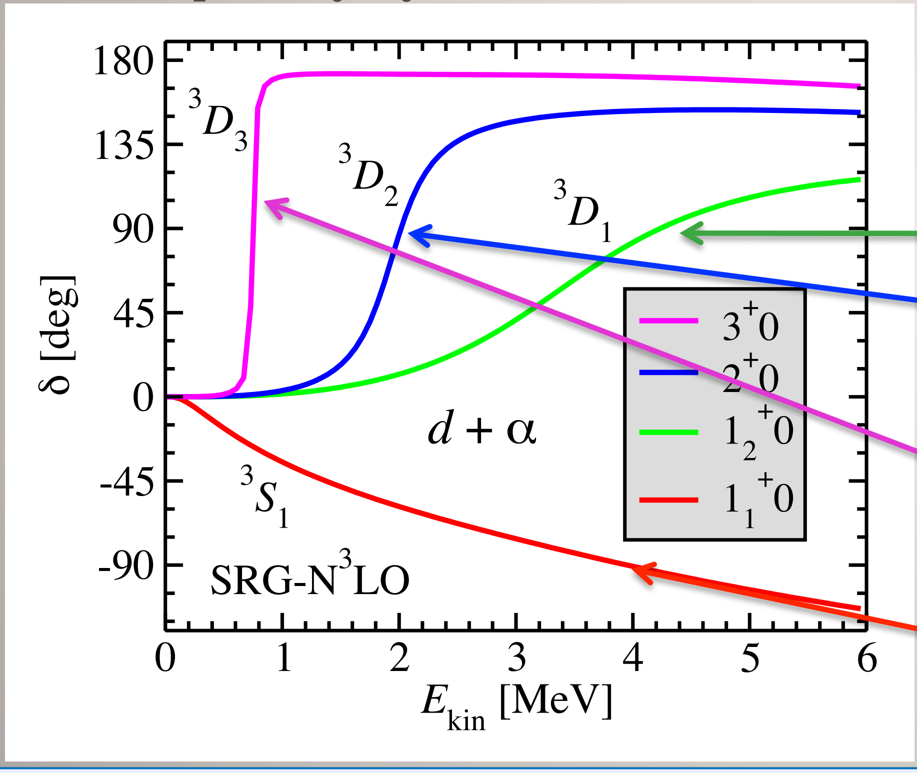
$$_{SD}\left\langle \psi_{\mu_1}^{(A-2)}\middle| a^+ a \middle| \psi_{\nu_1}^{(A-2)}\right\rangle_{SD}$$

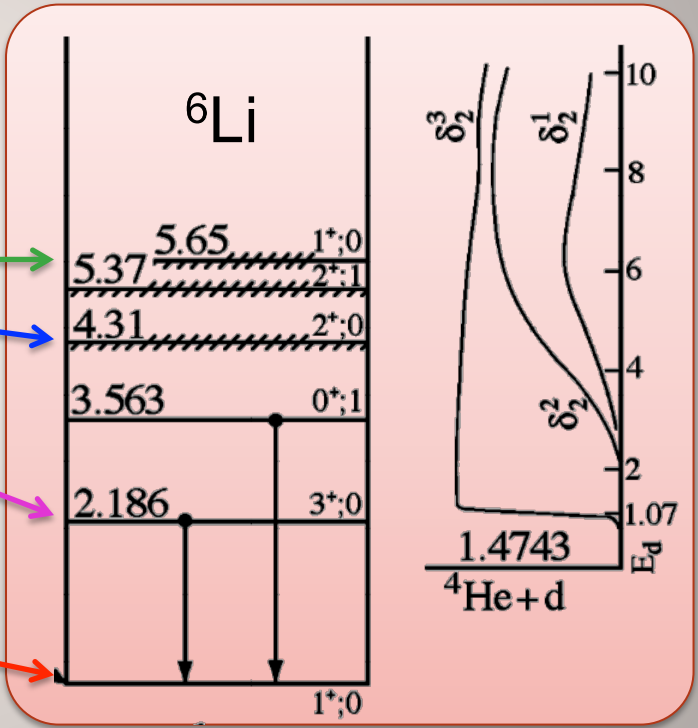
$$_{SD}\left\langle \psi_{\mu_1}^{(A-2)}\middle| a^+ a^+ a\, a \middle| \psi_{\nu_1}^{(A-2)}\right\rangle_{SD}$$

$$_{SD}\left\langle \psi_{\mu_1}^{(A-2)}\middle| a^+ a^+ a^+ a\, a\, a \middle| \psi_{\nu_1}^{(A-2)}\right\rangle_{SD}$$

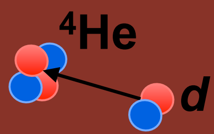
NCSM/RGM *ab initio* calculation of d-^{4}He scattering

- NCSM/RGM calculation with d + ^{4}He(g.s.) up to N_{max} = 12
  - SRG-N^{3}LO potential with Λ = 1.5 fm^{-1}
  - Deuteron breakup effects included by continuum discretized by pseudo states in $^3S_1$-$^3D_1$, $^3D_2$ and $^3D_3$-$^3G_3$ channels

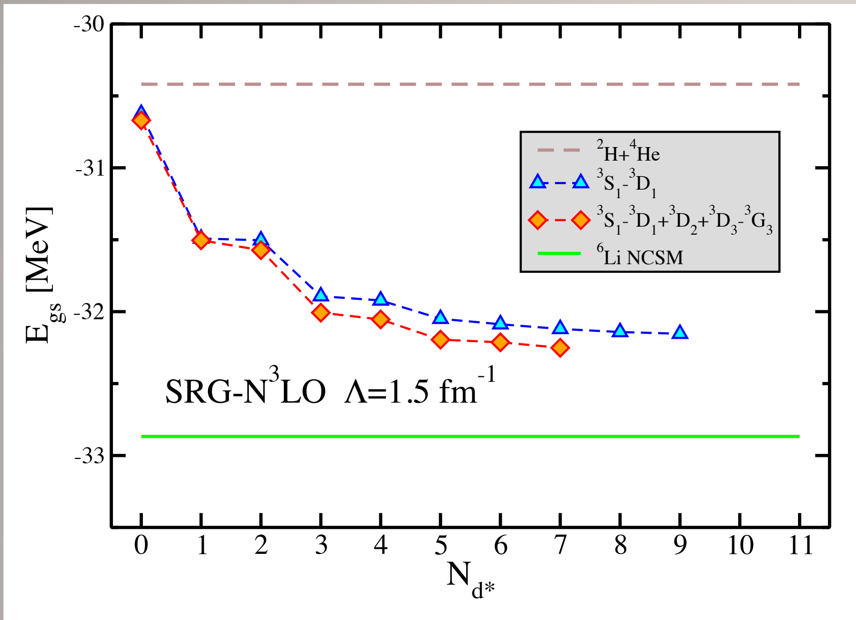


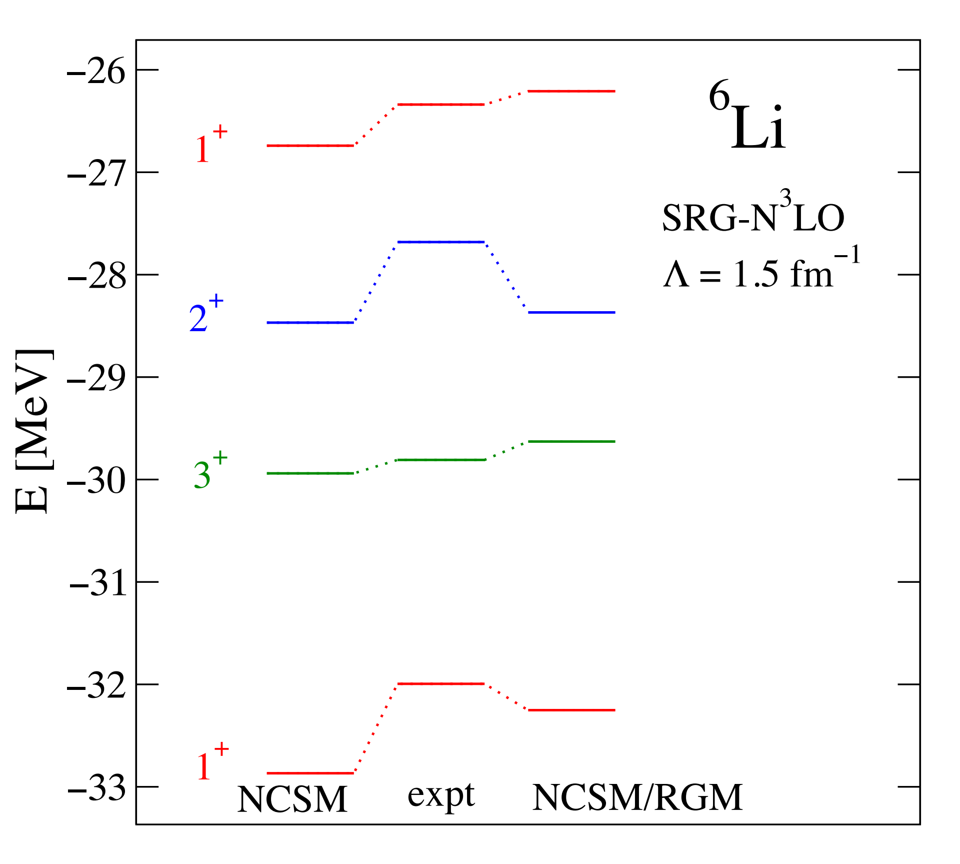


- The 1^+0 ground state bound by 1.9 MeV (expt. 1.47 MeV)
- Calculated T=0 resonances: 3^+, 2^+ and 1^+ in correct order close to expt. energies

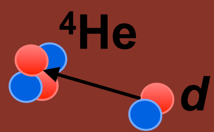


NCSM/RGM *ab initio* calculation of *d*-⁴He scattering

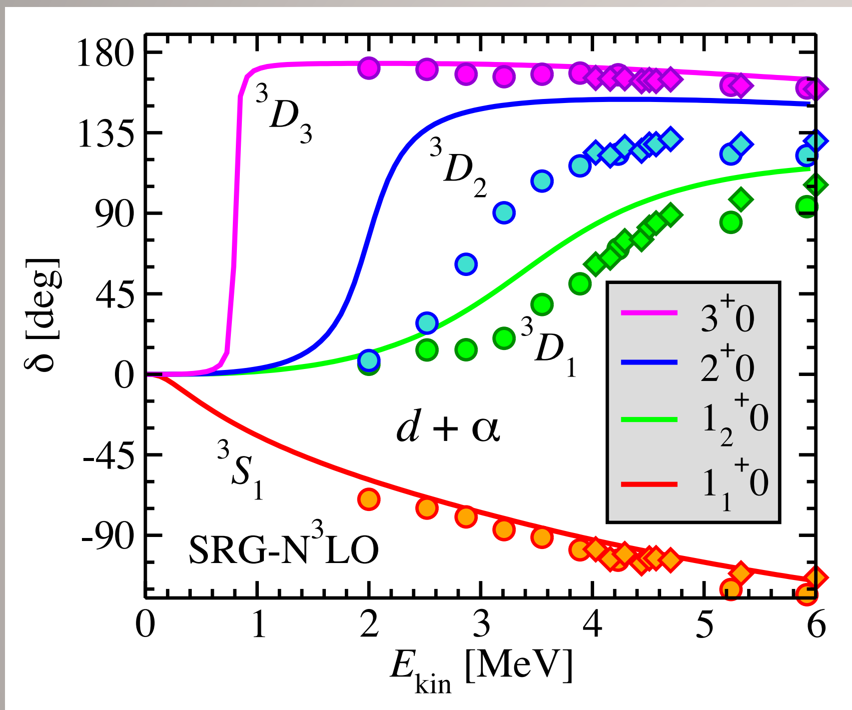


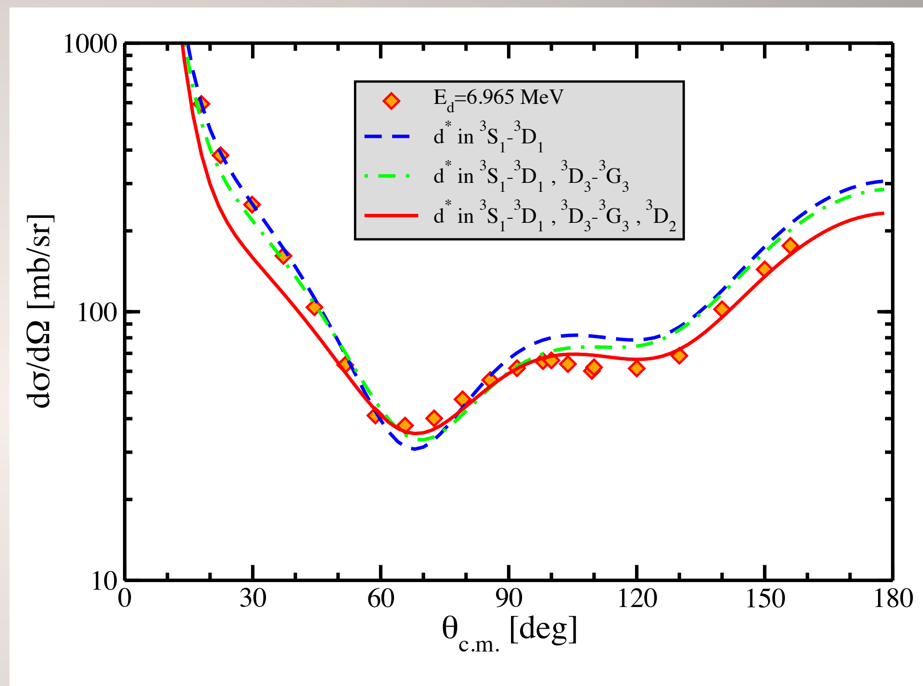


- The deuteron polarization and virtual break up must be taken into account
- NCSM/RGM calculates bound states as well as excited states…



NCSM/RGM *ab initio* calculation of *d*-^{4}He scattering

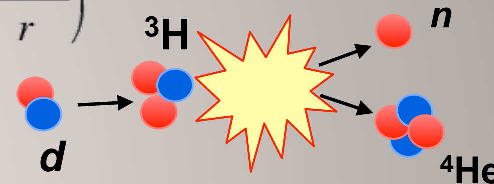


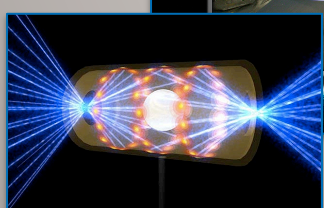


- NCSM/RGM a superior theory: Bound states, resonances, scattering
- NCSM efficiently accounts for many-nucleon correlations: **Coupling of the NCSM and the NCSM/RGM basis desirable**
- Scattering provides a strict test of NN and NNN forces

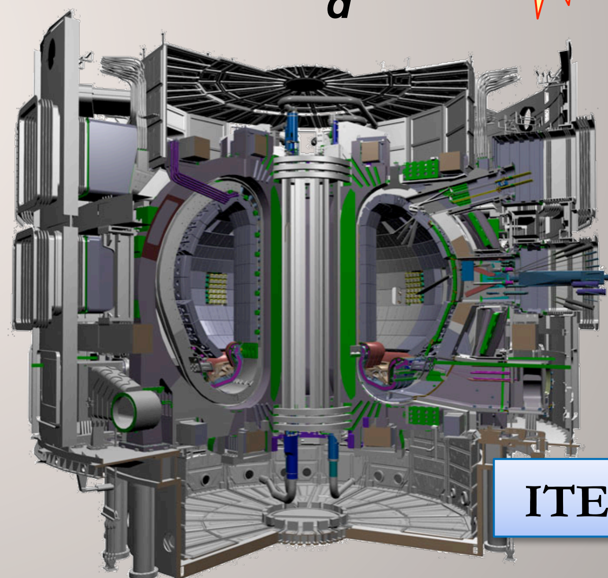
Toward the first *ab initio* calculation of the Deuterium-Tritium fusion

$$\int dr\, r^2 \begin{pmatrix} \left\langle \overset{r'}{n\;\alpha} \right| \hat{A}_1(H-E)\hat{A}_1 \left| \overset{r}{\alpha\;n} \right\rangle & \left\langle \overset{r'}{n\;\alpha} \right| \hat{A}_1(H-E)\hat{A}_2 \left| \overset{r}{{}^3\mathrm{H}\;d} \right\rangle \\ \left\langle \overset{r'}{d\;{}^3\mathrm{H}} \right| \hat{A}_2(H-E)\hat{A}_1 \left| \overset{r}{\alpha\;n} \right\rangle & \left\langle \overset{r'}{d\;{}^3\mathrm{H}} \right| \hat{A}_2(H-E)\hat{A}_2 \left| \overset{r}{{}^3\mathrm{H}\;d} \right\rangle \end{pmatrix} \begin{pmatrix} \frac{g_1(r)}{r} \\ \frac{g_2(r)}{r} \end{pmatrix} = 0$$





NIF

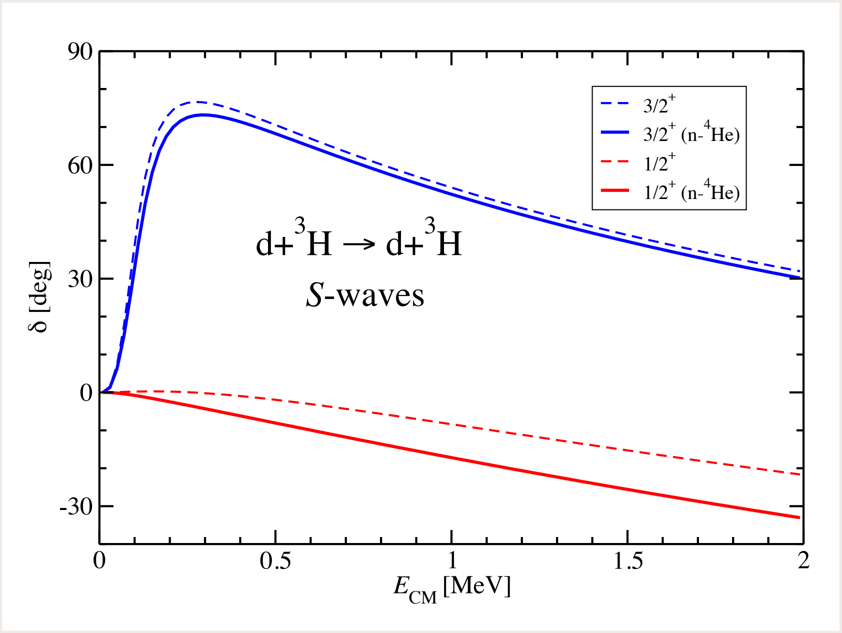


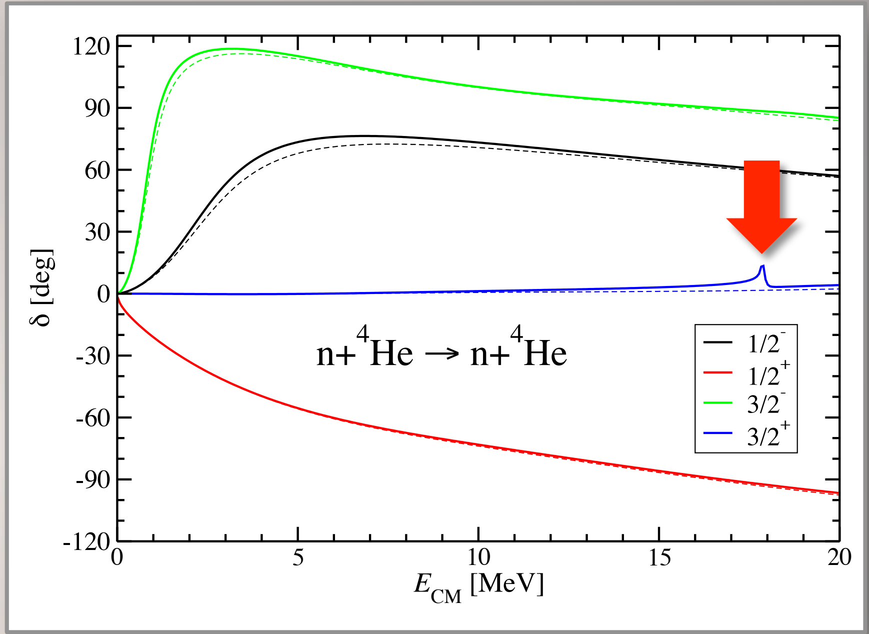
ITER

energy generation

TRIUMF

d+^{3}H and n+^{4}He elastic scattering: phase shifts



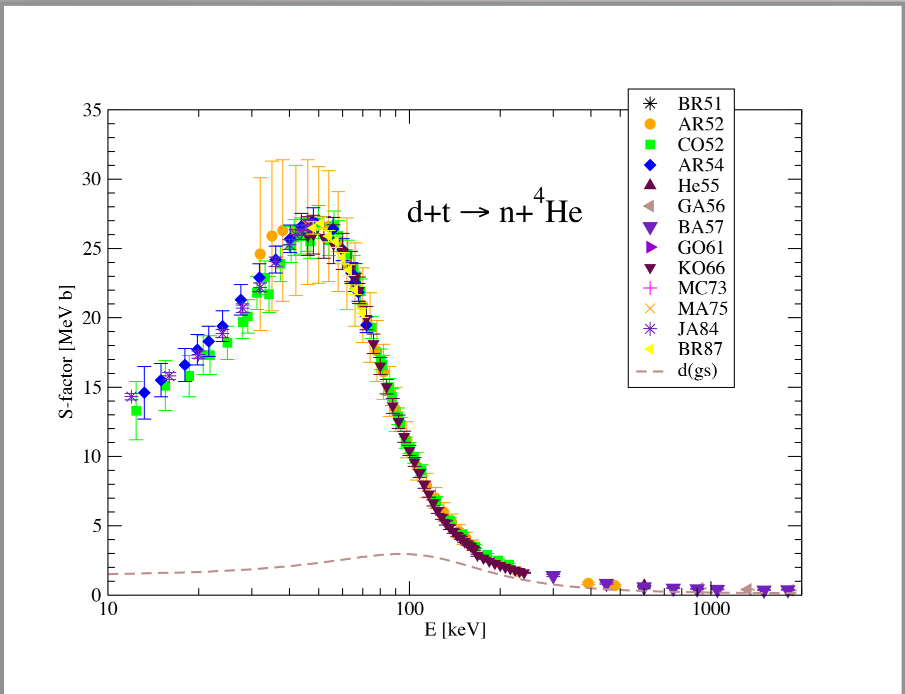


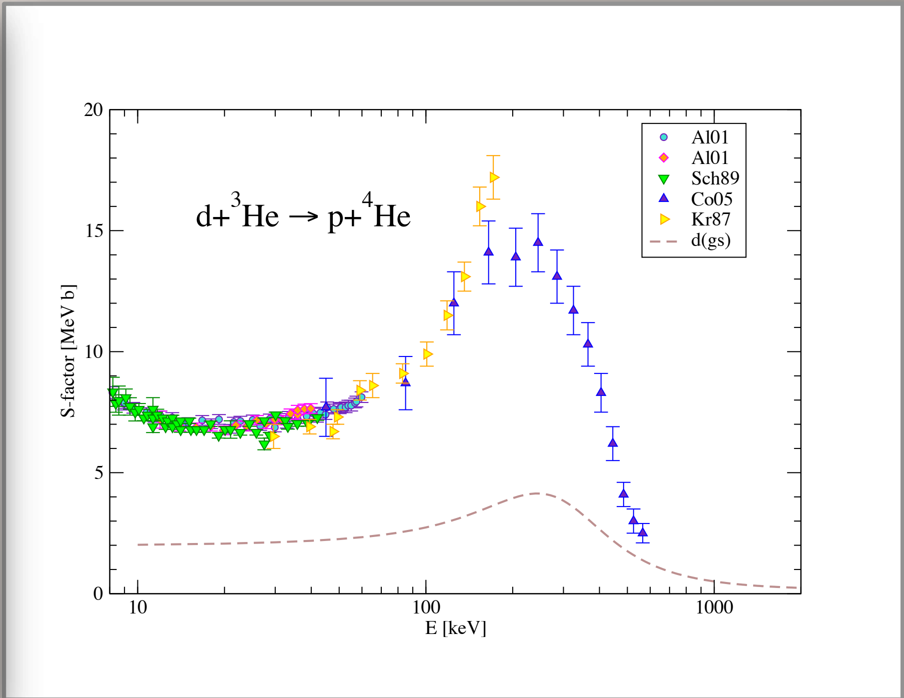
- d+^{3}H elastic phase shifts:
  - Resonance in the $^4S_{3/2}$ channel
  - Repulsive behavior in the $^2S_{1/2}$ channel ➔ Pauli principle

- n+^{4}He elastic phase shifts:
  - d+^{3}H channels produces slight increase of the P phase shifts
  - Appearance of resonance in the 3/2$^+$ D-wave, just above d-^{3}H threshold

The D-T fusion takes place through a transition of d+^{3}H is S-wave to n+^{4}He in D-wave

$^3H(d,n)^4He$ and $^3He(d,p)^4He$ cross sections



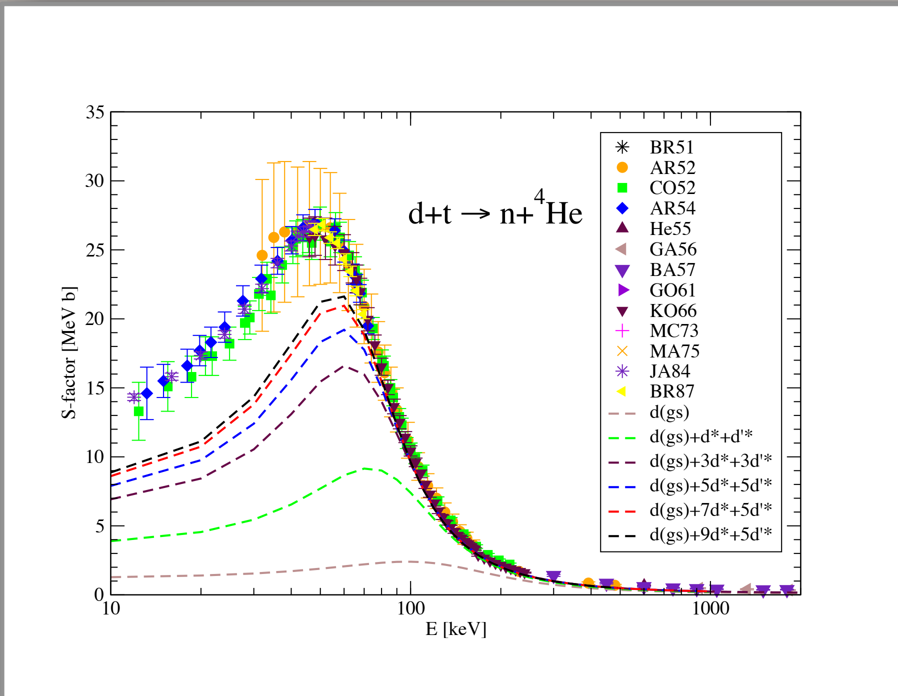


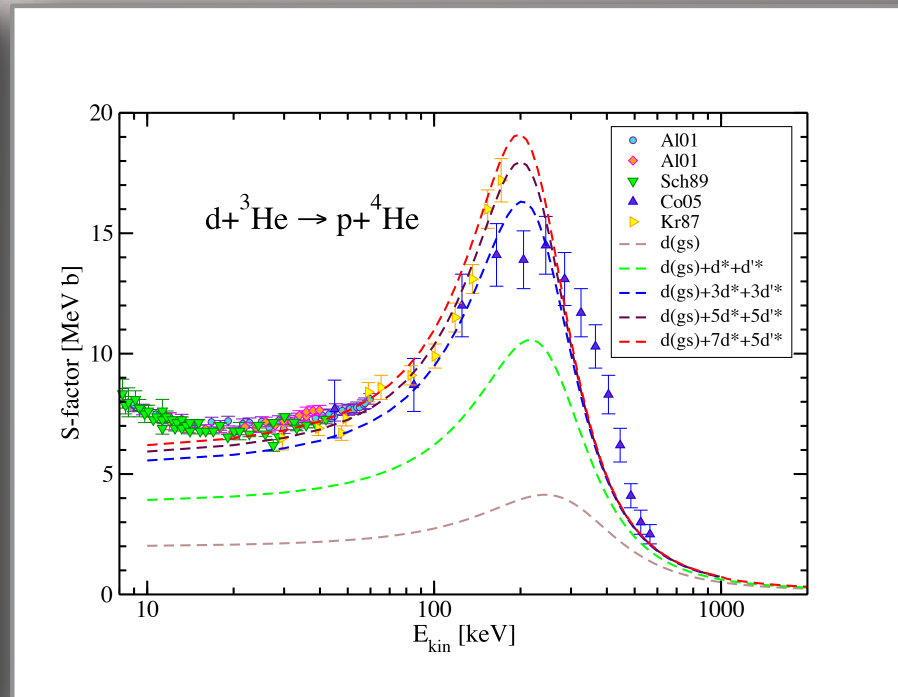
- The first results, still preliminary:
  - N_{max}= 13
  - SRG-N^{3}LO NN (Λ=1.5 fm^{-1}) potential
  - NNN interaction interaction effects for *A*=3,4,5 partly included by the choice of Λ
  - Only **g.s.** of *d*, ^{3}H, ^{4}He included above

$$S(E) = E\sigma(E)\exp\left(\frac{2\pi\ Z_1 Z_2 e^2}{\hbar\sqrt{2mE}}\right)$$

TRIUMF

$^{3}H(d,n)^{4}He$ and $^{3}He(d,p)^{4}He$ cross sections





- The cross section improves with the inclusion of virtual breakup of the deuteron
  - Deuteron weakly bound: easily gets polarized and easily breaks
  - These effects included below the breakup threshold with continuum discretized by excited deuteron pseudo-states

First *ab initio* results for *d*-T and *d*-^{3}He fusion:
Very promising, correct physics, can become competitive with fitted evaluations …

TRIUMF

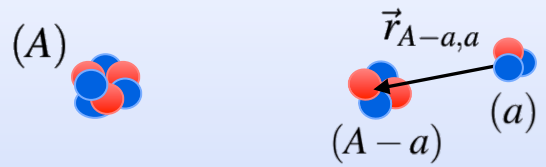
Conclusions and Outlook

- With the NCSM/RGM approach we are extending the *ab initio* effort to describe low-energy reactions and weakly-bound systems

- The first ^{7}Be(p,γ)^{8}B *ab initio* S-factor calculation
  - Both the bound and the scattering states from first principles
  - No fit
  - SRG-N^{3}LO *NN* potential selected to match closely the experimental threshold (Λ≈1.8~2 fm^{-1})
  - Prediction of new ^{8}B resonances

- New results with SRG-N^{3}LO *NN* potentials:
  - Initial results for ^{3}H(d,n)^{4}He & ^{3}He(d,p)^{4}He fusion and d-^{4}He scattering
  - First steps towards ^{3}He+^{4}He scattering
    - Wataru Horiuchi

- To do:
  - Inclusion of NNN force
  - Alpha clustering: ^{4}He projectile
  - NCSM with continuum (NCSMC)
  - Three-cluster NCSM/RGM and treatment of three-body continuum



$$\left|\Psi_A^J\right\rangle = \sum c_\lambda \left|A\lambda J\right\rangle + \sum \int d\vec{r}\, \varphi_\nu(\vec{r})\, \hat{\mathcal{A}}\, \Phi_{\nu\vec{r}}^{(A-a,a)}$$

$$\begin{pmatrix} H & h \\ h & \mathcal{H} \end{pmatrix} \begin{pmatrix} c \\ \varphi \end{pmatrix} = E \begin{pmatrix} 1 & g \\ g & \mathcal{N} \end{pmatrix} \begin{pmatrix} c \\ \varphi \end{pmatrix}$$

TRIUMF

Collaborators

Sofia Quaglioni (LLNL)

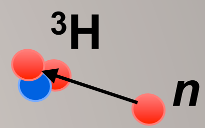
Robert Roth (TU Darmstadt)

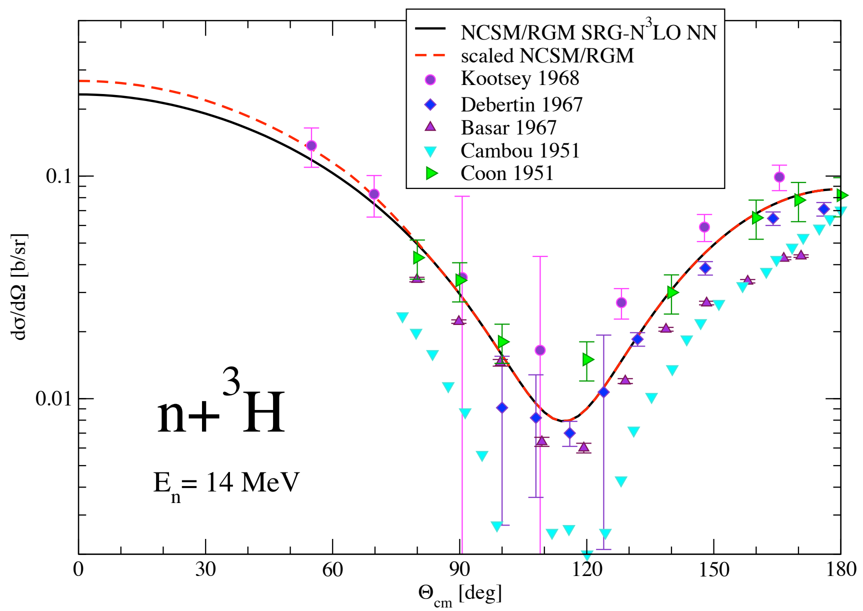
E. Jurgenson (LLNL), Dick Furnstahl (OSU)

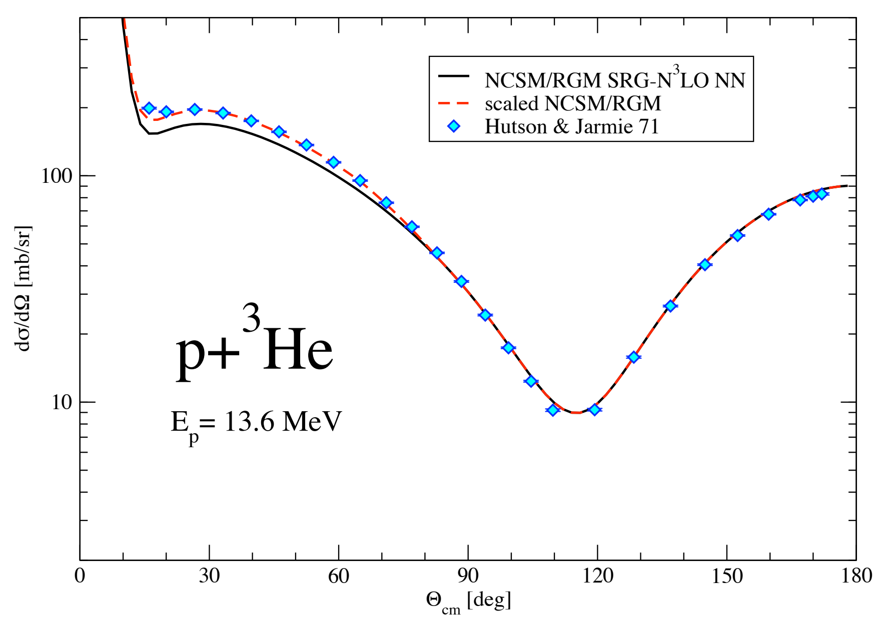
V. Gueorguiev (UC Merced), J. P. Vary (ISU),
W. E. Ormand (LLNL), A. Nogga (Julich)

Connection to the real world: neutron-triton elastic scattering at 14 MeV

- Important for the National Ignition Facility physics
  - deuteron-triton fusion generates 14 MeV neutrons
- Experimental situation confusing
- Good data for *p*+^{3}He elastic scattering



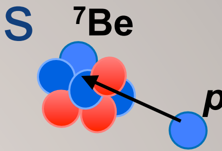


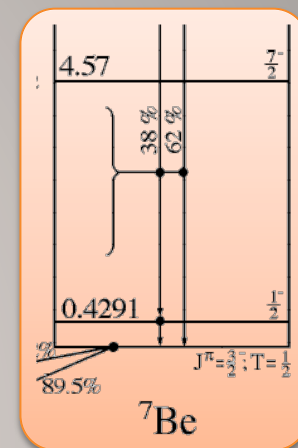


Use NCSM/RGM calculation to relate the two reactions and predict *n*+^{3}H cross section

p-^{7}Be scattering

- NCSM/RGM coupled channel calculations
  - ^{7}Be states $3/2^-$, $1/2^-$, $7/2^-$
  - Soft NN potential (SRG-N^{3}LO with Λ = 1.8 fm^{-1})



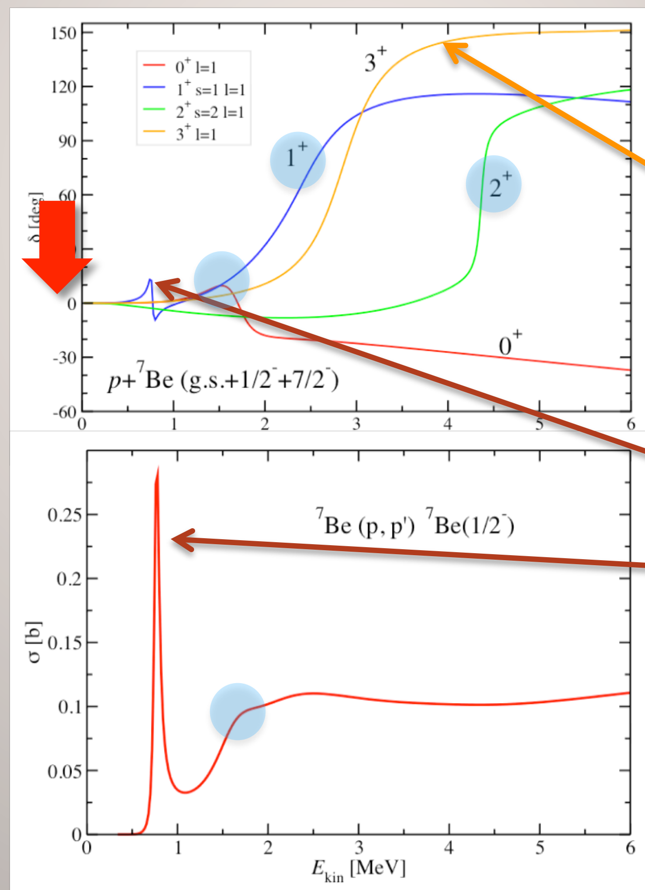


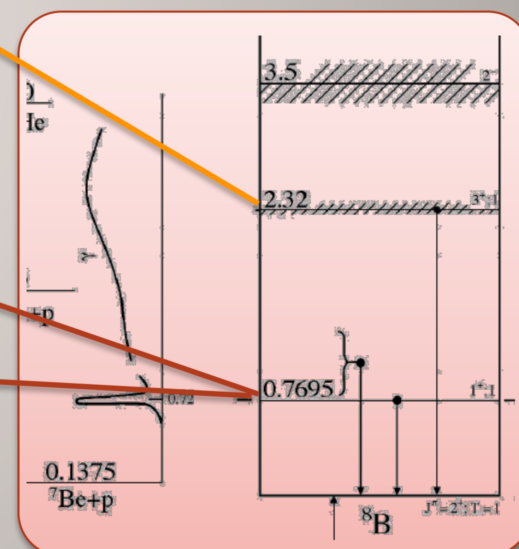
^{8}B 2$^+$ g.s. **bound** by 126 keV (expt. bound by 137 keV)

New 0$^+$, 1$^+$, 2$^+$ resonances **predicted**

Scattering length:
Expt: a_{02}= -7(3) fm
Calc: a_{02}= -10.2 fm
(Λ =2.02 fm^{-1})

P. N., R. Roth, S. Quaglioni, PRC **82**, 034609 (2010)

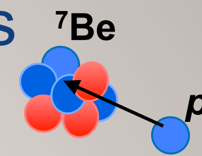


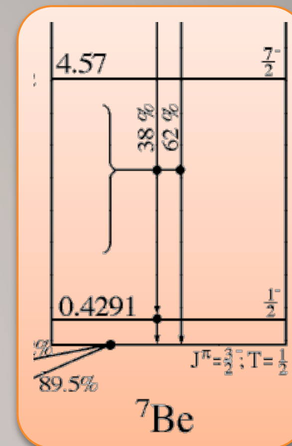


TRIUMF

$^{7}Be(p,\gamma)^{8}B$ radiative capture S-factor

- NCSM/RGM coupled channel calculations
  - ^{7}Be states $3/2^{-}$, $1/2^{-}$, $7/2^{-}$
  - Soft NN potential (SRG-N^{3}LO with Λ = 1.8 fm^{-1})

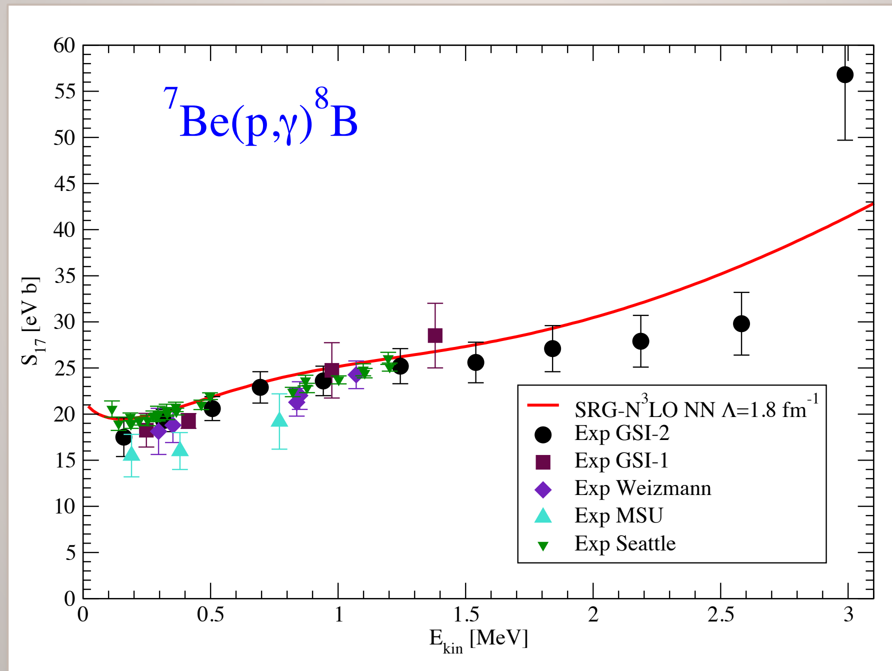


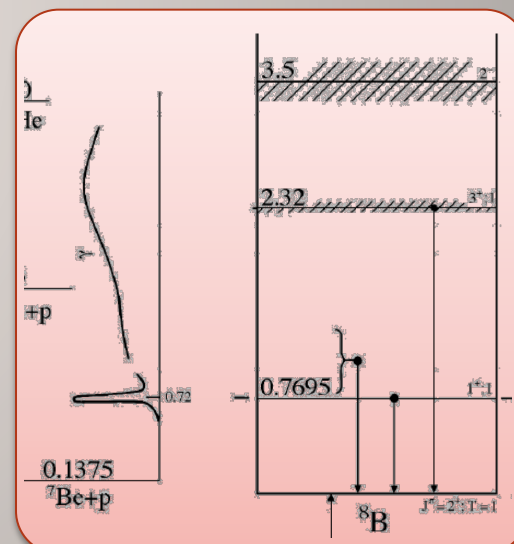


^{8}B 2^{+} g.s. **bound** by 126 keV

(expt. 137 keV)

S(0) ~ 21.5 eV b

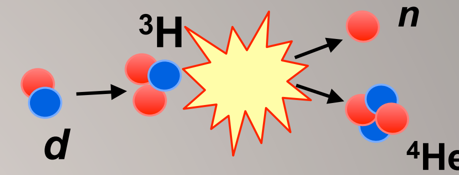




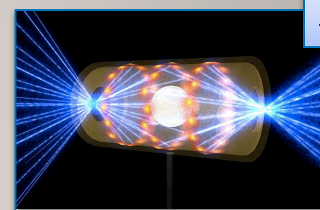
The first ever *ab initio* calculations of $^{7}Be(p,\gamma)^{8}B$ (still preliminary)

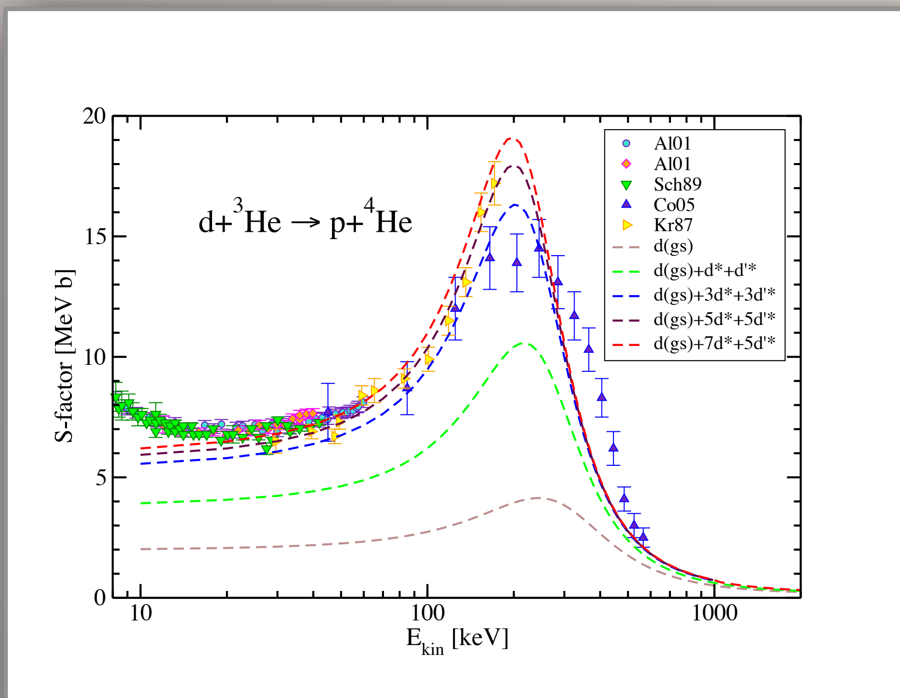
Toward the first *ab initio* calculation of the Deuterium-Tritium and *d*-^{3}He fusion

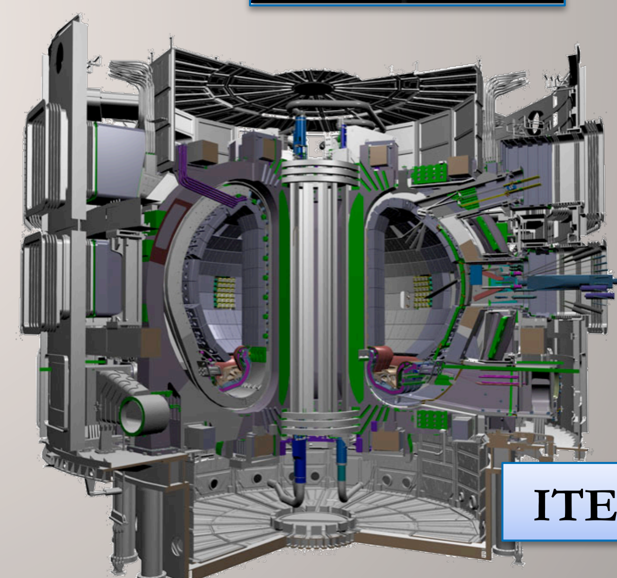
$$\int dr\, r^2 \begin{pmatrix} \left\langle {}^{r'}_{n\ \alpha} \middle| \hat{A}_1 (H-E) \hat{A}_1 \middle| {}^{r}_{\alpha\ n} \right\rangle & \left\langle {}^{r'}_{n\ \alpha} \middle| \hat{A}_1 (H-E) \hat{A}_2 \middle| {}^{r}_{{}^3\mathrm{H}\ d} \right\rangle \\ \left\langle {}^{r'}_{d\ {}^3\mathrm{H}} \middle| \hat{A}_2 (H-E) \hat{A}_1 \middle| {}^{r}_{\alpha\ n} \right\rangle & \left\langle {}^{r'}_{d\ {}^3\mathrm{H}} \middle| \hat{A}_2 (H-E) \hat{A}_2 \middle| {}^{r}_{{}^3\mathrm{H}\ d} \right\rangle \end{pmatrix} \begin{pmatrix} \frac{g_1(r)}{r} \\ \frac{g_2(r)}{r} \end{pmatrix} = 0$$



NIF







ITER

Predictive theory useful

Low energy: Electron screening problem. Resonance energy: Shape of the peak

P. Navratil et al., arXiv:1009.3965