



# Thermal Equilibrium, Entropy Injection and a Cross-section

## 1 Decoupled Species Revisited

*This part is a rerun of the last bit of Tutorial 1. Please skip this part if you have already finished tutorial 1.*

Suppose a massless relativistic particle species  $V$  in equilibrium with the cosmological plasma at time  $t_{fo}$  suddenly stops interacting at all with itself or anything else. After this happens, the expansion of spacetime will dilute the number density of  $V$  particles as  $a^{-3}$  energy of each  $V$  particle will redshift as  $a^{-1}$ .

- What is the contribution of  $V$  to the energy density of the Universe today?
- Rewrite this result in terms of temperatures and  $g_{*s}$  factors assuming entropy conservation after  $t_{fo}$ .
- One can show that the distribution function for  $V$  particles tracks the equilibrium form, but with a different effective temperature for the  $V$  particles. What is this effective temperature today in terms of the photon temperature and the number of effective degrees of freedom  $g_*$ ?  
*Hint:  $\rho(T) = \pi^2 g_*(T) T^4 / 30$  for this distribution function.*

## 2 Entropy Injection

Consider a Universe consisting of a thermal plasma of light particles together with a heavy particle  $\psi$  of mass  $M$  and density  $n_\psi$ . Suppose  $\psi$  is long-lived, and decays instantaneously at time  $t_\psi$  when the radiation temperature is  $T_\psi \ll M$ .

- a) Using energy conservation, compute the radiation temperature of the plasma immediately after the decay in terms of  $T_\psi$  and  $n_\psi(t_\psi)$ . You may assume that  $g_*$  remains constant.
- b) What does this do to the entropy density of the Universe?
- c) Suppose we also have a decoupled DM species  $\chi$  with freeze out temperature  $T_{fo}$ , and  $T_\psi$  is much smaller than  $T_{fo}$  and  $m_\chi$ , and whose contribution to the energy density is negligible until long after  $t_\psi$ . If the temperature of the plasma after the decay is also below  $T_{fo}$ , what is the relic density of  $\chi$  relative to what it would have been if  $n_\psi(t_\psi) = 0$ ? You should assume that no  $\chi$  particles are produced by the  $\psi$  decays.
- d) If the temperature after the decay is much larger than  $T_{fo}$  and  $m_\chi$ , what is the  $\chi$  density today compared to what it would have been with  $n_\psi(t_\psi) = 0$ ?  
*Hint: if  $\chi$  thermalizes, any initial relic density reaches the thermal value.*

### 3 A Cross-section

Consider the following interaction lagrangian between dark matter fermion  $\chi$ , a scalar field  $\phi$  and some other fermion  $f$

$$\mathcal{L} \supset \lambda \phi \bar{f} \chi + \text{h.c.}$$

Assuming that  $m_f = 0$  compute the averaged and spin-summed (amplitude)<sup>2</sup> for the following diagram in the limit  $m_\phi^2 \gg m_\chi^2$ :

