Lecture Note #9: Axions and Dark Matter

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Axions are very light pseudoscalar particles that are assumed to interact only very weakly with the SM at low energies. They can arise naturally as the pseudo Nambu-Goldstone bosons of a spontaneously broken approximate global symmetry. Coherent oscillations of an axion field redshift like matter, and as such, axions can be a candidate for dark matter. In this note we describe a specific type of axion motivated by a puzzle in QCD, we discuss its cosmological properties, and we describe some of the experimental limits on such axions.

1 Axions in QCD

An excellent reason to expect the existence of at least one axion field is the *strong CP problem* of QCD [1]. Adding an axion field to the SM can fix this problem in a very elegant way [2, 3]. We begin by describing the problem, introducing a QCD axion to fix it, and working out the effective couplings of the new field to matter at low energy.

1.1 The Strong CP Problem

As a warm up to formulating the strong CP problem, let us first consider "pure" QCD with no quarks. At the renormalizable level, there are two terms that can be written in the Lagrangian (before gauge fixing):

$$\mathscr{L} = -\frac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu} + \left(\frac{\alpha_s}{8\pi}\right)\Theta\,\widetilde{G}^a_{\mu\nu}G^{a\,\mu\nu}\ , \tag{1}$$

where $\alpha_s = g_s^2/4\pi$ is the strong coupling, and

$$\widetilde{G}^a_{\mu\nu} = \frac{1}{2}\,\epsilon_{\mu\nu\rho\sigma}\,G^{a\,\rho\sigma}\ . \tag{2}$$

The second term in Eq. (1) is called the *theta term*, where the corresponding parameter $\Theta \in [-\pi, \pi]$ is finite in its extent.

After some fiddling, one can show that $G\tilde{G} = \partial_\mu K^\mu$ for some four vector K^μ that is a function of the gluon fields [1]. As a result, it would seem that we could reduce the theta term to a boundary term in the action ($S = \int d^4x\,\mathscr{L}$), and removing it once and for all. However, this turns out not to be possible in non-Abelian gauge theories for a technical reason related to the fact that such theories have many degenerate vacua that are distinguished by their relative behaviour at the spacetime boundary. Each distinct physical vacuum state can be labelled by a real number $\Theta \in [-\pi, \pi]$, and the net result of including this fact in the quantum formulation of the theory is to add a theta term with the corresponding value of Θ to the Lagrangian.

Let us now add quarks to get a more realistic QCD theory. The Lagrangian with one $SU(3)_c$-triplet quark flavour ψ is

$$
\begin{aligned}
\mathscr{L} &= -\frac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu} + \left(\frac{\alpha_s}{8\pi}\right)\Theta\,\widetilde{G}^a_{\mu\nu}G^{a\,\mu\nu} \\
&\quad +\bar{\psi}_L i\gamma^\mu D_\mu\psi_L + \bar{\psi}_R i\gamma^\mu D_\mu\psi_R - (m\bar{\psi}_L\psi_R + m^*\bar{\psi}_R\psi_L)\ ,
\end{aligned}
\tag{3}
$$

where D_μ is the covariant derivative and we have split the Dirac fermion ψ into its chiral components. This should look familiar, except for maybe the complex mass parameter m.

To interpret the m parameter as a mass, we would like to make it real. We can seemingly achieve this by just redefining our field variables:[1]

$$\psi_L \to e^{i\phi_L}\psi_L\ , \tag{4}$$
$$\psi_R \to e^{i\phi_R}\psi_R\ . \tag{5}$$

If the phase in the original m parameter is

$$m = |m|e^{i\phi}\ , \tag{6}$$

choosing $(\phi_L - \phi_R) = \phi$ should will the mass parameter in the transformed Lagrangian real. However, this is not the only result of the change of variables in the quantum version of the theory. When quantum effects are included, one finds that under the transformations of Eqs. (4,5), we also end up with

$$\Theta \to \Theta + (\phi_L - \phi_R)\ . \tag{7}$$

When there are multiple quark flavours with a quark mass matrix M_q, this generalizes to

$$\Theta \to \bar{\Theta} = \Theta + Arg(\det M_q)\ . \tag{8}$$

In both cases, not all the phases can be removed entirely from the theory.

With both quarks and gluons, the presence of a non-zero theta term has an important physical implication – P and CP violation. These are already present in the SM by way of the weak interactions, but the new CP violation from a non-zero theta term is particularly dangerous. It can give rise to a permanent electric dipole moment (EDM) for the neutron. Searches for a neutron EDM have not found anything so far, and they imply that

$$|\bar{\Theta}| \lesssim 10^{-11}\ . \tag{9}$$

The strong CP problem is that do not know why $\bar{\Theta}$ should be so numerically small when it can take values in a much wider range.

[1] Changing field variables is just like changing the generalized variables in the Lagrangian in classical mechanics. It should have no physical effect

1.2 An Axion Solution

The QCD axion can solve this puzzle in a very nice way. Suppose there exists a new pseudoscalar field $a(x)$ that couples to the SM according to

$$- \mathscr{L} \supset \frac{\alpha_s}{8\pi}\frac{a}{f_a}\, G^a_{\mu\nu}G^{a\,\mu\nu} \ , \tag{10}$$

with all other couplings involving only derivatives $(\partial_\mu a)$. It looks like the axion field has no mass term at all. However, when low-energy QCD effects are taken into account (for $\Lambda_{QCD} \ll f_a$) an effective potential for $a(x)$ is generated of the form

$$V(a) \simeq m_a^2 f_a^2 \left[1 - \cos\left(\bar{\Theta} + a/f_a\right)\right] \ . \tag{11}$$

This is minimized for

$$\langle a \rangle = -\bar{\Theta}\, f_a \ . \tag{12}$$

Expanding $\tilde{a} = \tilde{a} + \langle a \rangle$, we find that the vacuum expectation value (VEV) of a cancels off the primordial theta term to give $\Theta_{eff} = 0$. No more strong CP problem! The cost of this solution is that we are left with a new physical particle $\tilde{a}$ that we call the axion.

1.3 (QCD) Axion Mass and Couplings

Our next job is to figure out the observational implications of the axion. For this, we need to figure out its mass and couplings to the rest of the SM. The mass of the axion can be obtained from the same effective potential that generates its VEV, given in Eq. (11). Expanding this result in $\tilde{a}/f \ll 1$, we find

$$V(\tilde{a}) = m_a^2 f_a^2 \left[1 - \left(1 - \frac{1}{2}\frac{\tilde{a}^2}{f_a^2} + \frac{1}{4}\frac{\tilde{a}^4}{f_a^4} - \ldots\right)\right] \tag{13}$$

$$= \frac{1}{2} m_a^2\, \tilde{a}^2 - \frac{1}{4}\frac{m_a^2}{f_a^2}\, \tilde{a}^4 + \ldots \tag{14}$$

and so the suggestively-named parameter m_a corresponds to the physical axion mass. In terms of QCD quantities, it can be shown that [2]

$$m_a = K\left(\frac{z}{1+z^2}\right)\frac{m_\pi f_\pi}{f_a} \simeq \text{eV}\left(\frac{10^7\ \text{GeV}}{f_a}\right) \ , \tag{15}$$

where K is a number of order unity, $z = m_u/m_d \simeq 0.48$, $m_\pi \simeq 135$ MeV, and $f_\pi \simeq 130$ MeV.

Axions couple to the SM with generic strength $1/f_a$, although the specific couplings depend on where the axion comes from. Besides the gluon coupling given above, the other possible terms relevant for axion physics at low energies are [3]

$$- \mathscr{L}_a = \frac{\alpha}{8\pi} C_{a\gamma\gamma}\frac{a}{f_a}\, a\, F_{\mu\nu}\widetilde{F}^{\mu\nu} + \frac{C_e}{f_a}\,(\partial_\mu a)\,\bar{e}\gamma^\mu\gamma^5 e + \frac{C_n}{f_a}\,(\partial_\mu a)\,\bar{n}\gamma^\mu\gamma^5 n \ , \tag{16}$$

where $F_{\mu\nu}$ is the photon field strength, e is the electron field, n denotes a nucleon field ($n = n, p$), and $C_{a\gamma\gamma}$, C_e, and C_n are of order unity.

The photon coupling in Eq. (16) allows the massive axion to decay to pairs of photons. The decay rate is

$$\Gamma_{a\to\gamma\gamma} \simeq \frac{1}{64\pi}\left(\frac{\alpha}{8\pi}\right)^2 C_{a\gamma\gamma}^2 \frac{m_a^3}{f_a^2} \tag{17}$$

$$\simeq (10^{-24}\,\mathrm{s}^{-1})\, C_{a\gamma\gamma}^2 \left(\frac{m_a}{1\,\mathrm{eV}}\right)^5 , \tag{18}$$

This is much longer than the age of the Universe for $m_a \lesssim 25\,\mathrm{eV}$. Despite their very low mass, metastable axions can be a viable candidate or cold dark matter.

2 Axions in the Early Universe

The cosmological evolution of the QCD axion is closely related to the cosmological evolution of QCD itself [4]. At high temperatures, $T \gg \Lambda_{QCD} \sim 300$ MeV, the QCD part of the cosmological plasma can be thought of as a soup of nearly free quarks and gluons. However, as T falls below Λ_{QCD}, colour-neutral bound states form and the proper degrees of freedom to use are baryons and mesons. This is called the QCD phase transition, and the number of relativistic degrees of freedom in the plasma changes drastically when it occurs.

With a QCD axion in the theory, a periodic potential of the form of Eq. (11) is also generated by the QCD phase transition. If the axion field doesn't happen to be sitting at $a = 0$ when this occurs, it will start rolling towards the minimum in the newly-formed potential.[2] The equation describing the rolling is approximately

$$\ddot{a} = -m_a^2 a , \tag{19}$$

which just gives oscillatory solutions. It can be shown that the energy density of such oscillations redshifts as a^{-3} (where this a is the scale factor), just like matter.

When the oscillation energy density is large enough, it can make up the dark matter. This is how axions can be the source of DM even though they are very light – the energy is carried in coherent axion field oscillations rather than the individual energies of a bunch of incoherent particles. In this sense, axion DM is highly non-thermal. The relic density of axions is approximately [4]

$$\Omega_a h^2 \simeq (0.1)\,(\Delta\Theta_i)^2 \left(\frac{10\mu\mathrm{eV}}{m_a}\right)^{7/6} \simeq (0.1)\,(\Delta\Theta_i)^2 \left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)^{7/6} , \tag{20}$$

where $\Delta\Theta_i = (\Theta + a_i/f)$ is the initial displacement of the axion field from the minimum, which we expect to be of order unity.

[2] The full story is a bit more complicated than this.

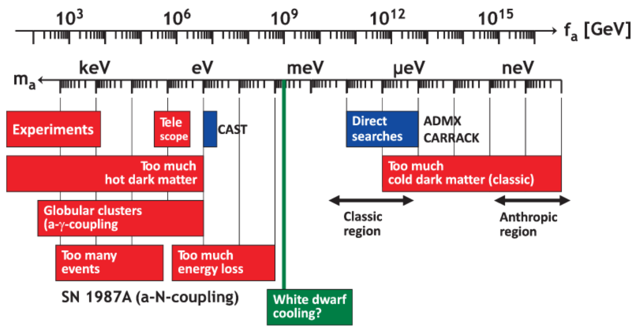

Figure 1: Astrophysical, cosmological, and direct bounds on axions. Figure from Ref. [5]

3 Limits on Axions

Axions can contribute to the DM density, but this can also be a problem if the density they generate is too large. For $\Delta\Theta_i \sim 1$, this forces the axion constant $f_a \lesssim 10^{12}$ GeV. Larger values are possible if the initial displacement is small for some reason (or by accident) [4].

Limits can also be placed on f_a from below, since smaller values of this quantity correspond to larger interactions with SM particles. The axion-photon coupling in Eq. (16) can give rise to the *Primakoff process*, where a photon in a strong electric or magnetic field is converted into an axion. This process can occur in stars, including the Sun, with the axion produced escaping the star in most cases. Several experiments have searched for axions coming from the sun, typically by using a cavity with a large internal magnetic field to try to convert the solar axions back into photons by the inverse Primakoff process. Axion production in stars can also help them radiate energy more quickly, which has the effect of speeding up their evolution. Limits as large as $f_a \gtrsim 10^7$ GeV can be obtained by comparing the population of stars that are seen to the expected population when there is additional axion cooling [5].

The axion-nucleon couplings in Eq. (16) can lead to an even stronger lower bound. Axions can be created in the violent environment of a supernova. If f_a lies in the right range, not too large and not too small, axions can be created efficiently but they will not interact so strongly that they won't be able to escape the supernova. When f_a falls into this window, they will help the supernova cool off much more quickly than it would otherwise. Using observations of the cooling rate of the nearby supernova SN 1987 A, a limit of 10^6 GeV $\lesssim f_a \lesssim 10^9$ GeV can be obtained[5].

Putting the limits together gives an exclusion summarized by Fig. 1. Only the range 10^9 GeV $\lesssim f_a \lesssim 10^{12}$ GeV is unconstrained. New laboratory experiments are underway to probe it.

References

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