Lecture Note #4: Non-Thermal Dark Matter Creation

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We have discussed thermal DM creation extensively. There are also many non-thermal ways for the DM density to be created, and we will cover some of the most popular mechanisms here. Before discussing such mechanisms, however, we will begin by discussing gravitinos in supersymmetry as a specific example of non-thermal dark matter creation.

1 Gravitinos and Dark Matter

Global supersymmetry is an extension of the Poincaré symmetries of flat space [1]. In the same way that we can elevate these Poincaré symmetries to an invariance under local coordinate transformations to obtain general relativity (GR), we can elevate global supersymmetry to a local symmetry called supergravity that extends GR [2]. In supergravity, the spin $s' = 2$ graviton obtains a $s' = 3/2$ superpartner called the gravitino, Ψ_μ. The gravitino can be a viable candidate for the DM, but only if it is produced non-thermally. Even when gravitinos are unstable and not DM, they can still contribute non-thermally to the DM abundance.

1.1 Properties of the Gravitino

In the limit of exact supersymmetry, the gravitino is degenerate with the graviton and therefore massless. When supersymmetry is broken, the gravitino can acquire a non-zero mass. However, the massless $s' = 3/2$ only has two physical polarization states, two less than the four polarization states of a massive $s = 3/2$ particle. The additional degrees of freedom are acquired by eating the would-be massless *goldstino* $s = 1/2$ fermion, which is the supersymmetry analogue of a Goldstone boson. This *super Higgs* mechanism works just like the regular Higgs mechanism, where the gauge boson of a spontaneously broken gauge symmetry acquires a mass and a longitudinal component by eating the would-be Goldstone boson [2].

Supersymmetry breaking is thought to occur in a *hidden sector* that does not couple directly to the SM or its superpartners. This breaking is then thought to be communicated to the SM superpartners by messenger particles of mass M_*. To quantify the amount of supersymmetry breaking, it is conventional to use an order parameter F with mass dimension equal to two. In terms of M_* and F, the scale of soft supersymmetry breaking in the MSSM m_{soft} and the gravitino mass $m_{3/2}$ are given by [1]

$$m_{soft} = C_* \frac{F}{M_*}, \qquad m_{3/2} = \frac{F}{\sqrt{3} M_{\rm Pl}} \ . \tag{1}$$

The constant C_* depends on the details of the messengers. Some examples are:

$$\begin{array}{lll} \text{gravity mediation} & C_* \sim 1 & M_* = M_{\rm Pl} \\ \text{gauge mediation} & C_* \sim g^2/(4\pi)^2 & M_* < M_{\rm Pl}\,(4\pi)^2/g^2 \\ \text{anomaly mediation} & C_* \sim g^2/(4\pi)^2 & M_* = M_{\rm Pl} \end{array} \tag{2}$$

This implies that $m_{soft} \sim m_{3/2}$ in gravity mediation, $m_{soft} \gg m_{3/2}$ in gauge mediation, and $m_{soft} \ll m_{3/2}$ in anomaly mediation. Thus, in either gauge or gravity mediation, the gravitino can be the LSP and a candidate for the DM.

The gravitino couples to SM particles and their superpartners. The general form of these couplings are [1]

$$-\mathscr{L} \supset \frac{1}{M_{\rm Pl}}(\partial_\mu \widetilde{f})\bar{f}\gamma^\mu\gamma^\nu\Psi_\nu \;+\; \frac{i}{8M_{\rm Pl}}\bar{\Psi}_\mu[\gamma^\nu,\gamma^\rho]\gamma^\mu\widetilde{A}\,F_{\nu\rho} \;+\; \text{h.c.}\,, \tag{3}$$

where $\tilde{\Psi}_\mu$ is the graviton field, f is a SM fermion and $\tilde{f}$ is its sfermion superpartner, and $F_{\nu\rho}$ is a vector boson field strength and $\tilde{A}$ is its gaugino superpartner. The factors of $1/M_{\rm Pl}$ signal that this coupling is gravitational in origin.

In some cases, however, the effective strength of the gravitino coupling can be much larger than gravitational. This occurs because the longitudinal components of the massive gravitino come from the Goldstino, which couples to the SM and its superpartners with strength $1/F$. When computing matrix elements of gravitinos, these potentially enhanced coupling emerge from gravitino polarization sums.[1] For processes with characteristic energies $E \gg m_{3/2}$, this effect can be handled by making the substitution [1]

$$\Psi_\mu \to \sqrt{2/3}\,\partial_\mu\psi/m_{3/2}\ , \tag{4}$$

where ψ represents the $s = 1/2$ Goldstino field. Note that $m_{3/2}M_{\rm Pl} \sim F$, so this substitution does indeed produce a factor of $1/F$ when inserted in Eq. (3). When $E \lesssim m_{3/2}$ the full expression of Eq. (3) should be used.

1.2 Gravitino as Dark Matter: $m_{3/2} < m_{soft}$

Gravitinos are neutral, and they can be stable if they are the lightest superpartner, $m_{3/2} < m_{soft}$. This can be both good and bad, depending on the predicted size of the gravitino density today relative to the observed matter density. When these are equal, the gravitino can be the DM. In contrast, a gravitino abundance much larger than the matter abundance is firmly ruled out, while a gravitino abundance less than the matter abundance can be acceptable if there are other contributions to the DM.

[1] A similar thing happens when massive vector bosons interact at high energies. The vector polarization sums give a factor of $(-\eta_{\mu\nu} + p_\mu p_\nu/m_V^2)$, and the second term in this expression can become numerically large for $E \gg m_V$.

The very weak couplings of the gravitino imply that if it was ever in equilibrium with the cosmological plasma, it would have decoupled from the plasma while it was still relativistic. Assuming no other production modes, this implies [3, 4]

$$\Omega_\Psi h^2 \simeq (0.1)\left(\frac{m_{3/2}}{100\,\mathrm{eV}}\right) \ . \tag{5}$$

Such a small mass corresponds to warm dark matter, and would lead to the semi-relativistic relic gravitinos streaming out of overdense regions and washing out matter density perturbations [5]. The amount of washout is too large to be consistent with observations unless $m_{3/2} \gtrsim 1$ keV or $m_{3/2} \lesssim 10\,\mathrm{eV}$. In the first case, the gravitino relic density would be too large, and so is ruled for a different reason. In the second case, with a very light gravitino below 10 eV, the density is small enough that it is unable to do much damage to the matter fluctuations [5]. Putting these cases together, we can conclude that if a gravitino exists, it must never have thermalized in the early Universe unless its mass is less than about 10 eV. Since gravitinos require high temperatures to thermalize, this requirement is equivalent to an upper bound on the *reheating temperature* T_{RH}, the largest temperature attained by the cosmological plasma (after inflation) while it was radiation-dominated.

Gravitinos can make up the DM if they are created by a non-thermal mechanism. Possible mechanisms include decays of the MSSM superpartners to gravitinos, production by thermal scattering that is too weak to cause equilibration, and the decay of very heavy and long-lived particles. We will describe the first two cases here, and leave the more general third case to the next section.

When the gravitino is the LSP, the lightest MSSM superpartner $\widetilde{X}$ will be the next-to-LSP (NLSP). It will decay to the gravitino at the rate [3, 4]

$$\Gamma(\widetilde{X} \to X\Psi) \simeq \frac{1}{48\pi}\frac{m_{\widetilde{X}}^5}{m_{3/2}^2 M_{\mathrm{Pl}}^2} \ . \tag{6}$$

The lighter the gravitino, the faster the decay. For $m_{\widetilde{X}} \sim 100$ GeV, these decays will occur after $\widetilde{X}$ undergoes thermal freeze out (assuming it had once equilibrated) for $m_{3/2} \gtrsim 100\,\mathrm{keV}$. This gives two qualitatively different cases.

When the NLSP $\tilde{X}$ decays after freezing out, $m_{3/2} \gtrsim 100\,\mathrm{keV}$, the would-be $\tilde{X}$ abundance is transformed into a gravitino abundance. Since each NLSP decay produces one stable gravitino, this gives

$$\Delta\Omega_\psi h^2 = \left(\frac{m_{3/2}}{m_{\widetilde{X}}}\right)\Omega_{\widetilde{X}} h^2 \ , \tag{7}$$

where $\Omega_{\widetilde{X}} h^2$ is the relic density the $\widetilde{X}$ NLSP would have produced if it were stable. This is just a specific example of the superWIMP scenario [6] discussed in `notes-3`. For many NLSP varieties, this particular realization of the scenario is very strongly constrained (or ruled out) by the fact that the NLSP decay also occurs during primordial nucleosynthesis; the energetic decay products can destroy the light elements that have been created, altering their abundances.

When $m_{3/2} < 100$ keV, gravitinos are produced copiously by $\widetilde{X}$ (and possibly other superpartner) decays that occur before freeze out [3, 4]. This almost always creates too much dark matter unless the MSSM itself was never thermalized, which can be arranged if the temperature of the Universe never exceeded $T_{RH} \simeq 100$ GeV while it was radiation-dominated.

Gravitinos can also be created by thermal scattering, even if they never quite attained thermodynamic equilibrium. The relevant Bolztmann equation for this is identical to the one for thermal freeze out. The dominant contribution to gravitino production usually comes from gluino processes such as $\widetilde{g}g \to \widetilde{\Psi}g$ and $gg \to \widetilde{g}\widetilde{\Psi}$. The net contribution to the gravitino relic density is [7]

$$\Delta\Omega_\Psi h^2 \simeq (0.1)\left(\frac{100 \text{ keV}}{m_{3/2}}\right)\left(\frac{\text{TeV}}{M_3}\right)\left(\frac{T_{RH}}{2 \text{ TeV}}\right) , \tag{8}$$

where M_3 is the mass of the gluino and T_{RH} is again the maximal temperature the Universe attained while it was dominated by radiation. Note that this formula only works for T_{RH} less than the temperature at which gravitinos thermalize with the plasma. When they do, Eq. (5) should be used instead.

In Fig. 1 we summarize the upper limit on the reheating temperature assuming a gravitino LSP and an MSSM NLSP with mass close to $m_{\widetilde{X}} \sim 100$ GeV. This plot is somewhat out of date for several reasons. First, it only applies the condition $\Omega_\Psi h^2 < 1$. Thus, in the right region ($m_{3/2} > 100$ keV) the current limit on T_{RH} is about an order of magnitude stronger. For $m_{3/2} > 10$ GeV, there are additional limits from NLSP decays to gravitinos during nucleosynthesis. The exclusion in the plot also cuts off at $m_{3/2} \simeq 1$ keV. With the more recent limits obtained from the washout of large-scale structure, the exclusion line should be extended approximately horizonatally from $m_{3/2} = 1$ keV all the way down to $m_{3/2} = 10\,$eV, at which point the bound disappears.

1.3 Gravitinos Decaying to Dark Matter: $m_{3/2} > m_{soft}$

When the gravitino is heavier than at least one of the other superpartners, it will decay to that superpartner and its SM counterpart. The rate for this decay is

$$\Gamma(\Psi_\mu \to \tilde{X}X) \simeq \frac{1}{48\pi}\frac{m_{3/2}^3}{M_{\text{Pl}}^2} , \tag{9}$$

corresponding to a lifetime of

$$\tau_{3/2} \simeq 1\,\text{s}\left(\frac{10 \text{ TeV}}{m_{3/2}}\right)^3 . \tag{10}$$

These decays can occur very late in the history of the Universe and can cause problems.

The strongest bounds on decaying gravitinos frequently come from primordial nucleosynthesis. When the temperature of the Universe fell below a few MeV, the free protons and

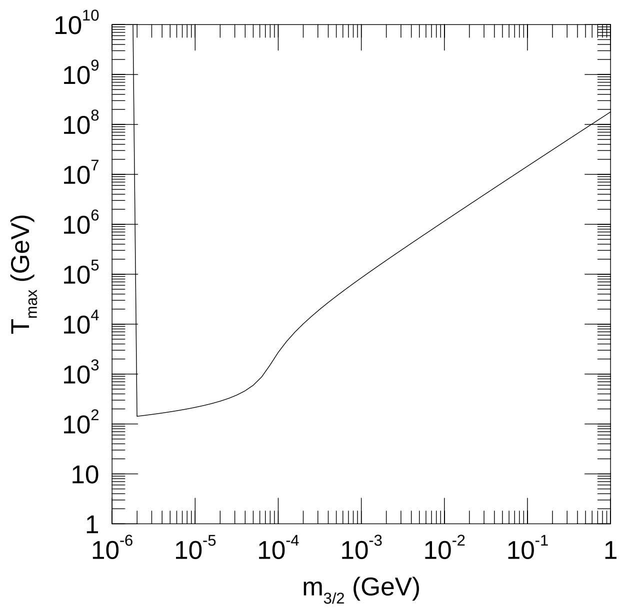

Figure 1: Upper limit on T_{RH} as a function of $m_{3/2}$ if there is gravitino LSP. Figure taken from Ref. [4]. Note that there are updated constraints that make the exclusion somewhat stronger, as discussed in the text.

neutrons in the plasma began to bind into light nuclei. The relative abundances of light nuclei predicted by the standard cosmological model match quite well with observations. When energy is injected into the plasma at temperatures below a few MeV, such as from the late decay of a gravitino, some of the light elements can be destroyed, changing their abundances. This can be used to put limits on the abundance of the decaying gravitino, which is usually set by the reheating temperature as in Eq. (8). A recent compilation of such limits is shown in Fig. 2.

2 Massive Particle Decays

A general way for DM to be created non-thermally is from the decays of a long-lived, massive particle. Consider a heavy particle P with mass m_P much larger than the DM mass m_χ. If P interacts very weakly with the SM (or any other states lighter than it), it can freeze out with a very large initial abundance and decay at a much later time to the SM or DM. If the P lifetime is very long and it becomes non-relativistic before it decays, the energy density in the abundance of P particles can come to dominate the Universe. In this case, the Universe goes from radiation domination to matter domination (by P), and later returns to

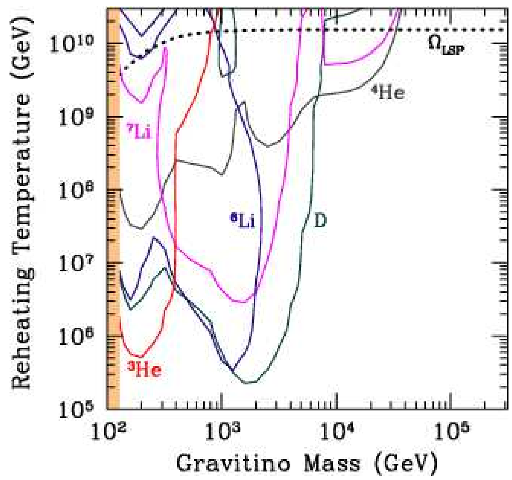

Figure 2: Upper limit on T_{RH} as a function of $m_{3/2}$ from decays of the gravitino to a lighter MSSM LSP of mass close to 100 GeV. The different lines show where each light element abundance is unacceptably modified by the gravitino decay products. Figure taken from Ref. [8].

radiation domination when the densit of P particles decays to energetic SM or other states. This process is called *reheating*. DM can also be created by P decays during reheating, and the DM density will be very different from what it would obtain from thermal freeze out if the reheating temperature is far below the freeze out temperature.

To illustrate this process qualitatively, suppose the partial decay width of P to DM is $\epsilon\Gamma_P$ and the partial decay width to radiation is $(1-\epsilon)\Gamma_P$, with $\epsilon \ll 1$. This implies that P decays mostly to radiation, and only a little bit to DM. Assume further that P decays while both it and the DM particle χ are non-relativistic. The transfer of energy from one species to another is then described by the following set of differential equations [9]: [2]

$$\frac{d\rho_P}{dt} + 3H\rho_P = -\Gamma_P \rho_p \tag{11}$$

$$\frac{d\rho_R}{dt} + 4H\rho_R = +(1-\epsilon)\Gamma_P \rho_P \tag{12}$$

$$\frac{dn_\chi}{dt} + 3Hn_\chi = +\epsilon\Gamma_P(\rho_P/m_P) - \langle\sigma v\rangle\left(n_\chi^2 - n_{\chi_{eq}}^2\right) \tag{13}$$

$$H = \left(\frac{\dot{a}}{a}\right) = \sqrt{\frac{8\pi G}{3}}(\rho_P + \rho_R + \rho_\chi)^{1/2} \tag{14}$$

Solving these equations, one finds that the temperature at which radiation takes over again after the period of P domination is

$$T_{RH} \simeq g_*^{-1/4}\sqrt{(1-\epsilon)M_{\rm Pl}\Gamma_P}\ . \tag{15}$$

[2]These equations assume that g_* remains constant throughout the process. A more general treatment can be found in Ch.5.3 of Ref. [9].

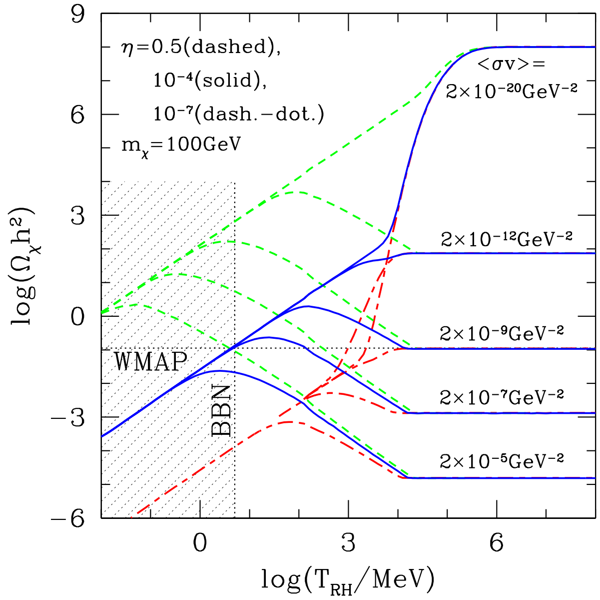

Figure 3: Relic densities of χ including production from the decays of a heavy particle P. The results are shown for various values of the annihilation cross section and $\eta = \epsilon(100\ \mathrm{TeV}/m_P)$. Figure taken from Ref. [10].

This occurs at time $t \simeq \tau_P = \Gamma_P^{-1}$.

The DM density created by this process depends on the value of T_{RH} relative to the thermal freeze out temperature T_{fo}. For $T_{RH} \ll T_{fo}$, the annihilation term in Eq. (13) can be neglected and the resulting yield is

$$Y_\chi \simeq \epsilon \left(\frac{T_{RH}}{m_P}\right) \ . \tag{16}$$

This yield does not include the small contribution from the initial DM density before P decays, which is strongly diluted by the entropy created by the decays. In the other case, $T_{RH} \gg T_{fo}$, the DM particles created in the decay will typically have enough time to re-equilibrate with the SM plasma, and their final relic density will be thermal. The general result for the intermediate case interpolates between these two limits [10]. We show their result in Fig. 3, where $\eta = \epsilon(100\ \mathrm{TeV}/m_P)$.

3 Freeze In of Dark Matter

Freeze in of DM corresponds to the production of DM by thermal scattering that is too weak to fully equilibrate the DM species [11]. This is analagous to the creation of gravitinos by gluino-gluon scattering discussed above. The difference between gravitino and the scenario discussed in Ref. [11] is that the production cross section for a freezing-in massive particle

(FIMP) is assumed to increase at lower temperatures, whereas the gravitino production rate decreases at higher temperatures.

FIMP DM can occur if three conditions are met:

i) The DM particle χ starts with a negligibly small ($n_\chi \ll n_{\chi_{eq}}$) initial abundance.
ii) The DM particle interacts so weakly with the cosmological plasma that it never attains thermodynamic equilibrium.
iii) The interaction of the DM with the plasma is governed by a renormalizable coupling.

With these conditions, one can show that the yield of χ, Y_χ/s, increases with time from the production of χ by the thermal scattering of particles in the plasma (such as $SM{+}SM \to \chi\chi$) until $T \sim m_\chi$. At this point, the thermal scattering production becomes kinematically disfavoured and the yield of χ levels off to a constant value, given approximately by

$$Y_\chi \sim \lambda^2 \frac{M_{\rm Pl}}{m_\chi} \,, \tag{17}$$

where λ is the very small renormalizable coupling to the SM. In contrast to thermal freeze out, larger couplings produce larger yields (and larger relic densities).

4 Asymmetric Dark Matter

Asymmetric dark matter (ADM) is DM that develops a cosmological abundance from an excess of particles over antiparticles, similar to how baryons develop their cosmological abundance [12]. Recall that baryons, protons and neutrons, are comprised of quarks and gluons. As a result, baryons annihilate with antibaryons very efficiently. If the densities of baryons and antibaryons were equal, they would have annihilated down to a relic abundance that is about 10^{10} times smaller than what is observed. Instead, an unknown mechanism in the early Universe created an asymmetry between baryons and antibaryons, producing more baryons than antibaryons. As the Universe cooled further, the baryons annihilated with the antibaryons until all the antibaryons were used up. Since baryon number is (mostly) conserved, baryons are unable to annihilate with themselves, and the extra leftover baryons have stayed with us until today to produce the observed baryon abundance. In the same way, a DM asymmetry can induce a relic density.

Just like for baryons, ADM requires that the DM particle χ have a distinct antiparticle $\bar{\chi}$, with each carrying an equal but opposite conserved charge. If more of χ is created in the early Universe than $\bar{\chi}$, and if χ annihilates very efficiently with $\bar{\chi}$, the final abundance of χ will be set by the asymmetry in the densities between χ and $\bar{\chi}$ rather than ordinary thermal freeze out.

An interesting possibility related to ADM is that the mechanism responsible for creating the baryon asymmetry also created the χ asymmetry. This can occur in a natural way if the conserved charge carried by χ is related to baryon number. In this case, we expect $n_\chi \sim n_B$

today. Given that the DM density is about five times larger than the baryon density, this implies that

$$m_\chi \sim m_p \left(\frac{\Omega_{DM} h^2}{\Omega_b h^2} \right) \sim 5 \text{ GeV} . \tag{18}$$

One of the implications of ADM (in its minimal form) is that there will be no DM annihilation today.

5 Axions as Dark Matter

An axion is a light pseudoscalar particle corresponding to the Nambu-Goldstone boson of an approximate global symmetry. They can arise in many ways, but the most popular realization is related to a solution of the *strong CP problem* of QCD. Axions can make up the DM if they are created non-thermally [13]. We shall defer our discussion of axions to later in the course.

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