Lecture Note #1: Dark Matter in the Cosmos

David Morrissey
March 19, 2013

1 Evidence for Dark Matter

Observations of the Universe over astronomical distances suggest there is much more matter than what we can see [1, 2, 3]. From the rotational dynamics of single galaxies, to the motion of galaxies within galactic clusters, to the distribution of stars across the observable sky, to the pattern of temperature fluctuations in the cosmic microwave background (CMB), much more matter seems to be required than what we can account for using the physics we know. This missing matter is called *dark matter* (DM).

1.1 DM and the CMB

The pattern of temperature fluctions in the $T \simeq 2.725\,K$ cosmic microwave background (CMB) carry a great deal of information about the evolution of the Universe between *recombination* and today [4]. At recombination, which occurred at a redshift of $z \simeq 1089$, free electrons and protons came together to form neutral hydrogen and the Universe become essentially transparent to photons.[1] The CMB radiation we see today consists of the photons left over at the end of recombination. Therefore the CMB gives us a snapshot of the Universe at $z \simeq 1089$, but reprocessed by the expansion history since then.

By examining the pattern of CMB temperature fluctuations, it is possible to deduce the fractions of dark energy, total matter, baryonic matter, and radiation in the Universe today. In Fig. 1, we show the dependence of the CMB temperature power spectrum on different values of the baryon density and the total matter density. In Fig. 2, we show we show the implications of the CMB data on the energy contents of the Universe together with complementary data from supernova (SN) surveys and measurements of baryon-acoustic oscillations (BAO).

Defining the energy fractions according to $\Omega_i = \rho_i/\rho_c$, where $\rho_c = 3H_0^2/8\pi G \simeq 8.1\,h^2 \times 10^{-47}\ \mathrm{GeV}^4$ ($h \simeq 0.704 \pm 0.014 = H_0/100\mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$) is the critical density today, the combined data gives [5]

$$\begin{aligned}
\Omega_{tot} &= 1.0023 \pm 0.0055 \\
\Omega_\Lambda &= 0.728 \pm 0.016 \\
\Omega_m h^2 &= 0.1383 \pm 0.0035 \\
\Omega_b h^2 &= 0.0260 \pm 0.00053
\end{aligned} \tag{1}$$

where Λ refers to dark energy, m to the total (non-relativistic) matter, and b to the baryonic matter (which includes all known particles that aren't relativistic at recombination, including

[1] Recall that we define the redshift z according to $z = \lambda_0/\lambda_1 - 1$, where λ_0 is the wavelength today and λ_1 is the wavelength at emission.

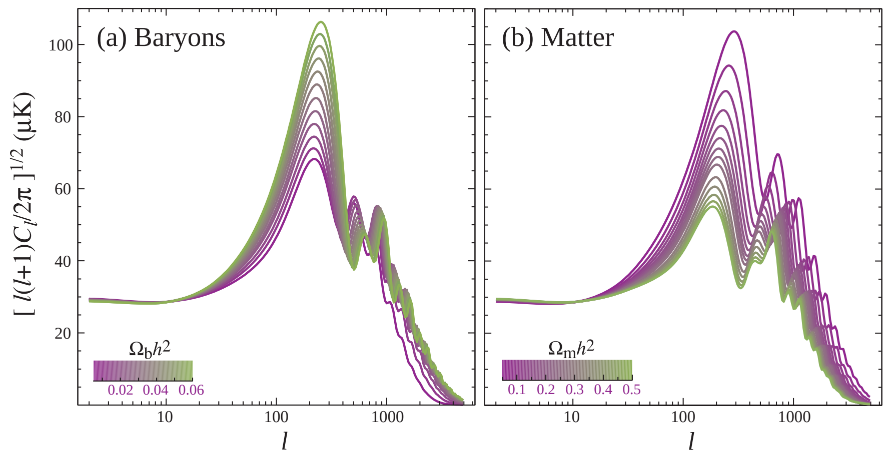

Figure 1: The effect of varying the density of baryons(left) or total matter (right) on the CMB temperature power spectrum, from Ref. [4].

electrons). Not surprisingly, the density of dark matter is defined to be the difference between Ω_m and Ω_b:

$$\Omega_{DM}h^2 = \Omega_m h^2 - \Omega_b h^2 = 0.1123 \pm 0.0035\ . \tag{2}$$

Note that this is quite a precise determination.[2]

As a cross-check, we can also compare the density of baryons determined from the CMB (and SN+BAO) with the value that is consistent with Big-Bang Nucleosynthesis (BBN) [9]. In BBN, free protons and neutrons bind to form the light elements. The underlying nuclear physics is understood very well, and starting from the total density of baryons, the relative densities of hydrogen, helium, deuterium, and lithium can be predicted. The value of $\Omega_b h^2$ found this way is consistent with the CMB value (although some of the lithium isotope densities seem to be a bit off).

1.2 DM from Astrophysics

Evidence for dark matter can also be found in many astrophysical systems. This includes the rotation curves of galaxies, the motion of galaxies within galaxy clusters, surveys of galaxy clusters by gravitational lensing, and even the overall distribution of visible matter over very large distances in a *filaments and voids* pattern [10, 11, 12]. Relative to the evidence from the CMB described above, the apparent signals of DM in these cases correspond to effects over much smaller distance scales. It is the multitude of hints pointing towards the existence of DM that make this story so compelling.

[2] These results have improved even further recently, but I didn't have a chance to update the numbers. See Ref. [6] for details. An even better set of data from measurements of the CMB by the Planck collaboration is also expected in the next few days [7].

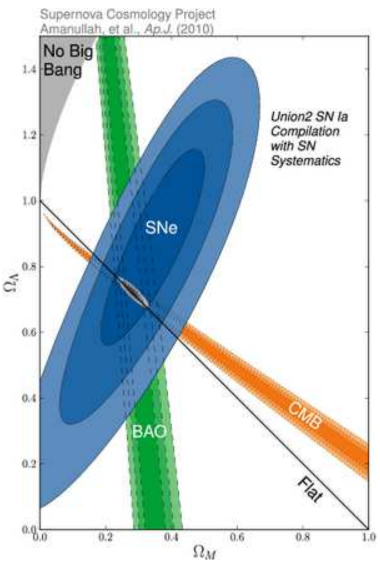

Figure 2: Combined fits to the CMB spectrum, supernova surveys, and BAO. From Ref. [8].

In many of the galaxies we can see, the material within them is rotating. The net rate of rotation $v(r)$ can be deduced by measuring the redshift of light coming from them, while the total visible luminosity (and surveys in other frequencies that are more sensitive to dust) is expected to trace the mass of the system. Applying what we know about gravity to such systems, we can compare the observed rotation velocity to what we would predict based on the visible mass distribution. A typical result is shown in Fig. 3, from which it is evident that the visible matter is unable to account for the observed rotation.

Let us assume instead that the galaxy is embedded within a spherical *halo* of dark matter. Applying Newtonian gravity, one finds

$$v(r) = \sqrt{\frac{GM_<(r)}{r}}\ , \tag{3}$$

where $M_<(r)$ is the total mass within the radius r. For a mass distribution $\rho(r) \propto r^n$, we find $v(r) \propto r^{n/2-1}$. Observationally, the rotation curves typically approach a constant value, implying $n = 2$ and $\rho(r) \propto 1/r^2$ at large radii.

Individual galaxies are frequently found within self-gravitating galaxy clusters, in which the constituent galaxies move around under their mutual gravitational influence. While this motion is very complicated, its mean properties can be described using the virial theorem. Applied here, it gives

$$v^2 \sim \frac{GM}{R}\ , \tag{4}$$

where v^2 and R represent typical values of the velocity of a galaxy and its distance to the centre-of-mass, and M is the total mass of the cluster. Using only the visible mass in this estimate, the predicted value of v^2 is much less than what is observed. In fact, it was precisely this observation that led Fritz Zwicky to first propose DM in 1933.

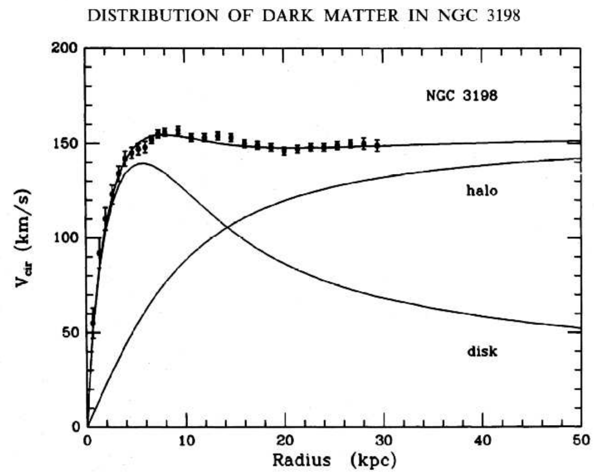

Figure 3: Observed and predicted rotation curves for the spiral galaxy NGC 3198.

The amount of matter within galaxy clusters can also be probed through gravitational lensing. By surveying the distortion of light from sources behind a cluster, its total mass distribution can be mapped out. These surveys also find much more matter than what can be accounted for by ordinary matter.

A particularly spectacular example of a lensing survey is the bullet cluster [13], shown in Fig. 4. This system is thought to be the aftermath of a collision between a pair of galaxy clusters. Here, the blue areas trace out the total matter distribution, while the red areas show the map of ordinary matter based on x-ray emission. The interpretation is that the dark matter components of the clusters passed right through each other, while the baryonic components collided in the middle to create shock waves.

1.3 DM and Structure

The distribution of matter in the Universe is found to be very uniform over large distances, above about 100 Mpc. We illustrate this using a map by the 2dF galaxy survey in Fig. 5. Even so, the local fluctuations in the density of matter carry a lot of information, much like the CMB. This fluctuation spectrum can be measured from galaxy surveys, lensing surveys, and dust maps made by observing the absorption of light by the Lyman alpha line in the hydrogen spectrum. It is found that DM is needed to account for the observed spectrum.

1.4 DM and Other Possibilities

All the evidence we have for DM is based on its gravitational influence on visible matter. A very good description of the data is obtained if we assume the existence of new species of particle with the following properties:

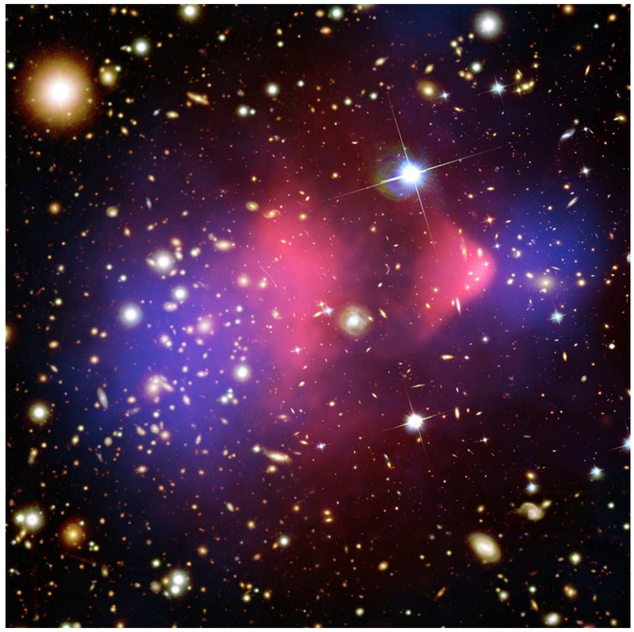

Figure 4: Matter (blue) and baryon (red) distributions within the bullet cluster.

1. non-luminous (uncharged, uncolored)
2. non-relativistic and stable (or very long-lived)
3. very weakly interacting with itself and with visible matter

No known elementary particle has these properties, and therefore the existence of DM points towards new and interesting particle physics.

Unfortunately, all our evidence for DM comes from its gravitational influence on visible matter. A logical alternative to DM is that gravity does not work the way we think it does (which is General Relativity). Many attempts have been made to formulate theories of modified gravity, but it has proven difficult to find one that can explain all the data. For this reason, DM has been investigated much more thoroughly by cosmologists.

In this course we will focus on the DM hypothesis and its implications for elementary particle physics. One of the main themes will be searches for DM using interactions other than gravity. A discovery in DM in this way would both confirm the DM hypothesis and tell us about the microscopic properties of the DM particle.

2 Distributions of DM

Dark matter plays a key role in determining the distribution of visible matter in the Universe. In this section we will discuss how DM is distributed throughout our Universe.

Before getting into specifics, let us first outline how a smooth initial distribution of DM evolved into the less-than-homogeneous pattern we seem to see today. Within the ΛCDM

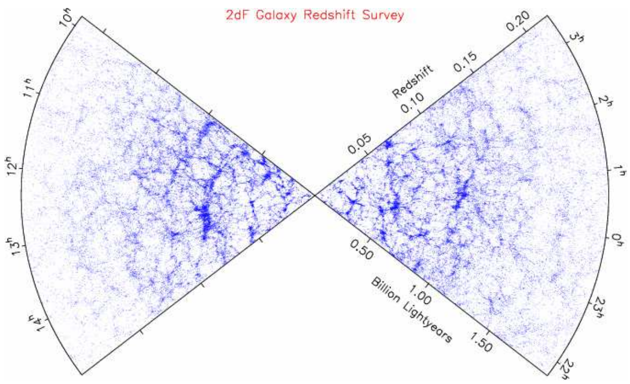

Figure 5: Galaxy survey by the 2dF Collaboration.

model and assuming an early period of inflation, quantum fluctuations in the inflaton field gave rise to small fluctuations in the density of matter in the Universe over very small and very large (super-horizon) distances. After inflation, these fluctuations evolved under the influence of the Hubble expansion and particle interactions. Fluctuations in the density of DM began growing in earnest when matter becomes the dominant source of energy in the Universe ("matter-radiation equality") at redshift $z_{eq} \simeq 3200$. On the other hand, fluctuations in the density of baryons are delayed until recombination at $z_{rec} \simeq 1100$ because of their strong coupling to photons.

When a fluctuation grows large enough, it begins to self-gravitate and is said to become *non-linear* (because we can no longer describe it reliably using linearized equations). This process happens first for DM since the baryons remain coupled to the relativistic photon fluid until a bit later. Gravitational collapse creates clumps of DM that act as potential wells for the baryons and pull them in. Thus, DM acts as a scaffolding upon which baryons can attach, cluster, and form stars and galaxies. In particular, we expect that the visible matter in the Universe should trace approximately the distribution of DM.

2.1 Large-Scale (DM) Structure

As mentioned above, the spectrum of fluctuations in the density of visible matter observed today contains a lot of information. This information is similar but complementary to the information in the spectrum of temperature fluctuations in the CMB. In particular, since the density of baryons today is thought to track the density of dark matter, the spectrum of fluctuations of visible matter tell us how dark matter is distributed in the Universe.

We can characterize the density fluctuations of visible matter by a quantity called $P(k)$, corresponding to the fluctuation amplitude power at the wavenumber k (wavelength k^{-1}) [3]. To compute this, an initial spectrum of fluctuations after inflation is assumed (which turns out to be pretty much a universal function of k for most reasonable models of inflation), and the resulting evolution after inflation is computed to obtain $P(k)$ today. This involves computing a complicated set of differential equations that describe the evolution of the metric

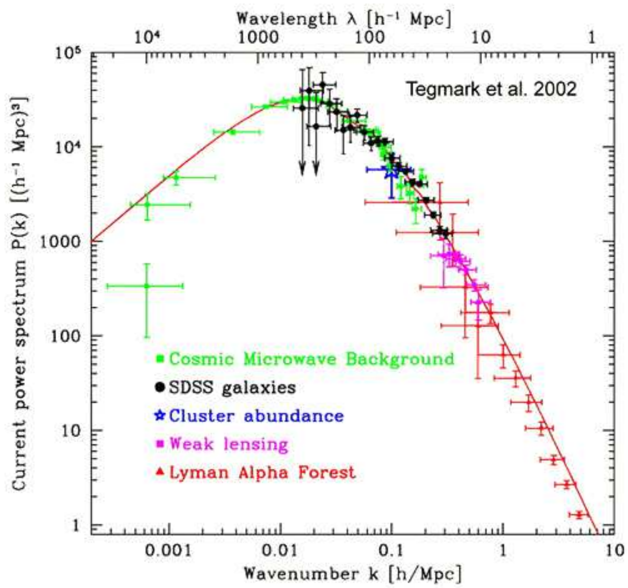

Figure 6: Galaxy survey by the 2dF Collaboration.

and the densities of DM, baryons, and radiation. To make these tractable, the equations are usually evaluated perturbatively in small deviations from a constant background. This treatment works well for perturbations at larger distances (smaller k) that have not grown too large.

The net result of these calcualtions, assuming a reasonable intial fluctuation spectrum from inflation and much more dark matter than baryons, is approximately

$$P(k) \propto \left\{ \begin{array}{ccl} k & ; & k \ll k_{eq} \\ k^{-3}\ln^2\left(\frac{k}{k_{eq}}\right) & ; & k \gg k_{eq} \end{array} \right. , \tag{5}$$

where k_{eq}^{-1} corresponds to the size of the Universe at matter-radiation equality. The change in the behaviour corresponds to a different evolution of fluctuations during the radiation- and matter-dominated epochs. As shown in Fig. 6, this prediction is consistent with observations over many different length scales. Without DM, the result would be very different.

2.2 Galactic DM

Galaxies are agglomerations of stars (and dust) moving about under their mutual gravitational attraction. Based on galactic rotation curves, we think that the visible stuff in galaxies is surrounded by a much larger *halo* of DM. Using rotation curves, we can estimate the matter density distribution within other galaxies. The density and velocity distributions of matter within galaxies are also estimated from large N-body simulations.

Of all the galaxies we know of, we are most interested in our own, at least as far as searching for DM goes. Since we are inside our own galaxy, it is not possible to measure the rotation curve so we must rely on simulations. The results of these simulations are frequently approximated by simple analytic fits. Some of the most popular fits for the density distribution $\rho(r)$ include:

- (α, β, γ)

$$\rho(r) = \rho_\odot \left(\frac{r_\odot}{r}\right)^\gamma \left[\frac{1+(r_\odot/r_s)^\alpha}{1+(r/r_s)^\alpha}\right]^{(\beta-\gamma)/\alpha} \tag{6}$$

  where $\rho_\odot \simeq 0.3$ GeV/cm^3 and $r_\odot = 8.5\,\text{kpc}$, along with:

Profile	α	β	γ	r_s(kpc)
NFW	1	3	1	20
Moore	1	3	1.16	30
Iso-Core	2	2	0	5

(7)

- Einasto:

$$\rho(r) = \rho_\odot\, \exp\left(-\frac{2}{\alpha}\left[\left(\frac{r}{r_\odot}\right)^\alpha - 1\right]\right)\ , \tag{8}$$

  with $1.0 \lesssim \alpha \lesssim 2.0$.

For the velocity distribution, the simulations point towards something that is close to a Maxwellian distribution:

$$f(\vec{v}) \simeq N\left(\frac{1}{\pi v_0^2}\right)^{3/2} e^{-v^2/v_0^2}\ \Theta(v_{esc} - v)\ , \tag{9}$$

where $v_0 \simeq 220\,km/s$ and the Θ is the step function that cuts the distribution off at $v = v_{esc} \simeq 600\,km/s$, the velocity above which DM particles can escape the halo. The factor of $N \simeq 1$ modifies the normalization so that $\int d^3v f(\vec{v}) = 1$ with the Θ function. Note that this velocity distribution applies to DM particles relative to the halo. Relative to the halo, the disk in our galaxy (which includes us) is rotating.

3 Challenges to the DM Hypothesis

As we have just seen, the hypothesis that the apparently missing matter consists of a new species of very feebly-interacting massive particle can account for a wide variety of observations. Even so, there are a handful of specific observations that provide a challenge to this picture of DM, which we will summarize briefly here. Note, however, that in all theses cases there is a significant theoretical uncertainty on what the precise predictions of DM are. These uncertainties are large enough that these discrepancies could very well be just a reflection of our lack of understanding of DM dynamics. More calculations and more observations are needed to sort things out.

3.1 Missing Satellites

Simulations of DM in galaxies are simple in principle but very complicated in practice. A system of N "particles" representing cold DM is allowed to evolve under a mutual gravitational attraction. As discussed above, these simulations make predictions for the DM density distribution $\rho(r)$ and velocity distribution $f(\vec{v})$ within our galaxy. These simulations are also found to predict a large amount of *substructure*, consisting of smaller clumps of DM within the larger galactic halo. The number of such *sub-haloes* or *satellites* predicted by N-body simulations seems to be considerably larger than what is observed. This is called the "Missing Satellite Problem" [14].

Many attempts to explain this apparent discrepancy have been made in the last few years, and a few of them seem like they could be correct. A DM-based explanation is to assume that some or all of the DM is *warm*, meaning that it was somewhat non-relativistic until fairly late.[3] The non-trivial thermal velocities of warm DM particles would prevent it from clumping effectively over short distances. A second suggestion is that baryons (and thus dust and stars) do not accumulate as well in small DM sub-haloes as has been assumed, implying that there are many more sub-haloes than have been observed [15].

3.2 Galactic Cusp vs. Core

Simulations of DM over galactic scales tend to predict a central cusp, with the density at small radii near the inner core going like $r^{-\gamma}$ with $\gamma \sim 1$. On the other hand, galactic observations point towards a flatter *cored* inner profile with $\gamma \sim 0$ [16]. Like the missing satellite problem, a number of proposals have been made to account for this result within the context of DM models. One class of ideas is that DM could have non-trivial self-interactions [17], which would prevent it from clumping too densely. A more pedestrian explanation is that baryons could have a significant effect on the distribution of DM near the galactic centre [18]. Baryons have only recently been included in simulations of galactic dynamics, and their effects are still being determined.

3.3 Galactic Matter Distributions

A further puzzle related to the standard picture of DM is the baryonic Tully-Fisher (BTF) relation [19]. This is an empirical result found for rotating circular galaxies of the form

$$M_b = \mathcal{A}\, v_c^x \ , \tag{10}$$

where M_b is the total mass of baryons in the galaxy, v_c describes the net rotation velocity of the galaxy, and $\mathcal{A}$ and x are constants. Fits to many such galaxies give $\mathcal{A} \simeq 50\, M_{\odot}\mathrm{km}^{-4}s^4$ and $x = 4$ [20], with individual galaxies deviating very little from the best-fit line.

[3] Regular non-relativistic DM is said to be *cold*, and DM that was relativistic until very recently is called *hot*.

The puzzle this relation presents for DM presented is that the galactic rotational velocity v_c should be set by the total matter density (DM and baryons), rather than just the baryons alone. While the baryonic mass of a galaxy is expected to trace the total mass on average, significant deviations between individual galaxies are expected. Thus, one would expect larger individual deviations of circular galaxies from the BTF relation than is observed. While the total baryonic mass is not large enough to explain the magnitude of v_c for conventional gravity, proponents of modified theories of gravity that can explain galactic rotation curves have pointed out that such theories often also predict the BTF relation [21].

It should be noted, however, that there is some controversy over how well the BTF relation actually works. Refs. [22, 23] suggest that the baryonic mass used in Refs. [20, 21] did not include the large amount of ionized gas expected to be present in many such galaxies, while Ref. [24] criticized other aspects of the fitting procedure. A strongly-worded response soon followed [25].

References

[1] E. W. Kolb and M. S. Turner, "The Early universe," Front. Phys. **69**, 1 (1990).

[2] S. Dodelson, "Modern cosmology," Amsterdam, Netherlands: Academic Pr. (2003).

[3] D. Baumann, "Cosmology", http://www.damtp.cam.ac.uk/user/db275/Cosmology/Lectures.pdf .

[4] For a nice review of CMB physics, see:
W. Hu, "Lecture Notes on CMB Theory: From Nucleosynthesis to Recombination," [arXiv:0802.3688 [astro-ph]].

[5] E. Komatsu *et al.* [WMAP Collaboration], "Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation," Astrophys. J. Suppl. **192**, 18 (2011) [arXiv:1001.4538 [astro-ph.CO]].

[6] G. Hinshaw, D. Larson, E. Komatsu, D. N. Spergel, C. L. Bennett, J. Dunkley, M. R. Nolta and M. Halpern *et al.*, [arXiv:1212.5226 [astro-ph.CO]].

[7] Scientific data release by the Planck collaboration, http://www.rssd.esa.int/index.php?project=planck.

[8] R. Amanullah, C. Lidman, D. Rubin, G. Aldering, P. Astier, K. Barbary, M. S. Burns and A. Conley *et al.*, Astrophys. J. **716**, 712 (2010) [arXiv:1004.1711 [astro-ph.CO]].

[9] A nice review of BBN:
G. Steigman, "Primordial Nucleosynthesis in the Precision Cosmology Era," Ann. Rev. Nucl. Part. Sci. **57**, 463 (2007) [arXiv:0712.1100 [astro-ph]].

[10] L. Bergstrom, "Nonbaryonic dark matter: Observational evidence and detection methods," Rept. Prog. Phys. **63**, 793 (2000) [hep-ph/0002126].

[11] G. Bertone, D. Hooper and J. Silk, "Particle dark matter: Evidence, candidates and constraints," Phys. Rept. **405**, 279 (2005) [hep-ph/0404175].

[12] G. D'Amico, M. Kamionkowski and K. Sigurdson, "Dark Matter Astrophysics," [arXiv:0907.1912 [astro-ph.CO]].

[13] M. Markevitch, A. H. Gonzalez, L. David, A. Vikhlinin, S. Murray, W. Forman, C. Jones and W. Tucker, "A Textbook example of a bow shock in the merging galaxy cluster 1E0657-56," Astrophys. J. **567**, L27 (2002) [astro-ph/0110468].

[14] L. E. Strigari, J. S. Bullock, M. Kaplinghat, J. Diemand, M. Kuhlen and P. Madau, "Redefining the Missing Satellites Problem," Astrophys. J. **669**, 676 (2007) [arXiv:0704.1817 [astro-ph]].

[15] A. M. Brooks, M. Kuhlen, A. Zolotov and D. Hooper, "A Baryonic Solution to the Missing Satellites Problem," Astrophys. J. **765**, 22 (2013) [arXiv:1209.5394 [astro-ph.CO]].

[16] W. J. G. de Blok, "The Core-Cusp Problem," Adv. Astron. **2010**, 789293 (2010) [arXiv:0910.3538 [astro-ph.CO]].

[17] A. Loeb and N. Weiner, "Cores in Dwarf Galaxies from Dark Matter with a Yukawa Potential," Phys. Rev. Lett. **106**, 171302 (2011) [arXiv:1011.6374 [astro-ph.CO]].

[18] F. Governato, A. Zolotov, A. Pontzen, C. Christensen, S. H. Oh, A. M. Brooks, T. Quinn and S. Shen *et al.*, "Cuspy No More: How Outflows Affect the Central Dark Matter and Baryon Distribution in Lambda CDM Galaxies," Mon. Not. Roy. Astron. Soc. **422**, 1231 (2012) [arXiv:1202.0554 [astro-ph.CO]].

[19] S. S. McGaugh, J. M. Schombert, G. D. Bothun and W. J. G. de Blok, "The Baryonic Tully-Fisher relation," Astrophys. J. **533**, L99 (2000) [astro-ph/0003001].

[20] S. S. McGaugh, "The Baryonic Tully-Fisher relation of galaxies with extended rotation curves and the stellar mass of rotating galaxies," Astrophys. J. **632**, 859 (2005) [astro-ph/0506750].

[21] S. McGaugh, "The Baryonic Tully-Fisher Relation of Gas Rich Galaxies as a Test of LCDM and MOND," [arXiv:1107.2934 [astro-ph.CO]].

[22] N. Y. Gnedin, "On the Baryonic Contents of Low Mass Galaxies," Astrophys. J. **754**, 113 (2012) [arXiv:1108.2271 [astro-ph.CO]].

[23] S. Chakraborti and S. Khedekar, "Neutral Hydrogen Tully Fisher Relation: The case for Newtonian Gravity," [arXiv:1109.0529 [astro-ph.CO]].

[24] S. Foreman and D. Scott, "What do gas-rich galaxies actually tell us about modified Newtonian dynamics?," Phys. Rev. Lett. **108**, 141302 (2012) [arXiv:1108.5734 [astro-ph.CO]].

[25] S. McGaugh, "Missing the Point - a Brief Reply to Foreman & Scott and Gnedin," [arXiv:1109.1599 [astro-ph.CO]].