PSI Dark Matter Homework #3

Due: Apr. 12, 2013

1. **Dark Matter Decay**
Consider a DM particle χ that is not completely stable, but that can decay with a lifetime τ_χ much longer than the age of the Universe. These decays will only have a small effect on the DM density for $\Gamma_\chi^{-1} = \tau_\chi \gg t$, but they can lead to observable indirect detection signals. For decaying DM (with no significant annihilation), the rate of change of the number density at late times is

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\Gamma_\chi n_\chi \ . \tag{1}$$

For comparison, the rate of annihilation for a stable self-annihilating DM particle long after freeze out is

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma v\rangle n_\chi^2 \ . \tag{2}$$

   a) Formulate a formula for the flux of gamma rays from dark matter decay analogous to Eq. (1) in `notes-6` and explain what all the terms in it mean and how you got them. Rewrite your result in the form

$$\Phi_\gamma = \frac{r_\odot}{4\pi} Q_{dec}(\odot)\ J_{dec}\ , \tag{3}$$

   for some appropriately defined new particle physics and astrophysics quantities $Q_{dec}(\odot)$ and J_{dec}.

   b) Observational limits on annihilating DM can be converted to limits on annihilating DM directly to limits on decaying DM. Suppose the limit on an annihilating DM species with photon spectrum $f_\gamma(E)$ derived from observations in a region near the galactic centre is:

$$\langle\sigma v\rangle < (3\times 10^{-26}\,\text{cm}^3/\text{s}) \left(\frac{m_\chi}{100\ \text{GeV}}\right)^2 \ . \tag{4}$$

   Use this result to derive the corresponding limit on the lifetime τ_χ of a decaying DM particle with the same photon spectrum $f_\gamma(E)$ per decay under the assumption that $J/J_{dec} = 100$, where J and J_{dec} are the astrophysics factors relevant for the region observed for annihilating and decaying DM.

2. **Axion Origins**
A very simple way for an axion to be generated is from the following theory of a complex scalar ϕ and an exotic quark $\Psi = (\Psi_L, \Psi_R)^t$ that is a triplet under $SU(3)_c$:

$$\mathscr{L} = |\partial\phi|^2 - V(\phi) - y\left(\phi\bar{\Psi}_L\Psi_R + \phi^*\bar{\Psi}_R\Psi_L\right)\ , \tag{5}$$

with $V(\phi) = \lambda(|\phi|^2 - f^2)^2$.

a) Show that $\langle\phi\rangle = f$ minimizes the potential.

b) Expand about the vacuum by writing $\phi(x) = \frac{1}{\sqrt{2}}(f + h(x))e^{ia(x)/f}$. Show that this gives canonical kinetic terms for both a and h by plugging the expansion back into the Lagrangian.

c) Derive the masses induced for h, a, and Ψ. You should find that a is massless. *Hint: expand* $e^{ia/f} = 1 + ia/f + \ldots$.

d) For $f \gg m_W$, we can integrate out the heavy particles from the theory to get an effective theory with only the light states. Show that by redefining the fields Ψ_L and Ψ_R by

$$\Psi_L(x) \;\rightarrow\; \Psi_L\, e^{iQ_L a(x)/f} \;, \tag{6}$$

$$\Psi_R(x) \;\rightarrow\; \Psi_R\, e^{iQ_R a(x)/f} \;, \tag{7}$$

we can remove the direct coupling of a to the fermions if we take $(Q_L - Q_R) = 1$. Show as well that this induces a coupling of a to $G^a_{\mu\nu}\widetilde{G}^{a\,\mu\nu}$ and find the coupling coefficient of the effective operator.
Hint: think of this as a generalization of the mass rephasing performed in `notes-9`.

e) This result can also be obtained from a loop calculation. Draw a Feynman diagram that links the a field to a pair of gluons through a loop of the heavy quarks (but don't try to calculate it). Count the number of factors of g_s in the diagram and show that it matches the result for the coupling you found in d).

f) If the Ψ state is also charged under other gauge groups, axion couplings to the corresponding gauge bosons can also be generated. To see this, we need to generalize the mass shift formula of `notes-9`, where we considered a colour triplet ψ. In general, if ψ has charge Q_x under the gauge group $U(1)_x$, the transformation of ψ_L and ψ_R given in Eqs. (4,5) of `notes-9` also generates a new term of the form

$$\Delta\mathscr{L} = \frac{\alpha_x}{4\pi}\, 3Q_x^2\, X_{\mu\nu}\widetilde{X}^{\mu\nu} \;, \tag{8}$$

where $X_{\mu\nu} = (\partial_\mu X_\nu - \partial_\nu X_\mu)$ is the field strength of the $U(1)_x$ gauge boson, and the factor of three accounts for the three colours of ψ. Use this result to show that if the Ψ fermion in our axion theory also carries a hypercharge equal to Y (but is a singlet under $SU(2)_L$, a coupling of a to the photon operator $F_{\mu\nu}\widetilde{F}^{\mu\nu}$ will also be generated. Derive the coupling coefficient as well.
Hint:
The hypercharge gauge field B^μ mixes into the photon (A^μ) and the Z boson (Z^μ) after electroweak symmetry breaking, with the decomposition

$$B_\mu = c_W A_\mu - s_W Z_\mu \;, \tag{9}$$

where $c_W = \cos\theta_W$ and $s_W = \sin\theta_W$ refer to the weak mixing angle. Also, $Q_{em} = Y$ for fields that are singlets under $SU(2)_L$ (like Ψ here), and the hypercharge coupling is related to the electromagnetic coupling by $\alpha_Y = c_W^2\,\alpha$.