PSI Dark Matter Homework #1

Due: Thursday, March 28, 2013

1. Dark Matter Profiles:

   a) Make plots (using a plotting program of your choice) of the NFW, Iso-Core, and Einasto ($\alpha = 1.7$) galactic DM profiles using the functional forms listed in the notes. Put them on the same set of (logarithmic) axes.

   b) We will see that the cosmic ray signal strength from given point in the sky from DM annihilation in our galaxy is proportional to $\langle\sigma v\rangle n_\chi^2$, where n_χ is the local DM number density. Assuming a constant cross section everywhere, make plots of the contributions to the signal strengths (as a function of the distance from the galactic center) for the NFW and Einasto profiles relative to the Iso-Core profile.

2. Boltzmannology:

   a) The collision term for the Boltzmann equation for n_χ we derived in class for the $\chi\chi \to f\bar{f}$ process was

   $$-\int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\, g_\chi^2\, \sigma v\, \left(f_1 f_2 - f_{1_{eq}} f_{2_{eq}}\right)\,, \tag{1}$$

   where "1" and "2" refer to the two χ particles. Assuming $f(E,t) = \xi(t) f_{eq}(E)$, with $E = \sqrt{p^2 + m^2}$, re-express this quantity in terms of n_χ, $n_{\chi_{eq}}$, and $\langle\sigma v\rangle$.

   b) Do the integral over p^0 in the Lorentz-invariant quantity $\int d^4p\, \delta(p^2 - m^2)\theta(p^0)$. What does this imply for how the quantity $(d\Pi) = d^3p/(2\pi)^3 2E$ changes under Lorentz transformations?

   c) Derive for yourself the rewriting of the freeze-out equation (Eq. (21) in `notes-2`) in terms of $Y_\chi = n_\chi/s$ as a function of $x = m/T$ (Eq. (25) of `notes-2`).

3. Cross Sections

   a) Consider the interaction

   $$-\mathscr{L} \supset \lambda\, \phi \bar{f} \chi + (\text{h.c.})\,, \tag{2}$$

   where χ is a DM fermion, f is a SM fermion of charge $Q = -1$, and ϕ is a complex scalar also of charge $Q = -1$. This interaction, together with its Hermitian conjugate, can mediate the annihilation $\chi\bar{\chi} \to f\bar{f}$ Compute the squared matrix element for this process, averaged over both initial and final spins. Write your answer in terms of dot products involving the two incoming momenta p_1 and p_2, and the outgoing momenta p_3 and p_4.
   Hint: consult your notes from the tutorial.

   b) Evaluate this squared matrix element in the centre-of-mass frame, but do not assume that the incoming particles are non-relativistic. You may assume that the fermion is massless.

c) Turn this matrix element into $4E_1E_2\,\sigma v$ by integrating over the final-state phase space ($\int d\Pi_3 d\Pi_4 (2\pi)^4 \delta^{(4)}(...)$) and multiplying by the appropriate factors.

d) Assume the collision happens with both χ and $\bar{\chi}$ highly non-relativistic, and expand the result in powers of $u = p/E$, and keep terms in the expansion up to u^2.

e) Use this to compute the thermal average $\langle \sigma v \rangle$ following the approximate prescription described in class.