Address hierarchy by bringing fundamental Planck scale near M_W. ", M_*"

$\rightarrow M_* \ll M_{Pl}$

This can be achieved by "dilution".

XD help here: dilution by volume or localization.

XD also arise in string theory, attempts to merge gauge and gravity.

$\rightarrow$ do scalar KK expansion on $w \in [0, 2\pi R]$.

1. LED = Large Extra Dimension.

Suppose we have $1+3+n = d$ dimensions, $\eta^{MN} = \mathrm{diag}(+1,-1,-1,-1,-1,\dots,-1)$ flat

gravity $\Leftrightarrow$ spacetime, so must be present in all.

For static, weak (Newtonian) field, $\underbrace{\vec{\nabla}^2 \Phi}_{\text{grav. pot.}} \sim \frac{1}{M_{Pl,d}^{2+n}} \rho$ " E/V, dimensions of $M^{1+(d-1)} - M^d$.

$$\Rightarrow \quad F(r) \sim \frac{1}{M_{Pl(4+n)}^{2+n}} \frac{m_1 m_2}{r^{2+n}} \qquad [F] = [Ma] = 1+1$$

This is not what we see!

But suppose the n XD are periodic, $w \sim w + \overbrace{2\pi R}^{"L}$, with radius R.

For $r \gg L$, XD part disappears, flux lines "channelled"

$$\Rightarrow \quad F(r) \sim \frac{1}{M_{Pl(d)}^{n+2}} \frac{m_1 m_2}{L^n r^2} \sim \frac{1}{M_{Pl}^2} \frac{m_1 m_2}{r^2}$$

Matches observation for: $M_{Pl}^2 = M_{Pl(d)}^{n+2} L^n \rightarrow M_{Pl(d)}^{n+2} V_n$ "volume" of XD.

Suppose $M_{Pl(d)} \sim$ TeV.

$\Rightarrow L \sim 10^{30/n} 10^{-17}$ cm $\sim \begin{cases} 10^{15} \text{ cm} & ; n=1, \text{ ruled out} \\ 1 \text{ mm} & ; n=2, \text{ ruled out recently } (L \lesssim 200\,\mu m) \\ 1\,\mu m & ; n=3, \text{ fine} \end{cases}$ $\rightarrow$ volume dilution!

Could this be the case for our Universe?

Gravity (i.e. graviton) must live in all dim.

What about SM fields?

$\Phi(x, w)$, can be rewritten as an infinite set of 4d fields with $m \sim \sqrt{\left(\frac{n\pi}{R}\right)^2 + m_{SM}^2}$.

Such KK modes have not been seen.

$\left(L \sim \underset{n=2}{10^{-13}\,\text{GeV}},\ \underset{n=3}{10^{-8}\,\text{GeV}},\ \ldots\ \underset{n=6}{10^{-2}\,\text{GeV}},\ \ldots\right)$

Fix: confine SM to a $d'=4$ subsurface of the total spacetime

(e.g. D3 brane in string theory)

$d=4+n$ gravity $+$ $d'=4$ SM is the LED theory, flat spacetime.

No hierarchy problem, but also no explanation for large R.

What are the consequences?

Let $M_* = M_{Pl\,(d)}$ be the fundamental Planck scale.

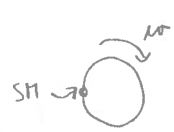
Write: $x^M = (\underset{0,1,2,3}{x^\mu}, \overset{4,5,\ldots,3+n}{x^a})$, $n = \#$ XD.

$$\eta^{MN} = (\underbrace{+1,-1,-1,-1}_{0,\ldots,3}, \underbrace{-1,\ldots,-1}_{4,\ldots,n+3})$$

g_{MN} = general bulk metric

Assume: - n XD all have $x^a \in [0, 2\pi R]$, periodic ($\mathbb{R}^{1,3} \times T^n$ spacetime)

- SM confined to $d=4$ surface at $x^a = 0$.



$$S_{grav} = \int d^4x \int d^n w \sqrt{-G}\, R^{(4+n)} \left(\tfrac{1}{2} M_*^{n+2}\right)$$

" Ricci tensor from G.

Expand $G_{MN} = \eta_{MN} + h_{MN}/2M_*^{1+n/2}$

↑ canonical kinetic

$g_{\mu\nu}(x) = G_{\mu\nu}(X, w=0)$, the induced metric where the SM lives

$$S_{SM} = \int d^4x \sqrt{-g}\, \mathcal{L}_{SM}(g_{\mu\nu}, \dots)$$

$$\sqrt{-g}\, T^{\mu\nu} := \frac{\delta S_{SM}}{\delta g_{\mu\nu}}, \text{ the SM energy momentum tensor.}$$

Expanding out, $S_{SM} = S(g_{\mu\nu} \to \eta_{\mu\nu}) + \int d^4x\, T^{\mu\nu} h_{\mu\nu} \dfrac{1}{M_*^{1+n/2}} + \dots$

" leading coupling

How do we deal with the graviton field?

For $n=1$, $h_{MN}(X, w)$ is periodic in w.

$\Rightarrow$ expand in orthogonal eigenfunctions, $\frac{1}{\sqrt{2\pi R}} e^{inw/R}$

$$h_{MN}(X, w) = \sum_{n=-\infty}^{\infty} \underbrace{\frac{1}{\sqrt{2\pi R}} e^{inw/R}}_{\text{wavefunction}} \underbrace{h_{MN}^{(n)}(X)}_{\text{KK mode}}, \quad \text{reality gives } h_{MN}^{(-n)} = h_{MN}^{(n)*}$$

Aside: real scalar $\phi(x, w)$ in $n=1$.

$$\phi(x, w) = \sum_n \frac{1}{\sqrt{2\pi R}} e^{inw/R} \phi^{(n)}(x)$$

$$S = \int d^4x \int_0^{2\pi R} dw \left[\tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2} m^2 \phi^2\right] = \int d^4x \left(\left[\tfrac{1}{2}(\partial\phi^{0})^2 - \tfrac{1}{2} m^2 \phi^2\right] + \sum_{n=1}^{\infty} \left[|\partial_\mu \phi^{(n)}|^2 - \left(m^2 + \tfrac{n^2}{R^2}\right) |\phi^{(n)}|^2 \right] \right)$$

$\Rightarrow$ n-th mode has $m_n = \sqrt{m^2 + n^2/R^2}$.

For $n > 1$, get an exp. for each dimension, $\vec{n} = (n_1, n_2, n_3, \dots, n_n)$

$$h_{MN}(X, w) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \cdots \sum_{n_n=-\infty}^{\infty} \frac{1}{\sqrt{V_n}} e^{i n^a w^a/R} h_{MN}^{(\vec{n})}(X).$$

$$\Rightarrow \quad S_{int} = \int d^4x\, T^{\mu\nu} \underbrace{\frac{1}{\sqrt{V_n}} \cdot \frac{1}{M_*^{1+n/2}}}_{1/M_{Pl}} \sum_{\vec{n}} h_{\mu\nu}^{(\vec{n})} T^{\mu\nu}$$

$$h^{(\vec{n})}_{MN}(x) \to \underset{\substack{\parallel \\ \text{4d tensor}}}{h^{(\vec{n})}_{\mu\nu}},\ \underset{\substack{\parallel \\ \text{4d vector}}}{h^{(\vec{n})}_{\mu a}},\ \underset{\substack{\parallel \\ \text{4d scalar}}}{h^{(\vec{n})}_{ab}} \quad \text{KK modes}$$

Counting: h_{MN} is $d \times d$ symmetric, so $\frac{d(d+1)}{2}$

General coordinate invariance allows $2d$ constraints to be imposed by "gauge choice".

$$\Rightarrow \frac{d(d+1)}{2} - 2d = \frac{d(d-3)}{2} \text{ ind. components at each KK level}$$

Decomposing: 0: $\underset{2}{h^{(0)}_{\mu\nu}}$, $\underset{n\cdot 2}{V^{(0)}_{\mu a}}$, $\underset{n(n+1)/2}{S^{(0)}_{ab}}$ all massless

$$2 + 2n + \tfrac{1}{2}n^2 + \tfrac{1}{2}n = \tfrac{1}{2}(n^2 + 5n + 4) = d(d-3)/2$$

$\vec{n} \neq 0$: $\underset{5}{h^{(\vec{n})}_{\mu\nu}}$, $\underset{3\cdot(n-1)}{V^{(\vec{n})}_{\mu a}}$, $\underset{n(n+1)/2 - n}{S^{(\vec{n})}_{ab}}$ $\quad M_{\vec{n}} = \sqrt{\vec{n}^2}/R$

$$5 + 3(n-1) + n(n+1)/2 - n$$

$\vec{n}_a V^{(\vec{n})}_{\mu a} = 0$ $\quad \vec{n}_a S^{(\vec{n})}_{ab} = 0$

$$= \tfrac{1}{2}n^2 + \tfrac{1}{2}n + 2n + 2 = \tfrac{1}{2}(n^2 + 5n + 4)$$

Of these, $V^{(\vec{n})}_{\mu}$, $S^{(\vec{n})}_{ab} - \mathrm{tr}(S^{(\vec{n})}_{ab})$ do not couple to the SM at $w = 0$.

$\Rightarrow h^{(\vec{n})}_{\mu\nu}$ couples to $T^{\mu\nu}/M_{Pl}$

$H^{(\vec{n})} = \sum_a S^{(\vec{n})}_{aa}$ couples to $T^{\mu}{}_{\mu}/M_{Pl}$, the "radion".

If we integrate out KK modes, $\to 0$, we get

$$S_{grav} = \frac{M_*^{2+n}}{2} \int d^4x \int d^n w \underset{\substack{\downarrow \\ \sqrt{g}}}{\sqrt{G}}\ \underset{\substack{\downarrow \\ R^{(4)}}}{R^{(4+n)}}$$

$$= \underbrace{\frac{M_*^{2+n}}{2} V_n}_{=\frac{1}{2}M_{Pl}^2} \int d^4x \sqrt{g}\, R^{(4)}$$

At order $\vec{n}$, $(\partial^2 + \vec{n}^2/R^2)(\cdot) = 0 + (\text{int})$

Zero mode graviton, decoupled zero mode vectors and scalars, zero mode dilaton.
KK modes are split by $1/R$.

What are the observational consequences?

Want $M_* \sim$ TeV, but may be bounded to be higher for a given n.

Limits: - modifications to $1/r^2$ gravity at "short" distances

- collider searches, precision searches

- astrophysics

Gravity: $V(r) = -G_N \frac{m_1 m_2}{r} \rightarrow -G_N \frac{m_1 m_2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$ modification

Here, modifications will come from massive KK modes exchanged.

$$\Rightarrow \begin{cases} \lambda = R \lesssim 37\,\mu m & \text{for } \alpha = 1 \\ \alpha = 1 \end{cases}$$

$n=1$ is ruled out

$n=2$ is borderline, $M_* \gtrsim 1.4$ TeV, a bit large relative to m_W

$n>2$ is fine

Precision + Colliders: M_* sets quantum gravity

$\Rightarrow$ expect higher-dim. ops suppressed by M_*

B, L violation $\Rightarrow M_* \gtrsim 10^{16}$ GeV

Flavor $\Rightarrow M_* \gtrsim 10^{6}$ GeV

Electroweak $\Rightarrow M_* \gtrsim 10^{4}$ GeV

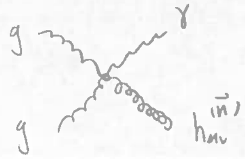
Must assume QG doesn't generate these, much

KK gravitons can also be created at colliders.

Coupling : $\frac{1}{M_{Pl}} \sum_{\vec{n}} h_{\mu\nu}^{(\vec{n})} \cdot T_{SM}^{\mu\nu}$

Each KK mode couples as $\frac{1}{M_{Pl}}$, but there are a lot of them.

Consider



$\Rightarrow$ monophoton, since don't see gravitons in detector ($1/M_{Pl}$)

$$\sigma(gg \to \gamma + h_{\mu\nu}^{(\vec{n})}) := \sigma(\vec{n}) \sim \frac{1}{M_{Pl}^2}$$

$$\sigma_{tot} = \sum_{\vec{n}} \sigma(\vec{n})$$

Since these are closely spaced, $\sum_{\vec{n}} \to \int dN = \underbrace{\Omega_{n-1}}_{\text{area of } n\text{-sphere of unit rad.}} k^{n-1}\, dk$

$$= \Omega_{n-1}\, m^{n-1}\, dm \cdot R^n \quad , \quad m = k/R$$

$$\Rightarrow \frac{d\sigma}{dm} = \Omega_{n-1}\, m^{n-1}\, \sigma(m) \cdot R^n$$

$$\sigma_{tot} \sim \Omega_{n-1} \underbrace{R^n \cdot \frac{1}{M_{Pl}^2}}_{=\frac{1}{M_*^{n+2}}} \int dm \cdot m^{n-1} \qquad \to M_* \gtrsim \text{TeV}$$

$$\sim \Omega_{n-1} \frac{1}{M_*^{n+2}} \int_0^{\cdots} dm\, m^{n-1} \longrightarrow \frac{1}{M_*^2}.$$

↳ diverges, but cut off at $E_{max} \lesssim M_*$

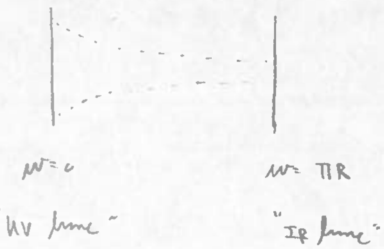
It might also be possible to make black holes!

Strongest direct bounds come from stellar / SN cooling for $n = 2, 3$.

$$M_* \gtrsim \begin{cases} 14 \text{ TeV} ; & n=2 \\ 1.5 \text{ TeV} ; & n=3 \\ \vdots & \vdots \end{cases}$$

2. Warped XD: RS 1...

(X, w), one XD



$$ds^2 = e^{-2kw}\eta_{\mu\nu}dx^\mu dx^\nu - dw^2 = G_{MN}\,dx^M dx^N.$$

This geometry solves Einstein's eqn's with a bulk cc, non-zero brane tensions.

Can arise in stringy constructions: "warped throat".

Called a "warped" metric.

Gravity propagates everywhere.

SM fields may be confined to branes or allowed in the bulk.

$$S_{grav} = M_*^3 \int d^4x \int_0^{\pi R} dw\, \sqrt{G}\, R^{(5)}$$

For only 4d fluctuations, $G_{MN} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix}$, $g_{\mu\nu} = e^{-2kw}\bar{g}_{\mu\nu}(x)$

$$\Rightarrow \sqrt{-G}\, R^{(5)} = \sqrt{-g}\, R^{(4)} = \underbrace{\sqrt{-\bar{g}}\, \bar{R}^{(4)}}_{w\text{-ind.}}\, e^{-2kw}$$

$$S_{grav} \to 2\cdot\frac{M_*^3}{2}\int d^4x \int_0^{\pi R} dw\, \sqrt{-\bar{g}}\, \bar{R}^{(4)}\cdot e^{-2kw}$$

$$= \underbrace{\frac{1}{2}\frac{M_*^3}{k}\left(1 - e^{-2\pi R k}\right)}_{\frac{M_{Pl}^2}{2}}\ \underbrace{\int d^4x\, \sqrt{-\bar{g}}\, \bar{R}}_{4d\ GR}$$

We typically expect $M_* \sim k \sim R^{-1}$.

Things get interesting when we add a Higgs localized on the IR brane. (8)

$$S_{Higgs} = \int d^4x \int_0^{\pi R} dw \sqrt{-G} \left[G^{\mu\nu} \partial_\mu H^\dagger \partial_\nu H - V(H) \right] \delta(w - \pi R)$$

↳ brane localisation

$$= \int d^4x \cdot \int_0^{\pi R} dw \cdot \left(e^{-2kw}\right)^{4/2} \cdot \left[e^{+2kw} \eta^{\mu\nu} \partial_\mu H^\dagger \partial_\nu H - V(|H|) \right] \delta(w - \pi R)$$

$$= \int d^4x \cdot e^{-4\pi kR} \left[e^{2\pi kR} |\partial H|^2 - V(|H|) \right]$$

$$= \int d^4x \left[|\partial \tilde{H}|^2 - e^{-4\pi kR} V(|e^{\pi kR} \tilde{H}|) \right] \qquad H = e^{\pi kR} \tilde{H}$$

↳ canonical

For $V(|H|) = -\mu^2 |H|^2 + \frac{\lambda}{2} |H|^4$

$$e^{-4\pi kR} V(|e^{\pi kR} \tilde{H}|) = e^{-4\pi kR} \left(-\mu^2 e^{2\pi kR} |\tilde{H}|^2 + \frac{\lambda}{2} |\tilde{H}|^4 e^{4\pi kR} \right)$$

$$= -\left(\mu^2 e^{-2\pi kR}\right) |\tilde{H}|^2 + \frac{\lambda}{2} |\tilde{H}|^4$$

↳ warped!

Naturalness suggests $\mu \sim M_*$, after including quantum corrections.

But $\tilde{\mu} = \mu \cdot e^{-\pi kR}$ due to warping, stable under quantum effects.

We expect $M_{Pl} \sim M_* \sim k$.

For $\mu \sim M_* \sim M_{Pl}$, $\tilde{\mu} \sim \text{TeV}$ if $kR \sim 11$, not too big.

This gives a soln to the hierarchy problem!

In this case, localization has pushed down the scale.

(Note: $\mu \sim M_* e^{-\pi kR}$ is a bit like $\Lambda_{QCD} \sim M_{Pl}\, e^{-2\pi / \alpha_s(M_{Pl})\, b_{QCD}}$)

The graviton must travel in all five dimensions. We'll come back to it.

RS1 ⇒ all of SM on the IR brane with the Higgs.

More general scenarios have SM vectors and fermions in the bulk

For this, it is helpful to make a change of variables.

$$e^{-k\mu r} = \frac{1}{kz}, \quad \text{so} \quad \begin{cases} \mu r = k^{-1}\ln(kz) \\ d\mu r = \frac{1}{kz} \end{cases} \quad \text{with } z \in [L_0, L_1]$$

$$L_0 = k^{-1} = \text{tiny}, \qquad L_1 = k^{-1} e^{+\pi k R} = \text{TeV}^{-1}$$

$$ds^2 = \left(\frac{L_0}{z}\right)^2 \underbrace{\left(\eta_{\mu\nu}\, dx^\mu dx^\nu - dz^2\right)}_{\text{"conformally flat"}} \quad \Rightarrow \begin{cases} G_{MN} = \left(\frac{L_0}{z}\right)^2 \eta_{MN} \\ G^{MN} = \left(\frac{z}{L_0}\right)^2 \eta^{MN} \end{cases}$$

Consider a bulk U(1) gauge field with Neumann B.C.'s: $\partial_z A^M = 0$ at L_0, L_1.

$$S = \int d^4x \int_{L_0}^{L_1} dz \sqrt{-G}\; G^{MA} G^{NB} F_{MN} F_{AB} \left(-\frac{1}{4}\right)$$

$$F_{MN} = \partial_M A_N - \partial_N A_M$$

$$A_M \to A_M - \frac{1}{e}\partial_M \alpha, \quad \text{for any } \alpha(x, \mu r)$$

$$\Rightarrow \text{choose } \alpha \text{ s.t. } A_5 \to 0$$

$$F_{MN} \to F_{\mu\nu}, \quad F_{\mu z} = -\partial_z A_\mu$$

$$S = \int d^4x \int_{L_0}^{L_1} dz \left(\frac{L_0}{z}\right)^5 \left(\frac{z}{L_0}\right)^2 \left(\frac{z}{L_0}\right)^2 \left(-\frac{1}{4}\right)\left(\eta^{\mu\alpha}\eta^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} + 2\eta^{\mu\alpha} F_{\mu z} F_{\alpha z}\right)$$

$$= \int d^4x \int_{L_0}^{L_1} dz \left(\frac{L_0}{z}\right) \underbrace{\left[\frac{1}{2} A^\mu\left(\eta_{\mu\nu}\partial^2 - \partial_\mu \partial_\nu\right) A^\nu + \frac{1}{2} A_\mu\left(\partial_z^2 - \frac{1}{z}\partial_z\right) A_\nu \cdot \eta^{\mu\nu}\right]}_{\text{integrate by parts, use } \partial_z A_M|_{L_0, L_1} = 0}$$

To simplify, find an orthonormal set w.r.t. the z-diff. eq.:

$$A_\mu(x,z) = \sum_{n=0}^{\infty} A_\mu^{(n)}(x)\cdot f^{(n)}(z).$$

$$\left(\partial_z^2 - \frac{1}{z}\partial_z + z^2 m_n^2\right) f^{(n)}(z) = 0$$

$$\int dz \left(\frac{L_0}{z}\right) f^{(n)} f^{(m)} = \delta^{mn}$$

Bessel functions

The general sol'n is $f^{(n)}(z) = N_n\left(\frac{z}{L_0}\right)\left[J_1(m_n z) + \beta_n Y_1(m_n z)\right]$

Applying Neumann B.C.'s:

$$\begin{cases} \beta_n = -\dfrac{J_0(m_n L_0)}{Y_0(m_n L_0)} = \dfrac{\pi}{2}\Big/\left[\gamma - \ln(m_n L_0/2)\right] & ; \; z = L_0 \\ \beta_n = -\dfrac{J_0(m_n L_1)}{Y_0(m_n L_1)} & ; \; z = L_1 \end{cases}$$

Set these equal to get

$$\begin{cases} m_n = \dfrac{\pi}{L_1}\left(n - \frac{1}{4}\right) & ; \; n \gg 1 \\ N_n = \dfrac{1}{\sqrt{n-\frac{1}{4}}}\cdot m_n\sqrt{L_0} & ; \; n \gg 1 \end{cases}$$

Recall that:

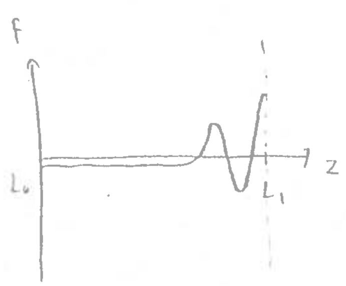
$$J_\alpha(x) = \begin{cases} \dfrac{1}{\Gamma(\alpha+1)}\left(\dfrac{x}{2}\right)^\alpha & ; \; 0 < x \ll \sqrt{\alpha+1} \\ \sqrt{\dfrac{2}{\pi x}}\cos\left(x - \dfrac{\alpha\pi}{2} - \dfrac{\pi}{4}\right) & ; \; x \gg \left|\alpha^2 - \frac{1}{4}\right| \end{cases}$$

$$Y_\alpha(x) = \begin{cases} -\dfrac{\Gamma(\alpha)}{\pi}\left(\dfrac{2}{x}\right)^\alpha & ; \; 0 < x \ll \sqrt{\alpha+1} \\ \sqrt{\dfrac{2}{\pi x}}\sin\left(x - \dfrac{\alpha\pi}{2} - \dfrac{\pi}{4}\right) & ; \; x \gg \left|\alpha^2 - \frac{1}{4}\right| \end{cases}$$

Modes: $m_0 = 0$, $f^{(0)} = 1\Big/\sqrt{L_0 \ln(L_1/L_0)}$, flat profile

$m_n \simeq \frac{\pi}{L_1}\left(n - \frac{1}{4}\right)$, so masses $\sim L_1^{-1} \sim k^{-1} e^{-\pi k R} \sim$ TeV

$\Rightarrow$ TeV-scale KK modes!



$$f^{(n)}(z) \sim \begin{cases} \sqrt{z}\cdot m_n \cdot (\text{oscillation}) & ; \; z \sim L_1 \\ -\sqrt{L_0}\cdot m_n / \ln(m_n L_0) & ; \; z \ll L_1 \end{cases}$$

"constant, small

$\Rightarrow$ localized towards the IR brane!

We can do the same thing with the graviton.

$$f^{(n)}(z) = N_n \left(\frac{z}{L_0}\right)^2 \left[J_2(m_n z) + \beta_n Y_2(m_n z)\right] \quad , \text{ and } f^{(0)} = 1/\sqrt{L_0} \text{ zero mode}$$

$$\begin{cases} \beta_n = \dfrac{-J_1(m_n L_0)}{Y_1(m_n L_0)} = (\#)(m_n L_0)^2 = \text{tiny} \\[2ex] \beta_n = \dfrac{-J_1(m_n L_1)}{Y_1(m_n L_1)} \end{cases}$$

$$h_{\mu\nu}(x,w) = \sum_{n=0}^{\infty} h^{(n)}_{\mu\nu}(x)\, f^{(n)}(z).$$

$$\Rightarrow m_n = \text{zeroes of } J_1 = \frac{\pi}{L_1}\left(n + \frac{1}{4}\right).$$

$$N_n = \ldots$$

KK modes are very localized near IR brane!

$$S_{SM} = \int d^4x \int_{L_0}^{L_1} dz \sqrt{-G}\; \mathcal{L}_{SM}(G)\; \delta(z - L_1)$$

$$\supset \int d^4x \Big[\underbrace{-\frac{1}{M_{Pl}} T^{\mu\nu}_{SM} h^{(0)}_{\mu\nu}(x)}_{\text{usual grav. coupling}} \; \underbrace{- \frac{1}{M_*} T^{\mu\nu}_{SM} \cdot \sum_{n=1}^{\infty} h^{(n)}_{\mu\nu}(x)}_{\text{enhanced coupling due to localization}} \Big]$$

Models: RSI: SM on IR brane, new graviton KK excitations only

Variation #1: H + fermions on IR brane, gauge + gravity in bulk

$\Rightarrow$ really big couplings to IR-localized KK modes

$\Rightarrow$ ruled out by precision electroweak

Variation #2: H on IR brane, everything else in the bulk

Works!

Fermions are slightly tricky to define in curved space

$\Gamma^A = (\gamma^\mu, -i\gamma^5)$ = flat space Dirac matrices (5D)

$$S_F = \int d^5x \sqrt{G} \left[\frac{i}{2} \bar{\Psi} e^M_A \Gamma^A D_M \Psi - \frac{i}{2} (D_M \Psi)^\dagger \gamma^0 e_A{}^M \Gamma^A \Psi - M \bar{\Psi}\Psi \right]$$

spin connection

gauge + gravity covariant derivative

$G^{MN} e^A_M e^B_N = \eta^{AB}$, connects curved to locally flat

Write $M = c L_0$.

The KK decomposition connects the L and R components of Ψ

$$\Psi_{L,R}(x,z) = \sum_n \psi^{(n)}_{L,R}(x) f^{(n)}_{L,R}(z)$$

$$O_c f^{(n)}_L = m_n f^{(n)}_R$$

$$O_c = \partial_z - \frac{1}{z}(2-c)$$

This implies $O_{-c} O_c f^{(n)}_L = -m_n^2 f^{(n)}_L$

Solution: $f^{(n)}_L(z) = A^{(n)}_F \cdot \left(\frac{z}{L_0}\right)^{5/2} \left[J_{c+\frac{1}{2}}(z) + \beta_n Y_{c+\frac{1}{2}}(z) \right]$

$A^{(n)}_F$ = normalization

$$S = \int d^4x \int dw \left(\frac{L_0}{z}\right)^4 \left[\bar{\Psi}_L i\gamma^\mu \partial_\mu \Psi_L + \bar{\Psi}_R i\gamma^\mu \partial_\mu \Psi_R + \frac{1}{2}\left(\bar{\Psi}_R^c \partial_w \Psi_L - \bar{\Psi}_L \overleftarrow{\partial_5} \Psi_R^c\right) + \frac{c}{z}\left(\bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L\right) \right]$$

Get coupled eq'n-s for Ψ_L, Ψ_R

$$\begin{cases} \Psi_L = \sum_n f_L^{(n)}(z) \psi_L^{(n)}(x) \\ \bar{\Psi}_R = \sum_n f_R^{(n)}(z) \psi_R^{(n)}(x) \end{cases} \Rightarrow \begin{cases} f_R'^{(n)} + m_n f_L^{(n)} - \left(\frac{c+2}{z}\right) f_R^{(n)} = 0 \\ f_L'^{(n)} - m_n f_R^{(n)} + \left(\frac{c-2}{z}\right) f_L^{(n)} = 0 \end{cases}$$

What about B.C.'s ?
$$\begin{cases} f_R(L_0, L_1) = 0 \Rightarrow \left[f_L'^{(n)} + \left(\frac{c-2}{z}\right) f_L^{(n)} \right]_{L_0,L_1} = 0 \\ f_L(L_0, L_1) = 0 = \left[f_R'^{(n)} - \left(\frac{c+2}{z}\right) f_R^{(n)} \right]_{L_0,L_1} = 0 \end{cases}$$

$f_R|_{L_0,L_1} = 0 \Rightarrow f_L^{(0)} = N_0 \left(\frac{z}{L_0}\right)^{2-c}, \quad f_R^{(0)} = 0$

$f_L|_{L_0,L_1} = 0 \Rightarrow f_R^{(0)} = \tilde{N}_0 \left(\frac{z}{L_0}\right)^{2+c}, \quad f_L^{(0)} = 0$

$\Rightarrow$ chiral zero modes by choice of BC!

Beyond $n=0$, $$\begin{cases} \left(\partial_z^2 - \frac{4}{z}\partial_z + \left[m_n^2 - \frac{(c-3)(c+2)}{z^2}\right]\right) f_R^{(n)} = 0 \\ \left(\partial_z^2 - \frac{4}{z}\partial_z + \left[m_n^2 - \frac{(c+3)(c-2)}{z^2}\right]\right) f_R^{(n)} = 0 \end{cases}$$

$$\hookrightarrow \begin{cases} f_L^{(n)}(z) = A_F^{(n)}\left(\frac{z}{L_0}\right)^{5/2}\left[J_{c+\frac{1}{2}}(m_n z) + \beta_n Y_{c+\frac{1}{2}}(m_n z)\right] \\ f_R^{(n)}(z) = A_F^{(n)}\left(\frac{z}{L_0}\right)^{5/2}\left[J_{c-\frac{1}{2}}(m_n z) + \beta_n Y_{c-\frac{1}{2}}(m_n z)\right] \end{cases}$$

Σ related ..

Imposing BC's fixes $m_n \sim n\pi/L_1$..

B.C.'s determine chirality of zero modes, KK modes non-chiral.

$(+,-) \Rightarrow f_R|_{L_0, L_1} = 0 \Rightarrow f_L^{(0)} = N_0\left(\frac{z}{L_0}\right)^{2-c}$

To see localization, must compare to integration measure of the action.

$$\underbrace{\int_{L_0}^{L_1} dz \left(\frac{L_0}{z}\right)^4 f_L^{(0)\,2}}_{=1 \text{ for normalization}} \cdot \underbrace{\bar{\Psi}_L^{(0)} i\gamma^\mu \partial_\mu \Psi_L}_{z\text{-ind}}$$

$$\Rightarrow \int_{L_0}^{L_1} dz\, N_0^2\left(\frac{z}{L_0}\right)^{4-2c}\cdot\left(\frac{L_0}{z}\right)^4 = \int_{L_0}^{L_1} dz\, N_0^2\cdot\left(\frac{z}{L_0}\right)^{-2c}$$

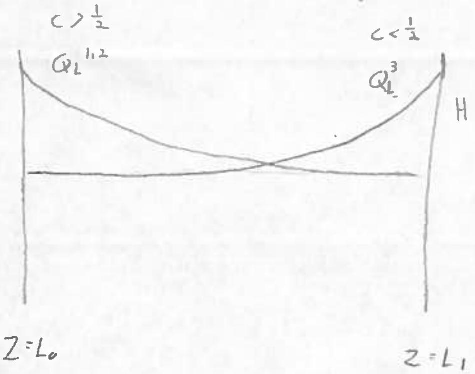
$$\begin{cases} c > \frac{1}{2} \Rightarrow \text{localized near } L_0 \quad (\text{diverges as } L_0 \to 0) \\ c < \frac{1}{2} \Rightarrow \text{localized near } L_1 \quad (\text{diverges as } L_1 \to 0) \end{cases}$$

RS with bulk SM:

Higgs on IR brane.

Gauge fields and fermions in bulk, chirality of zero modes from B.C.'s.

Flavor via fermion localization:



KK modes have masses on the order of $n/L_1 \sim$ TeV.
These are typically localized near the IR brane.

Fermion masses: $\int d^4x \cdot \int dw \cdot \left(\frac{L_0}{z}\right)^4 \bar{Q} H u_R \cdot f_{Q_L}^{(0)}(z) \cdot f_{u_R}^{(0)}(z)\, \delta(z - L_1)$

$$= \int d^4x \cdot \bar{Q} H u_R\, (\#) \cdot \underbrace{\left(\frac{L_1}{L_0}\right)^{2 - c_{Q_L}} \cdot \left(\frac{L_1}{L_0}\right)^{2 + c_{u_R}}}_{\text{bigger for IR localization!}}$$

This gives a nice explanation for the range of fermion masses.

3. Warped vs. Strong Coupling

AdS - CFT ("holography")

IIB String Theory on $AdS_5 \times S^5$ (→ SUSY; ↳ with gravity) $\overset{\text{dual}}{\longleftrightarrow}$ $N=4$ SU(N) SUSY YM in $d=4$ (→ SUSY; ↳ without)

S^5 isometries = SU(6) ~ $SU(4)_R$

AdS_5 isometries = conformal ~ conformal

$$\frac{R_{AdS}^4}{l_s^4} = 4\pi g^2 N$$

$R_{AdS} \sim k^{-1}$

l_s ~ string length where stringy excitations emerge.

For $R_{AdS} \gg l_s$, can neglect stringy states and use an XD EFT description of gravity.

$\Rightarrow$ $4\pi g^2 N \gg 1$, $N \gg 1$, strongly-coupled in this limit.

It is thought that this would hold more broadly

RS = slice of AdS_5, dual to a strongly coupled approximate CFT?

Interpret UV, IR branes as breaking conformal.

(Aside: conformal ↔ scale invariant

QCD: $S = \int d^4x \; \bar{q}(i\gamma^\mu D_\mu - m)q - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$

Rescale: $\begin{cases} x \to \lambda^{-1} x \\ q \to \lambda^{3/2} q \\ A_\mu \to \lambda A_\mu \end{cases}$ $\Rightarrow$ $S \to \int d^4x \cdot \lambda^{-4} \cdot \lambda^4 \left[\bar{q}\left(i\gamma^\mu D_\mu - \lambda^{-1} m\right) q - \frac{1}{4} G_{\mu\nu} \cdot G^{\mu\nu}\right]$

$\Rightarrow$ invariant under rescaling for $m \to 0$.

This is broken by: $\frac{d\alpha_s}{dt} = \beta_{QCD} = -\frac{b}{2\pi}\alpha_s^2$, $b = \frac{11}{3}N - \frac{2}{3}N_f$ (N_f = # light flavors, $m_q < \mu$.)

$\Rightarrow$ scaling invariance broken by quantum corrections.

Get back conformal for $b \to 0$ (+ higher loops) for right value of α_s.)

RS bulk ⇒ approximate CFT

UV brane ⇒ explicit breaking of conformal from heavy threshold at L_0^{-1}.

eg. Q = heavy flavour with mass M

N_F light flavours with $\frac{11}{3}N - \frac{2}{3}N_F \to 0$

$\mu > M$: $b_+ = \left(\frac{11}{3}N - \frac{2}{3}N_F\right) - \frac{2}{3}$, not conformal, $b_+ \neq 0$.

$\mu < M$: $b_- = \left(\frac{11}{3}N - \frac{2}{3}N_F\right) \to 0$, so flows to conformal (maybe)

IR brane ⇒ spontaneous breaking of conformal at scale L_1^{-1}

RS ⇒ get massless radion mode from h_{55}, associated with brane separation

CFT ⇒ get NGB massless state for the spontaneous breaking of conf

We also have: IR-brane field ⇔ completely composite operator

IR-localized field ⇔ mostly composite

UV-localized field ⇔ mostly fundamental

bulk gauge sym. G broken to H on UV brane ⇔ CFT global sym. G with gauge subgroup H.

These correspondences are useful for building models.

Suggest that RS is dual to a theory of walking TC.

Problem: we don't know exactly what the WTC theory is!