PHYS 528 Homework #7

Due: Mar.9, 2017

1. The narrow width approximation.

   Consider a theory consiting of a massless "electron," a fermion f of mass m, and a real scalar of mass $M > m$, with interactions

   $$-\mathscr{L} \supset g_e \phi \, \bar{e} e + g_f \phi \, \bar{f} f \ .$$

   In this theory:

   a) Compute the decay widths of ϕ into $e\bar{e}$ and $f\bar{f}$, and the total decay width Γ.

   b) Calculate the cross section for $e\bar{e} \to \phi$ in the CM frame. You should get a leftover delta function. Rewrite it as a delta function on the variable $s = (p_1 + p_2)^2$, where p_1 and p_2 are the initial momenta, and express the rest of the cross section in terms of of the decay width $\Gamma(\phi \to e\bar{e})$ and the mass M.

   c) Find the total cross section for $e\bar{e} \to f\bar{f}$ in the CM frame. In doing so, use the width-corrrected propagator

   $$\text{Prop} = \frac{i}{p^2 - M^2 + iM\Gamma} \ .$$

   d) For small couplings g_e and g_f, we will have $\Gamma \ll M$. In this limit, we can apply the *narrow width approximation*,

   $$\lim_{\Gamma/M \to 0} \frac{1}{(s - M^2)^2 + M^2\Gamma^2} = \frac{\pi}{M\Gamma} \delta(s - M^2) \ .$$

   Use this approximation to rewrite the $e\bar{e} \to f\bar{f}$ cross section in terms of $\sigma(e\bar{e} \to \phi)$ and $\text{BR}(\phi \to f\bar{f}) = \Gamma(\phi \to f\bar{f})/\Gamma$.

2. Z-pole asymmetries.

   a) Prove the expression for the left-right asymmetries in terms of the effective Z couplings given in `notes-06` (*i.e.* Eq. (16) starting from Eq. (15)).

   b) Compute the numerical values of the left-right asymmetries A_f ($f \neq e$) and compare to data.
   Hint: look up "Asymmetry Parameters" under the Z listing here: pdglive.lbl.gov/.

   c) Prove the relation between the forward-backward asymmetries and the left-right asymmetries at the Z pole discussed in class. At the pole, you may neglect the photon contribution.

3. A toy model of regularized and renormalized integrals.

a) Evaluate

$$I_2(m^2) = \int_0^\Lambda dx\, x^3 \frac{1}{x^2+m^2} \ .$$

b) Compute

$$I_4(m^2) = \int_0^\Lambda dx\, x^3 \left(\frac{1}{x^2+m^2}\right)^2 \ .$$

c) Define "renormalized" functions by

$$\begin{aligned} \widetilde{I}_2(m^2) &= I_2(m^2) + M^2\,\delta_2 + (m^2 - M^2)\,\tilde{\delta}_2 \\ \widetilde{I}_4(m^2) &= I_4(m^2) + \delta_4 \ , \end{aligned}$$

for some constants δ_2, $\tilde{\delta}_2$, δ_4 and the fixed mass parameter M. Now choose δ_2, $\tilde{\delta}_2$, and δ_4 such that $\widetilde{I}_2(M^2) = 0$, $\widetilde{I}_4(M^2) = 0$, and $d\widetilde{I}_2/dm^2(M^2) = 0$. These correspond to "renormalization conditions" at the renormalization scale $m^2 = M^2$. With these choices, find the expressions for $\widetilde{I}_2(m^2)$ and $\widetilde{I}_4(m^2)$ at general values of m^2 assuming that $\Lambda^2 \gg m^2$, M^2. Show that these are finite as $\Lambda \to \infty$, and look at what happens to them when m^2 becomes much larger than M^2